Bilateral Bargaining with Externalities*

by

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This paper provides an analysis of a non-cooperative but bilateral bargaining game between agents in a network. We establish that there exists an equilibrium that generates a cooperative bargaining type of division of the reduced surplus that arises as a result of non-pecuniary externalities between agents. That is, we have a non-cooperative justification for a cooperative division of a non-cooperative surplus. In so doing, we provide a non-cooperative foundation for the Myerson-Shapley value as well as a new bargaining outcome that contains properties making it particularly useful and tractable in applications. *Journal of Economic Literature* Classification Numbers: C78.

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1. Introduction

There are many areas of economics where market outcomes are best described by an on-going sequence of interrelated negotiations. When firms negotiate over employment conditions with individual workers, patent-holders negotiate with several potential licensors, and when competing firms negotiate with their suppliers over procurement contracts, a network of more or less bilateral relationships determines the allocation of resources. To date, however, most theoretical developments in bargaining have either focused on the outcomes of independent bilateral negotiations or on multilateral exchanges with a single key agent.

The goal of this paper is to consider the general problem of the outcomes that might be realised when many agents bargain bilaterally with one another and where negotiation outcomes are interrelated and generate external effects. This is an environment where (1) surplus is not maximised because of the existence of those external effects and the lack of a multilateral mechanism to control them; and (2) distribution depends upon the precise position of agents in the graph of network relationships. While cooperative game theory has developed to take into account (2) it almost axiomatically rules out (1). In contrast, non-cooperative game theory embraces (1) but restricts the environment considered – symmetry, two players, small players, etc. – to avoid (2).

Here we consider a set of agents who are linked by a network – describing which pairs can negotiate bilaterally. Our environment is such that pairs of agents negotiate over variables that are jointly observable. This might be a joint action – such as whether trade takes place – or an individual action undertaken by one agent but observed by the other (e.g., effort or an investment). We specify a non-cooperative game whereby each pair of agents in a network bargains bilaterally in a pre-specified sequence (although the order does not matter for the equilibrium we focus on). Pairwise negotiations utilise an alternating offer approach where offers and acceptances are made in anticipation of deals reached later in the sequence. Moreover, those negotiations take place with full knowledge of the network structure and the ability to make terms contingent upon that structure should it change. Specifically, the network may become “smaller” should other pairs of agents fail to reach an agreement.

We consider a situation where the precise agreement terms cannot be directly observed outside a network and focus on an attractive equilibrium outcome of the incomplete information game. That outcome involves agents negotiating actions that maximise joint surplus (as in Nash bargaining). Hence, with externalities, outcomes are what might be termed “bilaterally efficient” rather than socially efficient. Nonetheless, the characterisation of those outcomes involves a simple Nash equilibrium of a game where actions are chosen to maximise each pair’s utility.

The equilibrium set of transfers based on the same equilibrium that generates bilaterally efficient actions also gives rise to a precise structure; namely, a payoff that depends upon the weighted sum of values to particular coalitions of agents. This has a cooperative bargaining structure but with several important differences. First, the presence of externalities alongside the restricted communication space gives rise to an outcome that includes the Shapley value, the Myerson value (on a restricted graph) and the Shapley value in partition function space as special cases. Second, those externalities mean that, in certain circumstances, the equilibrium outcome is the Shapley value allocation but over a surplus that is characterised by bilateral rather than social efficiency. Thus, we have a non-cooperative foundation for a cooperative bargaining division of a non-cooperative surplus; both of which are easy to utilise in applied settings. To our knowledge, no similar simple characterisation exists in the literature for a multi-agent bargaining environment.

The paper proceeds as follows. In the next section, we review the current literature on non-cooperative foundations of the Shapley and Myerson values. Section 3 then introduces our action space which is the principal environmental restriction in this paper. Our extensive form game is introduced in Section 4. The equilibrium outcomes of that game are characterised in Sections 4 and 5; first with the equilibrium outcomes as they pertain to actions and then to distribution. Section 6 then considers particular economic applications including the resource trading environment of Gul (1989), Stole and Zwiebel's (1996) wage bargaining environment, and buyer seller networks. A final section concludes.

2. Literature Review

Winter (2002, p.2045) argues that "[o]f all the solution concepts in cooperative game theory, the Shapley value is arguably the most 'cooperative,' undoubtedly more so than such concepts as the core and the bargaining set whose definitions include strategic interpretations." Despite this, the Shapley value has emerged as an outcome in a number of non-cooperative settings. Harsanyi (1985) noted the emergence of the Shapley value in games that divide surplus based on unanimity rules. However, recent attempts to provide a non-cooperative foundation for the Shapley value have focused, for the most part, on the outcomes of a series of bilateral negotiations.

Gul (1989) proposed a game where a single agent can generate utility from consuming resources that are initially dispersed. His trading game has individual agents meeting randomly to conduct bilateral trades. Each bilateral negotiation involves one agent being selected at random to make a take-it-or-leave-it purchase offer to the other agent. Successful trades result in the seller leaving the game with their earnings. Essentially, a trade is equivalent to a seller agreeing to join the buyer's coalition. Eventually, sufficient trades occur that the grand coalition is formed with, for sufficiently patient players, the unique stationary subgame perfect outcome (with no delay) having each agent receive (in expectation) their Shapley value. The economic environment is quite specialised here, however, as it essentially amounts to a sequence of discrete trades.

Stole and Zwiebel (1996) examined an environment where a firm bargains bilaterally with a given set of workers. While their treatment is for the most part axiomatic, focusing on a natural notion of stable agreements, they do posit an extensive form game for their environment. In this extensive form game, there is a fixed order in which each worker bargains with the firm. Any given negotiation has the worker and firm taking turns in making offers to the other party that can be accepted or rejected. Rejected offers bring with them an infinitesimal probability of an irreversible breakdown where the worker leaves employment forever. Otherwise, a counter-offer is possible. If the worker and firm agree to a wage (in exchange for participation in production), the negotiations move on to the next worker. The twist is that, agreements are not binding in the sense that, if there is a breakdown in any bilateral negotiation, this automatically triggers a replaying of the sequence of negotiations between the firm and each remaining worker. This new subgame takes place as if no previous wage agreements had been made (reflecting a key assumption in Stole and Zwiebel's axiomatic treatment that wage agreements are not binding and can be renegotiated by any party at any time).

Stole and Zwiebel (1996, Theorem 2) claim that this extensive form game gives rise to the Shapley value as the *unique* subgame perfect equilibrium outcome (something they also derive in their axiomatic treatment). However, in their proof of this, they assume that the Binmore, Rubinstein and Wolinsky (1986) bargaining outcome holds for each negotiating pair even though a deviation from the equilibrium outcome in a previous negotiation may not yield that outcome.[1] We demonstrate below that if the informational structure between different bilateral negotiations is more precisely specified (Stole and Zwiebel implicitly assume that the precise wage that is paid to a worker is not observed by other workers) and 'out of equilibrium' beliefs specified, their result holds. Nonetheless, as will be apparent below, our extensive form bargaining game – consisting of a sequence of bilateral negotiations based on the Binmore, Rubinstein and Wolinsky outcome – is a natural extension of theirs to more general economic environments.

Finally, we note the influential contribution of Hart and Mas-Colell (1996) to this literature. They do not model an extensive form game based on bilateral offers and negotiations but instead consider rounds where players have opportunities to make offers to all 'active' players (i.e., players who have no had a proposal rejected). If this is accepted by all 'active' players, the game ends. If it is not accepted by one player there is a chance that the proposer will be excluded from the game. Hart and Mas-Colell (1996) demonstrate that there is a unique subgame perfect equilibrium of this game that results in each active player receiving its Shapley value. As Winter (2002) surveys, this game has given rise to a variety of extensions but in general the institutional environment requires the ability of proposers to make offers to all, for

[1] Indeed, we demonstrate below that deviations do, indeed, result in an alternative outcome to an independent Binmore, Rubinstein and Wolinsky (1986) outcome.

single rejections to nullify agreements and for a commitment to cause proposers to risk exit following rejection.

In summary, while significant, prior extensive form games that generate Shapley value outcomes as equilibrium outcomes have been based in somewhat restrictive economic environments. Either the set of choices is restricted – as in Gul (1989) and Stole and Zwiebel (1996) – to decisions to join coalitions or not or alternatively, the institutional environment involves communication structures and commitment not present in many important economic environments.

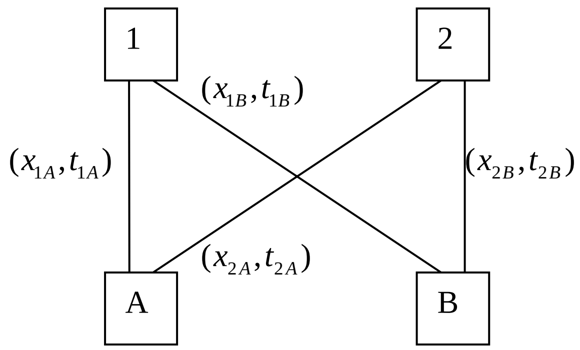
3. Observability of Actions

Because we consider an environment where all negotiations are bilateral, we similarly restrict the observability of actions to no more than two agents. We assume below that individual actions (such as effort expended or an investment) may be observable, and hence negotiable, with one other agent. Similarly, a joint action (such as exchange of goods, services or assets) may be observed and negotiated by the two agents concerned. However, in each case, other agents cannot observe the action taken. Importantly, what this means is that agents cannot negotiate agreements contingent upon negotiations that one or neither of them is a party to. To assume otherwise would be inconsistent with our restriction to bilateral bargaining and would suggest instead that a multilateral bargaining protocol might be more appropriate.

As an example, consider an environment where there are 2 buyers (1 and 2) and 2 sellers (A and B) of a product. Each buyer and seller can negotiate over the quantity of the product traded between them; e.g., 1 and A negotiate over x_{1A} and so on. The buyers' values are $b_1(x_{1A}+x_{1B})$ and $b_2(x_{2A}+x_{2B})$, respectively. Assume that the sellers have no costs. In exchange for the product, buyers pay the sellers a transfer; for example, 1 pays A, t_{1A}. Each pair trades a quantity and pays a transfer between them.

The network of bilateral negotiations is as depicted in Figure 1. Notice that the two buyers and the two sellers are assumed here not to negotiate with one another. Our observability requirements will also presume that 2 will not be able to observe (x_{1A}, t_{1A}) or (x_{1B}, t_{1B}). This means that agreements with 2 cannot be made contingent upon these outcomes even when 2 negotiates with A or B respectively.

Figure 1: Buyer-Seller Network



To formalize this, consider a set of agents, $N = \{1, 2, ..., n\}$. There are three types of actions:

1. Individually observable actions by *i*: let $\mathbf{a}_i$ be the vector of such actions with individual component, a_i^m.

2. Jointly observable actions by *i* and *j* (*i* < *j*): let $\mathbf{x}_{ij}$ be the vector of such actions with individual component, x_{ij}^m.

3. Transfers between i and j: without loss of generality, we will assume there is only one of these, t_{ij} (that may be positive or negative) between each pair (i, j).

As noted earlier, it is clear that $\mathbf{a}_i$, $\mathbf{x}_{ij}$ and t_{ij} are observed by i. (A1) formalizes our unobservability assumption. Let $A \equiv \{\mathbf{a}_i\}_{i \in N}$, $X \equiv \{\mathbf{x}_{ij}\}_{(i,j) \in N}$ and $T \equiv \{t_{ij}\}_{(i,j) \in N}$.

> **(A1) (Unobservable Actions)** *During negotiations, agent i cannot observe* $A / \mathbf{a}_i$, $X / \{\mathbf{x}_{ij}\}_{j \in N}$ *and* $T / \{t_{ij}\}_{j \in N}$.

In particular, this means that even if it is negotiating with j, j cannot directly communicate to i the outcomes of a previous negotiation with k. Instead, i must form beliefs over those actions it cannot observe and expectations about outcomes in the future. We let i's beliefs over a particular action be superscripted with i and marked with a tilde. That is, i's beliefs regarding $\mathbf{x}_{kl}$ would be $\tilde{\mathbf{x}}_{kl}^{i}$.

The results that follow do not explicitly depend upon the individually observable actions, $A \equiv \{\mathbf{a}_i\}_{i \in N}$, although these might prove important in applications. For that reason, we suppress reference to them in what follows so as to simplify the exposition.

4. Bargaining Game

We begin by stating some additional notation, before defining our extensive form bargaining game.

Set-up and notation

The most natural way to describe the set of bilateral negotiations is by a graph (N, L) which has the set of agents as its vertices each connected by a set of edges or links, $L \subseteq L^N = \left\{ \{i, j\} \middle| \{i, j\} \subseteq N, i \neq j \right\}$. Thus, the potential number of links in a *complete* graph (N, L^N) is $n(n$-1)/2. An individual link between i and j will be denoted ij (or symmetrically ji). L describes the state space of potential agreements. If a link, ij, is in L, then agents i and j can still come to a bilateral agreement. If ij is not in L, then agents have reached a disagreement state. If a pair $ij \in L$ were to disagree, the new state is denoted: $L - ij$. Finally, for any sub-graph, $K \subseteq L$, let $S(K) \equiv \left\{ i \middle| \exists j \text{ s.t. } ij \in K \subseteq L \right\}$. Note that $S(L) = N$.

Starting with a network (N, L), agents i and j negotiate bilaterally over choices, $x_{ij}(K) \in \mathbf{R}$ and payments $t_{ij}(K) \in \mathbf{R}$ for each $K \subseteq L$ where $ij \in K$. There are potentially $n(n$-1)/2 choice and payments variables; the (n x n) matrices of which are $\mathbf{X}^N$ and $\mathbf{T}^N$, respectively. We also define $\mathbf{X}^{N,L}$ the matrix of choice variables where $x_{ij} = 0$ for $ij \notin L$. Thus, if there is a disagreement, we normalise by setting the relevant choice variable at 0.[2] Similarly, we define $\mathbf{T}^{N,L}$ where $t_{ij} = 0$ for $ij \notin L$. That is, no bilateral payments are made if there is a disagreement.

Given the agreed $\mathbf{X}^{N,L}$ and $\mathbf{T}^{N,L}$, an agent's payoff is $v_i(\mathbf{X}^{N,L}, \mathbf{T}^{N,L}) = u_i(\mathbf{X}^{N,L}) - \mathbf{T}^{N,L}\mathbf{I}_i$ where $\mathbf{I}_i$ is an n by n matrix with a 1 in each row of column i and a 0 otherwise. This implies that $\mathbf{T}^{N,L}\mathbf{I}_i = \sum_{ij \in L} t_{ij}$. Thus, we are assuming a transferable utility environment where total surplus generated is not affected by T. That is, $\sum_i v_i(\mathbf{X}^{N,L}, \mathbf{T}^{N,L}) = \sum_i u_i(\mathbf{X}^{N,L})$.

This notation also allows us to define what we mean by a (constrained) efficient set of agreements.

[2] Note that this does not necessarily mean that following a disagreement, no action is taken. For instance, a jointly observed action may be within the discretion of one party. In the event of a breakdown, that party may take that action to maximise their own payoff. We set the index of such individually optimized actions to zero.

Definition (Efficiency): *For a given K, a set of agreements* $\left\{\mathbf{x}_{ij}^{*}(K)\right\}_{ij\in K}$ *is efficient if:*

$$\mathbf{x}_{ij}^{*}(K) \in \arg\max_{\left\{\mathbf{x}_{ij}(K)\right\}_{ij\in K}} \sum_{i\in N} u_i(\mathbf{X}^{N,K}) \text{ for all } ij \in K .$$

with $v(N,K) \equiv \max_{\left\{\mathbf{x}_{ij}(K)\right\}_{ij\in K}} \sum_{i\in N} u_i(A,\mathbf{X}^{N,K})$.

Thus, an agreement is efficient for a given network (*N*, *K*), if the choices agreed upon maximise the sum of utilities over all agents whether they are party to an agreement or not. Note also that for a subgraph, *S*: $v(S,K) \equiv \max_{\left\{\mathbf{x}_{ij}(K)\right\}_{ij\in K}} \sum_{i\in S} u_i(\mathbf{X}^{S,K})$.

Finally, for some analysis that follows it will be convenient to partition the set of agents. $P = \{P_1,...,P_p\}$ is a partition of the set *N* if and only if (i) $\bigcup_{i=1}^{p} P_i = N$; and (ii) for all $j \neq k$, $P_j \cap P_k = \varnothing$. We define *p* as cardinality of *P*. The set of all partitions of *N* is P^N. For a given network (*N*, *K*), we can also define a graph, K^P, imputed from a partition, *P*. That is, $K^P = \{ j,k \mid jk \in K \text{ and } j \in P_i, k \in P_i \}$. In other words, K^P is a graph partitioned by *P*.

All of these concepts can be illustrated by returning to our buyer-seller network example. In this situation, Figure 1 depicts the set of links, $L = \{1A, 1B, 2A, 2B\}$ and we have assumed that $u_1(\mathbf{X}^{L,N}) = b_1(x_{1A} + x_{1B})$, $u_2(\mathbf{X}^{L,N}) = b_2(x_{2A} + x_{2B})$ and $u_A(\mathbf{X}^{L,N}) = u_B(\mathbf{X}^{L,N}) = 0$. An efficient outcome would involve $v(L,N) = \max_{x_{1A},x_{1B},x_{2A},x_{2B}} b_1(x_{1A} + x_{1B}) + b_2(x_{2A} + x_{2B})$. If, however, 1 and A could no longer negotiate or trade with one another, the network would become $K = \{1B, 2A, 2B\}$ and (x_{1A}, t_{1A}) would be set equal to $(0,0)$ with $v(K,N) = \max_{x_{1B},x_{2A},x_{2B}} b_1(x_{1B}) + b_2(x_{2A} + x_{2B})$. Finally, if we were to partition the set of agents into *P* = {(1, *A*), (2, *B*)}, $L^P = \{1A, 2B\}$ and $K^P = \{2B\}$.

In terms of what the parties negotiate over, recall that these are contingent outcomes. So 1 and *A* could negotiate, say, a quantity $x_{1A}(L)=3$ and transfer $t_{1A}(L)=2$ as well as $x_{1A}(1A,1B,2B)=4$ and $t_{1A}(1A,1B,2B)=5$ and so on. That is, they consider all possible states that could emerge and they can negotiate different quantity and transfers that would be payable upon the final realisation of any particular network. In principle, the transfers and quantities paid under each network contingency could be the same. That is, the contract could be a full commitment. However, in the equilibrium we focus on below, this will not be the case.

Information regarding the bargaining network

Earlier we stated (A1) which confined the observability of actions to those agents undertaking them. Also of importance is the set of bilateral negotiations that can take place. These are described by the network, (*N*, *L*). That network connects sub-groups of agents or perhaps all agents. More precisely,

Definition (Connectedness). *Agents i and j are connected in network* (N,L) *if there exists a sequence of agents* $(i_1,i_2,...,i_t)$ *such that* $i_1=i$ *and* $i_t=j$ *and* $\{i_l,i_{l+1}\}\in L$ *for all* $l\in\{1,2,...,t-1\}$. *i is directly connected to j if* $ij\in L$.

Definition (Component). *A component of network* (N,L) *that contains i is* $C_i(L)=\{j\in N \mid j=i \text{ or } j \text{ is connected to } i\}$. *Let N/L be the set of components of network* (N,L).

Notice that all agents in a network (*N*, *L*) are connected if, for all $j\in L$, $j\in C_i(L)$. All agents are connected if, or for all $j\in N$, $j\in C_i(L)$. Finally, all agents are directly connected if $L=L^N$ (the complete graph).

Importantly, in what follows, a *breakdown* in bargaining between *i* and *j* is a situation where the network changes from (N,L) where $ij\in L$ to $(N,L-ij)$; implying that $\{\mathbf{x}_{ij}(K),t_{ij}(K)\}_{K\subseteq L-ij}=\{0,0\}$. It will also be considered irreversible as the link between *i* and *j* can never subsequently be restored. Thus, as breakdowns are possible, the network will potentially move from one with many links to graphs that are subsets of the original network. For convenience, we will sometimes describe networks in terms of states with a current state and potential future states.

A key assumption here is:

(A2) (Knowledge of the Bargaining Network) *The state of the network* (N,K) *is common knowledge.*

This assumption is necessary in order for agents to negotiate contracts that are contingent upon the network state. As we will also see below, this assumption would be necessary if, rather than writing contracts contingent upon networks that may arise, agents negotiated contracts based only on the current network and renegotiated them in the event a new network arose (following a breakdown). Analytically, we demonstrate below that (A2) simplifies the ultimate structure of the solution to our bargaining game.

Extensive form

We are now in a position to define the full extensive form game. Given (*N*, *L*), fix an order of pairs, $\{ij\}_{ij\in L}$. The precise order will not matter to the solution that follows. Bargaining proceeds as follows. Each pair negotiates in turn. A bilateral negotiation takes the following form: randomly select *i* or *j*. That agent, say *i*, makes an offer $\left\{\mathbf{x}_{ij}(K),t_{ij}(K)\right\}_{K\subseteq L,ij\in K}$ to *j*. *j* either accepts the offer or rejects it. If *j* accepts it, the offer $\left\{\mathbf{x}_{ij}(K),t_{ij}(K)\right\}_{K\subseteq L,ij\in K}$ is fixed and we proceed to the next pair. If *j* rejects the offer, with probability 1-σ negotiations end and the bargaining game recommences over a new network $(N,L-ij)$. Otherwise negotiations continue with *j* making an offer to *i*. Notice that offers are made *contingent* upon the potential agreement state (*K*).

This specification of an individual bilateral negotiation is essentially the same as that of Binmore, Rubinstein and Wolinsky (1986) for stand-alone bilateral negotiations. Here, however, bilateral negotiations are not isolated and are embedded within a sequence of negotiating pairs.

Belief structure

Given that our proposed game involves incomplete information, to demonstrate the existence of certain equilibrium outcomes in the game, we will need to impose some structure on 'out of equilibrium' beliefs. This is an issue that has drawn considerable attention in the contracting with externalities literature (McAfee and Schwartz, 1994; Segal, 1999; Rey and Verge, 2003).

It is not our intention to revisit that literature here. Suffice it to say that the most common assumption made about what players believe about actions that they do not observe, or that have not yet happened is the simple notion of "passive" or "market to market" beliefs. We will utilise it through this paper. To define it, let $\{(\hat{\mathbf{x}}_{ij}(K),\hat{t}_{ij}(K))\}_{ij\in L,K\subseteq L}$ be a set of equilibrium agreements between all negotiating pairs.

Definition (Passive Beliefs). *When i receives an offer from j of* $\mathbf{x}_{ij}(K)\neq\hat{\mathbf{x}}_{ij}(K)$ *or* $t_{ij}(K)\neq\hat{t}_{ij}(K)$, *i does not revise its beliefs regarding any other outcome in the game.*

At one level, this is a natural belief structure that mimics Nash equilibrium reasoning. That is, if *i*'s beliefs are consistent with equilibrium outcomes – as they would be in a perfect Bayesian equilibrium, then under passive beliefs, it holds those beliefs constant off the equilibrium path as well. At another level, this is precisely why passive beliefs are not appealing from a game-theoretic standpoint. Specifically, if *i* receives an unexpected offer from an agent it knows to be rational, a restriction of passive beliefs is tantamount to assuming that *i* makes no inference from the unexpected outcome; even if it were to accept this offer based on its current beliefs. Nonetheless, as we demonstrate here, passive beliefs plays an important role in generating tractable and interpretable results from our extensive form bargaining game; simplifying the interactions between different bilateral negotiations.

Feasibility

The non-cooperative bargaining game presented above will have an equilibrium whose convenient characterisation will at times rely upon agreements being reached in all bilateral negotiations in a network (*N*, *L*). However, an equilibrium with this property may not exist. For instance, as Maskin (2003) demonstrates, when an agent may be able to free ride upon the contributions and choices of other agents, that agent may have an incentive to force breakdowns in all their negotiations so as to avoid their own contribution. Maskin demonstrates that this is the case for situations where there are positive externalities between groups of agents (as in the case of public goods).

The idea that an agent or group of agents may not wish to participate in a larger coalition is related to the existence of the core. Here, the usual definition of the core will not, in general, apply as the actions agreed upon in bilateral negotiations may not maximise the value of a coalition. For that reason, we make an assumption equivalent to core existence in our bilateral context.

For this purpose, we first need a definition of bilateral efficiency:

Definition (Bilateral Efficiency). *For a given network (S, K), a set of actions,* $\hat{X}(K) = \{\hat{\mathbf{x}}_{ij}(K)\}_{ij \in K}$ *satisfies bilateral efficiency if:*

$$\hat{\mathbf{x}}_{ij}(K) \in \arg\max_{\mathbf{x}_{ij}} u_i(\mathbf{x}_{ij}, \{\hat{\mathbf{x}}_{kl}(K)\}_{kl \neq ij}) + u_j(\mathbf{x}_{ij}, \{\hat{\mathbf{x}}_{kl}(K)\}_{kl \neq ij}) \text{, } for\ all\ ij \in K .$$

Consistent with this definition, we define: $\hat{v}(S,K) \equiv \sum_{i \in S} u_i(\hat{X}(K))$ where $\hat{X}(K)$ are bilaterally efficient. Given this, we can assume:

(A3) (Bi-Core Existence) The Bi-Core as defined by:

$$\text{Bi-Core}(N,L) \equiv \left\{ \{v_i\}_{i \in N} \middle| \text{for all } S \subseteq N, \sum_{i \in S} v_i \geq \hat{v}(S, L^S) \right\}$$

is non-empty.

This assumptions states that given any set of payoffs to all agents, any subset of agents will be jointly better off with those payoffs than with the joint payoff they would receive if all existing links (given L) were severed with agents outside of that subset; assuming that joint payoff is bilaterally efficient.

Suppose that $v(K,N) \geq v(K - ij, N)$ for all $ij \in K$ and $K \subseteq L$. This is a weak form of superadditivity. Then the following condition clearly implies a non-empty core.

Definition (No Component Externalities). *$u_i(\mathbf{X}^{N,L})$ is independent of x_{kl} for any $k \notin C_i(L)$.*

This condition says that if *i*'s utility is not affected by actions of agents that it is not connected to. It is related to the concept of component decomposability in cooperative game theory that is an axiom on *i*'s realised payoff rather than condition on a primitive of the model.

Notice that our buyer-seller network example satisfies this condition as b_1 and b_2 are independent of the purchases of the other buyer. However, if these buyers were competitors in some other market, then it is possible that their purchases could enter into the utilities of each other. In this case, a component externality would be present.

5. Equilibrium Outcomes: Actions

In exploring the outcomes of this non-cooperative bargaining game, it is useful to focus first on the equilibrium actions that emerge before turning to the transfers and ultimate payoffs. Of course, an equilibrium described is one in which actions and transfers are jointly determined. It is for expositional reasons that we focus on each in turn.

The precise equilibrium transfers do not directly determine overall surplus generated. In this regard, we can demonstrate the following:

Theorem 1. *Suppose that all agents hold passive beliefs regarding the outcomes of negotiations they are not a party to. Given* (N,L) *, as* $\sigma \to 1$*, in any perfect Bayesian equilibrium outcome,* $(\hat{X}(L),\hat{T}(L))$*, is bilaterally efficient.*

All proofs are in the appendix. This result says that individual actions are chosen to maximise own utility given expectations about unobserved actions while joint actions are chosen to maximise joint utility under the same expectations. It is easy to see that in general the outcome will not be efficient.

The intuition behind the result is subtle. Consider a pair, *i* and *j*, negotiating in an environment where they have agreed to the equilibrium choices in any past negotiation and there is one more additional negotiation still to come and that this negotiation involves *j* and another agent, *k*. Given the agreements already fixed in past negotiations, the final negotiation between *j* and *k* is simply a bilateral Binmore, Rubinstein, Wolinsky bargaining game that would ordinarily yield the Nash bargaining solution if *j* and *k* had symmetric information regarding the impact of their choices on their joint utility, $u_j(x_{ij}, x_{jk},.)+u_k(x_{ij}, x_{jk},.)$. This will be the case if *i* and *j* agree to the equilibrium $\hat{x}_{ij}$. However, if *i* and *j* agree to $x'_{ij} \neq \hat{x}_{ij}$, *j* and *k* will have different information. Specifically, while, under passive beliefs, *k* will continue to base its offers and acceptance decisions on an assumption that $\hat{x}_{ij}$ has occurred, *j*'s offers and acceptances will be based on x'_{ij}. That is, *j* will make an offer, $(t'_{jk}, x'_{jk}(x'_{ij}))$, that maximises $u_j(x_{ij}, x_{jk}, .) - t_{jk}$ rather than $u_j(\hat{x}_{ij}, x_{jk}, .) - t_{jk}$ subject to *k* accepting that offer. Moreover, we demonstrate that *j* will reject offers made to it by *k*.

In this case, the question becomes: will *i* and *j* agree to some $x'_{ij} \neq \hat{x}_{ij}$? If they do, this will alter the equilibrium in subsequent negotiations. *j* will anticipate this, however, the assumption of passive beliefs means that *i* will not. That is, even if they agreed to $x'_{ij} \neq \hat{x}_{ij}$, *i* would continue to believe that $\hat{x}_{jk}$ will occur. For this reason, *i* will continue to make offers consistent with the proposed equilibrium. On the other hand, *j* will make an offer, (t'_{ij}, x'_{ij}), that maximises $u_j(x_{ij}, x'_{jk}(x_{ij}), .) - t'_{ij}(x_{ij})$ rather than $u_j(x_{ij}, \hat{x}_{jk}, .) - t_{ij}$ subject to *i* accepting that offer. We demonstrate that this is equivalent to *j* choosing:

$$x'_{ij} \in \arg\max_{x_{ij}} u_j(x_{ij}, x'_{jk}(x_{ij}), .) + u_i(x_{ij}, \hat{x}_{jk}, .) + u_k(\hat{x}_{ij}, x'_{jk}(x_{ij}), .)$$

which, by the envelope theorem applied to x'_{jk}, has $x'_{ij} = \hat{x}_{ij}$.

When the negotiation between *i* and *j* is not the second last negotiation, there is an additional complication in that deviations by them will trigger a cascade of deviations throughout subsequent negotiations. Nonetheless, in the proof we demonstrate that, taking this into account, *i* and *j* will still not to deviate from the conjectured equilibrium. Essentially, even if they are the first negotiating pair, then a deviation will impact on every subsequent negotiation through a connected graph. However, in this case, they take into account all agents' utilities in an additive fashion so that the envelope theorem continues to apply in the same manner as in the 'two negotiation' case.

Finally, it is useful to state a case where the perfect Bayesian equilibrium outcome under passive beliefs is efficient. Consider the following definition:

Definition (No Non-Pecuniary Externalities). *$u_i(\mathbf{X}^{N,L})$ is independent of $\mathbf{x}_{jk}$ for all* $\{jk | ij \notin L \text{ and } ik \notin L\}$.

That is, *i*'s utility is only affected by joint actions made by agents it is directly connected to. Notice that pecuniary externalities can still exist here through the transfers that are agreed upon in other bilateral negotiations that themselves impact on the value of an agreement between a particular pair. Given this we have the following result:

Corollary 1. *Assume the conditions of Theorem 1 and that there are no non-pecuniary externalities for all i. Then given (N,L), as $\sigma \to 1$, the unique perfect Bayesian equilibrium agreements are efficient.*

6. Equilibrium Outcomes: Transfers and Payoffs

We are now in a position to consider the equilibrium transfers and payoffs. As was determined above, when there are externalities present, sequential bilateral bargaining does not lead to a maximised surplus. Instead, under passive beliefs, it yields a Nash equilibrium where actions are taken ignoring externalities on other agents. In this sense, the outcome is very different than what might emerge from cooperative bargaining.

However, we demonstrate here that while surplus is determined in a non-cooperative manner, under the same passive beliefs assumption, division arising from the same underlying non-cooperative game takes on a form attractively similar to cooperative bargaining outcomes. In particular, depending upon the nature of externalities and the network of bilateral negotiations, the division of whatever surplus is created gives agents variants of their Myerson-Shapley value on that reduced surplus. As such, payoffs have an appealing coalitional structure even if surplus mirrors a non-cooperative determination.

Some definitions

It is useful at this point to state some additional definitions from cooperative bargaining theory using our notation.

Definition (Shapley Value). *The Shapley value of agent i, in a given coalition,* $S \subseteq N$ *,* $\Phi_i(S)$ *is:*

$$\Phi_i(S, L^S) = \sum_{T \subseteq S : i \notin T} \frac{|T|!(|S|-1-|T|)!}{|S|!}\left(v(T \cup i, L^{T \cup i}) - v(T, L^T)\right).$$

This is the definition introduced by Aumann and Dreze (1974) and it becomes the value derived by Shapley (1953) when the relevant coalition is the grand coalition, *N*. In contrast, Myerson (1977) provides a related value that is defined over a network.

Definition (Myerson Value). *Suppose that* $v(S, L) = \sum_{K \in S/L} v(S, K)$. *The Myerson value of agent i,* $\Psi_i(S, L)$ *is a function that satisfies: (i)* $\sum_{i \in G(K)} \Psi_i(S, K) = v(S, K)$ *for all* $K \subseteq L$ *and* $G \in S/K$ *; and (ii)* $\Psi_i(S, K) - \Psi_i(S, K - ij) = \Psi_j(S, K) - \Psi_j(S, K - ij)$ *for all* $K \subseteq L$ *and* $ij \in K$ *.*

Note that the condition on $v(S, L)$ (termed component efficiency in the cooperative game theory literature) is automatically satisfied if agents' utilities satisfy the no component and no non-pecuniary externalities conditions. This definition of the Myerson value comes from Jackson and Wolinsky (1996) which allows the value derived from a coalition to depend upon the network underlying the coalition. In relevant examples in the literature, considered below, the network itself plays a critical role in productivity. Myerson had assumed that a coalition would result in the same total value regardless of how agents in the coalition were connected.

The Myerson value is somewhat restrictive in that it is not defined in situations where different groups of agents impost externalities upon one another.

Myerson (1977b) generalised the Shapley value to consider this by defining it for games in partition function space. Here we provide a further generalised definition of the Myerson value to allow for a partition function space as well as a graph of potential communications.

Definition (Generalised Myerson Value). *The Generalised Myerson value of agent i, in a given coalition,* $S \subseteq N$ *,* $\Upsilon_i(S,L)$ *is:*

$$\Upsilon_i(S,L) = \sum_{P \in P^N} \sum_{T \in P} (-1)^{p-1}(p-1)! \left[\frac{1}{|S|} - \sum_{\substack{i \notin T' \in P \\ T' \neq T}} \frac{1}{(p-1)(|S|-|T'|)} \right] v(T, L^P) .$$

It is easy to demonstrate that when there are no component externalities, this value is equivalent to the Myerson value and, in addition, if it is defined over a complete graph, it is equivalent to the Shapley value.

Some Issues: An Illustrative Example

Before turning to consider these results, it is useful to highlight some important technical issues by way of an illustrative example. Consider a situation in which there are three agents (1, 2 and 3), each of whom can negotiate bilaterally with one another; that is, our starting point is a complete graph. We will denote this initial network by 123. If there is a breakdown in negotiations between one pair that will result in a network of 1-2-3, 1-3-2 or 2-1-3 respectively; with the middle agent the agent who has not had a breakdown with any of the other two agents. If there are two breakdowns in negotiations, the networks may become 12, 13 or 23. Finally, if all three negotiations breakdown, the state becomes 0.

We suppose also that there are only joint actions and, using the result in Theorem 1, those actions will lead to a payoff to agent *i* of $u_i(K)$; for example, if network 1-2-3 occurs, the expected negotiated actions are such that $u_1(1-2-3)$ is generated to agent 1.

To see how payoffs and transfers are determined in equilibrium, note that, as the probability of a breakdown anywhere, σ, goes to 0, we can treat negotiations over transfers in each state as separate bilateral negotiations between each negotiating pair. If this is 12, then, then our BRW bargaining game results in the Nash bargaining solution:

$$\begin{aligned} &u_1(12) - t_{12} - u_1(0) = u_2(12) + t_{12} - u_2(0) \\ &\Rightarrow t_{12}(12) = \tfrac{1}{2}\left(u_1(12) - u_2(12) + u_2(0) - u_1(0)\right) \end{aligned} \tag{1}$$

For 1-2-3 these are:

$$\begin{aligned} &u_1(1-2-3) - t_{12}(1-2-3) - u_1(23) \\ &= u_2(1-2-3) + t_{12}(1-2-3) - t_{23}(1-2-3) - \left(u_2(23) - t_{23}(23)\right) \end{aligned} \tag{2}$$

$$\begin{aligned} &u_1(2-1-3) - t_{12}(2-1-3) - t_{13}(2-1-3) - \left(u_1(13) - t_{13}(13)\right) \\ &= u_2(2-1-3) + t_{12}(2-1-3) - u_2(13) \end{aligned} \tag{3}$$

And for 123, these are:

$$\begin{aligned} &u_1(123) - t_{12}(123) - t_{13}(123) - \left(u_1(1-3-2) - t_{13}(1-3-2)\right) \\ &= u_2(123) + t_{12}(123) - t_{23}(123) - \left(u_2(1-3-2) - t_{23}(1-3-2)\right) \end{aligned} \tag{4}$$

$$\begin{aligned} &u_1(123) - t_{12}(123) - t_{13}(123) - \left(u_1(1-2-3) - t_{12}(1-2-3)\right) \\ &= u_3(123) + t_{13}(123) + t_{23}(123) - \left(u_3(1-2-3) - t_{23}(1-2-3)\right) \end{aligned} \tag{5}$$

$$\begin{aligned} &u_2(123) + t_{12}(123) - t_{23}(123) - \left(u_2(2-1-3) - t_{12}(2-1-3)\right) \\ &= u_3(123) + t_{13}(123) + t_{23}(123) - \left(u_3(2-1-3) - t_{13}(2-1-3)\right) \end{aligned} \tag{6}$$

With the total number of transfer prices over all contingent negotiations being 12. While solving for transfers would appear to be possible with 12 bargaining equations and 12 unknowns, equations (4), (5) and (6) are linearly dependent. For there are many consistent transfer prices -- $t_{12}(123)$, $t_{13}(123)$ and $t_{23}(123)$ -- that will satisfy those equations. In other cases, the transfer prices are uniquely determined. It is for this reason, that we refer in theorems to equilibrium outcomes rather then equilibria themselves. Nonetheless, even though particular transfer prices are not uniquely determined in some networks, payoffs are.

In this game, it is straightforward to demonstrate that in equilibrium, agents receive:

$$\begin{aligned}\Phi_1(123) = {} & \tfrac{1}{3}\left(u_1(123)+u_2(123)+u_3(123)\right)+\tfrac{1}{3}\left(2u_1(23)-u_2(23)-u_3(23)\right)\\ & +\tfrac{1}{6}\left(u_1(12)+u_2(12)-2u_3(12)\right)+\tfrac{1}{6}\left(u_1(13)-2u_2(13)+u_3(13)\right)\\ & +\tfrac{1}{6}\left(-2u_1(0)+u_2(0)+u_3(0)\right)\end{aligned}$$

$$\begin{aligned}\Phi_2(123) = {} & \tfrac{1}{3}\left(u_1(123)+u_2(123)+u_3(123)\right)+\tfrac{1}{6}\left(-2u_1(23)+u_2(23)+u_3(23)\right)\\ & +\tfrac{1}{6}\left(u_1(12)+u_2(12)-2u_3(12)\right)+\tfrac{1}{3}\left(-u_1(13)+2u_2(13)-u_3(13)\right)\\ & +\tfrac{1}{6}\left(u_1(0)-2u_2(0)+u_3(0)\right)\end{aligned}$$

$$\begin{aligned}\Phi_3(123) = {} & \tfrac{1}{3}\left(u_1(123)+u_2(123)+u_3(123)\right)+\tfrac{1}{6}\left(-2u_1(23)+u_2(23)+u_3(23)\right)\\ & +\tfrac{1}{3}\left(-u_1(12)-u_2(12)+2u_3(12)\right)+\tfrac{1}{6}\left(u_1(13)-2u_2(13)+u_3(13)\right)\\ & +\tfrac{1}{6}\left(u_1(0)+u_2(0)-2u_3(0)\right)\end{aligned}$$

These outcomes are, in fact, each agent's Shapley values. We demonstrate below that this is a general outcome in environments where the set of bilateral negotiations comprises a complete graph.

Notice that these payoffs do not depend on network states where there are two bilateral negotiations despite that fact that $\sum_i u_i(123)$ does not equal $\sum_i u_i(1-2-3)$ as it does in Myerson (1977). Jackson and Wolinsky (1995) demonstrate a similar outcome for the Myerson-Shapley value. Here, the outcome arises for the same reason as each pair of Nash bargaining equations represents a condition of balanced contributions. This is a property that makes these bargaining outcomes particularly useful in applications as we do not need to solve for non-cooperative action outcomes in beyond the network with most links in any connected coalition.

General Result

We are now in a position to state our main result.

Theorem 2. *Given* (N,L), *as* $\sigma \to 1$, *there exists a perfect Bayesian outcome of our extensive form bargaining game with each agent i receiving:*

$$\hat{\Upsilon}_i(N,L) = \sum_{P \in P^N} \sum_{T \in P} (-1)^{p-1}(p-1)! \left[\frac{1}{n} - \sum_{\substack{i \notin T' \in P \\ T' \neq T}} \frac{1}{(p-1)(n-|T'|)} \right] \hat{v}(T, L^P) .$$

Thus, in equilibrium, we have a generalised Myerson value type division of a reduced surplus. That surplus is generated by a bilaterally efficient outcome where each bilateral negotiation maximising their own sum of utilities while ignoring the external impact of their choices on other negotiations (as in Theorem 1).

As in Theorem 1, the proof relies upon the agents holding passive beliefs in equilibrium. For this reason, Theorem 2 is an existence proof. Without passive beliefs, the equilibrium outcomes are more complex and do not reduce to this simple structure. That simplicity is, of course, the important outcome here. What we have is a bargaining solution that marries the simple linear structure of cooperative bargaining outcomes with the easily determined actions based on bilateral efficiency. As we demonstrate below, that allows it to be of practical value in applied work.

To that end, directly following on from Theorem 2, are the following corollaries:

Corollary 2. *Suppose that for all* $i \in N$, $u_i(\mathbf{X}^{N,L})$ *satisfies no component externalities. Given* (N,L), *as* $\sigma \to 1$, *there exists a perfect Bayesian outcome of our extensive form bargaining game with each agent receiving:*

$$\hat{\Psi}_i(N,L) \equiv u_i(\hat{\mathbf{X}}^{N,L}) - \mathbf{P}^{N,L}\mathbf{I}_i \text{ where } \hat{\mathbf{X}}^{N,L} = \{\hat{x}_{ij}\}_{ij \in L} ,$$

where (a) $\sum_{i \in S(K)} \hat{\Psi}_i(S,K) = \hat{v}(S,K)$ *for all* $K \subseteq L$ *and* $G \in S/K$; *and (b)* $\hat{\Psi}_i(S,K) - \hat{\Psi}_i(S,K-ij) = \hat{\Psi}_j(S,K) - \hat{\Psi}_j(S,K-ij)$ *for all* $K \subseteq L$ *and* $ij \in K$. *If, instead, we have* (N,L^N), *each agent receives:*

$$\hat{\Phi}_i(N,L^N) = \sum_{S \subseteq N : i \notin S} \frac{|S|!\left(|N|-1-|S|\right)!}{|N|!} \left(\hat{v}(S \cup i, L^{S \cup i}) - \hat{v}(S, L^S) \right).$$

Thus, with no component externalities, we obtain the Myerson (or Shapley value) type division of a reduced surplus based on bilateral efficiency. On the other hand, with a stronger condition, we have a non-cooperative foundation for the Myerson-Shapley value: DEFINE NON-PECUNARY

Corollary 3. *Suppose that for all* $i \in N$, $u_i(\mathbf{X}^{N,L})$ *satisfies no non-pecuniary externalities. Given* (N,L)*, as* $\sigma \to 1$*, there exists a perfect Bayesian outcome of our extensive form bargaining game with each agent receiving their Myerson value.*

Non-Binding Agreements

It is possible, however, that in some environments agents will not be able to make agreements that are contingent upon the state *K*. This is a central assumption in, for example, Stole and Zwiebel (1996) who assume that labour supply contracts are non-binding and so can be unilaterally broken if there is a change in a publicly observed state.

To explore this, suppose that, given *K*, a sequence of pairs $\{ij\}_{ij \in K}$ is fixed and agent pairs make alternating offers to one another regarding a single choice and payment pair. If they agree, for example, to $\left(x_{ij}(K), t_{ij}(K)\right)$, the next pair in the sequence negotiates. However, if a breakdown occurs, then the state changes to $K - ij$ and a new subgame occurs in which a sequence of pairs in $K - ij$ is fixed and bilateral negotiations take place in sequence. On the other hand, if there is no breakdown in a sequence then the agreements $\left\{x_{ij}(K), t_{ij}(K)\right\}_{ij \in K}$ stand and each agent's payoff is determined.

This case involves *non-binding* agreements. An interpretation of this is that while each pair might arrive at an agreement, if there is a change in circumstance – that is, the state of agreements, *K* – then any individual agent can re-open negotiations with any other agent it is still linked to in K. This is precisely the generalisation of the Stole-Zwiebel bargaining game to our more general environment.

It is straightforward to demonstrate that the proofs of all results – in particular, Theorems 1 and 2 – are unchanged by this. The reason is that in those proofs we focus on an equilibrium where agreements contingent upon a state maximise the joint payoffs of the parties concerned. This is precisely what would happen if, in fact, the parties were to re-negotiate contract terms *following* the observation of a state (K) rather than prior to it. Indeed, this simplifies the belief structure considerably as they are the subgame perfect outcomes following a breakdown whereas in our contingent contract case they are the expectation of agreements signed by others.

7. Applications

We now consider how our basic theorems apply in a number of specific contexts where cooperative game theoretic outcomes have played an important role.

Gul's Resource Accumulation Game

Gul (1989) considers the following economic environment. Each of N agents has a valuable resource. The resources can be combined into bundles of $M \leq N$ resources that would give their owner $V(M)$ in utility. Moreover, $\max_M V(M) = V(N)$ so it is efficient for all resources to be combined. Gul (1989) uses this environment and a specific extensive form bargaining game (based on random matching) to demonstrate that there exists an equilibrium (specifically a stationary subgame perfect equilibrium) where each individual agent receives their Shapley value.

Here we examine the same environment but using our non-cooperative bargaining game. As in Gul (1989), it is assumed that all agents can transact with any other agent. However, unlike Gul, we assume that there is only a single opportunity for this as each pair bargains in a previously defined sequence. Thus, here $x_{ij} = M + 1$ if i purchases a resource bundle of size M from j and $x_{ij} = 0$ if i sells its resources to j. Agent i's utility is $V(\sum_{j \neq i} x_{ij})$ where $V(0) = 0$.

Observe first that utilities here involve no pecuniary externalities as they depend only directly on trades with agents they are directly connected to. Thus, Corollary 3 immediately applies and each agent receives it Shapley value. Moreover, we can naturally extend the environment to a situation where agent utilities differ and there is an efficient owner of all resources. In this situation, there exists an equilibrium outcome where that owner receives the resources.

Stole and Zwiebel's Wage Bargaining Game

Stole and Zwiebel (1996) develop a model of wage bargaining between a number of workers and a single firm. The workers cannot negotiate with one another or as a group. Thus, the relevant network has an underlying 'star' graph with links between the firm and each individual worker. A key feature of Stole and Zwiebel's model is that bargaining over wages is non-binding; that is, following the departure of any given worker (that is, a breakdown), either the firm or an individual worker can elect to renegotiate wage payments. As noted earlier, while Stole and Zwiebel posit an extensive form bargaining game as a foundation for their axiomatic treatment of bargaining, the equilibria in this game are not really characterised. Nonetheless, Theorem 2 now provides that characterisation; confirming their Shapley value outcome.

Interestingly, Theorem 2 now demonstrates that an assumption that wage contracts are non-binding is not necessary to motivate the Stole-Zwiebel wage bargaining outcome. Instead, wage contracts could be made contingent upon the number of workers employed by the firm. The result would be the same payoffs to the firm and each worker. Moreover, Stole and Zwiebel's key conclusions regarding how anticipation of this wage outcome impacts upon the firm's ex ante choices of employment, capital and technology will all be preserved for the contingent contract case. Thus, the economic driving force behind Stole and Zwiebel's labour market results is an environment that gives individual workers some bargaining power in ex post wage negotiations rather than the non-binding nature of wage contracts.[3]

Nonetheless, what is significant here is that, when a firm cannot easily expand the set of workers it can employ ex post, there will be a Myerson value wage bargaining outcome (as in Corollary 2). This happens if workers are not identical, differ in their outside employment wages, and have variable work hours. Moreover, if there were many firms, each of whom could bargain with any available worker ex post, each firm and each worker will receive their Myerson value over the broader network. As such, our results demonstrate that a Myerson value outcome can be employed in significantly more general environments than those considered by Stole and Zwiebel.

General Buyer-Seller Networks

Perhaps the most important application of the model presented here is to the analysis of buyer-seller networks. These are networks where buyers purchase goods from sellers and engage in a series of bilateral transactions; the joint actions between them being the total volume of trade. Significantly, it is often assumed – for practical

[3] This is also true of the results of Wolinsky (2000) who uses an axiomatic argument to justify a Shapley value wage bargaining outcome. de Fontenay and Gans (2003) examine a situation where a breakdown in negotiations causes a link with one worker (the insider) to be severed and a link to be established, if possible, with a new worker. The above results do not apply to breakdowns that build links as well as remove them.

and antitrust reasons – that the buyers and sellers do not negotiate with others on the same side of the market. Hence, the analysis takes place on a graph with restricted communication and negotiation options.

In this literature models essentially fall into two types. The first assumes that there are externalities between buyers (as might happen if they are firms competing in the same market) but that there is only a single seller (e.g., McAfee and Schwartz, 1994; Segal, 1999; de Fontenay and Gans, 2004a) while the second assumes that there are no externalities between buyers but there are multiple buyers and sellers (Cremer and Riordan, 1987; Kranton and Minehart, 2001; Inderst and Wey, 2003; Bjonerstedt and Stennek, 2002).[4] In each case, however, the underlying bargaining or market game differs from the model here ranging from a series of take it or leave it offers (McAfee and Schwartz, 1994) to auction mechanisms (Kranton and Minehart, 2001) to a simultaneous determination of bilateral negotiations (Inderst and Wey, 2003).

Nonetheless, regardless of the type of model, this literature is predominantly focused upon whether bilateral transactions between buyers and sellers can yield efficient outcomes. The broad conclusion is that where there are externalities between buyers, the joint payoff of buyers and sellers is only maximised when those externalities are not present.

Our environment here encompasses both of these model types – permitting externalities between buyers (and indeed sellers) as well as not restricting the numbers or set of links between either side of the market. In so doing, we have demonstrated that when there are no non-pecuniary externalities – i.e., the only externalities for variables that are bilaterally contractible between agents occur through prices – then

[4] Horn and Wolinsky (1988) permit externalities between buyers but sellers are constrained to deal with a single buyer.

surplus is maximised (Corollary 1). Thus, it provides a general statement of the broad conclusion of the buyer-seller network literature.

In effect, Corollary 1 can be viewed as a generalisation of the results of Segal (1999, Proposition 3) that when there are no externalities, contracts are efficient. We can also characterise the equilibrium outcomes and their relationship with the efficient set of outcomes; generalising Segal (1999, Proposition 4).

Theorem 2. *Suppose that each x_{ij} is measured in the same increments. Then if* $M^* = \left\{ \sum_{ij \in L} \mathbf{x}^*_{ij} \middle| \{\mathbf{x}^*_{ij}\}_{ij \in L} \text{ is efficient} \right\}$ *and* $E = \left\{ \sum_{ij \in L} \hat{\mathbf{x}}_{ij}(L) \right\}$, *then if each* $u_i(A, \mathbf{X}^{N,L} \left| \mathbf{x}_{ij} = \mathbf{x}^*_{ij} \text{ for all } j \text{ s.t. } ij \in L \right)$ *is non-decreasing (non-increasing) in each* $\mathbf{x}_{jk}$ *($k, j \neq i$), then* $E \cup M^* \leq (\geq) M^*$ *by the strong set order.*

As there are possible interactions between choices, as in Segal (1999), we can only make comparisons (using the strong set order) between the sets of equilibrium and efficient choices. For two sets, *A* and *B*, $A \leq B$ if whenever $a \in A$, $b \in B$ and $a \geq b$, this implies that $a \in B$ and $b \in A$. The proof follows Segal (1999) directly as $\hat{\mathbf{x}}_{ij}(K) \in \arg\max_{\mathbf{x}_{ij}} \left\{ u_i(A, \mathbf{X}^{N,K}) + u_j(A, \mathbf{X}^{N,K}) \middle| \mathbf{x}_{kl} = \hat{\mathbf{x}}_{kl} \text{ for all } kl \in K \right\}$. The significant generalisation is that we do not consider a principal-agent structure (or 'star' graph with links from a single agent to each other agent and no links between them) and we allow each agent to have some bargaining power (Segal considers situations where a single agent has all of the bargaining power and can make take-it-or-leave-it offers).

Ultimately, the framework here allows one to characterise fully the equilibrium outcome in a buyer-seller network where buyers compete with one another in downstream market. Significantly, this solution can be used to analyse the effects of changes in the network structure of a market. For example, Kranton and Minehart (2001) explore the formation of links between buyers and sellers while de Fontenay and Gans (2004b) explore changes in those links as a result of changes in the ownership of assets. The cooperative game structure of payoffs – in particular its linear structure – makes the analysis of changes relatively straightforward.

8. Conclusion and Future Directions

To be done.

Appendix

Proof of Theorem 1

As we are solving for a perfect Bayesian equilibrium with passive beliefs, we need only consider the incentives for one player, *i*, to deviate. Let $(\hat{A}(L), \hat{X}(L), \hat{T}(L)) = (\{\hat{\mathbf{a}}_i(L)\}_{i\in N}, \{\hat{\mathbf{x}}_{ij}(L)\}_{ij\in L}, \{\hat{t}_{ij}(L)\}_{ij\in L})$ be the equilibrium outcome and also agents' beliefs regarding unobserved actions. Let us assume for simplicity that *i* always gets to make the first offer, noting that if this were not the case, as σ approached 1, player *i* would simply reject any offer that differed from the offer that they would have made.

Suppose *i* is involved in *s* negotiations, and re-name the agents that *i* negotiates with as "1 to *s*". When *i* comes to negotiate with player *s*, in her final round, if *i* has deviated in previous negotiations, *i* can offer a deviation that *s* will accept in this round.

Without loss in generality, suppose that *i* has deviated in only a single past negotiation, agreeing to $(\mathbf{x}'_{ij}, t'_{ij})$ rather than $\hat{\mathbf{x}}_{ij}(L) = \tilde{\mathbf{x}}^s_{ij}(L)$ and $\hat{t}_{ij}(L) = \tilde{t}^s_{ij}(L)$. The agreements between I and j in all other contingencies remain at their equilibrium value. By passive beliefs, this will mean that the negotiations over contingencies between *i* and *s* will also be unchanged. Hence, here we focus on their negotiations over actions and transfers in the original state *L*.

In making the first offer to *s*, *i* solves the following problem:

$$\max_{\mathbf{x}_{is}, t_{is}} u_i(\hat{A}(L), \mathbf{x}_{is}, \mathbf{x}'_{ij}, \hat{X}(L)/\{\hat{\mathbf{x}}_{is}(L), \hat{\mathbf{x}}_{ij}(L)\}) - t_{is} - \sum_{j=1}^{s-1} \hat{t}_{ij}$$

$$\text{subject to } u_s(\hat{A}(L), \mathbf{x}_{is}, \hat{X}(L)/\{\hat{\mathbf{x}}_{is}(L)\}) + t_{is} \geq \sigma V_s + (1-\sigma)\Omega_{si}$$

where V_s is *s*'s expectation of their payoff if it makes a counter-offer, and Ω_{si} is *s*'s payoff if there is a breakdown in negotiations between *i* and *s* and contingent contracts come into force; because of passive beliefs, neither of these values is affected by the current offer. The transfer payment provides a degree of freedom that allows *i* to make the constraint bind; therefore:

$$t_{is} = \sigma V_s + (1-\sigma)\Omega_{si} - u_s(\hat{A}(L), \mathbf{x}_{is}, \hat{X}(L)/\{\hat{\mathbf{x}}_{is}(L)\})$$

and *i* solves:

$$\begin{aligned}\max_{\mathbf{x}_{is}} \; & u_i(\hat{A}(L), \mathbf{x}_{is}, \mathbf{x}'_{ij}, \hat{X}(L)/\{\hat{\mathbf{x}}_{is}(L), \hat{\mathbf{x}}_{ij}(L)\}) \\ & + u_s(\hat{A}(L), \mathbf{x}_{is}, \hat{X}(L)/\{\hat{\mathbf{x}}_{is}(L)\}) \\ & - \sum_{j=1}^{s-1} \hat{t}_{ij} - \sigma V_s - (1-\sigma)\Omega_{si}\end{aligned}$$

where the last three terms of the expression do not depend on $\mathbf{x}_{is}$. Nonetheless, a past deviation may cause a deviation in future negotiations. Let us call this new value $\mathbf{x}'_{is}(\mathbf{x}_{ij})$.

The issue becomes, anticipating this, will that past deviation actually occur. Consider *i*'s negotiation with *j*. Without loss in generality, we will assume that *j* is *i*'s (1-*s*)th negotiation. Under passive beliefs, *j*'s offers will not change even following an alternative offer from *i*; as it does not use this information to revise $\tilde{\mathbf{x}}_{is}^{j} = \hat{\mathbf{x}}_{is}^{j}$. *i* does anticipate this and when making an offer to *j*, solves:

$$\max_{\mathbf{x}_{ij},t_{ij}} u_i(\hat{A}(L),\mathbf{x}'_{is}(\mathbf{x}_{ij}),\mathbf{x}_{ij},\hat{X}(L)/\{\hat{\mathbf{x}}_{is}(L),\hat{\mathbf{x}}_{ij}(L)\}) - t_{ij} - t'_{is}(\mathbf{x}_{ij}) - \sum_{j=1}^{s-2}\hat{t}_{ij}$$

$$\text{subject to } u_j(\hat{A}(L),\mathbf{x}_{ij},\hat{X}(L)/\{\hat{\mathbf{x}}_{ij}(L)\}) + t_{ij} \geq \sigma V_j + (1-\sigma)\Omega_{ji}$$

Substituting in the constraint and *i*'s expected $t'_{is}(\mathbf{x}_{ij})$, we have:

$$\begin{aligned}\max_{\mathbf{x}_{ij}} & u_i(\hat{A}(L),\mathbf{x}'_{is}(\mathbf{x}_{ij}),\mathbf{x}_{ij},\hat{X}(L)/\{\hat{\mathbf{x}}_{is}(L),\hat{\mathbf{x}}_{ij}(L)\}) \\ & + u_j(\hat{A}(L),\mathbf{x}_{ij},\hat{X}(L)/\{\hat{\mathbf{x}}_{ij}(L)\}) \\ & + u_s(\hat{A}(L),\mathbf{x}'_{is}(\mathbf{x}_{ij}),\hat{X}(L)/\{\hat{\mathbf{x}}_{is}(L)\}) \\ & - \sigma V_j - (1-\sigma)\Omega_{ji} - \sigma V_s - (1-\sigma)\Omega_{si} - \sum_{j=1}^{s-2}\hat{t}_{ij}\end{aligned}$$

where again the terms in the last line do not depend on $\mathbf{x}_{ij}$. Note that, by the envelope theorem and the fact that $\mathbf{x}'_{is}(\hat{\mathbf{x}}_{ij}(L)) = \hat{\mathbf{x}}_{is}(L)$, this maximisation problem gives the same solution as:

$$\max_{\mathbf{x}_{ij}} u_i(\hat{A}(L),\mathbf{x}_{ij},\hat{X}(L)/\{\hat{\mathbf{x}}_{is}(L)\}) + u_j(\hat{A}(L),\mathbf{x}_{ij},\hat{X}(L)/\{\hat{\mathbf{x}}_{ij}(L)\}).$$

Thus, there is no deviation from the equilibrium negotiations with *j* and hence, no deviation in subsequent negotiations.

The proof for individual actions proceeds along similar but simplified lines.

Proof of Theorem 2

The proof of this theorem has two parts. First, we need to establish the set of conditions that characterise the unique cooperative game allocation in a partition function environment when the communication structure is restricted to a graph. Second, we will demonstrate that an equilibrium of our non-cooperative bargaining game satisfies these conditions.

Part 1: Conditions Characterising the Generalised Myerson Value

Myerson (1977a) examines a communication structure restricted to a graph – something that is extended by Jackson and Wolinsky (1996) – and demonstrates that the Myerson value is the unique allocation of the surplus under a fair allocation condition and a component balance condition. Myerson (1977b) defines a cooperative value for a game in partition function space but does not examine this on a restricted communication structure nor does he provide a characterisation of that outcome based on conditions such as fair allocation and component balance. Given our general environment here, we first fill these gaps.

Let $v(S, K^P)$ be the underlying value function of a game in partition function form with total number of agents (S) and graph of communication (K). Here are some definitions important for the results that follow.

Definition (Allocation Rule). *An allocation rule is a function that assigns a payoff vector,* $\mathbf{Y}(N,v,L) \in \mathrm{R}^N$ *, for a given (N, v, L).*

Definition (Component Balance). *An allocation rule,* $\mathbf{Y}$ *, is component balanced if* $\sum_{i\in C}\Upsilon_i(N,v,L) = v(C)$ *for every* $C \in N/L$ *, where* $v(C) = \sum_{i\in C} u_i$ *.*

Definition (Fair Allocation). *An allocation rule,* $\mathbf{Y}$ *, is fair if* $\Upsilon_i(N,v,L) - \Upsilon_i(N,v,L-ij) = \Upsilon_j(N,v,L) - \Upsilon_j(N,v,L-ij)$ *for every* $ij \in L$ *.*

The final two conditions are amendments of similar conditions imposed in Myerson (1977a) and Jackson and Wolinsky (1996) but for the notation in this paper.

The method of proof will be the following. First, Lemma 1 establishes that under component balance and fair allocation, there is a unique allocation rule. Second, we that the generalized Myerson value satisfies fair allocation and component balance. Thus, using Lemma 1, this implies that the generalized Myerson value is the unique allocation rule for this type of cooperative game.

First, we can demonstrate that:

Lemma 1. *For a given cooperative game (N, v, L), under component balance and fair allocation, there exists a unique allocation rule.*

PROOF: Suppose there are two allocations $\mathbf{Y}^1$ and $\mathbf{Y}^2$ that are different, and let g be the *minimal* graph for which the two rules are different: for some i, $\Upsilon_i^1(g,v) \neq \Upsilon_i^2(g,v)$. If i is not linked to any j, i must have the same payoff under both graphs, by component balance.

Therefore, i must be linked to some j, and the two graphs must be the same after any link is broken $\Upsilon_i^1(g-ij,v) \neq \Upsilon_i^2(g-ij,v)$. Fair allocation implies that for all i and j that are linked:

$$\begin{aligned}
&\Upsilon_i^1(g,v) - \Upsilon_i^1(g-ij,v) = \Upsilon_j^1(g,v) - \Upsilon_j^1(g-ij,v) \\
&\Rightarrow \Upsilon_i^1(g,v) - \Upsilon_j^1(g,v) = \Upsilon_i^1(g-ij,v) - \Upsilon_j^1(g-ij,v) \\
&\qquad\qquad\qquad\qquad\quad = \Upsilon_i^2(g-ij,v) - \Upsilon_j^2(g-ij,v) \\
&\qquad\qquad\qquad\qquad\quad = \Upsilon_i^2(g,v) - \Upsilon_j^2(g,v)
\end{aligned}$$

Therefore, $\Upsilon_i^1(g,v) - \Upsilon_i^2(g,v) = \Upsilon_j^1(g,v) - \Upsilon_j^2(g,v)$ = some Δ (different from zero by the first assumption), for any i and j that are connected, and, therefore, by extension, for any i and j in the same component h; with set of constituent agents, $N(h)$. Therefore, if there are n_h agents in the component, $\sum_{i\in N(h)} \Upsilon_i^1(g,v) - \sum_{i\in N(h)} \Upsilon_i^2(g,v) = \Delta n_h \neq 0$.

Notice however that by component balance the payoffs to all agents in a component have to sum up to the same thing: $\sum_{i\in N(h)} \Upsilon_i^1(g,v) = v(N(h)\,|\,g) = \sum_{i\in N(h)} \Upsilon_i^2(g,v)$; therefore we have a contradiction. □

Next we demonstrate that the generalized Myerson value satisfies fair allocation. Let *i* and *j* be linked together by a graph *L*, where payoffs to groups are described by a component additive payoff function *v*(.|*L*). Suppose that each agent *i* receives their generalized Myerson value from the game (*N*, *v*, *L*) in partition function form:

$$\Upsilon_i(N,L) = \sum_{P\in P^N} \sum_{T\in P} (-1)^{p-1}(p-1)!\left[\frac{1}{|N|} - \sum_{\substack{i\notin T'\in P \\ T'\neq T}} \frac{1}{(p-1)(|N|-|T'|)}\right] v(T,L^P).$$

We aim to show that $\left(\Upsilon_i(L,v)-\Upsilon_i(L-ij,v)\right)-\left(\Upsilon_j(L,v)-\Upsilon_j(L-ij,v)\right)=0$.

$$\begin{aligned}&\left(\Upsilon_i(L,v)-\Upsilon_i(L-ij,v)\right)-\left(\Upsilon_j(L,v)-\Upsilon_j(L-ij,v)\right)\\ &= \sum_{P\in P^N}\sum_{S\in P}(-1)^{p-1}(p-1)!\left(\begin{aligned}&-\left(\sum_{\substack{i\notin S'\in P\\ S'\neq S}}\frac{1}{(p-1)(|N|-|S'|)}\right)\left(v(S,L^P)\ -v(S,(L-ij)^P)\right)\\ &+\left(\sum_{\substack{j\notin S'\in P\\ S'\neq S}}\frac{1}{(p-1)(|N|-|S'|)}\right)\left(v(S,L^P)\ -v(S,(L-ij)^P)\right)\end{aligned}\right)\end{aligned}$$

Consider any partition *P*, and any set S' of that partition. If *i* and *j* are members of S', it does not appear in the summation. If neither *i* nor *j* are members of S', it appears in the top and the bottom line of the parenthesis, and cancels out. Thus the only relevant case is when *i* is a member of S' and *j* is not, or vice versa; but if *i* and *j* are not members of the same set of the partition, then $L^P = (L-ij)^P$, and therefore $v(S,L^P) = v(S,(L-ij)^P)$, and the term disappears. □

Third, we demonstrate that the generalized Myerson value satisfies component balance. Let *i* and *j* be linked together by a graph *L*, where payoffs to groups are described by a component additive payoff function *v*(.|*L*). Suppose that each agent *i* receives their generalized Myerson value from the game (*N*, *v*, *L*) in partition function form. We will show that for every component, *C*(*L*), $\sum_{i\in C(L)} \Upsilon_i(N,L) = v(C(L),L)$.

To do this, we first show that component balance is implied by two of the properties that Myerson (1977b) proved for the extension of Shapley values to games in partition function form: Value Axiom 2, that carriers get all the value, and Value Axiom 3, that adding two partition function games gives an addition of their values. Let $\mathbf{Y}$ be the allocation under the game (*N*, *v*, *L*). For a given component, *C*, let $\mathbf{Y}^1$ be the allocation so that for all $i\in C$, $\Upsilon_i^1 = \Upsilon_i(N,L)$ and for all $i\notin C$, $\Upsilon_i^1 = 0$.

Similarly, let $\mathbf{Y}^2$ be the allocation so that for all $i \in C$, $\Upsilon_i^2 = 0$ and for all $i \notin C$, $\Upsilon_i^2 = \Upsilon_i(N,L)$. By Axiom 2, the set of agents in *C* gets all the value in allocation 1, and *N\C* gets all the value in allocation 2. By Axiom 3, the vector of payoffs in 1 and 2 sum up to $\mathbf{Y}$.

Given the same *C*, consider a partition of *N* into *C* and *N\C*. Then define two games in partition function form with value functions, (a) $v(C, L^{\{C,N\setminus C\}})$ and (b) $v(N \setminus C, L^{\{C,N\setminus C\}})$. Let Υ_i^a and Υ_i^b be the Myerson values (in partition function space) for an agent associated with the first and second games respectively. By the carrier axiom,

$$\sum_{i\in C} \Upsilon_i^a(C, L^{\{C,N\setminus C\}}) = v(C,L)$$
$$\sum_{i\in N\setminus C} \Upsilon_i^b(C, L^{\{C,N\setminus C\}}) = v(N\setminus C,L)$$

Now we add these two games (a) and (b) together, obtaining the original game in partition function form. By Axiom 3, the payoff to each agent is the sum of their payoffs under (a) and (b). But agents in *C* only have a non-zero payoff in game (a), therefore:

$$\sum_{i\in C} \Upsilon_i(N,L) = \sum_{i\in C} \Upsilon_i^a(C, L^{\{C,N\setminus C\}}) + \sum_{i\in C} \Upsilon_i^b(C, L^{\{C,N\setminus C\}}) = \sum_{i\in C} \Upsilon_i^a(C, L^{\{C,N\setminus C\}}) = v(C,L).$$

Part 2: The non-cooperative bargaining game satisfies these conditions.

We want to show that the non-cooperative bargaining game satisfies fair allocation and component balance over a cooperative game with value function $\hat{v}(N,L)$ as determined by bilateral efficiency. Note that Theorem 1 demonstrates that an equilibrium of the bargaining game involves achieving bilateral efficiency. This defines an imputed value function. We now want to show that for this equilibrium the two conditions are satisfied for the game with this value function.

Let $\hat{t}_{ij}$ be the equilibrium transfer between each pair, $ij \in L$, that bargain. When *i* and *j* bargain together, let t_{ij}^i be the transfer that *i* offers, which would give a payoff $\hat{v}_i^i$ and $\hat{v}_j^i$ to *i* and *j* respectively; *j*'s offer t_{ij}^j would, if accepted, lead to payoffs $\hat{v}_i^j$ and $\hat{v}_j^j$ respectively. Given that the transfers are chosen to make the incentive constraint bind, the offers satisfy:

$$\begin{aligned} \sigma\hat{v}_i^i + (1-\sigma)\Upsilon_i(N, L-ij) &= \hat{v}_i^j \\ \sigma\hat{v}_j^j + (1-\sigma)\Upsilon_j(N, L-ij) &= \hat{v}_j^i \end{aligned} \qquad (7)$$

where $\Upsilon_i(N, L-ij)$ is the payoff to *i* after a breakdown with *j*.

Given that none of the transfer is wasted, there is also a summing up condition:

$$\hat{v}_i^i = \hat{u}_i + \sum_{k=1}^{i-1}\hat{t}_{ki} - \sum_{k=i+1}^{j-1}\hat{t}_{ik} - t_{ij}^i - \sum_{k=j+1}^{n}\hat{t}_{ik}$$

(where transfer t_{ij} is zero if i and j do not have a bargaining link).

$$\hat{v}_i^i + \hat{v}_j^i = \hat{v}_i^j + \hat{v}_j^j = \hat{u}_i + \sum_{k=1}^{i-1}\hat{t}_{ki} - \sum_{k=i+1}^{n}\hat{t}_{ik} + \hat{u}_j + \sum_{k=1}^{j-1}\hat{t}_{kj} - \sum_{k=i+1}^{n}\hat{t}_{jk} \tag{8}$$

Using (7) to substitute out $\hat{v}_i^j$ and $\hat{v}_j^i$ in the first part of (8):

$$\hat{v}_i^i + \sigma\hat{v}_j^j + (1-\sigma)\Upsilon_j(N, L-ij) = \sigma\hat{v}_i^i + (1-\sigma)\Upsilon_i(N, L-ij) + \hat{v}_j^j$$
$$\Rightarrow (1-\sigma)\hat{v}_i^i + (1-\sigma)\Upsilon_j(N, L-ij) = (1-\sigma)\hat{v}_j^j + (1-\sigma)\Upsilon_i(N, L-ij)$$
$$\Rightarrow \hat{v}_i^i + \Upsilon_j(N, L-ij) = \hat{v}_j^j + \Upsilon_i(N, L-ij)$$

Note from (7) that in the limit, as σ tends towards zero, payoffs $\hat{v}_i^i$ and $\hat{v}_i^j$ become the same payoff $\hat{v}_i$, and therefore:

$$\hat{v}_i + \Upsilon_j(N, L-ij) = \hat{v}_j + \Upsilon_i(N, L-ij)$$

which is the balanced contributions condition.

Now consider condition (8) and its analogue for every bargaining link in the component that includes i and j. In the limit, as σ tends towards zero, the condition becomes:

$$\hat{v}_i = u_i + \sum_{k=1}^{i-1}\hat{t}_{ki} - \sum_{k=i+1}^{n}\hat{t}_{ik}$$

for each i, where transfer t_{ij} is zero if i and j do not have a bargaining link. Therefore, for a given component, $C_i(L)$:

$$\sum_{i\in C_i(L)}\hat{v}_i = \sum_{i\in C_i(L)}\left(u_i + \sum_{k=1}^{i-1}\hat{t}_{ki} - \sum_{k=i+1}^{n}\hat{t}_{ik}\right) = \sum_{i\in C_i(L)}u_i$$

because there are no transfers to agents that you do not bargain with. The non-zero transfers in this summation term are all between agents in $C_i(L)$, and, therefore, the summation includes both $\hat{t}_{ij}$ and (-$\hat{t}_{ij}$), which cancel out. This demonstrates component balance.

References

Binmore, K., A. Rubinstein and A. Wolinsky (1986), "The Nash Bargaining Solution in Economic Modelling," *RAND Journal of Economics*, 17, 176-188.

de Fontenay, C.C. and J.S Gans (2003a), "Organizational Design and Technology Choice under Intrafirm Bargaining: Comment," *American Economic Review*, 93 (1), pp.448-455.

de Fontenay, C.C. and J.S. Gans (2003b), "Can Vertical Integration by a Monopsonist Harm Consumer Welfare?" *Working Paper*, No.2003-03, Melbourne Business School.

Hart, O. and J. Moore (1990), "Property Rights and the Theory of the Firm," *Journal of Political Economy*, 98 (6), pp.1119-1158.

Inderst, R. and C. Wey (2003), "Bargaining, Mergers and Technology Choice in Bilaterally Oligopolistic Industries," *RAND Journal of Economics*, 34 (1), pp.1-19.

Jackson, M.O. and A. Wolinsky (1996), "A Strategic Model of Social and Economic Networks," *Journal of Economic Theory*, 71 (1), pp.44-74.

Kranton, R.E. and D.F. Minehart (2001), "A Theory of Buyer-Seller Networks," *American Economic Review*, 91 (3), pp.485-508.

McAfee, R.P. and M. Schwartz (1994), "Opportunism in Multilateral Vertical Contracting: Nondiscrimination, Exclusivity and Uniformity," *American Economic Review*, 84 (1), 210-230.

Myerson, R. (1977a), "Graphs and Cooperation in Games," *Mathematics of Operations Research*, 2, pp.225-229.

Myerson, R. (1977b),

Myerson, R. (1980), "Conference Structures and Fair Allocation Rules," *International Journal of Game Theory*, 9, pp.169-182.

Rey, P. and J. Tirole (2003), "A Primer on Foreclosure," *Handbook of Industrial Organization*, Vol.III, North Holland: Amsterdam (forthcoming).

Rey, P. and T. Verge (2002), "Bilateral Control with Vertical Contracts," *mimeo.*, Toulouse.

Segal, I. (1999), "Contracting with Externalities," *Quarterly Journal of Economics*, 114 (2), pp.337-388.

Shaked, A. and J. Sutton (1984), "Involuntary Unemployment as a Perfect Equilibrium in a Bargaining Model." *Econometrica*, *52* (6), pp.1351-64.

Stole, L. and J. Zwiebel (1996), "Intra-firm Bargaining under Non-binding Contracts," *Review of Economic Studies*, 63 (3), 375-410.

Stole, L. and J. Zwiebel (1998), "Mergers, Employee Hold-Up and the Scope of the Firm: An Intrafirm Bargaining Approach to Mergers," *mimeo*., Stanford.

Wolinsky, A. (2000), "A Theory of the Firm with Non-Binding Employment Contracts." *Econometrica*, *68* (4), pp.875-910.