**Achieving Intertemporal Efficiency and Symmetry through Intratemporal Asymmetry: (Eventual) Turn Taking in a Class of Repeated Mixed-Interest Games ***

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November 2003

**Abstract**

Turn taking is observed in many field and laboratory settings. We study when and how turn taking can be supported as an equilibrium outcome in a class of repeated games, where the stage game is a symmetric two-player mixed-interest game with asymmetric joint-payoff-maximizing outcomes that may or may not be Nash equilibria. We consider the "turn taking with independent randomizations" (TTIR) strategy that achieves the following three objectives: (a) helping the players get onto a joint-payoff-maximizing turn-taking path, (b) resolving the question of who gets to start with the good turn first, and (c) deterring defection. The TTIR strategy is simpler than those time-varying strategies considered in the Folk Theorem for repeated games. We determine conditions under which a symmetric TTIR subgame-perfect equilibrium exists and is unique. We also derive comparative static results, and study the welfare properties of the TTIR equilibrium.

*Key Words:* Conflict, Coordination, Randomization, Turn Taking, Repeated Games

*JEL Classifications*: C70, C72

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* We are grateful to Stephen Chiu, Simon Grant, Yew-Kwang Ng, and Xianming Zhou, seminar participants at the Australian National University, the City University of Hong Kong, McGill University, Monash University, and the University of Hong Kong for their helpful comments and to Daniel Kelly and Felix Leung for their excellent research assistance.

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## 1 Introduction

Turn taking is widely observed in both field and laboratory settings. This paper studies when and how turn taking can be supported as an equilibrium outcome when the players interact repeatedly. An example of the importance of turn-taking behavior concerns the use of common-pool resources (CPRs) such as fisheries, irrigation systems, and forests. In communities that depend heavily on such resources for their economic livelihood, failure to resolve problems related to the use and preservation of these resources can lead to significant welfare loss, violent conflicts, and even murder. One conflict of this type is illustrated by the game of CPR assignment in Ostrom et al. (1994, pp. 58-61). This game captures, in the simplest fashion, a situation in which two fishermen independently decide to go to one of two fishing spots in their community. The good spot has a value of $h$, and the bad spot has a value of $l$, where $h > l > 0$. If the two choose different spots, each will obtain the respective value of the spot. If they choose the same spot, they will split the value of the spot equally. In this situation, which spot will each fisherman choose if they interact repeatedly?

In the situation described above, there are two asymmetric outcomes—(Good Spot, Bad Spot) and (Bad Spot, Good Spot)—that maximize the sum of the two players' payoffs. One might expect that eventually some sort of turn-taking scheme, in which the fishermen take turns going to the good spot, will develop as a solution mitigating the CPR dilemma. In fact, Berkes (1992) reports that fishermen in Alanya, Turkey, employ a turn-taking scheme to allocate fishing spots. A similar rotation scheme in the use of an irrigation system has also been adopted in Spain and the Philippines (Ostrom, 1990).

Turn-taking schemes that mitigate conflict and enable people to engage in intertemporal sharing of the gain from cooperation are also observed in other settings. For example, faculty members in a department may use turn taking to resolve the question of who will serve as the departmental representative on a university committee. Soldiers in a military operation often take turns to perform the more dangerous tasks (see, for example, Bergerud, 1993). Politicians may engage in vote-trading by taking turns voting for each other's preferred choice (Riker, 1982).

Different researchers have also observed turn-taking behavior in laboratory repeated games. In an experiment motivated by the observed importance of turn taking in the field discussed in Ostrom (1990), Prisbrey (1992, chapter 1) reports that in a repeated symmetric game that has two asymmetric joint-payoff-maximizing outcomes, twenty-one out of twenty-four pairs of subjects succeed in establishing (and subsequently maintaining) turn taking to achieve joint-payoff maximization within five periods. In a study of intergroup conflicts that involve different versions of a repeated public-good game, Bornstein et al. (1997) observe that a significant percentage of subjects use the turn-taking strategy.

Although turn taking has been observed in a variety of settings, there appears to be surprisingly little systematic analysis of this phenomenon. [1] This paper represents a step toward such analysis. Our objective is to provide a theoretical analysis of a benchmark model of equilibrium turn-taking behavior. However, it is not immediately clear what are the ingredients of the benchmark model in understanding turn-taking behavior, given that we observe people taking turns in a wide variety of settings. After searching for common elements in various examples, we propose the following characteristics for the benchmark model. First, it is obvious that turn taking happens in an intertemporal setting. Second, turn taking is less likely to be observed in situations in which joint-payoff maximization in a particular period requires that all players take the same action but is more likely to be observed when joint-payoff maximization requires that players take different actions (such as a player going to the good fishing spot while her opponent going to the bad spot in the example presented above). Third, the study of turn taking becomes particularly interesting in settings that involve conflicts, that is, in settings in which the turns that the players take include good and bad turns.

Capturing these observations, the benchmark model we consider involves a setting in which (a) two symmetric players interact repeatedly and each player chooses between two possible actions in every period, (b) a player's current-period payoff depends only on the current actions of both players, [2] and (c) two asymmetric outcomes of the stage game are joint-payoff-maximizing. In short, we consider a class of infinitely repeated games where the stage game is a symmetric two-player game with a pair of asymmetric joint-payoff-maximizing outcomes. From now on, we refer to these joint-payoff-maximizing outcomes as efficient outcomes and to the class of repeated games mentioned above as "repeated games with asymmetric efficient outcomes." A game with asymmetric efficient outcomes is an example of the type of game that has been referred to as a "mixed-motive game" by Schelling (1960) and as a "game of mixed interests" by Friedman (1994). In subsequent sections, we further

[1] There is, however, some related work in the repeated game literature that considers strategies that have turn-taking features. We discuss these papers in Section 6.

[2] That is, current action does not affect future payoffs through some kinds of "state variables" (such as capital or the amount of a natural resource remaining) in the model we consider. Of course, as is typical in the repeated game literature (e.g., Fudenberg and Maskin, 1986), any player can still take the history of actions of both players into account when setting her strategy. Thus, a player's current action can affect future actions (of both players), and thus has an indirect effect on the players' future payoffs.

classify these games according to whether the asymmetric efficient outcomes are Nash equilibria of the stage game. There are interesting differences in the analyses of these two cases.

Specifically, we consider infinitely repeated games where the stage game is given in the left-hand panel of Table 1. Let $T$ (Tough) and $S$ (Soft) be the two possible actions of a player in any period. Each player obtains the same payoff $t$ if both choose the same action $T$, and each obtains payoff $s$ if both choose $S$. On the other hand, a player obtains payoff $h$ if she chooses $T$ and the other player chooses $S$. In this case, the other player obtains a lower payoff $l$. Reflecting that the two asymmetric outcomes of the stage game are efficient, we assume $h + l > \max\{2t, 2s\}$. Examples of games belonging to this class include the battle of the sexes, the game of chicken, the best-shot public-good game (studied in Harrison and Hirshleifer, 1989),[3] (a particular version of) the prisoner's dilemma, and the game of CPR assignment.

At first glance, one may think that the question of whether turn taking can be supported as some kind of subgame-perfect equilibrium in this class of repeated games is relatively straightforward—the Folk Theorem in repeated games with discounting suggests that the answer to this question is a resounding "yes" (see, for example, Friedman, 1971; Fudenberg and Maskin, 1986; Abreu, 1988.) However, the analysis of equilibrium turn-taking behavior is not as simple as it appears. In particular, while either one of the two turn-taking sequences $X = \{(T, S), (S, T), (T, S), (S, T), ...\}$ and $Y = \{(S, T), (T, S), (S, T), (T, S), ...\}$ in which the sum of the two players' payoffs is maximized can be supported as an equilibrium when the discount factor is sufficiently high, the players have conflicting preferences between these sequences, with each player preferring the one in which she takes the good turn first. In this setting, the well-known problem of multiple equilibria associated with the Folk Theorem in repeated games manifests itself in the form of being silent regarding how the players may resolve their conflict regarding who gets to start with the good turn. Furthermore, as we demonstrate later, one has to investigate whether any player will defect from the equilibrium turn-taking path, especially when she is supposed to take the bad turn.

In the class of repeated games with asymmetric efficient outcomes, three major issues have to be resolved. First, how do the players get onto a joint-payoff-maximizing turn-taking path? Second, who gets to start with the good turn first along this path? Third, how are potential deviations deterred? In this paper, we consider a simple strategy—to be called the "turn taking with independent randomizations" (TTIR) strategy—that resolves these three issues.

In the TTIR strategy that we describe in more detail later, players randomize

[3] In a best-shot public-good game, the level of public good produced equals the maximum contribution made by the players.

between $T$ and $S$ in the beginning period and also if a symmetric outcome is observed in the period preceding the current one, or they rotate between the two asymmetric outcomes if one of them is observed in the period preceding the current one. An important feature of the TTIR strategy is that a player's behavior in any period depends on only a relatively small set of important variables instead of on the entire history of play. Moreover, because the mixed-interest game considered in this paper is symmetric, it is reasonable to expect that the equilibrium of this game satisfies the symmetry-invariance principle suggested in Harsanyi and Selten (1988), which requires that the equilibrium be invariant to re-labeling of the players; see also Bhaskar (2000). Thus, we consider the symmetric equilibrium when the players use the TTIR strategy.

The TTIR strategy has a number of attractive features. First, it ensures *ex ante* equality in the two (symmetric) players' payoffs. Second, for the class of repeated games with asymmetric efficient outcomes, the punishment in the TTIR strategy is less severe than the punishment in some other subgame-perfect equilibrium strategies, such as the grim strategy employed in Friedman (1971). Third, and most important, by restricting our attention to the TTIR strategy, we are able to show that the subgame-perfect equilibrium, if it exists, is unique. This uniqueness result enables us to derive interesting and easily interpretable comparative static results.

In the TTIR strategy, randomization serves the multiple roles of getting the players onto a joint-payoff-maximizing turn-taking path, resolving the question of who gets to start with the good turn first along this path, and deterring players from deviating from equilibrium behavior. When a player is supposed to take the bad turn according to the equilibrium strategy, she may have the incentive to defect so as to capture a short-term gain. To balance the *incentive* consideration and the *efficiency* consideration in an intertemporal setting, the value of the probability of randomization has to satisfy a "fixed point" requirement. Characterizing the equilibrium turns out to be a nontrivial problem. Despite the fact that the mapping that determines the fixed point of the probability of randomization is not always a contraction mapping, we find that whenever a TTIR subgame-perfect equilibrium exists, it is unique.

This paper is organized as follows. Sections 2 introduces the model and discusses the assumptions. Section 3 describes the TTIR strategy and the associated incentive conditions. It also presents results useful for subsequent analysis. Section 4 considers the repeated game in which the two asymmetric joint-payoff-maximizing outcomes are Nash equilibria of the stage game. Section 5 performs the corresponding analysis for the repeated game in which the two joint-payoff-maximizing outcomes are not Nash equilibria of the stage game. It also characterizes players' behavior along the equilibrium turn-taking path and performs comparative static analysis. In Section 6, we discuss related work, compare the TTIR strategy to other subgame-perfect equilibrium strategies, and investigate the welfare properties of the TTIR equilibrium. Section 7 provides a summary and suggests some directions for future research. Some proofs are relegated to the Appendix.

## 2 The Model

We consider a symmetric two-by-two infinite-horizon repeated game with discounting. In every period of the game, each of the two players (called 1 and 2) chooses (perhaps randomly) between two actions: playing $T$ or playing $S$. When making a new decision, say in period $n$ ($n \geq 0$), player $i$ ($i = 1, 2$) maximizes her intertemporal payoff (which is defined as the discounted sum of the stream of her current and future stage-game payoffs):

$$\sum_{m=n}^{\infty} \delta^{m-n} U_i \left(x_{1m}, x_{2m}\right), \tag{1}$$

where $\delta \in (0, 1)$ is the common discount factor, $x_{im}$ ($x_{im} = T$ or $S$) is the choice of player $i$ at period $m$, and $U_i \left(x_{1m}, x_{2m}\right)$ is the current-period payoff of player $i$ when player 1 chooses $x_{1m}$ and her opponent chooses $x_{2m}$. The players' payoffs in the stage game, $U_i \left(x_1, x_2\right)$, given in Table 1, are represented by

$$\begin{aligned} U_i \left(T, T\right) &= t, & U_i \left(S, S\right) &= s, \\ U_1 \left(T, S\right) = U_2 \left(S, T\right) &= h, & U_1 \left(S, T\right) = U_2 \left(T, S\right) &= l, \end{aligned} \tag{2}$$

where $i = 1, 2$, and $h$, $l$, $s$, and $t$ are finite real numbers. (Whenever there is no confusion, the time subscript is ignored.) It is assumed that each player observes both players' actions (but not strategies) in every period.

In this paper, we analyze repeated games with the following assumptions:

$$h > l, \tag{3}$$

$$h + l > 2s, \tag{4}$$

and

$$h + l > 2t. \tag{5}$$

Since interesting turn-taking behavior consists of good and bad turns, we assume condition (3), which states that if the two players choose different actions, their payoffs will be different. Without loss of generality, we assume that $h$ (which may be interpreted as "high") is larger than $l$ (which may be interpreted as "low"). When assumption (3) holds, then among the two asymmetric outcomes $(T, S)$ and $(S, T)$, each player prefers choosing $T$ while the other player is choosing $S$. The specification in (3) implicitly defines the labels $T$ and $S$ for any given game. For example, $T$ represents Good Spot and $S$ represents Bad Spot for the game of CPR assignment in the above framework.

Assumption (3) eliminates the pure coordination game (Case 1 of Table 2) from our analysis. An example of a pure coordination game is driving on one particular side of the road. It should be clear that while in principle the turn-taking strategy can be used to support an efficient outcome of this game (at least in an environment without uncertainty, as in Section 2 of Crawford and Haller, 1990), there are other "more natural" mechanisms (such as always driving on the left-hand side of the road) leading to an equally efficient outcome. Imposing assumption (3) eliminates this game from our analysis, but all the remaining examples in Table 2 are consistent with this assumption.

The other two assumptions, (4) and (5), compare the sum of the players' payoffs in the symmetric outcomes with the sum of those in the asymmetric outcomes. These assumptions ensure that the players' total payoff in the two asymmetric outcomes is higher than that in the two symmetric outcomes.[4] If one of these assumptions is not satisfied, then turn taking is less likely to be supported as an equilibrium outcome even if the players are patient enough. For example, assumption (4) is not satisfied in an assurance game (Case 2 of Table 2), and the efficient stage-game outcome is $(S, S)$. For the repeated assurance game, turn taking between $(T, S)$ and $(S, T)$ is an inferior outcome when compared with reaching $(S, S)$ in every period (if that is achievable). As our purpose in this paper is to analyze the turn-taking phenomenon, assumptions (4) and (5) are natural ones to be imposed.

Summing up, the focus of our analysis is repeated symmetric games with asymmetric efficient outcomes, that is, games with assumptions (3) to (5). As illustrated in Table 2, depending on the relative ranking of the payoff parameters, the specification with assumptions (3) to (5) captures many well-known games that have been analyzed in the literature.

Before analyzing this class of repeated games, it is helpful to summarize well-known results about these games when the players interact once. There are two different cases: $t < l$ and $t > l$. (While the analysis of the borderline case $t = l$ is different from the above two cases for the stage game, the analysis of this case is the same as that of $t < l$ for repeated interaction. Thus, we do not consider this borderline case when the players interact once.)

When $t < l$, the two asymmetric joint-payoff-maximizing outcomes $(T, S)$ and

[4] Alternatively, one can rewrite assumption (4) as $\frac{h+l}{2} > s$ and interpret it as saying that a player prefers being equally likely to reach the two asymmetric outcomes to reaching the symmetric outcome $(S, S)$ with certainty. A similar alternative interpretation holds for assumption (5).

$(S,T)$ are pure-strategy Nash equilibria of the stage game. There is also a symmetric mixed-strategy equilibrium with each player choosing Tough with probability $\frac{h-s}{h+l-s-t}$. Key examples for the $t < l$ case of games with asymmetric efficient outcomes include (a) the battle of sexes, (b) the game of chicken, which has been used by Farrell (1987) to study issues of market entry, (c) the best-shot public-good game studied by Harrison and Hirshleifer (1989), and (d) a particular version of the game of CPR assignment studied by Ostrom et al. (1994) when $h < 2l$, that is, when the bad spot is not too inferior compared to the good spot.

On the other hand, when $t > l$, there is a unique Nash equilibrium, $(T,T)$, for the stage game, and the two asymmetric joint-payoff-maximizing outcomes are not Nash equilibria. Examples for the $t > l$ case of games with asymmetric efficient outcomes include (a) a particular version of the prisoner's dilemma employed by Dixit and Skeath (1999, Figure 11.2) to study the issue of collective action in building an irrigation project,[5] (b) another version of the game of CPR assignment studied by Ostrom et al. (1994) with $h > 2l$, that is, when the good spot is "sufficiently more attractive" than the bad spot, and (c) the stage game of the laboratory repeated game studied by Prisbrey (1992) described above.

## 3 Turn Taking with Independent Randomizations

We analyze the equilibrium of a symmetric two-by-two infinitely repeated game with asymmetric efficient outcomes (i.e., with conditions (1) to (5).) This game will be denoted as $G_\infty$, and the one-shot version will be denoted as $G$. The equilibrium concept adopted is subgame perfection. The analysis turns out to be partially similar but also partially different for the $t \leq l$ and $t > l$ cases. This section looks at issues common to both cases, and the next two sections focus on the two cases individually.

We now make clear three major assumptions in our analysis. First, we assume in this paper that there is *no communication* between the players in any period, either before or after they take actions. Turn taking is observed in experimental

[5] Dixit and Skeath (1999, Figures 11.1 and 11.2) consider two games, each of which satisfies the two usual key properties of the prisoner's dilemma, namely, that Defect is the dominant strategy for both players, and the outcome (Cooperate, Cooperate) Pareto dominates (Defect, Defect). In the first game, (Cooperate, Cooperate) is the joint-payoff-maximizing outcome. This is the case upon which most analyses of the prisoner's dilemma concentrate, and we refer to it as the "standard" prisoner's dilemma. In the other case (their Figure 11.2), the two asymmetric outcomes—that is, when one player defects and the other cooperates—maximize the players' total payoff.

and field settings in which there is no communication possibility between the players, but it is also observed in environments in which the players are able to communicate; for example, Berkes (1992) describes how fishermen draw lots to assign fishing spots, which can be modeled as the use of correlated strategies (Aumann, 1974). In general, coordination problems can be mitigated by useful precedents when the players interact repeatedly (as in Crawford and Haller, 1990; Bhaskar, 2000; Lau, 2001) and/or by some forms of communication (such as nonbinding preplay communication in Farrell, 1987). As there are a large number of possible ways to model communication in this setting, it is useful to first study equilibrium turn-taking behavior in an environment in which the players cannot communicate or correlate their strategies. (Crawford and Haller (1990) and Bhaskar (2000) also make this assumption.) In this environment, any benefit accrued to the players of game $G_\infty$ in mitigating conflict is purely due to turn taking in repeated interaction and not to communication. [6] After analyzing turn-taking behavior in this benchmark case, we can introduce cheap talk or correlated strategies and examine their implications. [7]

Second, we consider a simple strategy—the TTIR strategy—that we believe is a natural one when there is no communication between the players. As mentioned in the introduction, the TTIR strategy possesses several desirable features in game $G_\infty$. Section 3.1 describes the TTIR strategy in detail, and Section 6 compares it to other strategies and provides justification for its use.

Third, we analyze the symmetric subgame-perfect equilibrium of this repeated symmetric game (as in Crawford and Haller, 1990; Bhaskar, 2000) when the players use the TTIR strategy. The equilibrium is referred to as the symmetric TTIR subgame-perfect equilibrium or simply the *TTIR equilibrium*.

### *3.1 The TTIR Strategy*

The TTIR strategy specifies the following: (a) In the beginning period, the players will independently randomize between $T$ and $S$. Denote the probability of choosing $T$ as $p$. [8] For meaningful TTIR strategy, $p$ is restricted to lie in the

[6] For similar reasons, we do not consider how reciprocal fairness (e.g., Rabin, 1993, Fehr and Gächter, 2000) could affect turn-taking behavior in this paper. We believe it is better to pursue such line of investigation in another project, and then compare its findings to those obtained in our benchmark model.

[7] In an ongoing project, we examine the implications of cheap talk or correlated strategies in the repeated game of CPR assignment.

[8] More generally, one can define $p_i$ $(i = 1, 2)$ as the probability that player $i$ chooses $T$ in the randomization phase. In the symmetric subgame-perfect equilibrium, the equilibrium values of $p_1$ and $p_2$ are equal. To avoid heavy use of notations, we specify from the beginning that both players use the same strategy. There is no loss

open interval $(0, 1)$, since the players cannot reach an asymmetric outcome if both choose $p = 0$ (i.e., action $S$) or $p = 1$ (i.e., action $T$). (b) As long as the randomization yields the symmetric outcome of either $(T, T)$ or $(S, S)$, the randomization phase will continue. (c) Whenever randomization "succeeds" in getting the players to the asymmetric outcome of either $(T, S)$ or $(S, T)$, the game will switch to the turn-taking phase in which each player chooses her opponent's action in the previous period. If no player defects from this strategy, the turn-taking phase will continue. (d) Any defection by a single player (or by both players) during the turn-taking phase will trigger a switch back to the randomization phase, and this randomization phase will continue until randomization succeeds in getting the players to the asymmetric outcome of either $(T, S)$ or $(S, T)$ again. (e) Once randomization succeeds in getting the players to either asymmetric outcome, the players will again behave according to steps (c) and (d).

In many existing papers examining the strategies supporting subgame-perfect equilibria of repeated games (such as the standard prisoner's dilemma), the authors analyze two phases of the game: the cooperative phase and the punishment phase (see, for example, Friedman, 1971; Fudenberg and Maskin, 1986). The underlying idea is that the strategies in the cooperative phase induce behavior leading to an efficient outcome, if the players cooperate. To prevent the players from deviating from the cooperative phase, the low payoff in the punishment phase is used as a deterrent.

In a broad sense, the TTIR strategy includes the cooperative and punishment features as well. However, there is a major difference. In the cooperative phase of the repeated standard prisoner's dilemma, the efficient outcome is usually reached immediately. In that game, the efficient path that the players want to sustain involves $(S, S)$ in each period, where $S$ represents Cooperate. It is a unique path, and it is in both players' interest to attain it. Thus, it is not surprising to see that the authors focus on strategies in which the two players reach $(S, S)$ immediately in the cooperation phase. On the other hand, for the repeated game with asymmetric efficient outcomes analyzed in this paper, an efficient path involves the asymmetric outcomes $(T, S)$ and $(S, T)$ in the stage game and there are multiple efficient paths. Even though the two players may want to "cooperate" to reach an efficient path, they have conflicting preferences regarding the possible efficient paths. For example, a player prefers a turn-taking path in which she takes her good turn (choosing $T$, with the opponent choosing $S$) first to a path in which the other player takes the good turn first. There are thus both coordination and conflict problems in this repeated game with asymmetric efficient outcomes ($G_\infty$).

Because of the need to deal with this *coordination-cum-conflict problem* in the

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of generality since we consider only symmetric equilibrium in this paper.

initial periods, it is helpful to extend the familiar cooperation-and-punishment framework to one with three phases for game $G_\infty$: the initial "getting to efficient outcome" phase (or simply the initial phase), the cooperative phase, and the punishment phase.

For the TTIR strategy studied in this paper, the players use the turn-taking strategy in the cooperative phase and independent randomizations in both the initial and punishment phases. In particular, in the symmetric equilibrium, both players use the same randomized strategy in the initial phase as well as after any player deviates. (A similar idea has been employed by Crawford and Haller, 1990; Bhaskar, 2000; and Lau, 2001.) Whether the TTIR strategy constitutes an equilibrium depends on the incentive conditions, which we consider in the next two subsections.

### *3.2 Players' Behavior in the Turn-Taking Phase*

Define $V_H$ as the player's intertemporal payoff at a period in which the player plays Tough and her opponent plays Soft, with the expectation that the equilibrium TTIR strategy (if it exists) will be chosen by the players forever. Similarly, define $V_L$ as the player's intertemporal payoff at a period in which the player plays Soft and her opponent plays Tough, with the expectation that the equilibrium TTIR strategy will be chosen by the players forever. Finally, define $V^*$ as a player's (expected) intertemporal payoff at the initial period or any period such that both players' actions were the same in the previous period, with the expectation that the equilibrium TTIR strategy will be chosen by the players forever. Note that $V^*$, which is the same for the two symmetric players and which is determined endogenously (according to (10) below), is referred to as the value of the game.

The two value functions of the turn-taking phase are given by

$$V_H = h + \delta V_L = \frac{h + \delta l}{1 - \delta^2}, \tag{6}$$

and

$$V_L = l + \delta V_H = \frac{l + \delta h}{1 - \delta^2}. \tag{7}$$

To ensure that (6) and (7) hold, we need to verify that the players will not deviate from the equilibrium strategy. Because of the stationary structure of the infinite-horizon repeated game, it is necessary to check only two no-deviation conditions for the turn-taking phase, one at the player's good turn and the other at her bad turn.

First, if the actions of players $i$ and $j$ were $S$ and $T$, respectively, in the previous period, then, assuming that the equilibrium TTIR strategy will be used by both players in the future (and also by her opponent in the current period), player $i$ will not deviate from the equilibrium strategy in the current period when

$$V_H - (s + \delta V^*) = (h - s) + \delta (V_L - V^*) > 0. \quad (8)$$

The no-deviation condition (8) can be understood as follows. If player $i$ chooses $T$ now and both players continue to use the equilibrium strategy, then her intertemporal payoff is given by $V_H$, which can be decomposed as the sum of current and future payoffs $(h + \delta V_L)$ according to (6). On the other hand, if player $i$ chooses $S$ now, the current payoff is $s$, since both players will end up playing Soft. Moreover, both players choosing the same action in the current period will trigger them to use the strategies of the randomization phase in the next period. As a result, player $i$'s intertemporal payoff by deviating in the current period is given by $s + \delta V^*$. Player $i$ will not deviate from the strategy of the turn-taking phase if (8) is satisfied. [9]

Similarly, if the actions of players $i$ and $j$ were $T$ and $S$, respectively, in the previous period, then assuming that the equilibrium TTIR strategy will be used by both players in the future (and also by her opponent in the current period), player $i$ will not deviate from the equilibrium strategy in the current period when

$$V_L - (t + \delta V^*) = (l - t) + \delta (V_H - V^*) > 0. \quad (9)$$

### *3.3 Players' Behavior in the Randomization Phase*

Now, examine the beginning of the game (or any period such that both players' actions in the previous period were the same). If both players use the TTIR strategy forever, it is easy to see that the game will "re-start" in the next period if and only if both players choose the same action in the current period. As a result, the intertemporal payoff matrix of the game (when viewed at period 0) is given by Table 3. For example, if both players choose action $T$ in period 0, then each player's current-period payoff is $t$. Moreover, each player's future payoff (when discounted back to the current period) is $\delta V^*$ since the game will re-start in the next period.

[9] It can be shown that if (8) holds, then choosing $T$ also dominates choosing any mixed strategy for player $i$ when player $j$ chooses $S$. Consider player $i$'s strategy of choosing $T$ with probability $r$ where $0 < r < 1$. The corresponding intertemporal payoff of player $i$ is $rV_H + (1 - r)(s + \delta V^*)$. It is easy to see that if (8) holds, then the above intertemporal payoff is strictly less than $V_H$.

Denote the equilibrium value of the probability of choosing $T$ in the randomization phase as $p^*$ (and the equilibrium probability of choosing $S$ as $1-p^*$). Note that $p^*$ is the equilibrium probability, in contrast to $p$, which is an arbitrary probability in the interval $(0,1)$. When both players use the equilibrium mixed strategy in the randomization phase, it can be deduced from Table 3 that, provided that (8) and (9) are satisfied, the equilibrium probability of randomization ($p^*$) and the value of the game ($V^*$) are jointly determined by

$$V^* = p^* \left(t + \delta V^*\right) + \left(1 - p^*\right) V_H = p^* V_L + \left(1 - p^*\right) \left(s + \delta V^*\right). \tag{10}$$

In a mixed-strategy equilibrium, a player chooses a strategy to make the other player indifferent between playing Tough and Soft. For example, player 1 chooses $p^*$ (in the randomization phase) to ensure that for player 2 the second equality of (10) holds. This equality leads to

$$p^* = \frac{V_H - s - \delta V^*}{\left(V_H - s - \delta V^*\right) + \left(V_L - t - \delta V^*\right)}. \tag{10a}$$

### *3.4 Useful Results*

This subsection groups together the results that are useful for the $t \leq l$ and $t > l$ cases of game $G_\infty$. Lemma 1 to Lemma 5 hold for all stage-game payoff parameters satisfying (2) to (5) and all $\delta \in (0,1)$.

**Lemma 1** *The two value functions of the turn-taking phases are related by*

$$V_H + V_L = \frac{h + l}{1 - \delta}, \tag{11}$$

*and*

$$V_H - V_L = \frac{h - l}{1 + \delta} > 0. \tag{12}$$

**Lemma 2**

$$\frac{h+l}{2} - U\left(p\right) = p^2 \left(\frac{h+l}{2} - t\right) + \left(1 - p\right)^2 \left(\frac{h+l}{2} - s\right) > 0, \tag{13}$$

*where*

$$U\left(p\right) = p^2 \left(t\right) + \left(1 - p\right)^2 \left(s\right) + p\left(1 - p\right)\left(h + l\right) \tag{14}$$

*is a player's expected current-period payoff in the randomization phase, if both players choose $T$ with probability $p$, an arbitrary number in the interval $(0,1)$.*

**Lemma 3**

$$\frac{V_H + V_L}{2} - V(p) = \frac{1}{\left\{1 - \delta\left[p^2 + (1-p)^2\right]\right\}}\left[\frac{h+l}{2} - U(p)\right] > 0, \tag{15}$$

*where*

$$V(p) = \frac{p^2 t + (1-p)^2 s + p(1-p)(V_H + V_L)}{1 - \delta\left[p^2 + (1-p)^2\right]} = \frac{U(p) + \delta p (1-p)(V_H + V_L)}{1 - \delta\left[p^2 + (1-p)^2\right]} \tag{16}$$

*is a player's intertemporal payoff at the randomization phase if both players choose $T$ with probability $p$.*

**Lemma 4** *For any $p \in (0,1)$,*

$$V_H > \frac{V_H + V_L}{2} > V(p). \tag{17}$$

**Lemma 5** *For the repeated game $G_\infty$,*

$$V_H - s - \delta V(p) > 0 \tag{8a}$$

*for all $\delta \in (0,1)$ and all $p \in (0,1)$. In particular, (8a) holds for $p = p^*$ (if it exists), and thus the no-deviation condition (8) at a player's good turn of the equilibrium path is satisfied for all $\delta \in (0,1)$.*

Lemma 1 relates $V_H$ and $V_L$ of the turn-taking phase of the TTIR strategy.

Lemma 2 has an intuitive interpretation. The term $U(p)$ represents each player's payoff when both players randomize independently, and $\frac{h+l}{2}$ represents each player's payoff if it is possible for them to correlate their strategies. The difference represents the loss of current-period payoff associated with independent randomizations when compared to correlated strategies. Independent randomizations are costly because there is a positive probability that the players may reach the symmetric outcomes $(T,T)$ or $(S,S)$ and may therefore fail to maximize their total payoff.

Lemma 3 is the (nontrivial) intertemporal analog of Lemma 2: the relative loss of intertemporal payoff in using independent randomized strategies when compared to using correlated strategies. When the players both play $T$ with probability $p$ and $S$ with probability $1-p$, then the randomization phase will continue with probability $p^2 + (1-p)^2$. Therefore, the effective discount factor will be $\delta\left[p^2 + (1-p)^2\right]$ in (15) during the randomization phase. Note that the value of the game, $V^*$, is related to function $V(p)$ by

$$V(p^*) = V^*. \tag{16a}$$

Lemma 4 is obtained by combining (12) and (15). With the players using the TTIR strategy, the repeated game $G_\infty$ includes, probabilistically, some of the beginning periods in which the two players turn out to choose the same action, $(T,T)$ or $(S,S)$. Therefore, $V(p)$ is smaller than $V_H$.

The no-deviation condition (8) is related to whether a player will defect when she is supposed to take the good turn. It is not surprising to see from Lemma 5 that (8) holds for all $\delta \in (0,1)$.

### *3.5 Key conditions that determine the TTIR equilibrium, and the remaining tasks*

As given in subsection 3.3, if both no-deviation conditions (8) and (9) are satisfied, then $p^*$ and $V^*$ are related according to (10). To examine the conditions under which turn taking arises as a subgame-perfect equilibrium of game $G_\infty$, we focus our attention on $p^*$ (the equilibrium probability of randomization).

There are two ways that we can eliminate $V^*$ to obtain a relationship with $p^*$ only. Both are useful in the analysis. First, substituting (16) and (16a) into (10a) lead to

$$p^* = \frac{V_H - s - \delta \left\{ \frac{(p^*)^2 t + (1-p^*)^2 s + p^*(1-p^*)(V_H+V_L)}{1-\delta\left[(p^*)^2+(1-p^*)^2\right]} \right\}}{V_H + V_L - s - t - 2\delta \left\{ \frac{(p^*)^2 t + (1-p^*)^2 s + p^*(1-p^*)(V_H+V_L)}{1-\delta\left[(p^*)^2+(1-p^*)^2\right]} \right\}}, \tag{10b}$$

where $V_H$ and $V_L$ depend on $\delta$, $h$, and $l$ only. Equation (10b) can be interpreted as follows. Think of $p^*$ on the left-hand side of (10b) as the probability that both players choose $T$ in the current period (at the randomization phase), and $p^*$ on the right-hand side as the probability that both players choose $T$ in the future if the game re-starts. When the no-deviation conditions (8) and (9) are satisfied, then it can be deduced from Table 3 that the equilibrium probability of randomization ($p^*$) in the current period is given by the right-hand term of (10a), which involves the value of the game, since the game may re-start in the future. As $V(p^*) = V^*$ in (16a) makes clear, a player's continuation payoff $V^*$ depends on (future) $p^*$. The equilibrium condition (10b) can be regarded as a consistency condition between current and future probabilities of randomization of this infinitely repeated game with discounting. Note that (10b) is non-linear in $p^*$.

Second, (10) leads to

$$V^* = \frac{p^* t + (1-p^*) V_H}{1-\delta p^*} = \frac{p^* V_L + (1-p^*) s}{1-\delta(1-p^*)}. \tag{10c}$$

Substituting (6) and (7) into the second equality of (10c) and simplifying give

$$a\left(p^*\right)^2 + bp^* + c = 0, \tag{18}$$

where the coefficients $a$, $b$, and $c$ are related to the fundamental parameters (stage-game payoff parameters and the discount factor) according to

$$a = \delta\left[(1+\delta)(t-s) - (h-l)\right], \tag{18a}$$

$$b = \left(1-\delta^2\right)t + (1+\delta)^2 s - h - (1+2\delta)\,l, \tag{18b}$$

and

$$c = h + \delta l - (1+\delta)\,s. \tag{18c}$$

In summary, for a given game $G_\infty$, a TTIR equilibrium exists if there exists a $p^* \in (0,1)$ that simultaneously satisfy (8), (9) and (10), where (8) and (9) are the no-deviation conditions (during the turn-taking phase), and (10) is the equilibrium randomization condition (during the randomization phase). Moreover, the TTIR equilibrium is unique if there exists only one $p^* \in (0,1)$ that satisfies these three conditions.

As the no-deviation condition (8) at the good turn always holds according to Lemma 5, our remaining tasks are to examine under what circumstances the no-deviation condition (9) at the bad turn holds, as well as whether $p^*$, defined in (10)—or equivalently, (10b) or (18)—exists in the interval $(0,1)$ and is unique. The analysis differs for the $t \leq l$ and $t > l$ cases, as the underlying structure of the game is different in these two cases.

## 4 The $t \leq l$ Case: The Asymmetric Outcomes are Nash Equilibria of the Stage Game

The analysis of no-deviation condition (9) for the $t \leq l$ case is straightforward. Because of $t \leq l$ and Lemma 4, it is easy to see that $l - t \geq 0$ and $\delta\left[V_H - V(p)\right] > 0$ for all $p \in (0,1)$. The result is summarized in the following Lemma.

**Lemma 6** *For the repeated game $G_\infty$ with $t \leq l$,*

$$V_L - t - \delta V(p) = (l-t) + \delta\left[V_H - V(p)\right] > 0 \tag{9a}$$

*for all $\delta \in (0,1)$ and all $p \in (0,1)$. In particular, (9a) holds for $p = p^*$ (if it exists), and thus the no-deviation condition (9) at a player's bad turn of the equilibrium path is satisfied for all $\delta \in (0,1)$.*

For the $t \leq l$ case of game $G_\infty$, a player will not defect when she is supposed to take the bad turn, because by adhering to the equilibrium strategy, she will have both a current gain of $l - t$ and a future gain of $\delta [V_H - V(p)]$. The fact that taking a bad turn still gives the player a higher (or at least the same) current-period payoff when compared to defecting is important for understanding why the turn-taking equilibrium can be supported for any discount factor.

Because (8a) and (9a) are satisfied for all $\delta \in (0, 1)$ and all $p \in (0, 1)$ when $t \leq l$, what remains in showing that a unique TTIR equilibrium exists is to show that there exists a unique $p^* \in (0, 1)$ satisfying (10). In the Appendix, we use the Brouwer's Fixed Point Theorem to show that when $t \leq l$, the TTIR equilibrium exists for all $\delta \in (0, 1)$. We also show that the TTIR equilibrium is unique, and obtain the closed-form solution for $p^*$ (the equilibrium probability of randomization).

**Proposition 1** *For the repeated game $G_\infty$ with $t \leq l$, the strategy profile in which both players adopt TTIR constitutes a subgame-perfect equilibrium for all $\delta \in (0, 1)$. Moreover, the TTIR equilibrium is unique. The unique value of $p^*$ in the interval $(0, 1)$ is related to the fundamental parameters—through the relationships in (18a) to (18c)—according to*

$$p^* = \frac{-c}{b} \tag{19}$$

*when $a = 0$, or according to*

$$p^* = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \tag{20}$$

*when $a \neq 0$.*

## 5 The $t > l$ Case: The Asymmetric Outcomes are Not Nash Equilibria of the Stage Game

The derivation of the conditions under which turn taking can be supported as the equilibrium for the repeated game $G_\infty$ in which the asymmetric efficient outcomes are not Nash equilibria of the stage game is more difficult but also, arguably, more interesting than the case in which the asymmetric efficient outcomes are Nash equilibria of the stage game.

### 5.1 *A Major Difference When Compared to the $t \leq l$ Case*

The earlier analysis of the equilibrium of the stage game for the $t > l$ case is helpful in understanding the turn-taking equilibrium of the corresponding repeated game. In a one-shot interaction, $T$ is the dominant strategy for both players. Thus, neither of the asymmetric outcomes $(T, S)$ and $(S, T)$ is a Nash equilibrium for the stage game, even though each outcome yields a larger total payoff than the Nash equilibrium $(T, T)$ under assumption (5).

The importance of the inequality $t > l$ is also reflected in the no-deviation condition (9) of the repeated game $G_\infty$. When considering whether to defect from a bad turn (choosing $S$ when her opponent chooses $T$), a player knows that there will be future gain (as $\delta [V_H - V(p)] > 0$, according to Lemma 4) in adhering to the equilibrium strategy. However, there is now a current loss if the player does so, as $t > l$. There is a trade-off between current loss and future gain. It is this trade-off that distinguishes the analysis of this case from that of the $t \leq l$ case.

### 5.2 *The Turn-Taking Equilibrium*

As the proof for the case $t \leq l$ (in Section 4) is relatively straightforward, it may appear that it would also be easy to obtain the conditions under which (9) holds for the $t > l$ case by expressing the equilibrium probability of randomization ($p^*$) and the value of the game ($V^*$) in terms of the discount factor ($\delta$) and the stage-game payoff parameters ($h$, $l$, $s$, and $t$). For example, one may think of using the closed-form solution for $p^*$ similar to (19) and (20) for the $t \leq l$ case. However, while (19) and (20) hold for all discount factors $\delta \in (0, 1)$ for the $t \leq l$ case, they will only hold for sufficiently high discount factors when $t > l$, as we shall show later. Therefore, we need to first determine the range of discount factors in which (19) and (20) hold when $t > l$.

In the analysis of game $G_\infty$ with $t > l$, we proceed as follows. We first conjecture that there exists a critical discount factor $\delta_{TT} \in (0, 1)$ such that for all $\delta \in (\delta_{TT}, 1)$, a unique TTIR equilibrium exists.[10] We then show that if a unique TTIR equilibrium exists for all $\delta \in (\delta_{TT}, 1)$, the function $p^*(\delta)$ must be strictly decreasing in $\delta$. These results enable us to determine the value of

[10] Conjecturing that the TTIR equilibrium exists in the interval $(\delta_{TT}, 1)$, instead of assuming more generally that the equilibrium may exist in a region consisting of several disconnected intervals, is consistent with most results in the repeated game literature. Had we initially allowed for the possibility of disconnected intervals, we could still subsequently use Lemma 7 to conclude that the TTIR equilibrium only exists in one interval and the interval is $(\delta_{TT}, 1)$.

$\delta_{TT}$ as a function of the payoff parameters of the stage game. We then close our proof by showing that for all $\delta \in (\delta_{TT}, 1)$, there in fact exists a unique $p^*(\delta)$ that simultaneously satisfy (8), (9), and (10). Using this approach, we are able to study how $p^*(\delta)$ behaves as a function of $\delta$ and understand its economic intuition. (Note that whenever appropriate, we state explicitly the dependence of $p^*$ on $\delta$.)

Assuming that a unique TTIR equilibrium exists for all $\delta \in (\delta_{TT}, 1)$, we now investigate how $p^*$ (the equilibrium probability of randomization) changes when the discount factor changes but the other parameters remain constant. The partial derivative $\frac{\partial p^*}{\partial \delta}$ is given in (22) below. Manipulating various terms in (22) leads to the following Lemma regarding the monotonicity of $p^*$ with respect to $\delta$.[11]

**Lemma 7** *For the repeated game $G_\infty$ with $t > l$, if a unique TTIR equilibrium exists for all $\delta \in (\delta_{TT}, 1)$, then $p^*$ satisfies*

$$0.5 < p^* < 1, \tag{21}$$

*and*

$$\frac{\partial p^*}{\partial \delta} = \frac{(1-p^*)\left[\frac{\partial V_H}{\partial \delta} - \delta\frac{\partial V^*(\delta,p^*)}{\partial \delta} - V^*\right] - p^*\left[\frac{\partial V_L}{\partial \delta} - \delta\frac{\partial V^*(\delta,p^*)}{\partial \delta} - V^*\right]}{(V_H - s - \delta V^*) + (V_L - t - \delta V^*) + \delta(1-2p^*)\frac{\partial V^*(\delta,p^*)}{\partial p^*}} \tag{22}$$

*is negative.*

Lemma 7, which is the most difficult result to prove in this paper, is a crucial step in understanding the turn-taking equilibrium for the $t > l$ case. The intuition behind this result is as follows. In the TTIR strategy, the probability of randomization performs multiple roles: helping the players get onto an efficient turn-taking path (i.e., $\{..., (T,S), (S,T), (T,S), (S,T), ...\}$ or $\{..., (S,T), (T,S), (S,T), (T,S), ...\}$) in the initial periods of the game, resolving the question of who gets to start with the good turn, and acting as a punishment device to deter defection from the equilibrium behavior in the turn-taking phase. For the role of getting the players onto an efficient turn-taking path, observe that the probability of reaching an asymmetric outcome in a period is $2p(1-p)$, which is highest at $p = 0.5$ and is decreasing when $p$ is farther from 0.5. According to (21), we know that the equilibrium $p^*$ must be in the range $(0.5, 1)$.[12] A lower probability of randomization in this range

[11] However, for the other case ($t \leq l$) of game $G_\infty$, we have found examples in which the monotonicity property of $p^*$ with respect to $\delta$ does not hold.

[12] Note that in the TTIR equilibrium, a player chooses $p^*$ to ensure that for the other player, the second equality of (10), or equivalently, $p^*(V_L - t - \delta V^*) = (1-p^*)(V_H - s - \delta V^*)$, holds. We show in the Appendix that when $t > l$, $(V_H - s - \delta V^*) > (V_L - t - \delta V^*)$, or equivalently, $h - l - (1+\delta)(s-t) > 0$ in

will increase the chance of getting onto a turn-taking path. However, a lower probability of randomization may not constitute a strong enough deterrent against defection. Lemma 7 shows that the equilibrium $p^*$ that balances these efficiency and incentive considerations is decreasing in $\delta$ in the relevant region. When players are more patient, a lower probability of randomization will be sufficient to deter the players from deviating.

According to Lemma 7, the function $p^*(\delta)$ decreases monotonically in $\delta \in (\delta_{TT}, 1)$, provided that a unique TTIR equilibrium exists. Moreover, it is clear from (10) that $p^*(\delta)$ is a continuous function in the relevant region. Combining these features, we conclude that

$$\lim_{\delta \to \delta_{TT}} p^*(\delta) = 1, \tag{23}$$

since otherwise we could have found a lower discount factor such that $p^*(\delta)$ is still less than 1. Note that if both players choose $T$ with certainty during the randomization phase, they will never get onto an efficient turn-taking path. Equation (23) says that at the limiting case when $\delta$ tends to the critical discount factor, the punishment of TTIR becomes most severe as $p^*(\delta)$ tends to 1. Figure 1 illustrates how $p^*$ behaves as a function of $\delta$.

Furthermore, from the second equality of (10), we have

$$V_L - t - \delta V^* = \frac{1-p^*}{p^*}\left(V_H - s - \delta V^*\right). \tag{10d}$$

Combining it with Lemma 5 and (23), we have

$$\lim_{\delta \to \delta_{TT}} \left[V_L(\delta) - t - \delta V^*(\delta)\right] = 0. \tag{24}$$

We conclude that at the critical discount factor $\delta_{TT}$, $p^*(\delta)$ tends to 1 and the no-deviation condition at the bad turn must be binding. Using these results, we can determine the critical discount factor as a function of the payoff parameters of the stage game. The result is given in Proposition 2.

**Proposition 2** *For the repeated game $G_\infty$ with $t > l$, if a unique* TTIR equilibrium exists for all $\delta \in (\delta_{TT}, 1)$, *then the critical discount factor $\delta_{TT}$ depends on the stage-game parameters as follows:*

$$\delta_{TT} = \frac{t-l}{h-t}. \tag{25}$$

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(A6). Therefore, each player chooses $T$ with a probability higher than 0.5 at equilibrium to ensure that the other player is willing to randomize between $T$ and $S$.

**Proof of Proposition 2.** Using (23) and the first equality of (10c), we have

$$\lim_{\delta\to\delta_{TT}} V^*(\delta) = \lim_{\delta\to\delta_{TT}} \frac{p^*(\delta)\,t + [1-p^*(\delta)]\,V_H(\delta)}{1-\delta p^*(\delta)} = \frac{t}{1-\delta_{TT}}. \tag{26}$$

Therefore, substituting (7) and (26) into (24) lead to

$$\lim_{\delta\to\delta_{TT}} V_L(\delta) = \frac{l+\delta_{TT}h}{1-\delta_{TT}^2} = \lim_{\delta\to\delta_{TT}} [t+\delta V^*(\delta)] = t+\delta_{TT}\left(\frac{t}{1-\delta_{TT}}\right) = \frac{t}{1-\delta_{TT}}. \tag{24a}$$

Simplifying (24a) gives (25). ■

What is the intuition of Proposition 2? According to Lemma 7, the endogenously determined $p^*$ of the TTIR strategy, which is used to strike a balance between the efficiency consideration and the incentive consideration, is strictly decreasing in $\delta$ (between $\delta_{TT}$ and 1). When $\delta$ tends to 1, the no-deviation condition (9) is non-binding and the TTIR strategy profile constitutes a subgame-perfect equilibrium. As $\delta$ decreases (and future payoffs become less important), to ensure that the no-deviation condition (9) holds, $p^*$ must increase to make deviation more costly. However, the maximum possible punishment is when $p^*$ tends to 1. This defines the critical discount factor $\delta_{TT}$. As $\delta$ tends to $\delta_{TT}$, $p^*(\delta)$ tends to 1, and the no-deviation condition at the bad turn becomes binding. Moreover, the punishment approaches the Nash punishment (of choosing $T$ with probability 1 at every period), as in Friedman (1971). Thus, $V^*(\delta)$ approaches $t+\delta_{TT}t+\delta_{TT}^2t+\ldots = \frac{t}{1-\delta_{TT}}$ in the limit, as given in (26). Using the above results, we can determine the critical discount factor as in (25). Because of (5) and $t>l$, it is easy to conclude from (25) that $\delta_{TT}\in(0,1)$.

Having determined $\delta_{TT}$ as a function of the payoff parameters of the game according to (25), we now show that for all $\delta\in(\delta_{TT},1)$, in fact there exists a unique $p^*(\delta)$ that simultaneously satisfy (8), (9), and (10). According to Lemma 5, the no-deviation condition (8) at the good turn always holds. According to (10d), if a $p^*\in(0,1)$ satisfies (8) and (10), it will satisfy (9) automatically. Therefore, to show that there exists a unique TTIR equilibrium, it is sufficient to show that there exists a unique $p^*(\delta)$ satisfying (10). This is given in Proposition 3.

**Proposition 3** *For the repeated game $G_\infty$ with $t>l$, the TTIR strategy constitutes a subgame-perfect equilibrium for all $\delta\in(\delta_{TT},1)$ where $\delta_{TT}$ is given by (25). The TTIR equilibrium is unique, and the unique value of $p^*$ in the interval $(0,1)$ is given by (19) if $a=0$ or by (20) if $a\neq 0$.*

Note that, unlike the $t\leq l$ case, condition (9a) does not hold for all $p\in(0,1)$ for the $t>l$ case (even though it does hold for $p=p^*$ when $\delta>\delta_{TT}$.) Therefore, our proof for Proposition 3 proceeds differently from that for Proposition 1, even though we focus on the equilibrium randomization condition (10) in both cases.

Proposition 3 shows that when $t > l$, for the TTIR strategy to constitute a subgame-perfect equilibrium, the discount factor must be larger than $\delta_{TT} = \frac{t-l}{h-t}$. This is in sharp contrast to the $t \leq l$ case, in which the TTIR strategy constitutes a subgame-perfect equilibrium for any discount factor. According to (9), a player who is supposed to take the bad turn according to the TTIR strategy will not deviate if $t - l < \delta (V_H - V^*)$, that is, if the current gain from deviating is smaller than the future loss from deviating. When $t \leq l$ (i.e., the two asymmetric efficient outcomes are Nash equilibria of the stage game), this condition is satisfied for any discount factor because deviation actually yields a current loss, not a gain. When $t > l$ (i.e., the two asymmetric outcomes are not Nash equilibria of the stage game), however, deviation yields a current gain. If the discount factor is too small, then (9) cannot be satisfied no matter how the probability of randomization (which affects the value of $V^*$) is chosen.

### *5.3 Comparative Static Results*

The characterization of the critical discount factor in (25) when $t > l$ allows us to derive comparative static results regarding some parameters of the stage game. Despite their simplicity, these comparative static exercises reveal some interesting commonalities in the various widely studied repeated games in which the asymmetric outcomes of the stage game are joint-payoff-maximizing.

As observed in (25), the critical discount factor does not depend on parameter $s$ but on the other three payoff parameters.[13] Note first that the left-hand term of (24a) is a player's intertemporal payoff of adhering to the equilibrium strategy when her bad turn comes up, whereas the right-hand term is the intertemporal payoff of defecting. An increase in $t$ (at an unchanged $\delta_{TT}$) will increase the current and future payoffs of defecting. To restore the equilibrium condition (24a), the critical discount factor has to increase. Formally, differentiating $\delta_{TT}$ with respect to $t$ gives

$$\frac{\partial \delta_{TT}}{\partial t} = \frac{h-l}{(h-t)^2} > 0, \tag{27}$$

because of (3) and (5). Holding the value of $h$ and $l$ constant, an increase in

[13] The analysis in Subsection 5.2 makes clear that the critical discount factor is determined by the limiting case of $V^*(\delta)$ tending to the intertemporal payoff associated with the strategy of always choosing Tough. When a player considers whether to deviate or not at her bad turn of the turn-taking path in this limiting case, payoff $s$ does not appear in (26) since the outcome $(S, S)$ will never be reached in the future. Consequently, the critical discount factor does not depend on $s$.

$t$ (up to $\frac{h+l}{2}$) makes it harder for the players to use TTIR to support turn taking as an equilibrium outcome.

For the other two parameters $h$ and $l$, instead of relating the critical discount factor to each of them, we find it more interesting to relate $\delta_{TT}$ to two concepts dependent on $h$ and $l$: the efficiency gain from succeeding in achieving (any one of) the asymmetric efficient outcomes and the degree of distributional conflict in the stage game. In the stage game, the maximum and minimum amounts of the players' total gain attained as a result of succeeding in reaching either of the two asymmetric outcomes are $h+l-\min\{2t, 2s\}$ and $h+l-\max\{2t, 2s\}$, respectively. Holding the value of $t$ and $s$ constant, an increase in $h+l$ increases both the maximum and the minimum gains that the players attain when they succeed in reaching an asymmetric outcome. Therefore, we define

$$\lambda = h + l \tag{2a}$$

as the index for the efficiency gain from succeeding in achieving either of the two asymmetric outcomes. We also define

$$\theta = \frac{h}{l} \tag{2b}$$

as the index for distributional conflict. Without loss of generality, we can normalize the payoffs so that $l > 0$. As a result, $\lambda > 0$ and $\theta > 1$ according to (2a) and (2b). When $\lambda = h + l$ is held constant, an increase in $\theta$ implies that there is a higher degree of conflict of interest in the stage game. Note that when $\theta$ tends to 1, $h = l$, and it is natural to say that there is no distributional conflict in this case.

From (2a) and (2b), we can obtain $h = \frac{\theta\lambda}{1+\theta}$ and $l = \frac{\lambda}{1+\theta}$. Therefore, the stage game $G$ can also be expressed as a game where the primitives are $t$, $s$, $\theta$, and $\lambda$. This is illustrated in the right-hand panel of Table 1.

In the following analysis, we study how changes in the degree of conflict and the efficiency gain affect the critical discount factor above which the players succeed in using TTIR as an intertemporal cooperation mechanism for the $t > l$ case.[14] The critical discount factor in (25) can now be expressed as a function of $t$, $\theta$ and $\lambda$ as follows:

$$\delta_{TT} = \frac{t-l}{h-t} = \frac{(1+\theta)\,t-\lambda}{\theta\lambda-(1+\theta)\,t}. \tag{25a}$$

[14] Note that Proposition 1 implies that so long as $t \le l$, the TTIR profile constitutes an equilibrium for all $\delta \in (0,1)$. Thus, changes in the degree of conflict and the efficiency gain have no effect on the players' ability to use TTIR to facilitate intertemporal cooperation when $t \le l$.

Therefore, we have

$$\frac{\partial \delta_{TT}}{\partial \theta} = \frac{\lambda (\lambda - 2t)}{[\theta\lambda - (1+\theta)\, t]^2} > 0, \tag{28}$$

and

$$\frac{\partial \delta_{TT}}{\partial \lambda} = \frac{-t\left(\theta^2 - 1\right)}{[\theta\lambda - (1+\theta)\, t]^2} < 0, \tag{29}$$

because $t > l > 0$ and $\lambda = h + l > 2t$.

Equation (28) says that $\delta_{TT}$ is increasing in $\theta$, which means that an increase in the degree of conflict makes it harder for the players to use TTIR to support turn taking as an equilibrium intertemporal cooperation mechanism. This result is intuitive. An increase in distributional conflict (when other parameters are constant) implies that the player's payoff at the bad turn along the equilibrium turn-taking path is relatively unattractive. As a result, the players have a greater incentive to defect (at a given discount factor); thus the turn-taking equilibrium can only be supported above a higher critical discount factor.

Equation (29) says that $\delta_{TT}$ is decreasing in $\lambda$, which means that a decrease in the gain from succeeding in achieving either of the asymmetric efficient outcomes (provided that (4) and (5) still hold) leads to an increase in the critical discount factor above which turn taking can be supported as an equilibrium outcome.

## 6 Discussion

In earlier sections, we analyze repeated games with asymmetric efficient outcomes when the players use the TTIR strategy. This strategy differs in a few important aspects from strategies that have appeared in several earlier papers. For expositional convenience, in preceding sections, we focus on deriving analytical results without highlighting these differences. In this section we systematically discuss the differences.

### *6.1 Related Papers*

Our analysis of equilibrium turn-taking behavior is related to, but differs from, some other contributions. In an analysis of learning in repeated pure coordination games, Crawford and Haller (1990) emphasize the role of repeated interaction in solving the coordination problem. Specifically, they show how players can use independent randomizations to achieve a coordinated outcome and can then maintain coordination either by repeating the achieved coordinated outcome or by alternating between the achieved outcome and the other joint-payoff-maximizing outcome.

In a study of the efficiency properties of different conventions in repeated symmetric two-player mixed-interest games, Bhaskar (2000) also considers strategies that involve independent randomizations and some kind of turn taking. His contribution is perhaps the most closely related to ours in the existing literature.

In the context of a mixed-interest game, Bhaskar (2000, p. 250) defines a convention as "a rule which achieves asymmetric coordination by conditioning upon history." A convention is "bourgeois" if it specifies that after an asymmetric outcome is reached as a result of randomization in some initial period, the players will always choose this particular outcome thereafter. A convention is "egalitarian" if the players' payoffs are equalized as far as possible. The egalitarian convention works as follows.[15] Suppose the realized outcome in some initial period is $(T, S)$, and player 1's current-period payoff exceeds player 2's. To implement the egalitarian convention, the players choose $(S, T)$ in succeeding periods until the intertemporal payoff of player 2 exceeds that of player 1. At this point the players switch to playing $(T, S)$ until player 1's intertemporal payoff exceeds player 2's, and so on.[16]

The key point of Bhaskar's (2000) analysis is that different conventions give rise to different incentives for the players' behavior in the randomization phase. Under the bourgeois convention, each player has a high stake in ensuring that coordination will be achieved in her preferred asymmetric outcome (that is, $(T, S)$ for player 1 and $(S, T)$ for player 2), and this causes the players to place a higher weight on playing $T$ in the randomization phase than they do under the egalitarian convention. As a result, coordination is less likely to be achieved in earlier periods under the bourgeois convention, and this convention is less efficient than the egalitarian convention.

In a contribution that provides a game-theoretic explanation for staggered decisions, Lau (2001) considers a repeated duopoly game in which the players

[15] See Bhaskar (2000, pp. 256-257). This method, which is an application of the results in Sorin (1986) and Fudenberg and Maskin (1991), is based on the idea of keeping track of the players' intertemporal payoffs at each period.

[16] Consider the following example that an asymmetric outcome, say $(T, S)$, is first achieved in period 0 by randomization in game $G_\infty$ with $h = 2$ and $l = 1$. It can easily be verified that if $\delta = 0.8$, the two players' actions in the first twenty periods under the egalitarian convention are as follows: The players will play $(T, S)$ in periods 0, 3, 5, 6, 9, 11, 12, 15, 16, and 18 and will play $(S, T)$ in other periods. If $\delta = 0.9$, the two players' actions in the first twenty periods under the egalitarian convention are as follows: The players will play $(T, S)$ in periods 0, 3, 5, 6, 9, 10, 12, 15, 17, and 18 and will play $(S, T)$ in other periods.

can always set the choice variable for one or two periods. That paper shows how, by avoiding head-to-head competition, nonsynchronized decisions help the players achieve intertemporal sharing of the gain from cooperation when strategic complementarity is present. Along the equilibrium path of staggered moves, the players set their choice variables in alternating periods.

Like the current paper, each of these three papers considers a repeated game in which a coordination (or coordination-cum-conflict) problem is present, and the players use randomization to mitigate this problem.

Despite these similarities, there are important differences among these papers. First, our goal is to understand the phenomenon of turn taking, which differs from the objectives of the other three papers. Second, while all four papers consider repeated symmetric two-player games, the stage game in each paper is different. Crawford and Haller (1990) consider a pure coordination game, which, by definition, has no conflict element. Bhaskar (2000) considers a class of mixed-interest game in which the asymmetric efficient outcomes are Nash equilibria of the stage game. This paper considers the mixed-interest game in which the asymmetric efficient outcomes may or may not be Nash equilibria of the stage game. The game considered in this paper nests those studied in Bhaskar (2000) as special cases.[17]

There are also interesting differences among the players' strategies and equilibrium payoffs in Crawford and Haller (1990), Bhaskar (2000), and this paper. In the pure coordination game in Crawford and Haller (1990), each player is indifferent in the two efficient outcomes. Thus, the crucial point is to achieve and then maintain the efficient outcome(s). Since there is no conflict element in this game, it is not surprising that the players' equilibrium payoffs are the same. On the other hand, both Bhaskar (2000) and this paper consider a game with coordination and conflict elements. In the infinite-horizon version of Bhaskar (2000), the conflict element present in the stage game (as $h > l$) can be completely removed by the egalitarian convention, which guarantees that the players' intertemporal payoffs are the same, even though the players' payoffs in each period are not.[18] While some kind of turn taking is observed under the egalitarian convention, the time-varying strategies (see, for instance, the examples in footnote 16) considered by Bhaskar (2000) are more complicated than the TTIR strategy considered here. On the other hand, in this paper, when the players use the TTIR strategy, only the players' *ex ante* payoffs—but not their *ex post* payoffs—are equalized.

Compared to the analyses in Crawford and Haller (1990) and Bhaskar (2000),

[17] On the other hand, Lau (2001) is more different from the other papers, in that the action space of the stage game is continuous rather than binary.

[18] In the finite-horizon version of Bhaskar (2000), the players' *ex post* payoffs usually cannot be completely equalized.

we emphasize analyzing the incentive conditions. In the $t \leq l$ version of game $G_\infty$, these conditions are not very important since the players have no incentive to deviate from the efficient path; see Lemma 5 and Lemma 6. Crawford and Haller (1990) and Bhaskar (2000) only consider the $t \leq l$ case, and it is therefore understandable that they do not explicitly analyze these conditions. In fact, Bhaskar (2000, p. 255) emphasizes that when the two asymmetric outcomes are both Nash equilibria, "any convention can be supported as the outcome path of a subgame perfect equilibrium—players can simply ignore deviations and continue with the convention." On the other hand, there is a current gain if a player deviates at her bad turn when the two asymmetric outcomes are not Nash equilibria of the stage game. It is extremely important to analyze the incentive condition at a player's bad turn. By focusing on it, we are able to show the existence and uniqueness of the TTIR subgame-perfect equilibrium for game $G_\infty$ with $t > l$.

### *6.2 Comparing the TTIR Strategy with Other Subgame-Perfect Equilibrium Strategies*

A useful way to organize the discussion of various possible strategies with turn-taking features is to look at the differences in the initial and punishment phases of game $G_\infty$. We discuss two questions: (a) Instead of using randomized strategies in the initial phase, are there other turn-taking subgame-perfect equilibria such as the one in which player 1 chooses $T$ and player 2 chooses $S$ in the first period and they take turns afterwards? (b) Instead of using randomized strategies in the punishment phase, can the turn-taking subgame-perfect equilibrium be supported by other punishment strategies?

We first look at the initial phase. As mentioned in Section 3.1, there are conflict and coordination problems in this game, which can be seen as follows. Provided that (8) and (9) are satisfied, there are two efficient equilibria at period 0: (play $T$ at period 0, play $S$ at period 0) and (play $S$ at period 0, play $T$ at period 0). These two equilibria correspond to the two outcome sequences $X = \{(T, S), (S, T), (T, S), (S, T), ...\}$ and $Y = \{(S, T), (T, S), (S, T), (T, S), ...\}$. According to the Folk Theorem, both sequences can be supported as subgame-perfect equilibrium outcomes when the players are sufficiently patient. However, each player prefers the outcome in which she is the first player to start with the good turn: player 1 prefers $X$ to $Y$, while player 2 prefers $Y$ to $X$. Any argument that may be offered in favor of one equilibrium is equally applicable to the other.

This coordination-cum-conflict problem is a repeated game analog of a similar problem in the (one-shot) game $G$ with $t < l$ such as the game used by Farrell (1987) to study market entry issues. Recall that in game $G$ with $t < l$, there are two pure-strategy Nash equilibria $(T, S)$ and $(S, T)$ and a (symmetric) mixed-strategy equilibrium. Analogous to the fact that the two players have conflicting preferences regarding $X$ or $Y$ in the repeated game $G_\infty$, the two players also have conflicting preferences regarding the two pure-strategy Nash equilibria $(T, S)$ and $(S, T)$ in the one-shot game $G$.

As several authors have argued, in the absence of communication, the logical prediction of the one-shot game $G$ is the mixed-strategy equilibrium (Dixit and Shapiro, 1986; Farrell, 1987; also see Cooper et al., 1989). Following this line of argument, we consider the subgame-perfect equilibrium in which the players use the same mixed strategies in the initial phase. Given the symmetric nature of the game, this strategy satisfies the symmetry-invariance principle and ensures that both players receive the same *ex ante* payoff. On the other hand, while either sequence $X$ or $Y$ may also be supported as subgame-perfect equilibria, these equilibria violate the symmetry-invariance principle.

The above discussion provides a justification for why randomization is used in the initial phase for game $G_\infty$. However, the TTIR strategy specifies more than the point made above. It also specifies that the players use the randomization strategy in the punishment phase. This assumption has also been used in Crawford and Haller (1990) and Lau (2001), but many repeated game papers use other punishment strategies, especially when they examine a repeated standard prisoner's dilemma (see, for example, Friedman, 1971, Maskin and Fudenberg, 1986). The following discussion focuses on the more interesting $t > l$ case of game $G_\infty$,[19] and it compares the use of mixed strategy with a commonly used punishment strategy—the grim strategy that involves the play of the Nash punishment forever when deviation occurs (Friedman, 1971).[20] For easy comparison, we assume that the strategies in the initial and turn-taking phases are the same as those of the TTIR. We refer to this strategy as the "turn taking with Nash punishment" strategy.

[19] The reason that the other case is not considered here is as follows. When $t \leq l$, it is shown in Section 4 that by adhering to the equilibrium strategy at the bad turn, a player will have both current and future gains. As a result, a subgame-perfect equilibrium can be supported by the TTIR strategy for all discount factors, according to Proposition 1. Because the continuation value (if a player deviates) under any reasonable punishment strategy is smaller than $V_H$, it is straightforward to show that when $t \leq l$, Proposition 1 will continue to hold for other punishment strategies.

[20] In general, the critical discount factor based on the Nash punishment strategy is higher than that based on a more elaborate punishment strategy involving other players minimaxing a deviator (see, for example, Fudenberg and Maskin, 1986), except in games in which the equilibrium of the stage game holds all players to the minimax values. In game $G_\infty$ with $t > l$, each player's minimax value and her payoff at the Nash equilibrium of the stage game are indeed equal.

For game $G_\infty$ with $t > l$, the use of the Nash punishment strategy means that both players will choose $T$ forever if one or both of them deviate in the turn-taking phase. At first glance, one may expect the Nash punishment strategy to be more severe than the randomized strategy and may therefore think that the strategy is able to support a turn-taking equilibrium even when the players are not sufficiently patient to ensure that a subgame-perfect equilibrium can be supported by the TTIR strategy. Interestingly, while the first part of this conjecture is correct (in a sense to be made precise), the second part is not.

Define $V^{NP}$ as the continuation value in the punishment phase and $\delta_{NP}$ as the critical discount factor above which the turn taking with Nash punishment strategy can support a subgame-perfect equilibrium. A comparison of the subgame-perfect equilibria supported by this strategy and by the TTIR is given in the following Proposition.

**Proposition 4** *In the repeated game $G_\infty$ with $t > l$, a subgame-perfect equilibrium can be supported by the turn taking with Nash punishment strategy when $\delta \in (\delta_{NP}, 1)$, where*

$$\delta_{NP} = \delta_{TT}. \tag{30}$$

*Moreover, when $\delta \in (\delta_{TT}, 1)$, we have*

$$V^* > V^{NP} = \frac{t}{1-\delta}. \tag{31}$$

A comparison of the proof of Propositions 2 and 4 suggests that the punishment of the TTIR strategy is endogenous (as it depends on $\delta$ through $p^*$), while that of the turn taking with Nash punishment strategy is not (as the probability of choosing $T$ in the punishment phase is always 1). However, at the limiting case when $\delta$ tends to the critical discount factor, the punishment of TTIR becomes most severe as $p^*$ tends to 1; see (23). But the punishment in this limiting case is exactly the same as the Nash punishment. Not surprisingly, the critical discount factors for the two punishment strategies are the same.

On the other hand, (31) shows that the continuation value of the TTIR strategy at the off-equilibrium path (which is the same as $V^*$, the value of the game) is higher than that of the Nash punishment strategy when the discount factor is strictly higher than the critical level. The underlying reason of (31) is that the punishment of the TTIR strategy is endogenously determined (to balance the efficiency and incentive considerations) and varies as a function of the discount factor $\delta$. The punishment does not have to be very severe if the players are more patient. The severity of the Nash punishment strategy, on the other hand, does not vary with the discount factor.[21] Another manifesta-

[21] Note also that the turn taking with Nash punishment strategy specifies different

tion of the endogenous punishment of the TTIR strategy is that any deviation behavior that triggers punishment will end (endogenously and stochastically) within a finite time period with probability 1.

### *6.3 Efficient Frontier of Game $G_\infty$ and Efficiency Loss of the TTIR Strategy*

In this subsection we study the welfare properties of the TTIR equilibrium. A convenient way to conduct welfare analysis is to convert the players' intertemporal payoffs into units of single-period payoffs. This can be achieved by defining a player's average payoff (i.e., the discounted average of the stream of single-period payoffs; see, for example, Fudenberg and Maskin, 1986) at the equilibrium as

$$W^* = (1-\delta)\, V^*. \tag{32}$$

In terms of the players' average payoffs, the efficient frontier of game $G_\infty$ is given by the line joining $(h, l)$ and $(l, h)$ in Figure 2.

In the TTIR equilibrium of the repeated game with asymmetric efficient outcomes, once the players succeed in achieving one of the two asymmetric outcomes, they will achieve joint-payoff maximization for the rest of the game and will engage in intertemporal sharing of the gain from cooperation along the equilibrium path. The players' equilibrium payoffs (given by points A and B in Figure 2, with point A representing the payoff vector in which player 1 takes the good turn and player 2 takes the bad turn, and point B representing the other case)—when evaluated at any period during the turn-taking phase—will lie on the efficient frontier but not along the 45-degree line. However, because initial randomization is used to solve the coordination-cum-conflict problem, the players' equilibrium payoffs (given by point C in Figure 2)—when evaluated at the initial period (or more generally, at any period when the players are in the randomization phase)—will lie strictly below the efficient frontier but will be on the 45-degree line.

Since an initial cost is associated with the TTIR equilibrium in game $G_\infty$, one may ask whether it is possible to use other subgame-perfect equilibrium strategies to implement the efficient and egalitarian outcome $\left(\frac{h+l}{2}, \frac{h+l}{2}\right)$, namely, point D in Figure 2. According to the Folk Theorem (e.g., Fudenberg and Maskin, 1986, 1991), this should be possible when the players are sufficiently patient. Because public randomizations are not available in the environment considered in this paper, the results in Fudenberg and Maskin (1991) are more

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strategies in the initial and punishment phases. For example, if $(T, T)$ occurred in the previous period during the initial randomization phase, then the players will continue using the randomized strategy. If $(T, T)$ occurred in the previous period because a player deviated from the equilibrium turn-taking strategy at her bad turn, then the players will choose $T$.

relevant. Following Sorin (1986), they show that even in the absence of public randomizations, any payoff vector in the convex hull of the outcomes of the stage game can be implemented by (possibly time-varying) deterministic sequences of pure strategies when the discount factor is close enough to 1. They then show that with suitable modification of the argument in Fudenberg and Maskin (1986), there exists a punishment strategy to ensure that players will in fact behave according to the particular sequence when the discount factor is large enough. Thus, any feasible, individually rational payoff vector of an infinitely repeated game can be supported as the outcome of a subgame-perfect equilibrium when the players are sufficiently patient, even when public randomizations are not available.

A possible algorithm to implement the efficient and egalitarian outcome is as follows. Suppose that player 1 chooses $T$ and her rival chooses $S$ in period 0. The players then choose $(S,T)$ in subsequent periods until player 2's discounted sum of payoffs exceeds player 1's. At that point the players switch to playing $(T,S)$ until player 1's discounted sum of payoffs exceed player 2's, and so on. This algorithm is modified from the egalitarian strategy used in Bhaskar (2000), except that the initial randomization phase is absent.

The punishment strategies to support this time-varying deterministic sequence exist if the discount factor is large enough, according to Lemma 2 in Fudenberg and Maskin (1991). (Roughly speaking, the suggested punishment strategy consists of (a) any player defecting from the proposed sequence will be minimaxed by other players, and (b) any player who deviates from the prescribed strategy in the punishment phase against the original defector will herself be minimaxed, and so on.) Therefore, it is possible to implement the efficient and egalitarian outcome in game $G_\infty$ by the above algorithm when the discount factor is large enough.

The advantage of the time-varying deterministic sequence of pure strategies suggested in Sorin (1986) and Fudenberg and Maskin (1991) is that it can support the efficient and egalitarian outcome as a subgame-perfect equilibrium when the discount factor is high enough. No efficiency loss occurs, and the two (symmetric) players get the same intertemporal payoff. However, such strategies are usually complex, and the exact strategy sequence is sensitive to changes in parameters such as $\delta$ (see, for example, footnote 16). Moreover, the problem of multiple equilibria reemerges when this kind of strategy sequence is used. It is easy to observe (say, from the example in footnote 16) that if the efficient and egalitarian outcome can be supported by a particular sequence, we can always obtain another subgame-perfect equilibrium supporting $\left(\frac{h+l}{2}, \frac{h+l}{2}\right)$ by changing the label of the two players for this sequence. These time-varying strategies do not satisfy the principle of symmetry invariance.

On the other hand, the merit of the TTIR strategy is its relative simplicity.

The TTIR equilibrium satisfies symmetry invariance and is unique. Moreover, even though there is an initial cost associated with the TTIR equilibrium, this cost is arbitrarily close to zero when the players are sufficiently patient. This is given in the following Proposition.

**Proposition 5** *For any $\varepsilon > 0$, there exists a critical discount factor depending on $\varepsilon$, $\underline{\delta}(\varepsilon)$, such that for all $\delta \in (\underline{\delta}(\varepsilon), 1)$, each player in game $G_\infty$ obtains an average payoff larger than $\frac{h+l}{2} - \varepsilon$ at the TTIR equilibrium.*

According to Proposition 5, the average payoff of each player at the TTIR equilibrium, $W^*$, is arbitrarily close to the efficient frontier when the discount factor is large enough. [22] In this sense, the TTIR strategy is almost as successful at achieving efficiency as is the time-varying strategies considered in the Folk Theorem for repeated games without public randomizations (Fudenberg and Maskin, 1991).

## 7 Concluding Remarks

Turn-taking behavior has been observed in many settings. However, a systematic investigation of such behavior has not been found in the literature. This paper represents a step toward such an investigation. Incorporating essential features of various turn-taking examples, we study a symmetric two-player repeated game such that (a) the total payoff of the two players at the asymmetric outcomes is higher than that in the symmetric outcomes, and (b) the turns that the players take include good and bad turns. The above specification is very general, and it includes a number of games widely studied in the literature.

In this class of repeated games, we show that when a symmetric subgame-perfect equilibrium supported by the TTIR strategy exists, it is also unique. When the two asymmetric efficient outcomes are Nash equilibria of the stage game, turn taking can be supported as an equilibrium by TTIR for any discount factor. When the two asymmetric efficient outcomes are not Nash equilibria of the stage game, the equilibrium probability of randomization decreases with respect to the discount factor, and a subgame-perfect equilibrium can be supported by TTIR when the discount factor is above a critical level that varies with the payoff parameters in an intuitive manner.

[22] A related question, which is quantitative in nature, is how large the initial cost associated with independent randomizations is at different discount factors. While preliminary results show that the initial cost is different for different games, our analysis of the repeated battle of sexes game (Lau and Mui, 2003) suggests that this cost is quite small in general.

In this paper we conduct an analysis of the repeated games with asymmetric efficient outcomes when there is no communication opportunity between the players. In this environment, the use of independent randomized strategies allows the players to mitigate the coordination-cum-conflict problem. After an initial randomization phase (which may be interpreted as a process of trial and error, similar to the experimental results reported in Prisbrey, 1992), turn taking eventually emerges. The TTIR strategy depends on only a small set of relevant variables, since it specifies actions based on whether a symmetric or asymmetric outcome occurred in the previous period. The TTIR strategy satisfies symmetry invariance, and is simpler than those time-varying deterministic sequences of pure strategies considered in the repeated game literature. We believe it is a very natural strategy to consider in this class of repeated mixed-interest games with no communication.

A natural direction of future research is to investigate how to extend this benchmark model to other environments in which turn-taking behavior is potentially important. For example, the results reported in Ostrom et al. (1994) suggest that nonbinding communication can be efficiency-enhancing in the laboratory repeated games that they consider, a result that is broadly consistent with findings that cheap talk can be efficiency-enhancing in static mixed-interest games such as the battle of sexes (Cooper et al., 1989). On the other hand, the results reported in Prisbrey (1992) suggest that asymmetric turn-taking schemes—for example, one in which a player is supposed to take the good turn for two periods and then take the bad turn for one period, with her opponent doing the opposite—are more difficult to sustain. In the future, we plan to investigate whether extending the model to incorporate different kinds of asymmetry and communication will make it more difficult or easier for (potentially more sophisticated) turn-taking strategies to achieve joint-payoff maximization and intertemporal sharing of the gain from cooperation.

Finally, the benchmark model considered in this paper rules out the possibility that a player may attempt to "modify the game" to her advantage. However, in certain environments—for example, in deciding whether turn taking can be used to determine who will be the chairperson of a department or an important committee—players may be reluctant to take the bad turn because they are concerned that a player who gets to take the good turn in a particular period may attempt to alter the game to her advantage. A possible direction for future research is to investigate when and how, in such an environment, some kinds of turn-taking strategies may still be able to mitigate the conflict-cum-coordination problems that are more difficult than the one considered in our benchmark model.

## 8 Appendix

The following result appears a number of times in the Appendix. From (3) and $\delta \in (0,1)$, we have

$$h + \delta l = (1+\delta)\left(\frac{h+l}{2}\right) + (1-\delta)\left(\frac{h-l}{2}\right) > (1+\delta)\left(\frac{h+l}{2}\right). \qquad \text{(A1)}$$

**Proof of Lemma 2.** For any $0 < p < 1$,

$$\frac{h+l}{2} = [p + (1-p)]^2\left(\frac{h+l}{2}\right) = \left[p^2 + (1-p)^2\right]\left(\frac{h+l}{2}\right) + 2p(1-p)\left(\frac{h+l}{2}\right).$$

Combining the above expression with (14), and using (4) and (5), we have (13). ■

**Proof of Lemma 3.** If both players choose $T$ with probability $p$ in the randomization phase, then

$$V(p) = p^2[t + \delta V(p)] + p(1-p)(h + \delta V_L) + (1-p)p(l + \delta V_H) + (1-p)^2[s + \delta V(p)]. \qquad \text{(A2)}$$

Rearranging (A2) and using (14), we have (16). On the other hand,

$$\left\{1 - \delta\left[p^2 + (1-p)^2\right]\right\}\left(\frac{V_H + V_L}{2}\right) = \{1 - \delta[1 - 2p(1-p)]\}\left(\frac{V_H + V_L}{2}\right)$$

$$= (1-\delta)\left(\frac{V_H + V_L}{2}\right) + 2\delta p(1-p)\left(\frac{V_H + V_L}{2}\right).$$

Combining the above expression with (16) and using (11) and (13), we obtain (15). ■

**Proof of Lemma 5.** Using (6), (7), (14), and (A2), we have

$$V_H - [s + \delta V(p)] = h + \delta l + \delta^2 V_H$$

$$-s - \delta U(p) - \delta^2\left\{\left[p^2 + (1-p)^2\right]V(p) + p(1-p)(V_H + V_L)\right\}.$$

First, (12) and (17) imply that

$$V_H = \left[p^2 + (1-p)^2 + 2p(1-p)\right]V_H > \left[p^2 + (1-p)^2\right]V(p) + p(1-p)(V_H + V_L).$$

Second, (4), (13), and (A1) imply that

$$h + \delta l - [s + \delta U(p)] > (1+\delta)\left(\frac{h+l}{2}\right) - [s + \delta U(p)] = \left(\frac{h+l}{2} - s\right) + \delta\left[\frac{h+l}{2} - U(p)\right] > 0.$$

Combining these three expressions, we obtain (8a). Since (8a) holds for all $p \in (0, 1)$, it holds at the equilibrium probability of randomization (if it exists). ■

**Proof of Proposition 1.** We are going to prove that, first, the solution to (10b) exists in the interval $(0, 1)$ and, second, it is unique.

To apply well-known mathematical results, we extend (slightly) the domain of $p$ from $(0, 1)$ to $[0, 1]$. We define the function

$$f(p) = \frac{V_H - s - \delta V(p)}{[V_H - s - \delta V(p)] + [V_L - t - \delta V(p)]} \tag{A3}$$

over $p \in [0, 1]$, where $V(p)$ is defined in (16). According to Lemma 5 and Lemma 6, for any $p \in (0, 1)$, $V_H - s - \delta V(p) > 0$ and $V_L - t - \delta V(p) > 0$ when $t \leq l$. Therefore, it can be concluded from (A3) that $0 < f(p) < 1$. Moreover, it is easy to extend the proof of Lemma 2 to Lemma 6 to show that they hold for $p = 0$ and $p = 1$ as well. Therefore, $f(.)$ is a continuous function from the compact set $[0, 1]$ to itself. Moreover, we can observe that the solution to (10b) is a fixed point of the function $f(.)$ in (A3).

Since the function $f(.)$ maps the interval $[0, 1]$ to itself, we can apply the Brouwer's Fixed Point Theorem and conclude that this function has a fixed point. That is, there exists a $p \in [0, 1]$ such that $f(p) = p$. Moreover, $f(p) = p$ does not hold at $p = 0$ or $p = 1$, since $f(0) > 0$ and $f(1) < 1$. Therefore, we conclude that the solution to (10b) exists in the interval $(0, 1)$. The solution is denoted by $p^*$.

To show the uniqueness of $p^*$, we know that (18) is a quadratic equation in $p^*$ and, therefore, that there are at most two real roots.[23] Together with the existence result above, there must be either one or two equilibrium $p^*$ in the interval $(0, 1)$.

From the standard results for quadratic equations, we know that if $a = 0$, then there is just one $p^*$ and it is given by (19), where $b$ must be negative in

[23] A common approach to show the uniqueness of $p^*$ is to attempt using the Contraction Mapping Theorem. We have, however, found some counterexamples for the $t > l$ case (such as $h = 160$, $l = 40$, $t = 80$, $s = 20$ and $\delta = 0.75$) that $f(p)$ in (A3) is not a contraction mapping. Hence, we use a different approach. One advantage of using the formulas for quadratic equations to prove the uniqueness of $p^*$ is that we also obtain the closed-form solution for $p^*$. The closed-form solution, given by (19) or (20), forms the basis for quantitative welfare analysis related to game $G_\infty$, such as those performed in Lau and Mui (2003).

this case.[24] If $a \neq 0$, the two roots to (18) are given by (20) and

$$p^* = \frac{-b + \sqrt{b^2 - 4ac}}{2a}. \tag{A4}$$

Since at least one equilibrium probability of randomization lies in $(0, 1)$, the term $(b^2 - 4ac)$ must be non-negative, and either (20) or (A4), but not both, may lie outside the interval $(0, 1)$.

From (18a) to (18c), we know that $c > 0$ but that $a$ and $b$ can be either positive or negative. If $a < 0$, then $b^2 - 4ac > b^2$. Therefore $\sqrt{b^2 - 4ac} > b$, and $p^*$ in (A4) is negative.

If $a > 0$, then $b^2 - 4ac < b^2$. We have two cases: $b > 0$ and $b < 0$. (It is easy to see that $b = 0$ is inconsistent with $b^2 - 4ac \geq 0$ and $a > 0$.) If $b > 0$, then $\sqrt{b^2 - 4ac} < b$ and $p^*$ in (A4) is negative. If $b < 0$, then $-b + \sqrt{b^2 - 4ac} > 0$ and $p^*$ in (A4) is positive. In this case, $p^*$ in (A4) is larger than 1 if and only if

$$-b + \sqrt{b^2 - 4ac} > 2a,$$

which, after simplification, is equivalent to

$$a + b + c < 0.$$

From (18a) to (18c), it can be shown that

$$a + b + c = (1 + \delta)\, t - (l + \delta h)\,. \tag{18d}$$

It is easy to see from (18d) that when $t \leq l$, $a + b + c < 0$ for all $\delta \in (0, 1)$. Therefore, whether $a$ in (18a) is positive or negative, $p^*$ in (A4) does not lie in the interval $(0, 1)$.

Consequently, there is only one solution to $0 < p^* < 1$ for all $\delta \in (0, 1)$ when $t \leq l$, and the solution is given by (19) if $a = 0$ or (20) if $a \neq 0$. ■

**Proof of Lemma 7.** From (8), (9), and (10a), we have

$$1 - p^* = \frac{V_L - t - \delta V^*}{(V_H - s - \delta V^*) + (V_L - t - \delta V^*)} > 0. \tag{A5}$$

[24] The pure coordination game in Crawford and Haller (1990) can be represented as $s = t$ and $h = l$ (and, thus, $t < l$, because of (5)), using the notation in Table 1. In this case, the no-deviation conditions (8) and (9) become the same, and it can be shown that our results (which holds for $h > l$) will also be applicable when $h = l$. It can further be shown that $a$ in (18a) is 0 for this game. Thus, for all $\delta \in (0, 1)$, $p^* = 0.5$ according to (19).

Also, (12) and (10a) imply

$$p^* - 0.5 = \frac{(V_H - s - \delta V^*) - (V_L - t - \delta V^*)}{2\left[(V_H - s - \delta V^*) + (V_L - t - \delta V^*)\right]}$$

$$= \frac{h - l - (1+\delta)(s-t)}{2(1+\delta)\left[(V_H - s - \delta V^*) + (V_L - t - \delta V^*)\right]}. \qquad \text{(A6)}$$

It is easy to see that the denominator of (A6) is positive. For the numerator, either $t - s \geq 0$ or $t - s < 0$ can be consistent with $t > l$. If $t - s \geq 0$, it is easy to conclude from (3) that the numerator of (A6) is positive. If $t - s < 0$, then (4), $t > l$ and $0 < \delta < 1$ imply

$$h - l - (1+\delta)(s-t) > h - l - 2(s-t) = 2\left[\left(\frac{h+l}{2} - s\right) + t - l\right] > 0. \qquad \text{(A7)}$$

Combining (A5), (A6), and (A7), we have (21).

From the second equality of (10), we have

$$p^*(\delta)\left[V_L(\delta) - t - \delta V^*(\delta, p^*(\delta))\right] = \left[1 - p^*(\delta)\right]\left[V_H(\delta) - s - \delta V^*(\delta, p^*(\delta))\right],$$

where the dependence of $p^*$ on $\delta$, and $V^*$ on $\delta$ and $p^*(\delta)$ are written explicitly. (Note that $V^*(\delta)$ in (24) and $V^*(\delta, p^*(\delta))$ are two ways to represent the dependence of $V^*$ on the underlying parameters.) Differentiating the above expression with respect to $\delta$, and rearranging, gives (22).

Consider the denominator of (22). Differentiating the first equality of (10c) with respect to $p^*$ and simplifying, we have

$$\frac{\partial V^*}{\partial p^*} = \frac{(1+\delta)t - (h + \delta l)}{(1 - \delta p^*)^2 (1+\delta)} < 0. \qquad \text{(A8)}$$

This is because the denominator of the middle term of (A8) is positive, and (5) and (A1) imply $h + \delta l > (1+\delta)\left(\frac{h+l}{2}\right) > (1+\delta)t$.

Combining (21) and (A8), the third term in the denominator of (22) is positive. The first two terms in the denominator of (22) are positive when (8) and (9) hold. Therefore, we have

$$(V_H - s - \delta V^*) + (V_L - t - \delta V^*) + \delta(1 - 2p^*)\frac{\partial V^*}{\partial p^*} > 0. \qquad \text{(A9)}$$

Consider the numerator of (22). First, (21) implies

$$p^* > 1 - p^* > 0. \qquad \text{(A10)}$$

Second, using (12), we have

$$\left(\frac{\partial V_L}{\partial \delta}-\delta\frac{\partial V^*}{\partial \delta}-V^*\right)-\left(\frac{\partial V_H}{\partial \delta}-\delta\frac{\partial V^*}{\partial \delta}-V^*\right)$$

$$=\frac{\partial V_L}{\partial \delta}-\frac{\partial V_H}{\partial \delta}=\frac{-\partial\left(V_H-V_L\right)}{\partial \delta}=\frac{h-l}{\left(1+\delta\right)^2}>0. \tag{A11}$$

Third, using (15), we have

$$\frac{\partial}{\partial \delta}\left\{\delta\left[\frac{V_H+V_L}{2}-V^*\left(\delta,p^*\left(\delta\right)\right)\right]\right\}=\frac{\partial}{\partial \delta}\left\{\frac{\delta\left[\frac{h+l}{2}-U\left(p^*\right)\right]}{1-\delta\left[\left(p^*\right)^2+\left(1-p^*\right)^2\right]}\right\}$$

$$=\frac{\left[\frac{h+l}{2}-U\left(p^*\right)\right]}{\left\{1-\delta\left[\left(p^*\right)^2+\left(1-p^*\right)^2\right]\right\}^2}>0. \tag{A12}$$

Fourth, (7) implies $\frac{\partial V_L}{\partial \delta}=\frac{\partial(\delta V_H)}{\partial \delta}=\delta\frac{\partial V_H}{\partial \delta}+V_H$, and (6) implies $\frac{\partial V_H}{\partial \delta}=\frac{\partial(\delta V_L)}{\partial \delta}=\delta\frac{\partial V_L}{\partial \delta}+V_L$. Using these relationships, (A11) and (A12), we have

$$\frac{\partial V_L}{\partial \delta}-\delta\frac{\partial V^*}{\partial \delta}-V^*=\frac{\partial}{\partial \delta}\left[\delta\left(V_H-V^*\right)\right]=\frac{\partial}{\partial \delta}\left[\delta\left(\frac{V_H+V_L}{2}-V^*\right)\right]+\frac{\partial}{\partial \delta}\left[\delta\left(\frac{V_H-V_L}{2}\right)\right]$$

$$>\frac{\partial}{\partial \delta}\left[\delta\left(\frac{V_H-V_L}{2}\right)\right]=\frac{1}{2}\left[\frac{\partial\left(\delta V_H\right)}{\partial \delta}-\frac{\partial\left(\delta V_L\right)}{\partial \delta}\right]=\frac{1}{2}\left(\frac{\partial V_L}{\partial \delta}-\frac{\partial V_H}{\partial \delta}\right)>0. \tag{A13}$$

Therefore, (A10), (A11), and (A13) imply that

$$p^*\left(\frac{\partial V_L}{\partial \delta}-\delta\frac{\partial V^*}{\partial \delta}-V^*\right)-\left(1-p^*\right)\left(\frac{\partial V_H}{\partial \delta}-\delta\frac{\partial V^*}{\partial \delta}-V^*\right)$$

$$>\left(1-p^*\right)\left(\frac{\partial V_L}{\partial \delta}-\delta\frac{\partial V^*}{\partial \delta}-V^*\right)-\left(1-p^*\right)\left(\frac{\partial V_H}{\partial \delta}-\delta\frac{\partial V^*}{\partial \delta}-V^*\right)>0. \tag{A14}$$

That is, the numerator of (22) is negative. Combining (A9) and (A14) gives Lemma 7. ■

**Proof of Proposition 3.** To show the existence of $p^* \in (0,1)$ satisfying (10) when $t>l$, we define the following continuous function

$$g\left(p\right)=\frac{V_H-s-\delta V\left(p\right)}{\left[V_H-s-\delta V\left(p\right)\right]+\left[V_L-t-\delta V\left(p\right)\right]}-p \tag{A15}$$

over $p\in[0,1]$, where $V\left(p\right)$ is defined in (16). It is easy to observe that $g\left(.\right)$ is a well-defined function for every $p\in[0,1]$, and the solution to (10b) is defined by $g\left(p\right)=0$.

Using (16) and (4) to (7), we can show that for all $\delta \in (\delta_{TT}, 1)$,

$$g(0) = \frac{V_H - s - \delta V(0)}{[V_H - s - \delta V(0)] + [V_L - t - \delta V(0)]} - 0 = \frac{V_H - s - \delta\left(\frac{s}{1-\delta}\right)}{V_H + V_L - s - t - 2\delta\left(\frac{s}{1-\delta}\right)} > 0, \tag{A15a}$$

and

$$g(1) = \frac{V_H - s - \delta V(1)}{[V_H - s - \delta V(1)] + [V_L - t - \delta V(1)]} - 1 = \frac{-\left[V_L - t - \delta\left(\frac{t}{1-\delta}\right)\right]}{V_H + V_L - s - t - 2\delta\left(\frac{t}{1-\delta}\right)} < 0. \tag{A15b}$$

Applying the Intermediate Value Theorem (see, for example, Rosenlicht, 1968, p. 82), we know that there exists a $p \in [0, 1]$ such that $g(p) = 0$. Moreover, $g(p) = 0$ does not hold at $p = 0$ or $p = 1$, as observed in (A15a) and (A15b). Therefore, we conclude that the solution to (10b) exists in the interval $(0, 1)$. The solution is denoted by $p^*$.

The proof of the uniqueness of $p^*$ is similar to that of Proposition 1, except that (18d) is negative only for $\delta \in (\delta_{TT}, 1)$ when $t > l$, whereas (18d) is negative for all $\delta \in (0, 1)$ when $t \leq l$. Consequently, we conclude that there is only one solution to $0 < p^* < 1$ for $\delta \in (\delta_{TT}, 1)$ when $t > l$, and the solution is given by (19) if $a = 0$ or (20) if $a \neq 0$. ■

**Proof of Proposition 4.** Since $(T, T)$ is reached in every period of the punishment phase if both players use the turn taking with Nash punishment strategy in game $G_\infty$ with $t > l$, it is easy to see that $V^{NP} = t + \delta V^{NP} = \frac{t}{1-\delta}$. Therefore, the no-deviation condition at the bad turn is[25]

$$V_L - \left(t + \delta V^{NP}\right) = (l - t) + \delta\left(V_H - V^{NP}\right) > 0. \tag{A16}$$

Using (6), it is straightforward to show that (A16) is satisfied when $\delta > \frac{t-l}{h-t}$. This proves (30).

To prove (31), we first show that $(1-\delta)\left(\frac{V_H+V_L}{2} - V^*\right)$ decreases monotonically with respect to $\delta$, when the stage-game parameters ($h$, $l$, $s$ and $t$) remain constant. When $h$, $l$, $s$ and $t$ remain constant, $V^*(\delta, p^*(\delta))$ depends on $\delta$ only—directly as well as indirectly through $p^*(\delta)$. Using (15), we have

$$\frac{\partial}{\partial\delta}\left\{(1-\delta)\left[\frac{V_H + V_L}{2} - V^*(\delta, p^*(\delta))\right]\right\} = \frac{\partial}{\partial\delta}\left\{\frac{(1-\delta)\left[\frac{h+l}{2} - U(p^*)\right]}{1 - \delta\left[(p^*)^2 + (1-p^*)^2\right]}\right\}$$

[25] It is easy to see that the no-deviation condition at the good turn, $V_H - \left(s + \delta V^{NP}\right) > 0$, is always non-binding.

$$= -\left[\frac{h+l}{2} - U\left(p^*\right)\right] \frac{\left[1-\left(p^*\right)^2-\left(1-p^*\right)^2\right]}{\left\{1-\delta\left[\left(p^*\right)^2+\left(1-p^*\right)^2\right]\right\}^2} < 0. \tag{A17}$$

Also,

$$\frac{\partial}{\partial p^*}\left\{(1-\delta)\left[\frac{V_H+V_L}{2} - V^*\left(\delta, p^*\left(\delta\right)\right)\right]\right\} = -\left(1-\delta\right)\frac{\partial\left[V^*\left(\delta, p^*\left(\delta\right)\right)\right]}{\partial p^*}. \tag{A18}$$

Using Lemma 7, (A8), (A17), and (A18), we know that for $t > l$,

$$\frac{d}{d\delta}\left[(1-\delta)\left(\frac{V_H+V_L}{2} - V^*\right)\right] = \frac{\partial}{\partial \delta}\left\{(1-\delta)\left[\frac{V_H+V_L}{2} - V^*\left(\delta, p^*\left(\delta\right)\right)\right]\right\}$$

$$+\frac{\partial}{\partial p^*}\left\{(1-\delta)\left[\frac{V_H+V_L}{2} - V^*\left(\delta, p^*\left(\delta\right)\right)\right]\right\}\frac{\partial p^*\left(\delta\right)}{\partial \delta} < 0. \tag{A19}$$

Since $(1-\delta)\left(\frac{V_H+V_L}{2}\right) = \frac{h+l}{2}$ is independent of $\delta$, we conclude from (A19) that for $\delta \in (\delta_{TT}, 1)$,

$$\frac{d}{d\delta}\left[(1-\delta)\, V^*\left(\delta, p^*\left(\delta\right)\right)\right] > 0. \tag{A20}$$

Finally, we know from (26) that $\lim_{\delta\to\delta_{TT}} (1-\delta)\, V^* = t$. Combining this expression with (A20), we conclude that $(1-\delta)\, V^* > t$ for $\delta \in (\delta_{TT}, 1)$. This proves (31). ■

**Proof of Proposition 5.** Using (6), (32), and the first equality of (10c), we have

$$W^* = \frac{(1-\delta)\left[p^*t + (1-p^*)\left(\frac{h+\delta l}{1-\delta^2}\right)\right]}{1-\delta p^*} = \frac{(1+\delta)(1-\delta)\,p^*t + (1-p^*)(h+\delta l)}{(1+\delta)(1-\delta p^*)}.$$

Since $\lim_{\delta\to 1} p^*\left(\delta\right) < 1$ according to (23) and Lemma 7, it is easy to show that

$$\lim_{\delta\to 1} W^* = \frac{h+l}{2}. \tag{A21}$$

Equation (A21), together with the fact that $W^*$ is continuous in $\delta$, leads to Proposition 5. ■

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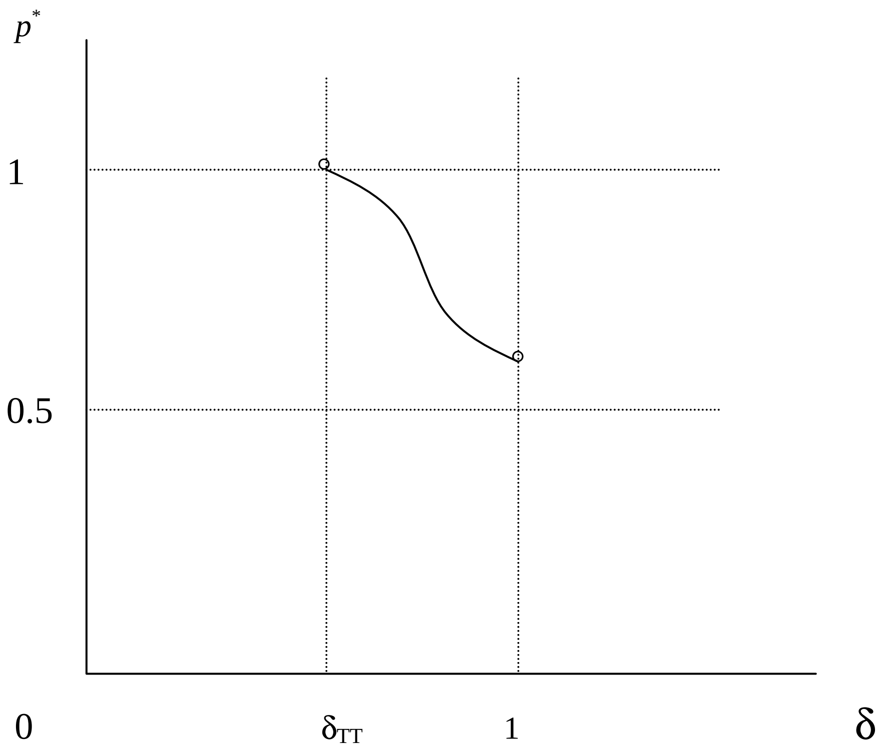
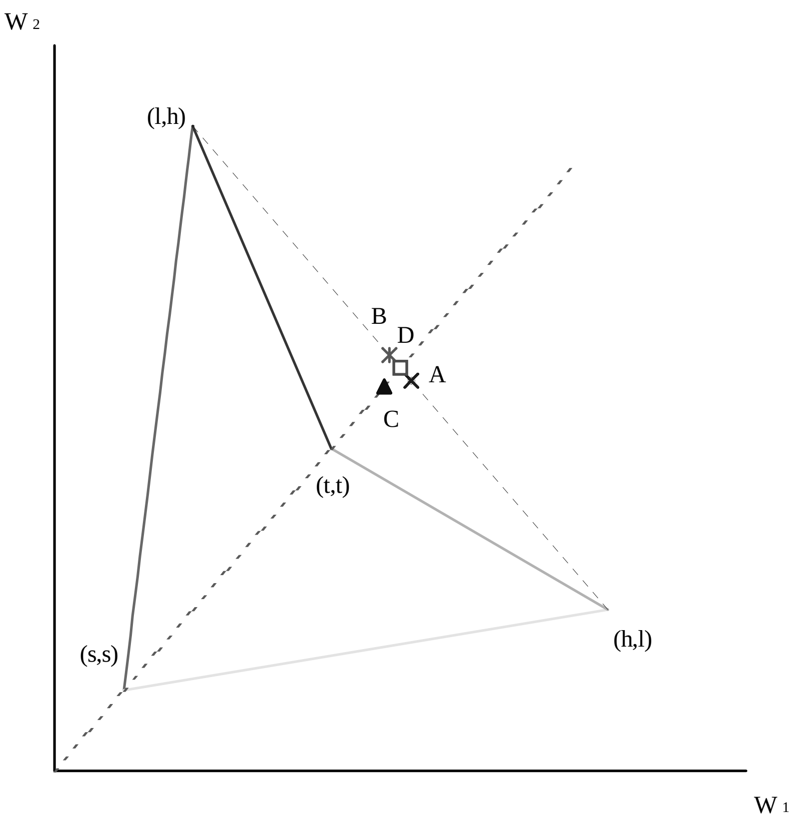



**Figure 1: The behavior of the $p^*(\delta)$ function (for the $t > l$ case)**




$A:\left((1-\delta)V_H,(1-\delta)V_L\right)$

$B:\left((1-\delta)V_L,(1-\delta)V_H\right)$

$C:\left(W^*,W^*\right)$

$D:\left(\frac{h+l}{2},\frac{h+l}{2}\right)$

Note: The figure is based on parameters $h=160$, $l=40$, $t=80$, $s=20$ and $\delta=0.9$.

**Figure 2: Efficiency loss of the TTIR strategy**

**Table 1: Strategies and Payoffs of the Stage Game**

(a) The $h$ and $l$ Specification

| 1\2 | Tough | Soft |
|---|---|---|
| **Tough** | $(t,t)$ | $(h,l)$ |
| **Soft** | $(l,h)$ | $(s,s)$ |

(b) The $\lambda$ and $\theta$ Specification

| 1\2 | Tough | Soft |
|---|---|---|
| **Tough** | $(t,t)$ | $\left(\frac{\theta\lambda}{1+\theta},\frac{\lambda}{1+\theta}\right)$ |
| **Soft** | $\left(\frac{\lambda}{1+\theta},\frac{\theta\lambda}{1+\theta}\right)$ | $(s,s)$ |

**Table 2: Different Symmetric Two-By-Two Games**

| Game | Parameter Restrictions | Equilibrium / Equilibria of the One-Shot Game | Example |
|---|---|---|---|
| (1) Pure Coordination Game | $h = l > s = t$ | $(S,T)$, $(T,S)$, mixed-strategy equilibrium | Crawford and Haller (1990), p. 573 |
| (2) Assurance Game | $s > h > t > l$ | $(S,S)$, $(T,T)$ | Arms Race game in Dixit and Skeath (1999), Figure 4.10 (*T*: Build; *S*: Refrain) |
| (3) Battle of the Sexes | $h > l > s = t$ | $(S,T)$, $(T,S)$, mixed-strategy equilibrium | Cooper at al. (1989), Figure 1 (*T*: Action 2; *S*: Action 1) |
| (4) Game of Chicken | $h > l >$ t; s > t | $(S,T)$, $(T,S)$, mixed-strategy equilibrium | Market Entry game in Farrell (1987) (*T*: In; *S*: Out) |
| (5) Prisoner's Dilemma (the non-standard version) | $h > s > t > l$; $h + l > 2s$ | $(T,T)$ | Dixit and Skeath (1999), Figure 11.2 (*T*: Not build; *S*: Build) |
| (6) Game of CPR Assignment (with $h > 2l$) | $h > t > l > s$; $t = \frac{h}{2}$; $s = \frac{l}{2}$ | $(T,T)$ | Ostrom et al. (1994), Figure 3.5(b) (*T*: Going to the good spot; *S*: Going to the bad spot) |

**Table 3: Strategies and Intertemporal Payoffs at the Beginning of the Repeated Game**

| 1\2 | Playing Tough at Period 0 | Playing Soft at Period 0 |
|---|---|---|
| **Playing Tough at Period 0** | $(t+\delta V^*, t+\delta V^*)$ | $(V_H, V_L)$ |
| **Playing Soft at Period 0** | $(V_L, V_H)$ | $(s+\delta V^*, s+\delta V^*)$ |