Optimal Environmental Protection and Environmental Kuzents Curve*

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Abstract

This paper explores the link between an environmental policy and economic growth employing an extension of the Neoclassical Growth Model. We include a state equation to renewable natural resources, and consider natural resources as a component of the aggregate productivity. It is assumed that the change of the environmental regulations induces costs and that economic agents also derive some utility from stock capital accumulation *vis-à-vis* environment. Using the Hopf bifurcation theorem, it can be shown that cyclical environmental policy strategies are optimal, providing a theoretical support to the Environmental Kuznets Curve.

Key-Words: Neoclassical Growth Model, Environmental Kuznets Curve, Hopf Bifurcation Theorem, Limit Cycles.

JEL Class: C61, C62, D62.

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1 Introduction

A great controversy has been generated regarding economic growth and environmental protection by empirical evidence suggested by Grossman and Krueger (1995, 1996), in which a relation between *per capita* GNP and the emission of pollutants assumes the form of an inverted U, receiving the name of the environmental Kuznets curve. The issue that is raised by such a stylized fact is: does economic growth in itself ensure the automatic protection of the environment?

The above question has received a positive answer by those who suggest that a growth policy is always the best course of action. In this sense, Jorgenson and Wilcoxen (1990) have provided estimates for environmental regulation fee costs with regard to the accumulation of capital and growth, and verified that during the 1974-1985 period, said costs reduced average annual growth in the US by 0.2 percentage points. These results corroborate those obtained by Hazilla and Kopp (1990). At the same time, Schmalensee (1994) and Jaffe et. al. (1995) have also suggested that said costs are underestimated, since environmental regulation costs would have a negative effect on product, investment and productivity.

The above statement has been refuted in several studies (e.g. El Serafy and Goodland (1996), and Clark (1996)) in which economic growth is considered to behave indiscriminately with regard to environmental protection, and have prescribed the need for direct governmental intervention by taxing the use of natural resources in order to protect the environment. In support of said hypothesis, Margulis (1992) has empirically pointed out, using data for Mexico, that pollution causes serious damage to the productivity of labor, while Pearce and Warford (1993) have produced a detailed accounting of the productivity losses regarding pollution in many countries. Complementarily, Stokey (1998) has formalized the environmental Kuznets curve in a growth model, in which environmental damage is regarded as a factor limiting long term growth, being also determinant for the inverted U format.

It is possible to observe in this discussion that the environmental Kuznets curve is frequently used to suggest that there is no need to tax the use of natural resources, since the growth process itself would automatically generate environmental protection. Therefore, the aim of this study is to suggest an alternative interpretation for the Kuznets curve by formalizing a growth model with micro-fundaments, in which the source of the relation between growth and environment in the inverted U format is given by the environmental regulation system itself. In this framework, the environmental Kuznets curve is obtained from the cyclical relation that exists between environmental regulation and the long-run accumulation of capital, resulting from the existence of regulatory policy adjustment costs, and the insertion of a utility gain hypothesis in stock capital accumulation *vis-à-vis* the environment.

In this context, the relaxing of the hypothesis, in which variables such as the accumulation of capital and institutional environmental protection norms adjust themselves instantaneously over time, seeks to make the model more realistic, by abandoning a certain theoretical simplification that makes the traditional model analytically convenient, as well as offering a reasonable explanation for the environmental Kuznets curve.

The analysis developed here is divided into two parts. The first one comprises an extension of the traditional neoclassical model with the insertion of the environment and the regulating agent, in which the productivity of the economy is directly affected by the environment. In this regard, the relation between stock capital and the environment remains linear over time. The second part of the analysis is developed with the insertion of regulatory policy adjustment costs, configuring a cyclical relation between growth and the environment, configuring behavior that is similar to the empirical suggestion observed by the environmental Kuznets curve.

2 Inserting Environment in Neoclassical Growth Model

The first model inserts the state equation for the environment in the neoclassical growth model, assuming an overall formulation of environmental dynamics given by,

$$\dot{E} = \beta R + \phi E - \varphi K \tag{1}$$

where $\dot{E}$ is the variation rate of natural resource stock, R is regulation rate imposed on the productive sector for the degradation of natural resources in time t, so that the alternative interpretation for this rate would be the environmental "reconstruction" rate imposed on the productive sector, β is the parameter that indicates the marginal recomposition of the environment with regard to the environmental regulation rate, E is the environmental stock in time t, ϕ is the natural recomposition rate of the environment, K is the capital stock in time t, and φ is marginal destruction rate of the environment related to the use of the capital stock. And, considering that the environment is an intrinsic factor to the productivity of factors, the capital stock variation rate is given by,

$$\dot{K} = A\left(K/E\right)K - C - R \tag{2}$$

where $\dot{K}$ is the physical capital stock variation, $A(K/E)$ is the productivity of the economy's factors, where this function is the stock capital - environment ratio, and C is consumption in time t.

At the same time, defining the following relations,

$$k = (K/E) \tag{3}$$

$$c = (C/E) \tag{4}$$

and

$$r = (R/E) \tag{5}$$

equations (1) and (2) may be synthesized by the following equation,

$$\dot{k} = k\left[A\left(k\right) - \beta r - \phi + \varphi k\right] - c - r \tag{6}$$

Lastly, considering that the utility of the agents depends on the relations between consumption and the environment, and the rate of regulation and the environment, we have reached the following intertemporal agents' optimization problem,

$$\begin{array}{ll} \max \int\limits_0^\infty e^{-\rho t} u\left(c, r\right) dt \\ s.t. \quad \dot{k} = k\left[A\left(k\right) - \beta r - \phi + \varphi k\right] - c - r \end{array} \tag{7}$$

where $\rho > 0$ is the temporal discount rate. Thus, we have reached a simple dynamics optimization model with two control variables, r and c, and a state variable, k.

The current Hamiltonian value is given by,

$$H = u\left(c, r\right) + \lambda\left[k\left(A\left(k\right) - \beta r - \phi + \varphi k\right) - c - r\right] \tag{8}$$

where λ is the co-state variable. The first order conditions are:

$$u_r = \lambda\left(\beta k + 1\right) \tag{9}$$

$$u_c = \lambda \tag{10}$$

and

$$\dot{\lambda} = \rho\lambda - \lambda\left[kA_k + A\left(k\right) - \beta r - \phi + 2\varphi k\right] \tag{11}$$

Differentiating (9) with regard to time, we have,

$$\frac{\dot{\lambda}}{\lambda} = \eta\frac{\dot{r}}{r} - \frac{\dot{k}}{k}\frac{\beta k}{\left(\beta k + 1\right)} \tag{12}$$

where $\eta = r\frac{u_{rr}}{u_r}$ is the elasticity of the marginal utility with respect to the regulation rate - environment ratio, that we assume here to be constant. Thus, equaling (12) and (11) we arrive at,

$$\frac{\dot{r}}{r} = \frac{\rho - kA_k - \frac{\beta k}{\beta k + 1}\left[\frac{c + r}{k}\right] + \left[\frac{1}{\beta k + 1}\right]\left[A\left(k\right) + \beta r + \phi - \varphi k\right] - \varphi k}{\eta} \tag{13}$$

at the same time, rewriting (6) we have that,

$$\frac{\dot{k}}{k} = \left[A\left(k\right) - \beta r - \phi + \varphi k\right] - \frac{c}{k} - \frac{r}{k} \tag{14}$$

Thus, equations (13) and (14) describe the optimal trajectory of r and k. These trajectories are illustrated in Figure 1. The $\left(\dot{r}/r\right) = 0$ function is negatively inclined in space $k - r$, since,

$$\left.\frac{dr}{dk}\right|_{\frac{\dot{r}}{r}=0} = \rho - 2k\left(A_k + \varphi\right) - k^2 A_{kk} - \frac{(kA_{kk} + 2\varphi)}{\beta} < 0 \tag{15}$$

At same time, the $(\dot{k}/k) = 0$ function is negatively inclined in space $k - r$, since,

$$\left.\frac{dr}{dk}\right|_{\frac{\dot{k}}{k}=0} = \frac{A_k + \varphi + c/k^2 + r/k^2}{\beta} < 0 \tag{16}$$

Therefore, we find the long term linear equilibrium between economic growth and the environment inserted in the traditional neoclassical growth model without altering the relation predicted by the model for the relation between consumption and the accumulation of capital, as demonstrated in Appendix A.

Based on the conditions made explicit by the theoretical model developed in this study, it is possible to synthesize the model's conclusions with the following two propositions:

Proposition 1 *If the environmental regulation rate in relation to the stock capital remains below the critical level* $(R/K)^c = [A(k) - \beta r - \phi + \varphi k] - (c/k)$ *the natural resources will depreciate monotonically until depletion.*

Proposition 2 *An environmental or stock capital accumulation shock will affect the optimal values of variables* r *and* k *in the short run, but not in the long run. In other words, environmental shocks are neutral in the long run.*

In this sense, the proof of proposition 1 comes directly from a simple rearrangement of the terms in equation (6), while the proof of proposition 2 comes from the stability of the system represented in Figure 1.

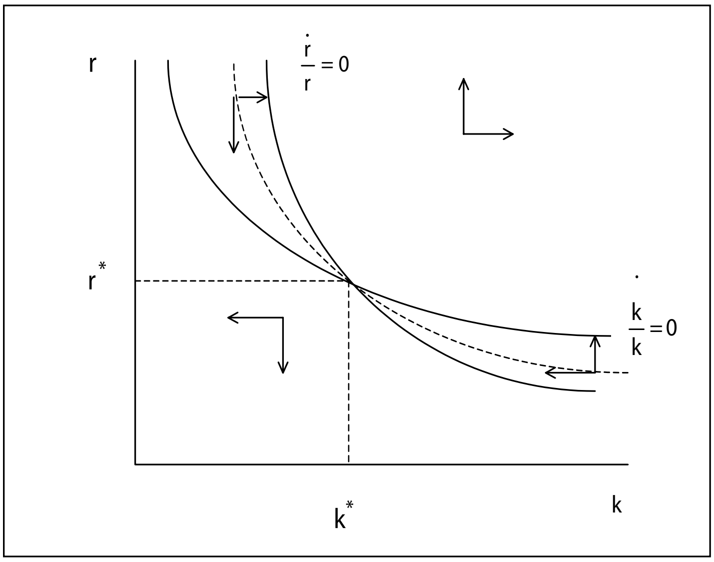
This way, proposition 1 succinctly points out the importance of environmental regulation for long-term macroeconomic activity, illustrating the ”the impossibility of environmental destruction” proposition for growth, and therefore contradicting the statement that ”an indiscriminate growth policy is always the best”.

On the other hand, proposition 2 leads us directly to the fact that, in the long run, an optimal steady-state (k^*, r^*) does not depend on the initial conditions $(k(0), r(0))$. Thus, exogenous accumulation of capital or environmental shocks do not affect long term equilibrium, so that the steady-state values assume approximately the same values, regardless of the these shocks.

3 A Non-Linear Model to Environmental Kuznets Curve

Although the analysis outlined in section 2 provides us with important propositions for our analysis, some stylized facts still need to be addressed such as, from a central perspective, the environmental Kuznets curve. In the meantime, the non-linearity of the

Figure 1:



relation between the environment and long-term economic growth becomes evident. Also, as made clear by the evidence presented by Grossman and Krueger (1995), it is probable that those countries that have reached the ”end” of the environmental Kuznets curve have once again manifested environmental misuse trends as per capita income increases. In other words, the relation between the environment and growth seems to assume cyclical behavior in the ultimate long run.

In an attempt to provide a theoretical answer to these facts, we suggest that there are adjustment costs in stock capital and in environmental regulation policies. In this context, the relaxing of the hypothesis stating that variables such as the accumulation of capital and institutional environmental protection rules are instantaneously adjusted over time seeks to make the model more realistic, by abandoning a certain theoretical simplification in order to make the traditional model more analytically convenient. Thus, the insertion of a stable cyclical relation between the accumulation of capital and the environment is obtained by applying the Hopf Bifurcation Theorem, following the methodology proposed by Feichtinger et. al. (1994).

By inserting adjustment costs for environmental regulation policies in section 2, problem (7) then becomes,

$$\begin{aligned} &\max \int_0^\infty e^{-\rho t}\left[u\left(c,r\right)+v\left(k\right)-z\left(\Phi\right)\right]dt \\ &s.a. \quad \dot{k} = k\left[A\left(k\right)-\beta r-\phi+\varphi k\right]-c-r \\ &\qquad \dot{r} = \Phi \\ &\lim_{t\to\infty} e^{-\rho t}\lambda_k k = 0 \qquad \lim_{t\to\infty} e^{-\rho t}\lambda_r r = 0 \end{aligned} \tag{17}$$

This way, the current Hamiltonian value of problem (17) is given by,

$$H = u(c, r) + v(k) - z(\Phi) + \lambda_k [k(A(k) - \beta r - \phi + \varphi k) - c - r] + \lambda_r \Phi \quad (18)$$

Thus, the first order conditions are,

$$u_c = \lambda_k \quad (19)$$

$$z_\Phi = \lambda_r \quad (20)$$

$$\dot{\lambda}_k = \rho\lambda_k - v_k - \lambda_k [kA_k + A(k) - \beta r - \phi + 2\varphi k] \quad (21)$$

$$\dot{\lambda}_r = \rho\lambda_r - u_r + \lambda_k (\beta k + 1) \quad (22)$$

To simplify, we shall consider the utility function as being additively separable, being given by, $u(c, r) = \zeta c + \xi r$, that function $v(k) = v_0 k$, and that adjustment is costly and quadratic, in accordance to what was suggested by Wirl (2000), being given by $z(\Phi) = 1/2\gamma\Phi^2$, and that $A(k) = a_0 k$. Thus, by substituting (19) and (20) in (21) and (22), and by applying the specifications of the functions suggested here, we have that the canonic equations are given by,

$$\dot{k} = k[a_0 k - \beta r - \phi + \varphi k] - c - r \quad (23)$$

$$\dot{r} = \frac{\lambda_r}{\gamma} \quad (24)$$

$$\dot{\lambda}_k = \rho\lambda_k - v_0 - \lambda_k [2ka_0 - \beta r - \phi + 2\varphi k] \quad (25)$$

$$\dot{\lambda}_r = \rho\lambda_r - \xi + \lambda_k (\beta k + 1) \quad (26)$$

So that the steady-state solutions obtained from the transversality conditions and from (23) to (26) are given by,

$$r^* = \frac{\left(\frac{\xi-\zeta}{\zeta\beta}\right)^2 (a_0 - \varphi) - \left(\frac{\xi-\zeta}{\zeta\beta}\right)\phi - c}{\left(\frac{\xi-\zeta}{\zeta} + 1\right)} \quad (27)$$

$$k^* = \left(\frac{\xi - \zeta}{\zeta\beta}\right) \quad (28)$$

$$\lambda_r^* = 0 \quad (29)$$

$$\lambda_k^* = \zeta \tag{30}$$

Thus, in order to apply the Hopf Bifurcation Theorem, we need to obtain the Jacobian of (23) to (26), whose evolution around the steady-state (27) to (30) is given by,

$$J = \begin{bmatrix} X & -(\beta k^* + 1) & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\gamma} \\ -(\lambda_k^* \alpha (a_0 + \varphi)) & \lambda_k^* \beta & \rho - X & 0 \\ \lambda_k^* \beta & 0 & (\beta k^* + 1) & \rho \end{bmatrix} \tag{31}$$

where $X = [2k^* a_0 - \beta r^* - \phi + 2\varphi k^*]$.

Also, according to Dockner and Feichtinger (1991), the eigenvalues of a Jacobian of type (25) are given by,

$$ {}_1^3\theta_2^4 = \rho/2 \pm \sqrt{(\rho/2)^2 - Y/2 \pm (1/2)\sqrt{Y^2 - 4\det(J)}} \tag{32}$$

where Y is the sum of the determinants,

$$\begin{vmatrix} X & 0 \\ -(\lambda_k^* \alpha (a_0 + \varphi)) & \rho - X \end{vmatrix} + \begin{vmatrix} 0 & 1/\gamma \\ 0 & \rho \end{vmatrix} + 2 \begin{vmatrix} -(\beta k^* + 1) & 0 \\ \lambda_k^* \beta & 0 \end{vmatrix} \tag{33}$$

However, this Jacobian has a pair of eigenvalues that are purely imaginary if, and only if, the conditions,

$$Y^2 + 2\rho^2 Y = 4\det(J) \tag{34}$$

and

$$Y > 0 \tag{35}$$

are met.

For our model, the constant Y and the determinant $det(J)$ are given by,

$$Y = X(\rho - X) \tag{36}$$

$$\det(J) = \frac{1}{\gamma}\left[(2X - \rho)\lambda_k^* \beta (\beta k^* + 1) - (\beta k^* + 1)^2 (2\lambda_k^* (a_0 + \varphi))\right] \tag{37}$$

By applying the bifurcation condition of (34) to (36) and (37), and by choosing γ as a bifurcation parameter, it is then possible to find the critical value γ_{crit} given by,

$$\gamma_{crit} = \frac{\left[(2X - \rho)\lambda_k^* \beta (\beta k^* + 1) - (\beta k^* + 1)^2 (2\lambda_k^* (a_0 + \varphi))\right]}{\frac{X(\rho - X)}{2}\left(\frac{X(\rho - X)}{2} + \rho^2\right)} \tag{38}$$

Note that the steady-state values for $(k, r, \lambda_k, \lambda_r)$ do not depend on parameter γ. Given these results, it is then possible to formulate proposition 3, as follows.

Proposition 3 *Considering the optimal control problem (17) and the equilibrium problem (27)-(30), then Hopf's bifurcation, using γ as a bifurcation parameter, whose critical value is determined by (38), assuming the validity of (34) and (35), leads to stable limit cycles.*

Proof: Given the choice of the other parameters of the model, and considering the validity of condition (34) and (35), the critical value may be calculated from (38). In such a case, the Jacobian arising around equilibrium assumes a purely imaginary pair of eigenvalues, with a non-null crossing velocity, so that it may be concluded that there are periodical solutions for both $\gamma > \gamma_{crit}$ and $\gamma < \gamma_{crit}$. Lastly, the proof of the stability conditions involves an extensive and tedious mathematical exercise. A similar proof has been obtained by Feichtinger et. al. (1994).

Therefore, proposition 3 establishes that the insertion of regulatory policy adjustment costs in the neoclassical growth model with environment, that was developed in section 2, generates stable cyclical behavior between the environment and the accumulation of stock capital. This theoretical formulation provides a plausible explanation for the stylized facts presented by Grossman and Krueger (1995), namely, the environmental Kuznets curve and the probable change in inclination of said curve after its "end", so that the environment is again increasingly depreciated, starting from a high level of per capital income.

Lastly, the theoretical suggestion offered by this model becomes relevant because it provides a formal answer to the statement that growth itself generates environmental protection mechanisms, thus justifying the need to protect the environment. It must be pointed out that this model suggests that the attention given to the environmental regulation problem ends up leveling off the environmental cycle, and that the environment is thus affected to a lesser degree. Said result is fundamental since there is evidence that most natural resources are not renewable, making the role of environmental protection all the more crucial.

4 Final Considerations

After empirical evidence produced by Grossman e Krueger (1995,1996) showed that the relation between the level of per capita income and the concentrations of certain pollutants assumes an inverted U format, the economic literature has offered a vast array of theoretical alternatives for the fact, and has triggered an intense debate regarding environmental policies to be adopted to address the issue.

Within this debate, this study seeks to investigate said relation by suggesting a micro-fundaments model for the environmental Kuznets curve, whose theoretical framework is based on an expansion of the traditional neoclassical growth model. In this framework, the environmental Kuznets curve is obtained from a cyclical relation that exists between environmental regulation and long-term accumulation of capital, due to the existence of regulatory policy adjustment costs, as well as to the insertion of the hypothesis that there is a utility gain in the capital stock formation *vis-à-vis* the environment.

In this context, we have sought to make the model more realistic by relaxing the hypothesis in which variables such as the accumulation of capital and institutional environmental protection regulations adjust instantaneously over time, thus abandoning a certain theoretical simplification aimed at making the traditional model more analytically convenient, besides providing a reasonable explanation for the environmental Kuznets curve.

Our analysis is divided into two parts. The first comprises an extension of the traditional neoclassical model with the insertion of the environment and a regulating agent, in which the environment has a direct effect on the productivity of the economy. In this regard, the relation between the capital stock and the environment remains linear over time. One of the fundamental results obtained was the importance of environmental regulation for long-term macroeconomic activity, illustrating the proposition of a "environmental destruction impossibility" for growth, and thus contradicting the statement that "a indiscriminate growth policy is always the best".

The second part of the analysis considers the insertion of regulatory policy adjustment costs, configuring a cyclical relation between the environment and growth, behaving similarly to the empirical suggestion observed by the environmental Kuznets curve. Thus, one of the conclusion of this study is the crucial emphasis of the fact that the environmental Kuznets curve, by itself, does not mean that economic growth leads automatically to environmental development, but that the environmental Kuznets curve is the result of a very long-term cyclical process between growth and the environment.

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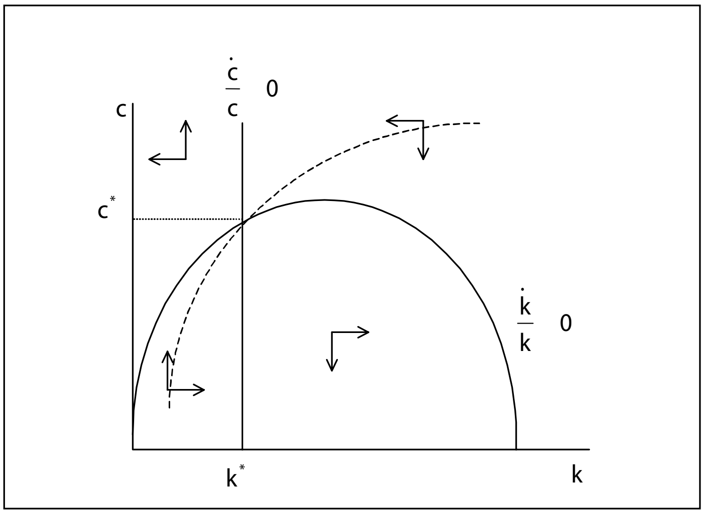
5 Appendix A

Differentiating (10) with respect to time, we have,

$$\frac{\dot{\lambda}}{\lambda} = \omega \frac{\dot{c}}{c} \tag{39}$$

where $\omega = \frac{u_{cc}}{u_c} c$ is the marginal utility elasticity with respect to the consumption rate-environment ratio, that we assume to be constant. Thus equaling (39) and (11) we arrive at,

Figure 2:



$$\frac{\dot{c}}{c} = \frac{\rho - kA_k - A\left(k\right) + \beta r + \phi - 2\varphi k}{\omega} \tag{40}$$

at the same time, by rewriting (6) we have that,

$$\frac{\dot{k}}{k} = \left[A\left(k\right) - \beta r - \phi + \varphi k\right] - \frac{c}{k} - \frac{r}{k} \tag{41}$$

Thus, the equations (40) and (41) describe the optimal trajectory of c and k. These trajectories are illustrated in figure 2 (considering that $A_k < 0$).The function $(\dot{c}/c) = 0$ is given in space $k - c$ by,

$$\left.\frac{dc}{dk}\right|_{\frac{\dot{c}}{c}=0} = 0 \tag{42}$$

where the function $\left(\dot{k}\middle/k\right) = 0$ is given in space $k - c$ by,

$$\left.\frac{dc}{dk}\right|_{\frac{\dot{k}}{k}=0} = \left[A\left(k\right) - \beta r - \phi + \varphi k\right] + k\left(A_k + \varphi\right) \tag{43}$$