COORDINATING MACROECONOMIC POLICY IN A SIMPLE *AK* GROWTH MODEL*

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ABSTRACT

Modern theories of government finance stress the importance of an economy=s fiscal deficits in determining the course of monetary policy. Modern growth theory stresses the role of monetary factors in economic growth. This paper explores how these two are interrelated, using a simple *AK* growth model, one with money, reserve requirements, and government debt. We provide a comprehensive look at the coordination of macroeconomic policy and its effects on long-run growth under three alternative coordinating arrangements. We uncover some unconventional results regarding the relationship between growth and a number of policy variables; these rest squarely on the constraint of the coordination process.

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ABSTRACT

Modern theories of government finance stress the importance of an economy=s fiscal deficits in determining the course of monetary policy. Modern growth theory stresses the role of monetary factors in economic growth. This paper explores how these two are interrelated, using a simple *AK* growth model, one with money, reserve requirements, and government debt. We provide a comprehensive look at the coordination of macroeconomic policy and its effects on long-run growth under three alternative arrangements. We uncover some unconventional results regarding the relationship between growth and a number of policy variables; these rest squarely on the constraint of the coordination process.

1 INTRODUCTION

It is widely recognized that a nation's fiscal deficits can play a critical role in determining the course of monetary policy. The government's consolidated budget constraint links the policies of its two fiduciaries (the monetary and fiscal authorities), preventing them from pursuing policies that are strictly independent of one another. Sargent (1999) refers to the reconciling of this budget constraint as *the coordination of monetary and fiscal policy*; how this is achieved and what are some of the consequences of the coordination arrangement for long-run growth, inflation, and monetary policy is a central theme of this paper.

Following the literature, we approach this problem assuming a dominant fiscal authority, one that sets an independent course for the nation's fiscal deficits. In the environment considered by Sargent and Wallace (1981) and others, the course of the money supply is then endogenous and the coordination of macro policy attains solely through changes in the inflation rate. This description eloquently captures the essence of the coordination process. It is, however, limited on at least two accounts: i) other macro variables, such as long-run economic growth or real interest rates, do not adjust along with the inflation rate, and ii) no other instruments of monetary policy, such open market operations, the reserve requirement, or discount policy, are considered as alternative means of achieving coordination.

This paper addresses these two concerns within the context of a simple endogenous growth model with money, reserve requirements, and government debt. Besides the process described above, which we refer to specifically as *coordination-by-inflation*, or more simply, an *inflation-coordinating arrangement,* we consider two alternative instruments of monetary policy that the central bank can use to achieve coordination. These include adjustments in the profile of government liabilities (an *open-market coordinating arrangement*) and changes in the required reserve ratio (a *reserve requirement coordinating arrangement*).[1] In all three cases, the growth rate adjusts simultaneously, along with the coordinating instrument, to satisfy market-clearing and the overall government budget constraint.

Why are these two issues important? Consider first the role of the growth rate in the coordination process. Of concern is the size of the growth rate relative to the real return on government debt. This

1. In our setting, a change in the required reserve ratio is equivalent to a change in the discount window when the monetary authority sets the nominal discount rate equal to zero.

determines whether long run, net debt issues provide positive real revenues for the government. Most, including Sargent and Wallace (1981), take both the growth rate and the real return as given, so either government debt provides sustainable revenues or it does not. It is in the latter case – which occurs when the growth rate is less than the return on debt – that the monetary and fiscal authorities cannot adopt long-run independent policies. Yet, the very conditions on growth and returns which determine whether long-run coordination is necessary seem to be ones that should originate from the model, not imposed from outside. Further, in light of the considerable evidence of the growth effects of monetary policy, there is good reason to believe that whichever monetary instrument the central bank uses to achieve coordination, it will affect an economy's growth rate (which in turn affects coordination). Our model makes a modest attempt at exposing this simultaneity between growth, monetary policy, and coordination.[2]

Growth and the real return on government debt also play an important role in determining how the coordinating instrument must change in response to an exogenous change in fiscal or monetary policy. Sargent and Wallace's 'unpleasant monetarist arithmetic' (tight money increases the inflation rate) provides a nice illustration. In their framework, an open market sale or increase in the debt-to-money ratio produces unpleasant monetarist arithmetic, provided the growth rate is less than the real return on debt. An increase in government debt, however, will likely impact negatively on growth in an endogenous growth model, simply by crowding out private capital. If so, unpleasant arithmetic may emerge even when the growth rate is greater than the return on debt. Lower growth may reduce net revenues from debt issues, forcing the monetary authority to raise the inflation tax in order to satisfy the government's budget. We devote part of our study of inflation coordination to the impact of open market operations on long-run growth and inflation, and we provide an example of this type of high-growth unpleasant monetarist arithmetic. Our model, of course, also displays other, more conventional arithmetics, similar to Espinosa-Vega and Russell (2001).

Our study also provides a comprehensive look at alternative ways by which the monetary authority can achieve coordination. As far as we know, ours is the only study examining these within the context of a

2. Our approach can be compared with Bhattacharaya *et al* (1997) and Espinosa-Vega and Russell (1998). Both studies allow long-run inflation and real returns to adjust in the coordination process. Espinosa-Vega and Russell (2001), Bhattacharaya and Kudoh (2002), and Nikitin and Russell (2003) consider long-run changes in the capital stock, but changes to the growth rate play no role in the long-run coordination process since steady state growth equals zero. Espinosa-Vega and Yip (1999) provide a model of an inflation-coordination arrangement and long-run economic growth with no government debt.

model with long-run growth.[3] Besides offering insight into the possible implications for policy that these alternatives offer, we are drawn to their study by the fact that many central banks have opted recently for greater autonomy, especially when setting money growth rates. Policies such as targeting the aggregate price level, or the inflation rate, seem to fly in the face of coordination-by-inflation. Reflecting on how these targeting policies square with policy coordination, one can simply supplant the assumption of a dominant fiscal authority for a dominant monetary one, though there is scant hard evidence to suggest governments have found ways to curb substantially their long-run fiscal appetites. On the other hand, even in a world with a dominant fiscal authority, money growth can be independent of fiscal policy – though overall monetary policy cannot. Using its other instruments for coordination, the monetary authority can achieve some degree of independence in setting its long-run inflationary goals. Our study of these alternative arrangements underscores this point, as well as the fact that a central bank's inflation target may not be entirely free or independent of the fiscal constraint. The alternatives to inflation coordination provide some monetary independence, but an overly ambitious inflationary target is incompatible with an equilibrium.

Our model provides some interesting comparative static results, two of which we spotlight here in this introduction. First, since both growth and inflation are determined endogenously under our inflation coordinating arrangement, our study can provide some insight into the sort of inflation-growth correlations one might observe in a cross section of economies. Varying government deficits, the reserve requirement, or the bond-money ratio – all exogenous determinants of a nation's inflation policy under this coordinating process – suggest a mix of both positive and negative inflation-growth correlations, not unlike the mixed observations on inflation and growth in the data. Second, while some of our results are straightforward, a few buck conventional wisdom. For example, an increase in the reserve requirement under an open market coordinating arrangement can *increase* long-run growth, counter to the notion that a higher reserve requirement is financially repressive and retards growth. The anomaly rests squarely on the type of monetary instrument we assume adjusts to achieve coordination and that the higher reserve requirement leads to a concomitant change in the coordinating instrument that enhances growth. The anomaly of an open market

3 Bhattacharya and Haslag (2003), Bhattacharya and Kudoh (2002), and Espinosa-Vega and Russell (2001) explore different aspects of these alternatives in models with no long-run growth. Espinosa-Vega and Yip (1999) compare a tax coordination arrangement with an inflation arrangement in a long-run growth model with no government debt.

sale associated with Sargent and Wallace's unpleasant monetarist arithmetic is rooted in the same principle – it is not so much the change in the policy itself that matters, as it is the change it evokes in the coordinating instrument. This is precisely why it is important to understand the coordinating process.

The rest of paper proceeds as follows. In Section 2, we discuss the general structure of the model. This includes a description of the decisions of households, firms, and intermediaries, the government=s budget constraint, and the market-clearing conditions of the model. The next three sections provide a comprehensive look at each coordinating process, starting with an inflation-coordinating arrangement. In each, we lay out the conditions for the existence of equilibrium and provide some comparative static results. Section 6 provides a short comparison of these arrangements. Concluding remarks are contained in Section 7.

2 THE MODEL

Agents

Time is discrete and at each date t, $t \geq 1$, a continuum (measure 1) of two-period lived agents is born. Agents of each generation are identical. Each is endowed with 1 unit of time when young, which is supplied inelastically to a competitive firm that produces the consumption good at each date using inputs of labor and capital. A portion of the agent=s wage earnings, w_t finances consumption in the current period, the rest is saved for future consumption. All saving is in the form of an intermediated bank deposit. At $t = 1$, the old are endowed collectively with M_0, B_0, and K_0 (cash, government debt, and capital, respectively).

Formally, a representative agent h born at date $t \geq 1$ saves $s_t^h = s\, w_t$ and consumes $c_t^h = (1 - s)\, w_t$ in the current period, where $0 < s < 1$ is the saving rate.[4] Letting r_t denote the gross return on intermediated deposits, the same agent consumes $x_t^h = r_t\, s_t^h$ at date $t + 1$. Aggregate saving, S_t, equals $s\,(1 - \alpha)\, Y_t$, where $1 - \alpha$ is labor's share of output and Y_t is date t *GNP*.

Firms

Firms hire young workers and rent capital from the intermediary. Factor markets are competitive and each factor is paid its marginal product. Growth is introduced by assuming a simple *Romer-type* model of

4. A constant saving rate is consistent with a model where agents have log-linear utility $\ln c_t^h + \beta \ln x_t^h$, where $0 < \beta < 1$. In this case, $s = \beta/(1+\beta)$.

capital externality; at the firm level, output is given by $Y_t^i = A\,\overline{K}_t^{1-\alpha}(K_t^i)^{\alpha}(N_t^i)^{1-\alpha}$, where $\overline{K}_t$ is the economy-wide average capital stock available at date t. Aggregate output is given by $Y_t = A\,K_t$. For simplicity, we assume capital depreciates fully each period.

Factor prices each period are $w_t = (1-\alpha)Y_t$ and $q_t = \alpha A$, where q_t is the rental price of capital.

Intermediaries

Intermediaries act as a go-between for agents and firms, accepting deposits from young agents and renting capital to firms. Each faces a reserve requirement, $\lambda, 0 < \lambda < 1$, which forces them to hold a minimum fraction of the deposits in the form of cash. When binding, the competitive return on deposits is $r_t = \lambda\, r_{m;t} + (1-\lambda)q_t$, where $r_{m;t} \equiv P_t / P_{t+1}$ is the real return on money (the inverse of gross inflation).[5]

Government

Two branches (a fiscal and a monetary authority) represent the government of this economy. The fiscal authority sets sequences for taxes, government spending, and issues nominal government debt. Denote the primary deficit as G_t, which we assume is a constant fraction of real *GNP*; $G_t = g\,Y_t$, where $0 < g < 1$. The amount of nominal bonds or debt at t is $P_{B;t}\,B_t$, where B_t is the number of bonds and $P_{B;t}$, $0 < P_{B;t} < 1$, is the nominal bond price. These bonds pay off at par at date $t+1$. To keep things simple, we assume government debt is subject to the same intermediation process as private capital – that is, we treat debt and capital as perfect substitutes, and each pays the same real return.[6]

The monetary authority sets the path of the money supply $\{M_t\}$ and the reserve requirement λ. It also manages the profile of the government's liabilities, through open market operations. These, together with fiscal policy, must satisfy the consolidated budget constraint of the government at each date,

5. Our treatment of intermediaries and reserve requirements is similar to Wallace (1984), Bhattacharya *et al* (1998), Bhattacharya and Haslag (2003), and Espinosa-Vega and Russell (2001). Alternatively, reserves can be determined endogenously. (See Schreft and Smith (1998), Bhattacharya *et al* (1997), or Espinosa-Vega and Yip (1999,2000)).

6. Wallace (1984) and McCandless (1995) make similar assumptions. Alternatively, one can assume government debt is not subject to intermediation, as in Bhattacharya *et al* (1998). There also may be a different (non-zero) reserve requirement for government debt. The assumption that debt and capital are intermediated in the same manner makes the model easier to present, but it does not change fundamentally many of the results. As an example, see Footnote 20.

$$G_t = \frac{M_t - M_{t-1}}{P_t} + \frac{P_{B;t}\ B_t - B_{t-1}}{P_t} \tag{1}$$

for $t = 1, 2, \dots$, with $M_0 > 0$ and $B_0 > 0$ given.

Market-Clearing

At each date, two markets must clear – the money market and the rental capital/bond market (the factor prices already incorporate clearing for the labor market). The clearing conditions for these markets are

$$\lambda\, s\,(1-\alpha)\,Y_t = \frac{M_t}{P_t}. \tag{2}$$

$$s\,(1-\lambda\,)(\,1-\alpha)\,Y_t = K_{t+1} + \frac{P_{B;t}B_t}{P_t}. \tag{3}$$

An equilibrium is a sequence for prices, interest rates, allocations of goods, capital, debt, and money and the coordinating instrument (money growth, reserve requirement, or debt-money ratio) such that:

- the allocations are optimal for agents and firms when faced with those interest rates and prices;
- the allocations satisfy the market-clearing conditions;
- the government budget constraint is satisfied at each date.

A *balanced growth (or stationary) equilibrium* is an equilibrium with a constant growth rate for output, capital, consumption, and is characterized by constant consumption-to-output ratios, capital-to-output ratios, and constant real returns to money and capital at each date.

Our strategy for establishing the existence of a balanced growth equilibrium in Sections 3-5 involves distilling two curves from the government budget constraint (1) and the clearing conditions (2), (3). These describe the relationship between growth and the coordinating instrument for the arrangement at hand, and determine jointly the (stationary) equilibrium values of the growth rate and coordinating instrument. We then use these curves in some comparative static exercises. Section 3 (coordination-by-inflation) contains the bulk of the discussion behind our procedure, though it readily extends to the other two coordinating arrangements discussed in Sections 4 and 5.

3 AN INFLATION-COORDINATING ARRANGEMENT

This section establishes the existence of a balanced growth equilibrium assuming the money growth rate adjusts to ensure the government's budget constraint holds for each date. We also provide a characterization of the equilibrium and explore some of the comparative static properties of this arrangement.

3.1 Equilibrium

Capital/bond market-clearing

Begin by rewriting the capital/bond clearing condition in terms of the return on money r_m, the (gross) growth rate $\gamma_t \equiv Y_{t+1} / Y_t$, and bond-money ratio B_t / M_t, which in this section we assume is constant and equal to θ for $t \geq 1$.[7] This assumption plays an integral role in our depiction of the inflation-growth relationship described below.

Since bonds and capital are perfect substitutes, government debt displays a standard 'Fisherian' property. Its nominal return, $1/P_{B;t}$, is an inflation-adjusted markup of the real return, αA: $1/P_{B;t} = \alpha A P_{t+1} / P_t$. Using this, along with (2) and the policy parameter θ, write (3) as

$$s(1-\lambda)(1-\alpha)Y_t = K_{t+1} + \frac{P_t}{P_{t+1}} \frac{\theta \lambda s (1-\alpha) Y_t}{\alpha A}. \tag{4}$$

Multiplying both sides of (4) by A and divide each by Y_t, we write

$$s A(1 - \lambda)(1 - \alpha) = \gamma_t + \frac{P_t}{P_{t+1}} \frac{\theta \lambda s (1 - \alpha)}{\alpha}. \tag{5}$$

In (5), we use the fact $Y_{t+1} = A K_{t+1}$ and $\gamma_t \equiv Y_{t+1} / Y_t$.

Rearranging (5), we can express growth as a decreasing, linear function of the real gross return on money, $r_{m;t}$ – our first of two curves describing the inflation-growth relationship:

$$\gamma_a(r_{m;t}) = s(1-\alpha)\left((1-\lambda)\alpha A - \lambda\theta r_{m;t}\right)/\alpha. \tag{6}$$

7. We interpret a *change* in θ under an inflation-coordinating arrangement as a once-and-for-all open market operation. Our approach here is consistent with Wallace (1984), McCandless (1995), and Espinosa-Vega and Russell (1998). Others interpret an open market operation as a once-and-for-all change in the ratio of the *market value* of debt to money, $P_{B;t} B_t / M_t$ rather than its *face value* B_t / M_t (see, for example, Bhattacharya *et al* (1998) or Schreft and Smith (1998)).

A higher inflation rate, all else the same, increases the nominal interest rate (bond prices fall). Given the ratio of bonds to money, θ, the real value of government debt issues declines, so a larger portion of saving is allocated to productive capital. This gives rise to the positive growth-inflation relationship found in (6).

Government budget constraint and money-market clearing

Turning to the government=s budget constraint, replace the lagged real balance term in (1) with $r_{m;t-1}\,\lambda s(1-\alpha)Y_{t-1}$, since $M_{t-1}/P_t = r_{m;t-1}\,M_{t-1}/P_{t-1} = r_{m;t-1}\,\lambda\, s\,(1-\alpha)\,Y_{t-1}$ for $t \geq 2$, from (2). Similarly, lagged debt outstanding at date t, B_{t-1}/P_t, is $(B_{t-1}/M_{t-1})(M_{t-1}/P_{t-1})(P_{t-1}/P_t) = \theta\,\lambda\, s\,(1\text{ - }\alpha)\,Y_{t-1}\,r_{m;t-1}$. Current debt issues are $P_{B;t}\;B_t/P_t = P_{B;t}(B_t/M_t)(M_t/P_t) = (r_{m;t}/\alpha A)\,\theta\,\lambda\, s\,(1-\alpha)\,Y_t$. Rewriting (1),

$$G_t = \lambda(1-\alpha)s\,Y_t - r_{m;t-1}\,\lambda(1-\alpha)s\,Y_{t-1} + r_{m;t}\,\theta\,\lambda(1-\alpha)s\,Y_t/\alpha A - r_{m;t-1}\,\theta\,\lambda(1-\alpha)s\,Y_{t-1}, \text{ for } t \geq 2.$$

$$G_1 = \lambda(1-\alpha)s\,Y_1 - M_0/P_1 + r_{m;1}\,\theta\,\lambda(1-\alpha)s\,Y_1/\alpha A - B_0/P_1, \text{ for } t = 1.$$

Since our point of focus is a stationary equilibrium, we set $r_{m;t-1} = r_{m;t} = r_m$ in the constraints above. Substituting $g\,Y_t$ for G_t and arranging terms, we have

$$g = \lambda\, s\,(1-\alpha)[1 - r_m/\gamma] + (r_m/\alpha A)\,\lambda\, s\,(1-\alpha)\,\theta[1-\alpha A/\gamma], \tag{7}$$

for $t \geq 2$, where γ is the balanced growth rate. This gives us our second expression describing the growth rate as a function of the real return on money,

$$\gamma_b(r_m) = \frac{\lambda\,\alpha\,A\,r_m\, s\,(1-\alpha)(1+\theta)}{s\,[\lambda(1-\alpha)\alpha\,A + \lambda(1-\alpha)\theta\, r_m] - \alpha\,A\,g}. \tag{8}$$

At date 1,

$$g = \lambda\, s\,(1-\alpha) - M_0/P_1\,Y_1 + r_m\,\lambda\, s\,(1-\alpha)\theta/\alpha A - B_0/P_1\,Y_1, \tag{9}$$

where M_0, B_0 are given, and $Y_1 = A\,K_1$ is given, since the initial capital stock, K_1, is given.

From (7), we identify two potential sources of government revenue under this arrangement – *money seignorage*, $ms \equiv \lambda\, s\,(1-\alpha)[1-r_m/\gamma]$, and *bond seignorage*, $bs \equiv (r_m/\alpha A)\lambda\, s\,(1-\alpha)\theta[1-\alpha A/\gamma]$. Money seignorage is non-negative in any monetary equilibrium, and, since the saving rate is constant, an increase in the inflation rate (a decrease in r_m) increases money seignorage revenues.[8] Depending on the growth rate and the return on debt, net revenues from bond issues can be positive or negative. Without sufficient growth, revenues from new bond issues are not enough to offset the real service of past debt.

With Assumption 1 below, $\gamma_b(r_m)$ describes a negative relationship between growth and inflation. This relationship stems from that fact that a marginal increase in r_m changes total seignorage, $ms + bs$, by $\lambda s(1-\alpha)[\theta/\alpha A - (1+\theta)/\gamma]$, which is negative, since the growth rate under this arrangement is less than $\alpha A(1+\theta)/\theta$.[9] To compensate for the reduction in revenues, the growth rate must increase in order to satisfy the budget constraint (7) (note money and bond seignorage both are increasing functions of the growth rate).

Existence of Equilibrium

Our strategy for establishing the existence of an equilibrium is as follows. First, we show we can find a r_m, $0 < r_m$ which satisfies $\gamma_a(r_m) = \gamma_b(r_m)$. This establishes a growth rate and return on money that satisfy market-clearing and the government=s budget constraint for all dates $t \geq 2$. Using (9), we then find the initial price level P_1 that satisfies market clearing and the government budget constraint for date 1. The initial price level depends on (given) values for the initial stocks of money and debt outstanding, (M_0 and B_0 respectively), output Y_1 (also given), and on r_m.

We focus on *binding equilibria*; by this we mean equilibria in which the reserve requirement is binding at each date, or, equivalently, the price of rental capital, q_t, is at least as great as the return on money, r_m. Assumption 1 and 2 below ensure a binding equilibrium exists.

Assumption 1. $\alpha + s(1-\alpha)[\lambda(1+\theta) - 1] > 0$.

Assumption 2. $\lambda s(1-\alpha) > g$.

8. The economy does not display an inflation-tax Laffer curve, in part because the monetary tax base is a constant function of the return on money r_m. Two values for r_m satisfy $\gamma_a(r_m) = \gamma_b(r_m)$; given Assumptions 1 and 2, only one is positive.

9. The fact the growth rate is less than $\alpha A(1+\theta)/\theta$ comes directly from (8); simple calculus shows (8) is increasing in r_m, and the limit of the right-hand side of (8) as r_m goes to infinity is $\alpha A(1+\theta)/\theta$.

Assumption 2 restricts the primary government deficit to be less than the monetary seignorage tax base, ensuring there exists an inflation tax that can finance the deficit. Wallace (1984) makes a similar assumption. With it, $\gamma_b(r_m)$ is increasing and concave over $(0,\infty)$, with $\gamma_b(0)=0$. Assumption 1 ensures bindingness.[10] An appendix detailing the significance of this assumption and containing the mathematical derivations of most of the results of the paper is available upon request.

A pivotal issue in the literature on unpleasant monetarist arithmetic and the effect of an open market operation on inflation turns on whether the growth rate in the economy is greater than or less than the real return on government debt. Our first result establishes the existence of equilibrium under an inflation-coordinating arrangement and characterizes the equilibrium growth rate relative to αA, the return on debt.

Result 1. Characterizing equilibrium growth under an inflation-coordinating arrangement.

A. Suppose $(1-\lambda)(1-\alpha)sA<\alpha A$. An equilibrium exists with a balanced growth rate less than αA.

B. Suppose $(1-\lambda)(1-\alpha)sA>\alpha A$.

i) If $\alpha+(1-\alpha)s[\lambda(1+\theta)-1]>\theta g$, an equilibrium exists and the growth rate is less than αA.

ii) If $\alpha+(1-\alpha)s[\lambda(1+\theta)-1]<\theta g$, an equilibrium exists and the growth rate is greater than αA.

3.2 Comparative Statics

The triplet (g,θ,λ) defines a country's inflation policy under this coordinating arrangement. A change in any component of the policy triplet changes the equilibrium growth and inflation rates. Below, we summarize how the steady-state changes for marginal changes in the two monetary policy parameters θ and λ. Since the primary deficit g is a parameter common to the triplets of all three coordinating arrangements, we defer until Section 6 our discussion of the effects of an increase in g on each arrangement. Section 6 also contains a table summarizing all the comparative static results of the model.

Result 2. Effect of an open market sale on growth and inflation. A permanent increase in the ratio of bonds-to-money, θ (a permanent, tighter monetary policy) has a negative effect on the equilibrium growth rate and an ambiguous effect on the inflation rate.

10. Assumption 1 is sufficient for bindingness; one can easily produce examples of binding equilibria where Assumption 1 does not hold.

This result admits two possible outcomes, due to the fact both γ curves shift with an increase in θ. We provide more definitive predictions for inflation by considering each of the following in turn.

Case A. ›Low-Growth= Cases. Tighter money increases the inflation rate and reduces the growth rate.

›Low-growth= refers to equilibria with a growth rate less than the real return, αA; it is the case considered by Sargent and Wallace (1981). Results 1A and 1B state conditions which support such equilibria. In each case, bond seignorage, $[r_m \lambda s(1-\alpha)\theta/\alpha A][1-\alpha A/\gamma]$, in equilibrium, is negative. All else the same, a marginal increase in the bond-to-money ratio θ increases the net debt service of the government. A higher θ also reduces the growth rate by (directly) crowding out private capital in the rental capital/government debt market. This has a secondary, reinforcing negative effect on both the seignorages. To compensate, the return on money must fall (the inflation rate increases) in order to satisfy the government budget constraint. Although a higher inflation rate reduces the price of new debt issues, and by consequence, has a positive effect on growth, a higher θ will not increase growth in this economy. (See Figures 1A, 1B).

Case B. ›High-Growth= Cases. Tighter money has an ambiguous effect on the equilibrium inflation rate and reduces the growth rate.

Under a high-growth case ($\gamma > \alpha A$), bond seignorage is positive. By itself, a marginal increase in θ increases the amount of revenue collected by the government. However, the increase in θ also crowds out some private capital, as described above for the low growth cases. Whether tighter money decreases or increases the equilibrium inflation rate depends critically on whether the positive impact of a higher θ on bond seignorage is larger or smaller than the negative effect of lower growth on the amount of bond and money seignorage collected. (See Figure 1C).

To illustration a case of unpleasant monetarist arithmetic with 'high growth' let $\alpha = .3, A = 3.45, s = .475, g = .0075, \lambda = .04$ and $\theta = 2$. Raising θ to $\theta = 2.5$ increases the net inflation rate from 106.87% to 113.53%. The net, equilibrium growth rate, for the lower and higher value of θ is 5.84% and 4.88% respectively; both are greater than the net real yield on government debt, 3.5%.

- Insert Figure 1 -

Despite the possibility that a tighter monetary policy leads to higher inflation, the model exhibits the conventional result that a tighter monetary policy reduces the initial price level. From (9), $P_1 = (M_0 + B_0)/Y_1 \left(\lambda(1-\alpha)s + \theta\lambda r_m (1-\alpha)s - g\right)$, where r_m satisfies $\gamma_a(r_m) = \gamma_b(r_m)$. It is easy to show $(\theta/r_m)d\,r_m/d\theta > -1$, so $r_m\theta\lambda(1-\alpha)s$ increases with an increase in θ and the price level is lower.

Result 3. Effect of a change in the reserve requirement on growth and inflation. A permanent increase in the reserve requirement λ reduces the balanced growth rate and has an ambiguous effect on the inflation rate.

We can describe the effect of a change in the reserve requirement parameter λ on the equilibrium growth and inflation rates in much the same terms as an open market operation. Unlike an open market operation, however, the initial or direct effect of an increase in λ increases the seignorage sum, $ms + bs$, regardless of whether $\gamma \gtreqless \alpha A$.[11] The increase in λ has two direct effects on the rental capital/debt market; both reduce growth. First, an increase in the reserve requirement forces the intermediary to hold more cash on reserve, so fewer resources are available to the market. Second, the real value of new government debt issues, $P_{B;t}B_t/P_t$, increases, since, for a given bond-to-money ratio θ, $P_{B;t}B_t/P_t = r_m\,\theta\,\lambda\,s(1-\alpha)Y_t/\alpha A$. As in open market sale, the negative effect on the sum, $ms + bs$, of a decrease in the growth rate may or may not outweigh the direct effect of λ on this sum. If the secondary growth effect does outweigh the direct effect, the inflation rate must rise (r_m falls) to satisfy the government budget constraint. Otherwise, a higher λ reduces the equilibrium inflation rate. The result is analogous to the two cases described in Figure 1C.

We close with some observations regarding possible inflation-growth diagrams that might emerge in a world consisting of economies with differing components for seignorage policy ($g, \theta, \text{or}\, \lambda$), but are otherwise identical. These illustrate the impact the policy has on the inflation-coordinating arrangement. Figure 2A shows a hypothetical inflation-growth cross-sectional diagram for a world comprised of similar economies, differing only as regards the ratio of bond-to-money, θ. Figures 2B and 2C show similar diagrams, the differences across economies being the reserve requirement λ, and fraction of government

11. From the definition of *ms* and *bs* and the government budget constraint, it is easy to see that the direct effect of an increase in λ on the sum *ms* + *bs* equals g/λ.

spending to GNP, respectively. [12]

Much of what we glean from these diagrams follows directly from the comparative statics. Figure 2A suggests a clear negative relation between inflation and growth, while Figure 2C suggests these variables are positively related.[13] Depending on the portion of the diagram one focuses on, Figure 2B shows a negative and positive relationship between the two. Collectively, these diagrams underscore the importance of understanding macro-policy coordination and the role it plays in the relationship between inflation and growth. Clearly, this relationship depends very much on the policy measures driving the difference in these variables across economies.[14] This point seems applicable, in light of the mixed empirical observations on inflation and growth in the data. (See, for example, McCandless and Weber (1995)).

-Insert Figure 2-

4 AN OPEN-MARKET COORDINATING ARRANGEMENT

In this section and the next, we examine two alternative coordinating arrangements at the disposal of a central bank. Under certain circumstances, these allow the bank to pursue the disparate goals of coordinating and inflation targeting – that is, under these arrangements, the inflation rate is an exogenous policy parameter.[15] Since the inflation target, among other factors, affects the coordinating process, a considerable part of our discussion is devoted to describing the comparative statics of a change in the inflation rate.

This section considers the bond-money ratio θ as a coordinating instrument. Here, the central bank sets the monetary growth rate (equivalently, the long-run inflation rate) and the reserve requirement, and the bond-money ratio θ adjusts, along with the economy=s growth rate, so as to satisfy the market-clearing conditions and the government=s budget constraint each period.

12. The diagrams in Figure 2 are illustrative examples, not calibrations of actual economies. We assume $\alpha = .27$, $A = 3.85$, $s = .44$; this yields a rental price of capital/return on government debt of 1.0395, or 3.95%. These values were selected, in part, because they support equilibria for a large variation in the policy parameters g, θ, and λ. In Figure 2A, we set $g = .025$ and $\lambda = .10$, and vary θ from $\theta = .60$ to $\theta = 6$. We assume $g = .025$ and $\theta = 4$, and vary λ from $\lambda = .08$ to $\lambda = .26$ in Figure 2B. In Figure 2C, we set $\lambda = .10$ and $\theta = 4$, and vary g from $g = .003$ to $g = .03$.

13. The diagram for variations in the open market operation parameter, θ can, theoretically, look like that of Figure 2B; for this parameterization of the model, however, it displays a distinct negative inflation-growth relationship.

14. The possibility that countries may operate under different coordinating arrangements compounds this problem.

15. In this model, inflation targeting is equivalent to targeting the nominal interest rate, since the real return is constant.

Throughout the section, we compare the central bank's target policy against a benchmarked inflation policy $1/r_m^o$, one where bond seignorage equals zero for all $\theta > 0$. When the bank's target is lower than $1/r_m^o$ ($r_m > r_m^o$), bond seignorage is always positive. An analogous argument holds for $r_m < r_m^o$.

4.1 Equilibrium

Let $r_m^o \equiv [\lambda(1-\alpha)s - g]\alpha A / \lambda\, s(1-\alpha)$. We characterize the inflation policy as a simple function of r_m^o, $r_m = \sigma\, r_m^o$, with $\sigma \gtreqless 1$.[16]

Capital/bond market-clearing

With the assignment $r_m = \sigma\, r_m^o$, the counterpart to (6) for this coordinating arrangement is

$$\gamma_a(\theta) = A[s(1-\alpha)(1-\lambda(1+\sigma\theta)) + \sigma\,\theta\, g], \tag{10}$$

which is decreasing and linear in θ, by Assumption 1. The intuition behind (10) is clear. Given a constant inflation rate, the nominal price of new debt issues is constant. A higher θ then represents a larger real debt obligation of the government, crowding out private capital and reducing the growth rate.

Government budget constraint and money-market clearing

The counterpart to eq.(8) is given by

$$\gamma_b(\theta) = \sigma\,\alpha\, A\,(1+\theta)/(1+\sigma\,\theta), \tag{11}$$

which is decreasing and convex if $\sigma > 1$, concave and increasing if $\sigma < 1$, and a constant function of θ if $\sigma = 1$. Regardless of the size of σ, $\lim_{\theta\to\infty} \gamma_b(\theta) = \alpha A$.

The intuition behind the γ_b curve (11) and its relationship with the inflation target is straightforward. By construction, bond seignorage is zero under the target $1/r_m^o$. Since $bs \equiv (r_m/\alpha A)\lambda\, s\,(1-\alpha)\theta[1-\alpha A/\gamma]$, growth must equal the real return αA for all $\theta > 0$ when the target equals $1/r_m^o$, and the government's deficit is financed solely by money seignorage, ms. A target set lower than $1/r_m^o$ ($r_m > r_m^o$ or $\sigma > 1$),

16. There is no loss in generality in using $\sigma r_m{}^o$ to characterize the targeted return r_m when establishing the existence of an equilibrium. It is, however, of limited use in comparative statics, since $\sigma\ r_m{}^o$ varies with λ and g. The condition for bindingness, $r_m < \alpha A$, implies, $\sigma < \alpha A/ r_m{}^o$. We assume this inequality holds throughout the discussion above.

reduces money seignorage for all $\theta \geq 0$. In turn, this implies bond seignorage under the lower target is positive for each $\theta > 0$ in order for $g = ms + bs$, as required by the government's budget constraint (7). A higher θ increases the bond seignorage tax base; given the government's revenue needs, g, the bond seignorage tax, $(1 - \alpha A/\gamma)$ must fall, i.e., the growth rate falls. The opposite occurs when $1/r_m > 1/r_m^o$.

Under this coordinating arrangement, the initial price, $P_1 = (M_0 + B_0)/[Y_1(1 + \sigma\,\alpha\,\theta\,A)(\lambda(1-\alpha)s - g)]$, is positive, provided Assumption 1 holds.

Existence of Equilibrium

Let $\sigma^* \equiv s(1-\alpha)(1-\lambda)/\alpha$. It turns out $\lim_{\theta \to 0} \gamma_a - \gamma_b \lesseqgtr 0$ whenever $\sigma \gtreqless \sigma^*$. Below, we use σ (policy) and σ^* (benchmark) to characterize the equilibrium growth paths.

Result 4. Characterizing Equilibria (Open-Market Coordinating Arrangement).

A. If $\sigma^* < \sigma < 1$, an equilibrium does not exist.

B. If $\sigma < 1$ and $\sigma < \sigma^*$, an equilibrium exists, it is unique, and the growth rate is less than αA.

C. If $1 < \sigma < \sigma^*$, an equilibrium exists and it is unique. The equilibrium growth rate is greater than αA.

D. If $\sigma > 1$ and $\sigma > \sigma^*$, coordination, if possible, admits two possible equilibria. In each case, the growth rate is greater than αA.

Figure 3 illustrates the four possibilities described above.

- Insert Figure 3-

The intuition behind Result 4 is as follows. The effective inflation tax is $(1 - r_m/\gamma)$. If the condition of 4A prevails, the growth rate is never high enough to support coordination. The conditions of 4B and 4C permit a growth rate high enough to allow coordination. In 4B, r_m is low enough (inflation target is high enough) that all revenues can be raised via money seignorage; bond seignorage is negative due to the fact the only θ that achieves coordination is one that leaves the equilibrium growth rate less than αA. In other words, the amount of monetary seignorage revenues raised when $\gamma = \alpha A$, $\lambda s(1-\alpha)(1 - r_m/\alpha A) > g$. Any $\theta > 0$ that leaves $\gamma > \alpha A$ produces even more monetary seignorage *and* positive bond seignorage, which is incompatible with the budget requirement $g = ms + bs$. The opposite is true in 4C. Together, these suggest that if $\sigma^* > 1$, all generations, aside from the initial old, benefit if the central bank adopts a lower inflation target (or, in other words, moves from 4B to 4C), as it permits higher growth and a higher effective return.[17]

4D reflects a Laffer curve relation for this economy, one that applies directly to bond, as oppose to monetary, seignorage. Here, the inflation rate is sufficiently low so that a portion of revenues must be raised through bond seignorage, $(r_m/\alpha A)\lambda s(1-\alpha)\theta[1-\alpha A/\gamma]$. This can be raised in two ways: i) levy a 'high' effective tax – one with a higher growth rate and 'low' base (as measured by a low θ), or ii) a 'low' tax (lower growth, high θ configuration). Note the proportion of money seignorage to total seignorage is higher under i), since the money tax $(1 - r_m/\gamma)$ is increasing in γ.

Result 4 also reveals limits to the coordination process, suggesting the budgetary needs of the government place some constraints on the central bank=s inflation target. 4A and D indicate the bank cannot be too ambiguous in targeting inflation without adjusting the other monetary instrument, the reserve requirement λ. How limited is the target? That's an open question. Obviously, the bank cannot be completely independent in setting $1/r_m$ – a target that sets $r_m = \gamma$, for example, raises no money seignorage, which is incompatible with an equilibrium.

4.2 Comparative Statics

Simple differentiation of (10) shows γ_a is decreasing in the target σ. Similarly γ_b of (11) is increasing in σ. These lead to our first comparative static result for this arrangement.

Result 5. The effects of inflation targeting under an open-market coordinating arrangement.

A. If the conditions of cases 4B or 4C hold, a lower inflation target $1/r_m$ increases the growth rate and decreases the equilibrium coordinating bond-money ratio.

B. If the conditions of case 4D hold and an equilibrium exists, a lower inflation target lowers the higher equilibrium growth rate and raises θ, and raises the lower equilibrium growth rate and reduces the ratio θ.

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17. The old benefit from a higher σ as well, in the form of a lower initial price P_1, but P_1 is also affected by the accompanying change in θ.

As noted, a lower $1/r_m$ reduces the total amount of seignorage collected. If bond seignorage is positive, (4C and D) an increase in the bond-money ratio θ can possibly offset the decline in revenues, though this has the effect of decreasing the growth rate, which effectively lowers the tax rate on money and on government debt. Alternatively, raising the tax rate on both seignorages – through an increase in growth – can always compensate for the decline in revenues resulting from a lower inflation target. This increase in the growth is accomplished through a lower θ – that is, the government relies relatively more on money versus bond seignorage. If bond seignorage is negative (4B), a lower θ increases growth (compensating in part for the direct impact the lower target has on total revenues) as well as reduce the government's overall financial burden by reducing the total amount it pays in net interest payments on debt.

We close with an example involving the effect of a higher reserve requirement under this arrangement.

Result 6. An increase in the reserve requirement and its effect on growth. A higher reserve requirement may increase the long-run growth rate under an open-market coordinating arrangement.

We highlight this result as it runs counter to the notion an increase in the reserve requirement is financially repressive and reduces long-run growth. This is, in fact, the conclusion reached in Result 3. The notion is generally correct, though it fails to recognize that an exogenous change in reserve policy can invoke other changes, as part of the coordination process, which may be growth-enhancing. In this case, specifically, the bond-money ratio θ falls. As noted in the Introduction, the anomaly is grounded in the same principle as unpleasant monetarist arithmetic. Here's an example: let $g = .0075; s = .475; \lambda = .05; r_m = .85; \alpha = .27; A = 3.85$. The economy supports two equilibria (4D). Growth in the steady-state with higher growth is 19.3%, and the bond-money ratio is 1.38. Increasing λ to .06 raises the growth rate to 22.7%, and θ falls to .426. Of course, growth in the other steady-state decreases with the increase in λ.

5 A RESEVE REQUIRMENT COORDINATING ARRANGEMENT

As an alternative to the two arrangements described above, the central bank can use the reserve requirement λ as a coordinating instrument of macro policy. As in Sections 3 and 4, we use (5), (2), and (7) to derive the appropriate γ_a and γ_b curves which summarize the relationship between growth and λ for this coordinating arrangement. This is similar to the steps in Section 3 and is not repeated here.

For this section, we replace Assumption 2 with:

Assumption 3 $\lambda > \underline{\lambda} \equiv \alpha\, A\, g / [s\,(1-\alpha)(\alpha\, A + \theta\, r_m)]$.

Assumption 3 ensures the initial price level, as well as the growth rate, are positive whenever the real rate of return, αA, is greater than 1. Any λ that satisfies Assumption 2 satisfies Assumption 3. Additionally, any $\lambda > 0$ with $\gamma_a(\lambda) = \gamma_b(\lambda) > 0$ will be larger than $\underline{\lambda}$, since $\gamma_b < 0$ for $\lambda < \underline{\lambda}$.[18]

5.1 Equilibrium

Capital/bond market-clearing

It is simple enough to show $\gamma_a(\lambda)$, the counterpart to (6) and (10), is decreasing and linear in the reserve requirement. A higher λ reduces the amount of intermediated saving channeled to the capital/bond market as well as increases the real value of government debt, by lowering the price level P_t . Both reduce growth.

Government budget constraint and money-market clearing

Given Assumption 3, the function summarizing the government budget constraint and money market-clearing, $\gamma_b(\lambda)$, is decreasing and convex in λ. All else the same, a marginal increase in λ increases revenues $ms + bs$ by $s\,(1-\alpha)(1 - r_m/\gamma) + s\,(1-\alpha)\,\theta(1 - \alpha A/\gamma)/\alpha\, A = g/\lambda$. Lowering the growth rate reduces these revenues in accord with the government budget constraint (7).

Existence of Equilibrium

Define the ancillary reserve requirement, $\lambda^*(r_m) \equiv \alpha\, A\, g / [s\;(1-\alpha)(\alpha A - r_m)]$ and target, $r_m^* \equiv \alpha A[(1-\alpha)s - g - \alpha]/[s\,(1-\alpha) + \theta\, g - \alpha]$. Evaluating γ_b at $\lambda^*(r_m)$ – for *any* target – we have, $\gamma_b(\lambda^*(r_m)) = \alpha A$, which equals γ_a , evaluated at $\lambda^*(r_m)$ at the target r_m^*. These benchmarks, along with fact $\gamma_a\,(\gamma_b)$ is decreasing (increasing) in r_m, are used to establish and characterize an equilibrium. Our characterization also provides insight into the effects of a change in the inflation target under this arrangement.

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18. An equilibrium reserve requirement satisfies $\underline{\lambda} < \lambda < 1$. The second inequality follows from the properties of the γ_a and γ_b curves, described below, and the fact that $\gamma_b > 0$ at the point (less than 1) where $\gamma_a = 0$.

Result 7. Characterizing Equilibria (Reserve Ratio Coordinating Arrangement).

A. If $r_m < r_m^*$, there exist two possible equilibria:

i) A low λ, high growth equilibrium (bond seignorage is positive).

ii) A high λ, low growth equilibrium (bond seignorage is negative).

B. If $r_m > r_m^*$, coordination, if possible, admits two possible equilibria; both either have growth rates less than αA (bond seignorage is negative in both case) or both have growth rates greater than αA (bond seignorage is positive in both cases).

Note it is possible $r_m^* < 0$, in which case all equilibria are of type 7B.

We show the three possibilities in Figure 4. The dashed curves in the figure represent the (λ, γ) loci implied by γ_a, γ_b when $r_m = r_m^*$. At $\lambda^*(r_m)$, both are equal to αA, as indicated above. When $r_m < r_m^*$, the γ_a curve (γ_b curve) lies to the right (left) of its dashed counterpart (these are shown as the solid curves in Figure 4). As shown in the figure, the economy supports two equilibria – a 'low' reserve requirement, high growth equilibrium, and a 'high' reserve requirement, low growth equilibrium. The intuition behind these is similar to our discussion of the Laffer-relationship for the bond-money ratio θ in the previous subsection.

A very similar situation holds when $r_m > r_m^*$, though in this instance, the two equilibrium growth rates are either both high ($\gamma > \alpha A$) or both low ($\gamma < \alpha A$), as shown in Figure 4. Moreover, as the case with an open market coordinating arrangement, an equilibrium may not exist if the targeted inflation rate is too low.

-Insert Figure 4-

5.2 Comparative Statics

The effects of a marginal change in the inflation target follows directly from our characterization of the equilibria in Result 7. We have

Result 8. The effects of inflation targeting under a reserve requirement arrangement. Regardless of whether $r_m \lesseqgtr r_m^*$, a marginal decrease in the inflation target will reduce the growth rate and raise the reserve requirement for the equilibrium with higher growth and the lower reserve requirement, and increase growth and lower the reserve requirement for the equilibrium with lower growth and a higher reserve requirement.

It is easy to infer the equilibrium effects of an increase in the inflation target on γ and λ from Figure 4, since an increase in r_m shifts the γ_a and γ_b curves in opposite directions.

The direct effect a marginal increase in r_m on total revenue is $-\lambda s(1-\alpha)\theta[\alpha A(1+\theta)/\theta-\gamma]/\alpha A<0$. Note that revenues will fall relatively more in the equilibrium with lower growth and a higher reserve requirement. The reduction in revenues can be offset by an increase in the effective inflation tax, $1-r_m/\gamma$, by increasing the growth rate. This occurs in the lower growth equilibrium, and is accomplished by a reduction in the reserve requirement. The latter does have the effect of reducing revenues, since it reduces the seignorage tax base, but the increase in the effective inflation tax more than offsets the reduction in the base. In the higher growth case, the equilibrium effective inflation tax falls. An increase in the reserve requirement compensates for the reduction in the tax rate.

Unfortunately, the comparative static results of a marginal change in the bond-money ratio θ on this coordinating arrangement are not as easy to portray graphically. This is due to two reasons. First, the shift in the γ_b curve for an increase in θ depends on the inflation target. Second, an increase in θ can shift the two γ curves in the same direction. However, definitive analytical results are available, which we present in Result 9 below.

Result 9. The effects of a marginal increase in θ on growth under a reserve requirement coordinating arrangement. A marginal increase in the bond-money ratio θ increases the growth rate for any equilibrium with lower growth and a high reserve requirement and lowers the growth rate for the equilibrium with higher growth and lower λ.

The direct impact of a higher θ on total revenues is $\lambda r_m s(1-\alpha)(1-\alpha A/\gamma)$, which is negative when growth is than αA. The tax base and/or the effective inflation tax rate $1-r_m/\gamma$ need to adjust in order to satisfy the budget constraint. In the case of the lower growth equilibrium, this is accomplished by an increase in the growth rate (which requires a lower λ, since a higher θ already has a negative crowding out effect on growth). For the higher growth equilibrium, the inflation tax base increases (λ increases); this has an additional crowding out effect on growth.

When $\gamma > \alpha A$, the increase in θ has a positive direct effect on revenues. The accompanying decrease in revenues is accomplished by a lower λ, which raises the growth rate in the lower growth equilibrium case. In the higher growth equilibrium, a decrease in the growth rate lowers the effective inflation tax. It is unclear, however, whether the reduction in the growth rate for this case must accompany an increase in the reserve ratio λ – a higher θ, by itself, reduces growth, so the equilibrium λ can rise or fall with an increase in θ.

As in the previous section, one result stands out from the rest. Here, it is possible that an open market sale can increase the growth rate, despite the fact that a higher bond-money ratio has the effect of crowding out private capital. As before, this result stems from the accompanying change in the instrument used to coordinate the government's budget constraint.

6 COMPARING COORDINATING ARRANGEMENTS

The policy triplet (g, θ, λ) defines inflation policy under inflation coordination. Similarly, (g, r_m, λ) and (g, r_m, θ) define open market policy and reserve requirement policy, respectively, under their corresponding coordination arrangements. Since each have the fiscal parameter g in common, we begin this section by presenting the comparative static results of the effects of an increase in g on the equilibrium obtained for each arrangement. We then provide some general statements comparing the coordinating arrangements. We close this section with a few brief comments regarding welfare.

6.1 Growth and deficit spending g under alternative coordination arrangements

Unlike Barro (1990), and more recently, Basu (2001), government spending in the model has no productive qualities. Instead, we assume g is the usual sort of 'dumped in the ocean' spending found commonly in macroeconomics. It may come of some surprise, then, that under each coordinating arrangement, an increase in g can have a *positive* effect on growth.

Where do these results originating, generally? In each case, it rests on the fiscal constraint and the imposition it places on the monetary coordinating instrument. An increase in the growth rate is one way to raise the additional revenue needed to finance higher deficit spending, since higher growth raises the effective tax on money and bond seignorages. Under inflation coordination, an increase in the equilibrium inflation rate must accompany the increase in government spending.[19] This in turn reduces the real value of government borrowing in the bond/rental capital market. With higher inflation, the price P_B of new government debt issues falls in order to retain the return equality $\left(1/P_{B;t}\right)\left(P_t/P_{t+1}\right)=\alpha A$, as required by the assumption that debt and capital are perfect substitutes. Since the central bank maintains the ratio of bonds to money θ, the real value of government debt issues declines, leaving more resources available for capital formation.[20] The growth effect, in this instance, very much depends on the fact that we assume the central bank sets the bond-money ratio θ in terms of its face value, rather than its market value. The higher inflation accompanying an increase in g, in effect, reduces the market value of the government=s debt, allowing for the increase in growth.

By contrast, a change in the bond-money ratio θ under the open market alternative considered in Section 5 has a direct (positive) impact on the face value of the debt (as well as its market value, since the inflation rate is constant under this arrangement).[21] If the inflation tax is large enough and bond seignorage is negative (5B above), a marginal increase in g can only be financed by reducing θ, the bond-money ratio. A lower θ permits greater capital formation and higher growth, which in turn raises the effective money seignorage tax rate, $\left(1-r_m/\gamma\right)$. On the other hand, if the deficit is financed by both money and positive bond seignorage, (5C and D), the increase in g can be financed by relying more on either form of seignorage, which is to say, equilibrium θ and growth may either rise or fall to achieve coordination. A similar result holds if

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19. If the inflation rate were to fall, revenues collected through seignorage would be insufficient to fund the additional spending. This is due to two features of the model – first, the tax collect on total seignorage, *ms* + *bs* would be lower, due directly to the lower inflation tax. Second, the growth rate falls, further reducing revenues.

20. This result holds, if alternatively, government debt is not subject to intermediation. In this case, the initial impact of an increase in g affects the real returns of both intermediated deposits and bonds. The concomitant increase in the inflation rate has a disproportionately (negative) impact on the return on bonds than on deposits since the latter is a weighted return on money and capital, and the return on rental capital is unaffected by inflation. The initial, marginal impact of an increase in inflation (lower r_m) on the real interest differential between deposits and bonds will equal $-(1-1/P_B) > 0$. Agents will then want to hold deposits instead of bonds, and the bond price must fall in order to restore the interest differential to 0, as required for an equilibrium. The remainder of the story - the impact on the long-run growth rate - follows the same logic as we describe above.

21. The analysis of an increase in g under an open-market coordinating arrangement is based on the government budget constraint/money market clearing relation described in (8) rather than (11), since (11) incorporates the assumption that $r_m = \sigma r_m^o$, which varies with g, as noted in Footnote 16. The curves for this coordinating arrangement, generated without the assumption $r_m = \sigma r_m^o$ share the same properties as those displayed in Figure 3.

coordinating by reserve requirement. (See the discussion in 6.2 and Table 1 below).

Result 8 formally states these findings.[22]

Result 8. The effect of an increase in g under each coordinating arrangement.

A. Inflation coordination: A marginal increase in deficit spending increases growth and inflation.

B. Open-market coordination:

i. A marginal increase in g increases growth and lowers θ in the low-growth equilibrium (5B) and in the case where the economy supports a unique high-growth equilibrium (5C).

ii. If the economy supports two equilibra (5D), a marginal increase in g increases growth and lowers θ for the lower growth equilibrium and lowers growth and increases θ for the equilibrium with higher growth.

C. Reserve requirement coordination: A marginal increase in g increases growth and lowers θ for the lower growth equilibrium and lowers growth and increases θ for the equilibrium with higher growth.

6.2 Comparing arrangements more generally

Note the policy parameter g is unique in the sense that it is the only element of the defining triplet – for each arrangement – that does not have a direct effect on the excess demand in the capital/bond market. That is, a marginal increase in g affects directly only the government's budget (7) (or the γ_b curve). Any change in the growth rate is the result of a change in the coordinating instrument. This is not the case for a marginal change in any other element of the triplets. Below are some general comments comparing the coordination arrangements, based on the comparative static results of the model.

An exogenous, marginal change in r_m, θ, or λ each have two direct effects in model. First, each has a direct, negative effect on growth via the impact on the excess demand in the capital/bond market. A higher value for r_m, like a higher bond-money ratio θ, increases the government's demand for capital/debt resources, directly crowding out private capital. A higher reserve requirement has a similar negative effect on growth (in this case, a higher λ affects real government debt through its effect on the price level P_t).

22. It is easy to see these results graphically. An increase in g shifts the γ_b curve upward (regardless of the coordinating arrangement) while having no impact on the γ_a curve.

Additionally, λ has a second, direct effect on excess demand and growth – a higher λ reduces the amount of intermediated saving allocated to the capital/bond market.

While each of these variables has, qualitatively, the same general effect in capital/bond market, a much different story unfolds as regards their direct impact on government revenues. A higher r_m always reduces total revenues; a higher λ always increases them. An increase in θ has a negative direct impact on revenues whenever $\alpha A > \gamma$, since bond seignorage in this case is negative. The opposite is true when $\alpha A < \gamma$.

What role do these two direct effects play in determining the change in the equilibrium values for growth and the coordinating instrument? First, the direct negative effect on growth emerging from the capital/bond market has a secondary, negative impact on government revenues, since a lower growth rate lowers the effective taxes on money and bonds. It follows that whenever the direct effect on revenues described above is also negative, the coordinating instrument must adjust in a manner that raises government revenues. When the coordinating arrangement supports two equilibria (θ and λ coordination), this is accomplished in one of two ways: increase the revenue base by increasing the coordinating instrument (which has a reinforcing negative impact on growth) or by raising the effective tax rate by raising the growth rate (which is accomplished by a lower value for the coordinating instrument). The former always applies to the equilibrium with higher growth, the latter with lower growth.

If, on the other hand, the equilibrium is unique and the growth rate is less than αA, the negative direct impact on revenues and growth solicits a change in the coordinating instrument that must be growth-enhancing. If this were not the case, the change in the coordinating instrument would repress growth further, which reduces government revenues. The two relevant cases – a marginal increase in θ under inflation coordination (where $dr_m/d\theta < 0$) and an increase in the inflation target under an open market arrangement where conditions of 4B are met ($d\theta/dr_m < 0$) – exhibit different overall impacts on the equilibrium growth rate. Evidently, in the former, the positive impact on growth accompanying the increase in the coordinating instrument (inflation) is not enough to overcome the negative direct impact higher θ has on growth, and the equilibrium growth rate falls. The opposite is true for the latter case.

When the change in the exogenous policy variable has a positive direct effect on government revenues, as in say, an increase in the reserve requirement under inflation coordination or an open market coordinating arrangement when $\gamma > \alpha A$, the change in the coordinating variable can either enhance or inhibit growth. The reasoning is straightforward. In net, the combined effect of the exogenous variable on revenues (the positive direct effect on (7) and the negative indirect effect coming from the impact on growth) cannot be determined without additional assumptions on the primitives. The net effect, in turn, plays a critical role in determining how the coordinating instrument and growth adjust to restore (7).

6.3 Welfare

As noted in Footnote 4, our assumption of a constant saving rate s is consistent with a model where agents have log-linear preferences and $s = \beta/(1-\beta)$. Given that returns and the growth rate are constant at each date in a stationary equilibrium, the utility of a member of Generation t, $t \geq 1$, is given by $u_t = \delta + (1+\beta)(t-1)\ln\gamma + \beta\ln r$, where $\delta \equiv \ln(1-\alpha)(1-s) + \beta\ln s(1-\alpha) + (1+\beta)\ln Y_1$ is a constant and $r = \lambda r_m + (1-\lambda)\alpha A$ is the return on the intermediated deposit. If we assume the government discounts the future at the same rate β as agents, the sum of (discounted) utility of all generations, at date 1, aside from the current old, is

$$U = \sum_{i=0}^{\infty} \beta^i u_{i+1} = \delta/(1-\beta) + \beta\ln r/(1-\beta) + (1+\beta)\ln\gamma/(1-\beta)^2 \ . \tag{12}$$

One difficulty in comparing welfare across the coordinating arrangements is the fact that each take, as given, a policy parameter that is endogenous to another arrangement. For example, when trying to assess whether U is higher under inflation coordination versus open-market coordination, what parameter value for θ is appropriate for studying the former and what value for r_m for the latter? (Setting the value of θ equal to the value that obtains under open-market coordination and likewise for r_m under inflation coordination, yields of course, the same value for U for the two arrangements).

Alternatively, one can consider the optimal policy choices for the given parameters for arrangement and then look at how welfare varies across arrangements. This is fairly easy; similar to Freeman's (1987) result for an inflation coordination with capital storage, the best outcome in all cases involves setting the reserve requirement to its limiting case ($g/s(1-\alpha)$).[23] By Assumption 2, the deficit can be financed with an equilibrium return on money equal to 0. The government simply defaults on any existing nominal obligations (M_0 and B_0), as the initial price, $P_1 = \infty$. The equilibrium growth rate, for this limiting case, is $A(s(1-\alpha)-g)$, and the return on deposits is $\alpha A(s(1-\alpha)-g)/s(1-\alpha)$, for each coordinating arrangement.

7 CONCLUSIONS

Fiscal policy often places constraints on the course of monetary policy. When such conditions prevail, the policies of the fiscal and the monetary branches of government must somehow be coordinated. This paper provides a comprehensive look at this process and its effects on long-run growth under three alternative coordinating arrangements.

The first of these, an inflation-coordinating arrangement, provides insight into the interrelationship that arises between inflation and growth in an environment with a dominant fiscal authority. Our results contribute to the literature on monetarist arithmetic by identifying a potentially important channel by which coordination is achieved. Within this section of the paper, we look at how changes in each component of a country=s seigniorage policy affects long-run inflation and growth rates. Both positive and negative nonlinear relationships between inflation and growth can be inferred from cross-sectional diagrams generated from numerical examples of the model.

Much less is understood of the other two coordinating arrangements we study. These alternatives provide the monetary authority with some degree of autonomy, in the sense it can, perhaps, pursue an inflation targeting policy that is seemingly independent of the fiscal constraint. However, as our results suggest, the central bank=s inflation target may not be entirely free of the fiscal constraint, as an overly ambitious target is incompatible with an equilibrium as we have envisioned it.

Missing here is any consideration of nonstationary equilibria. Most likely, a rich story of the dynamics of coordination and growth can be coaxed from our setting, complementing textbook treatments of money and government finance in models without endogenous growth (see, for example, Sargent (1987)).

23 . $\underline{\lambda}$ of Assumption 3 equals this value under the targeting policy $r_m = 0$.

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Exogenous policy parameter / Coordinating arrangement	A decrease in the inflation target $1/r_m$				An increase in the bond-money ratio θ				An increase in the reserve requirement λ				An increase in government spending g			
	Direct Impact		**Overall Impact**		**Direct Impact**		**Overall Impact**		**Direct Impact**		**Overall Impact**		**Direct Impact**		**Overall Impact**	
	Excess Demand Cap./Bond Market	*Net Govt. Revenues*	*Growth Rate*	*Coord. Instrmt.*	*Excess Demand Cap./Bond Market*	*Net Govt. Revenues*	*Growth Rate*	*Coord. Instrmt.*	*Excess Demand Cap./Bond Market*	*Net Govt. Revenues*	*Growth Rate*	*Coord. Instrmt.*	*Excess Demand Cap./Bond Market*	*Net Govt. Revenues*	*Growth Rate*	*Coord. Instrmt.*
Inflation	*NA*				↑	*LG:* ↓ *HG:* ↑	↓	*LG:* ↑ *HG:* ?	↑	↑	↓	*LG:* ↑ *HG:* ?	0	↓	↑	↑
Open-Market Operation	↑	↓	*LG* ↑ *HGU* ↑ *HGM* *LR:* ↑ *HR:* ↓	*LG* ↓ *HGU* ↓ *HGM* *LR:* ↓ *HR:* ↑	*NA*				↑	↑	*LG* ↓ *HGU* ↓ *HGM* *LR:* ↓ *HR:* ↑	*LG* ? *HGU* ? *HGM* *LR:* ? *HR:* ↓	0	↓	*LG* ↑ *HGU* ↑ *HGM* *LR:* ↑ *HR:* ↓	*LG* ↓ *HGU* ↓ *HGM* *LR:* ↓ *HR:* ↑
Reserve Requirement	↑	↓	*LR* ↑ *HR* ↓	*LR* ↓ *HR* ↑	↑	*LG:* ↓ *HG:* ↑	*LR* ↑ *HR* ↓	*LR* ↑ *HR* *LG* ↓ *HG* ?	*NA*				0	↓	*LR* ↑ *HR* ↓	*LR* ↓ *HR* ↑

Table 1. A summary of the comparative statics for each arrangement.

Key: LG = Low Growth ($\gamma < \alpha A$), HG = High Growth ($\gamma > \alpha A$), HGU = High Growth Unique Equilibrium, HGM = High Growth Multiple Equilibria, LR = Lower Growth Case, HR = Higher Growth Case. Lower and higher growth refers to the lower and higher growth equilibrium for the two coordinating arrangements that support multiple equilibria. Growth in a lower (higher) growth case need not be less (greater) than αA. See Section 4 and 5 for additional details.

Figure 1. Tighter Money Under Inflation Coordination.

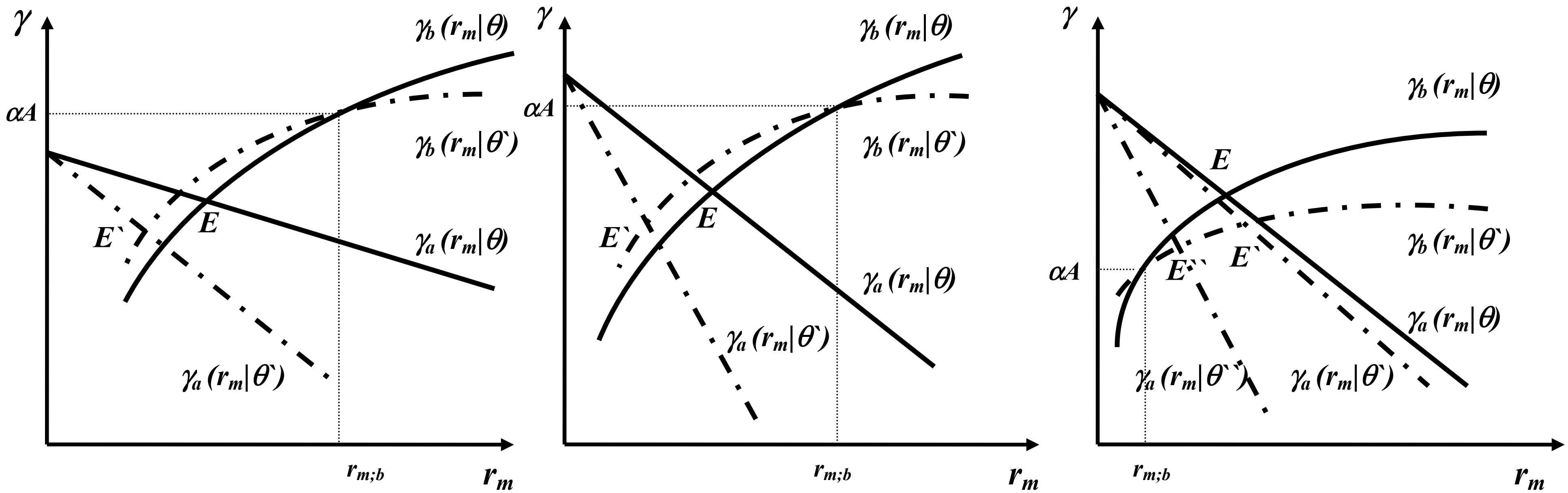


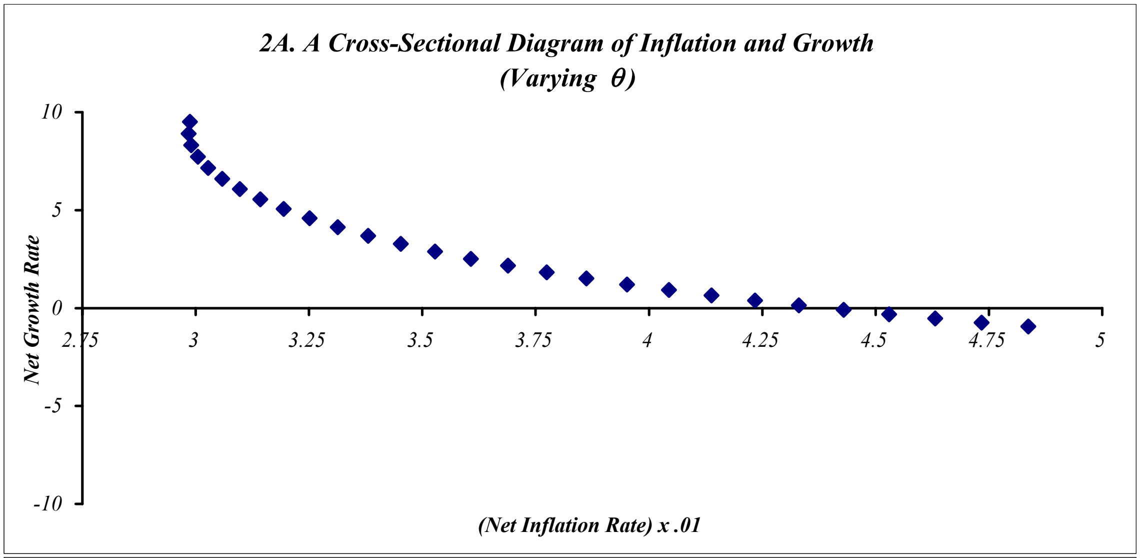
Fig.1A. A Permanent Open Market Sale** (θ` > θ). **Low Growth Case I.

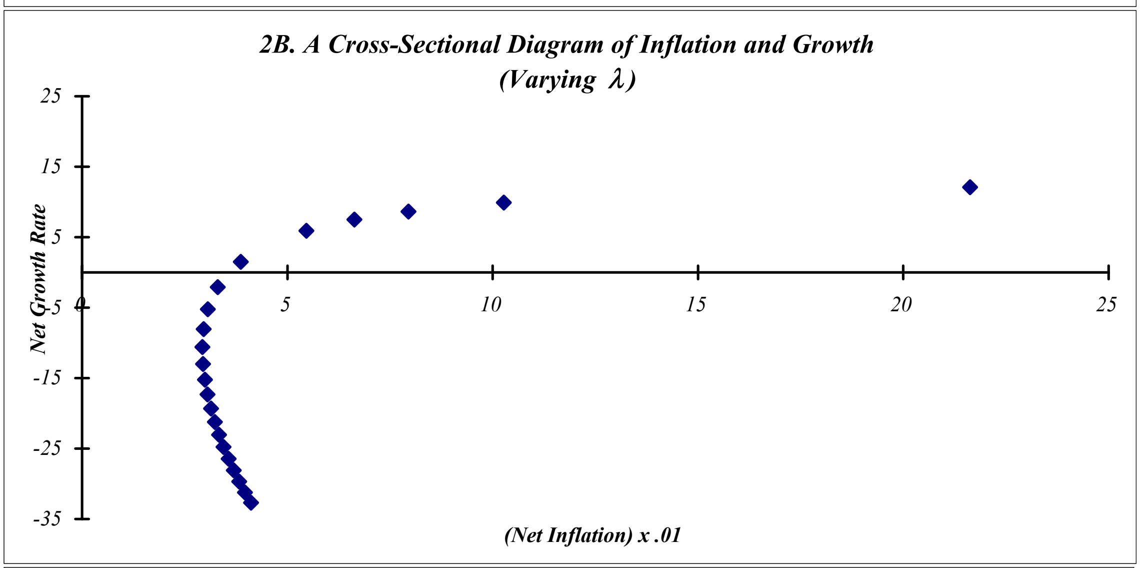
Fig.1B. A Permanent Open Market Sale** (θ` > θ). **Low Growth Case II.

Fig.1C. A Permanent Open Market Sale** (θ` > θ) **and** (θ`` > θ). **High Growth Case.

***Notes:** At the point $r_m = r_{m;b}$, $\gamma_a(r_m) = \gamma_b(r_m) = \alpha A$ for every $\theta > 0$.*

Figure 2. Hypothetical Cross-Sectional Diagrams





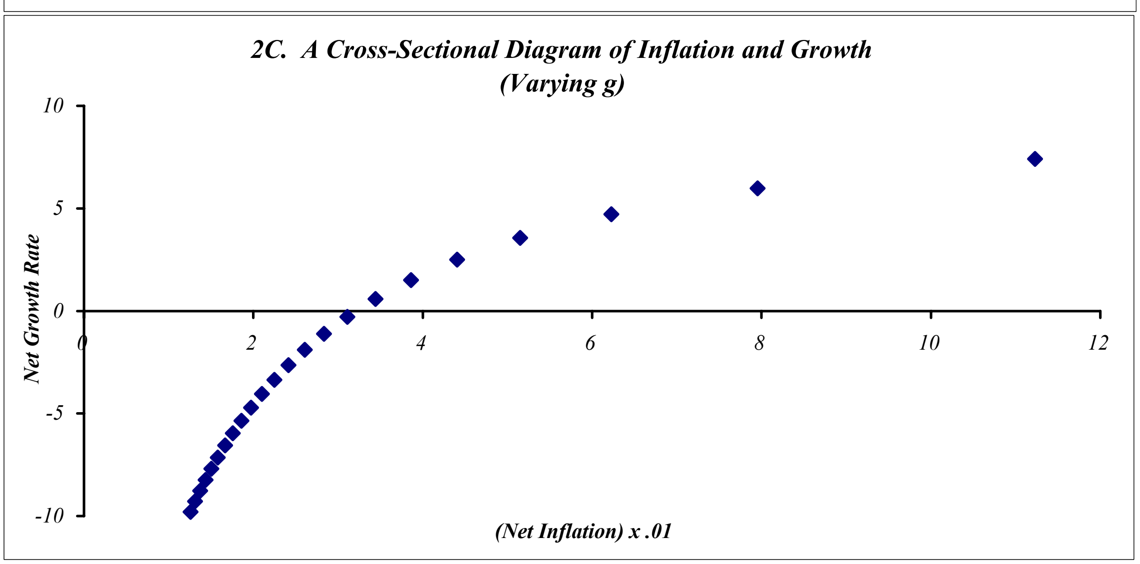


Figure 3. Open-Market Coordination.

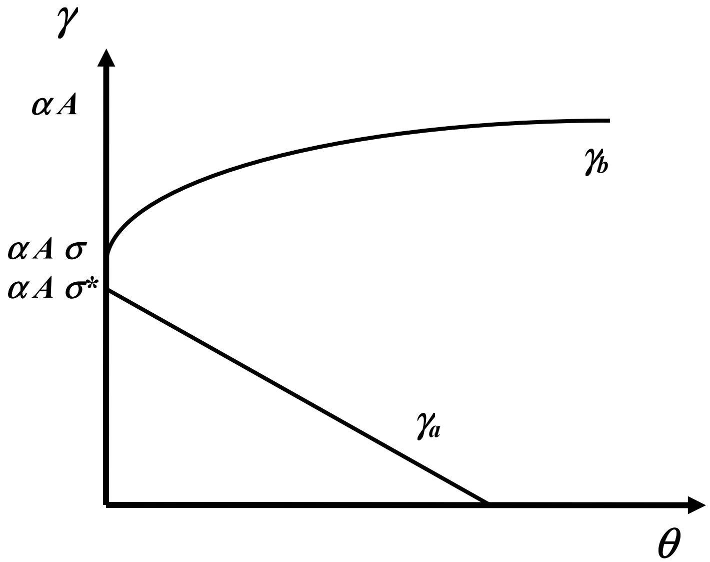


Fig. 3A. The Non-existence of an Equilibrium.

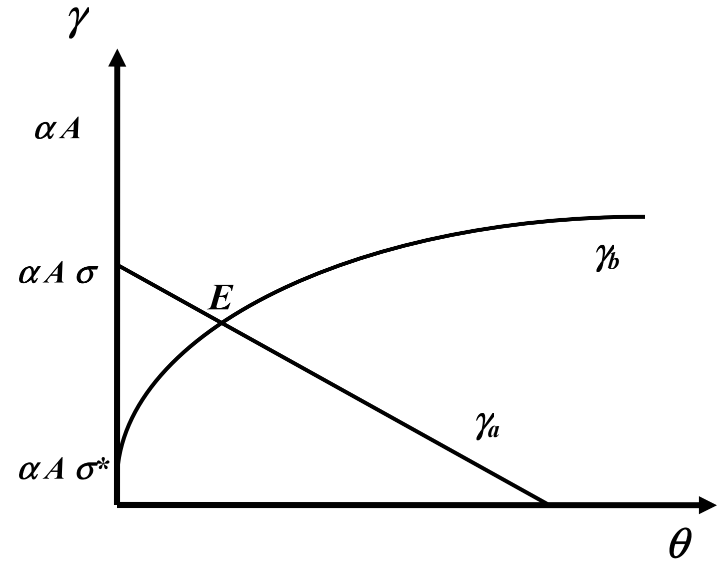


Fig. 3B. A Low Growth Equilibrium.

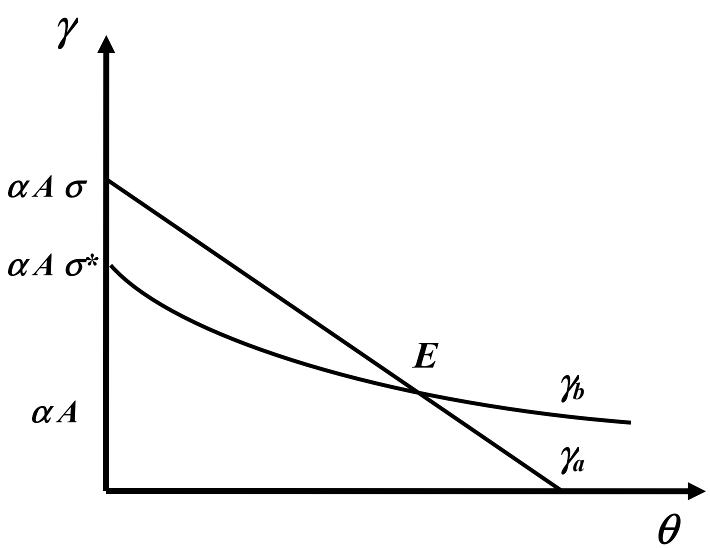


Fig. 3C. A High Growth Equilibrium.

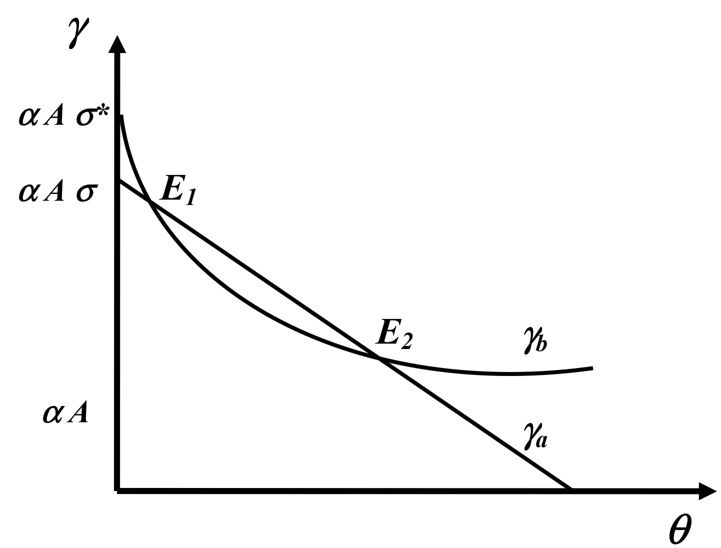


Fig. 3D. Multiple High Growth Equilibria.

Figure 4. Reserve Ratio Coordination.

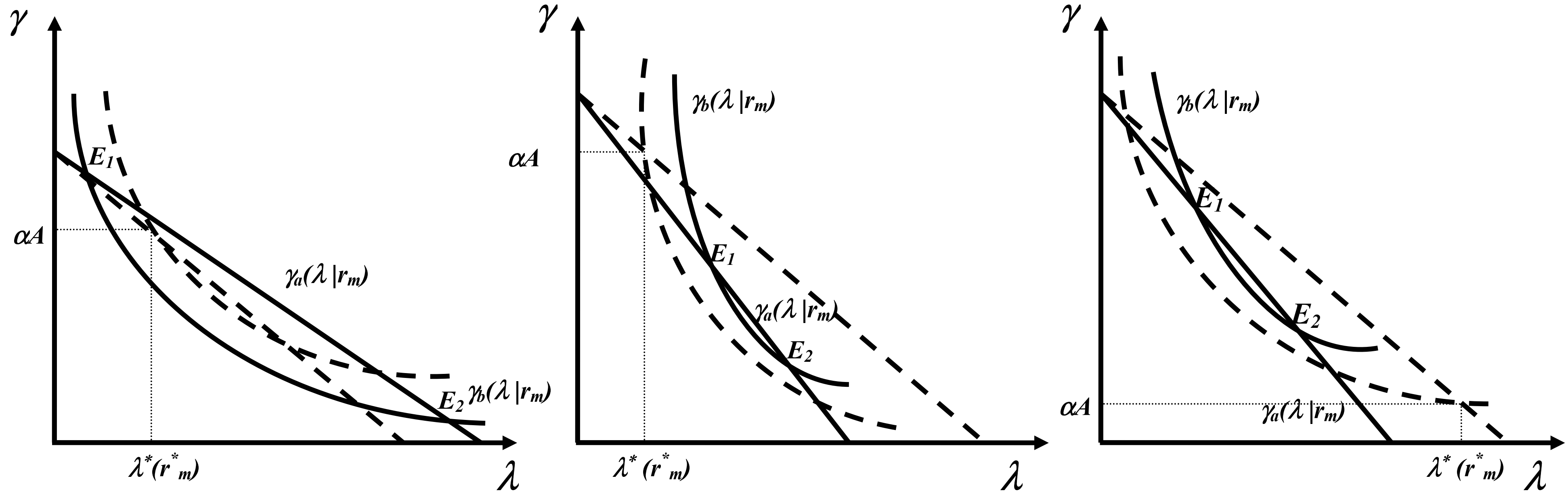


Fig.4A. Multiple Equilibria: Low and High Steady State Growth ($r_m < r^*_m$).

Fig.4B. Multiple Equilibria: Two Low Steady-State Growth Rates ($r_m > r^*_m$).

Fig.4C. Multiple Equilibria: Two High Steady-State Growth Rates ($r_m > r^*_m$).

Notes: The dashed curves represent γ_a and γ_b evaluated at the target r^*_m. At the point $\lambda^*(r^*_m)$, $\gamma_a = \gamma_b = \alpha A$.

TECHNICAL APPENDIX
(Not for Publication)

This appendix contains some technical notes that accompany the main text.

1. The impact of changes in r_m, g, θ, λ ***on the*** γ ***curves.***

Section 3 (Inflation-coordination).

i) $\partial\gamma_a/\partial r_m = -\lambda\theta s(1-\alpha)/\alpha < 0$ *and* $\partial^2\gamma_a/\partial r_m^2 = 0$.

ii) $\partial\gamma_b/\partial r_m = (\alpha A)^2 \lambda r_m s(1-\alpha)(1+\theta)[s\lambda(1-\alpha)-g]/[\lambda s(1-\alpha)(\alpha A+\theta r_m)-\alpha Ag]^2 > 0$ *and*

$\partial^2\gamma_b/\partial r_m^2 = -2(\alpha A)^2\lambda^2 s^2(1-\alpha)^2(1+\theta)[s\lambda(1-\alpha)-g]/[\lambda s(1-\alpha)(\alpha A+\theta r_m)-\alpha Ag]^3 < 0$.

iii) $\partial\gamma_a/\partial\theta = -\lambda r_m s(1-\alpha)/\alpha < 0$.

iv) $\partial\gamma_b/\partial\theta = \alpha A\lambda r_m s(1-\alpha)[\lambda s(1-\alpha)(\alpha A-r_m)-\alpha Ag]/[\lambda s(1-\alpha)(\alpha A+\theta r_m)-\alpha Ag]^2 \gtreqless 0$.

v) $\partial\gamma_a/\partial\lambda = -s(1-\alpha)(\alpha A+\theta r_m)/\alpha < 0$.

vi) $\partial\gamma_b/\partial\lambda = -(\alpha A)^2 g r_m s(1-\alpha)(1+\theta)/[\lambda s(1-\alpha)(\alpha A+\theta r_m)-\alpha Ag]^2 < 0$.

vii) $\partial\gamma_a/\partial g = 0$.

viii) $\partial\gamma_b/\partial g = (\alpha A)^2\lambda r_m s(1-\alpha)(1+\theta)/[\lambda s(1-\alpha)(\alpha A+\theta r_m)-\alpha Ag]^2 > 0$.

Section 4 (Open-market coordination).

i) $\partial\gamma_a/\partial\theta = -\sigma A(\lambda s(1-\alpha)-g) < 0$ *and* $\partial^2\gamma_a/\partial\theta^2 = 0$.

ii) $\partial\gamma_b/\partial\theta = \alpha A\sigma(1-\sigma)/(1+\sigma\theta)^2 \lesseqgtr 0$ *and* $\partial^2\gamma_b/\partial\theta^2 = -2\alpha A\sigma^2(1-\sigma)/(1+\sigma\theta)^3 \gtreqless 0$ *when* $\sigma \gtreqless 1$.

iii) $\partial\gamma_a/\partial\sigma = -\theta A(\lambda s(1-\alpha)-g) < 0$.

iv) $\partial\gamma_b/\partial\sigma = \alpha A(1+\theta)/(1+\sigma\theta)^2 > 0$.

To assess the impact of λ *and g on the* γ *curves for an open-market coordinating arrangement, set* $\sigma = r_m/r_m^o$ *in eqs. (10) and (11). The results are the same as shown in v)-viii) for inflation-coordination above.*

Section 5 (Reserve requirement coordination).

i) $\partial\gamma_a/\partial\lambda = -s(1-\alpha)(\alpha A + \theta r_m)/\alpha < 0$ *and* $\partial^2\gamma_a/\partial\lambda^2 = 0$

ii) $\partial\gamma_b/\partial\lambda = -(\alpha A)^2 g\, r_m\, s(1-\alpha)(1+\theta)/[\lambda s(1-\alpha)(\alpha A + \theta r_m) - \alpha Ag]^2 < 0$ *and*

$\partial^2\gamma_b/\partial\lambda^2 = 2(\alpha A)^2 g\, r_m\, s^2(1-\alpha)^2(1+\theta)(\alpha A + r_m\theta)/[\lambda s(1-\alpha)(\alpha A + \theta r_m) - \alpha Ag]^3 > 0$, *assuming*

$\lambda > \underline{\lambda} \equiv \alpha Ag/[\alpha Ag/s(1-\alpha)(\alpha A + \theta r_m)]$.

The impact of a change in r_m*, g, and* θ *on the* γ *curves for a reserve requirement coordination arrangement are the same as shown in i) – iv) and vii) and viii) for an inflation coordination arrangement above.*

2. Assumption 1 and its implications for bindingness.

In Section 3, we make the assumption that $\alpha + s(1-\alpha)[\lambda(1+\theta) - 1] > 0$ and we claim this assumption is sufficient to ensure $r_m < \alpha A$, as required for a binding equilibrium.

With no bond financing $(\theta = 0)$, $\gamma_a(r_m) = (1-\lambda)(1-\alpha)sA$ for all $r_m > 0$, and $\gamma_b(r_m)$ is simply a line extending from the origin. Since $\gamma_a(r_m)$ is everywhere below $(1-\lambda)(1-\alpha)sA$ when $\theta > 0$, financing a portion of the government=s deficit with debt issues necessarily results in an equilibrium growth rate less than $(1-\lambda)(1-\alpha)sA$.[1] With no bond financing of the deficit, the return on money is $A(1-\alpha)[\lambda(1-\alpha)s - g]/\lambda$.

It turns out the function γ_b for $\theta = 0$, $\gamma_b(r_m \mid \theta = 0)$, intersects the function $\gamma_b(r_m \mid \theta > 0)$ for *any* positive bond to money ratio θ at the point $r_{m;b} \equiv \dfrac{\alpha A[\lambda s(1-\alpha) - g]}{\lambda s(1-\alpha)}$ and growth rate αA. (See Figure 2). At this point, of course, bond seignorage equals 0, since the growth rate just matches the real rate of return on government debt. It is easy to verify, $r_{m;b} < \alpha A$.

If $(1-\lambda)(1-\alpha)sA > \alpha A$, we can find a $r_{m;a} > 0$, with $\gamma_a(r_{m;a}) = \alpha A$. Assumption 1 ensures $r_{m;a} < \alpha A$.

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15. Note that $\gamma_a(0) = (1-\lambda)(1-\alpha)sA$, and recall, $\gamma_a'(r_m) < 0$.

3. Comparative Statics

Section 3 (Inflation-coordination).

The rental capital/government debt market can be written as

$$s(1-\alpha)(1-\lambda)A = \gamma + \theta r_m s\,\lambda(1-\alpha)/\alpha \qquad \text{(A-1)}$$

Totally differentiating (A-1) for $\theta, \lambda, \gamma, r_m$, we have

$$-s(1-\alpha)A\,d\lambda = d\gamma + [\lambda r_m s(1-\alpha)/\alpha]d\theta + [\lambda\theta s(1-\alpha)/\alpha]d\,r_m + [\theta r_m\, s(1-\alpha)/\alpha]d\lambda \qquad \text{(A-2)}$$

The government constraint can be written as:

$$g = \lambda s(1-\alpha)[1 - r_m/\gamma] + [\lambda\ s(1-\alpha)\theta r_m/\alpha A][1-\alpha A/\gamma] \qquad \text{(A-3)}$$

Totally differentiating (A-3) with respect to $\theta, \lambda, \gamma, r_m$, and g

$$dg = [\,s(1-\alpha)[1-r_m/\gamma] + [s(1-\alpha)\theta r_m/\alpha A][1-\alpha A/\gamma]\,]\,d\lambda + [\lambda s(1-\alpha)r_m + \lambda s(1-\alpha)\theta r_m]d\gamma/\gamma^2 \qquad \text{(A-4)}$$

$$+[\,[\lambda\ s(1-\alpha)\theta/\alpha A][1-\alpha A/\gamma] - \lambda s(1-\alpha)/\gamma]d\,r_m + [\lambda\ s(1-\alpha)\theta r_m/\alpha A][1-\alpha A/\gamma]\,d\theta\,.$$

Effect of a change in θ on γ and r_m.

To solve for the effect of a change in θ on γ and r_m, set $d\lambda = dg = 0$ in (A-2) and (A-4) and solve for $d\,r_m/d\theta, d\gamma/d\theta$:

$$\frac{d\,r_m}{d\theta} = \frac{-r_m(A\lambda s(1-\alpha)(1+\theta)r_m - \gamma(\gamma-\alpha A))}{\lambda A s(1-\alpha)\theta(1+\theta)r_m + \gamma(\alpha A(1+\theta) - \theta\gamma)} \qquad \text{(A-5)}$$

$$\frac{d\gamma}{d\theta} = \frac{-\gamma A\lambda s\ (1-\alpha)r_m}{\lambda A s(1-\alpha)\theta(1+\theta)r_m + \gamma(\alpha A(1+\theta) - \theta\gamma)} \qquad \text{(A-6)}$$

The denominator in (A-1) (and (A-2)) is positive by the fact that $\gamma_b'(r_m) > 0$ and that $\gamma < \underset{r_m\to\infty}{Lim}\,\gamma_b(r_m) = \alpha A(1+\theta)/\theta$. Hence, $d\gamma/d\theta < 0$, $d\gamma/d\lambda < 0$, while $d\,r_m/d\theta \gtrless 0$. Clearly, $d\,r_m/d\theta < 0$ when $\gamma \le \alpha A$.

Effect of a change in λ on γ and r_m.

Solving for the effect of a change in λ on γ and r_m, we set $d\theta = dg = 0$ in (A-2) and (A-4):

$$\frac{d\,r_m}{d\lambda} = \frac{-\left(\lambda s(1-\alpha)(1+\theta)(\alpha A + \theta r_m)A r_m + \alpha A\gamma(1+\theta)r_m - (\alpha A + \theta r_m)\gamma^2\right)}{\lambda A s(1-\alpha)\theta(1+\theta)r_m + \gamma(\alpha A(1+\theta) - \theta\gamma)}\,. \qquad \text{(A-7)}$$

$$\frac{d\gamma}{d\lambda} = \frac{-\alpha A^2 s(1-\alpha)(1+\theta)\gamma}{\lambda A s(1-\alpha)\theta(1+\theta)r_m + \gamma(\alpha A(1+\theta) - \theta\gamma)} \ . \tag{A-8}$$

Again, the signs of the denominators of the right-hand sides of (A-7) and (A-8) are positive, so $d\gamma/d\lambda < 0$ while $d r_m/d\lambda \gtrless 0$.

Effect of a change in g on γ and r_m.

To solve for the effect of a change in g on γ and r_m, set $d\lambda = d\theta = 0$ in (A-2) and (A-4) and solve for $d r_m/dg$, $d\gamma/dg$:

$$\frac{d r_m}{dg} = \frac{-A\theta\gamma^2}{\lambda s(1-\alpha)(\lambda A s(1-\alpha)\theta(1+\theta)r_m + \gamma(\alpha A(1+\theta) - \theta\gamma))} \ . \tag{A-9}$$

$$\frac{d\gamma}{dg} = \frac{\theta A \gamma^2}{\lambda A s(1-\alpha)\theta(1+\theta)r_m + \gamma(\alpha A(1+\theta) - \theta\gamma)} \ . \tag{A-10}$$

In this case $d r_m/dg < 0$ and $d\gamma/dg > 0$.

Section 4 (Open-market coordination).

Begin by setting $\sigma = r_m/r_m^o$ in eqs. (10) and (11).

Effect of a change in r_m on θ and γ.

To solve for the effect of a change in r_m on γ and θ, set $d\lambda = dg = 0$ in (A-2) and (A-4) and solve for $d\theta/d r_m$, $d\gamma/d r_m$:

For $d\theta/d r_m$ see (A-5).

$$\frac{d\gamma}{d r_m} = \frac{\gamma A \lambda s\,(1-\alpha)}{\lambda A s(1-\alpha)\,(1+\theta)r_m + \gamma(\alpha A - \gamma)} \ . \tag{A-11}$$

Since $\partial\gamma_a/\partial r_m < 0$ and $\partial\gamma_b/\partial r_m > 0$, a marginal change in r_m will shift the curves in Figure 2 in opposite directions. It follows that $d\theta/d r_m < 0$ for Cases 4B) and 4C), and $d\gamma/d r_m > 0$ for Case 4C). (By inspection, (A-11) is positive for $\gamma \le \alpha A$, so $d\gamma/d r_m > 0$ for Case 4B) too). Using similar logic, $d\theta/d r_m < 0$ and $d\gamma/d r_m > 0$ for the lower growth equilibrium and $d\theta/d r_m > 0$, $d\gamma/d r_m < 0$ for the equilibrium with higher growth, in Case 4D).

Effect of a change in λ on θ and γ.

To solve for the effect of a change in λ on γ and θ, set $dr_m = dg = 0$ in (A-2) and (A-4) and solve for $d\theta/d\lambda$, $d\gamma/d\lambda$:

$$\frac{d\theta}{d\lambda} = -\frac{A\lambda r_m s\,(1-\alpha)(\alpha A + r_m\theta)(1+\theta) + \alpha A\gamma r_m(1+\theta) - (\alpha A + r_m\theta)\gamma^2}{\lambda A s\,(1-\alpha)(1+\theta)r_m + \gamma(\alpha A - \gamma)} \tag{A-12}$$

$$\frac{d\gamma}{d\lambda} = -\frac{\gamma A\lambda s\,(1-\alpha)(\alpha A - r_m)}{\lambda A s\,(1-\alpha)(1+\theta)r_m + \gamma(\alpha A - \gamma)} \tag{A-13}$$

Since $r_m < \alpha A$, $d\gamma/d\lambda$ is the opposite sign of $d\gamma/dr_m$ in (A-11). Hence, for Case 4B) and 4C), we have $d\gamma/d\lambda < 0$; for the lower growth equilibrium we have $d\gamma/d\lambda < 0$ and the higher growth equilibrium, $d\gamma/d\lambda > 0$, in Case 4D).

The sign of the denominator of (A-12) follows directly from our results for the sign of $d\gamma/d\lambda$ above. We can determine the sign of the numerator of (A-12) in the high growth equilibrium of Case 4D). Since the numerator of (A-13) is negative and $d\gamma/d\lambda > 0$ for any higher growth equilibrium, we have $\lambda A s\,(1-\alpha)(1+\theta)r_m + \gamma\alpha A < \gamma^2$. It follows that

$A\lambda r_m s\,(1-\alpha)(1+\theta) + \alpha A\gamma r_m(1+\theta)/(\alpha A + r_m\theta) - \gamma^2 < \alpha A\gamma r_m(1+\theta)/(\alpha A + r_m\theta) - \gamma\alpha A$.

But $\alpha A\gamma r_m(1+\theta)/(\alpha A + r_m\theta) - \alpha A\gamma = \alpha A\gamma(r_m - \alpha A)/(\alpha A + r_m\theta) < 0$, so the numerator of (A-12) is positive in this case. Hence, for the higher growth equilibrium in Case 4D), $d\theta/d\lambda < 0$.

Effect of a change in g on θ and γ.

To solve for the effect of a change in g on γ and θ, set $dr_m = d\lambda = 0$ in (A-2) and (A-4) and solve for $d\theta/dg$, $d\gamma/dg$:

$$\frac{d\theta}{dg} = -\frac{\alpha A\gamma^2}{\lambda r_m s(1-\alpha)[A\lambda s(1-\alpha)(1+\theta)r_m + \gamma(\alpha A - \gamma)]}. \tag{A-14}$$

$$\frac{d\gamma}{dg} = \frac{A\gamma^2}{A\lambda s(1-\alpha)(1+\theta)r_m + \gamma(\alpha A - \gamma)}. \tag{A-15}$$

From our work on the effect of a change in r_m on γ and θ above, we know the denominator of (A-14) is positive for Case 4B), Case 4C) and the lower growth equilibrium in Case 4D). Hence, for these cases, $d\theta/dg < 0$ and $d\gamma/dg > 0$. For the higher growth equilibrium in Case 4D), $d\theta/dg > 0$ and $d\gamma/dg < 0$.

Section 5 (reserve requirement coordination).

Effect of a change in r_m on λ and γ.

To solve for the effect of a change in r_m on γ and λ, set $d\theta = dg = 0$ in (A-2) and (A-4) and solve for $d\lambda/d\,r_m$, $d\,\gamma/d\,r_m$:

For $d\lambda/d\,r_m$, see (A-7).

$$\frac{d\gamma}{d\,r_m} = \frac{\alpha A^2 \gamma\lambda s(1-\alpha)(1+\theta)}{\lambda A r_m\, s(1-\alpha)(1+\theta)(\alpha A + \theta\, r_m) + \alpha A\gamma r_m(1+\theta) - \gamma^2(\alpha A + r_m\theta)}. \tag{A-16}$$

Since $\partial\gamma_a/\partial r_m < 0$ and $\partial\gamma_b/\partial r_m > 0$, a marginal change in r_m will shift the curves in Figure 3 in opposite directions. It follows that $d\lambda/d\,r_m < 0$ and $d\gamma/d\,r_m > 0$ for the equilibrium with lower growth, and $d\lambda/d\,r_m > 0$ and $d\gamma/d\,r_m < 0$ for the equilibrium with higher growth.

Effect of a change in θ on λ and γ.

To solve for the effect of a change in θ on γ and λ, set $d\,r_m = dg = 0$ in (A-2) and (A-4) and solve for $d\lambda/d\,\theta$, $d\,\gamma/d\theta$:

For $d\lambda/d\theta$, see (A-12).

$$\frac{d\gamma}{d\theta} = \frac{\lambda A\gamma r_m\, s(1-\alpha)(\alpha A - r_m)}{\lambda A r_m\, s(1-\alpha)(1+\theta)(\alpha A + \theta r_m) + \alpha A\gamma r_m(1+\theta) - \gamma^2(\alpha A + \theta r_m)}. \tag{A-17}$$

Using the comparative static results for the case for marginal change in r_m above, we can conclude that $d\gamma/d\theta > 0$ for any of the lower growth equilibrium for any of the cases, and $d\gamma/d\theta < 0$ for any of the equilibrium with higher growth. These follow from the sign of $d\gamma/d\theta$ in (A-16) above, which has the same denominator as (A-17).

Examining (A-12), we note that the numerator of (A-12) is the negative of denominator of (A-17). Hence, the numerator of (A-12) is negative for any of the lower growth equilibrium, and it is positive for any of the higher growth equilibrium. The denominator of (A-12) is clearly positive if $\gamma < \alpha A$, so we can concluded that $d\lambda/d\theta < 0$ for any of the lower growth equilibria with $\gamma < \alpha A$, and $d\lambda/d\theta > 0$ in any of the higher growth equilibria with $\gamma < \alpha A$.

Since the numerator of (A-17) is positive and $d\gamma/d\theta > 0$ for any lower growth equilibrium, we have $\lambda A r_m s(1-\alpha)(1+\theta)(\alpha A + \theta r_m) + \alpha A \gamma r_m (1+\theta) > \gamma^2 (\alpha A + \theta r_m)$, or $\lambda A r_m s(1-\alpha)(1+\theta) + \alpha A \gamma r_m (1+\theta)/(\alpha A + \theta r_m) > \gamma^2$. Given this inequality, $\lambda A s(1-\alpha)(1+\theta) r_m + \gamma(\alpha A - \gamma) > \alpha A \gamma - \alpha A \gamma r_m (1+\theta)/(\alpha A + \theta r_m) = \alpha A \gamma(\alpha A - r_m)/(\alpha A + \theta r_m) > 0$. From (A-12), it follows $d\lambda/d\theta < 0$.

Effect of a change in g on λ and γ.

To solve for the effect of a change in g on γ and λ, set $d r_m = d\theta = 0$ in (A-2) and (A-4) and solve for $d\lambda/d g$, $d\lambda/dg$:

$$\frac{d\lambda}{dg} = -\frac{\alpha A \gamma^2}{s(1-\alpha)\left(\lambda A r_m s(1-\alpha)(1+\theta) + \alpha A r_m \gamma(1+\theta) - (\alpha A + r_m \theta)\gamma^2\right)} \qquad \text{(A-18)}$$

$$\frac{d\gamma}{dg} = \frac{(\alpha A + r_m \theta) A \gamma^2}{s(1-\alpha)\left(\lambda A r_m s(1-\alpha)(1+\theta) + \alpha A r_m \gamma(1+\theta) - (\alpha A + r_m \theta)\gamma^2\right)} \qquad \text{(A-19)}$$

The signs of these derivatives are easy to determine. The sign of (A-19) is the same sign as (A-17). Given our work on $d\gamma/d\theta$ above, $d\gamma/dg > 0$ for any of the lower growth equilibrium for any of the cases, and $d\gamma/dg < 0$ for any of the equilibrium with higher growth. $d\lambda/dg$ shares the opposite sign of $d\gamma/dg$.