Capital Control, Speculation and Exchange Rate Volatility

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Abstract

This paper studies a plausible connection among rational speculators, exchange rate volatility and capital controls. When Krugman (1999) asserted that there should be appropriate controls on international capital movements to avoid currency volatilities from speculative activities, this paper shows whether to take capital controls depends on types of shocks and the risk preference of rational speculators.  
(1) If only current account shocks occur, the increase in the risk preference of rational speculators will decrease the conditional variance of exchange rates. In this case, the best policy is to let capitals freely move in the world.  
(2) If only interest rate shocks occur, the conditional variance of the exchange rate is monotonically increasing in the risk preference of rational speculators. Under such circumstance, the controls over international capital movements indeed decrease the exchange rate volatility.  
(3) When both current account and capital account shocks occur, then, if speculators are more risky, capital controls decrease exchange-rate volatility; however, if speculators are less risky, free capital movements can temper the exchange rates response to transitory shocks.

Key words: Capital controls, Speculation, Exchange Rate Volatility  
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This is a preliminary draft. Comments are welcome. Corresponding e-mail account is mercy@nccu.edu.tw.
1. Introduction

In 1990s, the capital flowed significantly into the developing countries as the international capital mobility increased. However, during this period, financial crisis exploded one after another in these developing countries: the Latin America crisis in 1994 and the Asian crisis in 1997. These two financial crises happened so closely that it means similar crisis will be likely to happen again in the future. The similarity in both financial crises was the speculative behavior playing a very important role. As a result, the relationships among the international capital mobility, speculative behavior and the currency crisis become the important topic for every government and authority, and turn out also as key issues for academic researches [Das (1999)].

The rapid movements of the international capital across countries have been regarded as the cause of unstable currency of the developing countries. The contagion effect occurred in the financial crisis leads to devaluation in those countries that originally adopt fixed exchange rate; as a result, the exchange rate is forced to float. On the other hand, those countries with floating exchange rates also face depreciation of their own currencies and large fluctuation in the exchange rates. Such fluctuation of exchange rates is one of the asset prices over-fluctuated phenomenon in the financial crisis [Crooke (1997)]. Calvo and Mendoza (2000) attributed the volatility of exchange rates to the herd behaviors of people when they re-allocate their assets in obtaining the optimum international portfolios. Such behavior is always treated as the speculative behavior, if it results to a dramatic fluctuation in the asset prices. However, Lagunoff and Schreft (1999) asserted that this is just a sensible reaction for investors when they attempt to maximize expected profits under uncertainties.

What are the effects caused by the speculative behavior on the exchange rate volatility? There have been various views. Friedman (1953) claimed that rational speculative activity must smooth exchange rates. On the other hand, Carlson and Osler (2000) criticized Friedman’s point because it does not take the interest-rate differentials into account in his interpretation of speculative behavior. As the difference between interest rates is what causes the speculative activity, speculation may increase the volatility of exchange rates. Whether rational speculators stabilize the exchange rate will depend upon the level of speculation. Rational speculation is stabilizing at low levels of speculative activity and destabilizing at high levels. Although the theoretical views are quite contradicting, the past experiences show the manipulation of hedge fund always leads to the great fluctuation in exchange rates.
during the financial crisis.

With an increase in international capital mobility and a decrease in the cost of transactions, the transactions of the foreign exchange have rapidly risen since the 1990s. Bartolini and Drazen (1997) showed that after loosening capital controls, large amount of foreign capital flowed into the developing countries. The short-term fluctuation of exchange rates will increase as foreign exchange transactions rise and the financial instruments developed [Tobin (1984)]. If this argument is true, then the free movements of capital obviously provide an opportunity to take arbitrage in either interest rates or exchange rates, and then result in volatile exchange rates. Krugman (1999) asserted that there should be appropriate controls on the international capital movements to avoid currency instability from speculative activities. Ito and Portes (1998) and Eichengreen (1999) also had similar views.

However, Edwards (1998) did not agree with capital control. By the empirical evidence from Latin American countries, he argued that there is no necessary relationship between the international capital mobility and the financial instability; therefore, we do not have to doubt those policies that help to liberalize the movement of capital across countries. In addition, Edwards (1999) further asserted that the short-run controls of capital movement might decrease the financial instability in the short run, but a country should not restrict the international capital movement in the long run. Instead, it should work hard to achieve a sound and integrated economic system and adopt proper policies.

From the literature that analyze the currency volatility, no matter it analyzes the changes in exchange rates through the behavior of speculators or from the view of controlling the movement of capital to achieve the stability of the currency, there has been no consensus. Furthermore, except Carlson and Osler (2000), the papers that investigate the relationship between the speculative behavior and the volatility of exchange rates do not explicitly study this issue theoretically. However, Carlson and Osler (2000) didn’t consider the consistency conditions required for profit-maximizing speculators, and whether restricting capital flows can stabilize exchange rates. Therefore, this paper is attempting to clarify the relationship between rational speculation, capital mobility and exchange rate volatility. There are two main aims of this paper: one is to build a stochastic foreign exchange model with rational speculators to analyze whether speculative behaviors cause the exchange rates volatility under the consideration of consistent conditions required for speculators. The second propose is to introduce capital controls to evaluate whether such policies reduce or increase the exchange rate volatility with the inclusion of rational speculators.
2. The Model

The stochastic foreign exchange model with rational speculators in this paper has the following characteristics:

(1) Under the assumption that rational speculators attempt to maximize the expected utility, and the utility is a function of speculative profits, we derive speculators’ demand for foreign exchanges. The first-order condition for optimum speculative behavior will be explicitly considered in this paper when we investigate the effects of both rational speculative activities and capital controls on exchange rate volatilities.

(2) There are two types of demands for foreign exchanges in each period of time: one is the need for actual transactions from international trades. This type of demand for foreign exchanges is assumed to be a linear function of exchange rates. The other is speculator’s demand from profit arbitrages.

(3) There are also two types of supplies for foreign exchanges in each period of time: one is the supply from international trades. This type of supply is also assumed to be a linear function of exchange rates. The other comes from the sales of speculators who bought in last period of time.

(4) In this paper, we assume that people’s expectations are rational. Therefore, the expectation is based on all information available in current period. In addition, the expected exchange rates have to satisfy the consistent condition required by rational expectation hypothesis [Lucas (1976)]. Since the expected variable appears in the asset demand function, such kind of asset demand for foreign exchanges by speculators would be rational.

First, we derive the speculators’ demand for foreign exchange. Assuming that $B_t$ is the amount of foreign exchange bought by speculators in $t$ period, and $\pi_t$ represents the expected profits for speculators, then

$$\pi_t = B_t (E_{t+1} e_{t+1} - e_t + \delta_t).$$  \hspace{1cm} (1)

The $e_t$, $E_{t+1} e_{t+1}$ in the above equation are the exchange rate in $t$ period and the
expected exchange rate for $t+1$ period in $t$ period respectively. Suppose that
$\delta_t = i_t^* - i_t$ denotes the excess of own-currency returns on foreign securities, $i_t^*$ over
own-currency returns on domestic securities, $i_t$. It will be assumed that the only
domestic and foreign securities available are one-period bonds. Thus the returns $i_t^*$
and $i_t$ are interest rates known with certainty at time $t$, and $\delta_t$ represents the
interest differential.

Suppose that these speculators maximize the expected value of a one-period
utility function $U$ defined over the level of speculative profits. The utility value of
speculators is therefore given by

$$U_t = U(\pi_t).$$  \hspace{1cm} (2)

In the steady state equilibrium, the expected profit is zero. Expanding the above
expression in a Taylor series and neglecting terms higher than second order, we obtain

$$U_t \approx U(0) + U'(0)B_t(Ee_{t+1} - e_t + \delta_t) + \frac{1}{2} U''(0)B_t^2(Ee_{t+1} - e_t + \delta_t)^2. \hspace{1cm} (3)$$

Therefore, the expected utility for speculators is given by

$$E(U_t) \approx U(0) + U'(0)B_t(Ee_{t+1} - e_t + \delta_t) + \frac{1}{2} U''(0)B_t^2\sigma_{t,1}^2, \hspace{1cm} (4)$$

where $\sigma_{t,1}^2 = E(Ee_{t+1} - e_t)^2$ represents the one-period-ahead forecast variance of $e_t$.

Maximization of the above expression with respect to $B_t$ yields the first-order
condition

$$\frac{\partial E(U_t)}{\partial B_t} = U'(0)(Ee_{t+1} - e_t + \delta_t) + U''(0)B_t\sigma_{t,1}^2 = 0. \hspace{1cm} (5)$$

We can obtain speculators’ demand for foreign exchanges from the above first-order
condition as follows:
\[ B_t = -\frac{U'(0)}{U''(0)\sigma_{i,j}^2}(E_{t+1} - e_t + \delta_t) , \quad (6) \]

where \(-\frac{U'(0)}{U''(0)}\) is an inverse measure of speculator risk aversion. If let \(\theta = \frac{U''(0)}{U'(0)}\), and \(\alpha = -\frac{U'(0)}{U''(0)\sigma_{i,j}^2} = \frac{1}{\theta \sigma_{i,j}^2}\), therefore

\[ B_t = \alpha(E_{t+1} - e_t + \delta_t) . \quad (7) \]

As a result, the speculative demand for foreign exchanges is related to the speculators’ attitude toward the risk aversion and the one-period-ahead forecast variance of the exchange rates.

If the demand for and the supply of foreign exchanges from current account traders in \(t\) period are \(D_t\) and \(S_t\), and both are assumed to be linear functions of the current exchange rate respectively as follows:

\[ D_t = -\beta e_t + v_t , \quad (8) \]

\[ S_t = \gamma e_t + u_t , \quad (9) \]

where \(v_t\) and \(u_t\) represent unexpected shocks from the demand and supply of foreign exchanges, respectively. Both are called as current account shocks later in this paper and assumed to be serially uncorrelated with \(E(v_t) = E(u_t) = E(v_t u_t) = 0\), \(E(v_t^2) = \sigma_v^2\), and \(E(u_t^2) = \sigma_u^2\).

3. Market Solution

Combining the purchases and sales of speculators to the foreign exchange supplies and demands of current account traders, we derive the period \(t\) market clearing condition:
\[ B_t + D_t = B_{t-1} + S_t; \]  

(10)

that is,

\[ (B_t - B_{t-1}) + (D_t - S_t) = 0. \]

Then, we obtain

\[
E_e_{t+1} - e_t - E_e_{t-1} + e_{t-1} = \left( \frac{\beta + \gamma}{\alpha} \right) e_t + \frac{1}{\alpha} (u_t - v_t) \cdot \Delta_t, \]

(11)

where the exogenous change in the interest rate differential, \( \delta_t - \delta_{t-1} \), has been denoted as \( \Delta_t \) and is called as capital account shock in this paper. This equilibrium condition is valid for any arbitrarily selected period of time. Looking \( j \) periods into the future, we therefore obtain

\[
E_e_{t+j+1} - e_{t+j} - E_e_{t+j} + e_{t+j-1} = \left( \frac{\beta + \gamma}{\alpha} \right) e_{t+j} + \frac{1}{\alpha} (u_{t+j} - v_{t+j}) \cdot \Delta_{t+j}. \]

(12)

Using all information set of period \( t-1 \), taking the expected value of the above expression, and recalling that \( E_i E_e_i = E_e_i \), and \( E_i u_{t+j} = E_i v_{t+j} = E_i \Delta_{t+j} = 0 \), for all \( j \geq 0 \), we obtain

\[
E_e_{t+j+1} - E_i E_e_{t+j+1} - E_i E_e_{t+j} + E_i E_e_{t+j-1} = \left( \frac{\beta + \gamma}{\alpha} \right) E_i e_{t+j}. \]

(13)

Use of the lag operator allows \( E_i e_{t+j+1} = L^{-1} E_i e_{t+j} \), \( E_i e_{t+j-1} = L E_i e_{t+j} \). We therefore obtain

\[
[L^2 - (2 + \frac{\beta + \gamma}{\alpha})L + 1]E_i e_{t+j} = 0. \]

(14)

To solve the solution of equation (15), either we require that \( E_i e_{t+j} = 0 \) for all \( j \) or we require that quadratic form in \( L \) be identically equal to zero. Therefore, a nontrivial solution requires that

\[
L^2 - (2 + \frac{\beta + \gamma}{\alpha})L + 1 = 0. \]

(15)

Assuming that \( \lambda_1 \) and \( \lambda_2 \) are the roots of the above quadratic function, then we can obtain
\[ \lambda_1 \lambda_2 = 1; \quad (16) \]

\[ \lambda_1 + \lambda_2 = 2 + \frac{\beta + \gamma}{\alpha}. \quad (17) \]

Therefore, the general solution to a homogenous difference equation of this type is given by

\[ E e_{t+j} = c\lambda_1^j + d\lambda_2^j, \quad (18) \]

where \( c \) and \( d \) are determined by an initial-value condition. Since \( E e_{t-1} = e_{t-1} \), then for \( j = -1 \), we obtain

\[ E e_{t-1} = c\lambda_1^{-1} + d\lambda_2^{-1} = e_{t-1}. \quad (19) \]

Since \( \lambda_1 \lambda_2 = 1 \), and \( \lambda_1 + \lambda_2 > 2 \), it must be true that each root is positive; moreover, one root is greater than 1, and the other root is less than 1. The equation in \( E e_{t+j} \) can be nonexplosive only if the constant corresponding to the root that is greater than 1 is equal to zero. Assume that \( \lambda = \lambda_1 < \lambda_2 \), where \( \lambda_1 \) is arbitrarily selected as the smaller of the two roots. In this case, we require that \( d = 0 \), and therefore, for \( j = -1 \),

\[ c\lambda_1^{-1} = e_{t-1}, \quad (20) \]

and thus

\[ c = \lambda e_{t-1}. \quad (21) \]

As a result, the rational exchange rate process is given by

\[ E e_{t+j} = \lambda^{j+1} e_{t-1}. \quad (22) \]

So, the expected exchange rates for period \( t \) and period \( t+1 \) are

\[ E e_t = \lambda e_{t-1}, \quad (23) \]

\[ E e_{t+1} = \lambda e_t. \quad (24) \]
Substituting the solved expected values of exchange rates into equation (11) to obtain

\[-(1-\lambda)e_i + (1-\lambda)e_{t-1} = \left(\frac{\beta + \gamma}{\alpha}\right)e_i + \frac{1}{\alpha}(u_t - v_t) - \Delta_t.\]  

(25)

Collecting terms, we therefore find that

\[e_i = \frac{\alpha(1-\lambda)}{(\beta + \gamma) + \alpha(1-\lambda)}e_{t-1} - \frac{u_t - v_t}{(\beta + \gamma) + \alpha(1-\lambda)} + \frac{\alpha \Delta_t}{(\beta + \gamma) + \alpha(1-\lambda)}.\]  

(26)

Calculate the expected value of the above equation to obtain

\[E_{t-1}e_i = \frac{\alpha(1-\lambda)}{(\beta + \gamma) + \alpha(1-\lambda)}e_{t-1}.\]  

(27)

Rationality therefore requires that

\[\lambda = \frac{\alpha(1-\lambda)}{(\beta + \gamma) + \alpha(1-\lambda)}.\]  

(28)

The above equation can be rewritten as

\[\alpha(1-\lambda)^2 = (\beta + \gamma)\lambda.\]  

(29)

Let us recall that the value of \(\alpha\) must be consistent with

\[\alpha = \frac{1}{\theta \sigma_{\varepsilon,1}^2}.\]  

(30)

From the reduced-form expression in equation (26), the one-period-ahead forecast for \(e_{t+1}\) conditional on all information available at \(t\) is given by

\[\sigma_{t,1}^2 = \frac{\sigma_u^2 + \sigma_v^2 + \alpha^2 \sigma_\Delta^2}{[(\beta + \gamma) + \alpha(1-\lambda)]^2}.\]  

(31)

Substituting equation (31) into equation (30) to obtain

\[\alpha = \frac{[(\beta + \gamma) + \alpha(1-\lambda)]^2}{\theta(\sigma_u^2 + \sigma_v^2 + \alpha^2 \sigma_\Delta^2)},\]  

(32)

which can be rearranged as:

\[\alpha(1-\lambda)^2 = \frac{(\beta + \gamma)^2}{\theta(\sigma_u^2 + \sigma_v^2 + \alpha^2 \sigma_\Delta^2)}.\]  

(33)

Using the required consistent conditions for rational expectation in equation (29) and the speculative behavior in equation (33), we obtain
\[
\lambda = \frac{\beta + \gamma}{\theta(\sigma_u^2 + \sigma_v^2 + \alpha^2 \sigma^2_\lambda)}. \tag{34}
\]

4. Rational Speculation and Exchange Rate Volatility

Lucas (1976) argued that the rules by which agents form their expectations vary as the structure of economy changes. Therefore, it was incorrect in the model of Muth (1961) to treat \(\lambda\) as a fixed parameter in performing comparative statics analysis involving expectational variables. From the result shown as in equation (34), \(\lambda\) and other parameters in the model have a specific relationship.

The following analyzes the effects caused by the change in the risk preference of rational speculators on the conditional variances of exchange rates. Differentiate \(\sigma_{i,t}^2\) with respect to \(\theta\) to obtain

\[
\frac{d\sigma_{i,t}^2}{d\theta} = 2[-(1-\lambda) + \frac{\lambda}{(1-\lambda)^2}\sigma_\lambda^2]d\lambda, \tag{35}
\]

Application of equation (29) and equation (33) allows derivation of

\[
\frac{d\lambda}{d\theta} = \frac{-1}{(\beta + \gamma)(\sigma_u^2 + \sigma_v^2 + \alpha^2 \sigma^2_\lambda) + \frac{2\theta(1+\lambda)}{(1-\lambda)^5(\beta + \gamma)^3(1-\lambda)^4 \sigma^2_\lambda}} < 0. \tag{36}
\]

From the above expressions, we can infer a number of important aspects of the relationship between speculative activity and exchange rate volatility:

(1) If only current account shocks occur, with \(\sigma_\lambda^2 = 0\), then

\[
\frac{d\sigma_{i,t}^2}{d\theta} = \frac{2(1-\lambda)}{(\beta + \gamma)\theta^2} > 0. \tag{37}
\]

Thus, the smaller \(\theta\), the more risky of speculators, the greater speculative activity, the smaller the conditional variance of the exchange rate is.

(2) If only interest rate shock occurs, with \(\sigma_u^2 = \sigma_v^2 = 0\), then
The outcome obtained in this case is contrasting with that derived in the first case. As speculative activity rises, the conditional variance of the exchange rate increases.

(3) If all types of shocks occur, then the sign of \( \frac{d\sigma_{t,1}^2}{d\theta} \) is ambiguous. Through numerical simulation, we can see that as \( \theta \) decreases, the conditional variance of the exchange rate first falls, and then rises from Table 1.

5. The Role of Rational Speculators

To understand why rational speculators either temper or strengthen the exchange rate’s response to transitory shocks, let’s examine the case of a transitory current account shock first.

The equation (26) can be rearranged as

\[
e_t = \lambda e_{t-1} + (1 - \lambda)\bar{e} - \frac{\lambda}{\alpha(1 - \lambda)} (u_t - v_t) + \frac{\lambda}{(1 - \lambda)} \Delta_t .
\]  

(39)

Suppose that \( u_t > 0 \) at period \( t \) with \( v_t = \Delta_t = 0 \). Assuming as well that the value of \( e_{t-1} \) initially equals to \( \bar{e} \), then in this case, equation (39) becomes

\[
e_t = \bar{e} - \frac{\lambda}{\alpha(1 - \lambda)} u_t .
\]  

(40)

When the speculators observe that \( e_t \) is below \( \bar{e} \) with \( u_t > 0 \), they would expect the exchange rate to rise back to \( \bar{e} \) in the next period. This implies a profit-making opportunity, and speculators would buy foreign currency to take this advantage. Those buys would put upward pressure on the exchange rate, as a result of which the exchange rate would initially decrease by less than the degree without speculators. The same logic can be applied to the case of \( v_t > 0 \) to find the stabilizing influence of rational speculators.

Now consider how the market reacts to a capital account shock. Assume that, up through period \( t - 1 \), the exchange rate was at its long-run equilibrium and the
interest differential was zero. Assuming as well that these are no current account shocks at period \( t \), but \( \Delta_t > 0 \), then the exchange rate’s dynamics can be written as

\[
e_t = \bar{e} + \lambda \frac{\Delta_t}{(1 - \lambda)}\]

(41)

Thus, for interest rate shocks, speculators would sell foreign exchange rates when they expect the exchange rate to decline to \( \bar{e} \) in the next period. According to equation (7), their adjustments in desired foreign currency holdings from an expected future fall in the value of foreign currency will be \( \alpha(E_{t+1}e_t - e_t) = -\alpha \lambda \Delta_t \).

However, a positive shock \( \Delta_t > 0 \) also causes speculators to increase their holdings of foreign assets due to the rise in foreign relative to domestic interest rates at period \( t \). The adjustments in this desired holdings are \( \alpha \Delta_t \). Then, the net change in the foreign currency position turns out to be \( \alpha(1 - \lambda)\Delta_t \). Thus, the overall response is to buy foreign exchange to drive the exchange rate far away from \( \bar{e} \) at period \( t \). In this case, the speculative activity is destabilizing.

6. The Capital Controls and Exchange Rate Volatility

As we deregulate the international capital controls in recent years, the increase in the capital mobility has been viewed as the cause of the fluctuation in exchange rates. Therefore, many contentions about controlling the speculative capital flows rise as well. This section introduces a capital control coefficient into the speculative foreign exchange demand function to study the relationship between capital controls and the fluctuation of exchange rates.

Assuming the foreign exchange demand function of speculators with capital controls is

\[
B_t = A \alpha(E_{t+1}e_t - e_t + \delta_t)
\]

(42)

In the above equation, \( A \) is the coefficient representing the degree of capital controls. If \( A = 1 \), this means there is no capital control and the international capital can move freely; if \( A = 0 \), this means there is a completed capital control: the movement of international capital is not allowed. Generally speaking, the capital controls of many countries among the actual economies fall between these two extreme cases. If \( A \) is between 0 and 1, this represents how capital movement is
controlled. The higher the value of $A$ is, the fewer the capital controls are.

If we introduce the above speculative foreign exchange demand into the equilibrium equation in the foreign exchange market, and resolve the expected exchange rates, and then the exchange rates and the conditional variance of exchange rates, the results are as follows:

$$e_t = \frac{A\alpha(1-\lambda)}{(\beta + \gamma) + A\alpha(1-\lambda)} e_{t-1} - \frac{u_t - v_t}{(\beta + \gamma) + A\alpha(1-\lambda)} + \frac{A\alpha\Delta_t}{(\beta + \gamma) + A\alpha(1-\lambda)}, \quad (43)$$

$$\sigma_{t,i}^2 = \frac{(\sigma_u^2 + \sigma_v^2 + A^2\alpha^2\sigma_{\Delta}^2)}{[(\beta + \gamma) + A\alpha(1-\lambda)]^2}. \quad (44)$$

The consistent conditions for both rational expectation and speculative behavior require

$$\lambda = \frac{A\alpha(1-\lambda)}{(\beta + \gamma) + A\alpha(1-\lambda)}, \quad (45)$$

$$\alpha = \frac{[(\beta + \gamma) + A\alpha(1-\lambda)]^2}{\theta(\sigma_u^2 + \sigma_v^2 + A^2\alpha^2\sigma_{\Delta}^2)}. \quad (46)$$

The following analyzes the effects caused by the change in the degree of capital controls on the conditional variance of exchange rates. Differentiating $\sigma_{t,i}^2$ with respect to $A$, we can obtain:

$$\frac{d\sigma_{t,i}^2}{dA} = 2\left[\frac{1-\lambda}{(\beta + \gamma)^2}(\sigma_u^2 + \sigma_v^2) + \frac{\lambda}{(1-\lambda)^2}\sigma_{\Delta}^2\right]\frac{d\lambda}{dA}. \quad (47)$$

Differentiating both equation (45) and equation (46) with respect to $A$, then we can solve these two equations together to obtain:

$$\frac{d\lambda}{dA} = \frac{\lambda(\beta + \gamma)}{A^2 \left\{ \frac{\beta + \gamma}{A} + \frac{2\lambda(1+\lambda)\sigma_{\Delta}^2}{\theta(1-\lambda)^2\left[\sigma_u^2 + \sigma_v^2\cdot (\beta + \gamma)^2 + \lambda^2\sigma_{\Delta}^2\right] + \lambda^2\sigma_{\Delta}^2\right\}} \right\} \right\}, \quad (48)$$

Substituting the result of equation (48) into equation (47), we can get
\[
\frac{d\sigma^2_{i,1}}{dA} = \frac{2[-(1-\lambda)(\sigma_u^2 + \sigma_v^2) + \frac{\lambda}{(1-\lambda)^3}\sigma^2_{\Delta}]\lambda(\beta + \gamma)}{A^2 \left[ \begin{array}{c}
\beta + \gamma \\
\frac{2\lambda(1+\lambda)\sigma^2_{\Delta}}{\theta(1-\lambda)^5} \frac{\sigma^2_u + \sigma^2_v}{(\beta + \gamma)^2} + \frac{\lambda^2}{(1-\lambda)^4}\sigma^2_{\Delta} \end{array} \right]}.
\]  

Since \( \lambda \) is between 1 and 0, the sign of \( \frac{d\sigma^2_{i,1}}{dA} \) is ambiguous. However, we can infer a number of important aspects as follows:

(1) If only current account shock occurs, with \( \sigma^2_{\Delta} = 0 \), then
\[
\frac{d\sigma^2_{i,1}}{dA} = \frac{-2\lambda(1-\lambda)(\sigma^2_u + \sigma^2_v)}{A} < 0.
\]

Thus, the conditional variance of the exchange rate is monotonically decreasing in the degree of capital control. Increasing international capital mobility would decrease the volatility of the exchange rate. Thus, the best policy in this case is to let international capital move freely.

(2) If only capital account shock occurs, with \( \sigma^2_u = \sigma^2_v = 0 \), then
\[
\frac{d\sigma^2_{i,1}}{dA} = \frac{2\lambda^2(\beta + \gamma)\sigma^2_{\Delta}}{A^2 (1-\lambda)^3 \left[ \begin{array}{c}
\beta + \gamma \\
\frac{2(1+\lambda)(1-\lambda)^3}{\theta\lambda^2\sigma^2_{\Delta}} \end{array} \right]} = \frac{2\lambda^2(\beta + \gamma)\sigma^2_{\Delta}}{A^2 (1-\lambda)^3 \left[ \begin{array}{c}
\beta + \gamma \\
\frac{2(1+\lambda)(1-\lambda)^3}{\theta\lambda^2\sigma^2_{\Delta}} \end{array} \right]} > 0.
\]

The outcome obtained in this case is contrasting with that derived in the first case. Increasing capital mobility would increase the volatility of the exchange rate. Thus, the controls over capital movements should be taken in this case.

(3) If all types of shocks occur, the sign is ambiguous. With numerical simulation, we find that as the risk preference rises, \( \frac{d\sigma^2_{i,1}}{dA} \) is negative first and then turns out to be positive. The resulting simulation outcomes are shown from Table 2 to Table 5. Whether we should take capital controls depends on the risk preference of rational speculators. If speculators are less risky, then the exchange-rate volatility would decrease with free capital flows internationally; however, if speculators are more risky, then capital controls would decrease exchange rate volatility.
7. Conclusion

This paper sets up a stochastic foreign exchange model with rational speculators. It first analyzes the influence of the rational speculative activity on the exchange rate volatility, and then introduces capital controls into the model to investigate the effects of such controls on the exchange rate volatility. The main points of this paper are as follows:

(1) If only current account shocks occur, with $\sigma^2 = 0$, the increase in the risk preference of rational speculators will decrease the conditional variance of exchange rates. As a result, rational speculators help stabilize the foreign exchange market. This view differs from the argument that the speculative activity leads to fluctuations in the asset prices. In this case, the best policy is to let capitals freely move in the world.

(2) If only interest rate shock occurs, with $\sigma^2 = \sigma^2 = 0$, the conditional variance of the exchange rate is monotonically increasing in the risk preference of rational speculators. As speculative activity rises, the conditional variance becomes large. This view is consistent with what speculators cause volatility in the assets market. Under such circumstance, the controls over international capital movements indeed decrease the exchange rate volatility.

(3) If both types of shocks occur, then as the risk preference increases, the conditional variance first falls, and then increases. Thus, whether we control capital movements also depends on the risk preference of rational speculators.

In order to simplify the analysis, both domestic and foreign interest rates are treated as exogenous policy variables in this paper. In the future, further investigation can be taken to consider the interest differentials as endogenously determined.
Table 1

The simulation results for $\frac{d\sigma_{i,i}^2}{d\theta}$ without capital control.

| $\theta$  | $\lambda$ | $\frac{d\sigma_{i,i}^2}{d\theta}$
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<td>0.000137</td>
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<td>-302895.698328</td>
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</tbody>
</table>

The simulation is taken by setting $\sigma_u = \sigma_v = \sigma_\lambda = 1, \beta = 0.4, \gamma = 0.6$. 
Table 2

The simulation results for $\frac{d\sigma^2_{\ell,1}}{dA}$ with $\theta = 0.945$.

| $A$  | $\lambda$  | $\frac{d\sigma^2_{\ell,1}}{dA}$ | $\sigma_1=\sigma_2=\sigma_3=1$  
$\beta=0.4, \gamma=0.6$  
$\theta=0.945$ |
<table>
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<td>-1.393412</td>
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<td>0.9</td>
<td>0.352181</td>
<td>-0.000088</td>
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</tr>
</tbody>
</table>
Table 3

The simulation results for \( \frac{d\sigma^2_{t,1}}{dA} \) with \( \theta = 0.944 \).

| \( A \) | \( \lambda \) | \( \frac{d\sigma^2_{t,1}}{dA} \) \( \big|_{\sigma_0=0.6,\sigma_1=1,\sigma_2=0.6,\theta=0.944} \) |
|---|---|---|
| 0.1 | 0.052874 | -1.929893 |
| 0.2 | 0.105029 | -1.690492 |
| 0.3 | 0.155226 | -1.393865 |
| 0.4 | 0.201752 | -1.058265 |
| 0.5 | 0.242988 | -0.729551 |
| 0.6 | 0.278160 | -0.452846 |
| 0.7 | 0.307524 | -0.245209 |
| 0.8 | 0.331931 | -0.099246 |
| 0.9 | 0.352360 | 0.000696 |
Table 4

The simulation results for $\frac{d\sigma_{l,1}^2}{dA}$ with $\theta = 0.105$.

| $A$ | $\lambda$ | $\frac{d\sigma_{l,1}^2}{dA}$ $|_{\beta=0.4, \gamma=0.6, \theta=0.105}$ |
|-----|-----------|-------------------------------------------------|
| 0.1 | 0.352181  | -0.000793                                       |
| 0.2 | 0.450560  | 1.926899                                        |
| 0.3 | 0.495081  | 1.922263                                        |
| 0.4 | 0.523005  | 1.794486                                        |
| 0.5 | 0.543098  | 1.671133                                        |
| 0.6 | 0.558679  | 1.565375                                        |
| 0.7 | 0.574134  | 1.507340                                        |
| 0.8 | 0.581975  | 1.399678                                        |
| 0.9 | 0.591113  | 1.334016                                        |
Table 5

The simulation results for \( \frac{d\sigma^2_{t,1}}{dA} \) with \( \theta = 0.104 \).

<table>
<thead>
<tr>
<th>( A )</th>
<th>( \lambda )</th>
<th>( \frac{d\sigma^2_{t,1}}{dA} ) with ( \sigma_\alpha=\sigma_\beta=\sigma_\gamma=1 ) ( \theta=0.104 )</th>
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Reference


