Optimal Rules under Adjustment Costs and Infrequent Information

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Abstract

A large number of microeconomic decision variables such as investments, prices, inventories or employment are characterized by intermittent large adjustments. The behavior of those variables has been often modeled as following state-dependent rules. The optimality of such state-dependent rules depends crucially on the continuous observation of the relevant state, an assumption which is far from being fulfilled in practice. We propose an alternative model, where at least part of information about the relevant state variable is infrequent. We study several alternatives. We start with the special case where innovations are infrequent, but are readily observed. Only in this case are optimal rules state-dependent. We then explore the common case of infrequent and delayed information. It may arrive at deterministic times, like periodic macroeconomic statistics, or stochastically, when some events trigger announcements. Part of the relevant information may be continuously observed, while the other part is only observed infrequently. The resulting rules are time and state dependent, characterized by trigger and target points that are functions of the time spent since the last time of information arrival. We derive the conditions which characterize the optimal rules and provide numerical algorithms for each case.

1. Introduction

In the last decade, the macroeconomic literature paid considerable attention to the potential aggregate effects of intermittent large adjustments in microeconomic decision variables[1] such as investments, prices, inventories or employment.[2] Microeconomic decision rules were modeled as state-dependent rules, where it was assumed that the relevant state variable is continuously and perfectly observed. These rules were justified by the existence of kinked adjustment costs (Harrison, Selke and Taylor, Bertola and Caballero, 1990). For example, in pricing models economic agents observe continuously and at no cost the frictionless optimal level of prices and reevaluate constantly at which level they should set their price if they were to change it. Since adjusting their price entails a cost, they weigh this cost against the expected benefits from the change and end up adjusting their price infrequently. As pointed out by Woodford (2002), the assumptions underlying these models are not particularly realistic. Faced by the costs associated with information collection or decision-making, firms often reconsider pricing policy at a particular time of year. In this paper we reexamine individual optimal rules in realistic situations where information is infrequent but where deciding to change economic variables still involves adjustment costs. Information may arrive periodically or when some events occur. We analyze several variations which capture different economic settings and derive for each the optimal decision rule.

Examples of such exogenous intermittent flows of information are pervasive: macroeconomic statistics such as inflation, level of employment or GNP are pub-

[1] We are referring to models of state-dependent rules. In fact, ten years before the start of this literature, time-dependent pricing rules became an essencial ingredient for the Keynesian reaction to the rational expectations revolution. However, the only microeconomic decision variable of interest was price. Examples of macroeconomic models built on time-dependent pricing rules are Fischer (1977), Taylor (1979), and more recently, Ball (1994), Ireland (1997), and Bonomo and Carvalho (1999).

[2] For prices, see Caplin and Spulber (1987), Caplin and Leahy (1991), Caballero and Engel (1992 and 1993), Tsiddon (1993) and Almeida and Bonomo (2002); for investment, Caballero and Engel (1999); for inventories, Caplin (1985); for consumption of durables, Caballero (1993) and for employment, Caballero, Engel and Haltiwanger (1994).

lished periodically, dividends of firms are announced only at certain dates, information arrives in asset markets after regular closings on weekdays and holidays. In all these cases, agents do not observe continuously the variable of interest.

We start by introducing infrequent information in its simplest analytical setting. If we assume that the new information is generated infrequently, but is readily observed, the optimal rule is still state-dependent, characterized by an inaction region which is time invariant. This is similar to Ss rules used in the literature. The type of decision rule is modified when innovation is generated continuously but it is observed infrequently. Then, it is reasonable to assume that the amount of new information revealed to the decision maker is positively related with the time elapsed since she was last informed. From here on we will refer to this type of information release as delayed.

We first assume that this delayed information arrives at random rates, to compare more directly with the infrequent but immediate release case. Then the inaction region for the relevant state variable depends on the time elapsed since the last observation. Thus, the optimal rule is both time and state dependent. We then proceed by assuming that the delayed information arrives at deterministic times. Then, the inaction range is radically enlarged at instants before the revelation of information, implying that in these instances it is always optimal to wait[3].

A feature of optimal rules with delayed information is the possibility of uninformed adjustments, based only on the long run trend of the frictionless process for the control variable. This possibility happens if the trend is large enough

[3]In Bonomo and Garcia (2001), we also determine the optimal policy in the presence of both lump-sum adjustment costs and infrequent information about the value of the frictionless optimal level of the control variable but we assume that the stochastic process of the frictionless optimal value of the control variable has no drift, simplifying the problem considerably. We found that the optimal rule is for agents to adjust or not depending on the state at times of information arrivals. It is characterized by a single parameter s, which determined the inaction range $(-s, s)$ for the discrepancy between the frictionless optimal value of the control variable and its actual value, at times of information arrival. This simplicity allowed us to proceed further, by aggregating the optimal rules and deriving macroeconomic implications.

compared to the average period between information arrivals. Then, we can rationalize pricing rules with preset adjustments between optimal adjustments, used in the macro literature (see Fischer , Yun, 1996, and Woodford, 1999). This type of rule seems realistic if inflation is sufficiently high, which is an implication of our model. Conversely, according to our model, there will be no uninformed adjustment if the trend is small, which seems consistent with the evidence of pricing rules for low inflation economies (see Blinder et al., 1998, for US evidence).

Next we analyze an intermediate but more realistic situation where part of the information arrives continuously and freely to the agent, while another part arrives infrequently. We also assume in this case that the infrequent arrival of information is deterministic. In this situation, some adjustments may occur given the partial information acquired continuously, over and above the adjustments which take place given the whole information. If the free information is the price level, this rule will lead to adjustment by the inflation rate in between optimal adjustments. This indexation rule is a realistic representation of price and wage setting rules followed in high inflation countries, as it happened in Brazil, Israel and Chile. It was also used in the literature to analyze the consequences of indexation for the cost of disinflation (see Bonomo and Garcia , 1994 for price setting, and Jadresic, 2002, for wage setting applications).

All the above problems can be characterized by dynamic programming with two conveniently chosen state variables: the elapsed time since the last information arrival, and the conditional expectation of the discrepancy between the frictionless optimal value and the actual value of the control variable, given the current information of the agent. Finding an optimal rule in the case of infrequent and free information consists in finding trigger upper and lower barriers $\{l(\tau), u(\tau)\}$ and an optimal target point $c(\tau)$ for this expected discrepancy, as a function of the time elapsed since the last information arrival. The Bellman equation which characterizes our value function in case of no adjustment or information collection is rewritten as a partial differential equation in time and space. Boundary conditions which depend on the various cases regarding the arrival of information and the presence of adjustment or information costs are imposed. The solution, which depends on both the differential equation and the boundary conditions, is solved numerically using various algorithms based on finite difference methods.

The rest of the paper is organized as follows. In Section 2, we develop a basic framework for analyzing decisions under infrequent information and adjustment cost, which will be used throughout the models. In section 3 we present models with immediate information where the optimal rules are state-dependent. In section 4 we derive optimal rules in models of infrequent delayed information and adjustment costs. The last section concludes.

2. The basic problem under infrequent information

We start by setting up the problem in a context where the agent does not have access continuously to new information. As in the standard case, we assume that the agent controls a variable x_t and that the frictionless optimal value of this control variable, denoted x_t^*, follows a Markovian stochastic process.

When the control variable drifts away from its optimal level, the agent incurs an instantaneous flow cost which, for simplicity, is assumed to be equal to $l(x_t - x_t^*)^2 dt$, where l is an arbitrary constant. To reduce the discrepancy between the control variable and its optimal level, the agent has to pay a lump-sum adjustment cost k. Time is discounted at a constant instantaneous rate ρ.

Suppose now that the agent does not have access continuously to the information about x^*. Then, she must form a probabilistic assessment of its value at time t, x_t^*, from the time of the last information arrival, say s, in order to evaluate the expected flow cost of deviating from the frictionless optimal value, $E_s(x_t - x_t^*)^2$. We can then decompose the instantaneous expected flow cost as:

$$E_s(x_t - x_t^*)^2 = (x_t - E_s x_t^*)^2 + E_s(x_t^* - E_s x_t^*)^2 \qquad (2.1)$$

The first term in the right-hand side represents the known cost of deviating from the expected optimal level as the second represents the expected cost of not observing the optimal level continuously. Let us examine the first term. If there were no adjustment costs, x_t will be set equal to $E_s x_t^*$, reducing the first part of the deviation cost to zero. In the presence of an adjustment cost, the agent must optimally solve the trade-off between letting x_t drift away from its expectation and paying the cost to adjust. As for the second term we know that it is zero when information is always available and is free. If information is costly, but there is no adjustment cost, the agent can reduce the second term by paying information costs. The optimal policy will therefore account for the trade-off between letting the second term increase and paying more information costs. If information is exogenously infrequent, the agent cannot reduce immediately the second term but, as we will see, the adjustment decision will be influenced by it.

From the structure of the problem and from the assumption that the stochastic process for x^* is Markovian, it is clear that the value function at any time is determined by two state variables: the known deviation of x_t from its expected frictionless optimal level defined as

$$y_t = x_t - E_s x_t^* \tag{2.2}$$

and the time elapsed since the last information arrival[4]

$$\tau = t - s. \tag{2.3}$$

We can therefore express $E_s(x_t - x_t^*)^2$ as a function $f(y_t, \tau)$ which can be written as:

$$f(y_t, \tau) = y_t^2 + Var_s(x_{s+\tau}^*) \tag{2.4}$$

where $Var_s(.)$ denotes the variance conditional on the information at time s.

[4] To have a discrepancy $x_s - x_s^*$ at the time of information arrival s gives a value function starting at t that is identical to the value function starting at time v when information arrived at time $v-(t-s)$ and the discrepancy $x_{v-(t-s)} - x_{v-(t-s)}^* = x_s - x_s^*$. Therefore, the discrepancy $x - x^*$ is a sufficient state variable for the value function at times of information arrival.

With lump-sum adjustment costs, resetting of the control variable will be infrequent. Between two adjustments, the value function - the minimized value of the program of the agent - should obey the following Bellman equation:

$$V(y_t, \tau) = f(y_t, \tau)dt + e^{-\rho dt} V(y_{t+dt}, \tau + dt) \tag{2.5}$$

This Bellman equation will be valid for all cases analyzed in this paper, including the full information case. The cases will vary according to the assumptions made about the stochastic process of x_t^* (and therefore y_t) and the boundary conditions imposed.

3. Optimal rules with immediate information

We will present first the known case of continuous full information, which will serve as a benchmark. We will then introduce infrequent information in its simplest setting: the innovation in the frictionless stochastic process x^* follows a Poisson process with a fixed variance. In this particular setting of infrequent information there is no delay in the arrival of information. As a consequence, optimal control rules are still state-dependent. Indeed, at any time, only the known discrepancy between the optimal and the actual value of the control variable matters.

When information is immediate the setup is a particular case of our general framework of the last section where s is always equal to t. Hence, $E_s x_t^* = x_t^*$ and τ will always be equal to zero. Thus, there is only one state variable y_t, which becomes:

$$y_t = x_t - x_t^*$$

The deviation cost function will simplify to:

$$f(y_t, 0) = y_t^2$$

And the general Bellman equation will be reduced to:

$$V(y_t) = y_t^2 dt + e^{-\rho dt} V(y_{t+dt}) \tag{3.1}$$

3.1. Continuous innovation

This is a well-known case, and we report it for comparison purposes (see Dixit, 1993). We assume, for simplicity, that x_t^* follows a Brownian motion:

$$dx_t^* = \mu dt + \sigma dW_t \tag{3.2}$$

where W is a Wiener process.

Observe that, when no control is exerted y_t follows the process

$$dy_t = -\mu dt + \sigma dW_t$$

Applying Ito's lemma we can rewrite the Bellman equation.(3.1) as the following stochastic differential equation:

$$\frac{1}{2}\sigma^2 V''(y) - V'(y)\mu - \rho V(y) + y^2 = 0 \tag{3.3}$$

This is a well known case, and the general solution is given by:

$$V(y) = \frac{y^2}{\rho} + \frac{-2y\mu}{\rho^2} + \frac{\sigma^2}{\rho^2} + \frac{2\mu^2}{\rho^3} + Ae^{\alpha y} + Be^{\beta y} \tag{3.4}$$

where

$$\alpha = \frac{\mu - \sqrt{\mu^2 + 2\rho\sigma^2}}{\sigma^2}, \text{ and} \tag{3.5}$$

$$\beta = \frac{\mu + \sqrt{\mu^2 + 2\rho\sigma^2}}{\sigma^2} \tag{3.6}$$

The optimal rule is characterized by (l, c, u), where l and u are respectively the lower and upper barriers which trigger control and c is the optimal target point. Therefore, the value function should satisfy several conditions. First, an optimality condition:

$$V_y(c) = 0 \tag{3.7}$$

Then, two value matching conditions between l and c and between u and c, which express that the differences in the value function at the barriers and at the optimal target point should be equal to the adjustment cost:

$$V(l) = V(c) + k, \tag{3.8}$$

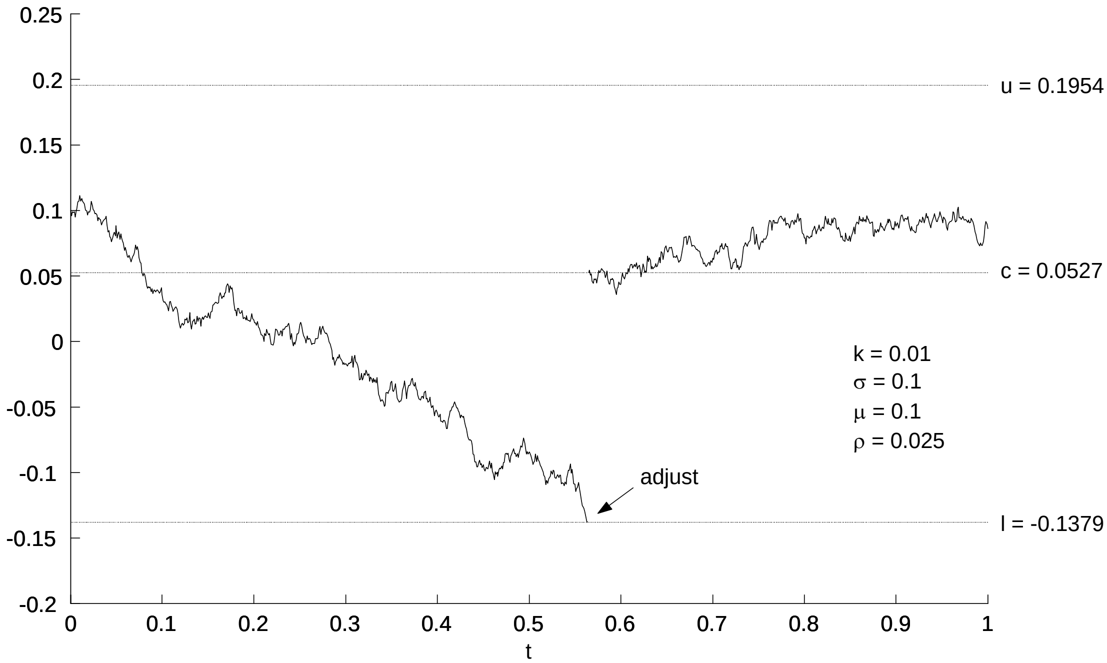
$$V(u) = V(c) + k \tag{3.9}$$

Thus, we must also have:

$$V_y(l) = 0 \tag{3.10}$$

$$V_y(u) = 0 \tag{3.11}$$

The conditions (3.7), (3.10) and (3.11), known as smooth pasting conditions, and the value matching conditions (3.8) and (3.9) imposed on the value function expression 3.4 allow us to determine the constants A, B and the policy parameters l, u and c. Figure 1 illustrates a trajectory of the state variable y_t with the following values of the parameters: $\mu = 0.1$, $\sigma = 0.1$, $k = 0.01$, $\rho = 0.025$. The figure shows that whenever y_t hits the lower or the upper barrier the agent pays the cost k and reduce the discrepancy to c. It should also be noticed that when $\mu > (<)0$, $c > (<)0$, since the drift in the frictionless optimal process will move the discrepancy in the opposite direction when there is no adjustment. The upper and the lower barriers u and l are not symmetric with respect to c either.



As mentioned above, this kind of rule has been used to model inventories, prices, employment, and investment. It has important features of the microeconomic behavior: there is inaction, and intermittent adjustments. However, it rests on the assumption that all the relevant information arrives continously and is immediately known.

3.2. Infrequent but immediate information

In this subsection we assume that we observe the desired level for the control variable x^*, but that the innovation is infrequent. Specifically, we assume that it follows a Poisson process with constant arrival rate λ, and that, conditional on the arrival, the innovation has zero mean and fixed variance distribution. Then:

$$dx^* = \mu dt + \sigma \varepsilon dq$$

where q is a Poisson arrival process with intensity λ, and ε is a standard normal random variable. When there is no control y will evolve according to the following stochastic differential equation:

$$dy = -\mu dt + \varepsilon\sigma dq$$

Then, the differential form of the Bellman equation (3.1) can be written as:

$$-V_y(y)\mu - (\rho+\lambda)V(y) + \lambda E\left[V(y+\sigma\varepsilon)\right] + y^2 = 0$$

The general solution is given by:

$$V(y) = Ae^{\beta_1 y} + Be^{\beta_2 y} + V_p(y) \tag{3.12}$$

where $e^{\beta y}$solves the homogeneous equation, which implies that the βs are the solutions to:

$$\frac{1}{2}\beta^2\sigma^2 = \ln\left(1+\frac{\rho}{\lambda}+\frac{\mu}{\lambda}\beta\right)$$

One particular solution would correspond to the case of never adjusting. This will be given by:

$$V_p(y) = \int_0^\infty \lambda e^{-\lambda\tau}\left[\int_0^\tau e^{-\rho s}\left(y^2 - 2y\mu s + \mu^2 s^2\right) ds + e^{-\rho\tau}\left[EV_p(y-\mu\tau+\sigma\varepsilon)\right]\right] d\tau$$

The solution to this equation can be obtained by the method of undetermined coefficients. We guess that $V_p(y) = ay^2 + by + e$, substitute it in the equation above and iterate to find:

$$a = \frac{1}{\rho} \tag{3.13}$$

$$b = \frac{-2\mu}{\rho^2} \tag{3.14}$$

As before the smooth pasting and value matching conditions (3.7), (3.10), (3.11), (3.8), and (3.9) are boundary conditions which should be satisfied, determining the constants A and B, and the policy parameters l, c, and u.

Figure 2 below shows various adjustment possibilities of the discrepancy between the control variable and its optimal level. First, a jump (denoted by a small circle in the figure) caused by the arrival of a lump of information brings the discrepancy outside the lower barrier triggering an adjustment to c. In the second instance of the Poisson arrival of information the discrepancy process jumps but stays within the barriers: there is no adjustment. There is also the possibility of an uninformed adjustment if the discrepancy reaches its lower level before an information arrival.

Instances of infrequent economic news which are known as soon as they happen can be a financial crash, a catastrophic event such as an earthquake, the discovery of an oil field, an accidental death of a company CEO, etc. Abstracting from innovations of other types which happen on a continuous basis, we can imagine that innovations are only of this random and lumpy type and compute the optimal rule as just described.

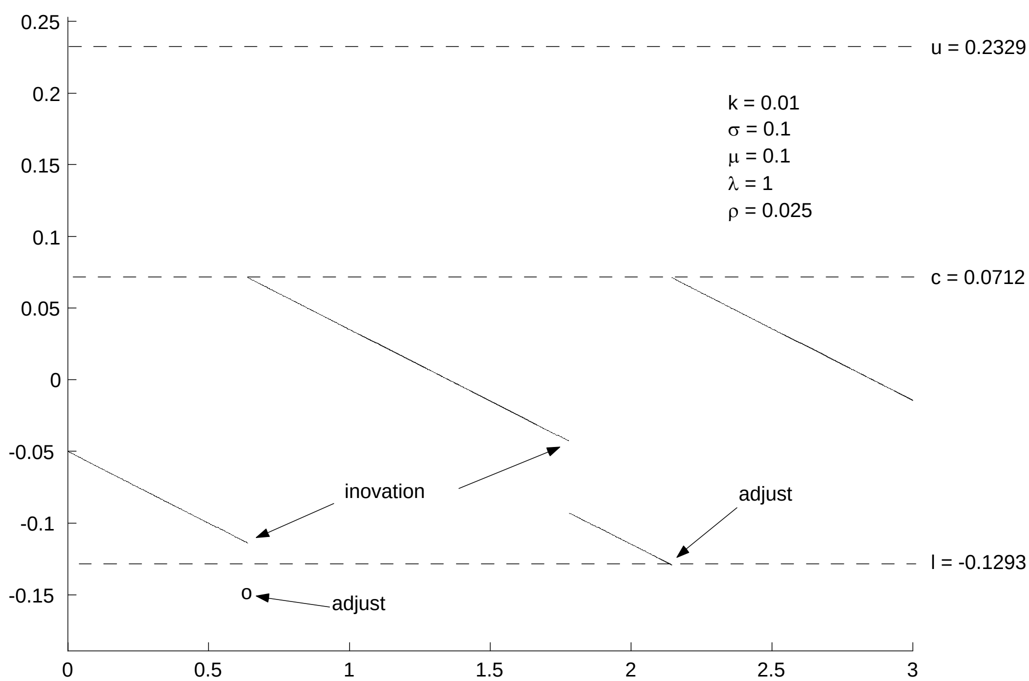


Table 1 below compares the values of the barriers and of the target level for the continuous Brownian case and two Poisson infrequent information cases with different intensities. To compare the continuous case to the infrequent Poisson cases we choose $\lambda = 1$ in order to equalize the conditional variances of the two discrepancy processes. The first result is that the barriers are wider under the infrequent information $\lambda = 1$ case than in the continuous case. The second result is that, as the Poisson intensity increases, the barriers become more symmetric around the target level. This is due to the symmetry of the Poisson process. Indeed, given a fixed μ, when λ is increased the Poisson process becomes relatively more important.

Table 1

Values of barriers and optimal target under continuous and infrequent immediate information

$\mu = 0.1$, $\sigma = 0.1$, $k = 0.01$, $\rho = 0.025$

	Continuous	Infrequent $\lambda = 1$	Infrequent $\lambda = 5$
u	0.1954	0.2329	0.2616
c	0.0527	0.0712	0.0343
l	-0.1379	-0.1293	-0.2144

Although we characterized the control rule as two-sided Ss rules in both the infrequent information case and in the standard continous information case, there are qualitative differences between the controlled processes. With continous information, adjustments in the same direction have always the same size, while in the infrequent information case this rarely happens. Adjustments may be triggered by new information or by the deterministic trend. In the latter case we have uninformed adjustments. The possibility of generating uninformed adjustments is an interesting feature of infrequent information models, since several decision rules described in the literature entail this type of adjustment (e.g. Fischer, 1976).

4. Optimal rules under Delayed Information

In the previous section, all information, continuous or infrequent, was readily available. In those cases, the optimal rules under adjustment costs were state-dependent. In this section, there will be lapses of time during which no information is available although we could think that some innovation is happening. Then each information arrival will encapsulate all the innovations about the optimal value of the control variable which occured during the last lapse. Examples are pervasive in economic life. The release of macroeconomic statistics and some economic news are examples of such infrequent and delayed information processes. In the former case arrival is deterministic while in the latter it can be seen as random. We show that in both cases the optimal rule becomes time-and-state dependent.

We start by the case of random information, which can be obtained by slightly changing the "immediately available but infrequent information" case of the previous section. Then, we explore the case where information about the optimal control level arrives at constant time intervals. Finally, we add more realism by recognizing that some flow of information arrives continuously between the discrete information arrivals while part of the information still arrives infrequently at constant time intervals.

All the problems with infrequent and delayed information will be solved numerically using finite difference methods. Thus, for each case, we first formulate the problem analytically and then present an algorithm for solving the problem numerically.

4.1. Random Information Arrival

4.1.1. Analytical formulation

We assume that x^* follows a Brownian motion as in (3.2), but it is only observed at a random time, which has a negative exponential distribution. Then, when there is no control, the expected discrepancy y will have a trend $-\mu$. If there is an information arrival, there will be an innovation with zero mean and variance proportional to the time elapsed since the last information arrival. Formally, when there is no control the expected discrepancy y will evolve according to the following differential equation:

$$dy = -\mu dt + \varepsilon\sigma\sqrt{\tau}dq \tag{4.1}$$

where q is a Poisson arrival process with intensity λ, and ε is a standard normal random variable.

Notice the similarity of this case with the immediate but infrequent information case of the previous section. However, the jumps in the state variable y have different sources: under immediate information, $y = x - x^*$ jumps whenever x^* jumps, while under the delayed information $y = x - E_s x^*$ jumps when a jump occurs at s (from a given value $s < t$ to t) due to the arrival of information about x^*.

An important difference is that the time since the last information arrival τ now matters. Although the probability of an information arrival does not depend on τ, the amount of information at each arrival (as measured by the variance of the accumulated innovation in x^* during this period) is proportional to τ . Thus, the flow costs of being uninformed about the optimal level x^* are also increasing with τ:

$$f(y_t, \tau) = y_t^2 + \sigma^2\tau \tag{4.2}$$

Therefore, the barriers will depend on τ.

We now look for rules $\{l(\tau), c(\tau), u(\tau)\}_{0\leq\tau<\infty}$ where $l(\tau)$, $u(\tau)$, $c(\tau)$ represent lower and upper trigger points and target point, respectively. Using (4.1) and (4.2), we can write the differential form of the Bellman equation (2.5) as:

$$-V_y(y,\tau)\mu + V_t(y,\tau) - (\rho+\lambda)V(y,\tau) + \lambda E\left[V(y+\sigma\varepsilon\sqrt{\tau},0)\right] + y^2 + \sigma^2\tau = 0 \tag{4.3}$$

Since adjustment costs are lump-sum, adjustment will be made to the point that minimizes intertemporal costs. Then:

$$c(\tau) = \inf_{y} \{V(y, \tau)\} \tag{4.4}$$

Since it is always possible to pay the adjustment cost k and to adjust to $c(\tau)$, we have that

$$V(y(\tau), \tau) = \min \{V(y(\tau), \tau), V(c(\tau), \tau) + k\} \tag{4.5}$$

It is clear that if $l(\tau)$ and $u(\tau)$ are trigger points, then:

$$\begin{aligned} V(l(\tau), \tau) &= V(c(\tau), \tau) + k \\ V(u(\tau), \tau) &= V(c(\tau), \tau) + k \end{aligned} \tag{4.6}$$

The time variable τ can take any positive value. Hence, we need to find the value function and the trigger and target points for each positive time. We use finite difference methods to solve the problem numerically.

4.1.2. Numerical algorithm

In order to find the optimal rule $\{l(\tau), c(\tau), u(\tau)\}$, we need to find the value function. We start by discretizing the partial differential equation(4.3), using the explicit difference method. We make the following approximations:

$$\begin{aligned} y &\approx i \triangle y \\ t &\approx j \triangle t \\ V_t &\approx \frac{v_{i,j+1} - v_{i,j}}{\triangle t} \\ V_y &\approx \frac{v_{i,j+1} - v_{i-1,j+1}}{\triangle y} \end{aligned} \tag{4.7}$$

and we obtain:

$$v_{i,j} = p^0.v_{i,j+1} + p^-.v_{i-1,j+1} + \frac{\lambda}{\rho + \frac{1}{\triangle t}} \sum_{k=-12}^{12} \pi(k).v_{i+k.\sigma,0} + \left(\frac{1}{\rho + \frac{1}{\triangle t}}\right) . \left[(i. \triangle y)^2 + \sigma^2.j. \triangle t\right] \tag{4.8}$$

where $\pi(.)$ is a discretization of the normal distribution, and

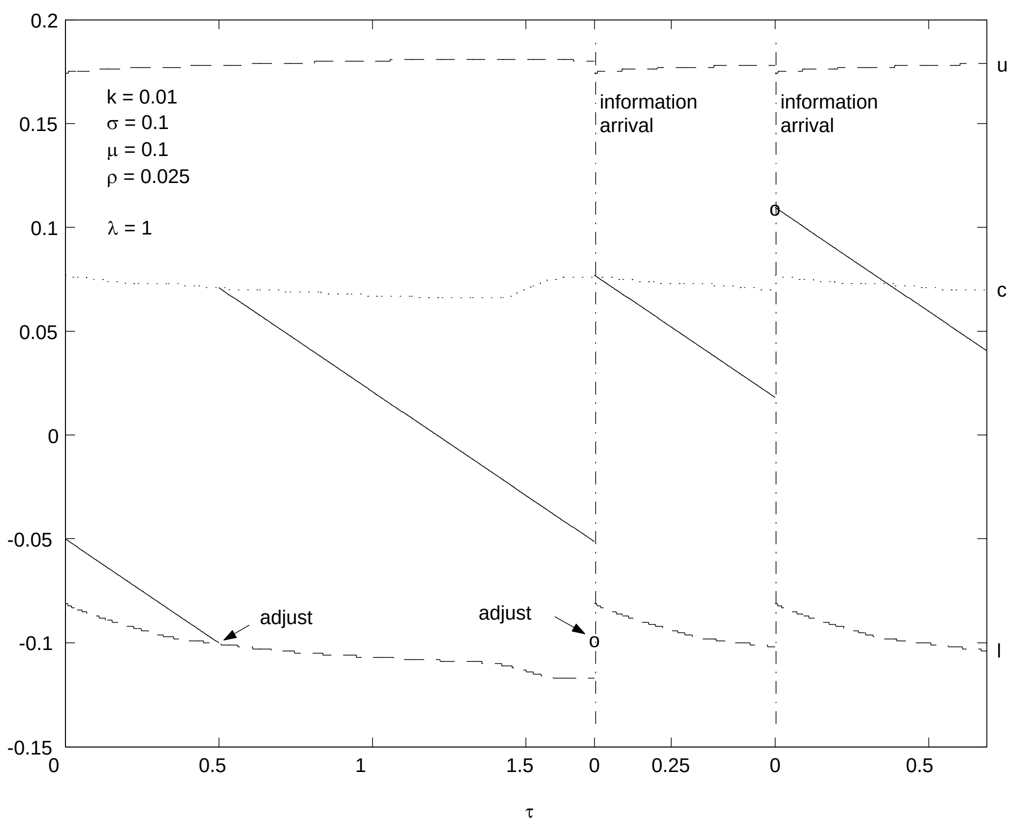
$$
\begin{aligned}
p^0 &= \left(\frac{1}{\rho + \frac{1}{\triangle t}}\right)\left(\frac{-\mu}{\triangle y} + \frac{1}{\triangle t}\right), \\
p^- &= \left(\frac{1}{\rho + \frac{1}{\triangle t}}\right)\frac{\mu}{\triangle y}.
\end{aligned}
$$

Thus, if we have the value function for all states at time $j+1$, we can use equation (4.8) to find the value function at time j.We start with an arbitrary value function for τ very large and proceed backwards using the difference equation in the same way as before until arriving at zero. If τ is large enough, the value function found should be a good approximation for τ small. Although the initial value function is arbitrary, if τ is large enough and if the discount rate ρ is not too big, it will have little importance for the value function evaluated at a small τ.

4.1.3. The optimal rule

Figure 3 shows the functions $l(\tau), c(\tau), u(\tau)$, which characterize the optimal rule. Since we have a negative drift in the process of y, uninformed adjustments are always upwards. As a consequence only the lower barrier is binding at times of no adjustment. The upper barrier is used only at time of information arrivals, where $\tau = 0$, and remains constant. The lower bound function, $l(\tau)$, is slightly decreasing. The reason is that the option value of waiting for an information arrival increases with τ due to a higher amount of innovations missed. We also depicts a sample trajectory for y, where we show two information arrivals. In the first one, y jumps to a point below the lower barrier triggering adjustment to c. In the second information arrival, y jumps upwards to a point inside the inaction range. Thus, there is no adjustment. Observe also in the figure that everytime there is an information arrival τ is reset to zero.

The general features of this case are that the rules are characterized by trigger and target functions of the time elapsed since the last information arrival. When there is no information, adjustment is only triggered by the trend and it moves the control variable in the same direction as the drift of the frictionless variable. Therefore, only the lower barrier (for a positive trend) or the upper barrier (for a negative trend) can be binding. The lower barrier (upper barrier) decreases (increases) with τ. Therefore, in the case of a positive drift, we have a decreasing lower barrier function and a decreasing target function for τ greater then zero. When information is revealed τ becomes zero and y may jump in any direction. Then, when τ is zero we have both an upper and a lower barrier. As a result, as in the case before, we can have uninformed adjustments between information arrivals and informed or "optimal" adjustments triggered by information arrivals. It is also possible that an information arrival will not trigger any adjustment.



4.2. Deterministic Information Arrival

4.2.1. Analytical formulation

As mentioned before, one important source of infrequent and delayed information is the release of economic statistics. This is done usually at regular intervals of times. How is the problem changed if now information arrives deterministically at regular intervals of time T?

TWhen no control is exerted, and there is no information arrival (for $0 < \tau < T$), the expected discrepancy y will have a deterministic trend μ and no innovation. Then, it will evolve according to the following differential equation:

$$dy = -\mu dt$$

The flow cost, $f(y, \tau)$ will still be given by (4.2). As a consequence, we can rewrite equation (2.5) as the following differential equation:

$$-V_y(y, \tau)\mu + V_t(y, \tau) - \rho V(y, \tau) + y^2 + \sigma^2 \tau = 0 \tag{4.9}$$

Again we look for rules $\{l(\tau), c(\tau), u(\tau)\}_{0 \leq \tau \leq T}$. But observe that now τ is bounded by T.

Since adjustment costs are lump-sum, we know that adjustment will be made to the point that minimizes intertemporal costs. Conditions (4.4), (4.5), and (4.6) are still valid.

However since information arrives deterministically, we need to tie the value function just before the arrival to the value fuction after the arrival. When information arrives, the known discrepancy will receive a shock with distribution $N(0, \sigma^2 T)$, and τ is reset to zero. Then we have the following additional condition:

$$V(y, T) = EV(y + \sigma \sqrt{T} \varepsilon, 0) \tag{4.10}$$

where ε is a random variable with distribution $N(0, 1)$.

4.2.2. Numerical algorithm

In order to find the optimal rule $\{l(\tau), c(\tau), u(\tau)\}$, we need to find the value function. We start by discretizing the partial differential equation (4.9) using the explicit difference method. Making the same approximations as in (4.7) we arrive at:

$$v_{i,j} = p^0.v_{i,j+1} + p^-.v_{i-1,j+1} + \left(\frac{1}{\rho + \frac{1}{\triangle t}}\right).\left[(i. \triangle y)^2 + \sigma^2.j. \triangle t\right] \tag{4.11}$$

where

$$p^0 = \left(\frac{1}{\rho + \frac{1}{\triangle t}}\right)\left(\frac{-\mu}{\triangle y} + \frac{1}{\triangle t}\right)$$
$$p^- = \left(\frac{1}{\rho + \frac{1}{\triangle t}}\right)\frac{\mu}{\triangle y}$$

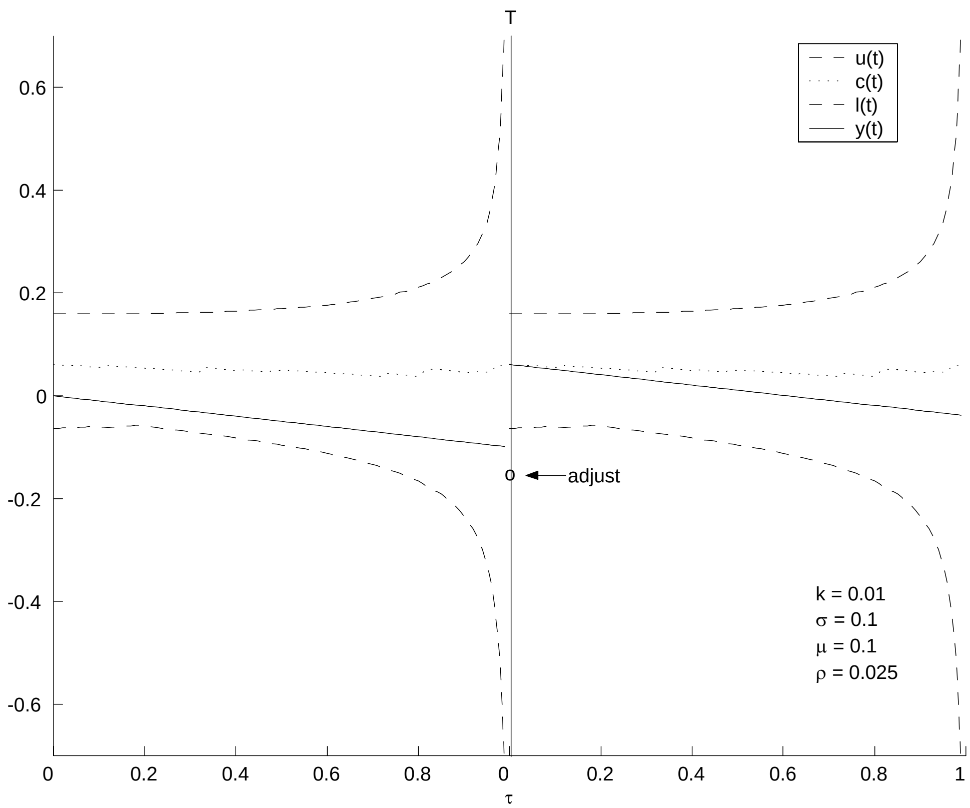
Thus, if we have the value function for all states at time $j+1$, we can use equation (4.11) to find the value function at time j. We use the following algorithm. We guess values for the function at time zero. It is important that it satisfies condition (4.5). We then use the expectation equation (4.10) to find the value function at time T. We find the y that minimizes the function at T, $c(T)$. We then use condition (4.5) to determine the new value at T. We use the difference equation (4.11) to find the value function at time $T - \triangle t$. We then find the value of y that minimizes the function at $T - \triangle t$, and so on, until we arrive at time zero. We then test if the value function just found at zero is close enough (according to some convergence criterion set a prioiri) to the value we had at the previous iteration before. If the value functions are different we begin another iteration of the same procedure, continuing in the same way until convergence.

After convergence we use conditions (4.4), and (4.6) for each τ to find $c(\tau)$, $u(\tau)$, and $l(\tau)$.

4.2.3. The Optimal Rule

Figure 4 illustrates the optimal rule, characterized by the lower barrier function $l(\tau)$, target function $c(\tau)$, and upper barrier parameter u. Observe that, again in this case, when there is no information flow to the agent, there are only upward adjustments. The upper barrier will be used only at times of information arrival. We illustrate a sample path for y. Initially y is close to zero, and arrives at time 1 inside the inaction region, but outside the inaction region for $\tau = 0$. Then, the accumulated shock is revealed and y jumps to the position marked with o, outside the new inaction range. An immediate adjustment is triggered to $c\,(0)$. Then, with no information, y decreases at a constant rate from $c(0)$, and so on.

Although in both random and deterministic information arrival cases the rules are similar, there is a distinguishing feature. In the deterministic case the inaction range becomes arbitrarily large just before information arrival. The reason is clear: the option value of waiting becomes very large when information is about to arrive. This is a testable implication. One should see less adjustments just before important announcements.



4.3. Part of information continuously observed

4.3.1. Analytical formulation

In fact there is almost a continuous flow of some information, although some important information arrives infrequently. We intend to capture this fact by extending the previous model in the following way. We assume that there are two kinds of information, one called idiosyncratic, which is continuously observed, and another one, called aggregate, which arrives at deterministic times.

In this case, when there is no adjustment and no aggregate information arrival, y changes continuously because of the idiosyncratic information, which has standard deviation parameter σ_i:

$$dy = -\mu dt + \sigma_i dw_i.$$

Aggregate information impacts also the expected costs of deviating from the optimal level. Then, the instantaneous cost function is given by:

$$f(y_t, \tau) = y_t^2 + \sigma_a^2 \tau$$

where σ_a is the standard deviation of aggregate shocks. Hence, the differential form of the Bellman equation is written as:

$$\frac{1}{2}\sigma_i^2 V_{yy}(y,\tau) - V_y(y,\tau)\mu + V_t(y,\tau) - \rho V(y,\tau) + y_t^2 + \sigma_a^2 \tau = 0 \tag{4.12}$$

The conditions that determine c (4.4), the adjustment option condition (4.5), and the conditions that determine l and u (4.6) remain the same. However, the condition that ties the value function at time 1 and at time 0 is altered:

$$V(y,T) = EV(y + \sigma_a \sqrt{T}\varepsilon, 0).$$

We apply the numerical procedures described in the previous subsection to the modified equations.

4.3.2. Numerical algorithm

Discretizing the partial differential equation using the explicit difference method, and making the same approximations as in (4.7), we arrive at:

$$v_{i,j} = p^0.v_{i,j+1} + p^-.v_{i-1,j+1} + p^+.v_{i+1,j+1} + \left(\frac{1}{\rho + \frac{1}{\triangle t}}\right).\left[(i.\triangle y)^2 + \sigma_a^2.j.\triangle t\right] \tag{4.13}$$

where

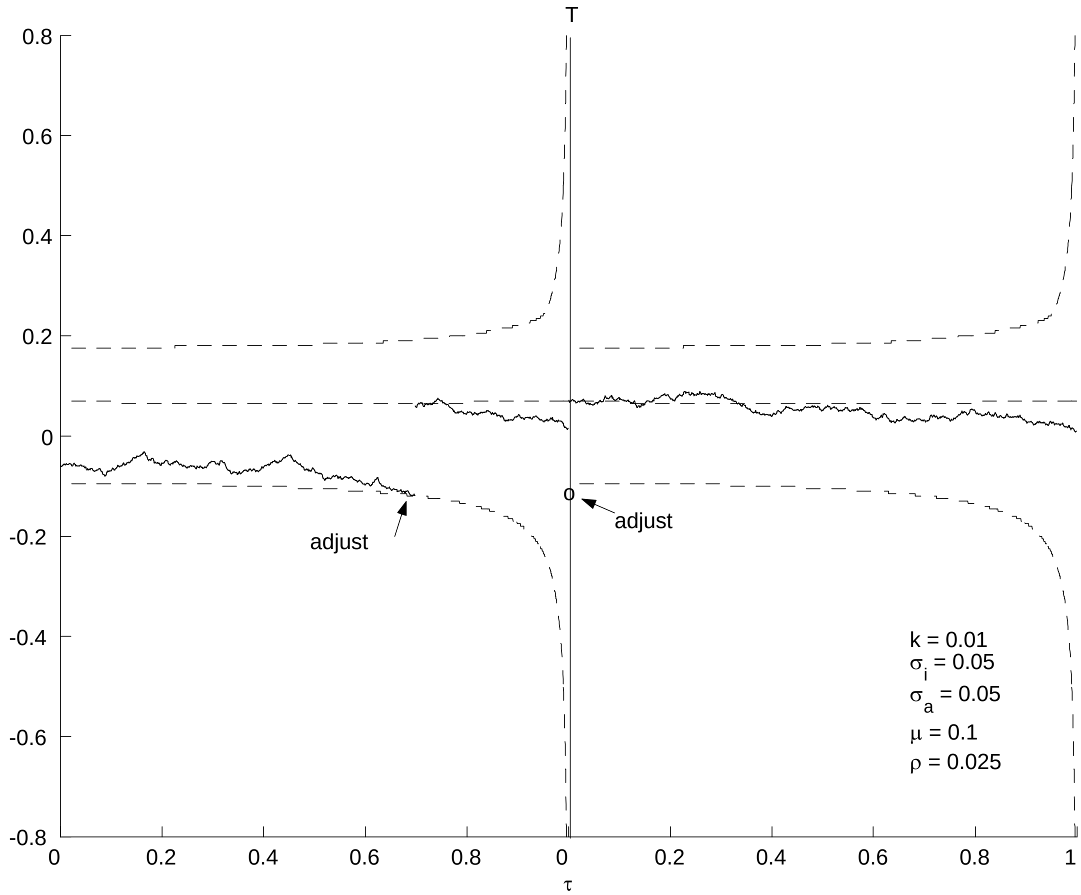
$$p^0 = \left(\frac{1}{\rho + \frac{1}{\triangle t}}\right)\left(-\left(\frac{\sigma_i}{\triangle y}\right)^2 + \frac{1}{\triangle t}\right)$$
$$p^- = 0.5\left(\frac{1}{\rho + \frac{1}{\triangle t}}\right)\left(\left(\frac{\sigma_i}{\triangle y}\right)^2 + \frac{\mu}{\triangle y}\right)$$
$$p^+ = 0.5\left(\frac{1}{\rho + \frac{1}{\triangle t}}\right)\left(\left(\frac{\sigma_i}{\triangle y}\right)^2 - \frac{\mu}{\triangle y}\right)$$

We use the same algorithm as in the section above to arrive at the solution.

4.3.3. The optimal rule

Figure 5 shows the rules and one sample path for y. For τ between zero and one, it oscillates according to the idiosyncratic stochastic component. When it reaches the lower barrier, adjustment is triggered to $c(.)$ of the respective τ. These adjustments take into consideration only part of the relevant information. When τ reaches 1 aggregate information arrives and y jumps. If it is outside the inaction range at zero, an adjustment is triggered to $c(0)$.

A feature of this setting is that no totally uniformed adjustment occurs. The agent uses the information readily available for adjustment if she evaluates that the discrepancy is large enough. If our control variable is individual price or wage, and if the continuous information is the price level we have a rule that generates adjustments by the price level between fully informed or "optimal " adjustments. In fact these price adjustments are state-dependent rather than time-dependent, being more similar to trigger clauses in wage contracts. Despite the continuous flow of information, the inaction range becomes again very large before the deterministic time of information arrival. Thus, the implication that one should not see adjustments before an important information announcement is not specific to that special setting.


T
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
0.2
0.4
0.6
0.8
0
0.2
0.4
0.6
0.8
1
τ
o
adjust
adjust
k = 0.01
σi = 0.05
σa = 0.05
μ = 0.1
ρ = 0.025

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5. Conclusion

In this paper we explore the consequences of infrequent information for decision making when there are adjustment costs. The rule followed by the agents are more complex than the usual state-dependent rules. In general, the inaction ranges depend on the time elapsed since the last information arrival.

There is room for uninformed adjustments when there is no new information arriving but the trend of the control variables must be large enough. When some important variable for the optimal control variable is observed, there can be adjustments based on its realization even when important information about the optimal level of the control variable has not been released. A robust fact is that there should be no adjustments just before the release of important information about the optimal level of the control variable.

We hope that the paper will stimulate applications in areas where microeconomic inaction seem realistic: pricing, investment, inventories, and employment. One natural extension to be made entails assuming that information is costly, and the information cost is separated from the adjustment costs[5]. The same state variables defined in this paper could be used, and the optimal rule should still be time-and-state dependent.

[5] When both adjustment cost and information costs cannot be separated the optimal rule is to set a fix level for the control variable for a predetermined amount of time. The rule is then time-dependent. See Bonomo and Carvalho (2003) for an application to nominal price rigidity.

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