tetrad: A Set of Commands for Confirmatory Tetrad Analysis

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Confirmatory Tetrad Analysis

- Method of testing fit of structural equation models
- Desirable properties
  - Some not-identified models can be tested
  - Some models are tetrad-nested but not nested for likelihood ratio tests
  - Can be performed on specific components of model
- Underutilized

Confirmatory Tetrad Analysis

Population

- Tetrads: differences in the product of pairs of covariances
- 4 variables contain 6 covariances that form 3 tetrads

$$\tau_{1234} = \sigma_{12}\sigma_{34} - \sigma_{13}\sigma_{24} \quad (1)$$

$$\tau_{1342} = \sigma_{13}\sigma_{42} - \sigma_{14}\sigma_{32} \quad (2)$$

$$\tau_{1423} = \sigma_{14}\sigma_{23} - \sigma_{12}\sigma_{43} \quad (3)$$

- with tetrad $\tau_{ghij} = \sigma_{gh}\sigma_{ij} - \sigma_{gi}\sigma_{hj}$ and covariance σ

Confirmatory Tetrad Analysis

Sample

- Tetrads: differences in the product of pairs of covariances
- 4 variables contain 6 covariances that form 3 tetrads

$$\hat{\tau}_{1234} = \hat{\sigma}_{12}\hat{\sigma}_{34} - \hat{\sigma}_{13}\hat{\sigma}_{24} \tag{4}$$

$$\hat{\tau}_{1342} = \hat{\sigma}_{13}\hat{\sigma}_{42} - \hat{\sigma}_{14}\hat{\sigma}_{32} \tag{5}$$

$$\hat{\tau}_{1423} = \hat{\sigma}_{14}\hat{\sigma}_{23} - \hat{\sigma}_{12}\hat{\sigma}_{43} \tag{6}$$

- with sample tetrad $\hat{\tau}_{ghij} = \hat{\sigma}_{gh}\hat{\sigma}_{ij} - \hat{\sigma}_{gi}\hat{\sigma}_{hj}$ and sample covariance $\hat{\sigma}$

Confirmatory Tetrad Analysis

- Hypothesized model structures imply some $\tau = 0$ (vanish)
- Set of implied vanishing tetrads can be simultaneously tested whether different than 0

$$T = n\hat{\boldsymbol{\tau}}' \left[\mathbf{D}(\hat{\boldsymbol{\sigma}})' \widehat{Cov}(\hat{\boldsymbol{\sigma}}) \mathbf{D}(\hat{\boldsymbol{\sigma}}) \right]^{-1} \hat{\boldsymbol{\tau}} \tag{7}$$

$$\xrightarrow{d} \chi^2_v \tag{8}$$

- $\hat{\boldsymbol{\tau}}$ is a vector of the sample implied vanishing tetrads
- $\mathbf{D}(\hat{\boldsymbol{\sigma}}) = \partial\hat{\boldsymbol{\tau}}/\partial\hat{\boldsymbol{\sigma}}$
- see Bollen (1990) for derivation

Confirmatory Tetrad Analysis General Procedure

- Identify model-implied vanishing tetrads
  - empirical approach using model-implied covariance matrix
- Select a set of nonredundant model-implied vanishing tetrads
  - sweep operator on asymptotic covariance matrix of vanishing tetrads
  - not necessary with a bootstrap test statistic
- Form the simultaneous test statistic for the set of (nonredundant) model-implied vanishing tetrads

tetrad Command

`tetrad` *varlist* [if] [in], `icm1`(*name*) [`icm2`(*name*) `reps`(#) `seed`(#) `tlist`(*1 = yes*)]

- *varlist* requires a minimum of four variables
- `icm1` is the name of an implied covariance matrix
- `icm2` is the name of an implied covariance matrix nested in the first model (optional)
- `reps` is the number of replications of the tetrad test with a random set of nonredundant vanishing tetrads
- `seed` allows users to specify a random number seed
- `tlist` reports a list of tetrads for a given model

tetrad_matrix Command

tetrad_matrix , obs(#) scm(*name*) icm1(*name*)
[icm2(*name*) reps(#) seed(#) tlist(*1 = yes*)]

- obs is the sample size
- scm is the sample covariance matrix

tetrad_bootstrap Command

tetrad_bootstrap *varlist* [if] [in], icm1(*name*) [reps(#) seed(#)]

- reps is the number of bootstrap replications

Example 1

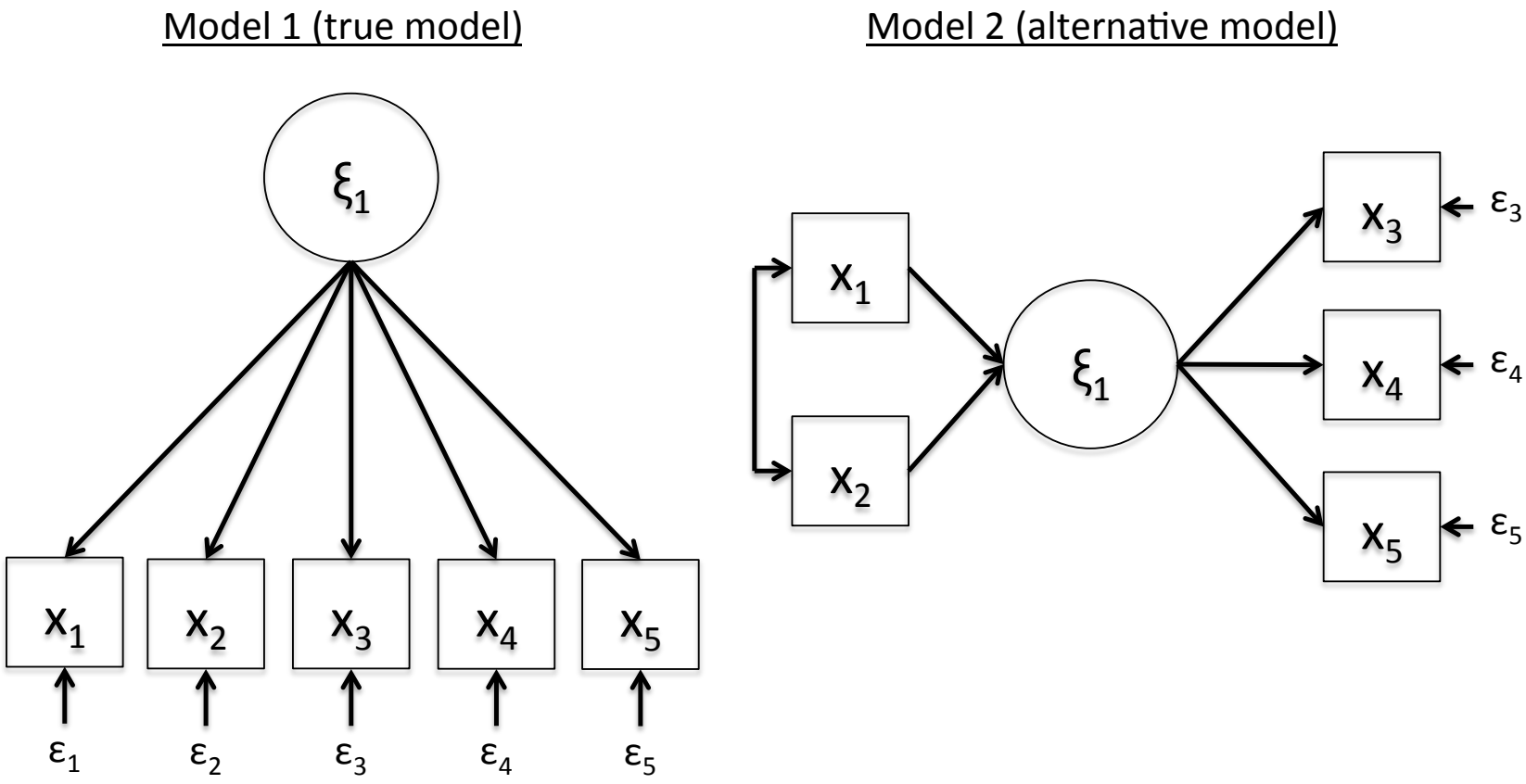

Figure : Models for example 1.

Example 1

Stata code for simulation and obtaining implied-covariance matrices

```
. qui set obs 500
. qui gen xi = rnormal(0,1)
. forval i = 1/5 {
.    qui gen x`i' = xi + rnormal(0, sqrt(0.5))
. }

. qui sem (Xi -> x1 x2 x3 x4 x5)
. qui estat framework, fitted
. mat sigma1 = r(Sigma)

. qui sem (Xi -> x3 x4 x5) (x1 x2 -> Xi)
. qui estat framework, fitted
. mat sigma2 = r(Sigma)
```

Example 1

Stata code invoking tetrad command

```
. tetrad x1 x2 x3 x4 x5, icm1(sigma1) icm2(sigma2) ///
        reps(5) tlist(1)

Model-Implied Tetrads
                        Model 1
tetrad    residual    AVar         t-value     vanish
-------------------------------------------------------
 1234     0.000000    0.000380      0.0000     1
...
 2534     0.000000    0.000388      0.0000     1
-------------------------------------------------------
```

Example 1

-output continued-

Model-Implied Tetrads

		Model 2		
tetrad	residual	AVar	t-value	vanish
1234	0.003192	0.000383	0.1632	0
1342	-0.000000	0.000383	-0.0000	1
...				
2534	-0.000000	0.000388	-0.0000	1

Example 1

-output continued-

```
Confirmatory Tetrad Analysis Results
       Model 1                Model 2                M1 - M2
rep Chi-sq  df  p-val    Chi-sq  df  p-val    Chi-sq  df  p-val
-------------------------------------------------------------
1   6.4887  5  0.2615   6.4455  4  0.1683   0.0432  1  0.8353
2   6.5036  5  0.2602   6.4730  4  0.1665   0.0307  1  0.8609
3   6.5413  5  0.2570   6.4122  4  0.1704   0.1291  1  0.7193
4   6.5859  5  0.2533   6.3444  4  0.1749   0.2415  1  0.6231
5   6.5512  5  0.2562   6.4192  4  0.1700   0.1320  1  0.7164
-------------------------------------------------------------
```

- non-significant test indicates model 1 is consistent with data
- non-significant test for nested model indicates that model 2 (unrestricted) is not a significant improvement over model 1 (restricted)

Example 2

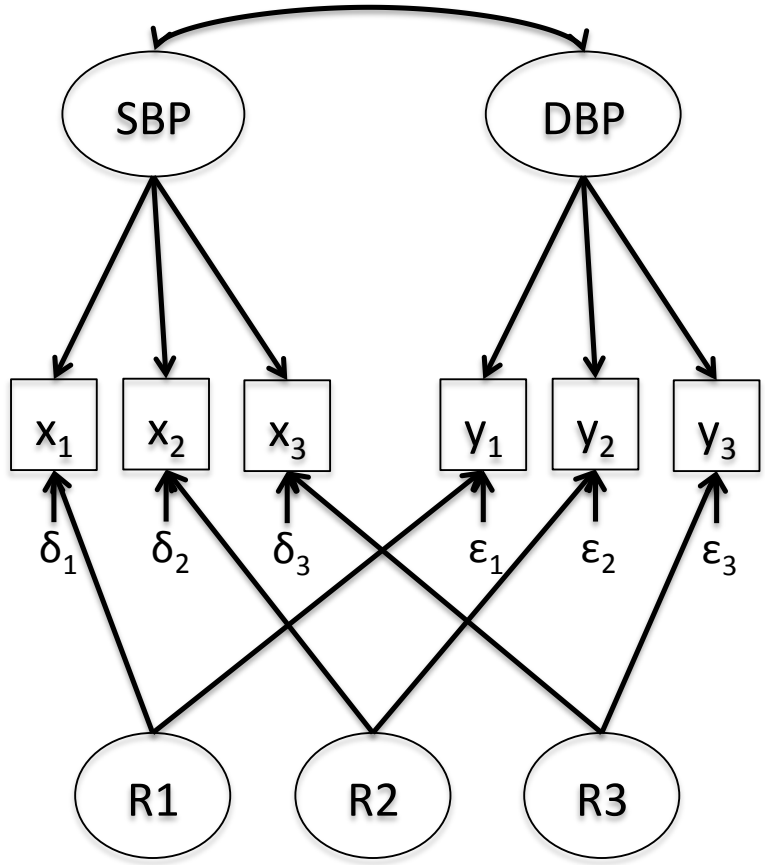

Figure : Multi-trait multi-method model for example 3. $x_1 - x_3$ are three readings of systolic blood pressure. $y_1 - y_3$ are three reading of diastolic blood pressure. $R1 - R3$ are latent method factors for each reading occasion.

Example 2

Stata code for model

```
. qui sem (SBP -> sbp13@1 sbp23@1 sbp33@1) ///
          (DBP -> dbp13@1 dbp23@1 dbp33@1) ///
          (R1 -> sbp13@1 dbp13@1) ///
          (R2 -> sbp23@1 dbp23@1) ///
          (R3 -> sbp33@1 dbp33@1), ///
          cov(SBP*R1@0 DBP*R1@0) ///
          cov(SBP*R2@0 DBP*R2@0) ///
          cov(SBP*R3@0 DBP*R3@0) ///
          cov(R1*R2@0 R1*R3@0 R2*R3@0)
. qui estat framework, fitted
. mat sigma1 = r(Sigma)
```

Example 2

Stata code invoking tetrad bootstrap command

```
. tetrad_bootstrap sbp13 sbp23 sbp33 ///
                   dbp13 dbp23 dbp33, ///
                   icm1(sigma1) reps(1000)

Confirmatory Tetrad Analysis Results

                          bootstrap
Chi-sq         df         p-value
-----------------------------------
  0.9792        6         0.9540
-----------------------------------
```

Conclusion

- Set of commands allows users to integrate Confirmatory Tetrad Analysis into a standard SEM analysis
- Beta versions of commands available at https://github.com/sbauldry/tetrad
- Can be installed within Stata "net from https://github.com/sbauldry/tetrad/raw/master"

Acknowledgement and References

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Additional sources for CTA

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