Forward looking information in S&P 500 options

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Abstract

Implied volatility generated from observed option prices reflects market expectations of future volatility. This paper determines whether or not, implied volatilities, and hence market expectations, contain any genuinely forward looking information not already captured by historical information. Historical information is represented by current levels of volatility and model based forecasts using a variety of volatility models. The VIX index, constructed from S&P 500 options data is the measure of implied volatility used in this study. Once accounting for historical information, VIX appears to contain no forward looking information regarding future S&P 500 volatility.

Keywords
Implied volatility, information, volatility forecasts, volatility models, realized volatility

JEL Classification C12, C22, G00

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1 Introduction

Estimates of the future volatility of asset returns are of great interest to financial market participants. Generally, there are two approaches which can be employed to obtain such estimates. First, predictions of future volatility can be generated from econometric models of volatility given historical information. For surveys of common modeling techniques see Campbell, Lo and MacKinlay (1997) and Gourieroux and Jasiak (2001). Second, estimates of future volatility can be derived from option prices using implied volatility (IV). IV should represent a market’s best prediction of an assets’ future volatility (see, amongst others, Jorion, 1995, Poon and Granger, 2003). To make an informed choice between these approaches, it is informative to examine whether IV incorporates any information that could not be obtained from historical information. Addressing this issue not only informs the choice of forecasting approach used, but also enhances our understanding of the operation of options markets.

Poon and Granger (2003) provide a wide ranging survey of literature examining the relative performance of the two approaches to forecasting volatility. There it was shown that the majority of previous research concludes that IV yields superior forecasts of future volatility. However, in many instances, a combination of forecasts from competing approaches is preferred. Therefore it appears as though option market participants derive option prices, and hence IV, from a wide-ranging information set. This information set could possibly contain both forward looking information and historical information captured by econometric models of volatility.

The central task of this paper is to determine whether IV contains any genuinely forward looking information, not already contained in historical information. Therefore it is useful to formally describe these sources of information that are reflected in option prices, and hence IV. Define a general information set, $\Theta_{IV}^t$ that reflects information pertaining to the option market’s expectation of future volatility. It can be postulated that information reflected in $\Theta_{IV}^t$ may be attributable to either historical or genuinely forward looking information (not attributable to historical information), denoted as $\Theta_{H}^t$ and $\Theta_{F}^t$ respectively. Af-
ter taking into account $\Theta^H_t$ and $\Theta^F_t$, any remaining information in $\Theta^{IV}_t$ would reflect random errors made by the options market when forming expectations regarding future volatility. These subsets of information would not intersect.

Therefore, when considering the informational content of IV, the question is to what degree do $\Theta^H_t$ and $\Theta^F_t$ enter into the decision making process of participants in the options market when determining option prices? From a practical viewpoint, such questions could be addressed from a number of perspectives. Recent works related to this issue are Fleming (1998), Blair, Poon and Taylor (2001, henceforth known as BPT) and Pong, Shackleton, Taylor and Xu (2004, henceforth known as PSTX).

Fleming (1998) examines whether or not information in IV subsumes all information contained in $\Theta^H_t$. While Fleming (1998) finds that IV produce biased forecasts of future actual volatility (as defined by squared average returns), forecast errors are orthogonal to historical information. These results indicate that IV subsumes all historical information.

PSTX establish that a number of econometric models of volatility produce volatility forecasts of a similar quality to those based on IV. They further show that a combination of both IV and model based forecasts produce the most accurate forecasts of future volatility. While not focusing on the issue of information directly, the results of PSTX would indicate that $\Theta^{IV}_t$ contains a portion of $\Theta^H_t$. BPT on the other hand, do not view these alternatives as competing approaches, but focus on the issue of whether including a wider set of historical information and IV improve on standard GARCH models for volatility.

A common theme amongst these articles is that they do not recognise that a portion of the information contained in IV, may simply be extracted from model based forecasts of volatility, given historical information. Therefore, this paper considers the content of $\Theta^{IV}_t$ from another perspective. The question of whether or not $\Theta^{IV}_t$ reflects information relevant to future volatility that cannot be extracted from predictions based on historical volatility, elements of $\Theta^H_t$, is addressed. To do so, the following framework is utilised,

$$\Theta^{IV}_t = g(\Theta^H_t) + \varepsilon_t, \quad \varepsilon_t \perp g(\Theta^H_t), \forall g(\cdot)$$

(1)
where after taking into account $\Theta_t^H$, the residual portion of $\Theta_t^{IV}$, $\varepsilon_t$ will be examined to see whether it is correlated with future volatility. If this is the case, $\Theta_t^{IV}$ contains some degree of $\Theta_t^F$ otherwise the residual portion of $\Theta_t^{IV}$ simply reflects random error.

This is quite a different view to that taken by Fleming (1998) in that formal predictions of volatility are also used to draw the link between historical information and future volatility (as defined by the realized volatility (RV) estimate of Andersen, Bollerslev, Diebold and Labys (2001, 2003, henceforth known as ABDL). In the context of Fleming (1998), $\Theta_t^H$ simply contains various measures of current and historical volatility, whereas in the current setting, $\Theta_t^H$ also contains model based forecasts of future volatility using historical data.

This paper proceeds as follows. Section 2 discusses the data relevant for this study. Section 3 outlines econometric models upon which elements of $\Theta_t^H$ are based. Section 4 presents empirical results that identify whether $\Theta_t^{IV}$ contains any forward looking information. Section 5 provides concluding remarks.

2 Data

This study is based upon data relating to the S&P 500 Composite Index, from 2 January 1990 to 17 October 2003 (3481 observations). Figure 1 shows plots of each of the series relevant to this study.

The top panel of Figure 1 plots daily logarithmic returns on the S&P 500 index. This shows that the magnitude of returns were relatively low (high) during the mid 1990’s (since 1997).

To formally test the informational content of IV, estimates of both IV and future actual volatility are required. The VIX index constructed by the Chicago Board of Options Exchange from S&P 500 index options constitute the estimates of IV utilised in this paper.¹ It is derived from a number of put and call options, which generally have strike prices close to the index value, and have maturities close to the target of 22 trading days. The binomial option pricing model is used to extract the estimates of IV, allowing for the possibility of early

¹For technical details relating to the construction of the VIX index, see CBOE (2003).
Figure 1: Daily S&P 500 index returns (top panel), daily VIX index (middle panel) and daily S&P 500 index RV estimate (bottom panel).

exercise and expected dividend payments. While the true process underlying option pricing in unknown, given the construction of the VIX, it is the most general measure of the market’s estimate of average S&P 500 volatility over the subsequent 22 trading days. VIX is believed to be a relatively unbiased estimate of the true, but unobservable IV (BPT, 2001, and Christensen and Prabhala, 1998).

The middle panel of Figure 1 plots daily VIX estimates for the relevant sample period\(^2\). Broadly, the behaviour of the VIX index reflects the overall changes in volatility in the S&P 500. IV estimates during much of the 1990’s were relatively low, while in more recent times IV has increased somewhat.

For the purposes of this study, estimates of actual daily volatility are obtained using the RV methodology outlined in ABDL (2001, 2003). ABDL (1999) suggest how to deal with practical issues relating to intra-day seasonality and

\(^2\)The daily volatility implied by the VIX can be calculated when recognising that the VIX quote is equivalent to 100 times the annualised return standard deviation. Hence \(\left(\frac{VIX}{100}\right)^2 \times 252\) represents the daily volatility measure (see CBOE, 2003).
sampling frequency when dealing with intra-day data. Based on this methodology, daily RV estimates are constructed using 40 minute S&P500 index returns. The bottom panel of Figure 1 contains estimates of daily S&P500 RV for the sample period considered. While the RV estimates exhibit a similar pattern when compared to the VIX, RV reaches higher peaks than the VIX. This difference would mainly be due to the fact that the VIX represents an average volatility measure.

It has previously been noted by many authors, amongst others ABDL. (2001, 2003), that RV measures are right skewed. The current sample of S&P 500 RV conforms to this pattern, skewness is found to be 8.13. This degree of skewness can be reduced by various transformations of RV, \( \sqrt{RV} \) and \( \ln \sqrt{RV} \) show skewness of 2.35 and 0.10 respectively. Logarithmic based transformations of RV produce series that are very close to normally distributed. A very similar pattern is observed when dealing with the VIX index, however \( \ln \sqrt{VIX} \) is somewhat platokurtic. Augmented Dickey-Fuller tests have also been applied, and reject the null hypothesis of a unit root in each of the RV and VIX series (including the transformed series).

3 Models of volatility

This section considers the econometric models upon which volatility forecasts are based, these forecasts representing possible subset of the historical information set, \( \Theta^H_t \). Defining \( \Theta^H_t \) in such a way, is designed to capture the manner in which option market participants may form views of future volatility based on historical information. While the true mechanism underlying the formation of estimates of future volatility is unknown, a number of plausible models to reflect this process exist. This study utilises models from the GARCH, Stochastic volatility (SV), and RV classes of models, in a similar manner to Koopman, Jungbacker and Hol (2004) and BPT (2001). In the current section, the specification of each competing model will be introduced, and estimated given the entire dataset. These models will then be utilised to generate volatility forecasts in the subsequent section.
GARCH style models utilised in this study are similar to those proposed by BPT (2001). The simplest model specification is the GJR (see Glosten et al., 1993, Engle and Ng, 1991) process,

\[ r_t = \mu + \varepsilon_t \]  
\[ \varepsilon_t = \sqrt{h_t}z_t \quad z_t \sim N(0, 1) \]  
\[ h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2 s_{t-1} \varepsilon_{t-1}^2 + \beta h_{t-1} \]

that captures the asymmetric relationship between volatility and returns. The indicator variable \( s_{t-1} \) takes the value of unity when \( \varepsilon_{t-1} < 0 \) and 0 otherwise. This process nests the standard GARCH(1,1) model when \( \alpha_2 = 0 \).

Following BPT (2001), this study utilises standard GARCH style models augmented by the inclusion of RV\(^3\). The most general specification of a GARCH process including RV is given by

\[ r_t = \mu + \varepsilon_t \]  
\[ \varepsilon_t = \sqrt{h_t}z_t \quad z_t \sim N(0, 1) \]  
\[ h_t = h_{1t} + h_{2t} \]  
\[ h_{1t} = \alpha_0 + \beta h_{t-1} + \alpha_1\varepsilon_{t-1}^2 + \alpha_2 s_{t-1} \varepsilon_{t-1}^2 \]  
\[ h_{2t} = \gamma_1 h_{2t-1} + \gamma_2 RV_{t-1} \]

and allows for two components to contribute to volatility, with each component potentially exhibiting persistence. For \( \gamma_1 = \gamma_2 = 0 \) this reduces to the GJR model in equation 2. Table 1 reports the parameter estimates from estimating equation 3 imposing various parameter restrictions.

Parameters for the GARCH and GJR models are similar to those commonly observed for GARCH models based on various financial time series, reflecting strong volatility persistence, and are qualitatively similar to those reported in BPT (2001). Furthermore, allowing for presence of asymmetric conditional volatility is important, irrespective of the volatility process considered. In all

\(^3\)While BPT (2001) also extend the GJR model to include the VIX index, this is not relevant to the current study. These models are to be used to extract information from VIX itself using forecasts based on historical data.
Table 1: Parameter estimates for GARCH style models. A 0 entry in a cell indicates that this parameter is restricted to be 0. A 0.0000* entry in a cell indicates that the parameter was estimated on the 0 boundary. Robust t-statistics are shown in parentheses.

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameters</th>
<th></th>
<th></th>
<th></th>
<th>$-\log L$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha_1$</td>
<td>$\alpha_2$</td>
<td>$\beta$</td>
<td>$\gamma_1$</td>
<td>$\gamma_2$</td>
</tr>
<tr>
<td><strong>GARCH</strong></td>
<td>0.0593</td>
<td>0</td>
<td>0.9375</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(15.05)</td>
<td></td>
<td>(2828)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>GJR</strong></td>
<td>0.0082</td>
<td>0.1060</td>
<td>0.9288</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(1.14)</td>
<td>(6.59)</td>
<td>(1338)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>GARCH+RV</strong></td>
<td>0.0499</td>
<td>0</td>
<td>0.9303</td>
<td>0</td>
<td>0.2199</td>
</tr>
<tr>
<td></td>
<td>(7.70)</td>
<td></td>
<td>(623)</td>
<td></td>
<td>(3.69)</td>
</tr>
<tr>
<td><strong>GJR+RV</strong></td>
<td>0.0038</td>
<td>0.1013</td>
<td>0.9211</td>
<td>0</td>
<td>0.1954</td>
</tr>
<tr>
<td></td>
<td>(0.52)</td>
<td>(6.03)</td>
<td>(526)</td>
<td></td>
<td>(3.50)</td>
</tr>
<tr>
<td><strong>GJR+RVG</strong></td>
<td>0.0000*</td>
<td>0.1000</td>
<td>0.7689</td>
<td>0.9345</td>
<td>0.0423</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(3.41)</td>
<td>(25.6)</td>
<td>(1022)</td>
<td>(11.5)</td>
</tr>
</tbody>
</table>

While not considered by BPT 2001, this study also proposes that an SV process may describe the formation of estimates of volatility based on historical information. Therefore, forecasts of future volatility based on SV style models will also be considered as elements of $\Theta_H$ when investigating the informational content of IV. SV models differ from GARCH models in that conditional volatility is treated as an unobserved variable, and not as a deterministic function of lagged returns. The simplest SV models describes zero-mean returns as

$$r_t = \sigma_t u_t \quad u_t \sim N(0, 1) \quad (4)$$

where $\sigma_t$ is the time $t$ conditional standard deviation of $r_t$. The SV models treats $\sigma_t$ as an unobserved (latent) variable, following its own stochastic path, the simplest being an AR(1) process,

$$\log (\sigma_t^2) = \alpha + \beta \log (\sigma_{t-1}^2) + w_t \quad w_t \sim N(0, \sigma_w^2). \quad (5)$$

Similar to Koopman et al. (2004), this study extends a standard volatility model to incorporate RV as an exogenous variable in the volatility equation. The standard SV process in equation 5 can be extended to incorporate RV in
the following manner

$$\log (\sigma_t^2) = \alpha + \beta \log (\sigma_{t-1}^2) + \gamma (\log (R_{V,t-1}) - E_{t-1} [\log (\sigma_{t-1}^2)]) + w_t. \quad (6)$$

Here, $RV$ enters the volatility equation through the term $\log (R_{V,t-1}) - E_{t-1} [\log (\sigma_{t-1}^2)]$. This form is chosen due to the high degree of correlation between $RV$ and the latent volatility process and represents the incremental information contained in the $RV$ series. It is noted that equation 6 nests the standard SV model as a special case by imposing the restriction $\gamma = 0$.

Numerous estimation techniques may be applied to the model in equations 4 and 5 or 6. In this instance the nonlinear filtering approach proposed by Clements, Hurn and White (2003) is employed. This approach is adopted as it easily accommodates exogenous variables in the state equation. As with the GARCH style models, the SV models are estimated on the entire data series with parameter estimates contained in Table 2.

SV parameter estimates appear to capture the same properties of the volatility process when compared to the GARCH results. In both instances, volatility is found to be a persistent process, and the inclusion of $RV$ as an exogenous variable is important. A test of the restriction, $\gamma = 0$ is clearly rejected as the LR statistic is 156.22.

In addition to GARCH and SV approaches it is possible to utilise estimates of $RV$ to generate forecasts of future volatility. These forecasts can be generated by directly applying time series models, both short and long memory, to daily $RV$. In following ADBL (2003) and Koopman et al. (2004) ARMA(2,1) and ARFIMA(1,d,0) process are utilised. Generally, these specification may be

<table>
<thead>
<tr>
<th>Model</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\sigma_w$</th>
<th>$-\log L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SV</td>
<td>-0.0046</td>
<td>0.9819</td>
<td>0</td>
<td>0.1693</td>
<td>4622.6</td>
</tr>
<tr>
<td></td>
<td>(-1.07)</td>
<td>(2245)</td>
<td>(7.00)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SV+RV</td>
<td>-0.0079</td>
<td>0.9916</td>
<td>0.1100</td>
<td>0.0941</td>
<td>4543.89</td>
</tr>
<tr>
<td></td>
<td>(-1.43)</td>
<td>(1394)</td>
<td>(5.98)</td>
<td>(3.58)</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Parameter estimates for the SV models. A 0 entry in a cell indicates that this parameter is restricted to be 0. Robust t-statistics are shown in parentheses.
Table 3: Parameter estimates for the RV models. A 0 entry in a cell indicates that this parameter is restricted to be 0. Robust t-statistics are shown in parentheses.

represented as

\[ A(L) (1 - L)^d (x_t - \mu_{x_t}) = B(L) \varepsilon_t. \]  

where \( A(L) \) and \( B(L) \) are coefficient polynomials and \( d \) is the degree of fractional integration. A general ARMA(p,q) process applied to \( x_t \) is defined under the restriction of \( d = 0 \).

Table 3 reports parameters estimates for the RV time series models. In the ARMA (2,1) case, parameter estimates reflect the common feature of volatility persistence. Allowing for fractional integration in the ARFIMA(1,d,0) case reveals that volatility exhibits long memory properties.

Forecasts of future volatility which may enter \( \Theta^H_t \) are based on these three classes of models (GARCH, SV and RV). Given these elements of \( \Theta^H_t \), the following section will consider the forward looking informational content of IV.

4 Empirical Analysis

This section presents empirical results addressing the question of informational content of IV, posed in Section 1. Section 4.1 outlines a preliminary investigation into the relevance of VIX for future RV. Section 4.2 formally examines the issue of whether \( \Theta_{IV} \) contains information relevant for future volatility beyond that contained in \( \Theta^H_t \).

4.1 Preliminary results

A very preliminary investigation into the information (in relation to future volatility) contained in \( \Theta_{IV} \), compares VIX and future RV. Irrespective of whether \( \Theta_{IV} \) contains information from within \( \Theta^H_t \) or \( \Theta^F_t \), this exercise high-
Figure 2: VIX index and average 22 ahead S&P500 RV.

lights the options market’s ability to forecast future S&P500 volatility. Figure 2 compares the VIX series with average RV over the next 22 trading days and reveals a striking result.

It appears as though both increases (decreases) in the VIX lag increases (decreases) in 22 day ahead averages of RV. Due to the RV series in figure ?? capturing a 22 day ahead average RV, this series will rise (fall) prior to the specific day on which RV does rise (fall). As the VIX series generally lags this forward looking RV series, is seems as though options markets do not anticipate such volatility changes.

This assertion can be examined more rigorously by testing whether RV Granger causes VIX and/or vice versa. Due to the persistence in volatility it is plausible to expect that RV Granger-causes VIX, given that recent values of RV are likely to prevail in the near future. If, however, VIX contains information regarding future volatility, which cannot be obtained from lagged values of RV, then VIX should also Granger cause RV. Table 4 illustrates that, while, as expected, RV clearly Granger causes the VIX index, there is no evidence that
Table 4: P-values for F-tests with null hypothesis of non Granger causality. Daily observations from 2 January 1990 to 17 October 2003. 25 lags, 3456 usable observations.

<table>
<thead>
<tr>
<th></th>
<th>RV, VIX</th>
<th>$\sqrt{RV}$, $\sqrt{VIX}$</th>
<th>log$\sqrt{RV}$, log$\sqrt{VIX}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>VIX gc RV</td>
<td>0.691</td>
<td>0.889</td>
<td>0.664</td>
</tr>
<tr>
<td>RV gc VIX</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

VIX Granger causes RV. These results highlight that after taking into account one possible element of $\Theta^H_t$, lagged values of RV, it seems as though the VIX series does not contain additional forward looking information. This issue will be more formally addressed in the subsequent section.

### 4.2 Forward looking information in VIX

This section formulates an empirical representation of the hypothesis outlined in Section 1 that IV does not contain any genuine forward looking information that could otherwise not be filtered from historical data. In doing so, the forward looking informational content of IV (VIX index in this context) will be revealed.

This in itself is a difficult task, as it is not clear which elements of $\Theta^H_t$ are used by option market participants to formulate expectations of future volatility. In reality, $\Theta^H_t$ may contain an possibly infinite amount of information. For the purpose of this study, it is assumed that the information in $\Theta^H_t$ can be represented by volatility forecasts based on econometric models along with current level of volatility measured by RV$_t$. Information contained in the VIX, which is not spanned by $\Theta^H_t$ may potentially be genuine forward looking information.

A linear projection is used to filter $\Theta^H_t$ from $\Theta^IV_t$, in so far as this linearity assumption is restrictive, this methodology is in fact biased toward rejecting the hypothesis that there is no forward looking information in VIX. The relevant methodology is now discussed.

To begin, let $\Theta^H_t$ be represented by $\omega_t$, a vector of S&P 500 volatility forecasts (relating to the subsequent 22 trading days) formed at time $t$. These forecasts are based on the models discussed in Section ???. All forecasts are generated on the basis of rolling window parameter estimates using 1,000 ob-
servations, the end of the window being the last observation before the 22 day
forecast period. When RV is included as an exogenous variable, a linear rela-
tion between volatility and RV was postulated in order to generate multi-period
forecasts.

Based on these forecasts, the information contained in $\Theta IV_t$ may be decom-
posed using the form set out in equation 1, which is restated here for conve-
nience,

$$\Theta IV_t = g(\Theta H_t) + \varepsilon_t. \quad (8)$$

To ascertain the forward looking informational content of $\Theta IV_t$, it is nec-
essary to test whether $\varepsilon_t$ are correlated with future RV. If no correlation is
evident then $\Theta IV_t$ appears to contain no forward looking information beyond
that explained by $\Theta H_t$. Given that $\omega_t$ is a representation of $\Theta H_t$, the simplest
way in which to operationalise equation 8 is invoke the assumption that $g(\Theta H_t)$
is a linear function and estimate the parameter vector $\gamma = (\gamma_0, \gamma_1)'$ in

$$\Theta IV_t = \gamma_0 + \gamma_1 \omega_t + \varepsilon_t \quad (9)$$

by OLS. However, inference in this context is not straightforward due the non-
normality and autocorrelation of the residuals. To provide accurate inference
a GMM framework is utilised which does not depend on the residual’s normal-
ity. Furthermore, the informational hypothesis is tested as a by-product of the
estimation procedure.

The GMM estimate of $\gamma$ minimises $V = MM'HM$, where $M = T^{-1} (\varepsilon_t (\gamma)' Z)$
is the $k \times 1$ vector of moment conditions, $H$ is a $k \times k$ weighting matrix and
$Z$ is a vector of instruments. In order to minimise coefficient variances, $H$
is chosen to be the variance-covariance matrix of the $k$ moment conditions
in $M$, where allowance is made for residual correlation (see Hamilton, 1994).
Whenever $k > \dim (\gamma)$, the test for overidentifying restrictions $J = TM'HM$, is
$\chi^2 (k - \dim (\gamma))$ distributed under the null hypothesis that the residuals in
equation 9 are uncorrelated with elements in $Z$. This test will be used to test
the hypothesis whether $\varepsilon_t (\gamma)$ is orthogonal to future RV.

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4For further details see BPT (2001), p 14.
To this end the instrument vector $z_t$ is defined to include such information, 

$$RV_t = \{RV_t, RV_{t+1\rightarrow t+5}, RV_{t+1\rightarrow t+10}, RV_{t+1\rightarrow t+15}, RV_{t+1\rightarrow t+22}\}$$

where $RV_{t+1\rightarrow t+j}$ is the average realised volatility in the days $t + 1$ to $t + j$. A number of instrument sets will be used, relying on combinations of elements of historical information, $\omega_t$ and future information $RV_t$. If $\varepsilon_t(\gamma)$ is correlated with future RV, significant nonzero elements of $M = T^{-1}(\varepsilon_t(\gamma)' Z)$ will be found. In this case, the $J$ statistic will be sufficiently large to reject the null hypothesis.

As the elements in $\omega_t$ are highly colinear, it is necessary to reduce the number of elements significantly. A general-to-specific strategy, eliminating the elements with the lowest p-values, leaves four significant elements, the GARCH, GJR+RVG, ARMA and ARFIMA forecasts. The parameter estimates for the ARMA and ARFIMA forecasts (being of opposite sign and approximately equal magnitude) suggest that the difference between the ARMA and ARFIMA forecasts, denoted here as DAR, captures important information.

To address the question of the informational content of $\Theta^IV_t$, two sets of estimation results are reported in Table 5. Results in Panel (a) are based on $\omega_t = \{GARCH, GJR+RVG, DAR\}$, whereas in Panel (b), $\omega_t = \{GARCH, GJR+RVG, DAR, RV_t\}$ where $RV_t$ is the current level of realised volatility.

Results for Model I indicate that the elements in $\omega_t$ have significant explanatory power in relation to the current level of VIX. Parameter estimates for all elements in $\omega_t$ are significant and $R^2$ is found to be 0.771. Model II, extends the set of instruments to include $RV_t$ which contains levels of future RV. Results from the J-test indicate that residuals, $\varepsilon_t(\gamma)$ are uncorrelated with future RV suggesting that $\Theta^IV_t$ contains no elements from $\Theta^E_t$.

Model III, is estimated on the premise that the instrument set is further extended to include $dRV_t = \{dRV_t, dRV_{t+1\rightarrow t+22}\}$. Here $dRV_t$ and $dRV_{t+1\rightarrow t+22}$, indicate the change in RV from the current level to the prevailing level during the next business day, and to the average level prevailing during the next 22 business days respectively. Model III results lead to quite a different conclusion.

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5While the VIX index is a measure of expected volatility during the next 22 business days it is reasonable to include RV over shorter horizons as VIX might have FLIC with respect to shorter horizons.
### Table 5: GMM estimates.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Panel (a)</th>
<th>Panel (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>const</td>
<td>$GARCH$</td>
</tr>
<tr>
<td>I: Instruments ${c, \omega_t}$</td>
<td>0.408</td>
<td>0.544</td>
</tr>
<tr>
<td></td>
<td>(7.05)</td>
<td>(7.75)</td>
</tr>
<tr>
<td>II: Instruments ${c, \omega_t, RV_t}$</td>
<td>0.389</td>
<td>0.556</td>
</tr>
<tr>
<td></td>
<td>(6.64)</td>
<td>(9.20)</td>
</tr>
<tr>
<td>III: Instruments ${c, \omega_t, RV_t, dRV_t}$</td>
<td>0.386</td>
<td>0.548</td>
</tr>
<tr>
<td></td>
<td>(6.33)</td>
<td>(9.11)</td>
</tr>
<tr>
<td>IV: Instruments ${c, \omega_t}$</td>
<td>0.416</td>
<td>0.521</td>
</tr>
<tr>
<td></td>
<td>(7.11)</td>
<td>(7.80)</td>
</tr>
<tr>
<td>V: Instruments ${c, \omega_t, RV_t, dRV_t}$</td>
<td>0.398</td>
<td>0.557</td>
</tr>
<tr>
<td></td>
<td>(6.88)</td>
<td>(10.17)</td>
</tr>
</tbody>
</table>

Table 5: GMM estimates. In parentheses, t-statistics for coefficient estimates and degrees of freedom for the J-test are reported. Significance test are performed using the Andrews-Monahan weighting matrix with pre-whitening.

Regarding the informational content of $\Theta_t^{IV}$. Here the null hypothesis of the J-test is rejected, indicating that there is some information in $\Theta_t^{IV}$ not captured solely by volatility forecasts. Inspection of the moment conditions reveals that the elements in $M$ associated with $dRV_t$ trigger the rejection.

As all elements included in $\omega_t$ contain some element of volatility smoothing, one might argue that the smoothing process eliminates some important information contained in the actual level of current volatility, $RV_t$. Hence panel (b) displays results for equation 9 where $\omega_t$ is extended to include $RV_t$. Parameter estimates for Model IV show that $RV_t$ is significantly related to VIX ($R^2$ is found to be 0.777). Model V addresses the question of information in $\Theta_t^{IV}$ when $\Theta_t^{H}$ is represented by not only volatility forecasts, but also $RV_t$. When including this measure, all evidence of forward looking information in $\Theta_t^{IV}$ disappears.

Although not of primary interest for the purposes of this study, the parameter estimates contain interesting information. The positive coefficients for $GARCH$ and $GJR + RVG$ indicate that VIX and the $GARCH$ and $GJR + RVG$ capture a significant amount of common information. Interestingly, the $RV_t$
parameter is significantly negative. To understand this it is important to acknowledge that \( R_{V_t} \), in comparison to the GARCH-model forecasts is a very noisy measure. Therefore it appears as if the negative coefficient represents an element of mean reversion in volatility. The reason for the negative coefficient on the \( DAR \) variable is not at all obvious. The difference between the \( ARMA \) and the \( ARFIMA \) forecasts is negatively correlated to all model based volatility forecasts, indicating that the \( ARFIMA \) forecast tends to be higher than the \( ARMA \) forecast whenever the volatility level is high. This may reflect the difference between long and short memory process underlying the \( ARMA \) and the \( ARFIMA \) models.

5 Concluding remarks

This paper has examined the informational content of IV, specifically whether IV offers any genuinely forward looking information not captured by historical information. Whilst numerous authors have considered the informational content of IV, none have sought to isolate the forward looking component of IV in the manner proposed here. Fleming (1998) for instance considers whether the information in IV completely subsume historical information.

To isolate forward looking information, it was first necessary to define information reflected in IV that was attributable to historical information. In this context such historical information included was not only current levels of volatility, but also forecasts of future volatility based only on historical data. These forecasts were included to reflect the possible process by which option market participants may form expectations of future volatility. Given these elements of historical information, the forward looking informational content of IV was then considered. To do so, the relationship between future volatility and information in IV not attributable to historical information was examined. Overall, the empirical results presented in Section 4 show that if the historical information set is correctly specified, S&P 500 option IV does not contain any information regarding future volatility not captured by historical information.

These findings reveal two important facts regarding the operation of the
S&P 500 options market. First, IV appears to be closely related to current levels of volatility. Therefore, option prices, and hence IV are strongly influenced by the prevailing level of volatility. Second, S&P 500 option market participants appear to have no foresight in relation to the future evolution of S&P 500 volatility.

References


