Intertemporal Quality Discrimination

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Abstract

We show that a durable goods monopolist that introduces its products sequentially will choose higher-than-optimum qualities. This result differs from traditional screening models, in which the qualities of a non-durable goods monopolist’s products never exceed the optimum.

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1 Introduction

New technology is usually expensive and it takes time for manufacturers to make the technology more accessible. In the stereo industry, the first Super Audio Compact Disk (SACD) player made by Sony,\(^3\) SCD-1, sold for \$5,000 in 1999; in 2002 the cheapest of Sony’s new SACD players, SCD-CE775, had a \$250 MSRP, while the SCD-1 continued to be Sony’s flagship model. The electrostatic speaker manufacturer MartinLogan developed a technology trade-marked ClearSpars for their Statement e2 speakers, which came to the market in 2000 with a list price of \$80,000 per pair. MartinLogan later applied the technology to their mid-price ($3300 per pair) Aeon i in 2003. The amplifier manufacturer Conrad-Johnson introduced in 2000 its current top pre-amplifier, ART Series 2, and in 2003 added to their product line a stripped-down version of the ART, the Premier 17LS, whose price is less than one-third the price of the ART. The four-wheel-drive vehicle manufacturer Land Rover introduced their mid-price model Discovery in 1986, after they remodeled their luxury line Range Rover in the early 80s.

In these examples, before the firms could scale down their new technologies for the mass markets, they sold only the high-end products; and after the more affordable low-end products became available, they sold both kinds of products. Furthermore, these products are durable goods, and so by the time the firms introduced the low-end products, the consumers who had bought the high-end products were no longer in the market.

In this paper we abstract from the inter-firm competition. That is, we assume that the durable goods market is monopoly, and study the quality decision and the pricing of the durable goods monopolist whose first-generation product has higher quality than the second-generation one, which is not available at the time the first-generation product is first introduced to the market.\(^4\) In addition to Coasian dynamics, or intertemporal price discrimination, the issue

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\(^3\)Sony Electronics, Inc. and Philips Electronics, Inc. jointly developed the SACD format to replace the compact discs.

\(^4\)Sometimes a firm is able to produce both high-end and low-end products from the beginning but decides not to. Publishers do not print paperbacks of new books until they have sold the hardcovers for some time. Some
involves intertemporal quality discrimination. Our analysis focuses on whether the monopolist would produce goods with qualities higher than the optimum.

In a static single-quality model without price discrimination, Spence (1975) compares the quality of non-durable good produced by a monopolist with the optimum, and finds that the monopoly’s quality could be higher or lower than the optimum, depending on the demand conditions. In particular, for a linear demand, the quality chosen by the monopolist is the same as the optimum. In a dynamic single-quality model with price discrimination, Chi (1999) discusses the quality choice of a durable goods monopolist, and shows that with linear demand, the intertemporal price discrimination makes a monopolist choose a quality at least as high as the optimum, and higher than the optimum when the discount factor is small.

In static quality (or quantity) discrimination models, where a monopolist can use several quality-price packages to screen consumers, it is well known that a monopolist would discriminate the consumers by offering the efficient quality only to the consumer with the highest valuation, and offering everyone else a quality less than the optimum. In no circumstances could the consumers get above-optimum quality in the static model. However, in our model of intertemporal quality discrimination, we find that the monopolist will produce goods of above-optimum qualities in its product line.

That the monopolist might offer above-optimum quality is new to the literature of quality discrimination. Moorthy and Png (1992) consider a monopolist who faces two types of consumers (high-demand and low-demand) and is able to introduce high and low qualities simultaneously. In some cases, the monopolist prefers sequential introduction: high-end product in the first period and low-end in the second. However, the qualities do not exceed the optimum in any equilibrium. Wang (2000) also uses a model with two types of consumers, and shows that when the monopolist is able to offer two quality-price packages each period, the result is the same as static quality discrimination. Therefore, no qualities can be higher than the optimum. Wilson and Norton (1989) focus on the timing for introducing the lower-quality product, and do not discuss how the firm chooses prices or qualities.

fashion designers offer mid-price lines of their clothes after they have established their brand names. Wilson and Norton (1989) and Moorthy and Png (1992) study when a monopolist should introduce the lower-quality version, given that its production is always feasible.

In the papers discussing product upgrades or product obsolescence, Bulow (1986) shows that the monopolist will reduce the durability of its product, and hence the quality is lower than the optimum. A firm can also make its current product obsoleted by introducing a better product. Levinthal and Purohit (1989) study the case. They focus on the timing of introducing the new product, the pricing, and the use of buyback policy, but assume that the quality difference between the two generations’ products is an exogenous variable. Fudenberg and Tirole (1998) use a very general framework to study product upgrades and the related marketing practices – tradeins and buybacks, but they too treat the qualities as exogenous variables.

Our paper is organized as follows. Section 2 describes the model. We adopt Bulow’s (1982) two-period framework. The monopolist and the consumers live for two periods and have the same discount factor. As in Mussa and Rosen’s (1978) vertical-differentiation model, consumers’ preference for quality are indexed by a parameter with a continuous distribution, which is simplified here to be uniform.

Section 3 establishes the efficient qualities of the product line chosen by a benevolent social planner, and shows that if the monopolist is able to commit to its second-period behavior, there is no distortion in qualities.

Section 4 solves the equilibrium when the monopolist cannot commit. We show that both the first-period and the second-period products have higher quality than the optimum.

2 The Model

There are two periods, \( t = 1, 2 \). At the start of period 1, the monopolist is endowed with a technology to produce a single perfectly durable good, denoted by \( H \), and to produce in period 2 a second-generation product with lower quality, denoted by \( L \). We assume that the technology takes time to become “mature,” and therefore it is not feasible for the monopolist to produce the lower quality product \( L \) in period 1. The monopolist makes the following choices. In period 1, it decides the quality of \( H \), \( q_h \), and then sets a price \( p_1 \). In period 2, after observing the quantity sold in period 1, it decides whether to introduce \( L \). If it does, it then decides the quality of \( L \), \( q_l \), and its price \( r \). Whether or not it introduces \( L \), it needs to decide the price of
$H$ sold in this period, $p_2$. To simplify the model, assume that the unit costs of the two products are $c(q_i) = bq_i^2/2$, $i = h, l$, where $b > 0$ is a parameter.

There is a continuum of consumers, whose marginal utility of income is constant and valuations of the products are indexed by $\theta$; the distribution of $\theta$ is uniform on $[0, 1]$. Consuming a product with quality $q$, a type-$\theta$ consumer gets per-period utility

$$\theta q + I,$$

where $I$ is his net income. Each consumer has unit demand for the monopolist’s product. In equilibrium, a consumer who purchases in period 1 will not purchase in period 2, since the second-period product’s quality is lower. The consumers have perfect information about the products’ qualities when the products are on the market, and have perfect foresight in the first period about the monopolist’s second-period strategy since the strategy has to be subgame perfect. The consumers and the monopolist have a common discount factor $\delta$.

A type-$\theta$ consumer could purchase $H$ in period 1, $H$ in period 2, $L$ in period 2, or nothing at all, depending on which decision gives him the highest net utility. Namely, he looks for the maximum of the following values:

$$\{(1 + \delta)\theta q_1 - p_1, \delta(\theta q_1 - p_2), \delta(\theta q_l - r), 0\}.$$

3 The Optimum and the Commitment Solution

We first consider the qualities chosen by a welfare-maximizing social planner. Then we assume that the monopolist can make commitment in period 1 to its period-2 actions: $q_l$, $p_2$, and $r$. Last we compare the commitment solution with the optimum.

3.1 The Optimum

Like the monopolist, the social planner can produce only one product in the first period. To maximize social welfare, the prices of $H$ is set to be the marginal cost $bq_i^2/2$ in both periods,
and the price of $L$ to be $bq_l^2/2$: $p_1^* = p_2^* = bq_h^2/2$, $r^* = bq_l^2/2$, where the asterisk stands for optimum. Since the price of $H$ equals marginal cost, all consumers who can get non-negative net utility from purchasing $H$ in period 2 will purchase it in period 1. Hence, there is no market for $H$ in period 2.

Let $\theta_h$ be the type of the consumer who is indifferent between purchasing $H$ in period 1 and purchasing $L$ in period 2:

$$(1 + \delta)\theta_h q_h - \frac{b q_h^2}{2} = \delta (\theta_h q_l - \frac{b q_l^2}{2}) \geq 0 .$$ (1)

Then consumers whose types are above $\theta_h$ purchase $H$ in period 1, and the demand for $H$ is $1 - \theta_h$.

Let $\theta_l$ be the type of the consumer who is indifferent between purchasing $L$ in period 2 and purchasing nothing:

$$\theta_l q_2 - \frac{b q_l^2}{2} = 0 .$$ (2)

All type-$\theta$ consumers, $\theta > \theta_l$, get positive net utility from purchasing $L$, and therefore the demand for $L$ is $\theta_h - \theta_l$.

The social planner chooses $q_h$ and $q_l$ to solve:

$$\max_{q_h, q_l} W = \int_{\theta_h}^{1} [(1 + \delta)\theta q_h - \frac{b q_h^2}{2}]d\theta + \delta \int_{\theta_l}^{\theta_h} (\theta q_l - \frac{b q_l^2}{2})d\theta ,$$ (3)

where $\theta_h$ and $\theta_l$ are defined in (1) and (2). The solution is:

$$q_h = \frac{(1 + \delta)(1 + \theta_h)}{2b} ,$$ (4)

$$q_l = \frac{\theta_h + \theta_l}{2b} ,$$ (5)
which, together with (1) and (2), lead to

\[
\theta_c^* = \frac{-9(1 + \delta)^2 + 6(1 + \delta)\sqrt{9 + 10\delta + 9\delta^2}}{27 + 22\delta + 27\delta^2}
\] (6)

\[
\theta_l^* = \frac{\theta_c^*}{3} = \frac{-3(1 + \delta)^2 + 2(1 + \delta)\sqrt{9 + 10\delta + 9\delta^2}}{27 + 22\delta + 27\delta^2}
\] (7)

\[
q_h^* = \frac{(1 + \delta)(1 + \theta_c^*)}{2b} = \frac{(1 + \delta)(9 + 2\delta + 9\delta^2) + 3(1 + \delta)^2\sqrt{9 + 10\delta + 9\delta^2}}{b(27 + 22\delta + 27\delta^2)}
\] (8)

\[
q_l^* = \frac{2\theta_c^*}{3b} = \frac{-6(1 + \delta)^2 + 4(1 + \delta)\sqrt{9 + 10\delta + 9\delta^2}}{b(27 + 22\delta + 27\delta^2)}
\] (9)

Note that the above four variables are all increasing in \(\delta\).

### 3.2 The Commitment Solution

Assume that the monopolist can commit itself in period 1 to its period-2 actions: \(q_l, p_2,\) and \(r\). There are four period-2 strategies to which the monopolist can make its commitment: selling \(L\) only, selling \(H\) only, selling both, and selling none. However, it is well known in the literature of the Coase conjecture, e.g., Bulow (1982), that the monopolist does worse from selling in period 2 only \(H\) than selling nothing. Furthermore, committing itself to selling both \(H\) and \(L\) is equivalent to the case in which the monopolist cannot commit. So we only need to consider the cases that the monopolist commits to selling in period 2 only \(L\) and selling nothing. And since the monopolist, when adopting the strategy of selling only \(L\), can mimic the strategy of selling nothing by setting the price \(r\) high enough, selling only \(L\) weakly dominates selling nothing.\(^6\) Therefore, if the monopolist can commit, it will sell in period 1 only \(H\) and in period 2 only \(L\).

To commit itself to selling only \(L\) in period 2, the monopolist will set \(p_2\) such that no consumers will buy \(H\) in period 2. Given \(p_1\), \(q_l\), and \(r\), the marginal consumer who purchases \(H\) in period 1 is type \(\theta_h^c\), where \(\theta_h^c\) satisfies

\[
(1 + \delta)\theta_h^c q_h - p_1 = \delta(\theta_h^c q_l - r).
\] (10)

\(^6\)Indeed, it can be shown that the profit from selling \(H\) in period 1 and \(L\) in period 2 is larger than the profit from selling \(H\) in period 1 and nothing in period 2, when \(\delta > 0\); and that the profits are equal if \(\delta = 0\).
And the marginal consumer who purchases \( L \) in period 2 is type \( \theta^c_i \), where \( \theta^c_i \) satisfies

\[
\theta^c_i q_l - r = 0 .
\]  

(11)

The demands for \( H \) and \( L \) are then \( 1 - \theta^c_h \) and \( \theta^c_h - \theta^c_l \), respectively. From (10) and (11), we get

\[
p_1 = (1 + \delta)\theta^c_h q_h - \delta(\theta^c_h - \theta^c_l)q_l
\]

(12)

\[
r = \theta^c_l q_l .
\]

(13)

The monopolist’s problem is then

\[
\max_{\theta^c_h, \theta^c_l, q_h, q_l} \Pi^c = (p_1 - \frac{b q^2_h}{2})(1 - \theta^c_h) + \delta(r - \frac{b q^2_l}{2})(\theta^c_h - \theta^c_l)
\]

s.t. (12), (13).

(14)

Denote by hat the solution, then

\[
\hat{\theta}^c_h = \frac{9 + 2\delta + 9\delta^2 + 3(1 + \delta)\sqrt{9 + 10\delta + 9\delta^2}}{27 + 22\delta + 27\delta^2},
\]

(15)

\[
\hat{\theta}^c_l = \frac{1 + \hat{\theta}^c_h}{3} = \frac{12 + 8\delta + 12\delta^2 + (1 + \delta)\sqrt{9 + 10\delta + 9\delta^2}}{27 + 22\delta + 27\delta^2},
\]

(16)

\[
\hat{q}^c_h = \frac{(1 + \delta)\hat{\theta}^c_h}{b} = \frac{(1 + \delta)(9 + 2\delta + 9\delta^2) + 3(1 + \delta)^2\sqrt{9 + 10\delta + 9\delta^2}}{b(27 + 22\delta + 27\delta^2)},
\]

(17)

\[
\hat{q}^c_l = \frac{4\hat{\theta}^c_h - 2}{3b} = \frac{-6(1 + \delta)^2 + 4(1 + \delta)\sqrt{9 + 10\delta + 9\delta^2}}{b(27 + 22\delta + 27\delta^2)}.
\]

(18)

And the profit is

\[
\hat{\Pi}^c = \frac{-(27 + 22\delta + 27\delta^2)(\hat{\theta}^c_i)^3 + 3(9 + 2\delta + 9\delta^2)(\hat{\theta}^c_i)^2 + 24\delta\hat{\theta}^c_i - 4\delta}{54b}.
\]

(19)

Comparing the commitment solution with the optimum, we see that

\[
\hat{q}^c_i = q^*_i, \quad \hat{\theta}^c_i > \theta^*_i, \quad i = h, l.
\]

(20)
Proposition 1. If the monopolist can commit itself in period 1 to selling only $L$ in period 2, to the quality of $L$, and to the price of $L$, then the qualities of $H$ and $L$ are both optimal, but the quantities sold are both less than the optimum.

4 The No-commitment Solution

Suppose now that the monopolist cannot commit itself to its period-2 behavior. Then its choices of $q_1$, $p_2$, and $r$ have to be subgame perfect, and therefore after observing $q_h$ and $p_1$, the consumers have perfect foresight in period 1 about the firm’s choices in period 2.

We solve the game by backward induction. Given $q_h$ and $p_1$, there is a $\theta_1^h$ such that all type-$\theta'$ consumers, $\theta' \geq \theta_1^h$, purchase $H$ in period 1. At the start of period 2, the types of consumers who are in the market are in the region $[0, \theta_1^h]$, which can be divided into two sections: $T_L = [\theta_l, \theta_1^h]$ and $T_H = [\theta_2^h, \theta_1^h]$, where the cutoff values are defined as follows:

\begin{align*}
\theta_l q_l - r &= 0, \quad (21) \\
\theta_h^2 q_h - p_2 &= \theta_l^2 q_l - r, \quad (22) \\
(1 + \delta)\theta_h^1 q_h - p_1 &= \delta(\theta_h^1 q_h - p_2). \quad (23)
\end{align*}

In period 2, those consumers whose types are in $T_L$ purchase $L$, and those in $T_H$ purchase $H$. Note that $p_2$ must be at least equal to $bq_h^2/2$, the marginal cost of $H$, and hence if $\theta_h^1 \leq bq_h/2$, then for all $\theta \leq \theta_h^1$, $\theta q_h \leq bq_h^2/2$, and the monopolist will not sell $H$ in period 2.

If $\theta_h^1 > bq_h/2$, then the monopolist’s problem in period 2 is:

$$\max_{\theta_h^2, \theta_h, q_l} \pi_2 = (p_2 - \frac{bq_h^2}{2})(\theta_h^1 - \theta_h^2) + (r - \frac{bq_h^2}{2})(\theta_h^2 - \theta_l). \quad (24)$$
subject to

\[ \theta_1^h \geq \theta_2^h \] \hspace{1cm} (25)

\[ \theta_2^h \geq \theta_l \] \hspace{1cm} (26)

(21), and (22).

Using (21) and (22), and ignoring the constraints (25) and (26), the first order conditions for \( \theta_2^h \), \( \theta_l \), and \( q_l \) are:

\[
(\theta_1^h - 2\theta_2^h)(q_h - q_l) + \frac{b(q_h^2 - q_l^2)}{2} = 0 \tag{27}
\]

\[
(\theta_1^h - 2\theta_l + \frac{bq_l}{2})q_l = 0 \tag{28}
\]

\[
bq_l + \theta_1^h - \theta_2^h - \theta_l = 0 \tag{29}
\]

and the solution is

\[
\theta_2^h = \frac{4\theta_1^h + 3bq_h}{8}, \quad \theta_l = \frac{4\theta_1^h + bq_h}{8}, \quad q_l = \frac{q_h}{2}. \tag{30}
\]

and therefore

\[
r = \frac{4\theta_1^h q_h + bq_h^2}{16}, \quad p_2 = \frac{2\theta_1^h q_h + bq_h^2}{4}. \tag{31}
\]

The period 2 profit is then

\[
\pi_2 = \frac{q_h[16(\theta_1^h)^2 - 16b\theta_1^h q_q + 5b^2 q_h^2]}{64},
\]

a function of \( q_h \) and \( \theta_1^h \).

From (30), \( \theta_2^h > \theta_l \) as long as \( \theta_2^h \) exists; and \( \theta_1^h \geq \theta_2^h \) if \( \theta_1^h \geq 3bq_h/4 \). If \( bq_h/2 < \theta_1^h < 3bq_h/4 \), then the constraint that \( \theta_1^h \geq \theta_2^h \) is binding, and the monopolist does not sell \( H \) in period 2.

Therefore, depending on the values of \( q_h \) and \( \theta_1^h \), there are two cases to consider:

(1) The monopolist sells both \( H \) and \( L \) in period 2: \( \theta_l < \theta_2^h \leq \theta_1^h \), which holds if \( \theta_1^h \geq 3bq_h/4 \).

(2) The monopolist sells only \( L \) in period 2: \( \theta_1^h < 3bq_h/4 \).
4.1 The monopolist sells both products in period 2

Suppose that $\theta_1^h \geq 3bq_h/4$, then $\theta_1 < \theta_2^h \leq \theta_1^h$. From (23) and (30), the firm’s first period price is

$$p_1 = \theta_1^h q_h + \delta \left( \frac{2\theta_1^h q_h + bq_h^2}{4} \right),$$

and the profit in period 1 is

$$\pi_1 = [\theta_1^h q_h + \delta \left( \frac{2\theta_1^h q_h + bq_h^2}{4} \right) - \frac{bq_h^2}{2}] (1 - \theta_1^h).$$

Then the monopolist in period 1 solves

$$\max_{\theta_1^h, q_h} \Pi = \pi_1 + \delta \pi_2$$

subject to the constraint that $\theta_1^h \geq \theta_2^h$, which, using (30), can be written as

$$\theta_1^h \geq \frac{3bq_h}{4}.$$  \hfill (33)

Let $\mu$ be the Lagrange multiplier, the first order conditions are:

$$\theta_1^h : \quad q_h [(-4 - \delta)\theta_1^h + (1 - \delta)bq_h + 2 + \delta] + 2\mu = 0,$$

$$q_h : \quad (-4 - \delta)(\theta_1^h)^2 + (4 + 2\delta)\theta_1^h + (4 - 4\delta)bq_h + (2\delta - 4)bq_h + \frac{15\delta^2q_h^2}{16} - 3b\mu = 0,$$

$$\mu (\theta_1^h - \frac{3bq_h}{4}) = 0; \quad \mu \geq 0.$$  \hfill (35)

First assume that the constraint is not binding, then $\mu = 0$ and the solution is:

$$\theta_1^h = \frac{160 - 88\delta + 106\delta^2 + 47\delta^3 - 4(1 - \delta)\sqrt{A}}{3(4 + \delta)(16 - 12\delta + 21\delta^2)},$$

$$q_{h,both} = \frac{4(16 + 4\delta^2 - \sqrt{A})}{3b(16 - 12\delta + 21\delta^2)},$$

where $A = 64 - 48\delta - 28\delta^2 - 216\delta^3 - 47\delta^4$, which is positive if $\delta < 0.51$.

The constraint (33) is not binding if $\delta < 2/7$. When $\delta \geq 2/7$, the constraint is binding. But
then the monopolist cannot sell a positive amount of $H$ in period 2, for $\theta_1^h - \theta_2^h = 0$; and the case is the same as the one that the monopolist sells only $L$ in period 2. When $\delta < 2/7$, denote by $\Pi^{both}$ the monopolist’s total profit from selling both products in period 2, then

$$\Pi^{both} = \frac{2(B + A^{3/2})}{27b(4 + \delta)(16 - 12\delta + 21\delta^2)^2}$$

(38)

where $B = 512 + 1152\delta + 1824\delta^2 + 5472\delta^3 + 1680\delta^4 + 1296\delta^5 + 314\delta^6$.

**Lemma 1.** The monopolist sells both $H$ and $L$ in period 2 only if $\delta < 2/7$.

### 4.2 The monopolist sells only $L$ in period 2

Suppose that $\theta_1^h \leq 3bq_h/4$, then the firm’s optimal strategy in period 2 is to sell only $L$. Since the price for $L$ is $r = \theta_lq_l$, the firm solves in period 2

$$\max_{\theta_l, q_l} \pi_2 = (\theta_lq_l - \frac{bq_l^2}{2})(\theta_1^h - \theta_l).$$

The solution is

$$\theta_l = \frac{2\theta_1^h}{3}, \quad q_l = \frac{2\theta_1^h}{3b}, \quad r = \frac{4(\theta_1^h)^2}{9b}$$

(39)

and the firm’s second period profit is

$$\pi_2 = \frac{2(\theta_1^h)^3}{27b}.$$ 

Given $\theta_1^h$, the first period price satisfies

$$(1 + \delta)\theta_1^h q_h - p_1 = \delta(\theta_1^h q_l - r)$$

and we have

$$p_1 = (1 + \delta)\theta_1^h q_h - \frac{2\delta(\theta_1^h)^2}{9b}.$$
The firm’s problem in period 1 is:

$$\max_{\theta^1_h, q_h} \Pi = [(1 + \delta) \theta^1_h q_h - \frac{2\delta (\theta^1_h)^2}{9b} - \frac{b q_h^2}{2}] (1 - \theta^1_h) + \delta \frac{2(\theta^1_h)^3}{27b}$$

s.t. $\theta^1_h \leq \frac{3bq_h}{4}$. \hfill (40)

Let $\lambda$ be the Lagrange multiplier, then the first order conditions are:

$$q_h : [(1 + \delta) \theta^1_h - bq_h] (1 - \theta^1_h) + \frac{3b\lambda}{4} = 0$$
$$\theta^1_h : [(1 + \delta)q_h - \delta \frac{4\theta^1_h}{9b}](1 - \theta^1_h) - (1 + \delta) \theta^1_h q_h + \delta \frac{4(\theta^1_h)^2}{9b} + \frac{b q_h^2}{2} - \lambda = 0$$
$$\lambda \left(\frac{3bq_h}{4} - \theta^1_h\right) = 0; \ \lambda \geq 0$$

First assume that the constraint is not binding and so $\lambda = 0$. The solution is

$$\theta^1_h = \frac{2(9 + 14\delta + 9\delta^2)}{27 + 38\delta + 27\delta^2} \hfill (42)$$
$$q_h = \frac{(1 + \delta) \theta^1_h}{b}$$
$$= \frac{2(1 + \delta)(9 + 14\delta + 9\delta^2)}{b(27 + 38\delta + 27\delta^2)}. \hfill (43)$$

If $\delta > 1/3$, then $\theta^1_h < \frac{3bq_h}{4}$ as required. If $\delta \leq 1/3$, the constraint is binding, and we have $\lambda > 0$ and $\theta^1_h = \frac{3bq_h}{4}$. The solution now becomes:

$$\theta^1_h = \frac{2 + 5\delta}{3 + 7\delta} \hfill (44)$$
$$q_h = \frac{4(2 + 5\delta)}{3b(3 + 7\delta)}. \hfill (45)$$

Denote by $\Pi^{L1}$ and $\Pi^{L2}$ the monopolist’s total profit from selling only $L$ in period 2 when $\delta \leq 1/3$ and $\delta > 1/3$, respectively. Then

$$\Pi^{L1} = \frac{2(2 + 5\delta)^3}{27b(3 + 7\delta)^2} \hfill (46)$$
$$\Pi^{L2} = \frac{2(9 + 14\delta + 9\delta^2)^3}{27b(27 + 38\delta + 27\delta^2)^2}. \hfill (47)$$
4.3 The Subgame Perfect Equilibrium

From lemma 1, we know that if $\delta \geq 2/7$, the monopolist will not sell both $H$ and $L$ in period 2. Using the results in section 4.2, we have the next lemma.

**Lemma 2.** If $\delta \geq 2/7$, the monopolist sells only $L$ in period 2, and the qualities of $H$ and $L$ are functions of $\delta$:

- If $2/7 \leq \delta \leq 1/3$, $q_h = q^{L1}_h = \frac{4(2 + 5\delta)}{3b(3 + 7\delta)}$, $q^{L1}_L = \frac{q^{L1}_h}{2}$;
- If $\delta \geq 1/3$, $q_h = q^{L2}_h = \frac{(1 + \delta)(18 + 28\delta + 18\delta^2)}{b(27 + 38\delta + 27\delta^2)}$, $q^{L2}_L = \frac{q^{L2}_h}{2}$.

If $\delta < 2/7$, we need to compare the monopolist’s total profit from selling both products in period 2, $\Pi^{both}$, with that from selling only $L$, $\Pi^{L1}$. Since $\Pi^{both} > \Pi^{L1}$ for all $\delta < 2/7$, we have the next result.

**Proposition 2.** If the monopolist cannot commit, then (a) when $\delta < 2/7$, it sells both $H$ and $L$ in period 2, and sets the quality of $H$ as $q^{both}_h$ of (37), and the quality of $L$ as $q^{both}_h/2$; (b) when $\delta \geq 2/7$, it sells only $L$ in period 2, and the qualities of $H$ and $L$ are as given in lemma 2.

We can now compare the monopolist’s quality choice with the optimal $q^*_h$ and $q^*_L$, given in (8) and (9), respectively. We find that the monopolist airways selects higher-than-optimum qualities.

**Proposition 3.** When the monopolist cannot commit and $\delta > 0$, the monopolist always selects a higher-than-optimum quality of $H$ and $L$.

Figure 1 plots the $q^*_H$ and the monopolist’s choices of $q_H$.

The “over-screening” result is due to the monopolist’s lack of commitment power. As we have shown, when the monopolist can commit, it will not distort the qualities of its products. But when it cannot commit, it needs to deviate its quality choices from the optimum. In static quality discrimination models, the monopolist keeps the quality for high-valuation consumers at the optimum and lowers the quality for low-valuation consumers. This strategy is not profit maximizing in our dynamic setup, for it does not stress the lack-of-commitment problem. In
In our case, the quality of $L$ is linked with the quality of $H$ in the subgame of period 2, and therefore the monopolist cannot distort $q_L$ only.

## 5 Conclusion

We have shown that when it takes time for a durable goods monopolist to extend its product line to the lower end, the qualities of both its high-end and low-end products will be higher than the optimum. This intertemporal quality discrimination problem differs from the static quality discrimination problems in that there is no commitment problem in static models. To counter the Coase problem, the monopolist raises the quality of its high-end product to exceed the optimum so that the consumers does not want to postpone their purchases.

## References


