Wage Bargaining under the National Labor Relations Act

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Abstract

Sections 8(a)(3) and 8(a)(5) of the National Labor Relations Act prevent a firm from unilaterally increasing the wage it pays the union during the negotiation of a new wage contract. To understand this regulation, we study a counterfactual negotiation model where the firm can temporarily increase compensation to its employees during wage negotiations. Comparing this to the case where the firm does not have this option, we show that the firm may strategically increase the union’s temporary wage to upset the union’s incentive to strike, decreasing the union’s bargaining power, and shrinking the set of permanent wage contracts that may arise in a perfect equilibrium. As the union becomes more patient, the best possible equilibrium contract to the union gets worse. In the limit, the uniqueness and hence the full efficiency of the perfect equilibrium are restored. We also demonstrate that allowing the union to refuse the firm’s temporary compensation does not affect the set of perfect equilibrium outcomes.

JEL Classification: C72 Noncooperative Games, C73 Stochastic and Dynamic Games, C78 Bargaining Theory

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1 Introduction

Created in 1935, the National Labor Relations Act (NLRA), also known as the Wagner Act, marked the federal government’s first comprehensive legislation supporting unionization and collective bargaining in the United States. Section 8(d) of the NLRA specifies that “the employer and the representative of the employees to meet at reasonable times and confer in good faith with respect to wages, hours, and other terms or conditions of employment.” The Supreme Court has interpreted this good faith provision of the NLRA as making unlawful for the management to alter any terms without the union’s consent during the negotiation (see pages 188-189 of Leslie, 2000). The Court’s reason for this is not to allow a situation where the management could undermine the union’s authority to negotiate and to represent the employees. In this vein, the Court even considers it unlawful for the management to temporarily increase the wages it pays labor while a new contract is being negotiated.

In this paper we examine the implications of this aspect of the NLRA that prevents a firm from offering additional compensation to the union during a wage negotiation. Aside from undermining the union’s authority, offering additional compensation has a strategic effect: additional compensation raises the union’s opportunity cost of striking. How does the NLRA affect the players’ bargaining behavior and the general efficiency of equilibrium outcomes? To address these issues, we start with the contract negotiation model of Haller and Holden (1990) and Fernandez and Glazer (1991) (also see Busch and Wen (1995), Houba (1997), and Muthoo (1999) for the general negotiation model). In contrast to the standard bilateral bargaining model of Rubinstein (1982), the players’ payoffs during disagreement are determined by a normal form game, called the disagreement game. This disagreement game captures the strategic relationship between the two players other than bargaining. For instance, in the contract negotiation model, the union may choose to either strike or work under the expired contract while bargaining over a new contract. It is quite common that workers continue to work under the expired contract, such as the recent negotiation between Verizon and its union. The negotiation model is also related to the money burning literature.
in non-cooperative bargaining; see Avery and Zemsky (1994), Busch, Shi and Wen (1998), and Manzini (1999). It has been shown that a negotiation game generally admits multiple perfect equilibrium outcomes, including inefficient outcomes with delayed agreements. In the contract negotiation model, in particular, the worst equilibrium contract for the union turns out to be the expired wage contract, which can be obtained if the union keeps working under the expired contract. In order to obtain the best equilibrium contract to the union, the union must adopt a non-stationary strategy, striking whenever the firm rejects its proposal, but working whenever it rejects the firm’s proposal. In doing so, the union would be able to impose the highest possible cost to the firm when the firm rejects the union’s offer. At the same time, this striking strategy minimizes the cost the union would have to bear if it rejects the firm’s offer.

The contract negotiation model does not consider the possibility that the firm may choose to temporarily increase compensation to the union. In order to conduct this research project, we will compare two non-cooperative bargaining models. We first study a bargaining model that generalizes the original contract negotiation model by allowing the firm to temporarily increase compensation before the union decides whether to strike during the current period of disagreement. Although both the union and the firm may strategically affect their disagreement payoffs, we demonstrate that this model cannot be analyzed under the framework of Busch and Wen (1995). The non-normal form disagreement game typically imposes additional restrictions on the equilibrium strategy profile. We find that in some situations, the firm does have an incentive to increase compensation to the union before reaching a new wage contract. By doing so, the firm would be able to lessen the union’s incentive to strike, and hence prevent the non-stationary striking behavior the union needs to obtain its best equilibrium outcome. This model may still have multiple equilibrium outcomes, including inefficient outcomes. However, the firm’s ability to temporarily raise compensation increases the efficiency of perfect equilibrium outcomes in general.

The NLRA permits a temporary wage increase by the firm if the union approves; it is the
unilateral wage increase that is deemed unlawful. In order to analyze the effects of the NLRA, we then study a second model where the union may refuse the firm’s additional compensation. We add the union’s consent to the firm’s compensation into our first model. The firm may still offer additional compensation to the union, but the additional compensation becomes effective only after the union’s approval.\footnote{The Court deems that it is the firm’s unilateral compensation that is at odds with the NLRA, see page 188-189 of Leslie (2000).} We show that, however, the union cannot credibly refuse the firm’s additional compensation if the firm chooses to offer in the best possible equilibrium to the union. Therefore, allowing the union’s consent will not alter the set of perfect equilibrium outcomes. The strategic effect of increasing temporary wage remains valid even when we allow the union to block the firm’s action.

In the next section we describe a non-cooperative bargaining model where the firm may unilaterally increase the union’s compensation in any period. In Section 3, we analyze perfect equilibrium outcomes, particularly the best and the worst equilibrium outcomes to the union. We characterize how the union’s best equilibrium can be drastically affected by the firm’s ability to temporarily increase wage, upsetting the union’s incentive to strike. We modify our model in Section 4 to allow the union to block the firm’s offer before choosing between striking and working in any period. We show that the set of perfect equilibrium outcomes is not affected by such a modification. Lastly, we summarize the paper and provide a few concluding remarks in Section 5.

2 A Model with Unilateral Compensation

Consider a situation where a union and a firm negotiate a new wage contract that specifies how to share the firm’s future gross profit, normalized to one per period over an infinite horizon. The expired wage contract is denoted as $w^0 \in [0, 1]$. The union must be paid at least $w^0$ per period if the union works during the contract negotiation.

The negotiation proceeds with alternating offers as in Rubinstein’s (1982) model. The union proposes in odd periods and the firm proposes in even periods. There are four stages
in every bargaining period. In any odd period, including the first period, such that no agreement has been reached, the union proposes a wage demand \( w' \in [0, 1] \) in the first stage, and then the firm decides whether to accept the union’s demand in the second stage. If the firm accepts the union’s demand, the negotiation ends. If the firm rejects the union’s demand, the negotiation proceeds to the third stage where the firm may offer a temporary compensation \( c' \geq w^0 \) to the union for the current period.\(^2\) In the fourth stage after observing the firm’s compensation \( c' \), the union decides whether to work in the current period (in which case the union receives \( c' \) and the firm receives \( 1 - c' \)), or to strike (in which case both the union and the firm receive 0). The negotiation then proceeds to the following even period.

Similarly, there are four stages in any even period. The firm offers an wage contract \( w'' \in [0, 1] \) in the first stage and the union decides whether to accept the firm’s offer in the second stage. The union’s acceptance concludes the negotiation. Otherwise, the union’s rejection leads the negotiation to the third stage, where the firm may offer a compensation \( c'' \geq w^0 \) for the current period. In the fourth stage, the union decides whether to work (in which case the union receives \( c'' \) and the firm receives \( 1 - c'' \)), or to strike (in which case both the union and the firm receive 0). The following Figure 1 illustrates this negotiation process in which the firm may unilaterally offer temporary compensation to the union during the contract negotiation:

![Diagram of contract negotiation with unilateral compensation](image)

Figure 1. Contract negotiation with unilateral compensation.

The model presented above has perfect information, so histories and strategies are defined

\(^2\)There is no upper bound for the firm’s compensation. Presumably the firm can borrow to finance the compensation. However, we will see that the firm will never compensate more than its gross profit in equilibrium.
in the usual fashion. For example, a history consists of all past contract proposals and rejections, all past compensations offered by the firm, and all past decisions by the union on whether to work or to strike. A strategy assigns a feasible action to the acting party after each possible finite history. Every strategy profile induces a unique probability distribution on the set of pure outcome paths. Denote a generic pure outcome path as \( \pi = (d^1, d^2, \ldots, d^{T-1}, a^T) \), where \( d^t \in \mathbb{R}^2 \) is the interim disagreement payoff vector in period \( t \) such that for \( 0 \leq t < T \),

\[
d^t = (d^t_u, d^t_f) = \begin{cases} 
(0, 0) & \text{if the union strikes in period } t, \\
(c, 1-c) & \text{if the union works for } c \geq w^0 \text{ in period } t,
\end{cases}
\]

and \( a^T \in \Delta^1 \) (the unit simplex in \( \mathbb{R}^2 \)) represents the agreement reached in period \( T \geq 1 \). An outcome with perpetual disagreement is represented by a infinite sequence of interim disagreement payoff vectors (or equivalently \( T = \infty \)). From such a generic outcome path \( \pi \), average discounted payoffs to the union and the firm are

\[
(1 - \delta_i) \sum_{t=1}^{T-1} \delta_i^{t-1} d^t_i + \delta^{T-1} a_i, \quad \text{for } i = u, f,
\]

where \( (\delta_u, \delta_f) \in (0, 1)^2 \) are the union’s and firm’s discount factors per bargaining period. In this paper, we will adopt the concept of subgame perfect Nash equilibrium, which induces a Nash equilibrium in every subgame after any possible finite history. Hereafter, we simply refer to a subgame perfect Nash equilibrium as an equilibrium.

Our model generalizes the contract negotiation model of Haller and Holden (1990) and Fernandez and Glazer (1991) by allowing the firm to offer temporary compensation to the union. In other words, if the firm is restricted to offer \( c = w^0 \) in every stage 3, then the model described here is equivalent to the contract negotiation model. Now we review some of the key results from the contract negotiation model. The following Proposition 1 asserts the lowest and the highest equilibrium contracts to the union in the contract negotiation model:

**Proposition 1** In the contract negotiation model (i.e., where \( c = w^0 \)),

(i) the lowest equilibrium contract in any period is \( w^0 \) for all \( (\delta_u, \delta_f) \in (0, 1)^2 \);
(ii) the highest equilibrium contract in any odd period is

\[ M_u = \begin{cases} w^0 & \text{if } (\delta_u, \delta_f) \not\in A, \\ \frac{w^0 (1-\delta_f)+\delta_f (1-\delta_u)w^0}{1-\delta_u \delta_f} & \text{if } (\delta_u, \delta_f) \in A, \end{cases} \]

and the highest equilibrium contract in any even period is

\[ 1 - m_f = (1 - \delta_u)w^0 + \delta_u M_u, \quad \text{where} \]

\[ A = \left\{ (\delta_u, \delta_f) \in (0,1)^2 : \delta_f \leq \delta_f^A(\delta_u, w^0) = \frac{\delta_u^2 - (1 - \delta_u + \delta_u^2)w^0}{\delta_u^2 - \delta_u w^0} \right\}. \]

**Proof:** See Lemmas 2 and 4 of Fernandez and Glazer (1991). Q.E.D.

This contract negotiation model has multiple equilibrium outcomes, including inefficient ones,\(^4\) if and only if \((\delta_u, \delta_f) \in A\) for any given \(w^0 \in [0,1]\). Note that \((\delta_u, \delta_f) \in A\) if and only if

\[ w^0 \leq \delta_u (1 - m_f) = \frac{\delta_u^2 (1 - \delta_f) + \delta_u (1 - \delta_u)w^0}{1 - \delta_u \delta_f}. \]

Condition (4) ensures the subgame perfection of a strategy profile in which the union strikes in any odd period after the firm rejects the union’s demand, but works in any even period after the union rejects the firm’s offer. The right side of (4) represents the union’s highest possible continuation payoff if the union strikes in an odd period, while the left side of (4) represents the union’s lowest possible continuation payoff if the union works in an odd period. More specifically, if the union works then the union will be punished by the lowest equilibrium contract \(w^0\) in the continuation game. If the union strikes then the union will be rewarded by the highest equilibrium contract \(1 - m_f\) in the continuation game. Condition (4) ensures that the union will strike in an odd period to punish the firm for rejecting its offer. Given the union’s alternating strategies between work and strike, the firm’s interim disagreement payoff is 0 when the firm responds to the union’s demand, and the union’s interim disagreement payoff is \(w^0\) when the firm responds to the firm’s offer. Equilibrium

\(^3\)By convention, \(M_u\) denotes the union’s highest equilibrium payoff in an odd period and \(m_f\) denotes firm’s lowest equilibrium payoff in an even period.

\(^4\)When there are multiple equilibria, they can be used to support equilibria with delayed agreement, see for example, pages 50-51 of Osborne and Rubinstein (1990).
contracts, $M_u$ in an odd period and $1 - m_f$ in an even period, correspond to the stationary equilibrium outcomes with interim disagreement payoff $w^0$ to the union and 0 to the firm.

As a special case when the union and the firm have a common discount factor $\delta \in (0, 1)$, Proposition 1 simplifies to:

**Corollary 1.1** If the firm is not allowed to offer any additional compensation, and the union and the firm have a common discount factor $\delta \in (0, 1)$, then

(i) the lowest equilibrium contract in any period is $w^0$ for all $\delta \in (0, 1)$;

(ii) the highest equilibrium contracts in an odd and an even periods are, respectively,

$$M_u = \begin{cases} w^0 & \text{if } 0 < \delta < \sqrt{w^0}, \\ \frac{1+\delta w^0}{1+\delta} & \text{if } \sqrt{w^0} \leq \delta < 1, \end{cases} \quad 1 - m_f = \begin{cases} w^0 & \text{if } 0 < \delta < \sqrt{w^0}, \\ \frac{\delta w^0}{1+\delta} & \text{if } \sqrt{w^0} \leq \delta < 1. \end{cases}$$

In Appendix A, we demonstrate that the model described here cannot be analyzed by the general negotiation model of Busch and Wen (1995), where the disagreement game is a static game given in normal form. The reason is that the disagreement game in our model, where the firm offers compensation and then the union decides to either work or strike, is not a static game but a dynamic game. In Appendix A, we show that treating this dynamic disagreement game in its normal form does not alter the set of equilibrium payoffs characterized by Proposition 1. However, when we treat the disagreement in its original extensive form, we show in this paper that the highest equilibrium contract in an odd period is sometimes strictly less than $M_u$. Subgame perfection imposes constraints on the additional subgames that begin during the dynamic disagreement game.

### 3 Equilibrium Analysis

In this section, we investigate equilibrium outcomes in our model where the firm may unilaterally compensate the union during the contract negotiation. We will derive a range of equilibrium contracts. Our model has multiple equilibria whenever the contract negotiation model has multiple equilibria. Comparing with the contract negotiation model, we identify
three sets of discount factors under which the firm behaves quite differently in the best equilibrium to the union. On one extreme when the union is sufficiently impatient relative to the firm, the firm does not have to compensate the union since there is a unique equilibrium that leads to the lowest equilibrium contract. On the other extreme when the union is sufficiently patient relative to the firm, the firm has incentive to offer additional compensation in order to induce the union to work in every odd period. The firm benefits from compensating the union since the highest equilibrium contract is actually less than that if the firm does not compensate the union. When the union’s discount factor is in an intermediate range, the firm chooses not to compensate the union since it is too costly to induce the union to work in every period. It is worthwhile to notice the firm’s different behavior when there are multiple equilibria. Given the firm’s discount factor, the highest equilibrium contract eventually falls with respect to the union’s discount factor, a result that is quite counter-intuitive. As the union becomes sufficiently patient, any equilibrium contract will be arbitrarily close to the expired contract, which is also the lowest equilibrium contract.

3.1 The Lowest Equilibrium Contract

We begin the analysis of our model by establishing the existence of a simple equilibrium for all discount factors \((\delta_u, \delta_f) \in (0, 1)^2\) and all expired wage contracts \(w^0 \in [0, 1]\). As in the contract negotiation model, \(w^0\) is the lowest equilibrium contract for all possible discount factors.

**Proposition 2** For all \((\delta_u, \delta_f) \in (0, 1)^2\) and \(w^0 \in [0, 1]\), there is an efficient equilibrium where the union and the firm agree on \(w^0\) in the first period.

**Proof:** See Appendix B. Q.E.D.

The equilibrium of Proposition 2 is supported by a simple and stationary strategy profile, in which the union always demands \(w^0\) and rejects any offer that is lower than \(w^0\), the firm always offers \(w^0\) and rejects any demand that is higher than \(w^0\), the firm never offers any additional compensation, and the union always works. The proof of Proposition 2 shows
that neither the union nor the firm has any incentive to deviate from this prescribed strategy profile. It is obvious that $w^0$ is also the lowest equilibrium contract since the union can choose to work and receive at least $w^0$ in every period during the course of contract negotiation. Now we state this result as

**Proposition 3** For all $(\delta_u, \delta_f) \in (0, 1)^2$ and $w^0 \in [0, 1]$, the union never receives less than $w^0$ in any equilibrium.

### 3.2 The Highest Equilibrium Contract: Conditions

With the existence of an equilibrium, we now turn our attention to the highest equilibrium contract. Let $M_u^*$ be the supremum of the union’s equilibrium payoffs in any odd period, and $m_f^*$ be the infimum of the firm’s equilibrium payoffs in any even period. We use $M_u^*$ and $m_f^*$ here to distinguish them from those in the contract negotiation model. The supremum of the union’s equilibrium payoffs in any even period is thereby $1 - m_f^*$. From the setup of the model and existence result of Proposition 2, both $M_u^*$ and $m_f^*$ are well defined functions of $(\delta_u, \delta_f)$ and $w^0 \in [0, 1]$. Proposition 3 implies that

$$M_u^* \geq w^0 \quad \text{and} \quad 1 - m_f^* \geq w^0.$$

Similar to the backward induction technique by Shaked and Sutton (1984), we now derive a set of necessary conditions for $M_u^*$ and $m_f^*$, imposed by subgame perfection. First, consider an even period where the firm makes an offer. By subgame perfection, since the union’s payoffs in next (odd) period cannot exceed $M_u^*$, and the union’s payoff during the current even period cannot exceed $c''$ (if the firm compensates $c''$ and the union works), the union’s payoff from rejecting a firm’s offer cannot exceed $(1 - \delta_u) c'' + \delta M_u^*$. This implies that the union will accept any offer that exceeds $(1 - \delta_u) c'' + \delta M_u^*$. Therefore in any equilibrium, the firm cannot receive less than (recall that the firm chooses $c'' \geq w^0$)

$$m_f^* = \max_{c'' \geq w^0} [1 - (1 - \delta_u) c'' - \delta_u M_u^*] = 1 - [(1 - \delta_u) w^0 + \delta_u M_u^*], \quad (5)$$
in any even period by making an offer sufficiently high (such as $1 - m_f^*$) to induce the union to accept while offering no additional compensation ($c'' = w^0$) if the union rejects.

Next consider an odd period where the union proposes a contract demand. Recall that there are four stages in an odd period. In the last stage after the union rejects the firm’s offer and the firm offers $c' \geq w^0$, the union decides to strike or work during the current (odd) period. The firm is able to induce the union to work by offering a sufficiently high compensation $c'$ such that

$$
(1 - \delta_u)c' + \delta_u w^0 \geq \delta_u (1 - m_f^*).
$$

The left hand side of (6) represents the union’s lowest possible continuation value if the union works under compensation $c'$, while the right hand side of (6) is the union’s highest possible continuation value if the union strikes. Condition (6) states that the union has a higher payoff from working than from striking.

If the firm chooses to induce the union to work with $c'$ that satisfies condition (6), the firm will receive at least

$$
(1 - \delta_f)(1 - c') + \delta_f m_f^*.
$$

Alternatively, the firm may choose not to offer any additional compensation to the union. As in the contract negotiation model, the union may chooses to strike during the current odd period. Therefore, if the firm chooses not to offer any additional compensation to the union, Proposition 1 applies and the firm’s equilibrium payoffs are not less than $1 - M_u$. To summarize, the firm chooses between these two alternatives and so the union’s equilibrium payoffs in an odd period are not higher than

$$
M_u^* = 1 - \max \left\{ 1 - M_u, \sup_{s.t.(6)} [(1 - \delta_f)(1 - c') + \delta_f m_f^*] \right\}
= \min \left\{ M_u, 1 - \sup_{s.t.(6)} [(1 - \delta_f)(1 - c') + \delta_f m_f^*] \right\}.
$$

Note that in order to induce the union to work during the current (odd) period, the firm’s compensation $c'$ must satisfy condition (6). Now we state these arguments as
Proposition 4 For all \((\delta_u, \delta_f) \in (0, 1)^2\) and \(w^0 \in [0, 1]\), \(M_u^*\) and \(m_f^*\) satisfy (5) and (7).

In other words, conditions (5) and (7) are necessary for the highest equilibrium contracts \(M_u^*\) in every odd period and \(1 - m_f^*\) in every even period. The remaining task is then to solve \(M_u^*\) and \(m_f^*\) from (5) and (7).

3.3 Incentive to Compensate

Instead of solving \(M_u^*\) and \(m_f^*\) directly from (5) and (7), we will utilize the results we have so far to pin down the values of \(M_u^*\) and \(m_f^*\) for all \((\delta_u, \delta_f) \in (0, 1)^2\) and \(w_0 \in [0, 1]\).

Proposition 3 states that \(w_0\) is the lowest equilibrium contract. When \((\delta_u, \delta_f) \notin A\), \(w_0\) is also the unique equilibrium contract if the firm does not offer any additional compensation. It is obvious then that when \((\delta_u, \delta_f) \notin A\), the firm should not offer any additional compensation to the union.

Lemma 1 When \((\delta_u, \delta_f) \notin A\), we have that \(M_u^* = 1 - m_f^* = w_0\).

When \((\delta_u, \delta_f) \in A\), Proposition 1 asserts that

\[
M_u = \frac{(1 - \delta_f) + \delta_f(1 - \delta_u)w^0}{1 - \delta_u \delta_f}. \tag{8}
\]

To obtain the \(M_u^*\) and \(m_f^*\), condition (5) states that the firm should not offer any additional compensation to the union and the union should work in every even period.

Suppose that the firm chooses to induce the union to work in an odd period with \(c' \geq w^0\) that satisfies condition (6), then the proposals that are consistent with the subgame perfection must satisfy the following equations:

\[
1 - M_u' = (1 - \delta_f)(1 - c') + \delta_f m_f', \tag{9}
\]

\[
1 - m_f' = (1 - \delta_u)w^0 + \delta_u M_u'. \tag{10}
\]

Equation (9) states that the firm is indifferent between accepting contract \(M_u'\) and rejecting it (after which collecting \(1 - c'\) in the current odd period and \(m_f'\) in the following even
Equation (10) states the union is indifferent between accepting contract $1 - m'_f$ and rejecting it (after which collecting $w^0$ in the current even period and $M'_u$ in the following odd period). Equations (9) and (10) yield

$$M'_u = \frac{(1 - \delta_f)c' + \delta_f(1 - \delta_u)w^0}{1 - \delta_u \delta_f}.$$  

(11)

Note that $M'_u$ by (11) is increasing with respect to $c'$ and is equal to $M_u$ at $c' = 1$. In the best possible equilibrium to the union, if the firm is able to induce the union to work in an odd period with $c' \geq w^0$, then $M'_u$ by (11) will be the highest equilibrium contract in an odd period. Comparing $M'_u$ in (11) and $M_u$ in (8) when $(\delta_u, \delta_f) \in A$, it is obvious that

$$M'_u \leq M_u \text{ if and only if } c' \leq 1.$$

This result is quite intuitive and important. Since it is always costly to the firm if the union strikes after firm’s rejection, the firm benefits if the firm can successfully induce the union to work in an odd period without compensating the union more than its gross profit. Otherwise, it is too costly for the firm to induce the union to work, and the firm is better off by not compensating the union more than $w^0$ in an odd period. Proposition 5 asserts that the threshold where the firm is just indifferent between offering additional compensation and not offering additional compensation, the necessary compensation needed to induce the union to work must be equal to 1.

**Proposition 5** If the firm can definitely induce the union to work with $c' \leq 1$ in an odd period, then the firm will do so in the union’s best possible equilibrium.

From condition (6), the optimal (the lowest necessary) compensation needed to induce the union to work in an odd period is

$$c^* = \frac{\delta_u(1 - m'_f - w^0)}{1 - \delta_u}.$$  

(12)

At the threshold where the firm is indifferent between compensating the union with $c^* = 1$ and not offering any compensation, the firm has the same (lowest) equilibrium payoff from
the either alternative. Setting \( c^* = 1 \), equation (12) yields

\[
m_f^* = \frac{2\delta_u - 1}{\delta_u} - w^0. \tag{13}
\]

The value of \( m_f^* \) given by (13) is the critical value such that if the firm’s lowest equilibrium payoff is higher than the right hand side of (13), the optimal compensation \( c^* \) will be less than 1, and so the firm will offer \( c^* \) to the union. Otherwise, the firm will not offer any additional compensation to the union. At such a threshold, the firm has the same (lowest) equilibrium payoff between the two alternatives: compensating and not compensating. This implies that

\[
1 - m_f^* = 1 - m_f = (1 - \delta_u)w^0 + \delta_u M_u,
\]

\[
\Rightarrow \quad \frac{1 - \delta_u}{\delta_u} + w^0 = (1 - \delta_u)w^0 + \delta_u (1 - \delta_f) + \delta_f (1 - \delta_u)w^0.
\tag{14}
\]

Solving \( \delta_f \) from (14) in terms of \( \delta_u \) and \( w^0 \), we obtain

\[
\delta_f^B(\delta_u, w^0) = \frac{(1 - w^0)\delta_u^2 + \delta_u - 1}{(2 - w^0)\delta_u^2 - \delta_u}. \tag{15}
\]

Define the set \( B \) as

\[
B = \left\{ (\delta_u, \delta_f) \in (0, 1)^2 : \delta_f \leq \delta_f^B(\delta_u, w^0) \right\}. \tag{16}
\]

We will show that the firm will choose to induce the union to work in every odd period if and only if \((\delta_u, \delta_f) \in B\). The following lemma asserts that the fact of \( B \subset A \) for all \( w^0 \in [0, 1] \), as well as a few other properties of sets \( A \) and \( B \):

**Lemma 2** Given \( w^0 \in [0, 1] \), we have

(i) \( \delta_f^A(\delta_u, w^0) \in (0, 1) \) for all \( \delta_u \in \left( \frac{\sqrt{5 - 4w^0} - 1}{2 - 2w^0}, 1 \right) \);

(ii) \( \delta_f^B(\delta_u, w^0) \in (0, 1) \) for all \( \delta_u \in \left( \frac{\sqrt{5 - 4w^0} - 1}{2 - 2w^0}, 1 \right) \);

(iii) \( \delta_f^A(\delta_u, w^0) > \delta_f^B(\delta_u, w^0) \) for all \( \delta_u \in \left( \frac{\sqrt{5 - 4w^0} - 1}{2 - 2w^0}, 1 \right) \);

(iv) \( \delta_f^A(1, w_0) = \delta_f^B(1, w_0) = 1 \), and

\[
\frac{\partial \delta_f^A(\delta_u, w^0)}{\partial \delta_u} = \frac{\partial \delta_f^B(\delta_u, w^0)}{\partial \delta_u} = 0 \quad \text{at} \ \delta_u = 1.
\]
Part (iii) of Lemma 2 implies that $A \subset B$, as illustrated in the following Figure 2:

![Figure 2. Sets A and B of $(\delta_u, \delta_f)$.](image)

**3.4 Values of $M_u^*$ and $m_f^*$**

To solve the value of $M_u^*$ in terms of $(\delta_u, \delta_f)$ and $w^0$, we first compute the corresponding value $M_u^*$ when the firm offers the optimal compensation $c^*$ by (6), then compare $M_u^*$ with $M_u$ to determine the value of $M_u^*$. If the firm offers $c^*$ in every odd period then the union will work in every odd period.\(^5\) Substituting $c' = c^*$ of (12) into (9), we obtain

$$
\tilde{M}_u^* = \frac{\delta_u + \delta_f - 2\delta_u\delta_f(1 - m_f^*) - \delta_u(1 - \delta_f)}{1 - \delta_u} w^0.
$$

Equations (5) and (17) yield the corresponding value of $M_u^*$ when the firm offers $c^*$ to the union and so the union works in every odd period, we have

$$
\tilde{M}_u^* = \frac{\delta_f - 2\delta_u\delta_f + 2\delta_u^2\delta_f}{1 - \delta_u - \delta_u\delta_f + 2\delta_u^2\delta_f} w^0.
$$

\(^5\)For the sake of argument, the firm could offer slightly higher than $c^*$ so that the union strictly prefers working over striking.
Equation (18) gives the highest equilibrium contract when the firm offers the optimal compensation $c^*$ to the union in every odd period.

From the construction, it is easy to see that on the boundary of set $B$ where $\delta_f = \delta_f^B(\delta_u, w^0)$, the firm has the same interim disagreement payoff of zero from either compensating the union with its entire gross profit in every odd period or not compensating the union at all so that the union will strike in every odd period. Recall that (18) gives the highest equilibrium contract when the firm provides just sufficient compensation to avoid the union’s striking in every odd period. To summarize, we have

**Proposition 6** Conditions (5) and (7) yield the union’s highest equilibrium contract in an odd period as

$$M^*_u = \begin{cases} M_u & \text{if } (\delta_u, \delta_f) \not\in B \\ M^*_u & \text{if } (\delta_u, \delta_f) \in B \end{cases} = \begin{cases} w^0 & \text{if } (\delta_u, \delta_f) \not\in A \\ \frac{1}{1-\delta_u^{\delta_f}} & \text{if } (\delta_u, \delta_f) \in A \setminus B \\ \frac{1}{1-\delta_u^{\delta_f} - \delta_u^{\delta_f} + 2\delta_u^{\delta_f}} w^0 & \text{if } (\delta_u, \delta_f) \in B \end{cases}$$

As a special case when the union and the firm have a common discount factor $\delta \in (0, 1)$, Proposition 6 simplifies to

**Corollary 6.1** When $\delta_u = \delta_f = \delta \in (0, 1)$, we have

$$M^*_u = \begin{cases} w^0 & \text{if } \delta \in (0, \sqrt{w^0}) \\ \frac{1+\delta w^0}{1+\delta} & \text{if } \delta \in [\sqrt{w^0}, \frac{1}{\sqrt{2-w^0}}] \\ \frac{2\delta^2 - \delta w^0}{2\delta^2 - 1} w^0 & \text{if } \delta \in ([\frac{1}{\sqrt{2-w^0}}, 1]) \end{cases}$$

The highest possible equilibrium contract $M^*_u$ can be supported by an equilibrium for all $(\delta_u, \delta_f) \not\in B$ in the same way as in the contract negotiation model. When $(\delta_u, \delta_f) \in B$, supporting $M^*_u$ in an equilibrium involves the inefficient continuation payoff, such as payoff vector $(w^0, m_f^*)$ in the subgame after the union works under optimal compensation $c^*$ in the previous odd period. If inefficient proposal is feasible then $M^*_u$ can be easily supported by equilibrium. Otherwise, Proposition 6 provides an upper bound of all equilibrium contracts.

Compared with the contract negotiation model, our Proposition 6 implies that the firm benefits from its ability to compensate when the union’s and the firm’s discount factors lie.
in set $B$. The lowest equilibrium contract is unaffected by the firm’s ability to compensate. This means that allowing the firm to compensate the union generally improves the efficiency of equilibrium outcomes, but in a somewhat lopsided way. The firm’s ability to compensate the union may limit the highest equilibrium contract to the union. This effect depends on the union’s discount factor and firm’s discount factor. Note that

$$\lim_{\delta_u \to 1} M^*_u = \lim_{\delta_u \to 1} \frac{\delta_f - \delta_u^2 - 2\delta_u \delta_f + 2\delta_u^2 \delta_f}{1 - \delta_u - \delta_u^2 - \delta_u \delta_f + 2\delta_u^2 \delta_f} w^0 = \frac{\delta_f - 1}{\delta_f - 1} w^0 = w^0.$$ 

It becomes so dramatic that as the union becomes more and more patient relative to the firm, any equilibrium contract will be sufficiently close to the expired contract $w^0$, which is the lowest equilibrium contract to the union.

**Proposition 7** For any given $\delta_f \in (0, 1) \text{ and } w^0 \in [0, 1]$, we have $\lim_{\delta_u \to 1} M^*_u = w^0$.

Figure 3 below illustrates $M_u$ and $M^*_u$ for a given value of $\delta_f$. Notice that $M^*_u = M^*_u$ for $\delta_u \leq \delta_u$, where $(\delta_u, \delta_f) \in B$ for all $\delta_u \geq \delta_u$. When $\delta_u \geq \delta_u$, $M_u$ is increasing (to one as $\delta_u$ goes to one), but $M^*_u$ is decreasing (to $w^0$ as $\delta_u$ goes to one).

![Figure 3. $M_u$ and $M^*_u$ for $\delta_f = 0.8$ and $w^0 = 0.4$.](image-url)
Propositions 6 and 7 (also Figure 3) suggest, as the union becomes more and more patient, the highest equilibrium contract is decreasing, which is quite different from the conventional result that patience is a virtue. The reason for this counter-intuitive result can be argued by one of our early results. As the union becomes more and more patient, the union needs less compensation to work in every odd period, which hurts the union in its best possible equilibrium outcome.

To conclude, we find that the firm may benefit from compensating the union in odd periods when the union is relatively more patient than the firm, namely when their discount factors lie in set $B$. However, the firm will not carry out the compensation since the union and the firm would agree on a new wage contract immediately. When the union is not more patient relative to the firm, the firm could not benefit from compensating the union. In this situation, the firm either does not have to compensate (when $(\delta_u, \delta_f) \notin A$), or does not want to compensate since the compensation needed to provide the union enough incentive to work is too high (when $(\delta_u, \delta_f) \in A \setminus B$).

4 Compensation with the Union’s Consent

As we have argued, the NLRA prohibits the firm from offering unilaterally additional compensation to the union during a contract negotiation. Such unilateral actions from the firm are considered to undermine the union’s authority to represent the workers. In the previous section, we showed that under certain conditions, the firm has an incentive to offer compensation to induce the union to work in every odd period in the reaching the highest possible equilibrium contract, so the firm’s unilateral ability to compensate can also hurt the workers economically. The NLRA does not completely prohibit the firm from compensating the workers, but rather provides power to the union to block the firm’s action.

Now we examine whether it is credible for the union to block firm’s compensation if the firm chooses to compensate in the best possible equilibrium to the union. Our answer is negative. In order to analyze this issue more formally, we modify our model studied in the
previous section so that the union needs to decide whether to approve the firm’s compensation before deciding whether to strike or to work in any period after disagreement.\footnote{Whether the union decides simultaneously or sequentially to approve/disapprove the firm’s compensation and to work/strike will not change our conclusions.}

The negotiation proceeds in the fashion of alternating-offer as in the previous model. There is one more stage where the union decides whether to approve the firm’s compensation offer. More specifically, in any odd period before reaching an agreement, the union proposes $w' \in [0, 1]$ in the first stage, the firm then decides whether to accept the union’s demand in the second stage. Acceptance concludes the negotiation. At the third stage after the firm rejects the union’s demand, the firm may offer compensation $c' \geq w^0$ to the union. Different from the previous model, the union now needs to decide whether to approve the firm’s compensation in stage four. In stage five, the union decides whether to work for $c' \geq w^0$ if the union has approved $c'$ or for expired contract $w^0$ if the union has disapproved $c'$, or whether to strike during the current period. Then the negotiation proceeds to the following even period, which is similar to an odd period except that the firm proposes a wage contract and the union responds, stages 3, 4 and 5 in an even period are identical to those in an odd period.

As in the model where the firm may unilaterally offer additional compensation, this modified model has perfect information. Histories, strategies and payoffs are defined in the usual fashion according to the additional element in the model. The union’s decision on whether to approve the firm’s compensation introduces new subgames so subgame perfection requires the strategy profile induced in these new subgames to be Nash equilibria as well.

Despite the union’s ability to block the firm’s compensation, $w^0$ is still the lowest equilibrium contract. In the rest of this section, we show that the union cannot credibly block the firm’s additional compensation when the firm offers in the best possible equilibrium to the union. When $(\delta_u, \delta_f) \notin B$, we know that the firm either does not have to or does not want to compensate the union. It will continue to be the case when the union can block the firm’s additional compensation. Suppose that the union always approves the compensation
offered by the firm. Then this modified model is virtually the same as our original model. On the other hand, if the firm does not offer any additional compensation, then the union’s approval decision becomes irrelevant.

We now concentrate our attention on the situation of \((\delta_u, \delta_f) \in B\). We adopt the same notation as before: \(M_u^*\) is a upper bound of the union’s equilibrium payoffs in any odd period and \(m_f^*\) is a lower bound of the firm’s equilibrium payoffs in an even period.

First, for the same argument as from our original model where the firm may unilateral increase compensation, condition (5) holds here as well. In the best possible equilibrium to the union, the firm will not offer any additional compensation and the union will work in any even period.

Next, we consider an odd period. Recall that the union may now block the firm’s compensation offer. In order for the firm’s compensation \(c'\) to be ineffective, it must be the case that the union strikes in the current odd period whether the union approves \(c'\) or not. Note that if the union chooses to strike, then its continuation payoffs will not be higher than \(\delta_u(1 - m_f^*)\). If the union chooses to work for the current period however, its continuation payoffs will not be less than \(w^0\) (after the union refuses \(c'\)) or \((1 - \delta_u)c' + \delta_u w^0\) (after the union approves \(c'\)). Figure 4 illustrates the situation that the union is most likely to strike in an odd period after the firm offers \(c' \geq w^0\), where the union is rewarded with the highest equilibrium contract in the continuation after it strikes and punished with the lowest equilibrium contract in the continuation after it works.

![Figure 4. The union’s four possible continuation payoffs.](image)

Restricting \(c' \geq w^0\) implies that the union never disapprove \((D)\) the firm’s compensation.
and then works for the current odd period, which is quite intuitive. It is not hard to see from Figure 4 that when condition (6) holds, it is not credible for the union to disapprove (D) c' and then to strike (S). In addition, any equilibrium outcome in our original model can also be supported in the current model by duplicating the continuation equilibrium in the subgames after the union approves or disapproves the firm’s compensation.

Proposition 8  

Allowing the union to block the firm’s additional compensation will not change the set of equilibrium payoffs.

Proposition 8 implies that suppressing the union’s ability to block the firm’s compensation makes no difference, the results obtained in Sections 2 and 3 are still valid. In particular, we have found that the firm can still upset the union’s incentive to strike when \((\delta_u, \delta_f) \in B\) even if the union can block the firm’s action.

5 Concluding Remarks

In this paper, we show that a firm may have an incentive to increase workers’ temporary wages during a contract negotiation. This could happen even when the firm needs to pay the workers almost all of its gross profit. Higher wages (either expired or temporary) will lower then the union’s incentive to strike. It is well known by now that the type of negotiation model we adopted in this study admits multiple equilibrium outcomes. We address the issues we concern by studying the range of wages that may arise in equilibrium. Our interpretation of the NLRA does not help the union in the sense that the union cannot credibly block the firm’s additional compensation when the firm decides to offer. From the point of view in non-cooperative bargaining literature, our model demonstrates what the firm can do legally to prevent the union’s strategic switching between work and strike during a contract negotiation. When parties are sufficiently patient, or alternatively when offer and counter-offer are made more rapidly, our model predicts a relatively small wage increase from a contract negotiation.
There are a number of issues we plan to investigate. One issue is the duration of the firm’s compensation if it is even offered. More specifically, what happens if the firm’s compensation remains in effect for more than one periods? It is quite intuitive that if the firm’s compensation is valid for an even number of periods, the firm cannot benefit from offering addition compensation at all. However, if the firm’s compensation is valid for an odd number of periods, then the firm could be better off by offering compensation strategically. Another direction we can consider is what if the firm can lower the wage paid to the union during a contract negotiation.
6 Appendix A

In this appendix, we argue that treating the dynamic disagreement game in its normal form does not alter the set of equilibrium payoffs characterized by Proposition 1. Consider the normal form representation of the disagreement, where the firm’s and union’s strategy spaces and payoff functions are

\[ A_f = [w^0, \infty) \]
\[ A_u = \{a_u(\cdot) : [w^0, \infty) \rightarrow \{\text{Strike, Work}\} \}, \]
\[ (d_u(a_u, a_f), d_f(a_u, a_f)) = \begin{cases} 
(0, 0) & \text{if } a_f = c \text{ and } a_u(c) = \text{Strike}, \\
(c, 1-c) & \text{if } a_f = c \text{ and } a_u(c) = \text{Work}. 
\end{cases} \]

Note that the union’s decision to work or strike depends on the firm’s compensation offer.

According to Busch and Wen (1995), in order to support the highest equilibrium contract, we need to find the firm’s lowest disagreement payoff and the union’s highest disagreement payoff supportable in an equilibrium. The firm’s lowest supportable disagreement payoff is the firm’s minimax value 0 in the disagreement game, achieved when the union strikes. The union’s highest supportable disagreement payoff is the highest difference between the union’s disagreement payoff and the firm’s gain from deviating:

\[ \max_{a_f, a_u} \left[ d_u(a_u, a_f) - \left( \max_{a_f} d_f(a_u, a_f) - d_f(a_u, a_f) \right) \right] = [c - ((1 - w^0) - (1 - c))] = w^0, \]

achieved when the union works even if the firm does not offer any additional compensation. This implies that Proposition 1 would continue to hold if one treated the normal form representation of the underlying game as the disagreement game. Therefore, allowing the firm to offer additional compensation would not change the results from the contract negotiation model of Haller and Holden (1990) and Fernandez and Glazer (1991).

7 Appendix B

Proof of Proposition 2: Consider the following strategy profile: In any odd period, the union demands \( w^0 \) and the firm accepts demands of no more than \( w^0 \). In any even period,
the firm offers \( w^0 \) and the union accepts offers of no less than \( w^0 \). The firm does not offer any additional compensation and the union chooses to work in any period. In what follows, we show that this strategy profile constitutes a subgame perfect Nash equilibrium.

Since the continuation payoffs are independent of the history in any stage of any period, it is optimal for the union to work for any compensation. Given that, the firm should not offer any additional compensation. In any odd period, the firm receives \( 1 - w^0 \) after rejecting the union’s demand so the firm will reject any wage demand higher than \( w^0 \). In any even period, the union’s payoff from rejecting the firm’s offer is \( w^0 \) so it is optimal to the union to reject any wage offer that is less than \( w^0 \). In summary, neither the union nor the firm has any incentive to deviate from the strategy profile described above.

Q.E.D.

**Proof of Lemma 2:** The proof is divided into four parts.

(i) Recall \( \delta^A_f(\delta_u, w^0) \) from (2), note that

\[
(1 - w^0)\delta_u^2 + w^0\delta_u - w^0 > 0 \quad \text{iff} \quad \delta_u < -\frac{\sqrt{(4 - 3w^0)w^0} + w^0}{2 - 2w^0} \quad \text{or} \quad \delta_u > \frac{\sqrt{(4 - 3w^0)w^0} - w^0}{2 - 2w^0},
\]

\[
\delta_u^2 - w^0\delta_u > 0 \quad \text{iff} \quad \delta_u < 0 \quad \text{or} \quad \delta_u > w^0.
\]

\[
\Rightarrow \quad \delta^A_f(\delta_u, w^0) > 0 \quad \text{iff} \quad \text{either } 0 < \delta_u < w^0 \quad \text{or} \quad \frac{\sqrt{(4 - 3w^0)w^0} - w^0}{2 - 2w^0} < \delta_u < 1.
\]

On the other hand, when \( \delta_u^2 - w^0\delta_u < 0, \delta^A_f(\delta_u, w^0) < 1 \) if and only if

\[
(1 - w^0)\delta_u^2 + w^0\delta_u - w^0 > \delta_u^2 - w^0\delta_u \quad \Leftrightarrow \quad w^0(1 - \delta_u)^2 < 0,
\]

which is impossible. When \( \delta_u^2 - w^0\delta_u > 0, \delta^A_f(\delta_u, w^0) < 1 \) if and only if \( w^0(1 - \delta_u)^2 > 0, \)

which is trivial.

(ii) Recall \( \delta^B_f(\delta_u, w^0) \) from (17), note that

\[
(1 - w^0)\delta_u^2 + \delta_u - 1 > 0 \quad \text{iff} \quad \delta_u < -\frac{1 + \sqrt{5 - 4w^0}}{2 - 2w^0} \quad \text{or} \quad \delta_u > \frac{\sqrt{5 - 4w^0} - 1}{2 - 2w^0},
\]

\[
(2 - w^0)\delta_u^2 - \delta_u > 0 \quad \text{iff} \quad \delta_u < 0 \quad \text{or} \quad \delta_u > \frac{1}{2 - w^0}.
\]

\[
\Rightarrow \quad \delta^B_f(\delta_u, w^0) > 0 \quad \text{iff} \quad \text{either } 0 < \delta_u < \frac{1}{2 - w^0} \quad \text{or} \quad \frac{\sqrt{5 - 4w^0} - 1}{2 - 2w^0} < \delta_u < 1.
\]
On the other hand, when \((2 - w^0)\delta_u^2 - \delta_u < 0, \delta_f^B(\delta_u, w^0) < 1\) if and only if

\[(1 - w^0)\delta_u^2 + \delta_u - 1 > (2 - w^0)\delta_u^2 - \delta_u \iff (1 - \delta_u)^2 < 0,
\]

which is impossible. When \((2 - w^0)\delta_u^2 - \delta_u > 0, \delta_f^B(\delta_u, w^0) < 1\) if and only if \((1 - \delta_u)^2 > 0\), which is trivial.

(iii) Note the following equivalency: If \(\delta_u \in \left(\frac{\sqrt{5 - 4w^0} - 1}{2 - 2w^0}, 1\right), \delta_f^A(\delta_u, w^0) > \delta_f^B(\delta_u, w^0)\) if and only if

\[\frac{(1 - w^0)\delta_u^2 + w^0\delta_u - w^0}{\delta_u(\delta_u - w^0)} > \frac{(1 - w^0)\delta_u^2 + \delta_u - 1}{\delta_u[(2 - w^0)\delta_u - 1]} \iff (1 - w^0)^2 \cdot (1 - \delta_u)^2 \cdot \delta_u > 0,
\]

which is trivial for all permissible values of \(\delta_u\) and \(w^0\).

(iv) This part of the proof is straightforward. For example

\[
\frac{\partial \delta_f^B(\delta_u, w^0)}{\partial \delta_u} = \frac{[2(1-w^0)\delta_u + 1][(2-w^0)\delta_u^2 - \delta_u] - [2(2-w^0)\delta_u - 1][(1-w^0)\delta_u^2 + \delta_u - 1]}{[(2-w^0)\delta_u^2 - \delta_u]^2}.
\]

At \(\delta_u = 1\), this derivative equals 0.

Q.E.D.

**Proof of Proposition 6:** From condition (7), we have that \(M_u^* = \min\{M_u, \tilde{M}_u^*\}\). For \((\delta_u, \delta_f)\), \(M_u\) is increasing with respect to \(\delta_u\) for any given \(\delta_f\). Now we show that \(\tilde{M}_u^*\) is decreasing with respect to \(\delta_u\) for any given \(\delta_f\) wherever it is well defined. Differentiating \(\tilde{M}_u^*\) with respect to \(\delta_u\), we have

\[
\frac{\partial \tilde{M}_u^*}{\partial \delta_u} = \frac{\partial}{\partial \delta_u} \left[ \frac{\delta_f - 2\delta_f \cdot \delta_u + (2\delta_f - 1) \cdot \delta_u^2}{1 - (1 + \delta_f) \cdot \delta_u + (2\delta_f - 1) \cdot \delta_u^2} \right]
\]

\[
= \frac{(\delta_f - 1)[\delta_f - 2(2\delta_f - 1) \cdot \delta_u + (2\delta_f - 1) \cdot \delta_u^2]}{[1 - (1 + \delta_f) \cdot \delta_u + (2\delta_f - 1) \cdot \delta_u^2]^2}.
\]

Observe that the second term on the numerator of (20) is

\[
\delta_f - 2(2\delta_f - 1) \cdot \delta_u + (2\delta_f - 1) \cdot \delta_u^2 = \begin{cases} 
\delta_f > 0 & \text{when } \delta_u = 0 \\
1 - \delta_f > 0 & \text{when } \delta_f = 1,
\end{cases}
\]

and

\[
\frac{\partial}{\partial \delta_u} [\delta_f - 2(2\delta_f - 1) \cdot \delta_u + (2\delta_f - 1) \cdot \delta_u^2] = 2(2\delta_f - 1)(\delta_u - 1),
\]

24
which is positive if and only if $\delta_f < 1/2$. We have shown that $\delta_f - 2(2\delta_f - 1) \cdot \delta_u + (2\delta_f - 1) \cdot \delta_u^2$ is monotonic with respect to $\delta_u$ for any given $\delta_f \in (0, 1)$, and it has positive values at $\delta_u = 0$ and 1. Therefore, it is always positive for all $(\delta_u, \delta_f) \in (0, 1)^2$.

Together with the fact the $(\delta_f - 1) < 0$ (the first term on the numerator of (20)), the right hand side of (20) is negative and so $\tilde{M}_u^*$ is decreasing with respect to $\delta_u$ for any given $\delta_f$. As we argued, $\tilde{M}_u^* = M_u$ on the boundary of set $B$ and $M_u$ is increasing with respect to $\delta_u$ for any given $\delta_f$. This implies that if $(\delta_u, \delta_f) \in B$, we must have $M_u > \tilde{M}_u^*$ and so $M_u^* = \tilde{M}_u^*$ if and only if $(\delta_u, \delta_f) \in B$. Q.E.D.
References


