Price-Neutral Tax Reform With an Informal Economy

M. Shahe Emran¹
George Washington University

Joseph E. Stiglitz
Columbia University

This Version: November, 2003

Abstract

A strand of recent literature shows that a reform of import tariff (export tax) and consumption tax (production tax) that keeps consumer (producer) price unchanged enhances welfare and increases revenue under plausible conditions. It has been argued that the results provide an ex post justification for the widely implemented reform policies in developing countries that reduce trade taxes and increase consumption tax like VAT for revenue. We demonstrate that the results derived so far critically depend on the unrealistic assumption that there is no informal sector in the economy, implying that each and every commodity in the economy can be taxed through VAT and production tax. Our results show that, when the feasibility restrictions on the tax instruments imposed by the presence of a large informal and shadow economy is taken into account, such consumer or producer price-neutral reform reduces both welfare and revenue under plausible conditions.

Keywords: Trade Tax, VAT, Production Tax, Welfare, Government Revenue

JEL Classification :H20, F13

¹We would like to thank D. Coady, S. Miller, T. N. Srinivasan, Forhad Shilpi and participants at NEUDC conference 2003 at Yale university for useful comments on earlier drafts. Standard disclaimers apply.
Price-Neutral Tax Reform With an Informal Economy

Introduction

The standard tax and tariff reform policies as advocated by the IMF and the World Bank favor a reduction in the trade taxes with a concomitant increase in some form of a consumption tax, usually a value added tax (henceforth VAT). It is widely believed, both by academic economists and policy practitioners, that such a coordinated reform in VATs and tariffs reduces the costs of distortionary taxation for financing government expenditure. The basic argument here is that the tariffs are extremely distortionary as instruments of raising government revenue, as they distort both consumer and producer prices.\(^2\) A consumption tax like VAT, on the other hand, has some well-known desirable features, like elimination of cascading and of undue protection to the domestic production of import substitutes. The relevant theoretical literature has focused on establishing sufficient conditions for ensuring a welfare improvement for the following three cases: (i) reform of tariffs and taxes (radial or selective)\(^3\) with an inactive government budget constraint (see, for example, Hatta (1977), Diewert et al. (1989)), (ii) tax and tariff reform when the government budget constraint is active\(^4\) (see, for example, Abe (1992, 1995), Panagariya (1992), Michael et. al. (1993), Anderson (1999)), (iii) a reduction in tariffs (export taxes) with an increase in the consumption taxes (production taxes) in a way to leave the consumer (producer) price unchanged. It is shown in the literature that this last type of reform can increase both welfare and revenue.\(^5\) Since trade taxes constitute a significant source of government revenue in the developing countries, and governments there are, in general, fiscally starved, the literature in the last two categories has gained prominence in recent years.\(^6\) However, the recent works on tax and tariff reform with an active government budget constraint have demonstrated that the results established

\(^2\)The wide prevalence of trade taxes at early stages of development is usually rationalized in terms of their well-known administrative advantages. Burgess and Stern (1993) summarize the consensus view: “..the main justification for using trade taxes is administrative feasibility. They are generally unattractive on efficiency grounds.” (p.779).

\(^3\)A radial reform is defined as the most comprehensive one and involves all of the commodities in the economy. A selective reform, on the other hand, covers only a subset of commodities, including the case of a single commodity widely analyzed in the literature.

\(^4\)The government budget constraint is said to be active when a reduction in the revenue due to a reduction in one distortionary tax can be offset only by an increase in another distortionary instrument, while it is inactive or passive if a lumpsum component of the budget can be adjusted instead.

\(^5\)See Hatzipanayotou et. al. (1994) for consumer price-neutral radial reform of import tariff and consumption taxes, and emran (2000) for producer price-neutral selective reform of export tax and production tax.

\(^6\)The trade taxes accounted for 27 percent of government revenue in Sub-Saharan Africa in 1992 (Devarajan et al. 1999). For Africa as a whole, the share of trade taxes in government revenue was 36.4 percent over the period 1975-79, 34.8 percent over 1980-84, and 32.5 percent over 1985-89. The corresponding numbers for Asia are 26.5 percent (1975-79), 24.8 percent (1980-84), and 23.8 percent (1985-89) (see Zee, 1996).
earlier in the literature in favor of the current consensus on tax policy reform are built on fragile grounds. Anderson (1999) shows that it is extremely difficult to guarantee a welfare improvement from a revenue-neutral radial reform when the existence of non-tradables in the economy is taken into account.\textsuperscript{7} The chapter 1 of this dissertation shows that a revenue-neutral selective reform in VAT and trade taxes reduces welfare under plausible conditions when one accommodates the existence of a large informal sector in the economy, which, by definition, escapes the VAT coverage. \textsuperscript{8} In this paper, we show that the results established in the literature regarding the last type of reform where tax and tariff reform leaves the consumer or producer price unchanged also depends critically on the assumption that there is no shadow economy or informal sector. \textsuperscript{9} The existence of a large informal and shadow economy implies that the increase in VAT (production tax) required to neutralize the changes in consumer (producer) price is feasible only if a commodity is produced and transacted in the formal part of the economy. Once this feasibility restriction on the choice of commodities for adjustments in VAT or production tax is taken into account, there are plausible sufficient conditions under which such consumer or producer price-neutral reform reduces both revenue and welfare. The results reported here thus add to the emerging skepticism about the wisdom of the widely implemented tax reform in developing countries under the policy conditionalities of IMF and the World Bank.

The rest of the paper is organized as follows. Section 1 describes the basic structure of the economy. The next section is devoted to the analysis of the consumer price-neutral reform. Section 3 discusses the results on the producer price-neutral reform. The paper concludes with a summary of the results and their implications for indirect tax reform in developing countries.

Section 1: The Model

We build the analysis on a simple model of the economy which has been the work-\textsuperscript{7} The empirical exercise in Anderson (1996) shows that the implementation of the standard indirect tax reform policies reduce welfare in case of Korea, even though the trade taxes are much higher than the consumption taxes at the initial position.

\textsuperscript{8} The production inefficiencies due to VAT can be important even for a developed country with a relatively small informal economy. The recent empirical work by Piggot and Whalley (2001) on GST in Canada show that a VAT base broadening induces significant substitutions in favor of inefficient informal and home productions. Their estimates indicate that such a base broadening of VAT is likely to reduce aggregate efficiency of Canadian economy.

\textsuperscript{9} The recent estimates show that the average size of the shadow economy over 1989–’93 as a percentage of GDP is 39 percent for developing countries and is 12 percent for OECD countries. When measured in terms of labor force employed in the shadow economy as a percentage of official labor force in 1997 – 98, the average is 50.1 percent for developing countries and 17.3 percent for OECD countries. In some developing countries like Nigeria and Egypt, the average size of the shadow economy over the period 1990 – 1993 is 68 to 76 percent of GDP (see Schneider and Enste, 2000). In most of the developing countries, small holder agriculture also belong to the informal segment of the economy, as consumption and production taxes are not feasible for numerous small producers transacting in local markets.
horse in the literature on tax and tariff policy reform. The economy, endowed with a vector of fixed factors \( (L) \), is a competitive small open economy. It is assumed that there are no non-tradable commodities.\(^{10}\) All of the commodities are consumed and produced domestically, and are also internationally traded.\(^{11}\) The set of commodities can be partitioned into four subsets depending on whether a commodity is produced in the formal or informal sector, and on whether it is an exportable or an importable. We use \( x \) for the set of exportables, \( m \) for the set of importables, \( f \) for the set of commodities produced in the formal sector, and \( s \) for the set of commodities produced in the informal sector. The set of all commodities, i.e., the union set of exportables and importables, is denoted as \( T \). The subset \( xf \) \((xs)\) consists of all the exportables produced in the formal \((\text{informal})\) sector. Analogously, \( mf \) \((ms)\) denotes the subset of importables produced in formal \((\text{informal})\) sector. There are some goods which are not taxable. For simplicity, we lump together all the untaxed goods into a single good and assume it to be an informal exportable. This non-taxable informal exportable serves as the numeraire, and is denoted as commodity ‘0’. There is a representative consumer who owns all the factors of production and maximizes a strictly quasi-concave utility function subject to the budget constraint. Let \( E(q_0, q, U) \) denote the expenditure function where \([q_0, q]\) is the vector of consumer prices. So \( E(.) \) is the minimum expenditure needed to achieve utility level \( U \) facing the consumer price vector \([q_0, q]\). The production side of the economy is represented by a revenue function \( G(p_0, p, L) \) which shows the maximum value of the national output produced with factors \( L \) and a convex technology when facing the producer price vector \([p_0, p]\). Pure profits, when they exist due to diminishing returns, are assumed to be untaxed.\(^{12}\) This implies that the assumption of an untaxed numeraire places restrictions on the set of admissible taxes. \( G(p_0, p, L) \) is assumed to be strictly convex in \( p \) and strictly concave in \( L \).\(^{13}\) Both the expenditure and revenue functions are assumed to be twice differentiable. The government raises revenue \( (R(\tau, v, t)) \) using the trade taxes \( (\tau) \), VAT \( (v) \) and production taxes \( (t) \). The government revenue is returned to the consumer in a lump-sum manner. The world prices of all the commodities are normalized to unity by suitable choice of units. The price relations in the economy before policy reform are

\(^{10}\)The assumption of a tradables-only economy, although widely used, is undoubtedly a strong one. We adopt the assumption on two grounds. First, it helps to compare and contrast our results with those of Hatzipanayotou et al. Second, as has been shown in the literature, accommodating nontradables in the model makes the case for such a reform even weaker (keen and Ligthart, 1999).

\(^{11}\)This implies, among other things, that the economy is not “marginally closed” with respect to any of the commodities; all commodities have non-zero international trade both at the initial position and after a reform.

\(^{12}\)For well-known reasons, governments in developing countries cannot impose 100 percent profits tax. For example, it is practically impossible to isolate any pure profit from quasi-rents \((\text{the returns to capital and entrepreneurship})\) in a typical small unincorporated business in developing countries (see Sah and Stiglitz, 1992).

\(^{13}\)The revenue function is strictly convex in \( p \) if there are some substitutability between untaxed numeraire and the taxed commodities (see Dixit, 1985, p.344).
as follows:

\[
q^f = 1 + \tau^f + v \\
p^f = 1 + \tau^f - t \\
q^s = 1 + \tau^s = p^s \\
p_0 = q_0 = 1
\]

where \( q^f \) is the vector of consumer prices, \( p^f \) the vector of producer prices, and \( \tau^f \) is the vector of trade taxes on commodities produced in sector \( l \), with \( l = f, s \), and \( v \) is the vector of VATs applicable only to the commodities produced in the formal sector.

The private budget constraint of the representative consumer equates the expenditure \( E(q_0, q, U) \) with the private revenue or GNP, \( G(p_0, p, L) \) plus the revenue transferred by the government.

\[
E(q_0, q, U) = G(p_0, p, L) + \Phi R(.) \tag{1}
\]

where \( \Phi \geq 1 \) is the shadow value of public expenditure or marginal cost of public funds.

The government revenue function is given by the following:

\[
R(\tau, v, t) \equiv \tau^f \left( E_{q^f} - G_{p^f} \right) + \tau^s \left( E_{q^s} - G_{p^s} \right) + v' E_{q^f} + t' G_{p^f} \tag{2}
\]

where the subscripts to the functions \( E(.) \) and \( G(.) \) denote the partial derivatives, the prime denotes a transpose of a vector or of a matrix, \( \tau \) with appropriate superscript denotes the vector of trade taxes on formal and informal tradables. In addition to the budget constraint of the consumer and the government revenue function, the equilibrium of the economy is characterized by the balance of trade condition which we can ignore by Walras law. So equations (1) and (2) constitute the basic framework for analyzing any tax reform in this economy.

Section 2: Consumer Price Neutral Reform of Import Tariff (Export Subsidy) and VAT

This section is devoted to the case of consumer price neutral reform of VAT and tariffs (or export subsidies). For simplicity and for ease of comparison with the extant literature, we assume that there are no production taxes, i.e. \( t = 0 \) implying \( t' G_{p,f} = 0 \) in the government revenue function. The policy experiment involves reducing the trade taxes uniformly or selectively and increasing the VAT to keep consumer price vector unchanged. Since VAT can be applied to only the formal commodities, this means the policy reform applies only to the formal part of the economy. We leave the prices and taxes unchanged in the informal part of the economy. We consider both the radial uniform and selective reforms.

Consumer Price-Neutral Radial Reform
Here we consider a reform that uniformly reduces the trade taxes on all formal commodities and the effects on the consumer price vector are offset by an increase in the VAT. More specifically, we focus on the following differential policy reform:

\[ dv = v d\alpha, \quad d\tau^f = -\tau^f d\beta, \quad dv + d\tau^f = 0 \text{ where } d\beta > 0, \text{ and } d\alpha > 0 \text{ are scalars.} \]

This is the policy reform experiment of Hatzipanayotou et. al. (1994) with the feasibility restrictions imposed due to the existence of an informal economy. As noted by Hatzipanayotou et. al. (1994) such a policy reform is feasible only under two sets of initial conditions: (i) both the trade taxes and consumption taxes are harmonized at the initial position, (ii) the trade tax and VAT rates are proportional to each other. The reform also requires that either there are no export tax or import subsidy at the initial position, or when they are allowed for a commodity, only a negative consumption tax (VAT) is admissible (i.e., if \( \tau_i < 0 \), it is necessary that \( v_i < 0 \)). Since a reduction in export tax increases the consumer price, it can only be offset by increasing the subsidy provided through a negative consumption tax (VAT).

**Government Revenue**

Taking total differential of the government revenue function (equation 2), we get:

\[
\begin{align*}
\frac{dR}{d\tau^f} &= \left\{ \left( E_{q^f} - G_{p^f} \right)' + \tau^f \left( E_{q^f q^f} - G_{p^f p^f} \right) + \tau^s \left( E_{q^f q^f} - G_{p^s p^f} \right) + v' E_{q^f q^f} \right\} d\tau^f \\
&\quad + \left\{ (\tau^f + v)'E_{q^f q^f} + E_{q^f} \right\} dv \\
&= \left( \tau^f G_{p^f p^f} \tau^f + G_{p^f} \tau^f + \tau^s G_{p^s p^f} \tau^f \right) d\beta \tag{4}
\end{align*}
\]

**Consumer Welfare**

Taking total differential of the budget constraint of the representative consumer, we have:

\[
E_{q^f}' \left[ dv + d\tau^f \right] + E_U dU = G_{p^f}' d\tau^f + \Phi \left[ \frac{\partial R}{\partial \tau^f} d\tau^f + \frac{\partial R}{\partial v} dv + \frac{\partial R}{\partial U} dU \right] \tag{5}
\]

where

\[
\begin{align*}
\frac{\partial R}{\partial \tau^f} &= \left\{ \left( E_{q^f} - G_{p^f} \right)' + \tau^f \left( E_{q^f q^f} - G_{p^f p^f} \right) + \tau^s \left( E_{q^f q^f} - G_{p^s p^f} \right) + v' E_{q^f q^f} \right\} \\
\frac{\partial R}{\partial v} &= \left\{ (\tau^f + v)'E_{q^f q^f} + E_{q^f} \right\} \\
\frac{\partial R}{\partial U} &= (\tau + \bar{v})' E_{q^f U}
\end{align*}
\]
Where $\hat{v}$ is a vector that assigns zero VAT on the informal commodities, i.e., $\hat{v}' \equiv [v' \ 0']$.

Under the policy reform $dv = vda$, $d\tau^f = -\tau^f d\beta$ and $dv + d\tau^f = 0$, equation (5) simplifies to the following:

$$\Omega dU = \Phi \left( \tau^f G_{p^f p^f} \tau^f + \tau^s G_{p^s p^s} \tau^f \right) d\beta$$

(6)

Where $\Omega = [E_U - \Phi(\tau + \hat{v})']E_{qu}$. Now, since $E_U$ is homogenous of degree one in $q$, Euler theorem implies that $q'E_{qu} = E_U$. Using this we have $\Omega = [1 + (1 - \Phi)(\tau + \hat{v})']E_{qu}$. Observe that if $\Phi = 1$, $\Omega = [1'E_{qu}] > 0$, under the standard assumption that there are no inferior commodities in the economy. But with $\Phi > 1$, the sign of $\Omega$ is not in general determinate. However the requirements of uniqueness and the Walrasian stability of the equilibrium dictate that $\Omega > 0$, and we assume it in what follows. With $\Omega > 0$, the sign of the welfare change is given by the sign of the right hand side of equation (6).

**Proposition 1:**

In an economy consisting of both formal and informal sectors, assume that all formal and informal commodities are pair-wise substitutes in production and there are no export tax or import subsidy. A consumer price-neutral radial uniform reform of VAT and trade taxes in such an economy reduces both welfare and revenue if the following conditions hold:

(i) the production substitution effects within the formal sector are low and that between formal and informal sectors are high enough,

(ii) the size of the formal sector is lower than a threshold.

**Proof**

The necessary and sufficient conditions for a reduction in government revenue is given by the following:

$$dR(.) < 0 \iff G'_{p^f} \tau^f < - \left( \tau^f G_{p^f p^f} \tau^f + \tau^s G_{p^s p^s} \tau^f \right)$$

(7)

Now the quadratic form $\tau^f G_{p^f p^f} \tau^f > 0$, because $G_{pp}$ is positive definite given strict convexity of the GNP function, and all principal sub-matrices of a positive definite matrix are also positive definite implying, in particular, that $G_{p^f p^f}$ is positive definite. The other part of the right hand side in inequality (7) ($\tau^s G_{p^s p^s} \tau^f$) is a bilinear form and convexity of the revenue function is of no help in pinning down its sign. However, under the assumptions that formal and informal commodities are substitutes in production, i.e., $G_{p_i p_j} < 0$, $\forall i \neq j$ and $i \in f$, $j \in s$, and that there are no export tax or import subsidy, we have $\tau^s G_{p^s p^f} \tau^f < 0$. So if the cross substitution effects in production between formal and
informal sectors are strong enough, then the right hand side of inequality (7) is positive. Now observe that \( \lim_{G_{pf} \to 0} G_{pf}' \tau_f = 0 \). So there always exists a small enough (positive) \( G_{pf} \) such that the inequality (7) is satisfied, and consequently, the coordinated reform of trade taxes and VAT reduces revenue if the size of the formal sector is smaller than a threshold. Under these conditions welfare also suffers, as is evident from equation (6).\(^1\)

Q.E.D.

The intuition behind the revenue reduction result is simple. Since the consumer price vector is kept unchanged, the reform works through production substitutions due to the decrease in the protection to the formal sector following the reduction in trade taxes. But when there is a large informal sector with close substitutes (in production) of formal sector goods, it results in significant resource outflow to the informal sector. As the informal sector production expands, the volume of competing imports decline (the domestic consumption is unchanged) pulling down with it the tariff revenue. Since there are no export taxes, only subsidies by assumption, a higher output of informal exportables entails higher government expenditure on subsidy as exports go up one for one given that the domestic consumption remains unchanged. The only positive effect on revenue comes from a lower expenditure on the subsidy to the formal producers as a group, because of lower import tax and export subsidy after the reform (given by the term \( G_{pf}' \tau_f \)). With a large enough informal economy and strong enough inter-sectoral production substitutions, the negative effects can dominate and result in a reduction in government revenue. Note that if the size of the formal sector is small, then the positive effect on the revenue due to the direct effect of lower trade taxes and subsidies will be insignificant. A large informal economy, on the other hand, implies that resources can flow easily out of the formal sector causing large production inefficiencies. A large informal economy thus makes it easier for the right hand side of inequality (7) to be positive. The above result is extremely important because the size of the informal and shadow economy is very large in most of the developing countries, and thus it is more likely to have a reduction in revenue and welfare from such a reform.

Now observe that in the absence of any informal economy, the revenue and welfare effects of a consumer price-neutral reform are as follows:

\[
\begin{align*}
\frac{dR}{(\cdot)} &= \left( \tau_f' G_{pf} \tau_f + G_{pf}' \tau_f \right) d\beta \\
\Omega dU &= \Phi \left( \tau_f' G_{pf} \tau_f \right) d\beta
\end{align*}
\]

The right hand sides of both the equations (8) and (9) are positive under the assumptions that the GNP function is strictly convex in producer prices and that the producers

\(^1\)Note that a negative revenue outcome is over-sufficient for a welfare reduction. It is possible to have a revenue increase but a welfare reduction when the substitution effects make the right hand side of inequality (7) positive but smaller than the subsidy to the producers.
enjoy subsidy as a group, implying $G'_{pf} \tau^f > 0$. This is the result derived by Hatzipanayotou et al. (1994).

Note that the condition in proposition 1 that there are no export tax or import subsidy is sufficient for $G'_{pf} \tau^f > 0$, but not necessary. But if we allow export tax or import subsidy, then the bilinear form $\tau^s G'_{p^p f} \tau^f$ can not be signed in general. However, as evident from the above discussion, the basic result remains valid even if both export tax and import subsidy are allowed up to a point, as long as the pattern of taxes and subsidies does not change the negative sign of the bilinear form $\tau^s G'_{p^p f} \tau^f$.

Consumer Price-Neutral Selective Reform

The above discussion deals with the radial reform that spans the entire formal sector. However, such comprehensive tax and tariff reform, especially in the context of developing countries, is rarely feasible. Moreover, as mentioned above, such a consumer price neutral radial uniform reform is feasible under stringent initial conditions only. This raises the question if a selective reform where the tax and tariff on a single commodity (or a subset of commodities) are reformed at a time and in a way to keep its consumer price unchanged can be a better approach. This section addresses this question.\(^{15}\) Note that the intuition about inter-sectoral production substitutions which is at the heart of the results on radial reform discussed above does not have any clean counterpart when a selective reform is considered. However, the following discussion shows that in the presence of an informal economy, there are plausible sufficient conditions such that a selective reform of VAT and import tariff reduces both welfare and revenue.

Selective Reform of Import tariff and VAT

Here we analyze the case where the import tax (or export subsidy) on commodity $k$, $\tau_k$ is reduced and the VAT $v_k$ is increased so that the consumer price $q_k$ remains unchanged. So we have

\[
\begin{align*}
dv_k &= v_k d\alpha, \\
d\tau^f_k &= -\tau^f_k d\beta, \\
dv_k + d\tau^f_k &= 0, \\
d\alpha, d\beta &> 0
\end{align*}
\]

Observe that, in contrast to the radial uniform reform, no additional initial conditions are required for feasibility of such a reform. It is easy to check that the revenue and welfare effects of this reform are given by the following equations:

\[
\begin{align*}
dR(\cdot) &= \left(\tau' G'_{p^p f} \tau^f_k + G'_{p^p f} \tau^f_k\right) d\beta \\
\Omega dU &= \Phi \left(\tau' G'_{p^p f} \tau^f_k\right) d\beta
\end{align*}
\]

\(^{15}\) The case of a selective reform is not treated by Hatzipanayotou et al. (op cit).
Now, since $G_{p_k}$ is homogeneous of degree zero in $[p, p_0]$, by Euler theorem, we have $G_{p_k} = -\frac{1}{p_k} \sum_{j \neq k} p_j G_{p_j}$. Using this we can rewrite $\tau' G_{pp_k}$ as follows:

$$\tau' G_{pp_k} = \tau'_{p_k} + \sum_{j \neq k} \tau_j G_{p_j}$$

$$= \frac{1}{p_k} \left\{ \sum_{j \in s} (\tau_j - \tau_k) G_{p_j} + \sum_{j \neq k, j \in f} (\tau_j - \tau_k) G_{p_j} \right\} \quad (12)$$

Utilizing equation (12), the revenue and welfare effects in equations (10) and (11) can be rewritten as follows:

$$dR(\cdot) = \tau'_{p_k} \left\{ \sum_{j \neq k, j \in T} (\tau_j - \tau_k) G_{p_j} + G_{p_k} \right\} d\beta \quad (13)$$

$$\Omega dU = \Phi \tau'_{p_k} \left\{ \sum_{j \neq k, j \in T} (\tau_j - \tau_k) G_{p_j} \right\} d\beta \quad (14)$$

Proposition 2:

(2.a) Assume that there is no informal segment in the economy and that commodity $k$ bears the highest production subsidy and is a pair-wise substitute of all other commodities in production. Then a selective reform of VAT and import tariff on commodity $k$ that keeps consumer price $q^f_k$ unchanged both enhances welfare and increases government revenue.

(2.b) In an economy with an informal sector, assume that commodity $k$ bears the highest production subsidy among the subset of formal commodities and is a pair-wise substitute in production of all other commodities. Then the implementation of a consumer price-neutral selective reform of consumption tax and import tariff (export subsidy) reduces both revenue and welfare if the following holds:

(2.b.i) the tariff rate (or export subsidy) on commodity $k$ is lower than a threshold;
(2.b.ii) the domestic production of commodity $k$ is smaller than a threshold.

Proof

Proof of (2.a)

In the absence of an informal economy, the necessary and sufficient condition for a welfare improvement from equation (14) is as follows:

$$\Omega dU > 0 \iff \left\{ \sum_{j \neq k} (\tau_j - \tau_k) G_{p_j} \right\} > 0 \quad (15)$$
The last inequality holds because commodity $k$ is a substitute of all other commodities in production implying $G_{pkpj} < 0$, $\forall j \neq k$, and $k$ also bears the highest production subsidy implying $(\tau_j - \tau_k) < 0$, $\forall j \neq k$. As $G_{pk}^f \tau_k^f > 0$, from equation (13) it follows that welfare increase is an over-sufficient condition for revenue increase from such a consumer price-neutral selective reform. Q.E.D.

The intuition for the above result is as follows. Since all other prices in the economy remain unchanged except for the decrease in the producer price of commodity $k$, the result critically depends on the induced resource reallocation effects. As $k$ is a pair-wise substitute of all other commodities in production, a lower producer price of $k$ reallocates resources to the rest of the economy. But $k$ enjoys the highest production subsidy, so a contraction of its production with a concomitant expansion of the production of all other commodities (with lower subsidies) reduces government’s expenditure on subsidies which increases revenue. This also reduces production inefficiencies in the economy.\(^{16}\)

Proof of (2.b)

When the economy consists of both formal and informal segments, it is easy to check from equation (13) that the necessary and sufficient condition for a reduction in revenue can be written as follows:

$$
\tau_k^f < \tilde{\tau}_k^f \equiv \frac{-\left(G_{pk}^f + \sum_{j \neq k} \tau_j G_{pkpj}^f\right)}{\left(G_{pk}^f - \sum_{j \neq k} G_{pkpj}^f\right)}
$$

(16)

Now the critical threshold $\tilde{\tau}_k^f > 0$ if the domestic production of commodity $k$ is small enough to satisfy the following inequality:

$$
G_{pk}^f < \tilde{G}_{pk}^f \equiv -\sum_{j \neq k} \tau_j G_{pkpj} > 0
$$

(17)

The last inequality above follows from the assumptions that there are no import subsidy or export tax implying that $\tau_j > 0$, $\forall j \in T$ and that $k$ is a pair-wise substitute of all other commodities in production, i.e., $G_{pkpj} < 0$, $\forall j \neq k$. It is obvious that a reduction in revenue is also sufficient for a decrease in consumer welfare. Q.E.D.

Observe that the assumptions of no export tax or import subsidy and pair-wise substitutability of $k$ with respect to all other commodities are over-sufficient for the above result. As long as the pattern of substitutability or complementarity in production and the initial structure of trade taxes are such that $\sum_{j \neq k} G_{pkpj}^f < 0$ and $\tilde{G}_{pk}^f > 0$, one can find a small enough value of domestic production that ensures the welfare- and revenue-reducing outcome emphasized in proposition (2.b).

\(^{16}\)This last statement depends on the assumption that there are no valid reasons (like learning externalities) to provide production subsidy to activity $k$. 
It is interesting to note an implication of the above result. Even if the consumer price neutral reform of trade tax and VAT on commodity $k$ enhances welfare and increases revenue given a high enough initial production subsidy, a policy reform that continually reduces the production subsidy will eventually drive down the subsidy below the critical threshold. So one can not rely on this strategy to eliminate the production subsidy completely. Also observe that as the tariff (export subsidy) is reduced, the domestic production of $k$ contracts as well, thus making it more likely that the condition (2.b.ii) will also be satisfied.

Section 3: Intermediate Inputs and Consumer Price-Neutral Tax Reform

The analysis of consumer price neutral tax reform presented so far implicitly assumes that the firms do not pay any VAT on the intermediate inputs. One of the important advantages of VAT is precisely the fact that it allows the producers to claim rebates for the taxes paid on the intermediate inputs and thus helps preserve production efficiency. In the presence of an informal sector, this production efficiency is, however, not preserved, as the informal sector firms can not claim rebates on their intermediate inputs purchase given that they do not file a VAT return. This also implies that there is an additional source of government revenue in this case; the taxes on the intermediate inputs not claimed by the informal sector. One might argue that, by ignoring this additional revenue from VAT, the model used in the previous sections is likely to overstate the case for a revenue and welfare reducing outcome when the consumer price neutral tax reform is implemented in an economy with an informal sector. In what follows we incorporate a VAT rebate system and derive sufficient conditions for welfare and revenue reducing outcome from a consumer price neutral tax reform. The results show that the conclusions reached earlier are, if anything, strengthened in an extended model with VAT rebates on the intermediate inputs for the formal firms.

We keep the model set-up as close as possible to the basic model described in section 1. In the model of section 1, the production side of the economy is represented by an aggregate revenue function (GNP function) which presumes production efficiency. However, as noted above, in the presence of intermediate goods under a VAT rebate system, production efficiency fails, as the firms in the formal and informal sector face different prices for the same intermediate inputs covered by VAT.\(^{17}\) The standard way of modeling the production side of the economy when producer prices are sector specific is to have two separate revenue functions for formal and informal sectors. This, however, complicates the algebra substantially as one needs to keep track of sectoral allocation of fixed factors $L$. We adopt a much simpler approach by redefining the commodity space for the production side of the economy that makes it possible to rely on an aggregate revenue function and

\(^{17}\)Similar production inefficiency can result from the duty drawback system where a set of privileged firms can claim duty drawback on the tariff on intermediate imports, as is the case in the widely used export promotion policies in developing countries.
thus allows us to ignore the inter-sectoral allocation of the fixed factors. The trick is to treat the same intermediate good as two different commodities in the aggregate revenue function depending on the location of the firm (formal vs. informal) producing or using the intermediate good. Denoting the subset of commodities used as intermediate inputs as $r$, for any intermediate input $k \in r, k \in m$, we have two producer prices in the private revenue (GNP) function

$$
\begin{align*}
p_{rf}^r &= 1 + \tau_k^r \\
p_{rs}^r &= 1 + \tau_k^r + v_k^r
\end{align*}
$$

where $p_{rf}^r$ is the price faced by the formal firms under the assumption that there is no duty drawback on tariff while the price faced by the informal firms is $p_{rs}^r$ that reflects both the tariff and the VAT. With duty drawback on tariff, the price faced by the formal firms is the world price, i.e., $p_{rf}^r = 1$. Note that an implicit assumption in this formulation is that there is no inter-sectoral trade in intermediate goods; in particular, the formal firms are assumed not to be able to resell the imported intermediates to the informal firms at a higher price (VAT inclusive price).\(^{18}\) If the formal firms can freely import for the purpose of reselling to the informal sector, then in equilibrium, the informal firms will also face the same price as the formal firms and there will be no government revenue from VAT on intermediate goods.\(^{19}\) We concentrate on the case where both formal and informal sector firms are net buyers of the intermediate inputs and there is no inter-sectoral arbitrage in intermediate inputs. We do not preclude the final consumption of these intermediate inputs, with the ‘pure intermediate inputs’ as a special case corresponding to zero final consumption. Our formulation also subsumes the case of ‘pure imported intermediate inputs’ \(\text{\`a la} \) Lopez and Panagariya (1992) where there is no domestic production (or final consumption) of the imported intermediates. The subsets of commodities $f$ and $s$ are now interpreted as consisting of only the final consumption goods produced in the formal and informal sectors respectively.\(^{20}\) As in the basic model, we ignore the non-tradables, and each commodity can be either an importable ($m$) or an exportable ($x$).

\[^{18}\text{Note that when a formal firm sells intermediate inputs to an informal firm, the latter does not have any incentive to insist on the VAT receipt, as, by definition, it does not file for VAT return. In such a transaction, it is highly likely that the transacting parties will share the ‘VAT revenue’ between themselves, thus adversely affecting government revenue.}\]

\[^{19}\text{If we allow intersectoral trade but the volume of intermediate imports by the formal firms is regulated, then the formal firms essentially face two prices for each intermediate good, one as the user of the input (1 + \tau_k) and another as a seller to the informal sector (1 + \tau_k + v_k). The model becomes more involved in this case.}\]

\[^{20}\text{Note that these final consumption goods are produced either by formal or informal firms, but not both. In contrast, the intermediate inputs are assumed to be produced both by formal and informal sector firms. The case where any given intermediate input is produced either by formal or informal firms but used by both types of firms can easily be handled in our model.}\]
side of the economy remains essentially unchanged, with the understanding that when
used for final consumption the location of the production of a commodity is immaterial,
and we can represent the expenditure function as \(E(q_0, q^f, q^s, q^r, U)\).

With the addition of the VAT on the intermediates unclaimed by the informal firms,
the total government revenue from taxes can be written as below:

\[
R(\tau, v^f, v^r) \equiv \tau^f (E_{q^f} - G_{p^f}) + \tau^s (E_{q^s} - G_{p^s}) + v^f E_{q^f} \\
+ v^r (E_{q^r} - G_{p^r}) + \tau^r \left( E_{q^r} - G_{p^r} - G_{p^r} \right)
\]

(18)

where \(v^r (-G_{p^r})\) is the revenue generated from the unclaimed VAT rebates from the
informal sector firms. Also, as in section (2) above, for simplicity, it is assumed that there
are no production taxes, i.e., \(t = 0\).

(3.1) Consumer price Neutral Radial Reform

Again we look at the revenue and welfare implications of a reform that keeps
the consumer price vector undisturbed. Since the producer price faced by the informal sector
firms is equal to the consumer price (pay both VAT and tariff), it also implies that such
a reform leaves the price of intermediate goods faced by informal firms unchanged. The
price of intermediate goods faced by the formal sector firms are, however, affected by the
reform in the absence of a duty drawback system.

Proposition 3

In an economy consisting of both formal and informal sectors, assume that all formal
and informal commodities are pair-wise substitutes in production and there are no export
tax or import subsidy. A consumer price-neutral radial uniform reform of VAT and trade
taxes in such an economy reduces both welfare and revenue if the following conditions
hold:

(i) the production substitution effects within the formal sector are low and that between
formal and informal sectors are high enough,

(ii) the size of the formal sector (final goods and intermediate inputs) is lower than a
threshold.

Proof:

We analyze a differential policy reform of the following form: \(dv^f = v^f d\alpha\), \(d\tau^f = \tau^f d\beta\), \(dv^r = v^r d\alpha\), \(d\tau^r = -\tau^r d\beta\), and \(dv^f + d\tau^f = 0, dv^r + d\tau^r = 0\). We now take total
differential of equation (18), keeping in mind that a change in tariff on intermediate good
\(k\) affects both the formal and informal sector prices \(p^f_k\) and \(p^s_k\) while a change in VAT
affects only the informal sector price due to the VAT rebate system.
The revenue effects of the reform can be written as following (for details, please see the appendix):

\[dR(\cdot) = \{\Psi_1 + \Psi_2\} d\beta \tag{19}\]

where

\[\Psi_1 = \tau f p f G p f p f \tau f + G_{p f}^r \tau^r + \tau s^r G p s p f \tau f + \tau r^r \left( G_{p f}^r p f + G_{p f}^s p f \right) \tau f + v r^r G_{p f}^s p f \tau f\]

\[\Psi_2 = \tau f p f p f \tau^r + \tau s^r G p s p f \tau^r + G_{p f}^r \tau^r + \tau r^r \left( G_{p f}^r p f + G_{p f}^s p f \right) \tau^r + v r^r G_{p f}^s p f \tau^r\]

The first term \((\Psi_1)\) in equation (19) represents the revenue effect of the reform due to the production substitutions following the reduction in the producer prices in the formal consumer goods sector as a result of the reduction in the import tariff or export subsidy, except for the term \(G_{p f}^r \tau f\) which shows the direct revenue effect of the reduction in import tariffs and export subsidies. The first term in \(\Psi_1\) is positive \((\tau f p f G p f p f \tau f > 0)\) because of the convexity of the GNP function, and \(G_{p f}^r \tau^r > 0\) given the assumption that all commodities are domestically produced and there are no import subsidy or export tax. However, all other terms in \(\Psi_1\) are negative under the assumption of universal substitutability of formal final goods and formal intermediate goods in production. So (i) if the cross substitution effects in production are strong enough, \((ii)\) the production substitutability within the formal consumption goods sub-sector is not strong, and \((iii)\) the size of the formal consumption goods production sector is small, then \(\Psi_1 < 0\). Analogously, an inspection of the other term \(\Psi_2\) shows that it is negative if \((i)\) the cross substitutability effects between formal intermediate goods sector and all other sectors are high enough in production, \((ii)\) the production substitution effects within the formal intermediate goods sub-sector is not strong, and \((iii)\) the size of the formal intermediate goods production is small. So under these conditions, a consumer price neutral reform will reduce revenue and thus worsen welfare (revenue reduction is sufficient for welfare worsening).

Again, the intuition for the above result is straightforward. Since the producer prices of the informal sector intermediate goods are not affected by the reform, the only new element is the production substitutions caused by the change in producer prices for intermediate goods faced in the formal sector. This effect is represented by the term \(\Psi_2\) above. A reduction in the tariff (or export subsidy) on the formal sector intermediate goods reduces their relative profitability, thus causing a reallocation of resources to all other sectors, including informal sector production of intermediate goods. As the production of competing sectors increase, their imports suffer and the tariff revenue also goes down. Also, the net demand for intermediate inputs by the informal sector is reduced resulting in a reduction in the unclaimed VAT rebates. If the cross substitutability effects are strong enough, these negative effects can dominate the positive effect on revenue due to a lower subsidy to the intermediate inputs used by the formal sector firms (given by the term \(G_{p f}^r \tau^r\)), and also a higher import volume and a higher tariff revenue because of a contraction in formal sector production of intermediate goods (given by the term \(\tau^r G_{p f}^r \tau^r\)).
In the above discussion, it is assumed that all of the firms pay import duty on the imported intermediate goods. This might, however, not be the most empirically relevant assumption, as the formal sector firms might have access to duty drawback, especially when they are exporters. This implies that there are three different groups of firms in the economy, the formal sector exporters facing the world price, the formal sector non-exporters facing \((1 + \tau_k)\) and the informal firms facing \((1 + \tau_k + v_k)\). In this case, the consumer price neutral reform of tariff and VAT do not have any effect on the producer price faced by the firms with access to duty drawback. This implies that the production substitution effects are due to the changes in prices of only those firms who do not have access to duty drawback. In the special case when all formal sector firms have access to duty drawback, both consumer and producer price vectors remain unchanged after the reform and no revenue can be raised. When most of the intermediate goods are “pure intermediate inputs” with no domestic production to begin with, the production substitution effects critical to the above discussion may not be forthcoming, especially when the producer prices are reduced as a result of the reform.

(3.2) Selective Reform of Import tariff and VAT

We now turn to the case where the tariff and VAT changes affect only a single commodity. We first discuss the case where the commodity \(k \in m\) under the consumer price neutral tax reform is not a pure imported intermediate input, i.e., \(k \notin r\). The other case where the reform focuses on a pure intermediate input is interesting only when there is no duty drawback on tariffs for the formal sector firms. In the presence of duty drawback for the formal sector firms, such a selective reform of intermediate inputs turns out to be rather uninteresting as it leaves the consumer and producer price vectors unchanged.

Consumer price neutral reform of an imported final consumption good

Proposition 4

(4.a) In an economy with both formal and informal sectors and a VAT rebate system for intermediate inputs, assume that commodity \(k \in f\) bears the highest production subsidy among all formal commodities and is a pair-wise substitute of all other commodities in production and that there are no import subsidy or export tax. A consumer price neutral reform of VAT and tariff on commodity \(k\) reduces welfare under the following conditions:

(4.a.i) the tariff rate (or export subsidy) on commodity \(k\) is lower than a threshold;
(4.a.ii) the domestic production of commodity \(k\) is smaller than a threshold.

(4.b) In the economy considered in proposition (4.a) above a consumer price neutral selective reform of tariff and VAT is more likely to be revenue reducing and welfare worsening when there is a VAT rebate system for the intermediate inputs, but there is no duty drawback system for the tariff on the intermediate inputs.

Proof:
With $k \notin r$, the government revenue effects of a reform of the form $dv_k = v_k d\alpha$, $d\tau_k^f = -\tau_k^f d\beta$, $dv_k + d\tau_k^f = 0$, $d\alpha, d\beta > 0$ is given by the following (for details see the appendix):

$$dR(\cdot) = (\Pi_1 + \Pi_2) d\beta$$

(20)

where

$$\Pi_1 = G_{p_k^f}^r \tau_k^f + \tau_k^f G_{p_k^f}^r \tau_k^f + \tau_k^s G_{p_k^s}^r \tau_k^f$$

$$\Pi_2 = \tau_k^r \left( G_{p_k^r}^r + G_{p_k^r}^s \right) \tau_k^f + \tau_k^s \left( G_{p_k^s}^r + G_{p_k^s}^s \right) \tau_k^f$$

Note that the revenue effects of the reform differs algebraically from the case where there is no taxes on the intermediate inputs by the term $\Pi_2$. Under the assumptions that there are no import subsidy or export tax, we have $\Pi_2 < 0$ if commodity $k$ is a substitute of all intermediate goods in production. This implies that, taking into account of the tariff on intermediate inputs and of the unclaimed VAT by the informal sector firms, in fact, strengthens our conclusions derived earlier in proposition (2.b) above, and it is more likely to have a revenue reducing and welfare worsening outcome from a consumer price neutral selective tax reform. The intuition behind the result is as follows. First consider the implications of the reform for net tariff revenue from intermediate inputs represented by the first term in $\Pi_2$, i.e., the sign of $\tau_k^r \left( G_{p_k^r}^r + G_{p_k^r}^s \right) \tau_k^f$. A reduction in tariff on commodity $k$ reduces its producer price $(p_k^f \downarrow)$ and thus reallocates resources to the production of all other commodities including the intermediate inputs. As the domestic production of the intermediate inputs expands, their net trade volume decreases (increases) in case of importables (exportables). This implies a lower tariff revenue on imported intermediates and a higher expenditure on subsidy to exported intermediates, both reducing government revenue.

The important thing here is that the first term in $\Pi_2$ is negative under the assumption of substitutability. Now we turn to the second term in $\Pi_2$, i.e., $v_r^r G_{p_k^r}^s \tau_k^f$ which represents the changes in the unclaimed VAT rebates on intermediate goods following the selective reform. Consider the effects of the selective reform on the unclaimed VAT rebates. As resources are reallocated to all other sectors including informal sector production of the intermediate goods, the net demand for the intermediates by the informal sector as a whole goes down, thus reducing the VAT revenue from unclaimed rebates. So the second term in $\Pi_2$ is also negative under the assumption of substitutability in production. As we have seen before, $\Pi_1 < 0$ under the conditions of proposition (4.a) (see the proof of proposition (2.a)). This completes the proof of part (4.a).

21 We again emphasize that the requirement that there is no import subsidy or export tax is oversufficient for the results in this paper. Both import subsidy and export tax can be allowed so long as they are not too high and affect only a relatively small subset of commodities. Also, the assumption seems broadly valid for a typical developing country at the beginning of 21st century.
The second part of the proposition (i.e., 4.b) follows from the observation that the cross production substitution effect is only partially operative in case of duty drawback by the formal firms implying that the change in revenue from tariffs on intermediate goods is now given by the production substitution effects only in the informal intermediate inputs sub-sector,

$$\tau^r \left( G_{p^r p^f k} \right) \tau^f_k.$$

Q.E.D.

Consumer Price Neutral Reform of an Imported Intermediate

We consider the case when there is no duty drawback for tariff on the intermediate input under price neutral reform. So we have \( k \in \mathbb{R} \), and \( \tau^r_k > 0 \) for both formal and informal sector firms. We also assume strictly positive final consumption of the intermediate goods.

When the tariff on intermediate input \( k \) is reduced marginally and VAT on it is raised to keep the consumer price unchanged, the revenue effects of such a reform is as follows (for details, see the appendix):

$$dR(.) = \left\{ \Lambda_1 + \tau^r \left( G_{p^r f p^r_k f} + G_{p^r f_k p^r_k} \right) \right\} \tau^r_k d\beta$$

$$= \left\{ \Lambda_1 + G_{p^r f_k} + \sum_{j \in \mathbb{R}, j \neq k} \tau^r_j G_{p^r f_j p^r_k} + \tau^r_k G_{p^r f_k p^r_k} \right\} \tau^r_k d\beta \quad (21)$$

where

$$\Lambda_1 = \tau^r G_{p^r f_k p^r_k} + \tau_s G_{p^r s p^r_k} + \tau^r G_{p^r s p^r_k} + v^r G_{p^r s p^r_k}.$$

Note that the first three terms within the bracket on the right hand side of equation (21) are negative under the assumptions that (i) intermediate good \( k \) is a substitute in production of all other commodities, (ii) the formal sector is a net user of intermediate input \( k \), and (iii) there are no import subsidy or export tax. As emphasized before, this last requirement is not critical and can be relaxed up to a point. The last term in equation (21), however, is positive and the net revenue effect depends on the relative strength of own and cross substitution effects in production and on the initial structure of the tariff. So the revenue effect of the consumer price neutral reform will be negative as long as the own supply response for commodity \( k \), \( \left( G_{p^r f_k p^r_k} \right) \), is not too high. Also note that it is more likely to have a revenue reducing outcome if (i) the net demand for the intermediate good by the formal sector is larger, (ii) the cross substitution effects between commodity \( k \) and others in production are stronger. We collect these conclusions in the following proposition.
Proposition 5

(5.1) In an economy with both formal and informal sectors, assume that commodity $k$ is an imported intermediate input which can be produced and used by both formal and informal firms and also can be used for final consumption. Also assume that there is a VAT rebate system for intermediate inputs but no duty drawback on tariff on intermediate goods and there are no import subsidy or export tax. A consumer price neutral reform of tariff and VAT on commodity $k$ reduces government revenue in this economy if (i) commodity $k$ is a pair-wise substitute of all other goods, and (ii) the own supply response of commodity $k$ in the formal sector $\left(G_{p_k^f}^{prf} p_k^f\right)$ is less than a threshold.

(5.2) In the economy described in proposition (5.1), a consumer price neutral reform of VAT and tariff on commodity $k$ is, ceteris paribus, more likely to reduce government revenue if (i) the net demand for intermediate input $k$ by the formal sector is larger, and (ii) the cross substitution effects in production between $k$ and all other goods are stronger.

An interesting special case of proposition (5.1) above obtains when the initial tariff on intermediate good $k$ is less than or equal to its world price (which is normalized to one), i.e., $\tau_k^f \leq 1$. The following Corollary gives a formal statement.

Corollary 1:
In an economy with both formal and informal sectors, assume that commodity $k$ is an imported intermediate input which can be produced and used by both formal and informal firms and also can be used for final consumption. Also assume that there is a VAT rebate system for intermediate inputs but no duty drawback on tariff on intermediate goods and there are no import subsidy or export tax. A consumer price neutral reform of tariff and VAT on commodity $k$ reduces government revenue in this economy if (i) the initial tariff on $k$ is less than or equal to its world price $\left(\tau_k^f \leq 1\right)$ (ii) commodity $k$ is a pair-wise substitute of all other goods, and (ii) the own supply response of commodity $k$ in the formal sector is less than the net demand for it by the formal sector, i.e., $\left(G_{p_k^f}^{prf} p_k^f < -G_{p_k^f}^{p}\right)$.

Proof:
Follows from equation (21).

Section 4: Producer price-Neutral Reform of Export tax (Import Subsidy) and Production Tax

Even in the absence of an informal economy, the appeal of a consumer price-neutral reform is severely compromised as it is not suitable for reforming export tax or import subsidy. A reduction in export tax increases the consumer price and thus to keep the consumer price vector unchanged a reduction in VAT is required. Since both the trade
tax and VAT need to be reduced in such a reform, the revenue implication is unambiguously negative, if the taxes are not on the wrong side of the Laffer curve to begin with. As mentioned before, there is an alternative strategy which is applicable for reforming export tax or consumption subsidy. It keeps the producer price unchanged by increasing production tax to offset the effects of a reduction in export tax or import subsidy. In the context of selective reform of export taxes, Emran (2000) has shown that such a producer price neutral reform can increase both revenue and welfare if there is no informal segment in the economy. In the following we consider both a radial and selective reform of export taxes and import subsidies that keep producer price unchanged with offsetting increases in production taxes. The main results mirror the results discussed above: the existence of a strategy of double dividend (or “win-win”) reform critically depends on the assumption that there is no informal segment in the economy. The price relations are given as follows:

\[
q^f = 1 - \tau^f \quad p^f = 1 - \tau^f - t
\]
\[
q^s = 1 - \tau^s = p^s \quad p_0 = q_0 = 1
\]

For simplicity, we assume that there are no consumption taxes (VAT). The government revenue function now becomes:

\[
R(\tau, t) \equiv \tau^f (G_{pf} - E_{qf}) + \tau^s (G_{ps} - E_{qs}) + t^f G_{pf}
\]  

(22)

Producer Price-Neutral Radial Uniform Reform

We first look at the radial uniform reform that keeps the producer price constant. We consider the following differential policy reform:

\[
dt = td\alpha, \quad d\tau = -\tau d\beta, \quad dt + d\tau = 0 \text{ where } d\beta > 0, \text{ and } d\alpha > 0 \text{ are scalars.}
\]

Proposition 6

(6.a) In an economy without any informal segment, assume that there are no export subsidy or import tax. A radial uniform reform in this economy that reduces the trade taxes and offsets the effects on the producer price vector by increasing production tax increases both revenue and welfare.

(6.b) When the above economy consists of both formal and informal sectors, a producer price neutral reform reduces both revenue and welfare if the following conditions hold:

(6.b.1) the consumption substitutability among the formal commodities are low and that between formal and informal commodities are high enough;
(6.b.2) the domestic consumption of formal commodities are smaller than a threshold.

Proof

Proof of (6.a)
In the absence of an informal economy, the revenue and welfare effects of such a producer price neutral reform are given as follows:

\[
dR(t, \tau) = \left[ E_q' \tau - \tau' E_{qq} \tau \right] d\beta > 0 \quad (23) \\
\Omega dU = \Phi \left( -\tau' E_{qq} \right) d\beta > 0 \quad (24)
\]

The last inequality in (23) and (24) follows because the quadratic form \( \tau' E_{qq} \tau < 0 \) given the strict concavity of the expenditure function. Q.E.D.

**Proof of (6.b)**

In the presence of an informal economy, the revenue and welfare effects of a producer price-neutral reform are given as below:

\[
dR(.) = \left( E_q' \tau^f - \tau^f E_{q^f q^f} \tau^f - \tau^s' E_{q^s q^f} \tau^f \right) d\beta \quad (25) \\
\Omega dU = -\Phi \left( \tau^f E_{q^f q^f} \tau^f + \tau^s' E_{q^s q^f} \tau^f \right) d\beta \quad (26)
\]

Now the right hand side of equation (25) is negative if the following holds:

\[
E_{q^f q^f} \tau^f < \tau^f E_{q^f q^f} \tau^f + \tau^s' E_{q^s q^f} \tau^f \quad (27)
\]

Note that the first term in the right hand side of (27) \( \tau^f E_{q^f q^f} \tau^f < 0 \), because \( E_{qq} \) is negative definite due to strict concavity of the expenditure function, and all the principal sub-matrices of a negative definite matrix are themselves negative definite which implies, in particular, that \( E_{q^f q^f} \) is negative definite. The second term is, however, positive under the assumption that formal and informal commodities are substitutes in consumption, given that there are no export subsidy or import tax. So if the consumption substitutability among the subset of formal commodities are low enough and the substitutability between formal and informal commodities high enough, the right hand side of inequality (27) is positive. As a result, given the tax structure, there always exists a small enough \( E_{q^f} \) such that the inequality (27) is satisfied. From equation (26) it is clear that a negative revenue outcome is sufficient for a welfare loss. Q.E.D.

The assumption that there are no import tax or export subsidy is used in proposition (6) to make clear the symmetry between the consumer price neutral reform of tariff and VAT discussed earlier and the producer price neutral reform of export tax and production tax as discussed in this section. It is obvious that this assumption can be relaxed up to a point. However, from an empirical point of view, the assumption that there are no import subsidy or export tax seems to do less violence to the reality in developing countries than the assumption that there are no import tariff or export subsidy. Most of the developing countries still rely substantially on import tariff as a source of revenue. The importance of export tax has, however, gone down significantly over the last couple
of decades. The most representative tax structure seems to be one where there is almost no import subsidy, limited export tax but substantial import tariff and a moderate use of export subsidy. But observe that even with such a tax structure, the results of proposition (6.b) are likely to remain valid. With such a tax structure, the vector of trade taxes $\tau$ contains both positive and negative elements. But if the tax structure is dominated by import tax and export subsidy, then most of the elements of the vector $\tau$ will be of same sign, and as a result the bilinear form $\tau^s E_{q,q'}^f \tau^f$ is likely to be positive under the assumption of substitutability between formal and informal final goods in production. To see it in simplest form, consider the case where only one commodity $k$ bears export tax and all other commodities are under either import tariff or export subsidy, and also assume that the consumption substitution effects do not vary across commodities. In this case, given the way price relations are defined in this section, we have $\tau_j < 0 \forall j \neq k$, and it is obvious that $\tau^s E_{q,q'}^f \tau^f > 0$ if the following holds:

$$\sum_{l \in s} \tau_l^f \tau_l < \sum_{j \in f, j \neq k} \sum_{l \in s} \tau_j \tau_l$$  \hspace{1cm} (28)

Moreover, with tariff on most of the consumer goods, the consumers as a group will be taxed implying $E_{q,q'}^f \tau^f < 0$ which makes it easier to satisfy inequality (27) and thus makes the revenue reducing outcome more likely.

**Producer Price-Neutral Selective Reform**

Here we analyze the revenue and welfare implications of the following reform:

$$dt_k = t_k d\alpha, \quad d\tau_k^f = -\tau_k^f d\beta, \quad dt_k + d\tau_k^f = 0, \quad d\alpha, d\beta > 0$$  \hspace{1cm} (29)

**Proposition 7:**

(7.a) In an economy with no informal sector assume that the consumption of commodity $k$ enjoys the highest subsidy and $k$ is a substitute of all other commodities. A marginal reform in this economy that reduces the trade tax on commodity $k$, $\tau_k^f$ and increases the production tax $t_k$ to keep the producer price $p_k$ unchanged both increases revenue and enhances welfare.

(7.b) In an economy with both formal and informal sectors, assume that the consumption of $k$ enjoys the highest subsidy among all formal commodities and $k$ is a substitute in consumption of all other commodities. A producer price-neutral reform of taxes on $k$ reduces both welfare and revenue under the following conditions:

(7.b.1) the consumption subsidy on $k$ is lower than a threshold;
(7.b.2) the domestic consumption of commodity $k$ is lower than a threshold.

**Proof**

(7.a)
It is straightforward to check that the revenue and welfare effects of the reform can be written as follows:

\[ dR(.) = \left( E_{q_k} \tau_k - \tau' E_{qq_k} \tau_k \right) d\beta \]  
(30)

\[ \Omega dU = \Phi \tau' E_{qq_k} \tau_k d\beta \]  
(31)

Using the homogeneity property of the expenditure function, we have (by Euler theorem)

\[ E_{q_k q_k} = -\frac{1}{q_k} \sum_{j \neq k} q_j E_{q_j q_j} \]. Substituting for \( E_{q_k q_k} \), we can rewrite \( \tau' E_{qq_k} \) as follows:

\[ \tau' E_{qq_k} = \frac{1}{q_k} \left[ \sum_{j \neq k} (\tau_j - \tau_k) E_{q_j q_j} \right] \]  
(32)

Now in the absence of an informal economy, we can choose commodity \( k \) as the one with the highest consumption subsidy implying \( (\tau_j - \tau_k) < 0, \forall j \neq k \). So \( \tau' E_{qq_k} < 0 \) under the assumption that \( k \) is a substitute of all other commodities in consumption. This guarantees a positive welfare outcome from equation (31). This also implies that the revenue increases, given that \( E_{q_k} > 0 \). Q.E.D.

(7.b)

Since the choice of commodity \( k \) for the reform is now restricted, we can not guarantee that commodity \( k \) will have the highest consumption subsidy among all commodities. Now it is easy to check that \( dR(.) < 0 \) if the following holds:

\[ \tau_k^f \left\{ E_{q_k^f} - \sum_{j \neq k} E_{q_j^f q_j} \right\} > \left\{ E_{q_k^f} - \sum_{j \neq k} \tau_j E_{q_j^f q_j} \right\} \]  
(33)

It is obvious that if \( E_{q_k^f} < \tilde{E}_{q_k^f} \equiv \sum_{j \neq k} \tau_j E_{q_j^f q_j} \), the right hand side of inequality (32) is negative. But \( E_{q_k^f} < \tilde{E}_{q_k^f} \) also implies that \( E_{q_k^f} < \sum_{j \neq k} E_{q_j^f q_j} \). This follows from observing that since all prices are positive and commodity \( k \) is assumed to be substitute of all other commodities in consumption, we have:

\[ \sum_{j \neq k} q_j E_{q_j^f q_j} \equiv \sum_{j \neq k} (1 - \tau_j) E_{q_j^f q_j} > 0 \Rightarrow \sum_{j \neq k} E_{q_j^f q_j} > \sum_{j \neq k} \tau_j E_{q_j^f q_j} \]  
(34)

So when the domestic consumption of commodity \( k \) is low enough, i.e., \( E_{q_k^f} < \tilde{E}_{q_k^f} \), both sides of the inequality (32) are negative and the revenue declines following the producer price-neutral reform if the consumption subsidy on \( k \) is low enough to satisfy the following:

\[ \tau_k^f < \tilde{\tau}_k^f \equiv \frac{\sum_{j \neq k} \tau_j E_{q_j^f q_j} - E_{q_k^f}}{\sum_{j \neq k} E_{q_j^f q_j} - E_{q_k^f}} \]  
(35)
The above result has important implications for attempts to reform export oriented industries. If the domestic consumption of an exportable commodity is miniscule and the export tax on it is not very high, a producer price neutral reform might actually yield a “double loss” rather than a “double dividend”.

Conclusions
In this paper, we consider the implications of a large informal and shadow economy for the consumer and producer price-neutral reform of trade taxes and domestic taxes (VAT and production taxes). The extant literature shows that, when there is no informal sector, such a reform can yield double dividend in the sense that it increases both revenue and welfare. It has been argued in the literature that these results provide a concrete justification for the tax and tariff reform policies pursued by a large number of developing countries under the policy conditionalities of IMF and the World Bank over the last two decades. We show that the conclusions in favor of a double dividend tax reform are fragile, and such a reform is as likely to reduce both welfare and revenue when the existence of a large informal and shadow economy in developing countries is taken into account. This strengthens the growing doubts about the wisdom of the standard tax and tariff reform policies implemented across a large number of developing countries that emphasizes the reduction and eventual elimination of trade taxes and shifts the burden of domestic revenue mobilization on to VAT.

Appendix

(1) Derivation of equation (19)
Taking total differential of equation (1), we get:

$$dR(\cdot) = \frac{\partial R(\cdot)}{\partial \tau^f} d\tau^f + \frac{\partial R(\cdot)}{\partial \tau^r} d\tau^r + \frac{\partial R(\cdot)}{\partial v^f} dv^f + \frac{\partial R(\cdot)}{\partial v^r} dv^r$$  \hspace{1cm} (36)

where

$$\frac{\partial R(\cdot)}{\partial \tau^f} = \left( E_{qf} - G_{pf} \right)' + \tau^f \left( E_{qf} - G_{pf} - G_{p^r p^f} \right) + \tau^f \left( E_{qf} - G_{p^r p^f} - G_{p^r p^s} \right) + v' \left( E_{qf} - G_{p^r p^f} \right)$$

$$+ \tau^r \left( E_{qf} - G_{p^r p^f} - G_{p^r p^s} \right) + v^r \left( E_{qf} - G_{p^r p^f} \right)$$ \hspace{1cm} (37)

$$\frac{\partial R(\cdot)}{\partial \tau^r} = \left( E_{qf} - G_{p^r f} \right)' + \tau^r \left( E_{qf} - G_{p^r f} - G_{p^r p^s} \right) + \tau^r \left( E_{qf} - G_{p^r f} - G_{p^r p^s} \right)$$

$$+ \left( E_{qf} - G_{p^r f} - G_{p^r p^s} \right)' + \tau^r \left( E_{qf} - G_{p^r f} - G_{p^r p^s} - G_{p^r p^r} - G_{p^r p^s} \right)$$

$$+ v' \left( E_{qf} - G_{p^r p^r} - G_{p^r p^s} \right) + v^r \left( E_{qf} \right)'$$ \hspace{1cm} (38)
\[
\frac{\partial R(.)}{\partial v^f} = \tau f' E_{q^f q^f} + \tau^v E_{q^v q^f} + \tau^r E_{q^r q^f} + v^f E_{q^f q^f} + v^r E_{q^r q^f} + E_{q^f q^f}
\]

\[
\frac{\partial R(.)}{\partial v^r} = \tau f' \left( E_{q^f q^r} - G_{p^f p^r} \right) + \tau^v \left( E_{q^v q^r} - G_{p^v p^r} \right) + \tau^r \left( E_{q^r q^r} - G_{p^r p^r} \right) + v^f E_{q^f q^r}
\]

Using equation (18), we get:

Utilizing equations (37)-(40) in equation (36) and imposing the restriction that \(dv^f + d\tau^f = 0, dv^r + d\tau^r = 0\) yields equation (19) in the text.

(2) Derivation of equation (20)

Using equation (18), we get:

\[
dR(.) = \frac{\partial R(.)}{\partial \tau_k^f} d\tau_k^f + \frac{\partial R(.)}{\partial v_k^f} dv_k^f
\]

where

\[
\frac{\partial R(.)}{\partial \tau_k^f} = \left( E_{q^f q^f} - G_{p^f p^f} \right) + \tau f' \left( E_{q^f q^f} - G_{p^f p^f} \right) - \tau^v \left( E_{q^v q^f} - G_{p^v p^f} \right) + v^f E_{q^f q^f}
\]

\[
\frac{\partial R(.)}{\partial v_k^f} = E_{q^f q^f} + \left( \tau^f + v^f \right) E_{q^f q^f} + \tau^v E_{q^v q^f} + \tau^r E_{q^r q^f} + v^f E_{q^f q^f}
\]

Substituting for \(\frac{\partial R(.)}{\partial \tau_k^f}\) and \(\frac{\partial R(.)}{\partial v_k^f}\) in equation (41), and imposing the restriction \(d\tau_k^f + dv_k^f = 0\), we get equation (20) in the text.

(3) Derivation of Equation (21)

Using equation (18), we get:

\[
dR(.) = \frac{\partial R(.)}{\partial \tau_k^r} d\tau_k^r + \frac{\partial R(.)}{\partial v_k^r} dv_k^r
\]

where

\[
\frac{\partial R(.)}{\partial \tau_k^r} = \tau f' \left( E_{q^f q^r} - G_{p^f p^r} \right) + \tau^v \left( E_{q^v q^r} - G_{p^v p^r} \right) + \tau^r \left( E_{q^r q^r} - G_{p^r p^r} \right) + v^f E_{q^f q^r}
\]

\[
\frac{\partial R(.)}{\partial v_k^r} = \tau f' \left( E_{q^f q^r} - G_{p^f p^r} \right) + \tau^v \left( E_{q^v q^r} - G_{p^v p^r} \right) + \tau^r \left( E_{q^r q^r} - G_{p^r p^r} \right) + v^f E_{q^f q^r}
\]
Plugging back into equation (43) and imposing the restriction that \( d\tau_k^r + dv_k^r = 0 \), yields equation (21) in the text.

References