Inflation Targeting and Q Volatility in Small Open Economies

G.C. Lim* and Paul D. McNelis†

February 27, 2004

Abstract

This paper examines the welfare implications of managing Q with inflation targeting by monetary authorities who have to "learn" the laws of motion for both inflation and the rate of growth of Q. Our results show that the Central Bank can achieve great success in reducing the volatility of GDP growth with basically the same inflation volatility, if it incorporates this additional target into its policy regime. However, the welfare effects are generally lower, in terms of consumption, when the monetary authority reacts to Q growth as well as inflation.

Key words: Tobin’s Q, learning, monetary policy rules, inflation targets
1 Introduction

The practice of inflation targeting - controlling changes in goods prices - is accepted by many Central Banks, but there is no consensus about the management of asset-price inflation, except in the sense that it is not desirable for asset prices to be too high or too volatile. In general, research indicates that central bankers should not target asset prices [see Bernanke and Gertler (1999), Gilchrist and Leahy (2002)]. But Cecchetti, Genberg and Wadhwani (2002) have argued that central banks should "react to asset price misalignments". In essence, they show that when disturbances are nominal, reacting to close misalignment gaps significantly improves macroeconomic performance. However, this stance of monetary policy is difficult to adopt. In practice, it is difficult to identify the degree of misalignment, since there is no clear agreement about the fundamental value of the asset.

In this paper, we consider the rate of growth of Tobin's Q as a potential target variable for monetary policy. Our reasoning is that Q-growth would be small when the growth in the market valuations of capital assets corresponds roughly with the growth of replacement costs. In other words, Q-growth can serve as a measure of asset price misalignment since there is a correspondence between volatility and high Q-growth with "excessive" share price volatility and inflated share prices. An advantage of the focus on the rate of growth Q is that it obviates the need to know the "fundamental value".

The focus on Q is also influenced by Brainard and Tobin (1968, 1977), who argued that Q plays an important role in the transmission of monetary policy both directly via the capital investment decision of enterprises and indirectly via consumption decisions. Thus Q has implications for inflation and growth. Large swings in Q can lead to systematic overinvestment, and moreover, in the open-economy context, to over-borrowing and serious capital account deficits.

We are concerned with comparing two monetary policy regimes - one in which the central bank reacts to inflation (when it exceeds the target range) and the other where the central bank reacts to inflation (when it exceeds the target range) as well as to Q-growth (when it rises above or falls below a specified range). We are also concerned with the welfare implications of inflation targeting in small open economies. Our interest in small open economies is a pragmatic one - about twenty countries now practice inflation targeting and they are small and open.

Following standard methodology we examine the effects of monetary policy in a stochastic dynamic general equilibrium framework. But, since our model is designed to reflect characteristics in small open economies, we pay specific attention to the traded-goods sectors and to price stickiness via the
pass-through method (see also Clarida, Gali and Gertler (2001)). We will also concentrate on shocks to the terms of trade since these tend to be more important than productivity as an underlying driving force for inflation and for economic fluctuations (see discussion in Mendoza (1995)). We have also opted to keep the comparative analysis simple and straightforward, in this paper by omitting "output gap" considerations in the policy rule.

We also introduce learning on the part of the monetary authority in that it does not know the "true laws of motion" of inflation generated by the private sector whose behavior is described by a stochastic dynamic, nonlinear general equilibrium model, with forward-looking rational expectations. Instead the central bank has to learn about the laws of motion of inflation from past data, through continuously updated least squares regression. This information is then used to obtain an optimal interest rate feedback rule based on linear quadratic optimization, using weights in the objective function for inflation which can vary with current conditions. Such a learning framework accords more closely with real life Central Bank policy setting behavior based on approximating models of the true economy. The monetary authority is thus "boundedly rational", in the sense of Sargent (1999), with "rational" describing the use of least squares, and "bounded" meaning model misspecification. The policy setting framework may also be viewed as an adaptation of the robust optimal control modelling framework of Hansen and Sargent (2002).

Thus, we present the implications for two monetary policy scenarios - inflation targeting with and without reacting to Q-growth - on the welfare of a small open economy with sticky prices and where the central bank learns about the nature of the shock and the degree of Q-growth. To anticipate results, we show that incorporating Q-growth targets with inflation targets can be very successful for reducing the volatility of GDP growth without much change in the volatility of inflation. However, for overall welfare, based on the utility of consumption, targeting Q-growth in addition to inflation leads to generally lower payoffs. Overall welfare has less volatility but is centered on a lower value than in the case of pure inflation targeting. We

1This issue of Q-growth targeting in smaller open economies may be even more pertinent than in industrialized economies. Pacharoni (2003) compared the volatility of the share market index of emerging market countries relative to G7 and a broader industrialized countries. He found that the volatility of the share prices in these countries to be much larger than that in the G7 and industrialized countries. He also found the cross-correlations of the rate of growth of share market indices with GDP growth to be small and positive for emerging market countries, while they are small and negative for the G7 and industrialized countries. Thus, targeting Q-growth may serve as an indirect means to stabilise output growth.
thus show that the specification of the interest-rate rule can influence not just the volatility of welfare, but also the average level of welfare. While this result may be provocative, we also note that the difference in the average level of welfare is not very large.

The paper is organized as follows. The model is described in Section 2, and the solution algorithm is presented in Section 3. Section 4 contains the simulation results. The concluding remarks are in Section 5.

2 Model Specification

The framework of analysis contains two modules - a module which describes the behavior of the private sector and a module which describes the behavior of the central bank.

2.1 Private Sector Behavior

The private sector is assumed to follow the standard optimizing behavior characterized in dynamic stochastic general equilibrium models.

2.1.1 Consumption

The utility function for the private sector “representative agent” is given by the following function:

\[ U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma} \]  

(1)

where \( C \) is the aggregate consumption index and \( \gamma \) is the coefficient of relative risk aversion. Unless otherwise specified, upper case variables denote the levels of the variables while lower case letters denote logarithms of the same variables. The exception is the nominal interest rate denoted as \( i \).

The representative agent as “household/firm” optimizes the following intertemporal welfare function, with an endogenous discount factor:

\[ W_t = E \left[ \sum_{i=0}^{\infty} \vartheta_{t+i} U(C_{t+i}) \right] \]  

(2)

\[ \vartheta_{t+1+i} = [1 + C_t]^{-\beta} \cdot \vartheta_{t+i} \]  

(3)

\[ \vartheta_t = 1 \]  

(4)

where \( E_t \) is the expectations operator, conditional on information available at time \( t \), while \( \beta \) approximates the elasticity of the endogenous discount factor \( \vartheta \) with respect to the average consumption index, \( \bar{C} \). Endogenous discounting
is due to Uzawa (1968). Such discounting is widely used in order to "close" open-economy models. Mendoza (2000) states that this type of discounting is needed for the model to produce well-behaved dynamics with deterministic stationary equilibria.\(^2\)

The specification used in this paper is due to Schmitt-Grohé and Uribe (2001). In our model, an individual agent’s discount factor does not depend on their own consumption, but rather their discount factor depends on the average level of consumption. Schmitt-Grohé and Uribe (2001) argue that this simplification reduces the equilibrium conditions by one Euler equation and one state variable, over the standard model with endogenous discounting, it greatly facilitates the computation of the equilibrium dynamics, while delivering “virtually identical” predictions of key macroeconomic variables as the standard endogenous-discounting model.\(^3\) In equilibrium, of course, the individual consumption index and the average consumption index are identical. Hence,

\[
C_t = \bar{C}_t
\] (5)

The consumption index is a composite index of non-tradeable goods \(n\) and tradeable goods \(f\):

\[
C_t = \left( C^f_t \right)^{\alpha_f} \left( C^n_t \right)^{1-\alpha_f}
\] (6)

where \(\alpha_f\) is the proportion of traded goods. Given the aggregate consumption expenditure constraint,

\[
P_tC_t = P^f_tC^f_t + P^n_tC^n_t
\] (7)

and the definition of the real exchange rate,

\[
Z_t = \frac{P^f_t}{P^n_t}
\] (8)

the following expressions give the demand for traded and non-traded goods as functions of aggregate expenditure and the real exchange rate \(Z\):

\[
C^f_t = \left( \frac{1 - \alpha_f}{\alpha_f} \right)^{-1+\alpha_f} Z_t^{-1+\alpha_f} C_t
\] (9)

\[
C^n_t = \left( \frac{1 - \alpha_f}{\alpha_f} \right)^{\alpha_f} Z_t^{\alpha_f} C_t
\] (10)

\(^2\)Endogenous discounting also allows the model to support equilibria in which credit frictions may remain binding.

\(^3\)Schmitt-Grohé and Uribe (2001) argue that if the reason for introducing endogenous discounting is solely for introducing stationarity, “computational convenience” should be the decisive factor for modifying the standard Uzawa-type model. Kim and Kose (2001) reached similar conclusions.
Similarly, we can express the consumption of traded goods as a composite index of the consumption of export goods, $C^x$, and import goods $C^m$:

$$C^f_t = (C^x_t)^{\alpha_x} (C^m_t)^{1-\alpha_x}$$  \hspace{1cm} (11)

where $\alpha_x$ is the proportion of export goods. The aggregate expenditure constraint for tradeable goods is given by the following expression:

$$P^f_t C^f_t = P^m_t C^m_t + P^x_t C^x_t$$  \hspace{1cm} (12)

where $P^x$ and $P^m$ are the prices of export and import type goods respectively.

Defining the terms of trade index $J$ as:

$$J = \frac{P^x}{P^m}$$  \hspace{1cm} (13)

yields the demand for export and import goods as functions of the aggregate consumption of traded goods as well as the terms of trade index:

$$C^x_t = \left( \frac{1-\alpha_x}{\alpha_x} \right)^{-1+\alpha_x} J_t^{-1+\alpha_x} C^f_t$$  \hspace{1cm} (14)

$$C^m_t = \left( \frac{1-\alpha_x}{\alpha_x} \right)^{\alpha_x} J_t^{\alpha_x} C^f_t$$  \hspace{1cm} (15)

### 2.1.2 Production

Production of exports and imports is by the Cobb-Douglas technology:

$$Y^x_t = A^x_t (K^x_{t-1})^{\theta_x}$$  \hspace{1cm} (16)

$$Y^m_t = A^m_t (K^m_{t-1})^{\theta_m}$$  \hspace{1cm} (17)

where $A^x$, $A^m$ represents the labour factor productivity terms\(^4\) in the production of export and import goods, and $(1-\theta_x)$, $(1-\theta_m)$ are the coefficients of the capital $K^x$ and $K^m$ respectively. The time subscripts $(t-1)$ indicates that they are the beginning-of-period values. The production of non-traded goods, which is usually in services, is given by the labour productivity term, $A^n_t$:

$$Y^n_t = A^n_t$$  \hspace{1cm} (18)

Capital in each sector has the respective depreciation rates, $\delta_x$ and $\delta_m$, and evolves according to the following identities:

$$K^x_t = (1-\delta_x)K^x_{t-1} + I^x_t$$  \hspace{1cm} (19)

$$K^m_t = (1-\delta_m)K^m_{t-1} + I^m_t$$  \hspace{1cm} (20)

where $I^x_t$ and $I^m_t$ represents investment in each sector.

\(^4\)Since the representative agent determines both consumption and production decisions, we have simplified the analysis by abstracting from issues about labour-leisure choice and wage determination.
2.1.3 Budget Constraint

The budget constraint faced by the household/firm representative agent is:

\[
P_tC_t = \Pi_t + S_t \left[ L^*_t - L^*_{t-1} (1 + i^*_{t-1}) \right] - \left[ B_t - B_{t-1} (1 + i_{t-1}) \right] \tag{21}
\]

where \( S \) is the exchange rate (defined as domestic currency per foreign), \( L^*_t \) is foreign debt in foreign currency, and \( B_t \) is domestic debt in domestic currency. Profits \( \Pi \) is defined by the following expression:

\[
\Pi_t = P^x_t \left[ A^x_t \left( K^x_{t-1} \right)^{1-\theta_x} - \frac{\phi^x_t}{2K^x_{t-1}} \left( I^x_t \right)^2 - I^x_t \right] + P^m_t \left[ A^m_t \left( K^m_{t-1} \right)^{1-\theta_m} - \frac{\phi^m_t}{2K^m_{t-1}} \left( I^m_t \right)^2 - I^m_t \right] + P^n_t A^n_t \tag{22}
\]

The aggregate resource constraint shows that the firm faces quadratic adjustment costs when they accumulate capital, with these costs given by the terms \( \frac{\phi^x_t}{2K^x_{t-1}} \left( I^x_t \right)^2 \) and \( \frac{\phi^m_t}{2K^m_{t-1}} \left( I^m_t \right)^2 \).

The household/firm may lend to the domestic government and accumulate bonds \( B \) which pay the nominal interest rate \( i \). They can also borrow internationally and accumulate international debt \( L^* \) at the fixed rate \( i^* \), but this would also include a cost of currency exchange.\(^5\)

The change in bond holdings and foreign debt holdings evolves as follows:

\[
P^n_t G_t = B_{t+1} - B_t (1 + i_t) \tag{23}
\]

\[
(P^x_t X_t - P^m_t M_t) = -S_t \left[ L^*_t - L^*_{t+1} [1 + i^*_t] \right] \tag{24}
\]

2.1.4 Euler Equations

The household/firm optimizes the expected value of the utility of consumption (2) subject to the budget constraint defined in (21) and (22) and the constraints in (19) and (20).

---

\(^5\)The time-varying risk premium is assumed to be zero.
$$\text{Max : } L = E_t \sum_{i=0}^{\infty} \vartheta_{t+i} \{ U(C_{t+i}) \}$$

$$-\Lambda_{t+i} C_{t+i} - \frac{P^{x}_{t+i}}{P_{t+i}} \left( A^{x}_{t+i} (K^{x}_{t-1+i})^{1-\theta_x} - \frac{\phi_x}{2K^{x}_{t-1+i}} (I^{x}_{t-i})^2 - I^{x}_{t+i} \right)$$

$$- \frac{P^{m}_{t+i}}{P_{t+i}} \left( A^{m}_{t+i} (K^{m}_{t-1+i})^{1-\theta_m} - \frac{\phi_m}{2K^{m}_{t-1+i}} (I^{m}_{t-i})^2 - I^{m}_{t+i} \right) - \frac{P^{n}_{t+i}}{P_{t+i}} A^{n}_{t+i}$$

$$- S^{*}_{t+i} \left( i^{*}_{t-i} (1 + i^{*}_{t-1+i}) \right) + \frac{1}{P_{t+i}} \left( B_{t+i} - B_{t-1+i} (1 + i_{t-1+i}) \right)$$

$$- Q^{x}_{t+i} \left[ K^{x}_{t+i} - I^{x}_{t+i} - (1 - \delta_x) K^{x}_{t-1+i} \right]$$

$$- Q^{m}_{t+i} \left[ K^{m}_{t+i} - I^{m}_{t+i} - (1 - \delta_m) K^{m}_{t-1+i} \right]$$

The variable $\Lambda$ is the familiar Lagrangean multiplier representing the marginal utility of wealth. The terms $Q^x$ and $Q^m$, known as Tobin’s Q, represent the Lagrange multipliers for the evolution of capital in each sector - they are the “shadow prices” for new capital. Maximizing the Lagrangean with respect to $C_t$, $L^*_t$, $B_t$, $K^{x}_t$, $K^{m}_t$, $I^x_t$, $I^m_t$ yields the following first order conditions:

$$U'(C_t) - \Lambda_t = 0$$

$$\vartheta_t \Lambda_t \frac{S_t}{P_t} - E_t \left[ \vartheta_{t+1} \Lambda_{t+1} \frac{S_{t+1}}{P_{t+1}} (1 + i^*_t) \right] = 0$$

$$- \vartheta_t \Lambda_t \frac{1}{P_t} + E_t \vartheta_{t+1} \Lambda_{t+1} \frac{1}{P_{t+1}} (1 + i_t) = 0$$

$$\left[ - \vartheta_t Q^x_t \right] + E_t \vartheta_{t+1} A^x_{t+1} \frac{P^x_{t+1}}{P_{t+1}} \left[ A^x_{t+1} (1 - \theta_x)(K^x_t)^{-\theta_x} + \frac{\phi_x(I^x_{t+1})^2}{2(K^x_t)^2} \right] = 0$$

$$\left[ - \vartheta_t Q^m_t \right] + E_t \vartheta_{t+1} A^m_{t+1} \frac{P^m_{t+1}}{P_{t+1}} \left[ A^m_{t+1} (1 - \theta_m)(K^m_t)^{-\theta_m} + \frac{\phi_m(I^m_{t+1})^2}{2(K^m_t)^2} \right] = 0$$

$$- \vartheta_t \Lambda_t \frac{P^{x}_{t}}{P_{t}} \left( \frac{\phi_x I^{x}_{t}}{K^{x}_{t-1}} + 1 \right) + \vartheta_t Q^{x}_{t} = 0$$

$$- \vartheta_t \Lambda_t \frac{P^{m}_{t}}{P_{t}} \left( \frac{\phi_m I^{m}_{t}}{K^{m}_{t-1}} + 1 \right) + \vartheta_t Q^{m}_{t} = 0$$
These equations can then be re-expressed as:

\[ \Lambda_t = U'(C_t) \]  

\[ \vartheta_t U''(C_t) = E_t \vartheta_{t+1} U'(C_{t+1})(1 + i_t - \pi_{t+1}) \]  

\[ E_t(s_{t+1} - s_t) = i_t - i_t^* \]

\[ \left[ -E_t \vartheta_{t+1} Q_t^x (1 - \delta_x) \right] = E_t \vartheta_{t+1} \Lambda_{t+1} \frac{P_{t+1}}{P_t} \left[ A_{t+1}^x (1 - \theta_x) (K_{t+1}^x)^{-\theta_x} \right. \]

\[ \left. + \frac{\phi_x (I_{t+1}^x)^2}{2(K_{t+1}^x)^{\gamma_x}} \right] \]  

\[ \left[ -E_t \vartheta_{t+1} Q_t^m (1 - \delta_m) \right] = E_t \vartheta_{t+1} \Lambda_{t+1} \frac{P_{t+1}}{P_t} \left[ A_{t+1}^m (1 - \theta_m) (K_{t+1}^m)^{-\theta_m} \right. \]

\[ \left. + \frac{\phi_m (I_{t+1}^m)^2}{2(K_{t+1}^m)^{\gamma_m}} \right] \]  

\[ I_t^x = \frac{1}{\phi_x} \left( \frac{P_t Q_t^x}{\Lambda_t} - 1 \right) K_{t-1}^x \]  

\[ I_t^m = \frac{1}{\phi_m} \left( \frac{P_t Q_t^m}{\Lambda_t} - 1 \right) K_{t-1}^m \]

where \( \Delta p_{t+1} = \log(P_{t+1}/P_t) \) is the per period inflation, \( s \) is the logarithm of the nominal exchange rate \( S \) and \((E_t s_{t+1} - s_t)\) is the expected rate of exchange rate depreciation.

Equation (26) is the typical Euler equation for consumption. Using the utility function in (1) yields the consumption function:

\[ C_t = E_t \left[ (1 + i_t - \pi_{t+1}) \vartheta_{t+1} C_{t+1}^{\gamma} \right]^{-\frac{1}{\gamma}} \]

which shows how current consumption depends on expectations of future values. Equation (27) describes the interest arbitrage condition and the forward-looking behavior of the exchange rate.

The above equations (28) and (29) also show that the solutions for \( Q_t^x \) and \( Q_t^m \), which determine investment and the evolution of capital in each sector, come from forward-looking stochastic Euler equations. The shadow price or replacement value of capital in each sector is equal to the discounted value of next period’s marginal productivity, the adjustment costs due to the new capital stock, and the expected replacement value net of depreciation.

Thus the model has four “forward-looking” stochastic Euler equations, which determine \( C_t, s_t, Q_t^x, Q_t^m \). These variables, together with (25), in turn
determine current investment $I^x_t$ and $I^m_t$ as described by the conditions in (30) and (31).

The Euler equation for investment, (30) and (31), shows clearly the importance of $Q$ in the economy. We note that when $\frac{P^x_t}{Q^x_t} > 1$ the economy builds up capital in the export-good sector and when $\frac{P^x_t}{Q^x_t} < 1$ it runs down capital. For a small open economy, with no constraints to international borrowing, "excessive" growth in investment leads to a build-up of foreign debt. Hence, it might be desirable to react to $Q$ and to use the policy instrument $i$ to control $P$ so that investment $I$ is kept within reasonable bounds.

2.1.5 Relative prices, exchange rate pass-through and stickiness

There are 7 prices (absolute and relative) to be determined ($P^x, P^m, J, Z, P^f_t, P^n_t, P$). The price of export goods is determined exogenously for a small open economy ($P^x$) and its price in domestic currency is $P^x = SP^x$. The price of import goods is also determined exogenously for a small open economy $P^m$, but, we assume that price changes are incompletely passed-through (see Campa and Goldberg (2002) for a study on exchange rate pass-through and import prices). Using the definition: $P^m = SP^m$, and assuming partial adjustment, we obtain:

$$p^m_t = \omega(s_t + p^{mx}_t) + (1 - \omega)p^m_{t-1}$$

where $\omega = 1$ indicates complete pass-through of foreign price changes.

Thus, given $P^x$ and $P^m$, we have $J = P^x/P^m$, and:

$$P^f_t = [(\alpha_x)^{-\alpha_x} (1 - \alpha_x)^{-1+\alpha_x}] (P^x_t)^{\alpha_x} (P^m_t)^{1-\alpha_x}$$

Finally, we obtain the aggregate consumption price deflator as:

$$P_t = [(\alpha_f)^{-\alpha_f} (1 - \alpha_f)^{-1+\alpha_f}] (P^f_t)^{\alpha_f} (P^n_t)^{1-\alpha_f}$$

2.1.6 Macroeconomic Conditions And Market Clearing

The national accounting equation is:

$$P^x_t \left( Y^x_t - \frac{\phi_x}{2K^x_t} (I^x_t)^2 \right) + P^m_t \left( Y^m_t - \frac{\phi_m}{2K^m_t} (I^m_t)^2 \right) + P^n_t Y^n_t$$

= $P^x_t (C^x_t + X^x_t + I^x_t) + P^m_t (C^m_t - M_t + I^m_t) + P^n_t (C^n_t + G_t)$

= $P_tC_t + (P^x_t I^x_t + P^m_t I^m_t) + (P^x_t X_t - P^m_t M_t) + P^n_t G_t$
Real gross domestic product is given as:
\[ y = \frac{1}{P_t} \left[ P_t^\pi \left( Y_t^\pi - \frac{\phi_\pi}{2K_t^\pi} (I_t^\pi)^2 \right) + P_t^m \left( Y_t^m - \frac{\phi_m}{2K_t^m} (I_t^m)^2 \right) + P_t^n Y_t^n \right] \] (36)

### 2.2 Monetary Authority

The Central Bank adopts practices consistent with optimal control models, specifically, the linear quadratic regulator problem. It chooses an optimal interest rate reaction function, given its loss function equation, and its perception of the evolution of the state variables, inflation and growth. The change in the interest rate is the solution of the optimal linear quadratic regulator problem, with control variable \( \Delta i \) solved as a feedback response to the lagged state variables.

We assume, perhaps more realistically, that the monetary authority does not know the exact nature of the private sector model, instead it “learns” and updates the state-space model equation, which underpins its calculation of the optimal interest rate policy period by period. In other words, at each period time \( t \), the Central Bank updates its information about the evolution of inflation and growth, and re-estimates the state-space system to obtain new estimates. The central bank then uses this information to determine the optimal interest rate.

- **Pure Inflation targeting**

In the pure inflation target case, the monetary authority estimates or “learns” the evolution of inflation as a function of its own lag as well as of changes in the interest rate.

\[ \Lambda_1 = \lambda_{1t} (\pi_t - \pi^*)^2 \] (37)
\[ \pi_t = \sum_{j=0}^{k} \Gamma_{1t,j} \pi_{t-j-1} + \Gamma_{2t} \Delta i_t + e_t \] (38)
\[ i_{t+1} = i_t + \sum_{j=0}^{k} h(\hat{\Gamma}_{1t,j}, \hat{\Gamma}_{2t}, \lambda_{1t}) \pi_{t-j-1} \] (39)

where \( \pi_t = \log(P_t/P_{t-4}) \), an annualized rate of inflation, \( \pi^* \) is the target for inflation, and \( k \) is the number of lags for forecasting the evolution of the state variable. The feedback function \( h \) is obtained by solving the linear quadratic regulator problem, as discussed in Sargent (1999).

The weight on the loss function, \( \lambda_t = \{\lambda_{1t}\} \) are chosen to reflect the Central Bank’s concerns about inflation and is shown in Table I.
In this pure anti-inflation scenario, if inflation is less than the target level $\pi^*$, the central bank does not optimize; in other words, the interest rate remains at its level: $i_{t+1} = i_t$. This is the “no intervention” case. However, if inflation is above the target rate, the monetary authority implements its optimal interest policy according to equation (39).

- Inflation and Q-Targets

In this policy scenario, the Central Bank practises inflation targeting as well as Q-growth targeting, where $Q_t$ is the average of $Q^x_t$ and $Q^m_t$. In this case, the monetary authority estimates or “learns” the evolution of inflation and Q-growth as a function of its own lag as well as of changes in the interest rate.

$$\Lambda_2 = \lambda_{1t}(\pi_t - \pi^*)^2 + \lambda_{2t}(\eta_t - \eta^*)^2$$

$$x_t = \sum_{j=0}^{k} \Gamma_{1t,j} \pi_{t-j-1} + \sum_{j=0}^{k} \Gamma_{2t,j} \eta_{t-j-1} + \Gamma_{3t} \Delta i_t + e_t$$

$$i_{t+1} = i_t + \sum_{j=0}^{k} h(t_{1t,j}, t_{2t,j}, t_{3t}, \lambda_{1t}, \lambda_{2t}) x_{t-j-1}$$

where $n_t = \log(Q_t/Q_{t-4})$, the annualized rate of growth of $Q$, and $\eta^*$ represents the target for Q-growth. In this case, we have a bivariate forecasting model for the evolution of the state variables, $\pi_t$ and $\eta_t$, with an equal number of lags; that is, the coefficient matrix $\Gamma_{1t,j}$, for $k$ lags contains two ($k \times 1$) recursively updated matrix coefficients, representing the effects of lagged inflation and growth on current inflation and growth.

<table>
<thead>
<tr>
<th>Table I: Policy Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inflation Only Target</strong></td>
</tr>
<tr>
<td>$\pi \leq \pi^*$</td>
</tr>
<tr>
<td>$\pi &gt; \pi^*$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table II: Policy Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inflation Targeting and Q reactions</strong></td>
</tr>
<tr>
<td>$Q$</td>
</tr>
<tr>
<td>$\pi \leq \pi^*$</td>
</tr>
<tr>
<td>$\lambda_2 = 0.9$</td>
</tr>
<tr>
<td>$\pi &gt; \pi^*$</td>
</tr>
<tr>
<td>$\lambda_2 = 0.5$</td>
</tr>
</tbody>
</table>
The weights for inflation and output growth in the loss function depend on the conditions at time $t$. The weights reflecting the Central Bank’s preference for inflation and growth in this policy scenario are summarized in Table II. In this second policy scenario, if inflation is below the target level $\pi^*$ and $Q$-growth below the target $\eta^*$, the Central Bank does not change the policy interest rate. When inflation is below target, but $Q$-growth is above target, the monetary authority puts greater weight on $Q$-growth than on inflation. In contrast, when inflation is above target and growth is below target, the central bank puts strong weight on the inflation target. Finally, if inflation is above its target and growth is above its target, the weights are set equally at 0.5.

Thus, corresponding to each scenario, the Central Bank optimizes a loss function $\Lambda$ and actively formulates its optimal interest-rate feedback rule. It also acts at time $t$ as if its estimated model for the evolution of inflation and output growth is true “forever”, and that its relative weights for inflation, or growth in the loss function are permanently fixed.

However, as Sargent (1999) points out in a similar model, the monetary authority’s own procedure for re-estimation “falsifies” this pretense as it updates the coefficients $\{\Gamma_{1t}, \Gamma_{2t}, \Gamma_{3t}\}$, and solves the linear quadratic regulator problem for a new optimal response “rule” of the interest rate to the evolution of the state variables at every point of time $t$.

### 3 Calibration and Solution Algorithm

In this section we discuss the calibration of parameters, initial conditions, and stochastic processes for the exogenous variables of the model. We then summarize the parameterized expectations algorithm (PEA) used for solving the model.

#### 3.1 Parameters and Initial Conditions

The parameter settings for the model appear in Table III.

<table>
<thead>
<tr>
<th>Table III: Calibrated Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption $\gamma = 1.5$, $\beta = 0.009$</td>
</tr>
<tr>
<td>$\alpha_x = 0.5$, $\alpha_f = 0.5$</td>
</tr>
<tr>
<td>Production $\theta_m = 0.7$, $\theta_x = 0.3$</td>
</tr>
<tr>
<td>$\delta_x = \delta_m = 0.025$, $\phi_x = \phi_m = 0.03$</td>
</tr>
</tbody>
</table>
Many of the parameter selections follow Mendoza (1995, 2001). The constant relative risk aversion $\gamma$ is set at 1.5 (to allow for high interest sensitivity). The shares of non-traded goods in overall consumption is set at 0.5, while the shares of exports and imports in traded goods consumption is 50 percent each. Production in the export goods sector is more capital intensive than in the import goods sector.

The initial values of the nominal exchange rate, the price of non-tradeables and the price of importable and exportable goods are normalized at unity while the initial values for the stock of capital and financial assets (domestic and foreign debt) are selected so that they are compatible with the implied steady state value of consumption, $\overline{C} = 2.02$, which is given by the interest rate and the endogenous discount factor. The values of $\overline{C}^x$, $\overline{C}^m$, and $\overline{C}^n$ were calculated on the basis of the preference parameters in the sub-utility functions and the initial values of $B$ and $L^*$ deduced.

Similarly, the initial shadow price of capital for each sector is set at its steady state value. The production function coefficients $A^m$ and $A^x$, along with the initial values of capital for each sector, are chosen to ensure that the marginal product of capital in each sector is equal to the real interest plus depreciation, while the level of production meets demand in each sector. Since the focus of the study is on the effects of terms of trade shocks, the domestic productivity coefficients were fixed for all the simulations.

Finally, the foreign interest rate $i^*$ is also fixed at the annual rate of 0.04. In the simulations, the effect of initialization is mitigated by discarding the first 100 simulated values.

### 3.2 Terms of Trade Shocks

The only shocks explored in this paper comes from the terms of trade. Specifically:

\[
\begin{align*}
\tilde{p}_t^x &= \tilde{p}^x_{t-1} + \varepsilon_t^{x}; \quad \varepsilon_t^{x} \sim N(0, 0.01) \\
\tilde{p}_t^m &= \tilde{p}^m_{t-1} + \varepsilon_t^{m*}; \quad \varepsilon_t^{m*} \sim N(0, 0.01)
\end{align*}
\]

where lower case denotes the logs of the respective prices. The evolution of the prices mimic actual data generating processes, namely that the variable is a unit-root autoregressive process, with a normally distributed innovation with standard deviation set at 0.01. The errors are assumed to be independent at this stage.

The simulations are also conducted assuming that the domestic price of export goods fully reflect the exogenously determined prices:

\[
\tilde{p}_t^x = s_t + \tilde{p}^x_t
\]
however, the domestic price of import goods are partially passed on:

\[ p_t^m = \omega(s_t + p_t^{m*}) + (1 - \omega)p^m_{t-1} \]  \hspace{1cm} (44)

where \( \omega \) is the coefficient of exchange rate pass-through, here set at 0.4.

Thus, this is a simulation study about the design of monetary policy for an economy subjected to relative price shocks. The log of the terms of trade \( (j) \) and the aggregate consumption price deflator \( (p) \) becomes respectively:

\[ j = s_t + p_t^{x*} - \omega(s_t + p_t^{m*}) - (1 - \omega)p^m_{t-1} \]

\[ p_t = \alpha_f \left[ \alpha_x (s_t + p_t^{x*}) + (1 - \alpha_x) \left[ \omega(s_t + p_t^{m*}) + (1 - \omega)p^m_{t-1} \right] \right] \]
\[ - \alpha_x \ln(\alpha_x) + (\alpha_x - 1) \ln(1 - \alpha_x) \]
\[ + (1 - \alpha_f)p_t^f - \alpha_f \ln(\alpha_f) + (\alpha_f - 1) \ln(1 - \alpha_f) \]

In our model, there are only two sources of stickiness, one from incomplete pass through, and the other from the Central Bank learning. The only noise comes from the terms of trade shocks. The rate of growth of \( Q \) captures the shadow price of installed capital and does not represent a "misalignment" in any sense.

### 3.3 Solution Algorithm and Constraints

Following Marcet (1988, 1993), Den Haan and Marcet (1990, 1994), and Duffy and McNelis (2001), the approach of this study is to parameterize the forward-looking expectations in this model, with non-linear functional forms \( \psi^s, \psi^c, \psi^Q^r \), and \( \psi^Q^f \):
\[ \partial_t U'(C_t) = \psi^C(x_{t-1}; \Omega_C) \]
\[ = E_t \partial_{t+1} U'(C_{t+1})(1 + i_t - \pi_{t+1}) \]  

(45)

\[ s_t = \psi^S(x_{t-1}; \Omega_S) \]
\[ = (i_t - i_t^*) - E_t(s_{t+1}) \]

(46)

\[ \partial_t Q_t^x = \psi^{Qx}(x_{t-1}; \Omega_Q^x) \]
\[ = E_t \partial_{t+1} \left[ \Lambda_{t+1} \frac{P_{x_{t+1}}}{\pi_{t+1}} \left( A_{t+1}(1 - \theta_x)(K_x^t)^{-\theta_x} + \frac{\phi_x (P_{x_{t+1}})^2}{2(K_x^t)^2} \right) + Q_{t+1}^x(1 - \delta_x) \right] \]

(47)

\[ \partial_t Q_t^m = \psi^{Qm}(x_{t-1}; \Omega_Q^m) \]
\[ = E_t \partial_{t+1} \left[ \Lambda_{t+1} \frac{P_{m_{t+1}}}{\pi_{t+1}} \left[ A_{t+1}(1 - \theta_m)(K_m^t)^{-\theta_m} + \frac{\phi_m (P_{m_{t+1}})^2}{2(K_m^t)^2} \right] + Q_{t+1}^m(1 - \delta_m) \right] \]

(48)

The symbol \( x_{t-1} \) represents a vector of observable instrumental variables known at time \( t \) — the variables are: consumption of import \( C^m \) and export goods \( C^x \), the marginal utility of consumption \( \lambda \), the real interest rate \( r \), the real exchange rate, \( Z \), and the shadow prices of replacement capital for the two sectors, \( Q^m \) and \( Q^x \), all expressed in deviations from the initial steady state:

\[ x_{t-1} = \{ C^m - C^m_{0}, C^x - C^x_{0}, \lambda - \lambda_0, r - r_0, Z - Z_0, Q^m - Q^m_0, Q^x - Q^x_0 \} \]

(49)

The symbols \( \Omega_\lambda, \Omega_S, \Omega_Q^x, \) and \( \Omega_Q^m \) represent the parameters for the expectation function, while \( \psi^C, \psi^S, \psi^{Qx} \) and \( \psi^{Qm} \) are the expectation approximation functions.

Judd (1996) classifies this approach as a “projection” or a “weighted residual” method for solving functional equations, and notes that the approach was originally developed by Williams and Wright (1982, 1984, 1991). The functional forms for \( \psi^S, \psi^C, \psi^{Qx}, \) and \( \psi^{Qm} \) are usually second-order polynomial expansions [see, for example, Den Haan and Marcet (1994)]. However, Duffy and McNelis (2001) have shown that neural networks can produce results with greater accuracy for the same number of parameters, or equal accuracy with fewer parameters, than the second-order polynomial approximation.

The model was simulated for repeated parameter values for \{\( \Omega_C, \Omega_S, \Omega_Q^x, \Omega_Q^m \}\} and convergence obtained when the expectational errors were
minimized. In the algorithm, the following non-negativity constraints for consumption and the stocks of capital were imposed:

\[ C_t^x > 0, \quad K_t^x > 0, \quad K_t^m > 0 \]  

The latter was achieved by assuming irreversible investment for capital in each sector, that is for \( i = X, M \):

\[
I_t^i = \begin{cases} 
\frac{1}{\phi_i} \left( \frac{P_t Q_t^i}{P_t N_t^i} - 1 \right) K_{t-1}^i & \text{if } \frac{P_t Q_t^i}{P_t N_t^i} > 1 \\
0 & \text{otherwise}
\end{cases}
\]  

The usual no-Ponzi game applies to the evolution of real government debt and foreign assets, namely:

\[
\lim_{t \to \infty} B_t \exp^{-it} = 0, \quad \lim_{t \to \infty} L_t^* \exp^{-(i^* + \Delta \kappa_{t+1})t} = 0
\]  

We keep the foreign asset or foreign debt to GDP ratio bounded, and thus fulfill the transversality condition, by imposing the following constraints on the parameterized expectations algorithm:\footnote{See Pacharoni (2003) for a suggestion for solving investment equations under parameterized expectations with irreversibility restrictions.}

\[
\left( \frac{|S_t L_t^*|}{y_t} \right) < \bar{L}, \quad \left( \frac{|B_t|}{y_t} \right) < \bar{B}
\]  

where \( \bar{L} \) and \( \bar{B} \) are the critical foreign and domestic debt ratios. In the simulation, the fiscal authority will exact lump sum taxes from non-traded goods sector in order to run a surplus and “buy back” domestic debt if it grows above a critical foreign or domestic debt/GDP ratio.

4 Simulation Analysis

4.1 Impulse-Response Analysis

Before proceeding to the full stochastic simulations, we first examine some dynamic properties of the model with impulse-response analysis. The model was set at its steady-state initial conditions and we assume that the monetary authorities simply set the interest rate at the steady state international interest rate.

\footnote{In the PEA algorithm, the error function will be penalized if the foreign debt/gdp ratio is violated. Thus, the coefficients for the optimal decision rules will yield debt/gdp ratios which are well belows levels at which the constraint becomes binding.}
The model was then subjected to a one-period only terms of trade shock. This was achieved by letting the process for the world price of exports, $p^x_t$, increase by one standard deviation ($p^x_t = \overline{p}^x + 0.01, t = T^*$) and then letting it revert back to its steady-state value thereafter ($p^x_t = \overline{p}^x, t \neq T^*$); the price for imported goods, $p^m_t$, remains at its steady-state value ($p^m_t = \overline{p}^m$). In effect, the terms of trades jumps at $T^*$ from its steady-state value of 1.00 to 1.0101 and then reverts back to its steady state value.

The effects of the shock on the economy accords with standard theory. Overall, there will be no real change and the effects of the temporary terms of trade shock are dissipated within four quarters. Figure 1 pictures the adjustment for three key variables, real exchange rate depreciation, the current account, and the rate of growth of Tobin’s Q. The variables oscillate reflecting the reversal effects of the temporary shock. The strongest effects take place after the shock in period 1, but the system stabilises quickly.

Consistent with Mendoza (1995) empirical findings, we see that terms
of trade effects have positive effects both on the current account and on the real exchange rate. [Mendoza (1995): p. 102]. However, the terms of trade increase also leads to an increase in the value of Q (see the Euler equation for the export goods which highlights the role of the export price on future marginal productivities). This result provides yet another reason for monitoring Q - changes in Q are indicators of developments in the current account, when economies are driven by terms of trade shocks.

4.2 Base-Line Results

The aim of the simulations, of course, is to compare the outcome for inflation, growth and welfare for the two policy scenarios - inflation targeting (π) and inflation and Q-growth targeting (π and η). To ensure that the results are robust, we conducted 1000 simulations (each containing a time-series of realizations of terms of trade shocks).

Figure 1 shows the simulated paths for one time series realization of the exogenous terms of trade index. This particular realization of the terms of trade shocks describes the case when there are improvements (upward trend) and deteriorations (downward trend), but where there are no export booms (almost all values are below one).

The simulated values for the key variables (inflation, consumption, investment, current account) are well-behaved. Figure 2 presents the evolution of these variables for the two scenarios. In general, despite the large swings in the terms of trade index, consumption is remarkably stable. But the introduction of Q-growth targeting for the monetary authority results in a lower level of consumption and investment. And more interestingly, the current
Figure 3: Time Series
account exhibits less fluctuations from surplus to deficits.

To ascertain which policy regime yields the higher welfare value, we examined the distribution of the welfare outcomes of the different policy regimes for 1000 different realizations of the terms of trade shocks. Before presenting these results, we evaluated the accuracy of the simulation results as well as the "rationality" of the learning mechanism.

4.3 Den Haan-Marcet Accuracy Test

The accuracy of the simulations is checked by the Den Haan-Marcet statistic, originally developed for the parameterized expectations solution algorithm but applicable to other procedures as well. This test makes use of the Euler equation for consumption, under the assumption that with accurate expectations, the path of consumption would be optimal, so that expectational term in the Euler equation may be replaced by the actual term and a random error term, $\nu_t$:

$$\theta_{t+1} \Lambda_{t+1} \frac{1}{P_{t+1}} (1 + i_t) - \theta_t \Lambda_t \frac{1}{P_t} = \nu_t$$

To test whether $\nu_t$ is significantly different from zero, Den Haan and Marcet propose a transformation of $\nu_t$ which has a chi-squared distribution under the hypothesis of accuracy. If the value of this statistic belongs to the upper or lower critical region of the chi-squared distribution, Den Haan and Marcet suggest that this is evidence "against the accuracy of the solution". [Den Haan and Marcet (1994): p. 5].

Table IV presents the percentage of realizations (out of 1000) in which the Den Haan-Marcet statistics fell in the upper or lower critical regions of the chi-squared distribution, for each policy regime.

<table>
<thead>
<tr>
<th>Policy Regime</th>
<th>Percentage in Upper/Lower Critical Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation Targeting</td>
<td>0.045/0.061</td>
</tr>
<tr>
<td>Inflation/Q Growth Targeting</td>
<td>0.045/0.020</td>
</tr>
</tbody>
</table>

4.4 Learning and Quasi-Rationality

In our model, the central bank learns the underlying process for inflation in the pure inflation-target regime and the underlying processes for inflation
and growth in the inflation-growth target regime. The learning takes place by updating recursively the least-squares estimates of a vector autoregressive model. Learning is thus a source of stickiness in the model.

Marcet and Nicolini (1997) raise the issue of reasonable rationality requirements in their discussion of recurrent hyperinflations and learning behavior. In our model, a similar issue arises. Given that the only shocks in the model are recurring terms of trade shocks, with no abrupt, unexpected structural changes taking place, neither to the stochastic shock process nor to the deep parameters of the model, the learning behavior of the central bank should not depart, for too long, from the rational expectations paths. The central bank, after a certain period of time, should develop forecasts which converge to the true inflation and growth paths of the economy, unless we wish to make some special assumption about monetary authority behavior.

Marcet and Nicolini discuss the concepts of “asymptotic rationality”, “epsilon-delta rationality” and “internal consistency”, as criteria for “boundedly rational” solutions. They draw attention to the work of Bray and Savin (1996). These authors examine whether the learning model rejects serially uncorrelated prediction errors between the learning model and the rational expectations solution, with the use of the Durbin-Watson statistic. Marcet and Nicolini point out that the Bray-Savin method carries the flavor of “epsilon-delta” rationality in the sense that it requires that the learning schemes be consistent “even along the transition” [Marcet and Nicolini (1997): p.16, footnote 22].

Following Bray and Savin, we use the Durbin-Watson statistic to examine whether the learning behavior is “boundedly rational”. Table V gives the Durbin-Watson statistics for the inflation and Q-growth forecast errors of the central bank, under both policy regimes. In the majority of cases, we see that the learning behavior does not violate the requirements of bounded rationality for inflation. However, there is evidence that learning is not boundedly rational for Q-growth, since the implied evolution of Q-inflation is not easily captured by our vector-autoregressive learning model.

<table>
<thead>
<tr>
<th>Table V: Durbin-Watson Statistics for Forecast Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage in Lower and Upper Critical Region</td>
</tr>
<tr>
<td>Policy Regime</td>
</tr>
<tr>
<td>--------------------------------</td>
</tr>
<tr>
<td>Inflation Targeting</td>
</tr>
<tr>
<td>Inflation/ Q-Growth Targeting</td>
</tr>
</tbody>
</table>
4.5 Comparative Welfare Results

This section summarizes the results for 1000 alternate realizations of the terms-of-trade shocks (each realization contains 150 observations). Table VI presents the first two moments of the 1000 sample means for consumption, inflation, growth, the changes in the policy instrument - the interest rate - and the intertemporal welfare index (based on the discounted utility function).\(^8\)

Figure 3 presents the kernel estimates of the distribution of the sample means from each of the 1000 realizations for inflation, consumption, investment and the relative welfare measure. The solid lines are for the case with inflation targeting, while the dashed lines are for the inflation and Q-growth targeting scenarios. The distributions show rather sharply that managing \(Q\) and inflation does not have much effect on inflation, while it reduces considerably the volatility of consumption (and hence welfare). These results show that the "gain" from including Q-growth as a monetary target lies in the reduction of volatility.

<table>
<thead>
<tr>
<th>Table VI: Summary Statistics (1000 Simulations)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First and Second Moments (in Parenthesis) of the sample means</td>
</tr>
<tr>
<td>Policy Regimes</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>Consumption ((c))</td>
</tr>
<tr>
<td>Investment ((inv))</td>
</tr>
<tr>
<td>Inflation Rate ((\pi)\text{%})</td>
</tr>
<tr>
<td>GDP Growth Rate ((\eta)\text{%})</td>
</tr>
<tr>
<td>Welfare ((W))</td>
</tr>
</tbody>
</table>

5 Concluding Remarks

This paper has shown that there are clear trade-offs when the rate of growth of Tobin’s Q is incorporated as an additional target to inflation targeting in the conduct of monetary policy. Our results show that the Central Bank can achieve a great deal of success if it adopts this additional target, for

\(^8\)We do not benchmark the welfare effects with respect to the steady state welfare, since the terms of trade realizations may lead to welfare outcomes either greater or less than the steady state welfare.

23
Figure 4: Distributions
reducing Q volatility while maintaining about the same degree of inflation volatility. However, across a range of realizations of terms of trade shocks, the welfare effects, measured in terms of the present value of consumption utility, are lower when Q-growth targets are incorporated with the inflation targets. Overall, the trade-off for monetary policy is clear: either a slightly higher mean welfare and high volatility or lower mean welfare and lower volatility.

The addition of Q-growth targets thus reduces the volatility of the welfare measure. Compared to a strict inflation targeting regime, monetary policy with inflation and Q-growth targets can marginally insulate an economy from adverse terms-of-trade shocks since growth and welfare measures do not fall as much when negative shocks are realized.

To be sure, we did not introduce "shocks" in this model in the form of asset price bubbles, or misalignments of the targeting share price from the fundamental Q value. We assumed that the driving force for Q growth comes from fundamentals. Given that the central bank has to learn the laws of motion of Q-growth as well as inflation, and set policy on the basis of longer-term laws of motion of these variables, it seems reasonable to start with Q driven solely by fundamentals. We leave to further research an examination of the robustness of our results to the incorporation of bubbles and other non-fundamental asset-price shocks.
References


