Contagion of Currency Crises across Unrelated Countries without Common Lender

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Abstract

I construct a micro-model to show that a currency crisis can spread from one country to another even when these countries are unrelated in terms of economic fundamentals and there is no capital linkage across countries through a common lender or an interbank market. The key to explaining contagious currency crises in the model lies in each speculator’s private information and learning behavior about other speculators’ type. Since the payoff of each speculator depends on other speculators’ behavior determined by their types, each speculator’s behavior depends on her belief about other speculators’ types. If a currency crisis in one country reveals the speculators’ types to some degree, it leads to an updating of each speculator’s belief about other speculators’ types and therefore a change in her optimal behavior, which in turn can cause a currency crisis even in another unrelated country without capital linkage. Although the presence of contagion itself is not new in the literature, there is an important implication difference between the literature and this paper. The model shows that the crisis with better economic fundamentals can be more contagious than that with worse economic fundamentals; this has not been shown in the literature. This is because the former conveys information about types of speculators while the latter does not. Even if country B does not suffer from a contagious crisis due to bad economic fundamentals from country A, it does not necessarily mean that it will never suffer contagion from some other country with better economic fundamentals than country A.

JEL classifications: F31; E58; D82; C72
Keywords: Contagion; Currency Crises; Global Game

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1 Introduction

The last decade has witnessed many contagious financial crises. They begin locally, in some region, country, or institution, and subsequently spread elsewhere. Example of these include the Mexican *Tequila* effect in 1994, the Asian *Flu* in 1997, the Russian *Virus* in 1998, the Brazilian *Sneeze* in 1999, and so on.

Following these detrimental events, recent international policy issues have revolved around questions on how to stop, mitigate, or prevent contagion. In order to answer these questions, it is important to clarify and pin down possible channels through which a financial crisis spreads from one country to another. This necessity has stimulated a significant body of research on possible channels of contagion. The literature is growing and our understanding is becoming better, but it is still unclear why a financial crisis in one country spreads to another even when these countries appear to be *unrelated* in terms of economic fundamentals. As an example, in the aftermath of the 1998 devaluation of the Russian ruble, the Brazilian stock market fell by over 50% and the sovereign spreads of Brazil rose sharply even though Russia and Brazil are considered to be countries that do not share correlated fundamental shocks. Could this be just a coincidence? To the best of my knowledge, all the empirical papers argue that it is not. The empirical literature agrees that the Brazilian stock market was indeed negatively affected by the Russian Virus. But it is not clear why the Russian Virus hit Brazil. Russia and Brazil have virtually no direct trade links; the two countries do not export similar goods which compete on third markets; they are hardly similar in terms of either economic situation or economic structure, and they have few direct financial links. These facts raise several important questions. Can contagion occur across countries that do not have any trade linkage and are not similar?

In the literature, only one channel of contagion across seemingly *unrelated* countries, without any direct or indirect trade linkage, has been explored so far: capital linkage across countries (or regions) through a common lender and/or an interbank market. The logic of capital linkage as the contagion channel is the following. If

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1. It is important to note that there is no universally accepted definition of contagion. Here I mean by contagion the negative influence of the crisis on other countries' financial variables such as the exchange rate, the stock index, sovereign spreads, and so on. I will give the definition of it in Section 2 in much more precise way. See Forbes and Rigobon (2000, 2002) for a further discussion of the definition of contagion.
2. The term *economic fundamentals* is sometimes ambiguous. Here I follow the common usage of the term in the literature of contagion: economic fundamentals are said to be *related* if there is either trade linkage or some similarity across countries.
4. If they trade each other, there is direct trade linkage. If they compete in trades through a third market, there is indirect trade linkage. I mean both by trade linkage.
the currency crisis occurs in a country, international investors and/or international banks would incur losses. Facing losses due to the crisis, they may pull out their money or call in their loans to re-balance their portfolio or meet the capital ratio requirement, not only from the originating crisis country but also from seemingly unrelated countries. As a result, their withdrawal may cause contagion. From the viewpoint of unrelated countries, they share international investors and/or international banks with the originating crisis country as the common lender. That is why this hypothesis is referred as the hypothesis of the common lender. Moreover, if the common lender incurs huge losses or goes bankrupt due to the crisis, this negative effect can spread to other investors or banks through the interbank market, which in turn can amplify contagion.

Although the logic of capital linkage is quite reasonable, unfortunately the empirical evidences are mixed.\(^6\) This is because such capital linkage can be not only the contagion channel but also a buffer against contagion. For example, suppose that the international bank, as the common lender, has accumulated plenty of loan loss reserves prior to the currency crisis. If the currency crisis in one country is not so significant for the international bank in that it can be dealt just within the existing loan loss reserves, the international bank does not need to call in loans from other countries to offset the loss of the currency crisis. Therefore, the negative effect of the crisis will not be transmitted to other countries by the international bank with enough loan loss reserves. Moreover, if the international bank provides liquidity to countries in the turmoil of crisis, it is the buffer against the negative shock of the crisis rather than the contagion channel. In fact, Baig and Goldfajn (2000) and Goldfajn (2000) point out that German banks, which had huge exposure to Russia and were badly affected by the Russian Virus, had a rollover rate far above the average in Brazil because they had set aside higher provisions for bad loans than other banks.\(^7\) This means that German banks, as the common lender, worked as the buffer in the turmoil of the Russian Virus, compared with other banks. So they conclude that the hypothesis of the common lender as the contagion channel does not find support from the data during the Russian Virus.\(^8\) By the same token, if the interbank market functions well, again it will be the buffer against the negative shock of the crisis by channelling liquidity to financial institutions in need of it.

\(^6\)Van Rijckeghem and Weder (2001) argue that there is an evidence in favor of the common lender effect in the Russian Virus.

\(^7\)It is noteworthy that German banks received a large number of guarantees on their Russian loans from the state-supported export-guarantee agency Hermes, which may have lessened their losses compared to other banks. Thus not only the provision of German banks for bad loans but also the guarantee might help German banks serve as the buffer against contagion.

\(^8\)Van Rijckeghem and Weder (2001) exclude German banks from the sample in their estimation because of the reason stated in footnote 7. They do not report whether or not there remains the evidence in favor of the common lender effect, even after introducing German banks in the sample. Moreover, they report in another paper (Van Rijckeghem and Weder (2000)), which excludes German banks from the sample as in Van Rijckeghem and Weder (2001), that they do not find the evidence in favor of the common lender effect in the Russian Virus when they use more disaggregated data of bank flows than in Van Rijckeghem and Weder (2001).
Indeed, Furfine (2002) argues the federal funds market performed well during the Russian Virus to mitigate liquidity shortage of financial institutions. These papers suggest that capital linkage is not a panacea to explain contagion across unrelated countries and there may be another contagion channel that has not yet been found. Thus it is still worthwhile to explore possible channels of contagion across unrelated countries.

This paper aims at complementing the growing literature by proposing a new channel of contagion. I show that the currency crisis can be contagious across unrelated countries even when there is no capital linkage among them. I consider a situation where economic fundamentals of countries are unrelated and there is no capital linkage through the common lender or the interbank market. None of the contagion channels in the literature work in this situation.

Needless to say, the contagion channels, including the one proposed in this paper, can be complementary. In other words, they can work at the same time in the contagious crisis. Therefore the contagious effect of the crisis will be amplified if several channels work simultaneously. The result of this paper then suggests that the contagious effect can be larger than the existing literature would anticipate.

There are two common aspects in the literature in considering contagious currency crisis: the extent of the negative shock of the crisis and who is hurt by it. The reasoning behind much of this literature can be classified into one of the following two categories. First, if economic fundamentals are related across countries, the negative shock of the crisis in one country will hurt related countries directly, resulting in contagion. Second, if economic fundamentals are not related, the negative shock will hurt the common lender who in turn causes contagion across unrelated countries. This paper departs from these two common aspects, which enables us to find a new channel of contagion: the channel arises because of each speculator’s private information and learning behavior about other speculators’ types. Since the payoff of each speculator depends on other speculators’ behavior in relation to their type, the optimal behavior of each speculator depends on her belief about other speculators’ types. If the currency crisis in one country reveals speculators’ types to some degree, it will lead her to update her beliefs and thereby change her optimal behavior, which in turn leads to contagion. In this channel, the negative shock per se does not matter for contagion and no speculator is hurt. Therefore, this channel can work even when economic fundamentals of each country are independent, there is no capital linkage across countries, and no speculator is negatively affected by the crisis.

While the existence of contagion itself is not new in the literature, there is an important implication difference of contagion between the literature and this paper. The implication of the literature is that the crisis with worse economic fundamentals will be more contagious than that with better economic fundamentals. But this

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9 Thus we need to interpret the finding of Van Rijckeghem and Weder (2001) with caution, besides the reason in footnote 8. This is because they do not take into account the possible role of the interbank market as the buffer against contagion.

10 As will be shown in the model, speculators do not lose money but earn profits in crises.
model shows that the crisis with better economic fundamentals can be more contagious than that with worse economic fundamentals. The literature implies that if country B does not suffer from a contagious crisis due to bad economic fundamentals from country A, then it will never suffer contagion from some other country with better economic fundamentals than country A. This is because the literature relies on the relationship of economic fundamentals or the common lender to explain contagion. The worse economic fundamentals in country A are, the more severely the related countries or the common lender is hurt. Thus the worse economic fundamentals in country A are, the more contagious the crisis in country A becomes. But according to the findings of this paper, the opposite could be true. This is because the crisis in country A with better economic fundamentals will convey more information regarding the types of speculators. If economic fundamentals are quite bad, the currency crisis will occur for sure irrespective of the type of speculators. So the currency crisis does not convey any information of the type. Therefore, nobody would update her belief about others’ type. But if the currency crisis happens in the country whose economic fundamentals are considered to be good, it reveals the type of speculators to some degree. This is because the crisis would happen in this case if and only if speculators are of a certain type. Therefore, the crisis makes speculators update their belief about others’ type and thereby change the behavior, which can lead to contagion. It implies that even if country B could get rid of the contagion of the crisis, it does not necessarily mean that it will not be vulnerable to contagion of crises that happens in the country with better economic fundamentals. This seems counterintuitive, but it is reasonable in fact. If the currency crisis happens in the country whose economic fundamentals are good, everybody will be surprised, which can make the crisis more contagious.

In the theoretical literature of currency crises, the closest papers to this paper are Morris and Shin (1998) and Metz (2002). They show how to derive the unique equilibrium in the models of currency crises where there are multiple equilibria. However, these papers are not concerned with the problem of contagion whereas the issue of contagion is central to this one.

This paper is organized as follows. In Section 2, the model is described. In Section 3, the empirical implications are considered. Section 4 concludes.

2 The Model

There are two countries, country A and country B. The government of each country pegs the currency at some level. The economy in each country is characterized by a state of underlying economic fundamentals, $\theta_j$ ($j = A, B$). A high value of $\theta_j$ refers to good fundamentals while a low values refers to bad fundamentals. I assume $\theta_j$ is randomly drawn from the real line, with each realization equally likely. Also, there is no linkage of economic fundamentals between country A and country B: $\theta_A$ and $\theta_B$ are independent. That is, there is no direct trade or financial linkage, no indirect trade linkage through a third market, or no indirect capital linkage through
Table 1: Payoff Matrix

<table>
<thead>
<tr>
<th></th>
<th>Success</th>
<th>Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attack</td>
<td>$D - t - \mu_1$</td>
<td>$-t - \mu_1$</td>
</tr>
<tr>
<td>Not Attack</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

a common lender or an interbank market.

There are two groups of speculators, group 1 and group 2. 11 Both groups consist of a continuum of small speculators, so that an individual speculator’s stake is negligible as a proportion of the whole. I index the set of speculators by the unit interval $[0, 1]$. There are two possible types of Group 1 with respect to aggressiveness: one type is the “bull” with probability $q$ while another type is the “chicken” with probability $1 - q$. That is, all the speculators in group 2 are bull (chicken) with probability $q (1 - q)$. I assume group 1’s type is their private information. Group 1’s type is always bull and is public information, which is just for simplicity. The size of group 1 is $\lambda$ while that of group 2 is $1 - \lambda$, where $0 \leq \lambda \leq 1$.

Receiving the possibly noisy private signal about economic fundamentals, a speculator decides whether to short-sell the currency, i.e., attack the currency peg, or not. I envisage the short-selling as consisting of borrowing the domestic currency and selling it for dollars. If the attack is successful (i.e., the peg is abandoned), she gets a fixed payoff $D (> 0)$. Attacking the currency, however, also leads to a cost $t + \mu_1 (> 0)$. The cost $t$ can be viewed largely as consisting of the interest rate differential between the domestic currency and dollars, plus the transaction cost. If a speculator refrains from attacking the currency, she is not exposed to any cost but she does not gain anything either. (See Table 1.) $\mu_1$ captures a difference of aggressiveness between the bull and the chicken, as specified below. To make the model interesting, I assume that successful attack is profitable for any speculator.

**Assumption 1** $D - t - \mu_1 > 0$.

I assume that the government defends the currency peg if the cost of this action is not too high. The cost of defending the peg depends on two factors: the proportion of speculators attacking the currency peg of country $j$, $l_j$, and the economic fundamentals of country $j$, $\theta_j$. I assume the cost is increasing in $l_j$ and decreasing in $\theta_j$. The intuition behind this is as follows. If, for instance, speculative pressure is very high (i.e., $l_j$ is so large), the government may need to increase interest rates quite sharply in order to defend the peg, which will be detrimental to the country. Thus the cost of defending is increasing in $l_j$. But if economic fundamentals are good, the government may have plenty of foreign reserves to defend the peg so that it may not have to raise the interest rates. It means the negative effect of defending

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11 Here I construct a model as the speculators game in order to make the new contagion channel at work as clear as possible: I exclude any other channels from the model. But the new channel can be embedded easily into other frameworks such as the common lender models. See the Appendix.
the peg on the country will be relatively mild. Therefore the cost of defending is decreasing in \( \theta_j \). More specifically, I assume the net cost of defending the peg is \( l_j - \theta_j \). The government’s optimization is as follows.

\textbf{Assumption 2 (Government’s Optimization)} The government defends the peg if \( l_j - \theta_j < 0 \). It abandons the peg if \( l_j - \theta_j \geq 0 \).

In what follows, I call \textit{crisis} if the government abandons the peg and \textit{no crisis} if the government defends the peg.

As regards speculators’ preferences, the expected utility of attacking the currency of the country is the following.

\[ U = \begin{cases} \text{Prob [Attack is successful]} D - t - \mu_1 & \text{if a speculator is the chicken} \\ \text{Prob [Attack is successful]} D - t & \text{if a speculator is the bull} \end{cases} \]

Since attacking the currency may fail depending on the government’s action or other speculators’ action (i.e., it will fail if \( l_j - \theta_j < 0 \)), it is a riskier action than not-attacking. Thus assume the chicken speculator feels some disutility when she takes the risky action. The term \( \mu_1(\geq 0) \) captures the disutility.

On the other hand, it is safe to refrain from attacking the currency in that the payoff is surely zero irrespective of the government’s action or other speculators’ action. Therefore, the utility of not attacking the currency is zero both for the chicken speculator and the bull speculator.

As will be shown, it is critical in this paper that the \textit{private} signal of economic fundamentals is \textit{noisy} to speculators, in deriving the unique equilibrium. The intuition behind the \textit{noisy private} information is that the relevant information in deciding whether or not to attack is not always accurate. Sometimes the information may have measurement error simply. Sometimes the government may announce the wrong information intentionally, trying to discourage speculators to attack.\(^{12}\) It in turn implies that there is some room for small discrepancies among speculators as to how the information is interpreted. In reality, it is also common that each speculator does not know exactly either which information other speculators have or how they interpret the information. This means that the signal is \textit{private} at least to some degree. The \textit{noisy private} signal in the model captures the above idea. Let \( x_{ji} \) be the speculator \( i \)'s private signal about economic fundamentals about country \( j \). I assume the property of \( x_{ji} \) as follows.

\textbf{Assumption 3 (Noisy Private Signal)}

\textit{When the true state is} \( \theta_j \), \textit{a speculator} \( i \) \textit{observes a signal} \( x_{ji} \) \textit{which is drawn uniformly from the interval} \([\theta_j - \epsilon, \theta_j + \epsilon]\), \textit{for some small} \( \epsilon \). \textit{Conditional on} \( \theta_j \), \textit{the signals are identical and independent across individuals}.

\(^{12}\)For example, the Bank of Korea announced that its international reserves were more than $30 billion in face of the Thai Baht collapse in the 1997 Asian currency crisis. The announcement was intended to restore foreigners’ confidence about Korean economy. But it turned out that the actual reserves the Bank of Korea could use in the crisis were considerably less than its announced official reserves. See ?).
Assumption 3 states the following. The signal is *private* and *noisy* if $\epsilon > 0$ and the noise is uniformly distributed. The noise in the private signal is independent of $\theta_j$ and across speculators. Moreover, the noise in the signal of country A is independent of the noise in the signal of country B.

The timing of the game among governments and speculators is structured as follows.

- **Period 1**
  - Nature chooses each value of $\theta_A$ and $\theta_B$ *independently*, as well as the type of group 1. Group 1 is chosen to be the bull with probability $q$ or the chicken with probability $1 - q$ ($0 < q < 1$). The value of $\theta_j$ is known to the government of country $j$. Group 1’s type is known to every speculator in group 1, but it is not known to any speculator in group 2.
  - Each speculator receives a *private* signal $x_{Ai} = \theta_A + \epsilon_{Ai}$.
  - Each speculator decides whether to attack the currency of country A or not individually.
  - The government of country A abandons the peg if and only if $l_A - \theta_A \geq 0$. It defends the peg otherwise.
  - Both the aggregate outcome in country A and the value of $\theta_A$ are known to every speculators. If the attack is successful, those who attacked get $D - t - \mu_1$. If the attack is not successful, their payoff is $-t - \mu_1$. The payoff of those who did not attack is zero irrespective of the result of attack.

- **Period 2**
  - Each speculator receives a private signal $x_{Bi} = \theta_B + \epsilon_{Bi}$.
  - Each speculator decides whether to attack the currency of country B or not individually.
  - The government of country B abandons the peg if and only if $l_B - \theta_B \geq 0$. It defends the peg otherwise.
  - Both the aggregate outcome in country B and the value of $\theta_B$ are known to every speculators. If the attack is successful, those who attacked get $D - t - \mu_1$. If the attack is not successful, their payoff is $-t - \mu_1$. The payoff of those who did not attack is zero irrespective of the result of attack. (See Figure 1)

For simplicity, I assume $\mu_1$ is some positive constant $\mu$ if a speculator is the chicken while it is zero if a speculator is the bull.

**Assumption 4**

$$\mu_1 = \begin{cases} 
\epsilon \mu & \text{if a speculator is the chicken.} \\
0 & \text{if a speculator is the bull.} 
\end{cases}$$
Nature chooses the type of group 1

$\theta_j$ are realized

$x_Ai$ is observed

Decides whether to attack country A

The aggregate outcome in country A is realized and $\theta_A$ is known to all speculators

$x_Bi$ is observed

Decides whether to attack country B

The aggregate outcome in country B is realized and $\theta_B$ is known to all speculators

Figure 1: Timing of the Game

From Assumptions 2 and 4, the expected utility of attacking the currency can be rewritten as follows.

$$U = \begin{cases} 
\text{Prob} [l_j \geq \theta_j] D - t - \mu & \text{if a speculator is the chicken} \\
\text{Prob} [l_j \geq \theta_j] D - t & \text{if a speculator is the bull}
\end{cases}$$

In sum, the information structure of the model in period 1 is the following, in period 1\textsuperscript{13} The government of country $j$ knows the value of $\theta_j$. Any speculator in group 1 knows his own type, the type of speculators in group 2 ($\mu_1 = 0$ for any speculator in group 2), the government’s optimization rule and his own private signal. Group 1’s type is private information. Each speculator in group 2 knows her own type, the probability that group 1 is chicken (bull) $q$ $(1 - q)$, the government’s optimization rule and her own private signal. In order to derive the model’s equilibrium, it is crucial to correctly define which elements of the game are common knowledge in period 1. These are the payoff $D$, the cost $t$, the value of $\mu_1$ ($\mu$ or 0), the distribution of the noise and its parameter $\epsilon$, the probability that group 1 is chicken, the type of speculators in group 2 and the government’s optimization rule. The value of $\theta_j$ is common knowledge if and only if $\epsilon = 0$. The intuition behind the assumption that group 1’s type is private information is the following. Assume that group 1 consists of offshore funds registered in so-called tax heavens and therefore regulation on them is less stringent than that of industrialized countries. Since information about such offshore funds is not typically open to the public, it is hard to figure out how much risk they can take. Therefore, some uncertainty exists about group 1’s attitude for risk. That is why group 1’s type is private information.

In Subsection 2.1, I show that there are multiple equilibria with self-fulfilling beliefs in the case $\epsilon = 0$ where the economic fundamentals are common knowledge. In Subsection 2.2, I show that we can obtain the unique equilibrium in the case $\epsilon > 0$ where the economic fundamentals are not common knowledge. Also, I show that a

\textsuperscript{13}I will argue in Subsection 2.2 that the information structure may change in period 2, depending upon the aggregate outcome in country A. This will be the key to explain contagion in the model, as shown later.
currency crisis in country A can spread to country B through the new channel of contagion: the learning of speculators about others’ type.

2.1 Multiple Equilibria

First assume \( \epsilon = 0 \). In this case, \( \theta_j \) becomes common knowledge because everyone receives the same signal. This is a situation where each speculator knows exactly what everyone else knows in deciding whether or not to attack the currency. It is similar to the situation investigated by Obstfeld (1986, 1996). Here I show there are multiple equilibria à la Obstfeld under certain conditions.

Clearly, the currency peg is stable if \( \theta_j > 1 \). This is because the net cost of defending the peg is always negative irrespective of the speculators’ actions. Thus the government always defends the peg. On the other hand, the peg is unstable if \( \theta_j \leq 0 \), since the government always abandon the peg regardless of speculators’ actions. If \( 0 < \theta_j \leq 1 \), the peg is said to be ripe for attack. In this region, the government is forced to abandon the peg if a sufficient proportion of speculators attacks the currency, whereas the peg will be kept if a sufficient proportion of speculators chooses not to attack. (See Figure 2) Therefore we have the following proposition.

Proposition 1 (Multiple Equilibria)

(i) If \( \theta_j > 1 \), the unique equilibrium is that no speculator attacks and the currency peg is sustained (No-Crisis Equilibrium).

(ii) If \( \theta_j \leq 0 \), the unique equilibrium is that every speculator attacks and the currency peg is abandoned (Crisis Equilibrium).

(iii) If \( 0 < \theta_j \leq 1 \), there are two possible equilibria: No-Crisis Equilibrium and Crisis Equilibrium.

The speculators will attack the currency only if they believe in success and will refrain from attacking otherwise. Thus in any of the equilibria, their actions
vindicate the initial beliefs. In other words, the equilibria have the self-fulfilling property.

In the ripe-for-attack region, we naturally come up with an important question. Out of two possible equilibria, which will indeed arise? How is one equilibrium selected over another? The typical approach in the literature is just to assume that it is determined by sunspot. In this approach, the important question is in effect unanswered since sunspot itself is given exogenously. More importantly from my perspective, this approach is silent on the issue of contagion because there is no equilibrium selection mechanism in the multiple equilibria model. I show in the next subsection that we can obtain the unique equilibrium even in the ripe-for-attack region, by relaxing the assumption of common knowledge of economic fundamentals (i.e., $\epsilon > 0$).

2.2 Unique Equilibrium

In Subsection 2.1, I have derived multiple equilibria. The multiplicity is due to strategic complementarities among speculators. In general, it is quite common that there are multiple equilibria in models of strategic complementarities. A major drawback of these multiple equilibria models is the ambiguity of possible outcomes, as the sole prediction to be made is that there exists a range of fundamentals for which a crisis is possible but does not have to occur necessarily. Because of the self-fulfilling property of the equilibrium, beliefs of speculators will determine the outcome. But beliefs themselves are indeterminate within the models. Consequently, it is hard to generate comparative statistics and therefore policy alternatives can hardly be derived.

Carlson and van Damme (1993) are the first to show that the indeterminacy of beliefs in multiple equilibria models is an artifact of the simplifying common knowledge assumption. This assumption is meant only to simplify the analysis in the literature but in fact delivers more than intended; non-common knowledge can generate a unique equilibrium in such class of models under certain conditions. Morris and Shin (1998) applies it to derive the unique equilibrium in the multiple equilibria model of currency crises à la Obstfeld (1986, 1996). I follow Morris and Shin’s approach. One of the contributions of this paper is to show a new contagion channel whereas their papers are not concerned with contagion. In the new channel, it is critical that economic fundamentals are not common knowledge, there are two possible types of speculators, the type of group 1 is not common knowledge in period 1, and each type behaves differently in period 1 under certain circumstance. The different behavior contingent on the type may reveal their type to some degree and

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14As regards other applications of non-common knowledge to obtain the unique equilibrium in various settings that have multiple equilibria under common knowledge, see an important series of works by Morris and Shin (1999, 2000, 7, 2001, 2002, 2003a, 2003b). However, non-common knowledge is not a panacea to obtain the unique equilibrium. There can be multiple equilibria under certain conditions even when we assume non-common knowledge. See Metz (2002), Chan and Chiu (2002) and Morris and Shin (2003b).
thereby speculators in group 2 update beliefs about the type of group 1. It in turn leads to a change in the behavior of speculators in group 2 in period 2, resulting in contagion across unrelated countries without any common lender.

Now assume $\epsilon > 0$. In this case, $\theta_j$ is no longer common knowledge because the signal about $\theta_j$ is noisy and private.\(^{15}\) This is a much more realistic situation where each speculator does not know exactly what everyone else knows in deciding whether or not to attack, as opposed to in the situation of the previous subsection.

Following Morris and Shin (1998) and Metz (2002), I concentrate on the switching strategy equilibrium. The switching strategy equilibrium consists of the following values conditional on the choice of nature and the information structure: a unique value of the economic fundamentals $\tilde{\theta}_j$ up to which the government always abandons the peg, and a unique value of the private signal conditional on the type of speculators $\tilde{x}_{ji}(\mu_1)$, such that every speculator who receives signal lower than $\tilde{x}_{ji}(\mu_1)$ attacks the currency peg.

The general intuition behind this equilibrium is the following: conditional on the information structure, there is a unique switching fundamental value of $\tilde{\theta}_j$ below which the government always abandons the currency peg. The unique value $\tilde{\theta}_j$ generates a distribution of private signals, such that there is exactly one switching signal $\tilde{x}_{ji}(\mu_1)$, for each type respectively, below which the speculator always attack the currency. The switching signal $\tilde{x}_{ji}(\mu_1)$ would make a speculator receiving it indifferent between attacking and not-attacking. If all speculators with signals smaller than $\tilde{x}_{ji}(\mu_1)$ decide to attack, the distribution of private signals, generated by $\tilde{\theta}_j$, would in turn generate a proportion $l_j = \tilde{\theta}_j$ of attackers that will be sufficient to force a devaluation of the currency peg.

Let $Y_A = I\{\tilde{\theta}_{A} \geq \theta_{A}\}$ where $I\{\bullet\}$ is the indicator function whose value is unity if the argument is true and zero otherwise. Thus $Y_A = 1$ if and only if the currency crisis happens in country A. Using $\tilde{\theta}_j$ and $Y_A$, I define contagion as follows.

**Definition 1** There is contagion if and only if $\tilde{\theta}_{B}(Y_A = 1) > \tilde{\theta}_{B}(Y_A = 0)$.

Definition 1 captures contagion precisely, in that it can distinguish contagion of crises from just a coincidence of crises. To see this, first pick any $\theta_B$ such that $\theta_B(Y_A = 0) < \theta_B < \tilde{\theta}_{B}(Y_A = 1)$. Given such $\theta_B$, the currency crisis happens in country B if and only if the crisis happens in country A. This is exactly contagion. Next, pick any $\theta_B$ such that $\theta_B < \tilde{\theta}_{B}(Y_A = 0)$. In this case, the currency crisis happens in country irrespective of the occurrence of the crisis in country A. Therefore, it is just a coincidence in the latter case if the currency crises happen in both countries.

I derive the unique equilibrium in this two-period game in the following way. First I derive the unique equilibrium in the one-shot game in each period separately. Next I show that the unique equilibrium for both periods is indeed the subgame-perfect equilibrium in the two-period game.\(^{16}\)

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\(^{15}\)In the context of industry investment, Caplin and Leahy (1994) explore a model to explain the timing of market crashes in which the private noisy signal plays the key role, while they are not concerned with contagion.

\(^{16}\)Morris and Shin (1999) show that if there is the unique value of $\tilde{x}(\mu_1)$, then the switching
2.2.1 The Equilibrium in Country A

The equilibrium values $\tilde{\theta}_j$ and $\tilde{x}_{ji}(\mu_1)$ belong to two situations of indifference: for $\theta_j = \tilde{\theta}_j$ the government is indifferent between defending the peg and abandoning it, whereas speculators with $\mu_1$ receiving a private signal $\tilde{x}_{ji}(\mu_1)$ are indifferent between attacking the peg and refraining from doing so. I consider the symmetric switching strategy equilibrium in which every speculator of the same type uses the same switching value: $\tilde{x}_{ji}(\mu_1) = \tilde{x}_j(\mu_1)$. Thus in what follows, I omit the subscript $i$.

After receiving the private signal, each speculator has to decide whether to attack the peg, which leads to costs of $t + \mu_1$ and an uncertain payoff of $D$, or not to attack the peg which is associated with a net profit of zero with certainty. The switching signal, $\tilde{x}_A(\mu_1)$, makes each speculator of type $\mu_j$ indifferent between these two choices. For indifference it is thus required that:

$$\text{Prob} \left[ \text{Attack is successful} \mid \tilde{x}_A(\mu_1) \right] D - t - \mu_1 = 0 \quad (1)$$

The government will abandon the peg if $\theta$ is smaller than or equal to $\tilde{\theta}_A$, so that the probability of a successful attack equals the probability that the realized value of $\theta$ is smaller than or equal to $\tilde{\theta}_A$, given $\tilde{x}_A(\mu_1)$. As will be shown, $\tilde{\theta}_A$ takes different values depending on whether speculators in group 1 are the bull or the chicken. So denote $\tilde{\theta}_A(\mu_1) \equiv \tilde{\theta}^*_A$ if speculators in group 1 are bull (i.e., $\mu_1 = 0$) and $\tilde{\theta}_A(\mu_1) \equiv \tilde{\theta}^{**}_A$ if speculators in group 1 are chicken (i.e., $\mu_1 = \mu$).

First, consider the behavior of speculators in group 1. They know their own type and the type of speculators in group 2, so that they know which value of $\tilde{\theta}_A(\mu_1)$, $\tilde{\theta}^*_A$ or $\tilde{\theta}^{**}_A$, would appear in the equilibrium. Noting $x_A = \theta_A + \epsilon_A$ and $\epsilon_A \sim U[-\epsilon, \epsilon]$, (1) can be rewritten as follows.

$$t + \mu_1 = \text{Prob} \left[ \text{Attack is successful} \mid \tilde{x}_{A1}(\mu_1), \tilde{\theta}_A(\mu_1) \right] D$$

$$= \text{Prob} \left[ \theta_A \leq \tilde{\theta}_A(\mu_1) \mid \tilde{x}_{A1}(\mu_1), \tilde{\theta}_A(\mu_1) \right] D$$

$$= \text{Prob} \left[ \tilde{x}_{A1}(\mu_1) - \epsilon_A \leq \tilde{\theta}_A(\mu_1) \mid \tilde{x}_{A1}(\mu_1), \tilde{\theta}_A(\mu_1) \right] D$$

$$= \text{Prob} \left[ \tilde{x}_{A1}(\mu_1) - \tilde{\theta}_A(\mu_1) \leq \epsilon_A \mid \tilde{x}_{A1}(\mu_1), \tilde{\theta}_A(\mu_1) \right] D$$

$$= \left\{ 1 - \text{Prob} \left[ \epsilon_A \leq \tilde{x}_{A1}(\mu_1) - \tilde{\theta}_A(\mu_1) \mid \tilde{x}_{A1}(\mu_1), \tilde{\theta}_A(\mu_1) \right] \right\} D$$

$$= \left\{ 1 - \frac{\tilde{x}_{A1}(\mu_1) - \tilde{\theta}_A(\mu_1)}{2\epsilon} \right\} D \quad (2)$$

strategy with $\tilde{x}(\mu_1)$ is the only equilibrium strategy that survives the iterated deletion of dominated strategy, i.e., there cannot be any other form of the equilibrium strategy. They prove it in the one-shot game where there is only one type of speculators. I am working to check whether it holds in the two-stage game of this paper where there are two types of speculators, but I have not completed yet. It is an interesting topic for future research.
Here the subscript of $\tilde{x}_{A1}$ stands for country A and group 1. In the symmetric equilibrium, every speculator in group 1 uses the same switching private signal. From (2), the switching signals $\tilde{x}_{A1} \equiv \tilde{x}_{A1} (\mu_1 = 0)$ and $\tilde{x}_{A1}^{**} \equiv \tilde{x}_{A1} (\mu_1 = \mu)$ have to satisfy the following.

\[
\begin{align*}
\frac{\tilde{x}_{A1}^* - \tilde{\theta}_A^*}{2\epsilon} &= \frac{\tilde{x}_{A1} (\mu_1 = 0) - \tilde{\theta}_A (\mu_1 = 0)}{2\epsilon} = 1 - \frac{t}{D} \quad (3) \\
\frac{\tilde{x}_{A1}^{**} - \tilde{\theta}_A^{**}}{2\epsilon} &= \frac{\tilde{x}_{A1} (\mu_1 = \mu) - \tilde{\theta}_A (\mu_1 = \mu)}{2\epsilon} = 1 - \frac{t + \mu}{D} \quad (4)
\end{align*}
\]

Second, consider the behavior of speculators in group 2. They know their own type, but they do not know the type of speculators in group 1. Hence they do not know which value of $\tilde{\theta}_A (\mu_1)$ would appear in the equilibrium. Let $q$ be their belief that speculators in group 1 are bull. (1) can be rewritten as follows.

\[
t = \text{Prob}[\text{Attack is successful} \mid \tilde{x}_{A2}] D \\
= \text{Prob}[\text{Attack is successful when } \mu_1 = 0 \mid \tilde{x}_{A2}] D \\
+ \text{Prob}[\text{Attack is successful when } \mu_1 = \epsilon \mu \mid \tilde{x}_{A2}] D \\
= q \times \text{Prob} \left[ \theta_A \leq \tilde{\theta}_A \mid \tilde{x}_{A2} \right] D + (1-q) \times \text{Prob} \left[ \theta_A \leq \tilde{\theta}_A^{**} \mid \tilde{x}_{A2} \right] D \\
= q \times \text{Prob} \left[ \tilde{x}_{A2} - \epsilon \Lambda \leq \tilde{\theta}_A \mid \tilde{x}_{A2} \right] D \\
+ (1-q) \times \text{Prob} \left[ \tilde{x}_{A2} - \epsilon \Lambda \leq \tilde{\theta}_A^{**} \mid \tilde{x}_{A2} \right] D \\
= q \times \left\{ 1 - \text{Prob} \left[ \epsilon \Lambda \leq \tilde{x}_{A2} - \tilde{\theta}_A^{**} \mid \tilde{x}_{A1} \right] \right\} D \\
+ (1-q) \times \left\{ 1 - \text{Prob} \left[ \epsilon \Lambda \leq \tilde{x}_{A2} - \tilde{\theta}_A^{**} \mid \tilde{x}_{A1} \right] \right\} D \\
= q \times \left\{ 1 - \frac{\tilde{x}_{A2} - \tilde{\theta}_A^{**}}{2\epsilon} \right\} D + (1-q) \times \left\{ 1 - \frac{\tilde{x}_{A2} - \tilde{\theta}_A^{**}}{2\epsilon} \right\} D \quad (5)
\]

Therefore, conditional on $q$, $\tilde{x}_{A2}$ has to satisfy the following.

\[
\frac{\tilde{x}_{A2} - q \tilde{\theta}_A^* - (1-q) \tilde{\theta}_A^{**}}{2\epsilon} = 1 - \frac{t}{D} \quad (6)
\]

Third, consider the behavior of the government. The government is indifferent between defending the currency peg and abandoning it, if the proportion of speculators attacking the peg $l_A = \theta_A$. The proportion of attacking speculators is equal to the proportion of speculators who get a private signal smaller than or equal to $\tilde{x}_{Aj} (\mu_1)$ $(j = 1, 2)$. However, notice that $\epsilon_A$ is assumed to be independent of the true value of $\theta_A$, there is a continuum of speculators in group 1 and group 2, the
size of group 1 is \( \lambda \), and the size of group 2 is \( 1 - \lambda \). This means that this proportion corresponds to the weighted average of the probability with which one single speculator in group 1 observes a signal smaller than or equal to \( \tilde{x}_{A1} (\mu_1) \) and the probability with which one single speculator in group 2 observes a signal smaller than or equal to \( \tilde{x}_{A2} \). Thus the proportion of attacking speculators conditional on \( \theta_A \) is the following.

\[
l_A (\theta_A) = \lambda \text{Prob} \left[ x_{A1} \leq \tilde{x}_{A1} (\mu_1) \mid \theta_A \right] + (1 - \lambda) \text{Prob} \left[ x_{A2} \leq \tilde{x}_{A2} \mid \theta_A \right]
\]

\[
= \lambda \text{Prob} \left[ \theta_A - \epsilon_A \leq \tilde{x}_{A1} (\mu_1) \mid \theta_A \right] + (1 - \lambda) \text{Prob} \left[ \theta_A - \epsilon_A \leq \tilde{x}_{A2} \mid \theta_A \right]
\]

\[
= \lambda \frac{\tilde{x}_{A1} (\mu_1) - \theta_A}{2\epsilon} + (1 - \lambda) \frac{\tilde{x}_{A2} - \theta_A}{2\epsilon}
\]

(7)

In RHS of the first line, the first term is the proportion of attacking speculators in group 1 while the second term is the proportion of attacking speculators in group 2. From (7) and the definition of \( \tilde{\theta}_A (\mu_1) \), we get the following.

\[
\tilde{\theta}_A (\mu_1) = l_A \left( \tilde{\theta}_A (\mu_1) \right) = \lambda \frac{\tilde{x}_{A1} (\mu_1) - \tilde{\theta}_A (\mu_1)}{2\epsilon} + (1 - \lambda) \frac{\tilde{x}_{A2} - \tilde{\theta}_A (\mu_1)}{2\epsilon}
\]

Therefore, \( \tilde{\theta}^*_A \) and \( \tilde{\theta}^{**}_A \) can be described as follows.

\[
\tilde{\theta}^*_A \equiv \tilde{\theta}_A (\mu_1 = 0) = \lambda \frac{\tilde{x}_{A1}^* - \tilde{\theta}^*_A}{2\epsilon} + (1 - \lambda) \frac{\tilde{x}_{A2} - \tilde{\theta}^*_A}{2\epsilon}
\]

(8)

\[
\tilde{\theta}^{**}_A \equiv \tilde{\theta}_A (\mu_1 = \mu) = \lambda \frac{\tilde{x}_{A1}^{**} - \tilde{\theta}^{**}_A}{2\epsilon} + (1 - \lambda) \frac{\tilde{x}_{A2} - \tilde{\theta}^{**}_A}{2\epsilon}
\]

(9)

There are five equations, (3), (4), (6), (8), and (9) and there are five unknowns, \( \tilde{x}_{A1}^*, \tilde{x}_{A1}^{**}, \tilde{x}_{A2}, \tilde{\theta}^*_A \) and \( \tilde{\theta}^{**}_A \). Solving these five equations for five unknowns, we get the following.

\[
\tilde{\theta}^*_A = \frac{1}{q} \left[ 1 - \frac{t}{D} - \frac{\mu}{D} \lambda (1 - q) - \frac{(1 - q)(1 - \lambda)}{2\epsilon + (1 - \lambda)} \left\{ \frac{2\epsilon}{1 - \lambda} + 1 - \frac{t}{D} \right\} \right.
\]

\[
- \frac{\mu}{D} \lambda (1 - q) - \frac{1}{2\epsilon + (1 - \lambda)} \left\{ \frac{2\epsilon}{1 - \lambda} + 1 - \frac{t}{D} \right\}
\]

\[
\tilde{\theta}^{**}_A = \frac{1 - \lambda}{2\epsilon + (1 - \lambda)} \left[ \frac{2\epsilon}{1 - \lambda} + 1 - \frac{t}{D} - \frac{\mu}{D} \lambda (1 - q) - \frac{1}{2\epsilon + (1 - \lambda)} \left\{ \frac{2\epsilon}{1 - \lambda} + 1 - \frac{t}{D} \right\} \right.
\]

\[
- \frac{2\epsilon \lambda}{1 - \lambda D} \mu \right]
\]

(10)

\[
\tilde{x}_{A1} = \frac{1}{q} \left[ 1 - \frac{t}{D} - \frac{\mu}{D} \lambda (1 - q) - \frac{(1 - q)(1 - \lambda)}{2\epsilon + (1 - \lambda)} \left\{ \frac{2\epsilon}{1 - \lambda} + 1 - \frac{t}{D} \right\} \right.
\]

\[
- \frac{\mu}{D} \lambda (1 - q) - \frac{1}{2\epsilon + (1 - \lambda)} \left\{ \frac{2\epsilon}{1 - \lambda} + 1 - \frac{t}{D} \right\}
\]

\[
\tilde{x}_{A2} = \frac{2\epsilon}{1 - \lambda D} \mu + 2\epsilon (1 - \frac{t + \mu}{D})
\]

(12)

\[
\tilde{x}_{A1}^{**} = \frac{1 - \lambda}{2\epsilon + (1 - \lambda)} \left[ \frac{2\epsilon}{1 - \lambda} + 1 - \frac{t}{D} - \frac{\mu}{D} \lambda (1 - q) - \frac{1}{2\epsilon + (1 - \lambda)} \left\{ \frac{2\epsilon}{1 - \lambda} + 1 - \frac{t}{D} \right\} \right.
\]

\[
- \frac{2\epsilon \lambda}{1 - \lambda D} \mu \right]
\]

(13)
\[ \tilde{x}_{A2} = 2\epsilon + 1 - \frac{t}{D} - \frac{\mu}{D} \lambda (1-q) - \frac{2\epsilon t}{D} \]  

(14)

By construction, switching private signals makes each speculator in each group indifferent between attacking the currency peg and refraining from doing so respectively, conditional on \( \tilde{\theta}^*_A \) and \( \tilde{\theta}^{**}_A \). To show that these values indeed consist of the switching strategy equilibrium, it needs to be verified that every speculator in each group strictly prefers to attack (refrain from) the currency peg given any private signal less than (greater than) the switching private signals conditional on \( \tilde{\theta}^*_A \) and \( \tilde{\theta}^{**}_A \). In order to verify this, it is sufficient to check if an individual speculator in each group finds it optimal to follow the switching strategy provided that every other speculator follows the switching strategy. So suppose every other speculator follows the switching strategy. Then an individual speculator in each group takes \( \tilde{\theta}^*_A \) and \( \tilde{\theta}^{**}_A \) as given.

From (2) and (5), the expected utility of attacking the currency peg is strictly decreasing in the private signal given \( \tilde{\theta}^*_A \) and \( \tilde{\theta}^{**}_A \). This is because the probability that an attack is successful is strictly decreasing in the private signal given \( \tilde{\theta}^*_A \) and \( \tilde{\theta}^{**}_A \). Therefore, for any private signal less than (greater than) the switching signal, the expected utility of attacking the currency peg is strictly greater than (smaller than) that of refraining from doing so. Thus it is optimal for any speculator to follow the switching strategy provided that everyone else follows the switching strategy. Moreover, it can be shown that \( \tilde{\theta}^*_A > \tilde{\theta}^{**}_A \). Intuitively speaking, the bull speculators are more likely to attack than the chicken speculators for any given private signal. It means the bull speculators are more likely attack for any given economic fundamentals, which in turn implies that the currency peg is more likely to be abandoned if speculators in group 1 are bull. That is why \( \tilde{\theta}^*_A > \tilde{\theta}^{**}_A \). Proposition 2 summarizes the above argument.

**Proposition 2 (Unique Equilibrium in Country A)**

Conditional on the type of speculators in group 1, the unique switching strategy equilibrium in country A consists of the switching private signal \( \tilde{x}_A(\mu_1) \) and the switching economic fundamentals \( \tilde{\theta}_A(\mu_1) \) as follows.

(i) Suppose speculators in group 1 are bull (\( \mu_1 = 0 \)) and their type is private information.

(a) Every speculator in group 1 attacks the currency peg if and only if she observes the private signal less than or equal to \( \tilde{x}^*_{A1} \).

(b) Every speculator in group 2 attacks the currency peg if and only if she observes the private signal less than or equal to \( \tilde{x}^*_{A2} \).

(c) The government of country A abandons the currency peg if and only if economic fundamentals are less than or equal to \( \tilde{\theta}^*_A \).

---

\[ ^{17} \text{The greater the private signal is, the more likely everyone else receives the greater signal also. It means the proportion of attacking speculators is likely to be small because everyone else is more likely to refrain from attacking the peg. So the probability that an attack is successful becomes smaller.} \]
(ii) Suppose speculators in group 1 are chicken \( (\mu_1 = \mu) \) and their type is private information.

(a) Every speculator in group 1 attacks the currency peg if and only if she observes the private signal less than or equal to \( \tilde{x}_{A1}^{**} \).

(b) Every speculator in group 2 attacks the currency peg if and only if she observes the private signal less than or equal to \( \tilde{x}_{A2} \).

(c) The government of country A abandons the currency peg if and only if economic fundamentals are less than or equal to \( \tilde{\theta}^*_{A} \).

(iii) The currency peg is more likely to be abandoned in (i) than in (ii), i.e., \( \tilde{\theta}^*_A > \tilde{\theta}^{**}_A \).

2.2.2 The Equilibrium in Country B

In period 2, every speculator observes not only what has happened in country A but also the exact value of \( \theta_A \). These observations convey information about the type of speculators in group 1 if speculators in group 1 follow the strategy described in Proposition 2. The strategy in Proposition 2 is the Nash equilibrium strategy in the one-shot game, so that it is not obvious whether speculators in group 1 follow it in the two-stage game or not. First, I analyze the switching strategy equilibrium in country B assuming speculators in group 1 follow the strategy described in Proposition 2 and then summarize the results in Proposition 3. Next, I will prove that Proposition 2 and Proposition 3 describe the subgame perfect equilibrium in the two-stage game.

Assume speculators in group 1 follow the strategy described in Proposition 2. There are two possible cases of \( \theta_A \) that is critical to the information structure in period 2. First, pick any \( \theta_A \notin [\tilde{\theta}^{**}_A, \tilde{\theta}^*_A] \). For any \( \theta_A \leq \tilde{\theta}^{**}_A \), the currency crisis will occur in country A for sure, irrespective of the type of speculators in group 1. For any \( \theta_A \geq \tilde{\theta}^*_A \), the currency crisis will never occur in country A irrespective of the type of speculators in group 1. Therefore, the type of speculators in group 1 will never be revealed for any \( \theta_A \notin [\tilde{\theta}^{**}_A, \tilde{\theta}^*_A] \). It means that there is no essential change in the information structure in period 2. Second, pick any \( \theta_A \in [\tilde{\theta}^{**}_A, \tilde{\theta}^*_A] \). Conditional on such \( \theta_A \), the currency crisis happens in country A if and only if speculators in group 1 are bull. In other words, whether or not the currency crisis happens in country A for such \( \theta_A \), the type of speculators in group 1 is revealed. If the currency crisis has occurred in country A for such \( \theta_A \), speculators in group 2 know that speculators in group 1 are bull. If the currency crisis has not occurred in country A for such \( \theta_A \), speculators in group 2 know that speculators in group 1 are chicken. In each case, the type of speculators in group 1 becomes common knowledge in period 2. So it changes the information structure of the game in period 2 critically. Thus, speculators in group 2 update their belief of the type of group 1 and thereby they change the optimal behavior. Therefore, even though economic fundamentals are totally unrelated across country A and country B, what has happened in country A affects the optimal behavior of speculators, which in
turn affects what would happen in country B. This is exactly the key to explaining contagion across unrelated countries without any common lender.

Now we can find the switching strategy equilibrium in country B, which is conditional both on what has happened in country A and on the value of $\theta_A$. Since there is no essential change in the information structure for any $\theta_A \notin [\hat{\theta}_A^*, \hat{\theta}_A^*]$, the switching strategy equilibrium in country B is exactly the same as the one in country A in Proposition 2. In other words, the switching strategy equilibrium in country B is described just by replacing the subscript A with B in Proposition 2 for any $\theta_A \notin [\hat{\theta}_A^*, \hat{\theta}_A^*]$. How about the switching strategy equilibrium in country B for $\theta_A \in [\hat{\theta}_A^*, \hat{\theta}_A^*]$? There are two possible cases. One is that the currency crisis has happened in country A while another is that the currency crisis has not happened in country A. In the former case, speculators in group 2 learns that speculators in group 1 are bull. Let us call this Case 1. In the latter case, speculators in group 2 learns that speculators in group 1 are chicken. Let us call this Case 2. First, I analyze the switching strategy equilibrium in country B in Case 1. Next, I analyze the switching strategy equilibrium in country B in Case 2.

In Case 1, every speculator is bull and this fact is common knowledge. It means every speculator is identical so that all the speculator will use the same switching signal, $x_B^*$. Thus the proportion of attacking speculators conditional on $\theta_B$ is the following.

$$l_B(\theta_B) = \text{Prob}[x_B \leq x_B^* | \theta_B]$$

$$= \text{Prob}[\theta_B + \epsilon_B \leq x_B^* | \theta_B]$$

$$= \frac{x_B^* - \theta_B}{2\epsilon} \quad \text{(15)}$$

Since the government is indifferent between defending the peg and abandoning it when $\theta_B = l_B$, we get the switching economic fundamentals $\theta_B^*$ from (15) as follows.

$$\theta_B^* = l_B(\theta_B = \theta_B^*) = \frac{x_B^* - \theta_B^*}{2\epsilon} \quad \text{(16)}$$

Observing the switching private signal $x_B^*$, the expected utility of attacking the currency peg must be zero.

$$t = \text{Prob}[\text{Attack is successful} | x_B^*] D$$

$$= \text{Prob}[\theta_B \leq \theta_B^* | x_B^*] D$$

$$= \text{Prob}[x_B^* - \epsilon_B \leq \theta_B^* | x_B^*] D$$

$$= \left(1 - \frac{x_B^* - \theta_B^*}{2\epsilon}\right) D \quad \text{(17)}$$

From (16) and (17), we can solve for $x_B^*$ and $\theta_B^*$ as follows.

$$\theta_B^* = 1 - \frac{t}{D} \quad \text{(18)}$$

$$x_B^* = (2\epsilon + 1) \left(1 - \frac{t}{D}\right) \quad \text{(19)}$$
It can be shown that (18) and (19) consist of the switching strategy equilibrium exactly same as in Proposition 2.

In Case 2, it is common knowledge that speculators in group 1 are chicken and speculators in group 2 are bull. Denote the switching signal of a speculator in group 1 by \( x^*_{B_1} \), that of a speculator in group 2 by \( x^*_{B_2} \) respectively. The proportion of attacking speculators conditional on \( \theta_B \) is the following.

\[
l_B(\theta_B) = \lambda \Pr [x_{B1} \leq x^*_{B1} \mid \theta_B] + (1 - \lambda) \Pr [x_{B2} \leq x^*_{B2} \mid \theta_B]
\]

\[
= \lambda \Pr [\theta_B + \epsilon_B \leq x^*_{B1} \mid \theta_B] + (1 - \lambda) \Pr [\theta_B + \epsilon_B \leq x^*_{B2} \mid \theta_B]
\]

\[
= \frac{\lambda x^*_{B1} - \theta_B}{2\epsilon} + (1 - \lambda) \frac{x^*_{B2} - \theta_B}{2\epsilon}
\]  

(20)

Since the government is indifferent between defending the peg and abandoning it when \( \theta^*_B = l_B \), we get the switching economic fundamentals \( \theta^*_B \) from (20) as follows.

\[
\theta^*_B = l_B (\theta_B = \theta^*_B) = \frac{1}{2} \frac{x^*_{B1} - \theta^*_B}{\epsilon} + \frac{1}{2} \frac{x^*_{B2} - \theta^*_B}{\epsilon}
\]  

(21)

Observing the switching private signal \( x^*_{B1} \), the expected utility of attacking the currency peg for the chicken speculator must be zero.

\[
t + \mu = \Pr [\text{Attack is successful} \mid x^*_{B1}] D
\]

\[
= \Pr [\theta_B \leq \theta^*_B \mid x^*_{B1}] D
\]

\[
= \Pr [x^*_{B1} - \epsilon_B \leq \theta^*_B \mid x^*_{B1}] D
\]

\[
= \left(1 - \frac{x^*_{B1} - \theta^*_B}{2\epsilon}\right) D
\]  

(22)

Similarly, observing the switching private signal \( x^*_{B2} \), the expected utility of attacking the currency peg for the bull speculator must be zero. Since the type of speculators in group 1 is common knowledge, speculators in group 2 do not need to take an expectation over \( \theta^*_B \) and \( \theta^*_B \) in their expected utility. They actually know that \( \theta^*_B \) would arise as the switching economic fundamentals. Thus the expected utility conditional on \( x^*_{B2} \) can be written as follows.

\[
t = \Pr [\text{Attack is successful} \mid x^*_{B2}] D
\]

\[
= \Pr [\theta_B \leq \theta^*_B \mid x^*_{B2}] D
\]

\[
= \Pr [x^*_{B2} - \epsilon_B \leq \theta^*_B \mid x^*_{B2}] D
\]

\[
= \left(1 - \frac{x^*_{B2} - \theta^*_B}{2\epsilon}\right) D
\]  

(23)

From (21), (22) and (23), we can solve for \( x^*_{B1} \), \( x^*_{B2} \) and \( \theta^*_B \) as follows.

\[
\theta^*_B = 1 - \frac{t}{D} - \frac{\lambda \mu}{D}
\]  

(24)

\[
x^*_{B1} = (2\epsilon + 1) \left(1 - \frac{t}{D}\right) - (2\epsilon + \lambda) \frac{\mu}{D}
\]  

(25)

\[
x^*_{B2} = (2\epsilon + 1) \left(1 - \frac{t}{D}\right) - \frac{\lambda \mu}{D}
\]  

(26)

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It can be shown that (24), (25) and (26) consist of the switching strategy equilibrium exactly same as in Proposition 2. Proposition 3 summarizes the above argument.

**Proposition 3 (Unique Equilibrium in Country B)**

Suppose speculators follow the strategy in Proposition 2. Conditional on $\theta_A$ and what has happened in country A, the unique switching strategy equilibrium in country B consists of the switching private signals and the switching economic fundamentals as follows.

(i) For any $\theta_A \notin [\tilde{\theta}_A^{**}, \tilde{\theta}_A^*]$, the unique switching strategy equilibrium in country B replicates the one in country A exactly: every speculator and the government follow the exactly same switching strategy as in Proposition 2.

(ii) If the currency crisis has happened for $\theta_A \in [\tilde{\theta}_A^{**}, \tilde{\theta}_A^*]$ in country A, it becomes common knowledge that speculators in group 1 are bull.

(a) Every speculator in each group attacks the currency peg if and only if she observes the private signal less than or equal to $x_B^*$.

(b) The government of country B abandons the currency peg if and only if economic fundamentals are less than or equal to $\theta_B^*$.

(iii) If the currency crisis has not happened for $\theta_A \in [\tilde{\theta}_A^{**}, \tilde{\theta}_A^*]$ in country A, it becomes common knowledge that speculators in group 1 are chicken.

(a) Every speculator in group 1 attacks the currency peg if and only if she observes the private signal less than or equal to $x_{B1}^{**}$.

(b) Every speculator in group 2 attacks the currency peg if and only if she observes the private signal less than or equal to $x_{B2}^{**}$.

(c) The government of country B abandons the currency peg if and only if economic fundamentals are less than or equal to $\theta_B^{**}$.

### 2.2.3 The Sub-Game Perfect Equilibrium in the Two-Stage Game

So far I have investigated the equilibrium in the one-shot game. In particular, I have derived Proposition 3 assuming that Proposition 2 holds not only in the one-shot game but also in the two-stage game. But it is not obvious whether Proposition 2 holds in the two-stage game or not. In other words, it may not be the case that the sequence of Proposition 2 and Proposition 3 is the sub-game perfect equilibrium. Here I prove this is indeed the case.

First, note that the sequence of Proposition 2 and Proposition 3 is at least a Nash equilibrium, if not the sub-game perfect equilibrium. If everyone follows the strategy in Proposition 2 and Proposition 3 in the two-stage game, no one has an incentive to change the behavior. This is because any individual speculator cannot change the switching economic fundamentals by herself alone so it does not pay to deviate. Second, notice that if speculators in group 2 believe speculators in group 1 are bull,
it is more likely for the currency peg to be abandoned. So the chicken speculators 
may have an incentive to make speculators in group 2 believe that group 1 were 
bully. In order for chicken speculators to deceive, they need to behave in country A 
as if they were bull by using the switching private signal of bull speculator, \( \tilde{x}^*_{A1} \). In 
that case, the switching economic fundamentals in country A would be \( \tilde{\theta}^*_A \) as long 
as all the chicken speculators use \( \tilde{x}^*_{A1} \). Then for \( \theta_A \leq \tilde{\theta}^*_A \), the currency crisis would 
happen in country A. So the expected utility of the deceiving chicken speculator 
conditional on \( \tilde{x}^*_{A1} \) is the following.

\[
\text{Prob [Attack is successful | } \tilde{x}^*_A \text{]} D - t - \mu \\
= \left\{1 - \frac{\tilde{x}^*_A - \tilde{\theta}^*_A}{2\epsilon}\right\} D - t - \mu \\
= -\mu 
\]  
(27)

The second equality is because \( \tilde{x}^*_A \) makes the bull speculator indifferent between 
attacking the peg and refraining from doing so, i.e., \( 1 - t/D - (\tilde{x}^*_A - \tilde{\theta}^*_A)/2\epsilon = 0 \). Thus \( \mu \) can be thought of as the cost of deceiving. Next suppose the chicken 
speculators succeed in deceiving group 2 by using \( \tilde{x}^*_{A1} \) as their switching private 
signal: it becomes more likely for the currency peg in country B to be abandoned 
since speculators in group 2 believe speculators in group 1 were bull. It means the 
expected utility in period 2 would increase. As long as the increase in the expected 
utility in period 2 is greater than or equal to the cost \( \mu \), deceiving seems to be the 
equilibrium. But actually it cannot be the equilibrium. To see this, suppose the 
increase in the expected utility in period 2 is greater than the cost \( \mu \). In this case, 
a chicken speculator has an incentive to refrain from attacking the peg in period A 
when she observes \( \tilde{x}^*_{A1} \), provided that every other chicken speculator attacks. This 
is because she can save the cost \( \mu \) by refraining from attacking, without changing 
the switching economic fundamentals in country A.\footnote{Since there is a continuum of speculators, an individual speculator cannot affect the switching 
economic fundamentals by herself alone.} Even if she does not pay the deceiving cost, she can enjoy the increase in the expected utility in period 2 because 
of other chicken speculators’ deceiving behavior. Put another way, she can free-
ride on other chicken speculators’ deceiving behavior. It in turn implies no chicken 
speculator has an incentive to pay the cost. Therefore, deceiving cannot be the 
equilibrium. In other words, there is no switching private signal other than those in 
Proposition 2 and Proposition 3 in the two-stage game. I have proven the following 
proposition.

**Proposition 4 (Unique Sub-Game Perfect Equilibrium)**

*The unique sub-game perfect switching strategy equilibrium consists of the switching 
private signals and the switching economic fundamentals as follows.*

(i) In period 1, every speculator and the government follows the switching strategy 
in Proposition 2.
(ii) In period 2, every speculator and the government follows the switching strategy in Proposition 3.

Now, I have shown the existence of contagion as the sub-game perfect equilibrium.

**Proposition 5 (Contagion)**

There exists contagion from country A to country B even if there is no economic fundamentals linkage or the common lender either: for any $\theta_A \in [\tilde{\theta}^{**}_A, \tilde{\theta}^*_A]$, $\tilde{\theta}_B(Y_A = 0) < \tilde{\theta}_B(Y_A = 1)$.

## 3 Empirical Implication

I have identified the new channel of contagion theoretically. The question arises as to how to measure it: what observable variables can we look at to check whether the new channel indeed works in recent contagious currency crises? Put another way, what is the unique prediction of the new channel that is observable in data?

The empirical literature on contagion has tried to construct an index of contagion to capture the specific channel: the trade linkage index, the capital linkage index, the geographical distance index, and so on. How should one construct an index to capture the channel of this paper? The index has to enable us to distinguish the channel proposed in this paper from other contagion channels. In reality, it is hard to estimate the type of speculators directly from data. But noting the difference of the empirical implication between the existing literature and this paper, I can construct an index to capture the unique implication of the channel proposed in this paper. The index will be constructed so as to contradict the common implication of the existing literature but fit only the one of this paper. Therefore I can estimate the contents of this paper at least indirectly. The key difference of the implication comes from the following Proposition.

**Proposition 6 (Crisis with Better Economic Fundamentals Are More Contagious)**

Pick any $\theta'_A \leq \tilde{\theta}^{**}_A$ and $\theta''_A \in [\tilde{\theta}^{**}_A, \tilde{\theta}^*_A]$. The currency crisis in country A with better economic fundamentals $\theta''_A$ is more contagious than that with worse economic fundamentals $\theta'_A$. That is, $\theta^{**}_B < \theta^*_B$ where $\theta^{**}_B$ is the switching value of economic fundamentals conditional on country A’s crisis for $\theta'_A$ while $\theta^*_B$ is the switching value of economic fundamentals conditional on country A’s crisis for $\theta''_A$.

The intuition for this result is simple. For any $\theta'_A \leq \tilde{\theta}^{**}_A$, the currency peg would be abandoned irrespective of the type of group 1. Thus the currency crisis would convey no information about the type, which would not cause contagion. In other words, economic fundamentals in country A are so bad that no speculator would be surprised at the crisis, which means no speculator would change the behavior. But for any $\theta''_A \in [\tilde{\theta}^{**}_A, \tilde{\theta}^*_A]$, the currency crisis would occur in country A if and only if speculators in group 1 are bull. Thus whether or not the crisis has happened in
country A does convey information about the type of group 1, which is the source of contagion. For any $\theta_A' \in [\tilde{\theta}_A^*, \tilde{\theta}_A^*]$, economic fundamentals in country A are not so bad in that the currency crisis would not have happened if speculators in group 1 were chicken. Thus the currency crisis would surprise speculators in group 2 in this case, which make them change the behavior so as to cause the contagion. Put another way, if the currency crisis happens where economic fundamentals are considered to be not so bad or seemingly good, it will lead to a big surprise and thereby the crisis could hit even unrelated countries. This is exactly the recent experience of the contagious crises we have seen. So far, such contagion across unrelated countries that is not due to capital linkage has been explained just as an international panic or treated as a puzzle. But this paper provides an explanation for why such contagion happens.

The common implication in the literature is that the worse economic fundamentals in the originating crisis country (country A) are, the more contagious the crisis is. This is because the literature has been exploring the transmission mechanism through which the negative effect of bad economic fundamentals of the originating crisis country would spread directly or indirectly. If countries are related in terms of economic fundamentals such as trade linkage, the negative effect of the crisis would hit them directly. If there are capital linkage through the common lender or the interbank market, the negative effect of the crisis would hit the common lender or the interbank market first and then would be transmitted to countries through capital linkage indirectly. In each case, the worse economic fundamentals of country A are, the larger the negative effect of the originating crisis is. The larger the negative effect of the originating crisis is, the more severe countries would be affected. This is the logic behind the common implication above. But Proposition 6 shows the opposite can be true. If the currency crisis happens where economic fundamentals are considered to be not so bad or seemingly good, it will lead to a big surprise and hence the crisis could hit even unrelated counties sometimes. It implies that even if country B could get rid of the contagion of the crisis, it does not necessarily mean that it will not be vulnerable to contagion of crises that happens in the country with better economic fundamentals than country A. This seems counterintuitive, but it is reasonable in fact. If the currency crisis happens in the country whose economic fundamentals are good, everybody will be surprised, which can make the crisis more

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19We could construct a model in which the common lender is the contagion channel to show similar result as Proposition 6. Suppose the international bank, as the common lender, has made huge loans to country A and has been too optimistic to set aside enough loan loss reserves because economic fundamentals in country A have been considered to be good ex ante. If the unexpected currency crisis happens, the international bank with huge loans is hit so severe since it does not have enough loan loss reserves. Then it may call in its loans from other unrelated countries to meet the required capital ratio, resulting in contagion. This scenario seems to lead to the conclusion exactly same as Proposition 6. But it is different in fact. To see the difference, first note that the worse economic fundamentals ex post are, the more non-performing loans there are. As a result, the worse economic fundamentals ex post are, the more contagious the crisis is. However, Proposition 6 states that the better economic fundamentals ex post are, the more contagious the crisis is. Thus they are different critically. Moreover, Proposition 6 is derived without the common lender.
It should be noted that the negative effect of the originating crisis itself does not matter in the contagion channel of this paper. Since economic fundamentals are independent between country A and country B, there cannot be direct transmission of the negative effect from country A to country B. Since no speculator incurs losses in the crisis and there is no capital linkage, the negative effect would never spread from country A to country B even indirectly. In sum, the negative effect of bad economic fundamentals of country A itself is not transmitted to country B in contagion of this model. What matters in this model is that the crisis in country A can convey speculators’ type.

In spite of the difference of the implication between the existing literature and this paper, all the channels of contagion including the one proposed in this paper, are not exclusive to each other. Actually they can work at the same time. So they are complementary in that the contagious effect of the currency crisis will be amplified when several channels work simultaneously. Therefore it is worthwhile to investigate the interactive effects of multiple channels.

Moreover, there is an important implication about the timing of financial crisis in the theoretical models of Morris and Shin (1998) that has as yet to be taken into account in empirical papers: that signals (regressors) are noisy may matter in determining the timing of financial crises. The key role of noisy signals is in their importance in getting rid of self-fulfilling property of financial crisis and thereby holding out the prospect of pinning down the timing of these crises. In other words, measurement error of regressors is important in estimating the timing of financial crises. To the best of my knowledge, all the empirical research of financial crises assumes no measurement error of regressors. Thus I would like to pursue this issue in future research.

4 Conclusion

A possible extension of the model presented in this paper is to consider the role of a single large speculator who can affect the whole market to some degree. In this paper, none of the speculators can affect the market as a whole by herself. But recent currency crises episodes suggest that large traders, like George Soros, can exercise a disproportionate influence on the likelihood and severity of a financial crisis by fermenting and orchestrating attacks against weakened currency pegs. Corsetti, Dasgupta, Morris, and Shin (2004) argue that the presence of the large speculator does make all other traders more aggressive in their selling. But they do not consider the implication of the presence of the large speculator for contagion. The implication for contagion is not so obvious. On the one hand, the currency crisis may be more contagious in the the presence of the large speculator, simply because the large speculator has larger market power in attacking the currency peg of country B than she does in the model presented in this paper. But on the other hand, the currency crisis may be less contagious because the type revealing effect, which is critical to
the contagion channel in this paper, may become small if all other traders become more aggressive in attacking the currency peg of *country A*. It is not so difficult to incorporate the large speculator into the model presented in this paper. Therefore I will be working on this issue in another paper.\textsuperscript{20}

\footnotesize{\textsuperscript{20}See Taketa (2003).}
Appendix

A A Rolling Over Game among Foreign Creditors

The contagion channel in this paper can work with other channels at the same time. Here I show it can work with the common lender. In particular, I show the model in this paper can describe a roll-over game among common lenders.

Just rename speculators in the model as foreign creditors. Foreign creditors have invested both in country A and in country B. They have financed a project in each country. So they are the common lenders. Observing the private signal, a creditor decides whether she rolls over her loan or not. If she decides not to roll over and asks to liquidate, her payoff is $t = \mu_1$ for sure. Not to roll over is a safe choice, which corresponds to refraining from attacking the peg in the model. If she decides to roll over, her payoff depends on two factors - the economic fundamentals $\theta_j$ and the degree of disruption caused to the project by the early liquidation by creditors. The latter is measured by the proportion of creditors who do not roll over, $l_j$. The project yields the payoff $D$ (i.e., roll over is successful) if $\theta_j \geq l_j$ and zero (i.e., roll over fails) if $\theta_j < l_j$. That is, if a sufficient proportion of creditors refuses to roll over relative to the economic fundamentals ($\theta_j < l_j$), the project is liquidated entirely and yields nothing.\(^{21}\) Roll over is a risky choice in that the payoff is uncertain, which corresponds to attacking the peg in the model. Notice the similarity of the payoff structure between the speculators game and the creditors game (see Table 1 and Table 2). Indeed, all the reasoning of the speculators game applies to the creditors game: the switching strategy equilibrium arises and contagion exists due to the learning the type of creditors of group 1. If creditors in group 1 are chicken and $\theta_A \in [\hat{\theta}_A^*, \tilde{\theta}_A^*]$, the sufficient proportion of creditors refuses to roll over so that the project would die in country A (i.e., the financial crisis in country A). Observing that, creditors in group 2 learns the type of creditors in group 1 and thereby change the switching signal for country B, resulting in contagion from country A to country B.\(^{22}\)

In terms of deriving the equilibrium, the speculators game and the creditors game are exactly the same. Thus we can derive similar Propositions in the creditors game as in the speculators game. A difference is that the crisis is beneficial for

\(^{21}\)This formulation is similar as Diamond and Dybvig (1983).

\(^{22}\)Obviously, contagion becomes more severe if economic fundamentals are correlated (i.e., $\text{Cov}(\theta_A, \theta_B) > 0$).

Table 2: Payoff Matrix

<table>
<thead>
<tr>
<th>Roll Over</th>
<th>Success</th>
<th>Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not Roll Over</td>
<td>$t + \mu_1$</td>
<td>$t + \mu_1$</td>
</tr>
</tbody>
</table>
speculators while no-crisis (successful roll over) is beneficial for creditors. Another
difference is that bull speculators play a central role in contagion in the speculators
game while chicken creditors play a central role in contagion in the creditors game.
References


