The Spirit of Capitalism and International Risk Sharing

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Abstract

We show that a model of the spirit of capitalism can generate a high degree of international risk sharing as measured by the discount-factor-based approach of Brandt, Cochrane, and Santa-Clara (2001), even when consumption and portfolio holdings exhibit “home bias”. We also show how portfolio externalities can arise in the model, and highlight the caution that one needs in interpreting discount-factor-based measures of international risk sharing: in the presence of portfolio externalities, even when the measured degree of risk sharing is perfect, it is still possible for government policies to induce investors to hold better-diversified portfolios and attain higher welfare.

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1 Introduction

Using consumption or portfolio data, as well as specific assumptions on investor preferences, a long literature concludes that the degree of international risk sharing is far from perfect. Since the benefit of risk sharing seems too high to be offset by observable costs of foreign investment, the lack of international diversification has often been called the “home bias puzzle”.¹

In contrast with this literature, Brandt, Cochrane, and Santa-Clara (2001) propose a novel approach that does not require any preference assumptions to measure the degree of international risk sharing. The authors use asset price data to infer the co-movement between different countries’ discount factors, and find that they are highly correlated. This result implies that the extent of international risk sharing is in fact much higher than that suggested by previous studies.

We show that a model that incorporates the “spirit of capitalism” can reconcile the seemingly contradictory results from the two approaches. By the “spirit of capitalism”, we are referring to Weber’s (1948) idea that investors accumulate wealth not only for consumption, but also for wealth-induced social status:

Man is dominated by the making of money, by acquisition as the ultimate purpose of his life. Economic acquisition is no longer subordinated to man as the means for the satisfaction of his material needs. This reversal of what we should call the natural relationship, so irrational from a naive point of view, is evidently a leading principle of capitalism.—Max Weber, The Protestant Ethic and the Spirit of Capitalism (1948, p. 53)

Keynes (1972) expresses a similar idea:

[Needs] fall into two classes—those which are absolute in the sense that we feel them whatever the situation of our fellow human beings may be, and those which are relative in the

¹See Lewis (1999) for a recent survey of this literature.
sense that we feel them only if their satisfaction lifts us above, makes us feel superior to, our fellows.—John M. Keynes, *Essays in Persuasion* (1972, p. 326)

Bakshi and Chen (1996) and Smith (2001) examine the spirit of capitalism in a closed-economy setting. Here, in an international environment, we postulate that the spirit of capitalism has an important country-related component. Due to closer interactions among residents of the same country and media coverage at the national level, the benchmark relative to which investors calculate their wealth status can be country specific. In this case, the portfolio that investors need to hold in order to hedge variations in the benchmark will also be country specific.

A key insight of the Brandt et al.’s approach is that regardless of the specific form that investor preferences take, perfect risk sharing implies that investors from different countries will optimize their consumption and portfolio choices until their marginal rates of substitution are equalized. In particular, if preferences are non-standard, it is possible for risk sharing to be perfect (in the sense that investors’ intertemporal marginal rates of substitution are equalized) even when investors’ consumption and portfolio exhibit bias relative to those obtained from the conventional, power utility setup. We show that this is indeed the case for our model of the spirit of capitalism.

We also show that the spirit of capitalism can lead to external effects in investors’ portfolio choice. Suppose some local agents are constrained from participating in international financial markets. To the extent that their wealth affects the domestic benchmark, their home bias can induce unconstrained investors from the same country to hold biased portfolios as well. In this way, the spirit of capitalism can serve as a channel through which portfolio biases

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2 These constrained agents can represent investors who are endowed with only human capital or entrepreneurial wealth (which behave like domestic stocks), and are borrowing-constrained for moral hazard reasons. They can also be investors who face high actual or perceived costs of foreign investment, such as transactions costs, information asymmetries, or psychological biases.
that directly affect only a subset of agents are transmitted to other members in the economy. The unconstrained investors from the home country choose to hold biased portfolios—even when they are sharing risks perfectly, in the marginal utility sense, with other unconstrained investors from abroad.

This result leads to an important implication for the interpretation of the discount-factor-based measure of international risk sharing. Specifically, the degree of risk sharing that this approach measures is subject to a given structure of portfolio externalities. We show that even when the discount-factor-based measure suggests that international risk sharing is perfect among the unconstrained investors, it is possible for government policies that aim at removing the participation constraints of the constrained investors to induce the unconstrained investors to hold better-diversified portfolios and attain higher welfare. In other words, even when risk sharing is already “perfect”, further improvements in risk sharing are still possible.

DeMarzo, Kaniel, and Kremer (2003) and Shore and White (2002) also examine the role of externalities in portfolio choice. Agents in DeMarzo et al.’s model have no direct concern for status, but portfolio externalities arise from agents’ competition for scarce local resources. Investors in Shore’s and White’s study exhibit external habit persistence, so that one agent’s consumption has an external effect on the utility of other agents. These consumption externalities lead to portfolio externalities. Duflo and Saez (2002) and Hong, Kubik, and Stein (2003) report empirical evidence of peer effects in portfolio choice. Dumas and Uppal (2001) examine the effect of the imperfect mobility of goods on international risk sharing.

The remainder of this paper is organized as follows. Section 2 presents our model of the spirit of capitalism. Section 3 discusses the model’s implications for international risk sharing. Section 4 examines the role of portfolio externalities, and their implications for the interpretation of discount-factor-based measures of international risk sharing. Section 5 concludes.
2 A Model of the Spirit of Capitalism

2.1 Preferences

The “spirit of capitalism” induces investors to be concerned with their relative wealth status. We model this concern with a specification that Bakshi and Chen (1996) suggest. For an investor in country $i$, her utility is given by

$$U(C_{i,t}, W_{i,t}, V_{i,t}) = C_{i,t}^{1-\gamma} W_{i,t}^{-\gamma} V_{i,t}^{-\lambda},$$

(1)

where $C_{i,t}, W_{i,t}$ are the investor’s own consumption and wealth at time $t$, and $V_{i,t}$ is a “social wealth index”—the benchmark relative to which the investor compares herself. When the investor prefers higher status, the permissible range for the parameters is $\lambda \geq 0$ when $\gamma \geq 1$, and $\lambda < 0$ when $0 < \gamma < 1$. The absolute value of the parameter $\lambda$ measures the extent to which the investor cares about relative wealth status. The difference between our specification and Bakshi’s and Chen’s is our international setting. In particular, we assume that the social wealth index $V_{i,t}$ is country-specific—investors in the same country compare themselves to the same norm or reference level.

2.2 Asset Returns

We consider a world with $N + 1$ constant-returns-to-scale technologies, the first $N$ of which are risky, have constant expected returns and volatilities, and are imperfectly correlated. The $N + 1$st technology is riskless, with a rate of return $r$. We assume that financial markets are complete, and none of the assets are redundant. The cumulative real value of a unit investment in risky technology $j$ follows a geometric Brownian motion:

$$\frac{dP_{j,t}}{P_{j,t}} = \mu_j dt + \sigma_j dz_{j,t},$$

(2)

for $j = 1, ..., N$. $\mu_j$ is the expected return on asset $j$, $\sigma_j^2$ its instantaneous variance, and $dz_{j,t}$ a standard Wiener process. The instantaneous correlation structure of the Wiener processes
is given by \( dz_j dz_k = \rho_{jk} dt \), and the variance-covariance matrix defined as \( \Sigma \equiv [\sigma_j \sigma_k \rho_{jk}] \) is symmetric, positive-definite, and invertible. We further define \( \mu \equiv [\mu_j] \), \( dz_t \equiv [\sigma_j dz_j,t] \), and \( dp_t \equiv \mu dt + dz_t \) as \( N \times 1 \) column vectors.

2.3 Consumption and Portfolio Choice

As in Bakshi and Chen (1996), we model the social wealth index as a geometric Brownian motion:

\[
dV_{i,t} = \mu_{v,i} dt + \sigma_{v,i} dz_{v,i,t}. \tag{3}
\]

Since markets are complete, it is possible to perfectly hedge the stochastic movements in \( V_i \) by holding a portfolio of the \( N + 1 \) assets. We call this portfolio the replicating portfolio of \( V_i \), and denote its weights in the \( N \) risky technologies as an \( N \times 1 \) column vector \( \alpha_i \). Specifically, \( \sigma_{v,i}^2 = \alpha_i' \Sigma \alpha_i \), and \( \mu_{v,i} = r + \alpha_i' (\mu - \mathbf{1}) - \theta_i \), where \( \theta_i \) is a non-negative constant.

Taking the \( V_i \) process as given, an investor from country \( i \) makes her consumption and portfolio decisions in order to

\[
\max E_0 \left\{ \int_0^\infty e^{-\beta t} U(C_{i,t}, W_{i,t}, V_{i,t}) \, dt \right\}. \tag{4}
\]

Denoting her portfolio weights as \( \alpha_i \), we can define the value function of this maximization problem as:

\[
J(W_{i,t}, V_{i,t}) = \max_{C_{t,s}, \alpha_{t,s}, s \in [t, \infty)} E_t \left\{ \int_t^\infty e^{-\beta(s-t)} U(C_{t,s}, W_{i,s}, V_{i,s}) \, ds \right\}, \tag{5}
\]

subject to \( dW_{i,t} = \{W_{i,t} [r + \alpha_i' (\mu - \mathbf{1})] - C_{i,t}\} dt + \sigma_{w,i} W_{i,t} dz_{w,i,t} \), where \( \sigma_{w,i}^2 = \alpha_i' \Sigma \alpha_i \), \( dz_{w,i,t} = \sum_{j=1}^N \frac{\alpha_i(j) \sigma_j dz_j,t}{\sigma_{w,i}} \), and \( \alpha_i(j) \) denotes the \( j \)th element of \( \alpha_i \). This value function gives the maximum expected lifetime utility attainable by a country \( i \) investor when her own private wealth and country \( i \)'s social wealth index are equal to \( W_{i,t} \) and \( V_{i,t} \) respectively.
The first-order condition for (5) with respect to $C_i$ and $\alpha_i$ yields

$$J_W(W_{i,t}, V_{i,t}) = U_C(C_{i,t}, W_{i,t}, V_{i,t})$$

$$0 = J_W(\mu - r1) + J_W W_i \Sigma \alpha_i + J_W \Psi_i.$$  

(6)

(7)

where $\Psi = (\sigma_{1\nu_i}, ..., \sigma_{N\nu_i})' \in \mathbb{R}^N$ is a vector of all the covariances of each of the $N$ risky assets with the social wealth index $V_i$. Solving (7) for $\alpha_i$, we obtain the vector of optimal portfolio weights:

$$\alpha_i = \frac{-J_W}{W_{i,t} J_{WW}} \Sigma^{-1} (\mu - r1) - \frac{J_{VW}}{J_{WW} W_{i,t}} \Sigma^{-1} \Psi + \frac{J_{VW}}{J_{WW} W_{i,t}} \Sigma^{-1} \alpha_i,$$

(8)

Even before solving explicitly for $J$, we can see that the concern for status creates a hedging demand in investors’ portfolio. This hedging component is given by the second term in equation (8), and motivates the dependence of individual portfolio holdings $\alpha_i$ on the replicating portfolio weights $\hat{\alpha}_i$ of country $i$’s social wealth index.

Before proceeding further, it is useful to derive more economic content from equation (8). First, note that $-\frac{J_W}{W_{i,t} J_{WW}}$ is the inverse of the coefficient of relative risk aversion (RRA), while $-\frac{J_{VW}}{J_{WW} W_{i,t}} = \frac{J_{VW}}{J_{WW} \text{RRA}}$, where $J_{VW} V_{i,t}$ is the elasticity of $J_W$ with respect to $V_i$.

The following proposition solves an investor’s consumption-portfolio problem explicitly:

**Proposition 1** Let investor utility be given by (1). Then, the solution to the consumption-portfolio problem in (5) is

$$C_{i,t}^* = \eta_i W_{i,t}^*$$

$$\alpha_{i,t}^* = \frac{1}{\gamma + \lambda} \Sigma^{-1} (\mu - r1) + \frac{\lambda}{\gamma + \lambda} \hat{\alpha}_i$$

$$J(W_{i,t}, V_{i,t}) = \frac{\eta_i \gamma W_{i,t}^{1-\gamma-\lambda}}{(1 - \gamma - \lambda)V_{i,t}^{-\lambda}}.$$  

(9)

(10)

(11)
where
\[
\eta_i = \frac{\gamma - 1}{\gamma (\gamma + \lambda - 1)} \left[ r (\gamma - 1) + \frac{\gamma + \lambda - 1}{2(\gamma + \lambda)} (\mu - r \mathbf{1})' \Sigma^{-1} (\mu - r \mathbf{1}) - \frac{\lambda}{\gamma + \lambda} (\mu - r \mathbf{1})' \hat{\alpha}_i + \frac{\lambda \gamma}{2(\gamma + \lambda)} \hat{\alpha}_i' \Sigma \hat{\alpha}_i + \lambda \theta_i + \beta \right],
\]
(12)
\[
\eta_i > 0, \; \gamma + \lambda > 1.
\]

**Proof.** See Appendix. ■

The restrictions \(\eta_i > 0, \; \gamma + \lambda > 1\) are derived from the transversality condition of the optimization problem. Since our solution requires that \(\gamma + \lambda > 1\), we restrict our analysis to the range of parameter values, \(\gamma \geq 1\) and \(\lambda \geq 0\). At the same time, the elasticity of \(J_{W_{i,t}}\) with respect to \(V_i\) is given by \(\lambda\).

Since the consumption-wealth ratio \(\eta_i\) is constant, we can express the \(W_{i,t}\) process as a geometric Brownian motion:
\[
dW_{i,t} = \mu_{w,i} dt + \sigma_{w,i} dz_{w,i,t},
\]
(13)
where \(\mu_{w,i} = r + \alpha_i'^{(1)} (\mu - r \mathbf{1}) - \eta_i\), \(\sigma_{w,i}^2 = \alpha_i' \Sigma \alpha_i\), \(dz_{w,i,t} = \sum_{j=1}^{N} \frac{\alpha_i(j) \sigma_{j,i} dz_{j,t}}{\sigma_{w,i}}\), and \(\alpha_i(j)\) denotes the \(j\)th element of \(\alpha_i\). A constant consumption-wealth ratio also implies that \(\frac{dC_{i,t}}{C_{i,t}} = \frac{dW_{i,t}}{W_{i,t}}\).

Thus, we can write
\[
\frac{dC_{i,t}}{C_{i,t}} = \mu_{c,i} dt + \sigma_{c,i} dz_{c,i,t},
\]
(14)
where \(\mu_{c,i} = \mu_{w,i}\), \(\sigma_{c,i} = \sigma_{w,i}\), and \(dz_{c,i,t} = dz_{w,i,t}\).

\[\text{3} \quad RRA = \frac{-W_{i,t}^{\mu_{W_{i,t}}}}{J_{W_{i,t}}} = \gamma + \lambda.\]

\[\text{4} \quad \text{The elasticity of intertemporal substitution between consumption at time } t \text{ and } s \text{ is given by } -\frac{u'(c_s)/u'(c_t)}{u'(c_s)/u'(c_t)} \frac{d(c_s/c_t)}{d(c_s/c_t)} = \frac{-\omega(c_t)/\omega(c_s)}{u'(c_s)/u'(c_t)} \frac{d(c_s/c_t)}{d(c_s/c_t)}. \text{ Taking the limit as } s \to t \text{ gives } EIS = \frac{-\omega(c_t)/\omega(c_s)}{u'(c_s)/u'(c_t)} \frac{d(c_s/c_t)}{d(c_s/c_t)}, \text{ and is equal to } \frac{1}{\gamma + \lambda}.\]
2.4 The Discount Factor

The discount factor
\[
\frac{d\Lambda}{\Lambda} = -rdt - \mu'\Sigma^{-1}dz_t
\]  
(15)

prices the set of assets defined by equation (2) above, as it satisfies the following pricing conditions:

\[
E\left(\frac{d\Lambda}{\Lambda}\right) = -r dt, \\
E\left(\frac{d\Lambda}{\Lambda}\frac{dP_t}{P_t}\right) = -\mu dt.
\]

Moreover, since markets are complete, this discount factor is unique.

3 International Risk Sharing

Equation (10) above shows the dependence of individual portfolio choice on the social wealth index. The first term in the equation, \( \frac{1}{\gamma + \lambda} \Sigma^{-1} (\mu - r1) \), represents an “unbiased” portfolio that a power-utility investor (with \( RRA = \gamma + \lambda \)) will hold. We denote this term as \( \overline{\alpha} \), and note that it is a common component in the portfolio of all investors, regardless of their country of origin. The second term in equation (10), \( \frac{\lambda}{\gamma + \lambda} \hat{\alpha}_i \), represents the incentive an investor from country \( i \) has in imitating \( \hat{\alpha}_i \), the replicating portfolio of her own country’s social wealth index.

In particular, if \( \hat{\alpha}_i \) is “biased” towards asset \( j \) (i.e., overweight asset \( j \) relative to \( \overline{\alpha} \)), \( \alpha_i \) will also be biased towards the same asset. In turn, this portfolio bias induces a bias in the processes for \( W_{i,t} \) and \( C_{i,t} \), in the sense that they are now affected even by diversifiable, idiosyncratic shocks that hit technology \( j \). By contrast, in a world with no concern for status (i.e., \( \lambda = 0 \)), all investors hold the same diversified portfolio \( \overline{\alpha} \), and there are no country-specific components in their consumption and wealth processes.
3.1 Portfolio Bias and Risk Sharing

When the social wealth indexes (and hence the corresponding replicating portfolios $\alpha_i$) are different across countries, investors’ portfolio and consumption allocations will contain country-specific components. In this case, traditional portfolio- and consumption-based measures of international risk sharing, by comparing $\alpha_i$ and $C_{i,t}$ with the full-diversification benchmark under power utility, will conclude that risk sharing is incomplete.

Yet, by examining the model’s discount factor, we obtain a markedly different conclusion. We show above that the discount factor given by equation (15) is not country specific. In other words, the growth rates of marginal utility are equalized across investors from different countries, and discount-factor-based measures will conclude that the degree of international risk sharing is perfect. This result holds regardless of whether there are country-specific variations in the social wealth index, and hence country-specific components in investors’ consumption and portfolio choice. In particular, “home bias” in consumption and portfolio choice can be consistent with perfect risk sharing (as measured by the discount factor).

3.2 From Biased Consumption to Perfect Risk Sharing: Inspecting the Mechanism

To obtain a deeper understanding of how the different consumption and portfolio choices across countries translate into perfect co-movements in marginal rates of substitution, we express the discount factor in the more familiar, discrete-time convention, and in terms of an investor’s consumption, wealth, and social wealth index.

Let $\{(C_{i,t}^*, \alpha_{i,t}^*) : t = 0, \Delta t, \ldots\}$ represent an optimal plan for (4). To derive the discrete-time Euler equation, we follow a variational argument in Grossman and Shiller (1982). For an investor from country $i$ who sells $s\Delta t$ units of asset $j$ at time $t$, consumes the proceeds in the same period, and buys the $s\Delta t$ units of asset $j$ back at time $t + \Delta t$ (by reducing her
consumption during $t + \Delta t$), her total reduction in expected utility is given by

$$
U(C_{i,t}^*, W_{i,t}^*, V_{i,t}) + E_t e^{-\beta \Delta t} U(C_{i,t+\Delta t}^*, W_{i,t+\Delta t}^*, V_{i,t+\Delta t}) \left[ U(C_{i,t}^* + sP_{j,t}, W_{i,t}^*, V_{i,t}) + E_t e^{-\beta \Delta t} U(C_{i,t+\Delta t}^* - sP_{j,t+\Delta t}, W_{i,t+\Delta t}^* - sP_{j,t+\Delta t}, V_{i,t+\Delta t}) \right].
$$

(16)

For $C_{i,t}^*$ and $\alpha_{i,t}^*$ to be optimal, (16) must be minimized at $s = 0$. This requirement implies that the derivative of (16) with respect to $s$ must be zero at $s = 0$, so that

$$
P_{j,t} U(C_{i,t}^*, W_{i,t}^*, V_{i,t}) = e^{-\beta \Delta t} E_t \left[ P_{j,t+\Delta t} \left( U(C_{i,t+\Delta t}^*, W_{i,t+\Delta t}^*, V_{i,t+\Delta t}) + \Delta t U_W \left( C_{i,t+\Delta t}^*, W_{i,t+\Delta t}^*, V_{i,t+\Delta t} \right) \right) \right].
$$

Setting $\Delta t = 1$, we obtain the discrete-time stochastic discount factor for investors from country $i$:

$$
M_{i,t+1} = e^{-\beta} \frac{U(C_{i,t+1}, W_{i,t+1}, V_{i,t+1})}{U(C_{i,t}, W_{i,t}, V_{i,t})} + U_W(C_{i,t+1}, W_{i,t+1}, V_{i,t+1})
$$

$$
= e^{-\beta} \left( \frac{C_{i,t+1}}{C_{i,t}} \right)^{-\gamma} \left( \frac{W_{i,t+1}}{W_{i,t}} \right)^{-\lambda} \left( \frac{V_{i,t+1}}{V_{i,t}} \right)^{\lambda} \left[ 1 + \frac{\lambda}{\gamma - 1} \left( \frac{C_{i,t+1}}{W_{i,t+1}} \right) \right]
$$

$$
= e^{-\beta} \left( \frac{W_{i,t+1}}{W_{i,t}} \right)^{-\gamma - \lambda} \left( \frac{V_{i,t+1}}{V_{i,t}} \right)^{\lambda} \left[ 1 + \frac{\lambda}{\gamma - 1} \left( \frac{C_{i,t+1}}{W_{i,t+1}} \right) \right],
$$

(17)

where the last equality follows from the fact that the consumption-wealth ratio is constant.

Using this expression for the discount factor, we compute Brandt et al.’s (2001) risk sharing index between countries $d$ and $f$, given by

$$
1 - \frac{\sigma^2 (\ln M_{f,t+1} - \ln M_{d,t+1})}{\sigma^2 (\ln M_{f,t+1}) + \sigma^2 (\ln M_{d,t+1})}.
$$

To show that risk sharing between the two countries is perfect, it suffices to demonstrate that the stochastic components of $\ln M_{f,t+1}$ and $\ln M_{d,t+1}$ are the same, so that $\sigma^2 (\ln M_{f,t+1} - \ln M_{d,t+1})$ equals zero. From equation (17), and the processes for $V_i$ and $W_i$ defined above in equations (3) and (13), we can express the stochastic component of $\ln M_{i,t+1}$ as

$$
-(\gamma + \lambda) \int_t^{t+1} \sigma_{w,i} d\bar{z}_{w,i,s} + \lambda \int_t^{t+1} \sigma_{v,i} d\bar{z}_{v,i,s}.
$$

(18)
But since \( \sigma_{w,i}dz_{w,i,s} = \sum_{j=1}^{N} \alpha_i(j) \sigma_jdz_{j,t} = \sum_{j=1}^{N} \left( \overline{\alpha}(j) + \frac{1}{\gamma+\lambda} \hat{\alpha}_i(j) \right) \sigma_jdz_{j,t} \), and \( \sigma_{v,i}dz_{v,i,s} = \sum_{j=1}^{N} \hat{\alpha}_i(j) \sigma_jdz_{j,t} \), where \( \overline{\alpha} \equiv \frac{1}{\gamma+\lambda} \Sigma^{-1} (\mu - r1) \) is the common component in the portfolio of all investors defined above, the expression in (18) reduces to
\[
-(\gamma + \lambda) \sum_{j=1}^{N} \left[ \overline{\alpha}(j) \sigma_j \int_{t}^{t+1} dz_{j,t} \right],
\]
which is identical across countries. Thus, indeed, for any two countries \( d \) and \( f \), the stochastic components of \( \ln M_{f,t+1} \) and \( \ln M_{d,t+1} \) are the same, and \( \sigma^2 (\ln M_{f,t+1} - \ln M_{d,t+1}) \) equals zero.

In other words, even though there are country-specific variations in consumption, wealth, and the social wealth index, they enter the discount factor in a way that exactly offsets one another, and only the component that is common across countries remains. The economics behind this result is simple. With complete markets, investors from different countries make their consumption and portfolio choice to share risk perfectly—in the marginal-utility sense—subject to variations in their country-specific social wealth index.

4 Country-Specific Variations in the Social Wealth Index: A Simple Example

Our analysis so far has focused on the effects of country-specific variations in the social wealth index on investors’ consumption and portfolio allocations. Here, we examine why there are country-specific variations in the social wealth index in the first place.

We consider a special case of our model that has two countries and two risky production technologies, where technology \( i \) is located in country \( i \), for \( i = 1, 2 \). In addition to the two risky technologies, there is also a riskless technology that can be used in both countries. We assume that the returns on the two risky technologies have the same mean and volatility, but are imperfectly correlated. We denote their common mean and volatility as \( \mu \) and \( \sigma \), and their covariance as \( \sigma_{12} \). As before, the rate of return of the riskless technology is given by \( r \).

To keep the analysis tractable, we assume that each country is populated by two agents: a
constrained agent (Agent C) and an unconstrained agent (Agent U). Agent C faces a binding participation constraint in international financial markets, and holds the domestic and the riskless assets only. Agent U faces no participation constraints, but has the spirit of capitalism preferences given by equation (1) above, and uses the wealth of Agent C as her social wealth index. Thus, even though Agent U does not face any direct cost of foreign investment, she will not hold the fully-diversified portfolio $\alpha^*$—since her social wealth index (the wealth of Agent C) contains the domestic and the riskless assets only. Instead, the “home bias” in Agent C’s portfolio (denoted by $\tilde{\alpha}$ above) induces Agent U’s portfolio to be home-biased as well.

Since it is only the unconstrained agents in each country who participate in international financial markets, they are the marginal investors whose preferences are reflected in the equilibrium discount factor. As we discuss above, in complete markets, these unconstrained agents from different countries share risks perfectly—subject to the country-specific variations in their social wealth indexes (i.e., the wealth of the constrained agents located in their own countries).

There are a number of possible interpretations of who the constrained agents represent in reality. First, they can represent those local residents who are endowed with only human capital or entrepreneurial wealth (which behave like domestic stocks), and are borrowing-constrained for moral hazard reasons. Second, they can represent investors who face high actual or perceived costs of international investment. Actual costs can include information asymmetries, transactions costs, taxes, and exchange-rate risks. Perceived costs include various behavioral biases, such as agents’ reluctance to participate in “unfamiliar” gambles, or hold “unfamiliar” assets.$^6$

$^5$With more agents, additional state variables have to be introduced to keep track of the relative levels of wealth among them.

4.1 Welfare Effects of Changing the Degree of Bias in the Social Wealth Index

From equation (10) above, it is easy to see that the removal of biases from the social wealth index induces Agent U to hold a fully-diversified portfolio, i.e., when $\hat{\alpha}$ is proportional to $\bar{\alpha}$, Agent U’s portfolio becomes proportional to $\bar{\alpha}$ as well. Less clear, however, is whether such a change improves Agent U’s welfare. Intuitively, there are two effects that go in opposite directions. The first is the usual diversification effect: the better-diversified portfolio that Agent U holds after the change leads to a more favorable risk-return trade-off that improves welfare. The second is the spirit of capitalism effect. Since Agent U receives utility from “outperforming” Agent C, and it becomes more difficult to outperform when Agent C’s portfolio is less biased, Agent U can become worse off as a result of the change.

We investigate the relative importance of these two effects by examining the value function $J$ given by equation (11) above. $W_{i,t}$ represents the wealth of Agent U in country $i$, and $V_{i,t}$ represents the wealth of Agent C in the same country—as Agent U uses Agent C’s wealth as her social wealth index. For given values of $W_{i,t}$, $V_{i,t}$, and preference parameters $\gamma$ and $\lambda$, Agent U’s welfare depends on her own consumption-wealth ratio $\eta_i$. The size of $\eta_i$, in turn, depends on $\hat{\alpha}_i$ and $\theta_i$ (see equation (12) above), which correspond in our example here to Agent C’s portfolio holdings and consumption-wealth ratio respectively. Thus, how these quantities change as a result of the removal of biases in Agent C’s portfolio determines Agent U’s welfare.

To know how these quantities respond, we have to specify Agent C’s preferences. Here, we assume that Agent C has power utility with $RRA = \gamma + \lambda$, i.e., her period utility is given by $U(C_t) = \frac{C_t^{1-\gamma-\lambda}}{(1-\gamma-\lambda)}$, where $C_t$ is her own consumption at time $t$. We choose this particular specification for the relatively simple consumption and portfolio rules that it generates. Our purpose is only to show that even for a simple specification such as this, whether the diversification or by Lewis (1999).
the spirit of capitalism effect dominates is still ambiguous, and depends on the relative size of certain preference parameters.

On Table 1, we report the portfolio choice ($\alpha_1$) and consumption-wealth ratio ($\theta_1$) of Agent C in country 1, both in the case when the agent’s participation constraint in international markets is binding, and in the case when the constraint is removed. Note that for this agent, asset 1 is her domestic asset. Since the derivation of these results is standard, we omit the proofs, which are available from the authors on request.

Using results from the table, we can express Agent U’s consumption-wealth ratio in terms of our model’s parameters, both when Agent C faces an international participation constraint, and when the constraint is removed. We denote these two consumption-wealth ratios as $\eta_{1,\text{constrained}}$ and $\eta_{1,\text{unconstrained}}$ respectively. To evaluate $\eta_{1,\text{constrained}}$ ($\eta_{1,\text{unconstrained}}$), we only need to substitute the values of $\alpha_{1,\text{constrained}}$ and $\theta_{1,\text{constrained}}$ ($\alpha_{1,\text{unconstrained}}$ and $\theta_{1,\text{unconstrained}}$) from Table 1 into equation (12) above. In particular, it is straightforward to show that

$$\eta_{1,\text{unconstrained}} - \eta_{1,\text{constrained}} = \frac{(\mu - r)^2 \sigma^2 - \sigma_{12}}{(\gamma + \lambda)(\sigma^2 + \sigma_{12})} \left[ \frac{\lambda ((\gamma + \lambda)^2 - 2(\gamma + \lambda) - \gamma)}{2(\gamma + \lambda)} \right].$$  \hspace{0.5cm} (19)

We are interested in this quantity because from equation (11) above, we see that for $\gamma$, $\lambda > 0$, and $\gamma + \lambda > 1$, the value function $J$ is increasing in $\eta_i$, and this observation implies that relaxing the participation constraint of Agent C is welfare-improving for Agent U if $\eta_{1,\text{unconstrained}} - \eta_{1,\text{constrained}} > 0$. From equation (19), this condition is satisfied if

$$(\gamma + \lambda)^2 - 2(\gamma + \lambda) - \lambda > 0.$$  \hspace{0.5cm} (16)

This expression shows quantitatively what we have discussed above intuitively—whether relaxing the participation constraint of Agent C is welfare-improving for Agent U depends on the strength of the spirit of capitalism effect. In particular, when $\lambda$ is small relative to $\gamma$ (so that the spirit of capitalism effect is relatively weak), the improved diversification associated with the change raises Agent U’s welfare. On the other hand, when $\lambda$ is large relative to $\gamma$ (so
that the desire for Agent U to outperform Agent C is strong), the change can actually lower Agent U’s welfare—as the removal of Agent C’s portfolio constraint makes it more difficult for Agent U to outperform Agent C.

4.2 Implications for the Interpretation of Discount-Factor-Based Measures of Risk Sharing

Our findings from the previous section suggests that one has to exercise caution when interpreting discount-factor-based measures of risk sharing. The first point is obvious. The equilibrium discount factor that researchers can infer from asset prices applies only to the unconstrained investors who participate in international financial markets. In our example above, even though the unconstrained investors are sharing risks perfectly with each other, the constrained investors are not.

The second point is more subtle. Even though the constrained investors’ intertemporal marginal rates of substitution do not price the different assets across countries, these investors can still play an important role in the economy’s equilibrium—as their portfolios can exert an external effect on those of the unconstrained investors. The externality arises when constrained investors only take into account their own benefit when deciding whether or not to participate in international financial markets, without internalizing the effects their portfolio allocations have on other agents in the economy.

Thus, even when discount-factor-based measures suggest that the prevailing degree of risk sharing is perfect, there can still be room for further risk sharing. This point is obvious for the constrained investors, whose intertemporal marginal rate of substitution is different from that of the market equilibrium. A more surprising result is that this point also holds for the unconstrained investors. Even though their discount factors are already equal to that of the market equilibrium to begin with, the unconstrained investors can still adjust
to better-diversified portfolios and attain higher welfare when the constrained investors become less biased. In the context of our example above, this scenario arises when the condition 
\[ \eta_{1, \text{unconstrained}} - \eta_{1, \text{constrained}} > 0 \]

This result implies that even the unconstrained investors can benefit from government policies (such as investor education or actual subsidies to encourage portfolio diversification) that aim at reducing the portfolio bias of the constrained investors. This conclusion holds even though discount-factor-based measures suggest that the existing degree of risk sharing among the unconstrained investors is already perfect. On the other hand, if 
\[ \eta_{1, \text{unconstrained}} - \eta_{1, \text{constrained}} < 0, \]

policies that remove portfolio biases in the constrained investors will actually lower the welfare of the unconstrained.

5 Conclusion

Even when investors are sharing risks perfectly in the marginal utility sense, the spirit of capitalism in investor utility together with country-specific variations in the social wealth index can give rise to “home bias” in investors’ consumption and portfolio choice. Thus, high degrees of international risk sharing according to discount-factor-based measures (as reported by Brandt et al. 2001) can be consistent with low correlations of cross-country consumption growth rates.

We also show that the degree of international risk sharing that researchers obtain from discount-factor-based measures is subject to the existing structure of portfolio externalities in the economy. For example, the home-biased portfolios of constrained investors can be an externality that induces unconstrained investors to hold biased portfolios as well. Even when researchers find that the degree of international risk sharing is perfect using the discount-factor approach, the result only implies that risk sharing is perfect conditional on existing portfolio externalities. If government policies can induce the constrained investors to hold less biased
portfolios, the unconstrained investors will also become better diversified, and can see their welfare improve as a result.
Appendix. Proof of Proposition 1

We solve the investor’s consumption-portfolio problem using the solution technique of Merton (1971). Using Ito’s lemma, the stochastic Bellman equation for (5) is given by:

\[
0 = \max_{C_i, \alpha_i} \left\{ \begin{array}{l}
U(C_{i,t}, W_{i,t}, V_{i,t}) + J_W \{ W_{i,t} [r + \alpha_i' (\mu - r 1)] - C_{i,t} \} \\
+ J_V \{ V_{i,t} [r + \hat{\alpha}_i' (\mu - r 1) - \theta_i] \} + \frac{1}{2} W_{i,t}^2 J_{WW} \alpha_i' \Sigma \alpha_i \\
+ J_{VW} W_{i,t} V_{i,t} \alpha_i' \Sigma \hat{\alpha}_i + \frac{1}{2} V_{i,t}^2 J_{VV} \hat{\alpha}_i' \Sigma \hat{\alpha}_i - \beta J \end{array} \right\}, \tag{20}
\]

the first-order conditions of which are stated in equations (6) and (7). For an investor with the utility function displayed in equation (1), we solve explicitly for her optimal consumption and portfolio decisions assuming the social wealth index \( V_{i,t} \) follows the process stated in equation (3) above. By first conjecturing that the value function \( J(W_{i,t}, V_{i,t}) \) is of the form

\[
J(W_{i,t}, V_{i,t}) = \eta_i^{-\gamma} W_{i,t}^{1-\gamma-\lambda} \left(1-\gamma-\lambda\right)V_{i,t}^{-\lambda},
\]

and then substituting it into equations (6), (7) and (20), we can solve jointly for \( C_{i,t}^*, \alpha_{i,t}^* \) and \( \eta_i \). \( \blacksquare \)
References


Table 1. Portfolio choice and consumption-wealth ratio of Agent C in Country 1

<table>
<thead>
<tr>
<th></th>
<th>With participation constraint</th>
<th>Without participation constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>$\hat{\alpha}_{1,\text{constrained}} = \left( \frac{\mu - r}{(\gamma + \lambda)\sigma^2}, 0 \right)$</td>
<td>$\hat{\alpha}<em>{1,\text{unconstrained}} = \left( \frac{\mu - r}{(\gamma + \lambda)(\sigma^2 + \sigma</em>{12})}, \frac{\mu - r}{(\gamma + \lambda)(\sigma^2 + \sigma_{12})} \right)$</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>$\theta_{1,\text{constrained}} = \frac{1}{\gamma + \lambda} \left( \beta + (\gamma + \lambda - 1) r + \frac{(\gamma + \lambda - 1)(\mu - r)^2}{2(\gamma + \lambda)\sigma^2} \right)$</td>
<td>$\theta_{1,\text{unconstrained}} = \frac{1}{\gamma + \lambda} \left( \beta + (\gamma + \lambda - 1) r + \frac{(\gamma + \lambda - 1)(\mu - r)^2}{(\gamma + \lambda)(\sigma^2 + \sigma_{12})} \right)$</td>
</tr>
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