Currency Mismatch, Uncertainty and Debt Structure *

Matthieu Bussière\textsuperscript{a}, Marcel Fratzschер\textsuperscript{a} and Winfried Koeniger\textsuperscript{b,c}

April 21, 2004

Abstract

The importance of currency and maturity mismatches in the debt structure of emerging markets is an issue that can hardly be overemphasized. Yet, while many papers have explained how imbalances in the asset and liability structure of emerging markets can cause currency and financial crises, the factors that may have triggered such imbalances in the first place have received relatively little attention so far. The aim of the paper is to contribute to fill this gap, both at a theoretical and empirical level. We show that if (i) a significant share of the debt is denominated in foreign currency -creating a currency mismatch- and (ii) borrowing is constrained by solvency, then currency mismatch can create and exacerbate a maturity mismatch. In particular, higher economic or political uncertainty tightens solvency constraints and tilts the debt profile towards short-term debt. The combination of currency mismatch and maturity mismatch increases the volatility of output growth in the short-run. Taking the model implications to the data, we find substantial empirical support for the model’s predictions using data for 30 open emerging market economies.

Keywords: Maturity Mismatch, Currency Mismatch, Debt, Emerging Markets.
JEL Nos: F34, F36

\textsuperscript{a} European Central Bank; \textsuperscript{b} IZA, Bonn; \textsuperscript{c} Finance and Consumption in the EU, European University Institute.

* We thank Galina Hale and seminar participants at the ECB for helpful comments.
1 Introduction

The importance of currency and maturity mismatches in the debt structure of emerging markets is an issue that can hardly be overemphasized. For example, sizeable currency and maturity mismatches have been found to be associated with the Asian crisis (see, for instance, Chang and Velasco, 2000, and Rodrik and Velasco, 1999). Yet, while many papers have explained how imbalances in the asset and liability structure of emerging markets can cause currency and financial crises, the factors that may have triggered such imbalances in the first place have received relatively little attention so far. In particular, few papers have considered the possibility that causality may be running in the opposite direction: high anticipated exchange rate volatility may tilt the debt profile towards short-term debt, thereby increasing the vulnerability of the borrowing countries to financial crises.

The objective of this paper is to contribute to filling this gap. First, we provide a model which links currency mismatch and maturity mismatch in open emerging market economies. Whereas these two types of phenomena are typically treated separately in the literature (a notable exception is the empirical work in de la Torre and Schmukler, 2003), we show theoretically how maturity mismatch depends on market uncertainty, such as exchange rate uncertainty because of currency mismatch, and how maturity mismatch increases output volatility in the short run. Second, we provide empirical results that support the predictions of the model.

More specifically, this paper provides a simple model that links the exchange-rate uncertainty inherent in foreign-currency debt to solvency and the choice of debt maturity. We show that a currency mismatch can exacerbate a maturity mismatch and increase output volatility. In the model, forward-looking and impatient risk-neutral agents choose whether to consume or to invest, financing their investment with short- or long-term foreign debt. We assume that debt: (i) can only be obtained in the international capital market, (ii) is denominated in foreign currency and (iii) is constrained by solvency, which requires that agents can always repay. In particular, we neglect equity or foreign direct investment as sources of external finance and focus on foreign private debt. Foreign private debt is an important component of capital inflows in non-OECD countries accounting for about 35% of GDP in the 1990s (see Hale, 2003, Table 9).

Agents face a simple trade-off in their choice of debt maturity. Since (exchange-rate) uncertainty tightens solvency constraints relatively more for long-term debt, borrowers have an incentive to raise the share of short-term debt. However, short-term debt is risky and the investment project can be liquidated before the investment return materializes so that agents have a smaller collateral if they borrow short term. As a consequence, a larger share of short-term debt raises the share of investment projects at risk, the likelihood of a substantial short-term drop in aggregate output, and thus output volatility. In our model, liquidation of the collateral, volatile short-run growth and a larger fraction of short-term debt are the result of optimal choices of individual agents.

We provide empirical evidence that substantially supports the main predictions of the model using a sample of 30 emerging market economies. We use yearly data from the World Economic Outlook (WEO)
for macro-economic variables, the Bank of International Settlements (BIS) for debt variables, and a private agency (International Country Risk Guide) for various measures of economic and political risk. Results are as follows. We find that more uncertainty—in particular higher exchange rate uncertainty—lowers the level of both long-term and short-term debt as a ratio of GDP. Moreover, the data reveal that higher exchange rate volatility (as well as various indicators for political risks) is associated with a stronger maturity mismatch. Furthermore, countries with a larger maturity mismatch of foreign debt have more volatile output, confirming the hypothesis that economies with a higher share of short-term debt are more likely to suffer from stronger boom-bust cycles and financial crises.

It is important to note that we abstract by-and-large from asymmetric information and moral hazard which are important for credit markets, the banking and maturity structure in the real world (see, e.g., Diamond, 1991, Jeanne, 2000, and Tirole, 2003). Although we recognize that asymmetric information and moral hazard may be important, we show that such model ingredients are not necessary to explain the joint phenomena of currency depreciation and asset liquidation accompanied by high short-term debt ratios. Thus, market distortions such as moral hazard might not be the only reason behind high short-term debt, and the removal of such distortions may not suffice to tilt the debt profile towards safer, long-term debt. Indeed, the model shows that even in the absence of such market distortions, perfectly rational agents may choose a high share of short-term debt, which is optimal for both lenders and borrowers.

Our model is related to a number of papers on bank runs and international liquidity crises where banks perform a debt-maturity transformation function. Chang and Velasco (2000) apply the model structure of Diamond and Dybvig (1983) to an open economy in order to analyze the optimal choice of debt maturity and the possibility of self-fulfilling bank runs in which banks with external debt partly default (see also Rodrik and Velasco, 1999). The term structure of interest rates is endogenous as agents take the possibility of default into account ex ante. Short-term debt is beneficial because it is available at a relatively lower interest rate; yet, it is also costly because it exacerbates the vulnerability to bank runs. In the model, forward-looking investors need to be sufficiently risk averse to take on short-term debt since short-term investors have a priority over long-term investors in the liquidation procedure. By contrast, Rodrik and Velasco (1999) and Jeanne (2000) show that short-term debt can be beneficial for risk neutral agents if it acts as a disciplining device for the government. In our model instead short-term debt can be beneficial because it allows impatient consumers to anticipate consumption relatively more compared with long-term debt: solvency constraints can be less binding for short-term debt because exchange rate uncertainty increases over longer time horizons which lowers the amount foreigners are willing to lend long term, ceteris paribus. Such an effect is not present in the model of Rodrik and Velasco (1999) since they allow for default and abstract from discounting. Moreover, the amount of foreign debt obtainable by domestic investors is endogenously determined by the solvency constraints in our model whereas it is exogenous in Chang and Velasco (2000).

Allen and Gale (2000) analyze optimal risk sharing between domestic bank depositors and the international bond market. In their model bankruptcy and liquidation of loans can be beneficial if banks cannot meet their promised non-state contingent payments in the case of adverse aggregate shocks: bankruptcy implicitly makes interest payments state contingent. Hence, bankruptcy and liquidation can be optimal although they involve a deadweight cost given that assets are liquidated prematurely. In our paper solvency
guarantees full repayment by domestic borrowers to foreign lenders. Liquidation is optimal ex ante to the extent that the use of investment goods as collateral for potential repayment allows impatient agents to borrow. Moreover, exchange rate volatility can improve risk sharing in the model of Allen and Gale whereas such volatility decreases welfare in our model as solvency constraints become tighter.

Building on Tornell and Westermann (2002), Rancière et al. (2003) show within an endogenous two-sector growth model that contract enforceability problems and bailout guarantees can imply growth paths with currency mismatch, credit crunches and volatile growth. Real exchange rates vary endogenously in their model since non-tradables are used as inputs both in the non-tradable and tradable sector. Instead, our much simpler three-period setup allows us to analyze in a tractable way how currency mismatch affects maturity mismatch through exchange rate volatility. We focus on the effect of such mismatches on short-run output volatility whereas Rancière et al. analyze the optimality of currency mismatch and risky growth paths.

Few papers have empirically studied the determinants of debt maturity. Using a panel of 32 emerging market economies, Rodrik and Velasco (1999) find that short-term debt is associated with high per-capita income levels and M2/GDP ratio's. They also test whether short-term debt is correlated with trade openness and a corruption index but find no statistically significant relationship with these two variables. Independently from the present paper, Valev (2004) has analyzed in a recent contribution the maturity structure of loans made by US banks in 44 countries over the period 1982-1996. His results suggest that (i) economic volatility is associated with debt maturity in a non-linear relationship (only a very high level of economic volatility is associated with shorter debt maturity), whereas (ii) non-economic uncertainty, as proxied in particular by some of the ICRG variables we are also using, contributes to shorter debt maturity. One needs to underline that there are differences in the composition of the country sets used in Valev (2004) and in our paper: his sample includes 21 developed economies out of 44 countries, whereas our sample is a more homogeneous set of 30 emerging markets. These differences could explain why the results of the two studies, while broadly consistent with each other, are slightly different concerning the effect of economic volatility on short-term debt.

The remainder of the paper is structured as follows. In Section 2 we present a simple model to show how currency mismatch affects maturity mismatch and growth volatility. We empirically test the key predictions of the model in Section 3. We discuss policy implications and conclusions in Section 4.

2 A Model

We build a partial equilibrium model that allows us to highlight interactions between currency risk, solvency and maturity choice in a tractable way. Although it would be interesting to extend the model, e.g., by allowing for general equilibrium feedbacks on the exchange rate, the focus of the paper is on short-term effects for which such feedbacks are likely to be less important. Moreover, the data do not allow us to identify general equilibrium effects in the empirical estimations.

We assume that agents are risk-neutral since risk aversion would make the solution of the model much more cumbersome also because of precautionary motives. Agents live for three periods and their utility is
defined by the function

\[ U = c_1 + \beta E_1c_2 + \beta^2 E_1c_3, \]

where \( E_t \) is the mathematical expectation operator conditional on time \( t \), \( \beta \) is the agent’s discount factor and \( c_t \) denotes consumption in period \( t \). As utility is linear in consumption, the solvency constraints introduced below would never be binding unless we assume that agents are impatient. In this case the optimal consumption profile is trivial since agents anticipate consumption as much as they can in the present and utility depends on credit supply. We will see below that this simple model structure usefully isolates the decision about optimal maturity and its interaction with exchange rate uncertainty.

Agents are endowed with investment goods of value \( K \) which they can consume immediately in period 1. But agents also have access to a production technology that produces \( Y \) units of output in period 3 with \( K \) units of input in period 1. If agents invest, they can borrow in foreign currency to finance consumption during the time of the project.\(^2\) One motivation not to lend in local currency is the lack of credibility and reputation of governments in developing countries that have the incentive to implement policies to reduce domestic liabilities (see, e.g., Allen and Gale, 2000, or the literature on currency mismatch and original sin in Eichengreen et al., 2003).\(^3\)

Agents have the possibility to borrow in two alternative ways: they can either take on debt with a maturity of one or two periods. We call the former short-term debt and the latter long-term debt. If agents borrow long term, the project income and debt are due in the same period. If agents decide to take on short-term debt instead, they need to roll-over the debt in period 2 in order to continue the project. Should they be unable to roll-over the debt, the project is liquidated and lenders appropriate parts or all the investment goods of value \( K \). In this case agents do not earn the project returns \( Y \) in period 3. The timing of events is summarized in Figure 1.

2.1 Exchange-rate risk and solvency

Before analyzing the agent’s optimal choice of debt maturity, we need to show how solvency constraints for foreign debt depend on exchange rate uncertainty and debt maturity. Exchange-rate risk is often poorly hedged in developing countries, as for example in pre-crisis Asia (see, e.g., Tirole, 2002, p. 5, or Eichengreen, 2003, p. 270 n.). Moreover, exchange-rate risk is an aggregate risk and currency risk is not distributed independently across developing countries if there is the potential of contagion. Given that exchange rates follow a non-stationary stochastic process as specified below, the law of large numbers does not ensure that foreign risk-neutral lenders can eliminate all risk and break-even at every point in time if borrowers can default.

We assume that the world interest rate \( r \) is not state contingent and foreign lenders impose solvency constraints on developing countries so that the debt is repaid with certainty. Since foreign lenders do not

\(^2\)We could allow for domestic borrowing opportunities where agents’ borrowing is constrained. In this case there would be additional feedbacks from foreign borrowing opportunities on the domestic interest rate. We neglect such feedbacks for simplicity.

\(^3\)Caballero and Krishnamurty (2003) show instead that foreign-currency denominated debt might exceed the social optimum especially for less-developed countries where domestic borrowing constraints are tighter.
control whether domestic borrowers hedge their risk or not, the solvency constraint guarantees repayment for the highest possible depreciation of the exchange rate contained in the support of the distribution: lenders assume that exchange-rate uncertainty is fully unhedged and borne as risk. The assumption of solvency constraints excludes that risk-neutral lenders offer credit contracts in which interest rates depend on the exchange-rate realization and the implied bankruptcy risk. Indeed such state-contingent pricing might not be optimal if market failures like moral hazard prevent more credit supply. Since government policy can affect exchange rates, lenders will be reluctant to offer credit contracts which allow for bankruptcy if the probability of bankruptcy (due to exchange-rate depreciation) depends on government policy. Thus, for the same reason why lenders do not lend in local currency to emerging market economies (see above), they also are reluctant to lend more than what can be repaid with certainty.

 Developing countries adopt different exchange-rate regimes. In the model we focus on flexible exchange-rate regimes. Note, however, that although the mechanism in our model relies on exchange-rate uncertainty, this does not imply that it is irrelevant for countries that peg their exchange rate: exchange-rate regimes change over time so that exchange rates are still uncertain. We assume that the natural logarithm of the (real) exchange rate is a martingale. The choice of the stochastic process and the partial equilibrium perspective of exogenous stochastic exchange rates are justified empirically since the natural logarithm of flexible exchange rates has stochastic properties similar to a random walk especially for time horizons up to two years (see, e.g., Mussa, 1979, for nominal exchange rates and Stockman, 1987, for real exchange rates and the special issue on exchange rate models edited by Engel et al., 2003). We define the (real) exchange rate $X$ in terms of foreign units in period $t + 1$ as

$$X_{t+1} = \mu e_t X_t,$$
where $\mu$ is the deterministic drift and $\varepsilon_t$ is a random variable which we assume to be uniformly i.i.d. on
the interval $[1, a; 1 + a]$. This implies that the currency depreciates in the next period with probability
$1 + a \mid \mu^1 \varepsilon / 2$. Normalizing the exchange rate in period $1$, $X_1 = 1$, the exchange rate can depreciate at
most to $X = \mu(1 + a)$ in the second period. In order to guarantee solvency, foreign investors will consider
this maximum exchange rate, where the highest possible level of the exchange rate at future maturity time $m$
is
$$X^m = (\mu(1 + a))^m.$$
Thus, the market’s discount factor applied to collateral for foreign debt with maturity $m$ is
$$(X R)^m,$$
where $R \cdot 1 + r$ and $r$ is the world interest rate.
Besides this discount factor, the maximum debt level depends on the collateral the lender can appropriate.
Without loss of generality we assume that the investment goods of value $K$ do not depreciate and that foreign
lenders can appropriate the full collateral. More realism could easily be introduced by adding parameters
to capture phenomena such as weaker enforcement of property rights, e.g., because of judicial ineﬁciencies.
Given the timing of events mentioned above, in period $1$ the maximum debt level for short and long term
debt are
$$B_{2,1} = \frac{K}{X R},$$
and
$$B_{3,1} = \frac{K + Y}{X^2 R^2},$$
where $B_{t+m,t}$ denotes the maximum debt in period $t$ with maturity $t + m$. Note that the project returns $Y$
cannot be used as collateral for short-term debt in period $1$ as long as the project is liquidated with positive
probability in period $2$ and the agent potentially never earns these returns. This implies a trade-off for the
agent: compared with long-term debt, short-term debt implies a smaller discount factor because uncertainty
is smaller until the debt matures, but short-term debt also implies a smaller collateral because of the risk of
liquidation.

Although what we call exchange-rate uncertainty could be any uncertainty attached to investment returns
that increases over time, exchange rate uncertainty is the most natural interpretation in our application.
Moreover, it implies realistically that developed countries have more access to foreign debt than less-developed
countries since the former are exposed to less exchange rate risk because of better . . . nancial institutions such
as well developed derivative markets, or relatively more sound government policy.

---

4Note that $a \cdot 1$. At the cost of more clumsy notation we could specify $X_{t+1} = \mu e^{\varepsilon_t} X_t$ with $\varepsilon_t$ uniformly distributed
on the interval $[1, a; a]$ so that no restriction needs to be imposed on $a$. 

7
2.2 Consumption and Maturity

In our simple three-period model the agent’s choice in the rst period is discrete: the agent decides whether to invest or not and if she invests she can borrow short or long term.\(^5\) We rst derive consumption and utility for the three possible choices as a function of the model’s parameters, before we analyze which of the alternatives is optimal.

As mentioned above, risk neutrality and impatience imply that agents borrow to anticipate as much consumption as they can in the rst period. Formally, the condition is

\[
\beta RE_t X_{t+1} < 1,
\]

where \(RE_t X_{t+1}\) is the expected market return for assets denominated in foreign currency and we focus on the case where the home currency is expected to depreciate, \(E_t X_{t+1} > 1\).

As we will see below, the model setup implies that the optimality of investment and debt maturity crucially depends on how much of the future resources can be consumed in the present. I.e., the respective utility derived from investment nanced with short or long term debt depends on the tightness of the respective solvency constraint: the consumption pro le and thus utility are determined by credit supply as agents demand credit until the solvency constraints are binding. Thus, the model’s structure usefully isolates the e ect of credit supply (determined by exchange rate uncertainty) on the optimality of investment and debt maturity structure.

We now proceed to characterize consumption pro les and utility for the different choices of the agent. We summarize the consumption pro les in Table 1. Note that risk neutrality implies that zero consumption can very well be optimal in some periods. This stark feature of the model is not crucial for the results and positive consumption in all periods could be generated by policy-induced consumption oors.

No investment, no debt If the agent decides not to invest in the project, she just consumes \(K\) in the rst period (recall that the exchange rate in the rst period, \(X_1 = 1\)). We normalize utility dividing by \(K\) so that utility over the course of the investment project is

\[
u_n = 1,
\]

where the subscript \(n\) denotes the case in which the agent does not take on any debt.

\(^5\)The assumption of risk neutrality implies that a mix of long and short-term maturity debt is never optimal unless utility derived from long-term and short-term debt is the same. In this knife-edge case, the maturity choice is not determined. In the analysis below we break the tie in favor of long-term debt.
Let us now determine the utility derived from debt with short or long-term maturity. Note that in the three-period model all uncertainty about ..nishing the project is eliminated if the agent chooses the debt contract with a maturity \( m = 2 \). In this case she has to repay the debt in period 3 when the project is ..nished so that she will earn \( Y \) with certainty. There is no risk of liquidation. Instead if the agent chooses a short-term debt contract so that the debt needs to be rolled over in period 2, unfavorable realizations of the exchange rate can force her to liquidate the project in order to service the debt. However, as long as it is possible that the home currency depreciates, \( X > 1 \), the solvency constraint can be tighter for long-term debt than for short-term debt because the market discount factor introduced above, \( \frac{X^m}{X^{m-1}} \), increases with maturity \( m \). This implies that the agent trades off tighter access to funds because of exchange-rate uncertainty against liquidation with positive probability and a smaller collateral.

### Investment, long-term debt

If agents invest \( K \) and take on debt that matures in period 3, we denote consumption in the rst period by \( c_{1l} \), where the subscripts denote the period and the decision to borrow long-term. The explicit expression is displayed in the rst row and second column of Table 1. In the rst period impatient risk-neutral agents consume the maximum amount they can borrow long-term. In the second period the exchange rate \( X_2 \) is known. The collateral is reevaluated at this exchange rate which allows the agent to borrow an additional amount for consumption if the exchange rate is less than its ex ante maximum value, \( X_2 < \bar{X} \). In this case additional consumption is feasible in period 2 (see Table 1, second row and column). This consumption is ..nanced with short-term debt which is completely riskless since the debt can be repaid with certainty: if the maximum depreciation of the exchange rate between period 2 and 3 realized, the repayment in period 3 would be

\[
R^2X_2\bar{X} c_{1l} + RX_2\bar{X} c_{2l} = K + Y .
\]

Finally, in period 3 the agent consumes what remains after paying the debt plus interest (see Table 1, third row and second column).

### Investment, short-term debt

If agents invest \( K \) and take on debt that matures in period 2, the consumption in the rst period is denoted by \( c_{1s} \). We have to distinguish two cases. We show in the Appendix
that if uncertainty is small enough \((a \cdot a^a)\), where we derive an explicit expression for \(a^a\) below), it is optimal for the agent to restrict consumption in the first period so that the project is never liquidated and the project income \(Y\) can be used as collateral. Instead if \(a > a^a\), optimal consumption implies that the project is liquidated with positive probability and \(Y\) can no longer be used as collateral. The explicit expressions for consumption in the first period are displayed in the first row and third and fourth column of Table 1.

In period 2 the agent has to repay and can take on new debt. For \(a \cdot a^a \) consumption in the first period is such that debt can always be rolled over. For \(a > a^a\), the agent can borrow against project income \(Y\) only in period 2. Debt is rolled-over in this case if

\[
\frac{K + Y}{R} X_2 \frac{X_2 K}{X} \cdot 0
\]

which can be rearranged to

\[
X_2 \cdot p \frac{R^{1(1+y)}}{2a}
\]

where \(y = Y/K\) is the project return on investment. Since \(X_2 = \mu \varepsilon_1\) and \(\varepsilon\) is distributed uniformly on the interval \([1 - a, 1 + a]\), the probability of successful debt roll-over is

\[
p = \frac{a + \mu^1 \frac{R^{1(1+y)}}{1} \cdot 1}{2a}.
\]

Equation (2) implies that the project is never liquidated, \(p = 1\), if

\[
a \cdot a^a \cdot \mu^1 \frac{R^{1(1+y)}}{1} \cdot 1.
\]

We show in the Appendix that the consumption profile displayed in Table 1 for the case \(a \cdot a^a\) always dominates the profile displayed for \(a > a^a\) as long as \(a < a^a\). Both profiles are identical at \(a = a^a\) so that \(u_s\) is continuous in \(a\).

It is straightforward to show that the probability of debt roll-over \(p \cdot 1/2\) and increases in \(y\) but decreases in \(\mu, R, a\). Note that \(p \cdot 1/2\) for any distribution of \(\varepsilon\) with \(E(\varepsilon) = 1\), as long as domestic investors exploit arbitrage opportunities so that

\[
1 + y = \mu^2 R^2,
\]

where \(\mu^2 R^2\) is the expected return of a bond denominated in foreign currency with a two-period maturity. The intuition is that for \(a > a^a\), \(y\) is relevant for short-term debt only in period 2: the effect of the additional collateral that can be used to borrow, outweighs the expected discount factor \(R^{1} \mu^1 \cdot 1\over a\) over a one-period horizon.

Inspection of equation (2) reveals that a higher expected cost of the debt (a higher \(R\) or \(\mu\)) decreases the probability that debt can be rolled over. Moreover, a higher project return \(y\) increases the collateral against which agents can borrow in period 2 so that it is more likely that debt can be rolled over. Finally, more uncertainty about exchange rates (a larger \(a\)) makes it less likely that debt can be rolled over. This result relies on the assumption of a uniform distribution and \(p \cdot 1/2\). Moreover, the marginal effect of uncertainty on \(p\) becomes smaller the higher is the initial uncertainty because \(\lim_{a \to 1} p = 1/2\).

\(^6\)Of course, \(a > 1\) is not a reasonable parametrization for the assumption of a symmetric uniform distribution around 1 so that the limit serves only illustration purposes.
If the project is liquidated, agents consume the rest of the collateral which remains after repayment of the debt in period 2 and nothing in period 3 (see columns 3 and 4 in Table 1). Liquidation entails a welfare cost because the project income of size $Y$ does not materialize in period 3 and impatient agents have postponed consumption in period 1 to invest. If the project is not liquidated instead, agents consume the additional amount of credit obtainable in period 2 and what remains after repayment in period 3. Note that if the project is not liquidated so that the project income realizes, consumption in the third period is the same whether the project is financed short or long-term. The two types of debt allow, however, different consumption profiles in the first and second period where the difference depends on the model’s parameter values.

Normalizing by $K$, the expected utility of the agent borrowing long or short-term, respectively is defined as

$$u_j = i c_{1j} + \beta E_1 c_{2j} + \beta^2 E_1 c_{3j}, \quad j = l, s.$$  \hspace{1cm} (4)

In the Appendix we derive explicit expressions for $u_l$ and $u_s$ as a function of the model’s parameters. In particular, we show that $\frac{\partial u_l(a, \mu)}{\partial \mu} < 0$ and $\frac{\partial u_l(a, \mu)}{\partial a} < 0$ if agents are impatient enough. The intuition for these results is simple. From above we know that $X = \mu(1 + a)$. I.e., a higher expected depreciation and uncertainty of exchange rates, increases the market discount factor and thus tightens the solvency constraints. Consumption is shifted to the future which decreases the expected utility of impatient agents.\(^7\)

Similarly we show in the Appendix that $\frac{\partial u_s(a, \mu)}{\partial \mu} < 0$ if agents are impatient enough, whereas $u_s$ can increase or decrease in $a$. The derivative $\frac{\partial u_s(a, \mu)}{\partial a}$ can be decomposed in three main effects: a negative effect on utility resulting from tighter solvency constraints; a negative effect on utility because of a higher probability of liquidation; and a positive effect due to $p > 1/2$ which implies that risk-neutral agents face a favorable gamble.\(^8\)

Finally, one can show that $u_s < u_l$ for $a < a^\#$. If optimal consumption implies that the project is never liquidated independent of debt maturity, borrowing long-term is at least as good as rolling over the debt in the second period.

2.3 Optimal maturity

We proceed to illustrate the optimal choice of maturity and its dependence on the model’s parameters graphically. The parameter values are displayed in Table 2. In the numerical example we assume agents

---

\(^7\)In the Appendix we show that the condition on impatience becomes more binding as $a$ increases. This is because Jensen’s inequality implies that $E_1 x_1 = (1 + a)^2 x_1$ which increases in $a$ and thus exerts a positive effect on the agent’s utility. This effect is rather mechanical so that in the numerical examples below we focus on parameters for which $u_l$ decreases in $a$, i.e., the condition on impatience is not violated.

\(^8\)Moreover, as for $u_s$, Jensen’s inequality implies a positive effect of $a$ on $u_s$ for rather mechanical reasons. In the numerical examples below, we focus on the region of $a$, where this effect is not dominant.
to be quite impatient for illustration purposes. It is straightforward to check that the parameter values imply that the conditions for impatience and the project returns used for signing the derivatives \( \partial u_l / \partial a \) and \( \partial u_s / \partial \rho, j = l, s \), are satisﬁed for not too large \( a \).

Table 2: Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>1</td>
</tr>
<tr>
<td>( R )</td>
<td>1.01</td>
</tr>
<tr>
<td>( \beta )</td>
<td>( (1 + .4)^{-1} )</td>
</tr>
<tr>
<td>( y )</td>
<td>( Y/K = .17 )</td>
</tr>
</tbody>
</table>

In Figure 2 we illustrate solutions for \( u_l \) and \( u_s \) as a function of \( a \) for the interval \( a \in [0; 3] \). The utility derived from no investment, \( u_n \), equals 1 and does not change with \( a \). For the chosen parameter values it is always optimal to borrow: \( u_l \) and \( u_s \) are larger than \( u_n \). Both, \( u_l \) and \( u_s \) are highly non-linear. For small values of \( a > 0 \), an increase in exchange-rate uncertainty (a higher \( a \)) does not change the optimal decision to borrow long-term. As \( a \) attains higher values, however, it becomes optimal to borrow short-term rather than long-term.\(^9\) The intuition is that higher uncertainty tightens the solvency constraint relatively more for long-term than for short-term debt in the rst period if \( a > a_0 \) and liquidation occurs with positive probability. Since agents are impatient, this makes them favor short-term over long-term debt. Moreover, as we have mentioned above uncertainty can increase utility derived from short-term debt because in this case

\(^9\)Note that both \( u_l \) and \( u_s \) increase if \( a \) increases in \( a \). In this case agents are no longer impatient enough so that the condition derived in the Appendix is violated for the model’s parameters. This is because Jensen’s inequality implies that \( E[1 - a^2] = (1 - a^2)^{\frac{1}{2}} \) increases in \( a \). Moreover, the unconditional expectation of \( z^2 \) increases faster than the expectation conditional on the debt roll-over (see the Appendix). Hence, for large enough \( a \), \( \partial u_l / \partial a > \partial u_s / \partial a > 0 \) so that a second crossing of the \( u_l \) and \( u_s \)-loci can occur.
agents face a favorable gamble given that the probability of rolling over the debt $p$, 1/2. The opposite sign and non-linearity of the effects of $\alpha$ on $u_s$ and $u_l$ explains the crossing of the $u_l$ and $u_s$-loci in the Figure.

Figure 3 illustrates how the solution of the model is affected by a change of the project return $y$ from .17 to .22 (numerical illustrations for other parameters of the model are in the Appendix). Analytic results are not readily available because $u_s$ is a highly non-linear function of the model’s parameters. To get some insight about the generality of the numerical results, we provide analytic results for the limit case $\alpha! \alpha^n$ in the Appendix.

In Figure 3, a larger project return increases the utility derived from short-term debt because it alleviates the borrowing constraint in the second period and decreases the probability of liquidation. However, it increases the utility derived from long-term debt relatively more because agents are impatient and the solvency constraint is alleviated in the first and not only in the second period. Thus, short-term debt is relatively less beneficial: in Figure 3 it is no longer optimal to borrow short-term for the considered parameter values. The analytic result reported in the Appendix for the limit case $\alpha! \alpha^n$ supports the illustration of the numerical example.

2.4 Short-run growth volatility and debt maturity

We now extend the simple model to analyze the relationship between debt maturity and growth volatility. In this paper we focus on the short-horizon growth impact of access to foreign capital and investigate how the interaction of currency risk with the optimal maturity structure determines growth volatility in the short-run. Although the economy has only three periods, the results can be used to analyze the effects of
the models' parameters on short-run output growth. Indeed, a longer time horizon could be analyzed by restarting the economy in period 3: instead of consuming the remaining resources, agents would decide as in period 1 whether to invest the resources another time or whether to consume them immediately. More resources are available for investment in period 3 than in period 1 if

\[ c_{3,j} > K, \ j = l, s \]

which in expectation (from the perspective of period 1) occurs in the limit \( a \rightarrow a^* \) if

\[ 1 + y > \frac{1 + a}{a}. \]

This inequality is intuitive: uncertainty (a larger \( a \)) tightens solvency constraints and tilts the consumption profile towards the future; and thus makes it more likely that the inequality holds (the right-hand side is decreasing in \( a \)).

To keep the analysis as simple as possible we keep the focus on the three-period model. In our model foreign capital alleviates borrowing constraints and allows agents to finance investment. If the project income is financed long-term, the amount of goods available for production/consumption will grow by \( 1 + y \) with certainty. However, if the project is financed short-term, it is possible that the project is liquidated so that the investment return is lost. Hence, it is short-term debt that induces growth volatility.

In order to derive the effect of currency mismatch on short-run output growth in the simplest possible way, assume that there exists a variety of projects which differ with respect to their return \( y_i \). Assume that \( y_i \) is uniformly distributed on the interval \([0, \bar{y}]\). As illustrated above, a higher return \( y \) makes it more likely that projects are financed with long-term debt. Hence, as \( y \) increases, the project return at which agents are indifferent whether to finance their project with long-term or short-term debt. I.e., \( y_{ls} \), \( y_{sn} \) implies that the project is financed with long-term debt. Similarly, let \( y_{sn} \) and \( y_{ln} \) denote the project return at which agents are indifferent between not taking on debt at all and financing the project with short-term or long-term debt, respectively. E.g., if \( y_{ls} > y_{ln} > y_{sn} \), projects with return \( y_{2} \) \([y_{sn}, y_{ls}]\) are financed long-term, projects with returns \( y_{2} \) \([0, y_{sn}]\) are not financed at all.\(^{10}\)

The thresholds \( y_{ls}, y_{sn} \) and \( y_{ln} \) are derived by the following equations (explicit expressions for \( y_{ls}, y_{sn} \) and \( y_{ln} \) are derived in the Appendix for the limit case \( a \rightarrow a^* \)):

\[ u_l(y_{ls}) = u_s(y_{ls}), \]

\[ u_s(y_{sn}) = u_n = 1 \]

and

\[ u_l(y_{ln}) = u_n = 1. \]

Figures 4 and 5 illustrate the solution numerically for the thresholds \( y_{ls}, y_{sn} \) and \( y_{ln} \) and the debt structure in the first period\(^{11}\). We use the benchmark parameter values of Table 2, and set \( \bar{y} = .2 \).\(^{12}\) Quite

\[^{10}\]The critical value of \( y \) at which projects start to be financed has to satisfy the arbitrage condition mentioned above. This is indeed the case for the numerical example considered below.

\[^{11}\]In the second period any additional debt will be short term since all debt is due in the third period.

\[^{12}\]Note that \( y_{sn} \) is only plotted for \( a \leq a^* \) since \( u_l \), \( u_s \) for all \( y \) if \( a \leq a^* \).
Figure 4: Numerical solution for project returns $y_i$ at which agents are indifferent between borrowing short or long-term, $y_{ls}$, short-term or not at all, $y_{sn}$, or long-term or not at all, $y_{ln}$.

Figure 5: The volume of debt in the first period normalized by $K$. 
intuitively, the project returns at which agents ... and it optimal to invest, \( y_{sn} \) and \( y_{ln} \), are higher if exchange rate uncertainty is larger. Higher returns need to compensate for tighter solvency constraints. Moreover, also \( y_{ls} \) increases: as mentioned above exchange-rate uncertainty makes short-term debt a relatively better deal and project returns need to rise in order to make long-term debt relatively more attractive. In the Appendix we show formally that \( \partial y_{ls}/\partial a > 0 \) for \( a > a^* \). In general, \( \partial y_{ls}/\partial a > 0 \) as long as \( \partial p/\partial y > 0 \) is not “too large” (the explicit restriction on the parameter space is messy and not insightful). I.e., a higher project return does not increase the probability of rolling over short-term debt in the second period so much to outweigh the effect of a looser solvency constraint for long-term debt in the first period. As is intuitive, the parameter restriction is more likely to be satisfied if agents are more impatient.

In Figures 4 and 5 it is apparent that all projects are financed with long-term debt if at all for \( a > a^* \) since \( u_s < u_l \). There is no uncertainty in the economy. For \( a > a^* \), however, some fraction of projects (with return \( y_i \in [y_{sn}; y_{ls}] \)) is financed with short-term debt as soon as \( y_{ln} > y_{sn} \). In Figure 4 no projects are financed short-term for the values of \( a \) where \( y_{ls} < y_{ln} < y_{sn} \). Thus, it is not the size of currency mismatch per se but its interaction with maturity mismatch which induces output volatility. Figure 5 illustrates that higher exchange rate uncertainty can induce the two types of mismatch to occur jointly.

In order to illustrate the effect of the debt structure on short-run growth volatility, define expected growth in the economy as

\[
g' = F(y_i; [y_{sn}; y_{ls}]) \sum_{y_{sn}} p(n)f_ydn + F(y_i; [y_{ls}; \bar{y}]) \sum_{y_{ls}} n f_ydn, \text{ if } y_{ls} > y_{sn},
\]

where \( f_y \) denotes the conditional density for the relevant interval of \( y_i \) and \( F(.) \) is the cumulative distribution function. The first term in equation (5) is the average growth resulting from projects financed with short-term debt. Returns \( y_i \) only materialize with probability \( p(y_i) \) for these projects. The second term is the average growth resulting from projects financed with long-term debt for which returns \( y_i \) realize with certainty. If \( y_{ln} < y_{sn} \) (and thus also, \( y_{ls} < y_{sn} \), given that a larger \( y_i \) makes long-term debt more beneficial),

\[
g' = F(y_i; [y_{ln}; \bar{y}]) \sum_{y_{ln}} n f_ydn,
\]

as long as \( y_{ln} < \bar{y} \).

In the Appendix we derive the explicit solution of the growth rate for the uniform distribution which we use to illustrate the effect of exchange-rate uncertainty on the growth rate in Figure 6: as exchange-rate uncertainty increases, the confidence interval of the growth rate widens substantially since many projects are financed with short-term debt. To sum up, the numerical examples just presented illustrate that if access to foreign debt creates a currency mismatch, output can increase in the short-run; but such growth may be quite volatile if exchange-rate uncertainty induces a substantial maturity mismatch. We now provide empirical evidence on key predictions of the theoretical model.

\[\text{13}\] The numerical result that all debt is financed short-term for \( a > 3 \) is not general and depends on the parameter \( \bar{y} = .2 \).
3 Empirical evidence

The purpose of this section is to provide an empirical test of the model's implications. In particular, what role does exchange rate and other types of uncertainty play in explaining the size and the maturity structure of international debt? And also, can the maturity structure of debt help us understand the real volatility in terms of economic growth? We should note at the outset that we cannot perform a structural estimation of the model. Instead, the aim of this section is rather to investigate whether the model contains an important insight into the functioning of international capital flows and debt dynamics.

The theoretical model entails that (exchange rate) uncertainty should have three implications for debt and growth volatility. First, higher uncertainty should decrease the total debt a country is able to raise on international financial markets. Second, uncertainty should increase the fraction of debt financed short-term. And third, overall uncertainty is projected to raise the short-run growth volatility of an economy, in particular through its effect on the debt structure. The theoretical model applies best to emerging market economies (EMEs) because the currency mismatch of debt is much less of a problem for developed countries. Thus, we test the predictions of the model using a sample of 30 mostly open economies EMEs, including 12 Asian economies, 8 EU acceding countries and 10 Latin American countries. We use annual data for the period 1985 to 2002 for most economies, a period during which most of the 30 economies liberalized their financial account. For Eastern European countries time series start in the early 1990s and the initial period of the transition to a market economy often has to be left out as it was characterized by statistical uncertainty. The panel is therefore unbalanced. The source for the debt data is the BIS, where debt is private sector bank debt, thus not including official debt flows which are likely to follow different dynamics.
from that implied by the model.

Before taking the model to the data, let us mention how we address three major issues. The first issue is that uncertainty is not directly observable so that we need to use proxies which are, by definition, imperfect. Since the empirical counterpart of uncertainty is hard to come by, we try three different strategies. First, we proxy exchange rate uncertainty via actual exchange rate volatility calculated over the past 1 to 3 years. The problem with this measure is that it assumes purely lagging expectations, whereas it is reasonable to assume that agents also consider forward looking indicators to form their expectations. As a second alternative, we take the other extreme case and proxy uncertainty with the exchange rate volatility of the future 1 to 3 years. However, this measure is also imperfect because we use realized instead of expected exchange rate volatility, assuming perfect foresight. As a third proxy, we use data on economic and political risk from the International Country Risk Guide (ICRG), which provides a quantitative assessment of political, economic, financial and investment risk for the great majority of the world’s countries, covering all of the 30 economies in our sample.\textsuperscript{14} The rationale for using this measure is that it covers a much broader range of sources of uncertainty. It therefore allows us to also test alternative hypotheses in that it may not necessarily be exchange rate uncertainty that drives a country’s debt dynamics, but also political risk or other types of financial and economic risk. This measure is imperfect too: it has the disadvantage that some of the indicators might also be based on the observed debt levels and maturity mismatch. This would imply that the risk indicators are correlated with the total debt and the fraction of short-term debt by construction.

A related concern is the role played by the de jure exchange rate regime (fixed or floating) in the framework developed in the theoretical section. As agents consider the maximum magnitude of the exchange rate depreciation, they do not limit themselves to the official exchange rate regime implemented by a given country, but also consider the de facto nature of the regime. On the one hand, a fixed exchange rate regime does not prevent a sharp depreciation if the peg is no longer sustainable, as demonstrated by the example of Argentina in 2001. On the other hand, a floating exchange rate regime may actually be relatively stable if the central bank intervenes in the foreign exchange market to dampen exchange rate fluctuations. This pattern of intervention, known as the “fear of floating” (Calvo and Reinhardt, 2002) seems to be very common among emerging markets (see also Bordo and Flandreau, 2003, for a historical perspective). As a consequence, we focus in the estimations on de facto measures of exchange rate uncertainty only.

The second issue is a possible omitted-variable bias since other variables than uncertainty matter for the debt structure. We address this issue by including in the regressions other determinants of debt as control variables. The first control variable is GDP per head, which we use as a proxy for the catching up potential of emerging markets. We expect a poor but growing country to borrow more (against future income) than a country that is already rich, following a standard consumption smoothing argument. Similarly as in Rodrik and Velasco (1999), we find that a higher GDP per head is associated with more short-term debt, expressed both as a percentage of GDP and as a fraction of total debt. However, contrary to Rodrik and Velasco, we do not find any statistically significant relation with the ratio M2/GDP. The second control variable is

\textsuperscript{14}The International Country Risk Guide (ICRG) is provided by the Political Risk Group (PRS). It consists of quantitative assessments of various risk components which are then aggregated into broader definitions. In all regressions reported in the empirical section, a higher number means a lower risk assessment. More documentation on the methodology can be found on-line at http://www.prsgroup.com/icrg/icrg.html.
the government budget balance, which we expect to be more or less negatively correlated with total debt depending on whether a significant proportion of economic agents follows a Ricardian behavior (this would for instance not be the case in the presence of liquidity constraints). The third variable is investment, which we expect to be positively correlated with total debt as access to international capital markets should allow countries to invest more by borrowing abroad instead of reducing saving. The fourth variable is a dummy that is equal to one if the financial account of the balance of payments is open and zero otherwise. The main source is Kaminsky and Schmukler (2002), which we complemented by other sources when data were missing (the IMF Annual Report on Exchange Arrangements and Exchange Restrictions and the EBRD’s Transition Report, various years). Financial account openness is expected to be positively correlated with the phenomenon of currency mismatch for at least two reasons: currency mismatch is only an issue if the financial account is open and most, if not all, foreign debt is denominated in foreign currency. One of the pitfalls of de jure openness measures is that a country can be formally open and yet receive little foreign capital, for instance because of other regulatory measures. We therefore complement this de jure variable by de facto measures, such as the share of capital inflows as a proportion to GDP. In particular, we use the share of FDI and portfolio inflows, separately and together. The ...th variable is a dummy variable for currency crises, based on a definition presented in detail in Bussière and Fratzscher (2002). The motivation for introducing this variable is that the debt ratio's jump up during crises due to a conversion factor which is mechanical and not directly related to the question we want to answer.

The third set of issues is related to the econometric methodology. We use panel data, which allows us to control for idiosyncratic (country specific) effects. Indeed, the debt structure can be different across countries for a host of unobservable reasons such as different degrees of risk aversion or differences in institutions that are not well captured by any of our right-hand side variables. However, given the characteristics of the data, in particular its dynamics over time, we need to use a dynamic panel data estimator (the dependent variables are autocorrelated). It is well known that fixed effect estimators are biased for dynamic models since the lagged dependent variable is correlated with the error term (see Nickell, 1981). Although this bias becomes unimportant as the number of time observations approaches infinity, we cannot assume this to be the case in our application since our sample has less than 20 time observations.

To address this problem, we use the methodology developed by Arellano and Bond (1991). This methodology relies on a large set of instruments, combining the lags of the dependent variable with the lags of the exogenous variables as instruments. One of the key advantages of this methodology is that it is also designed to tackle the problem of endogeneity of some of the right-hand side variables. However, this approach is not without problems either: in small samples the GMM estimator can be biased if the instruments are weak.  

### 3.1 Effects on the level of debt

The first set of results attempts to establish whether more uncertainty and openness are indeed associated with lower levels of total debt as implied by the model.

Table 2 shows the results for the effect on total debt as a ratio of GDP. Table 3 shows the same analysis but with short-term debt to GDP as the dependent variable. Each regression uses the same control variables:

---

15 As a check we also used a fixed effect (least-square dummy variable) model, which by and large yielded similar results.
Table 2: regression results; dependent variable is total debt/GDP (%)

<table>
<thead>
<tr>
<th>Exchange rate volatility defined over</th>
<th>coef.</th>
<th>std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Previous 3 years</td>
<td>-0.478</td>
<td>0.144 ***</td>
</tr>
<tr>
<td>Previous year</td>
<td>-0.271</td>
<td>0.111 ***</td>
</tr>
<tr>
<td>Next 3 years</td>
<td>-0.461</td>
<td>0.239 **</td>
</tr>
<tr>
<td>Next year</td>
<td>-0.188</td>
<td>0.143</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Risk indicator as computed by IRCG:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total composite risk</td>
</tr>
<tr>
<td>Composite financial risk indicator</td>
</tr>
<tr>
<td>Composite investment risk indicator</td>
</tr>
<tr>
<td>Composite political risk indicator</td>
</tr>
<tr>
<td>Quality of bureaucracy</td>
</tr>
<tr>
<td>Corruption</td>
</tr>
<tr>
<td>Democratic accountability</td>
</tr>
<tr>
<td>Government stability</td>
</tr>
<tr>
<td>Law&amp;Order</td>
</tr>
<tr>
<td>Socioeconomic conditions</td>
</tr>
</tbody>
</table>

| Trade openness                      | 0.144 | 0.022 ***  |
| FDI inflows                         | -0.119| 0.192      |
| Portfolio inflows                   | 0.074 | 0.088      |
| Total inflows (FDI+portfolio)       | 0.035 | 0.074      |

Notes:
Estimations carried out using the Arellano-Bond GMM estimator and including one lag of the dependent variable. All regressions use 5 control variables on the right-hand side.

*, **, *** denote significance at the 10%, 5%, 1% level, respectively.

The tables only report the coefficients and standard errors of the additional variables.16

First, a key finding is that higher exchange rate volatility is associated with lower total debt, both for the measure of backward-looking and forward-looking exchange-rate volatility: countries tend to have more external debt if they are considered to be less risky in terms of exchange rate uncertainty. This is an important result that supports the key argument of the theoretical model in that currency uncertainty is indeed linked to a country’s ability to raise funds from international debt markets. These results prove robust both across different regions of emerging markets and over different time periods. Concerning the link between exchange rate volatility and the short-term debt to GDP ratio, evidence is less conclusive. The exchange rate volatility measure defined over the next three years actually has the wrong sign and is significant at the 10% level. One possible explanation is the “fear of floating” effect mentioned in the introduction: if this pattern is sufficiently widespread, one might expect countries with large amounts of short-term debt to intervene in foreign exchange markets in order to limit exchange rate variability, thereby creating a negative correlation between the two variables. This result may therefore suggest that this effect is not fully accounted for by the econometric instrumentation we employed.

Second, there is some evidence that openness is linked to a higher ratio of short-term debt. Trade openness is positively linked to the ratio of debt to GDP (Table 2) and to the ratio of short-term debt to GDP (Table 3). An interesting finding is that FDI is negatively related to the level of debt, both total and
<table>
<thead>
<tr>
<th>Exchange rate volatility defined over</th>
<th>coef.</th>
<th>std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Previous 3 years</td>
<td>-0.042</td>
<td>0.042</td>
</tr>
<tr>
<td>Previous year</td>
<td>0.005</td>
<td>0.053</td>
</tr>
<tr>
<td>Next 3 years</td>
<td>-0.244</td>
<td>0.156 *</td>
</tr>
<tr>
<td>Next year</td>
<td>-0.05</td>
<td>0.095</td>
</tr>
</tbody>
</table>

| Risk indicator as computed by ICRG:           |       |            |
| Total composite risk                           | -0.119| 0.040 ***  |
| Composite financial risk indicator            | -0.061| 0.049      |
| Composite investment risk indicator           | -0.440| 0.133 ***  |
| Composite political risk indicator            | -0.073| 0.035 **   |
| Quality of bureaucracy                        | 0.652 | 0.426      |
| Corruption                                    | 0.147 | 0.281      |
| Democratic accountability                     | -0.173| 0.233      |
| Government stability                          | -0.241| 0.119 **   |
| Law&Order                                     | 0.041 | 0.273      |
| Socioeconomic conditions                      | 0.000 | 0.000      |
| Trade openness                                | 0.029 | 0.011 ***  |
| FDI inflows                                   | -0.172| 0.082 **   |
| Portfolio inflows                             | 0.034 | 0.081      |
| Total inflows (FDI+portfolio)                 | -0.065| 0.068      |

Notes:
- Estimations carried out using the Arellano-Bond GMM estimator and including one lag of the dependent variable. All regressions use 5 control variables on the right-hand side.
- Only the coefficient and std. error of the additional variable is reported.
- *, **, *** denote significance at the 10%, 5%, 1% level, respectively.

short-term. This finding may suggest that FDI and debt are substitutes rather than complements, but would require a more detailed investigation that goes beyond the scope of this paper.

Third, we find a negative relationship between lower political and economic risk of several indicators - i.e. a higher value of the ICRG risk indicator - and the level of debt a country raises from international financial markets. This may be counter-intuitive at first, but may suggest that countries with low risk may access international financial markets by raising FDI or portfolio investment, rather than bank debt. The significant negative relationship between FDI inflows and debt levels, as discussed above, may indeed provide support for this hypothesis.

Overall, we conclude that there is indeed a strong and significant empirical relationship between exchange rate uncertainty and the total level of debt, as well as between openness and debt. Both of these findings provide support for our theoretical model.

### 3.2 Effects on the maturity mismatch of debt

We next turn to testing one of the key implications of the theoretical model, namely that higher uncertainty and risk are associated with a higher maturity mismatch of foreign debt. Our proxy for the maturity mismatch is the ratio of a country’s short-term debt to its total debt. Clearly, a potential shortcoming of this proxy is that it does not capture the maturity structure of investment returns, which also plays a role. However, since no such data can be obtained or estimated reliably for a broad set of emerging market economies, we assume that the maturity structure of investment returns does not differ too strongly across countries and over time and employ the ratio of short-term to total debt as our proxy.
<table>
<thead>
<tr>
<th>Table 4: regression results; dependent variable is short-term/total debt (%)</th>
<th>coef.</th>
<th>std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exchange rate volatility defined over</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Previous 3 years</td>
<td>-0.024</td>
<td>0.083</td>
</tr>
<tr>
<td>Previous year</td>
<td>0.024</td>
<td>0.075</td>
</tr>
<tr>
<td>Next 3 years</td>
<td>0.135</td>
<td>0.181</td>
</tr>
<tr>
<td>Next year</td>
<td>0.172</td>
<td>0.107 *</td>
</tr>
<tr>
<td>Risk indicator as computed by IRCG:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total composite risk</td>
<td>0.052</td>
<td>0.046</td>
</tr>
<tr>
<td>Composite financial risk indicator</td>
<td>0.155</td>
<td>0.055 ***</td>
</tr>
<tr>
<td>Composite investment risk indicator</td>
<td>-0.281</td>
<td>0.150 *</td>
</tr>
<tr>
<td>Composite political risk indicator</td>
<td>-0.021</td>
<td>0.039</td>
</tr>
<tr>
<td>Quality of bureaucracy</td>
<td>0.418</td>
<td>0.483</td>
</tr>
<tr>
<td>Corruption</td>
<td>0.482</td>
<td>0.319</td>
</tr>
<tr>
<td>Democratic accountability</td>
<td>-0.369</td>
<td>0.263</td>
</tr>
<tr>
<td>Government stability</td>
<td>-0.201</td>
<td>0.136 *</td>
</tr>
<tr>
<td>Law&amp;Order</td>
<td>0.278</td>
<td>0.308</td>
</tr>
<tr>
<td>Socioeconomic conditions</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Trade openness</td>
<td>0.011</td>
<td>0.019</td>
</tr>
<tr>
<td>FDI inflows</td>
<td>0.072</td>
<td>0.189</td>
</tr>
<tr>
<td>Portfolio inflows</td>
<td>-0.059</td>
<td>0.131</td>
</tr>
<tr>
<td>Total inflows (FDI+portfolio)</td>
<td>-0.074</td>
<td>0.104</td>
</tr>
</tbody>
</table>

Notes:
- Estimations carried out using the Arellano-Bond GMM estimator and including one lag of the dependent variable. All regressions use 5 control variables on the right-hand side. Only the coefficient and std. error of the additional variable is reported.
- *, **, *** denote significance at the 10%, 5%, 1% level, respectively.

The first key finding is that higher exchange rate uncertainty is associated with a higher share of short-term debt (Table 4). The coefficient of exchange rate volatility is significant when we use the forward looking measure (calculated over the following year). This is to be expected given that short-term debt is defined as debt with maturity below one year. The result is robust across regions and time periods and provides support for the relevance of our theoretical model.

Second, we find that higher investment risk and political risk (government stability) are linked to a higher ratio of short-term debt. This confirms the theoretical model’s key finding that risk leads investors and borrowers alike to shift their borrowing increasingly towards short-term debt. Among the sub-components of political risk, it is found that in particular government stability is significantly related to the share of short-term debt. However, economic and financial risks are negatively associated with the ratio of short-term debt, which stands in contrast to the finding for political risk and to the predictions of the model.

Finally, there does not seem to be a significant correlation between openness, either de jure openness or trade openness, and the maturity mismatch. This finding may be quite intuitive as financial and trade openness allows countries to borrow more from international financial markets. However, as shown in the previous section, openness may not only raise short-term debt but also total debt, thus having an ambiguous effect on the share of short-term to total external debt.

In summary, the empirical evidence found that in particular higher exchange rate uncertainty and higher political risk raise the maturity mismatch of a country. This is very much in line with the predictions of our theoretical prior presented above. However, one must underline as well that for one of the variables we
Table 5: regression results; dependent variable is output volatility

<table>
<thead>
<tr>
<th>coef.</th>
<th>std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short-term to total debt ratio</td>
<td>2.851</td>
</tr>
<tr>
<td>Short-term debt to GDP ratio</td>
<td>0.014</td>
</tr>
<tr>
<td>Total debt to GDP ratio</td>
<td>0.004</td>
</tr>
<tr>
<td>Exchange rate volatility defined over</td>
<td></td>
</tr>
<tr>
<td>Previous 3 years</td>
<td>-0.145</td>
</tr>
<tr>
<td>Previous year</td>
<td>0.296</td>
</tr>
<tr>
<td>Next 3 years</td>
<td>0.312</td>
</tr>
<tr>
<td>Next year</td>
<td>0.316</td>
</tr>
<tr>
<td>Risk indicator as computed by IRCG:</td>
<td></td>
</tr>
<tr>
<td>Total composite risk</td>
<td>-0.089</td>
</tr>
<tr>
<td>Composite financial risk indicator</td>
<td>-0.089</td>
</tr>
<tr>
<td>Composite investment risk indicator</td>
<td>-0.076</td>
</tr>
<tr>
<td>Composite political risk indicator</td>
<td>-0.002</td>
</tr>
<tr>
<td>Quality of bureaucracy</td>
<td>-0.307</td>
</tr>
<tr>
<td>Corruption</td>
<td>-0.059</td>
</tr>
<tr>
<td>Democratic accountability</td>
<td>-0.224</td>
</tr>
<tr>
<td>Government stability</td>
<td>0.080</td>
</tr>
<tr>
<td>Law&amp;Order</td>
<td>0.048</td>
</tr>
<tr>
<td>Socioeconomic conditions</td>
<td>-0.064</td>
</tr>
<tr>
<td>Trade openness</td>
<td>0.124</td>
</tr>
<tr>
<td>FDI inflows</td>
<td>-0.019</td>
</tr>
<tr>
<td>Portfolio inflows</td>
<td>-0.012</td>
</tr>
<tr>
<td>Total inflows (FDI+portfolio)</td>
<td>-0.010</td>
</tr>
</tbody>
</table>

Notes:
Estimations carried out using the Arellano-Bond GMM estimator and including one lag of the dependent variable. All regressions use 5 control variables on the right-hand side.
Only the coefficient and std. error of the additional variable is reported.
*, **, *** denote significance at the 10%, 5%, 1% level, respectively.

tested, results are contrary to the predictions of the model.

3.3 Effects on growth volatility

The last step is to test whether uncertainty, openness and debt are linked to output/growth volatility. Our theoretical priors are that more uncertainty should lead to higher economic growth volatility. Moreover, the theoretical model implies that higher short-term debt and a larger maturity mismatch should also raise output volatility. Growth volatility is measured as the standard deviation of growth rates over the future 3 or 5 years. Hence note that the model regresses growth volatility on past independent variables (recall that in the results presented in the Tables, the variables are tested sequentially rather than together, to avoid multicollinearity problems). Results are presented in Table 5.

First, there is a strong significant link between a higher maturity mismatch - proxied by the share of short-term to total debt - and larger growth volatility. This concerns countries that have a relatively large share of short-term debt are more likely to go through a boom-bust cycle of economic growth as inflows in debt may boost growth initially, but the withdrawal of short-term debt may also lead to sudden capital-flow reversals and economic contractions.

Second, we find a robust link between higher political and economic/financial risks and larger growth
volatility. In particular, three of the political risk sub-components (quality of bureaucracy and democratic accountability) are significantly negatively related to output volatility.

Third, we do not find a statistically significant relationship between exchange rate uncertainty in future years and output volatility and between openness and output volatility. Both the de jure openness variable and the exchange rate volatility variable have the correct sign, but are not statistically significant. This suggests that exchange rate uncertainty and openness affect output volatility mainly through changes in the maturity debt structure.

Overall, this section has presented evidence that shows in particular a significant and robust relationship between maturity mismatch of debt and growth volatility and between various types of economic and political risk and growth volatility. Both of these findings provide further support for the relevance of our theoretical framework.

4 Conclusion

We have presented a model which shows how a currency mismatch can exacerbate a maturity mismatch, in particular, in countries with higher (exchange-rate) uncertainty. As a consequence, the share of investment projects at risk increases in these countries so that output becomes more volatile. It would be interesting in future research to incorporate moral hazard and asymmetric information into the model since these phenomena are important for capital markets in reality.

Taking the model to the data provides considerable support to the model’s predictions. We find that more exchange rate uncertainty - proxied with different exchange rate volatility measures, both backward and forward looking - unambiguously reduces the total level of debt. The effect of exchange rate uncertainty on the maturity mismatch is significant for the forward looking volatility measure which is most sensible for the definition of short-term debt in the data. Better data would allow to investigate the predictions of the model in more detail in future research. Moreover, it would be interesting to extend the model in a number of directions, allowing for risk aversion and general equilibrium determination of exchange and interest rates.
Appendix

Derivation of \( u_t \) as a function of the model’s parameters:
Inserting the expressions for consumption displayed in Table 1 into the utility function and dividing by \( K \), we get

\[
\begin{align*}
    u_t &= \frac{1 + y}{X} R^2 + \beta E_1 \frac{X_i X_2}{X_2X} (1 + y) + \beta^2 E_1 \frac{1}{X} + \frac{X_1 X_2}{X} X_3 (1 + y),
\end{align*}
\]

where \( y \) is the project return on investment. Plugging in

\[
    X_{1+m} = \mu^m \varepsilon_s \text{ and } \bar{X} = \mu(1 + a),
\]

we have

\[
    u_t = \frac{1 + y}{\mu^2(1 + a)^2R^2} + \beta E_1 \frac{\mu(1 + a) \varepsilon_1}{\mu^2 \mu^2(1 + a)^2R} (1 + y) + \beta^2 E_1 \frac{1}{\mu^2(1 + a)^2} \frac{\mu^2(1 + a) \varepsilon_2}{\mu^2 \mu^2(1 + a)^2} (1 + y).
\]

This can be simplified to

\[
    u_t = \frac{1 + y}{\mu^2(1 + a)^2R^2} + \beta R^1 \frac{1}{1 + a} E_1 \frac{\mu^2(1 + a) \varepsilon_1}{\mu^2 \mu^2(1 + a)^2} (1 + y) + \beta^2 E_1 \frac{X_1}{X_2} X_3 (1 + y),
\]

A1

where 2 equations follow because \( \varepsilon \) is i.i.d. with an expected value of 1 and

\[
    E_1 \varepsilon_1^1 = \frac{1}{2} \frac{1}{1 + a} + \frac{1}{1 + a} = \frac{1}{2} \varepsilon_1^{1}. \]

Derivation of \( u_s \) as a function of the model’s parameters:

Part 1: \( p < 1 \)

We insert the expressions for consumption displayed in the fourth column of Table 1 into the utility function so that

\[
    u_s = \frac{1}{R \bar{X}} + \beta(1 + p) E_{1L} \frac{\mu X_i X_2}{\mu^2 \mu^2} (1 + y) + \beta p E_{1L} \frac{1}{X_2X} X_3 (1 + y),
\]

where \( E_{1L}, i = l, r, \) denotes the expectation in the rst period conditional on liquidation and rolling over the debt in the second period, respectively. Denoting analogously \( \varepsilon_r \) and \( \varepsilon_t \) as the mean of \( \varepsilon \) conditional on rolling over the debt and liquidation,

\[
    u_s = \frac{R^3 \mu^5 (1 + a)^3}{3} + \beta(1 + p) E_{1L} \frac{1}{X_2X} X_3 (1 + y) + \beta^2 p E_{1L} \frac{1}{X_2X} X_3 (1 + y), (A2)
\]

We get

\[
    u_s = \frac{R^3 \mu^5 (1 + a)^3}{3} + \beta(1 + p) E_{1L} \frac{1}{X_2X} X_3 (1 + y) + \beta^2 p E_{1L} \frac{1}{X_2X} X_3 (1 + y),
\]

25
where \( p, \varepsilon_l, \varepsilon_r \) and \( E_{1r}e^1 \) are functions of the models parameters. The explicit expressions for \( \varepsilon_l \) and \( \varepsilon_r \) are

\[
\varepsilon_l \quad E_{1l}e = 1 + \frac{a^u + a}{2},
\]

\[
\varepsilon_r \quad E_{1r}e = 1 + \frac{a^u}{2} + \frac{a}{1 + a^u},
\]

and

\[
E_{1r}e^1 = \frac{1}{2} \frac{1}{1 + a} + \frac{1}{1 + a^u},
\]

where \( a^u \), defined in equation (3), is the critical value at which projects start to be liquidated. Note that \( E_{1r}e^2 = 1 \) since \( \varepsilon \) is i.i.d.

**Part II:** \( p = 1 \)

If \( a < a^u \), \( p = 1 \) and \( \varepsilon_r = 1 \). Agents can use the project return \( Y \) as collateral also if borrowing short-term. Formally, the project is not liquidated if

\[
1 + \frac{y}{RX X_2} \cdot \frac{1}{RX X_2} > 0
\]

which can be rearranged to

\[
(X_2)^2 \cdot \frac{1 + y}{RX X c^1} > 0.
\]

Plugging in the explicit expressions for \( X_2 \) and \( X \) we find that \( p = 1 \) as long as

\[
\mu^2(1 + a)^2 \cdot \frac{(1 + y)}{R^2 \mu(1 + a)^2} > 0.
\]

For impatient consumers this holds as equality and

\[
c^1 = \frac{K + Y}{R^2 \mu^2(1 + a)^2} \cdot c^u.
\]

The consumption levels \( c_2 \) and \( c_3 \), displayed in the third column of Table 1 can be derived in a straightforward manner as for the case of long-term debt. If \( p = 1 \) and agents borrow short-term, they consume less in the first period than in the case of long-term debt so that \( u_s \cdot u_l \) as long as they are impatient (for \( \mu > 1 \) or \( a > 0 \) this holds as strict inequality). The intuition behind this result is that agents have tighter solvency constraints when borrowing short-term and \( p = 1 \), as long as they roll-over the debt in the next period with certainty. Interestingly, agents are indifferent to forego the collateral \( Y \) and consume more in the first period if \( K + Y = \frac{K}{R^2 \mu^2} \) since consumption and thus \( u_s \) are exactly equal for the two cases at \( a = a^u \).

**Derivation of \( \partial u_l/\partial \mu \) and \( \partial u_l/\partial a \):**

Using equation (A1) we find that

\[
\text{sgn} \frac{\partial u_l}{\partial a} = \text{sgn} (A\beta^2 + B\beta + C).
\]

I.e., we have shown optimality of the respective consumption profiles (displayed in Table 1 for the cases \( a \cdot a^u \) and \( a > a^u \)). Moreover, \( u_s \) is continuous in \( a \) since consumption and thus \( u_s \) are exactly equal for the two cases at \( a = a^u \).
The argument on the right-hand side of the equation is a quadratic equation in $\beta$ where

\[
A = (1 + a)^{1/2}, \\
B = R_1^1 \mu_1^2 (1 + a)^{1} + 1 \, i \, 1 \, i \, a^2 \xi_1^i + 2a \, i \, 1 \, i \, a^2 \xi_1^i + 2(1 + a)^{1/2}, \\
C = 2R_1^2 \mu_1^2 (1 + a)^{1/2}.
\]

Thus the condition for $\partial u_t / \partial a < 0$ is

\[
\beta < \beta^a, \quad \text{where} \quad \beta^a = \frac{\partial^b}{\partial a^2} \cdot \frac{B + \frac{p}{2} R_1^1 4AC}{2A}.
\]

Abstracting from the effect of an increase in $E_{1L}^1$ for a small $a$ with $E_{1L}^1 \to 1$, the condition for $\partial u_t / \partial a < 0$ simplifies to

\[
\beta R_1^1 \cdot i \, R_1^1 \, 2 \xi_1^i \mu_1^2 (1 + a)^{1} + 3 \, i \, \beta^2 \, i \, \beta R_1^1 \cdot i \, 2 \mu_1^2 (1 + a)^{1} + 2 \mu_1^2 (1 + a)^{1/2} (1 + y) < 0
\]

which holds if

\[
\beta R < 1.
\]

Differentiating with respect to $\mu$ we find

\[
\frac{\partial u_t}{\partial \mu} = \frac{3}{2} \mu \cdot 2 \mu_1^2 (1 + a)^{1} \cdot 2 \beta R_1^1 \cdot i \, 1 \, 1 \, i \, a^2 \xi_1^i + 2 \mu_1^2 (1 + a)^{1/2} (1 + y) < 0
\]

if

\[
\beta R < 1.
\]

Derivation of $\partial a_u / \partial \mu$ and $\partial a_u / \partial a$:

From equation (A3) it follows that if $a > a^u$ and $p = 1$, $c_u$ falls relatively more than $c_L$ as $a$ and $\mu$ increase. Hence, the sufficient conditions derived for $u_t$ suffice for this case as well. If $a > a^u$ instead, equation (A2) implies that

\[
\frac{\partial u_t}{\partial \mu} = \frac{3}{2} \mu \cdot 2 \beta R_1^1 \cdot i \, 1 \, 1 \, i \, 2 \mu_1^2 (1 + a)^{1} \cdot 3 \, \beta \, \mu_1^2 E_{1L} \cdot i \, 1 \, \xi_1^i (1 + a)^{1} \cdot 1 \, 1 \, i \, 2 \mu_1^2 (1 + a)^{1} \cdot 2 \mu_1^2 (1 + a)^{1} \cdot 2 \mu_1^2 (1 + a)^{1} \cdot (1 + y)
\]

where the third line results from

\[
\frac{\partial \xi_1^i}{\partial \xi_1^i} \frac{\partial e_1^i}{\partial \mu} = \frac{1}{2} \mu_1^2 2P \cdot \frac{1}{R_1^1 (1 + y)} + \frac{1}{1 + a}
\]

and

\[
\frac{\partial i \, E_{1L}^1 \, i \, \xi_1^i}{\partial \xi_1^i} \frac{\partial e_1^i}{\partial \mu} = \frac{2P \cdot \frac{1}{R_1^1 (1 + y)}}{1 + a}.
\]

The terms in the first and third line of $\partial a_u / \partial \mu$ add up to a negative number since $p > 1/2$ and the arbitrage condition holds, $1 + y < \mu^2 R^2$. The expression in the second line is negative because $\frac{\partial}{\partial \mu} < 0$ and the term in brackets in the second line is positive: plugging in the explicit expressions for $\xi_1^i, \xi_1^i, E_{1L}^1$ and rearranging the term in brackets becomes

\[
\frac{\beta}{1 + a} \cdot 1 \cdot \mu_1^2 1 \, P \cdot \frac{1}{R_1^1 (1 + y)} \cdot \frac{1}{1 + a} \cdot 1 \, 1 \, i \, 1 \, + \frac{\beta^2 (1 + y) \cdot \frac{1}{1 + a}}{1 + a} > 0.
\]
if \( 1 + y > R\mu^2 \) which holds as long as the arbitrage condition is satisfied.

Differentiating \( u_s \) with respect to \( a \) we get

\[
\frac{\partial u_s}{\partial a} = \frac{Ri}{(1 + a)^2} + \frac{\mu(1 + a)^{1/2}}{(1 + a)^2} + \frac{\beta p 1 + y}{R\mu^2} E_{1r} i e_i \xi_i \epsilon_i \epsilon_r + \frac{\beta^2 p_1 + y}{(1 + a)^2} \xi_i
\]

where we use in the last line that

\[
\frac{\partial \epsilon_r}{\partial a} = \frac{1}{2}, \quad \frac{\partial \epsilon_i}{\partial a} = \frac{1}{2} \quad \text{and} \quad \frac{\partial E_{1r} i e_i \xi_i}{\partial a} = \frac{1}{2(1 + a)^2}.
\]

Line 1 of the \( \partial u_s / \partial a \) displays the direct partial effect of \( a \) on \( u_s \) resulting from the tightening of the solvency constraint. As long as agents are impatient enough this effect is negative. In line 2 we have the negative effect of \( a \) on \( u_s \) resulting from the decrease in probability of debt rollover (\( \partial p / \partial a < 0 \)). Finally, line 3 shows the positive effect of \( a \) on \( u_s \) because agents face a favorable gamble (\( p > 1/2 \)). It turns out that the opposite sign of these effects implies that in general

\[
\frac{\partial u_s}{\partial a} > 0.
\]

Comparative statics for \( a \) ! \( a^n \):

\[
u_i^n - \lim_{a^n \to 0} u_i^n = \mu i^{1/2} (1 + a^n) i^{1/2} R i^{1 + \beta} i^{1/2} 1 i^{1} i^{1} a^n q_i^{1} \xi_i i^{1} (1 + a^n)^{1/2} (1 + y)
\]

and

\[
u_s^n - \lim_{a^n \to 0} u_s^n = R i^{1 + \beta} i^{1} a^n^{1} i^{1} \mu i^{1/2} R i^{1/2} (1 + a^n)^{1/2} (1 + a^n)^{1/2} + \beta^2 i^{1} i^{1} i^{1} (1 + a^n)^{1/2} (1 + y),
\]

where

\[
\lim_{a^n \to 0} E_{1r} i e_i \xi_i = E_{1r} i e_i \xi_i = i^{1} a^n q_i^{1} \xi_i.
\]

Considering the limit preserves the difference in the collateral if agents borrow short or long-term, but allows us to derive simple analytic expressions because \( \lim_{a^n \to 0} p = 1 \).

We find that

\[
u_i^n > u_i^n
\]

if

\[
y > y_s,
\]

where

\[
y_s = \frac{\mu (1 + a^n) R (1 + \beta R \mu)}{(1 + \beta R)} i^{1} 1 > 0
\]

given the assumptions on impatience. We denote the locus on which agents are indifferent between long and short-term debt as

\[
u_s^n - \nu_i^n = 1 + y i \frac{\mu (1 + a^n) R (1 + \beta R \mu)}{(1 + \beta R)}
\]

28
Moreover,
\[ \frac{\partial u^n_{ts}}{\partial y} = 1 > 0 , \]
i.e., long-term debt becomes relatively more beneficial as the project return increases. Furthermore,
\[ \frac{\partial u^n_{ts}}{\partial \mu} = i \frac{(1 + a^n) R(1 + 2\beta \mu R)}{(1 + \beta R)} > 0 \text{ if } \beta \mu R > \frac{1}{2} , \]
\[ \frac{\partial u^n_{ts}}{\partial (1 + a^n)} = i \frac{\mu R(1 + \beta R \mu)}{(1 + \beta R)} < 0 \text{ if } \beta R \mu < 1 , \]
\[ \frac{\partial u^n_{ts}}{\partial \beta} = R^2 \mu (1 + a^n) (\mu R) i > 0 \text{ if } \mu > 1 , \]
\[ \frac{\partial u^n_{ts}}{\partial \beta} = \mu (1 + a^n) (R \beta (2 i R \beta) i) > 0 \text{ if } \beta < \frac{\mu i \rho (\mu i 1)}{\mu R} . \]
Note that a positive derivative implies that long-term debt becomes relatively more beneficial compared with short-term debt.

Similarly, define
\[ u^n_{ls} = R l + \frac{1}{2} (1 + a^n) i \cdot 1 + \beta \mu l (1 + a^n) i \cdot 1 l a^{n^2} q_i 1 R l (1 + y) i (1 + a^n) i \cdot 1 \]
\[ + \beta^2 1 l (1 + a^n) i \cdot 1 (1 + y) i 1 \]
which implies that
\[ y_{sn} = \frac{R(1 + a^n + \beta) i}{\beta \mu l (1 + a^n) i \cdot 1 + \beta^2 R a^n} i \cdot 1 \ref{3} . \]
Also,
\[ u^n_{ln} = 3 \mu l (1 + a^n) i \cdot 1 R l 2 + \beta R l 1 \mu l (1 + a^n) i 1 l a^{n^2} q_i 1 i l a^{n^2} q_i 1 i \cdot 1 \mu l (1 + a^n) i \cdot 1 (1 + y) \]
\[ + \beta^2 1 l (1 + a^n) i \cdot 1 (1 + y) i 1 \]
which implies
\[ y_{ln} = 3 \mu l (1 + a^n) i \cdot 1 R l 2 + \beta R l 1 \mu l (1 + a^n) i 1 l a^{n^2} q_i 1 i l a^{n^2} q_i 1 i \cdot 1 \mu l (1 + a^n) i \cdot 1 (1 + y) \]
\[ + \beta^2 1 l (1 + a^n) i \cdot 1 (1 + y) i 1 \]
We find that impatience implies that
\[ \frac{\partial y_{ls}}{\partial a^n} = \mu R(1 + \beta R \mu) \frac{(1 i \beta R)}{(1 + \beta R)} > 0 , \]
whereas the derivatives for \( y_{ln} \) and \( y_{sn} \) are messy and less insightful.

**Growth rate \( g \) for the uniform distribution:**
Given the assumption that \( y \) is uniformly distributed on the interval \([0; \overline{y}]\), the second term of equation (5) simplifies to
\[ \frac{y}{\overline{y}} \frac{y_{ls} Z}{y_{ls}} \cdot 1 \frac{1}{y_{ls}} dn = 1 \frac{y}{2} \frac{y_{ls}}{\overline{y}} (y_{ls} + \overline{y}) . \]

29
Using the explicit expressions for \( p(y) \), the first integral in equation (5) \( \int_{y_s}^{y_n} p(n)f_y dn \) can be written as

\[
\frac{1}{y_n - y_s} \int_{y_s}^{y_n} \left( \frac{1}{2a} \right) n f_y dn = \frac{1}{y_n - y_s} \left( \frac{\mu}{2a} \right) 2n^2 \int_{y_s}^{y_n} \left( \frac{1}{2a} \right) n f_y dn + \frac{1}{y_n - y_s} \left( \frac{\mu}{2a} \right) 2n^3 \int_{y_s}^{y_n} \left( \frac{1}{2a} \right) n f_y dn
\]

where the second integral is solved using integration by parts.

The conditional variance of the growth rate is

\[
\int_{y_s}^{y_n} p(n) n^2 f_y dn = \frac{1}{y_n - y_s} \int_{y_s}^{y_n} \left( \frac{\mu}{2a} \right) 2n^2 \int_{y_s}^{y_n} \left( \frac{1}{2a} \right) n f_y dn
\]

The last integral can be solved using integration by parts:

\[
\int_{y_s}^{y_n} (1 + n)^{1/2} n^2 dn = \frac{\mu}{3} n^2 (1 + n)^{3/2} \int_{y_s}^{y_n} \left( \frac{1}{2a} \right) n f_y dn + \frac{\mu}{3} n (1 + n)^{5/2} \int_{y_s}^{y_n} \left( \frac{1}{2a} \right) n f_y dn
\]

Numerical illustrations of comparative statics:

![Figure 7: Comparative statics: a higher expected depreciation \( \mu \)](image)
Figure 8: Comparative statics: a higher interest factor \( R \)

Figure 9: Comparative statics: a smaller discount rate \( \beta \)
Figure 7 illustrates the change of the solution if the expected depreciation rate $\mu$ rises from 0 to .02. Figure 8 plots the model’s solution if $R$ increases from 1.01 to 1.03. Finally, Figure 9 displays the results if the discount factor $\beta$ decreases from $(1.4)^{-1}$ to $(1.3)^{-1}$.

The results are very intuitive. A larger expected depreciation $\mu$ decreases the utility derived from both, long and short-term debt. In the parametric example it becomes optimal not to borrow for some parameter values. Utility decreases more for borrowing short-term because additionally to tightening the solvency constraint a larger $\mu$ also increases the probability of liquidation. Hence, borrowing short-term is optimal over a smaller range of values of the parameter $a$.

A higher interest rate shifts utility derived from borrowing downward. As shown above for $a = a^n$, whether $u_l$ or $u_s$ fall relatively more depends on the degree of impatience. In the numerical example short-term debt becomes a relatively better deal.

More patience increases the utility derived from borrowing and investment. This effect becomes relatively stronger for high $a$. This is because a higher $a$ shifts consumption into the future since it tightens borrowing. Long-term borrowing becomes optimal for a larger range of parameter values.

References


