Does Housing Wealth Make Us Less Equal? The Role of Durable Goods in the Distribution of Wealth*

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Abstract

We study the role an illiquid durable consumption good plays in determining the level of precautionary savings and the distribution of wealth in a standard Aiyagari model (i.e. a model with heterogeneous agents, idiosyncratic uncertainty, and borrowing constraints). Transactions costs induce an inaction region over which the durable stock and the associated user cost are not adjusted in response to changes in income, increasing, on average, the volatility of non-durable consumption. The volatility of total consumption is then a function of the share of the durable good in the utility function and the width of the inaction region. We are particularly interested in parameterizations which increase the precautionary motive for saving through an increase in "committed expenditure risk". We find, for an empirically

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relevant share of durable consumption and for all transaction costs below an upper threshold, that the level of precautionary savings is increasing in the transaction costs. Transaction costs have only a modest impact on the degree of wealth dispersion, as measured by the Gini index, as the associated increase in savings is close to linear in wealth. While we are unable to match the dispersion of wealth in the data, we increase the dispersion over a single asset model (Gini index of .71 for financial assets and .37 for total wealth) and we are able to match the relative dispersion of financial to durable assets, i.e. we find financial assets much more unequal than durable assets. We also match the ratio of housing wealth to total wealth for the median agent. We calibrate the model to data from the PSID, the CES, and the SCF.

Keywords: Precautionary Savings, Wealth Distribution, Durable Goods, Prices

1 Introduction

In recent years, beginning with Aiyagari (1994), economists have attempted to quantify the importance of precautionary saving and explain the pattern of asset holding using ex-ante identical agents with standard preferences who face uninsurable idiosyncratic risk. The standard model allows the agent one asset choice, liquid financial assets, and abstracts from the stock of durable consumption goods that compose a large proportion of total wealth in the data.[1] In that durable assets differ from financial assets by yielding utility directly and being relatively less liquid, it seems plausible that a model that incorporates durable goods would yield substantially different results than one that precludes non-financial wealth.

We consider an Aiyagari-style model modified to allow for consumption of a durable good subject to transaction costs, which we take to be representative of housing[2]. The consideration of housing, as both

[1] The importance of housing in the household decision problem has been acknowledged with increasing frequency in recent years. The affect durable housing has on savings and asset allocation has been studied by Krueger and Fernández-Villaverde (2001) and Diaz and Luengo-Prado (2003); while its importance for asset pricing, has been studied by Piazzesi et al. (2003), Kullman (2002) and Lustig and Van Nieuwerburgh (2002). We discuss these papers in the literature review below.

[2] While the durable consumption good in our model is quite general, the empirically relevant good is housing. Our results hold for any durable good which is subject to substantial transaction costs and which is an important aspect of the household portfolio. The ratio of housing to total wealth for the median household is around .7. In addition, the transaction costs for housing are quite large.

an illiquid asset and a relatively non-volatile consumption flow, allows us to study the interaction between the durable good, transaction costs, and the precautionary motive for asset accumulation. In addition, explicit modeling of the durable sector allows us to study an additional, and important, aspect of the wealth distribution. Specifically, the existence of the durable good allows us to study the portfolio allocation between financial and non-financial assets.

Transaction costs induce an inaction region over which agents do not adjust their stock of the durable good in response to income shocks. The high frequency volatility of durable consumption, and the associated user cost of durables, declines. However, the low frequency movement of the durable good are quite large (i.e. the household makes large changes their durable stock upon hitting the boundary of the inaction region). Hence, the affect on the precautionary motive is ambiguous and depends on the volatility of income and the discount rate (this trade-off is studied in the appendix on the precautionary motive). The transaction costs introduce a "committed expenditure risk"[3] which increases volatility on the part of non-durable consumption. The additional volatility of non-durable consumption increases the desire for a liquid asset buffer to smooth consumption. The dominant effect is a function of the share of the durable good in the utility function. Of course, whenever the durable good has an increasing effect on the precautionary motive the concept of dominance is second order. As a result, for a broad range of parameter values, we find that transactions costs increase the level of precautionary saving[4] over a model with a liquid consumption-yielding asset.

We allow households to borrow against their durable stock at the market interest rate and require no down payment. Households are free to borrow against their stock of housing so long as they can repay the debt with probability 1. In any period, households have their labor income and the net-of transaction costs value of the house available to pay off debt. We define committed expenditure in this context to arise from

[3] Fratantoni (2001) examines committed expenditure risk as an explanation for the low proportion of relatively high-yielding risky assets in household portfolios. He finds that the committed expenditure risk associated with a fixed mortgage payment schedule acts to shift household portfolios away from risky assets.

[4] Throughout the paper, we define the level of precautionary savings to be the increase in the stock of liquid assets over the stock of assets that would exist if there was no income risk. Also note, in the absence of risk, the durable good plays no role in determining the equilibrium interest rate. The Euler equation for the bond holds and is independent of the (constant) holdings of the durable good.

the idea that whilst in the inaction region the household finds it optimal to maintain their current stock of the durable. Hence, the household does not have direct access to the funds which are currently tied up in housing. Their ability to borrow against the stock of housing ameliorates this channel but does not eliminate it. We show in the results section (Table 4 versus Table 5) that this result is more severe if we set the borrowing constraint to zero.

We also consider the impact of transaction costs on the distribution of wealth. Standard Aiyagari-type models have a difficult time replicating the distribution of wealth in the United States, largely because wealthy agents do not have a strong precautionary motive to save.[5] The precautionary motive is concentrated among wealth poor agents, with a flattening effect on the distribution of wealth. Not only are fluctuations to productivity (labor income) a smaller proportion of income for the very wealthy, for households with a sufficient accumulation of wealth (and with CRRA utility), the value function is essentially linear. As a result, relatively poor agents save a much higher proportion of their income which in turn implies that poor agents accumulate assets, on average much faster than rich agents, decreasing the dispersion of the wealth distribution, a result opposite to that found in the data.

Our model alters the market structure and changes the period utility function by adding a separate durable consumption good. Transaction costs, by making durables adjustment more painful, function in a manner similar to the introduction of habits (see the discussion of Diaz et al. (2003) below). However, unlike habits, transactions cost increase the motive for precautionary saving across the wealth spectrum, as wealthy households increase their durable stock essentially in proportion to increases in wealth[6]. Transaction costs have only a modest impact on the degree of wealth dispersion, as measured by the Gini index, as the

[5] As noted in Kimball (1990) and Carroll (1992), the expected variance of consumption declines with wealth, and therefore so does the precautionary motive for accumulating wealth.

[6] The transaction costs in our model are a mixture of fixed and proportional costs. If the costs were proportional to the state, the model would be homogenous and the problem solution would be the same for all levels of the state variables (see for example Martin (2003)). The current model is not homogenous and as a result poor agents accumulate the durable asset much faster than they accumulate financial assets. However, once the agents reach a threshold in durable assets, financial assets and durable assets are accumulated essentially proportionally. This result is consistent to the data on asset accumulation and is similar in nature to the result found by Fernandez-Villaverde and Krueger (2001).

associated increase in savings is close to linear in wealth.

We will calibrate and test the model to U.S. survey data. Estimates of transaction costs are derived from the Survey of Consumer Expenditures. In that survey, households are asked to estimate the total financial costs of moving. While the households report a wide variety of associated relocation expenses, we use their estimates of taxes and agency fees. We find transactions costs account for approximately 7.5 percent of the value of the purchased home. We split these costs symmetrically between buying and selling. Our income process is estimated from the Panel Survey of Income Dynamics. We are able to estimate both the relative variance in the income process and the transition matrix directly from this data. Using the panel frees us from having to estimate the transition probabilities. Notice, the transition probabilities given from the data are quite different from those derived from estimating the transition probabilities from an AR(1) process. Finally, we test the model by comparing the distribution statistics in several dimensions with the Survey of Consumer Finances. We compare the Gini indices for total wealth, housing wealth, and financial wealth. We also match the relative dispersion and median holdings of financial wealth and housing wealth.

2 Literature Review

Since Aiyagari (1994) there have been many papers seeking to understand the distribution of wealth in the U.S. using the same general framework. As summarized by Quadrini and Rios-Rull (1997) and by Ljungqvist and Sargent (2000), in order to generate a wealth distribution which is roughly consistent with the data, we must either increase the level of uncertainty with increases in wealth accumulation, alter the utility function, or restrict the budget constraint in a way which increases the precautionary motive of wealthy agents relative to the standard model.

Cagetti and De Nardi (2003) use essentially the first approach. They explicitly acknowledge that a large portion of the apparent volatility in income is a result of including business owner income in the sample. They split the agents into two classes, entrepreneurs and workers. Entrepreneurs have income streams which have a high mean and high volatility relative to that of the workers. They introduce financial constraints into the model such that only wealthy agents select into entrepreneurship. Their model does a nice job of matching the distribution of both entrepreneurs and income workers in the data. In order to achieve distribution they also add bequests to their model. As shown in De Nardi (2002), bequests have important implications for the distribution of wealth in that bequest taxes are not proportional to the bequest but are instead a fixed amount Effectively, then the after-tax rate of return to bequests is much higher for wealthy agents leading to higher savings by wealthy agents. Hence, the extra accumulation of savings by wealthy households reflects less a precautionary motive and more a rate-of-return motive.

Castaneda, Diaz-Gimenez and Rios-Rull (2003) take a similar approach. They calibrate their income process to data from the SCF. While they do not explicitly model an entrepreneurial sector in their paper, their income process (particularly for wealthy agents) is highly influenced by the existence of business owners in the data. The higher volatility in the income process leads to a higher level of precautionary savings. However, because they do not have a worker class, in order to increase the dispersion in the distribution, they add a retirement state. Agents entering retirement receive a progressive social security payment. As a result, the retirement state is better for low wealth agents than for high wealth agents which reduces the precautionary motive for low wealth agents, increasing the dispersion of wealth.

A recent paper by Diaz, Pijoan-Mas and Rios-Rull (2003) takes the approach of altering the period utility function to increase the precautionary motive. They introduce habit formation into the period utility function. Since the presence of habits makes consumption fluctuations more painful, those households with a small level of self-insurance will try to increase it by holding higher asset stocks. Households with large asset stocks are sufficiently well self-insured that additional accumulation of asset stocks is not necessary to reduce consumption fluctuations. They find that the presence of habit formation significantly increases the amount of precautionary savings and decreases the risk free rate over an economy without habit formation. Not surprisingly, the increase in asset accumulation occurs primarily amongst asset-poor agents, flattening the wealth distribution in opposition to the data. In essence, the habit stock does not accumulate sufficiently quickly to increase the need for precautionary savings at all wealth levels.

A natural extension of the Diaz et al. paper is to change the habit stock in the utility function to a durable good. That is the approach taken by this paper and, in a similar framework, by Diaz and Luengo-Prado (2002). Diaz and Luengo-Prado focus on the increase in precautionary savings between models with and without the durable sector. As a result, they fix the parameters of the utility function such that the smoothing effect of the durable good dominates (see the precautionary appendix for details). They find a small decrease in precautionary savings. In this sense, their model is a special case of our model. They achieve high relative dispersion in assets by adopting the income process of Castaneda et at. In our study, we wish to focus on the effects of the transaction costs and the share of the durable good in the utility function on the level of precautionary savings.

A related and growing body of literature is one in which a durable housing good enters into the pricing kernel in order to help explain asset pricing puzzles. In particular, the work of Lustig and Van Nieuwerburgh (2002), Piazzesi et al. (2003), and Kullman (2002) are excellent examples. While these papers do not attempt to explain the distribution of wealth, the mechanism at work is essentially the same in both classes. The link between asset prices and the wealth distribution is demonstrated in Lustig (2001).

3 The Model Economy

We consider an environment with a continuum of infinitely-lived agents. We normalize the total mass of agents to 1. Each agent experiences idiosyncratic labor productivity shocks. The shocks are structured such that there is no aggregate uncertainty (i.e. we assume a law of large numbers applies such that the total labor endowment in the economy each period is non-stochastic). Agents may store value in productive capital which they rent, period by period, to an aggregate technology and a durable good which provides a utility flow to the agent The technology which produces the durable good is subject to construction and deconstruction costs. These costs must be paid whenever the household changes its stock of the durable good. The durable good is specific to the household and can not be traded or rented without first converting it back to productive capital. In our analysis, we restrict ourselves to steady states. In the following sections, we present a detailed description of the environment, the household's problem, and a formal definition of the steady state equilibrium.

3.1 Endowment

Each agent is endowed with a stochastic sequence of labor endowments, $\{e_t\}_{t=0}^{\infty}$. The labor endowments are thought of as productivity. Each e_t lives on a measure space (E, ξ). We make the following assumption on this space: 1) ξ is the $\sigma-$algebra consisting of all subsets of E. 2) E is compact and countable. 3) Let $e_{\min}$ $e_{\max}$ denote the smallest and largest element of E respectively, then $e_{\min} > 0$ and $e_{\max} < \infty$.

Each agent's labor endowment follows a Markov process with transition function Q. $Q(a, c)$ gives the probability of moving to a state c if the current state is a. We make the following assumption on Q. 1) Q is monotone and increasing. 2) There exists $e \in E$, $\varepsilon > 0$, and N$\geq$ 1 such that $Q^N(a, [e, b]) \geq \varepsilon$ and $Q^N(b, [a, e]) \geq \varepsilon$.

Each agent is endowed with an initial capital stock and an initial stock of the durable good. Since we study only steady states, the initial endowment of assets is not critical to the analysis.

3.2 Preferences

Households derive utility from consumption of a nondurable good and the service flow of a durable good. Let c denote consumption of the nondurable good, h denote the service flow associated with holding a stock, H, of the durable good. Let the function f map stocks of the durable good into flows: $f(H) = h$. We write the per period utility as $u(c, f(H))$, and total utility as $\sum_{t=0}^{\infty} \beta^t u(c_t, f(H_t))$. Preferences are time separable; however, the value function will inherent many of the properties of nonseparable utility since, in equilibrium, the level of the durable stock will change only infrequently as a result of the construction and deconstruction costs.

We assume for all parameterizations of the utility function an increase in h increases the utility from a given level of consumption; $\frac{\partial u(c,h)}{\partial h} > 0$. The durable stock also increases the utility of the agent indirectly by relaxing the agent's borrowing constraints. Because the durable can be transformed, albeit at a cost, into productive capital the agent is allowed to borrow against the durable stock. In other words, agents with positive holdings of the durable good may set their financial assets negative in an amount equal to the productive capital equivalent of their durable stock. In addition, agents will be allowed to borrow against the present value of their labor income. The agents may borrow with the constraint that they repay their debt with probability 1. This constraint is more general than allowing the agent to borrow a fixed percent of their durable stock. The latter requires the implicit constraint that the percent always be sufficiently small that the former constraint holds. In order to demonstrate the interaction between the borrowing constraint and the precautionary motive, in many of the simulations below, we will tighten the borrowing constraints of the agents.

3.3 Technology

Aggregate output, Y, is produced according to an aggregate neoclassical production function that takes as inputs capital, K, and efficient units of labor, L, $Y = Z(K^{\theta} L^{(1-\theta)})$. The aggregate labor input comes from aggregating all household's efficiency units of labor. The aggregate supply of labor is constant and normalized to 1. Output can be transformed into future capital, durable goods capital, and future consumption according to: $Y = C + K' + D(H, H') - (1 - \delta) K$. The function D maps the current durable stock and next period's choice of durable stock into current period productive capital. The details of the function are given below. δ is the depreciation rate of capital.

Investment in the durable good is only partially reversible. The durable good, H, is produced according to an individual production function which uses as inputs productive capital. The amount of capital devoted to the production of the durable good can be changed; however, changing the investment level is not loss less. In order to change the production of the durable good the household must incur a destruction expense, λ_s, which is proportional to the current investment in the durable sector and a construction expense, λ_b, which is proportional to his new level of investment in the durable sector. The production function is linear in capital investment, with marginal rate of transformation equal to 1, once the transaction costs are paid. Once the destruction costs are incurred, durable capital may be transformed to productive capital, consumed, or returned to durable production. The production function is identical across households. Production is specific to the household and is not transferable expect insofar as the initial investment of productive capital may be recovered and subsequently traded. The stock of durable capital depreciates at rate δ_h not necessarily equal to δ. For most of our simulations, we choose $\delta = .1$ and $\delta_h = 0$. We believe these numbers to be of the right order of magnitude to match the data.

We assume a constant returns-to-scale production technology in capital and labor. Given our restrictions on the aggregate labor supply, we have the following restrictions on Y, and K: 1) Y is a strictly concave function of aggregate capital. 2) There exists a maximum sustainable aggregate capital stock denoted $K_{\max}$. In order to bound preferences from below later on, we assume a lower bound for capital denoted $K_{\min}$. As a result, $K \in [K_{\min}, K_{\max}]$. The lower bound will never bind so long as we choose a production function which satisfies Inada conditions for fixed labor supply.

Given the linear technology on the production of the durable good, bounding the space for the durable is slightly more problematic. Durable capital accumulation lives on the interval $H \in \left[0, \frac{\max_K(F(K,L)-K)}{\max(\lambda_s+\lambda_b,\delta)}\right]$. If transaction costs and δ are all zero, H is unbounded above[7]. Hence, we restrict attention to cases where either transaction costs or the depreciation rate is strictly positive. In addition, in the same fashion as for productive capital, we assume a lower bound for durable capital denoted $H_{\min}$. This assumption will not

[7]If both δ and λ are zero, a feasible strategy is to allocate a fixed percent of production to the durable sector each period implying an infinite accumulation as $t \to \infty$.

bind for all preferences where the marginal utility of durable consumption is infinite at zero consumption, a standard assumption on the period utility function. Alternatively, we could have specified the function f to be affine with a positive constant: $f(0) > 0$. Either standard serves to bound the utility function from below.

3.4 Market Arrangements

There are no state contingent markets for the household specific shock, e. Households holds assets $a \in [\underline{a}, \infty)$ that pay an interest rate r. We assume that households are restricted by a lower bound on their asset holdings $\underline{a}$. The lower bound arises endogenously as the quantity that ensures the household is capable of repaying its debt in all states of the world, the natural limit. We allow households to borrow against their housing equity at the market interest rate without penalty. The natural limit in the model with durable goods is the minimum present value of labor income plus the present value of housing equity. The absence of state-contingent markets and the presence of a lower bound on asset accumulation are sufficient to produce a non-trivial distribution over assets.

Households rent capital and efficient labor units to the aggregate technology each period for which they receive interest r and wage w, determined by their respective marginal products in the aggregate production function.

The durable good, once produced, is indexed to the specific household. The durable good can not be traded without first converting the stock of the durable good into productive capital. The technology for production of the durable good can be thought of as location specific and we do not allow households to relocate. This market structure has the added benefit of giving us only a single market price rather than separate market prices for the durable good and productive capital. In addition, this market structure precludes a rental market in the durable good. It should be noted at this point, that a rental market would not change the general results of the paper so long as it was also subject to similar transaction costs. In addition, the ability to borrow against housing, perfect mortgage markets, would cause the rental market to be dominated by the purchase market. Note, in this world, rental units and owned housing would be perfect substitutes with the same equilibrium price. Only a model which priced rental housing and owned housing differently or in which the transaction costs differ significantly between the two sectors would produce substantivally different results.

3.5 The Household's Problem

We restrict ourselves to steady states. The individual household's state variables are its shock, its stock of the durable good, and its assets, $\{e, h, a\}$. The problem that the household solves is

$$v(e, H, a) = \max_{c,\ a',\ H'} \left\{ u(c, f(H')) + \beta \sum_{e'} \pi_{e,e'} v(e', H', a') \right\} \tag{1}$$

$$s.t.\ a' = ew + (1+r)a + D(H', H) - c$$

$$D(H', H) = \left\{ \begin{array}{c} 0 \text{ if } H' = \delta_h H \\ \\ (1-\lambda_s)\delta H - (1+\lambda_b)H' \text{ o/w} \end{array} \right\}$$

$$a' \in A = [\underline{\text{a}}, \infty),\ H' \in H = [0, \infty),\ c \in C = [0, \infty)$$

where r and w are the return on assets and the rental rate for efficiency units of labor. $D(H', H)$ takes choices of H' given H and converts them to current period assets. The function f takes H (a stock) and maps it into a flow. We require $f(0) = 0$, $f'(H) > 0$, and $f''(H) \leq 0$. In general, we choose $f''(H) = 0$. e, H, and a denote the agent's current state values of labor productivity, durable stock, and assets respectively. A prime denotes next periods value and c is current consumption, r is the interest rate net of depreciation, w is the wage rate, λ_s denotes the transaction costs associated with selling the durable stock and $\lambda_b$denotes those associated with buying. δ_h is the depreciation rate of the durable stock.

Under the condition set forth above and given the equilibrium conditions set forth below for r and w, the household's problem has a solution which we denote $a' = g^a(e,h,a)$, $h' = g^h(e,h,a)$, and $c = g^c(e,h,a)$ with an upper and lower bound on asset holdings, $\{\underline{a},\ \bar{a}\}$, and on durable holdings, $\{\underline{h},\ \bar{h}\}$, such that $\underline{a} \geq g^a(e,h,a) \geq \bar{a}$ and $\underline{h} \geq g^h(e,h,a) \geq \bar{h}$ for all $s \in S$. Where $\{s, S\}$ are the compact notation $s = \{e, h, a\}$ and $S = \{ExHxA\}$. With respect to assets the strict concavity of the aggregate production function is sufficient. With respect to the durable, we require either a positive depreciation rate or positive transaction costs which are proportional to either the current or the next period stock of durable capital.

It is possible to construct a Markov process for the individual state variables, from the Markov process on the shocks and from the decision rules of the agents (see Huggett(1993) or Hopenhayn and Prescott (1992) for details). Let B be the $\sigma-$algebra generated in S. Let $P(s,b)$ denote the probability that a household of type $\{s\}$ has of becoming a type $b \subset B$. P is then defined by the probability measure Q and the policy functions for h, and a as follows

$$\begin{aligned} P(s,b) &= Q(e, e' \in b) \text{ if } g^h \in b, \text{ and } g^a \in b \\ P(s,b) &= 0 \text{ o/w} \end{aligned} \tag{2}$$

P satisfies the following theorem

Theorem 1 *Let (S, B) be defined as above; let Q be a transition function on (E, ξ); and let $g : S \to \{HxA\}$ be a measurable function. Then P as described in 2 defines a transition function on (S, B).*

A probability measure μ over B exhaustively describes the economy by stating how many households are of each type. Note that the first moment of μ over e yields the aggregate labor input while the first moment over a yields aggregate capital and the first moment over h yields the aggregate durable stock.

Function P naturally describes how the economy moves over time by generating a probability measure for tomorrow μ' given a probability measure μ today. The exact way in which this occurs is

$$T^n\mu(b) = \int_S P^n(s,b)d\mu \tag{3}$$

If the process for the earnings shock is nice in the sense that it has a unique stationary distribution, then so has the economy. Specifically, we wish to apply the following theorem of Hoppenhayn and Prescott (1992).

Condition 1 (Monotone Mixing Condition (MMC)) *There exists a point $s \in S$ and an integer m such that $P^m(b,[a,s]) > 0$ and $P^m(a,[s,b]) > 0$.*

Theorem 2 *Suppose P is increasing, S contains a lower bound and an upper bound, and the MMC is satisfied: Then there is a unique stationary distribution μ^* for process P and for any initial measure μ, $T^n\mu = \int P^n(n,\cdot)\,\mu(ds)$ converges to μ^*.*

Since, Q is monotone and satisfies assumption $Q1$ and $Q2$, P inherits these properties and the economy has a unique stationary distribution (see Martin 1999 for details). Furthermore, this unique stationary distribution is the limit to which the economy converges under any initial distribution.

3.6 Equilibrium

We are now in a position to define a steady state equilibrium. We need only add the condition that marginal productivities yield factor prices as functions of μ. A steady state equilibrium for this economy is a set of function of the household problem $\{v, g^a, g^h, g^c\}$, and a measure of households, μ, such that: (i), Factor inputs are obtained by aggregating over households: $K = \int_S a d\mu$, and $L = \int_S e d\mu$; (ii), factor prices are factor marginal productivities, $r = F_1(K,L) - \delta$, and $w = F_2(K,L)$; (iii), given μ, K, and L, the functions $\{v, g^a, g^h, g^c\}$solve the household's decision problem described by 1; (iv), the goods market clears: $\int_S \left[g^c(s) + g^a(s) + g^h(s)\right] d\mu = F(K,L) + (1-\delta)K + \int_S D(g^h(s), h)d\mu$, and (v), the measure of households is stationary: $\mu(b) = \int_S P(s,b)\,d\mu$, for all $b \subset B$.

4 Calibration

Where possible, we will choose the key parameters of our model to those available in the data or in previous work. Since, our primary interest is in how transaction costs on durable stock adjustment effect consumption and saving decisions, our key calibration for the model will be in the type of transaction costs which we consider. There are basically two types of transaction costs - fixed transaction costs and proportional transaction costs. Fixed transaction cost models entail a fixed sum of assets, essentially a fee, which is independent of the size of the transaction. Proportional transaction cost models entail a fee which is determined by the amount the durable stock is changed. The implications of each model are very different in terms of the policy function for the durable.

The transaction cost structure which we feel best captures the structure of U.S. housing markets is one which incorporates both fixed and proportional transaction costs. Hence, the household pays a fee to sell their existing stock of housing and then pays a second fee to purchase the new stock. The fee on the sale of the existing stock of housing is a fixed cost because the household can not affect the size of the transaction in their current choice set. The fee on the purchase of new housing acts as an asymmetric proportional transaction cost. The transaction cost depends on the size of the transaction but is also decreasing in the size of the purchase. We make both of these costs proportional to the size of the relevant transaction, e.g. selling costs are proportional to the existing stock of the durable good. In order to determine the size of the transaction costs, we use CES data. The CES provides detailed information on the costs associated with the purchase and sale of housing. Total transaction costs in this data are highly variable with the median household paying costs on the order of 7 percent to sell their house and 2.5 percent to purchase. In our main simulations, we restrict ourselves to symmetric costs and choose 3 percent on sales and purchases as a conservative cost estimate. We also will provide results for a variety of other costs structures including asymmetric costs.

The parameter α determines the share of consumption allocated to the durable good. There has been some attempt in previous work to estimate α (for example Ogaki and Reinhart (1998)). However, the exact estimate of α is sensitive to choice of utility function and the existence of varying budget constraints. We choose to select our α by matching a particular point of the data in our equilibrium. Specifically, we will choose α such that the median household in our model holds the same ratio of durable stock to total wealth, the sum of durable and financial wealth, as that observed in the 2001 Survey of Consume Finances. We restrict ourselves to the consideration of homeowners, as our model does not allow for renters, aged 25 - 55 so as to only include working age households. We also eliminate households reporting business income, as we do not allow entrepreneurial assets or income. We find the median household has a ratio of durable stock to total wealth of .7, implying that the median household has negative financial assets when mortgages are classified as a financial asset. This treatment of mortgages is consistent with our model. A mortgage can be thought of as simply a short position in the bond market. The resulting α will differ depending on our choice of other parameters but is typically in the ball park of .7 which gives a thirty percent share to durable consumption in the period utility function

Next we come to the choice of income states and uncertainty. Most papers in this field follow Aiyagari and estimate a Markov process from an AR(1) process for income. The difficulty is that this process (at least using the techniques given in Tauchen (1986)) gives a transition matrix which except for a very large number of states does not well approximate the original process. While we give results for Aiyagari's original process, for our main simulations we use income states and transition matrices estimated directly from the PSID. One of the difficulties with PSID households is the existence of a subset of households which report very low (less than $100 per year) total family income. In order to avoid biasing our outcomes toward high Gini coefficients, we eliminate all low income households and to avoid retirement problems we subset to all households whose head is between 25 and 55 years of age. Finally, we use labor income as our benchmark rather than total family income in order to avoid picking up asset income and transfers.

We choose to approximate the income process from the PSID using a four state Markov process. We calculate the incomes process to be the same (in relative terms) as the median of each quartile (e.g. our first income state represents the median income in the first quartile etc.). We scale the incomes so that the aggregate labor supply is approximately 1. With the income states in hand we can compute transition probabilities.

The transition probabilities are computed as the average probability of transition between quartiles each year. We take the average over the years 1990-1994. That is, we find the probability that a household in the first quartile transitions to each quartile in the subsequent years and repeat this process for each quartile. We do this for the years 1990-1991, 1991-1992, 1992-1993, 1993-1994 then average the outcomes. The income process and transition matrix are reported in Table 1. Using the quartiles will prevent us from achieving the tails of the distribution. In effect, we are average away all high income households and as a result we reduce our ability to match the upper tails. Therefore, in actuality, the best test of our model will be how well the quartiles of our wealth distribution match that from the data rather than the tails.

The final elements to be calibrated are the risk aversion coefficient, σ,the time preference rate, β,and the depreciation rates on capital and housing. For our choices of risk aversion and time preference rates, we choose to tie our hands in the calibration by setting $\beta = .95$ and $\sigma = 4$. These numbers are fairly commonly used and maintaining these assumptions allows us greater comparability to previous work. In addition, changing the discount factor has significant implications in models with illiquid durable goods. As β decreases, the solution to the model moves towards one in which the durable is never adjusted. In other words, the agent discounts the future at a high enough rate that he no longer views the durable good as a choice variable affecting his other policy functions when he is sufficiently far from a boundary of the inaction region. The model in which this is true differs from a single sector model only in that the value function is multiplied by a constant. Hence, as one adjusts β, the transaction costs must also be adjusted or the problem is fundamentally different.

We choose the following per period utility function:

$$u(c, f(h)) = \frac{\left(c^{\alpha} (h)^{1-\alpha}\right)^{1-\sigma}}{1-\sigma}, \qquad \alpha \in [0,1], \ \ \sigma > 0$$

Notice, we have set the function f to $f(h) = h$. The results are sensitive to the choice of and to the fact that the curvature of f is zero. As always, the parameter α determines the share of durables in consumption. For $\alpha = 1$, the model is identical to Aiyagari's model above. The durable technology is dominated by the productive sector. The technology fixes the relative price of durables and non-durables, hence, if transaction costs were zero, the ratio of c to h would be constant over time.

The parameter α is crucial to our model. The parameter determines the share of durables in the period utility function The share is decreasing in α. We wish to calibrate this parameter so that the RA consumes approximately the same proportion of housing as the median household in the Survey of Consumer Finances. Under our maintained assumptions on f, this approach yields an α=.7 in our benchmark model. This value also gives us the intuitive value of spending approximately 1/3 of household income on housing on average over time. Sticking with this approach to calibrate some of our key parameters, we choose transaction costs of 3 percent on purchases and sales of housing. This percentage is on the low end of transaction reported in the Consumer Expenditure Survey as reported by Martin (2002).

5 Results

The results of our model are characterized by the distribution of the population of agents over holdings of durable goods and financial assets. To demonstrate how changes in the model parameters affect the solution of the model, we compute the Gini coefficient, and give the aggregate capital stock, and average durable stock in each version.

The effect of the durable good on the degree of precautionary savings is driven by two offsetting factors.

First, as the durable good becomes increasingly important to the agent, the agent ties up a larger portion of his wealth in the illiquid asset. Since this wealth is no longer available for the smoothing of nondurable consumption, the variation of nondurable consumption increases[8]. Second, the existence of transaction costs leads to increased local concavity of the value function or increased local risk aversion. Both of these factors increase the amount of precautionary savings undertaken by the agent. However, as α decreases short-term fluctuation of nondurable consumption are less costly to the agent. This is because the weight on nondurable consumption is less and because the agent consumes a smaller share of nondurable consumption. More important, however, is the fact that the agent only updates his durable stock infrequently. Therefore, period-to-period fluctuations in his income are much less important to him than the total fluctuation over say 20 periods. The size of the adjustment of the durable good makes the comparison non-trivial. The rate of convergence of the Markov transition matrix then determines how much more variable income is over the short run relative to the long run and the discount rate determines the relative importance of the fluctuation. In general, these two features combine to create a u-shaped response in precautionary savings to changes in the share of the durable good. A detailed analysis of this issue is given in the appendix on the precautionary motive.

We will start our simulations in the world where households are prohibited from borrowing. This world is actually quite close to that studied by Aiyagari (1994). For these simulations, which we will refer to as the base case, we set the durable share in the utility function to $.3 = 1 - \alpha$[9], the depreciation rate on capital to .1, and the depreciation rate on housing to 0. Throughout these simulations we will leave the technology and preference parameters as seen in Table 1. Because the Gini index is not bounded by 1 in the case in which some agents hold negative asset positions, we give, in addition to the Gini index, the distribution

[8] While the agent is free to borrow against his stock of the durable good, the user cost remains roughly constant and hence adjustments in total consumption to changes in permanent income must occur exclusively through the household's nondurable consumption.

[9] Because financial wealth is much more disperse than housing wealth. We can always increase the Gini index of total wealth by increasing the share of nondurable consumption in the utility function. The Gini index for total wealth is the weighted sum of the Gini index for housing and the Gini index for financial assets. Increasing the share, however, sacrifices the ratio of housing to total wealth.

of asset holdings as summarized by the wealth held in each quintile, the top 10, 5, and 1 percent of the population.

Table 4 gives the information for three different levels of the symmetric construction and deconstruction costs. We can see that the precautionary motive is increasing in the transaction costs and is substantially higher than in the risk free case. The stock of liquid assets increases between 7.8 percent and 17.5 percent over the model with no income risk and the same average labor supply. Increasing the transaction costs from 1 percent to 5 percent increases the stock of capital almost 9 percent. Notice, the distribution over capital is much more disperse than the distribution over holdings of the durable good an important empirical fact. The ratio of housing wealth to total wealth for the average agent is also very close to that found in the data. However, this parameter is pinned down, on average, by our choice of Cobb-Douglas utility and the share of durable goods in the consumption aggregator. Changes in transaction costs have a marginal effect on the dispersion in the distribution. The Gini index for wealth increases a mere 1.6 percent as we move from 1 to 5 percent transaction costs. The dispersion over housing is increased more but still only moves 2.7 percent.

The next set of simulations maintain all of the assumptions of the base case except that now the households are permitted to borrow against their durable stock[10]. The results are tabulated in Table 5. Notice the sharp increase in the Gini coefficient for financial assets. This increase occurs for two primary reasons. First, because some households hold negative assets (here just over 40 percent hold negative assets), positive asset households accumulate much more wealth before pushing down the interest rate. Therefore, rich households are richer and poor households poorer. This is not a statement on the welfare differences between the two setups. An agent given the choice of this economy or one without collateralized borrowing would always select this economy. The simulations yield the same basic interpretation with respect to

[10] In our set up, it is immaterial whether we keep track of loans explicitly collateralized by housing or if we just keep track of the single asset. As there is no default and all risk free assets must have the same return (i.e. there exists a single interest rate), this is without loss of generality. The only households for which the distinction matters, are those households with wealth below the present value of their labor income. For these households we can identify a portion of their assets as being specifically collateralized by their stock of the durable good.

increasing transaction costs. Notice, the level of precautionary savings falls in this model. This fall is a direct result of the relaxation of the household's borrowing constraint. There exists less risk in the economy when households have access to lending markets. Notice, the ratio of housing to total wealth is essentially unchanged between the two models.

6 Conclusion

We have studied the role illiquid housing markets play in determining the amount of precautionary savings and the wealth distribution in heterogeneous agents model economies with idiosyncratic uncertainty. Incorporating illiquid housing into a model with income risk increases the need for precautionary savings at all levels of wealth, and increases the level of precautionary savings substantially over the model without income risk. The level of precautionary savings is increasing in the size of the transaction costs and is decreasing in the household's ability to borrow against their durable stock. We find a larger effect of uncertainty on precautionary savings than the typical model. In a pure Aiyagari-style model, the level of precautionary savings is about 3 % of the aggregate capital stock. Our model raises the aggregate capital stock an additional 7-17% over that found in a model without risk.

Previous studies have examined models with a single type of capital. The relevant comparison between our model and those studies is in the Gini index for financial assets. In this dimension, we do quite well. Using an income process with much smaller volatility (in comparison to Castaneda et al) and a mean (aggregate labor supply of 1) comparable to Aiyagari's, we achieve a Gini index for financial assets on the order of .7. This is very close to that observed in the data once business wealth is excluded from the sample (see Table 3). Our Gini index for total wealth is only slightly higher than that found for financial assets in earlier work. The low Gini index for total wealth is a function of the very equal distribution of housing wealth in our model.

This model does not explain in its entirety the distribution of wealth in the United States. It does however make important strides in that direction. This paper shows that durable goods subject to transaction costs are likely to be critically important in explaining many of the aggregate variables. Notice, that in addition to increasing the dispersion of wealth in the economy, increasing the importance of the durable good, pushes down the risk free rate and increases the valuation of the durable stock. Therefore the premium for durables is very large in this model. While this model does not have a risky asset, this result shows that a model of this class may be very important for resolving the excess return on risky assets observed in the data. In addition, a similar model with aggregate risk may be able to explain the cyclical pattern of house price in the United States. The cyclicality of housing prices is quite different than that of other assets (Davis and Heathcote 2003).

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8 Computational Procedures

The difficulty with solving this problem is the nonconvexities in the constraint space induced by the transaction costs on the durable good. The nonconvexities prevent the use of Euler equation approximation techniques or policy function iteration. Hence, we solve the households problem using successive approximations to the value function. The household's problem contains two continuous state variables, productive and durable capital, and one discrete state variable, the current realization of the labor endowment shock. The labor endowment shock is naturally discrete and we need only discretize the other two variables. Since, we have only the ability to evaluate the value function a finite number of times, the difficulty comes in assigning vales to asset choices which lie between grid points. There are many ways to do this[11].

[11] Judd (1998) gives a very good treatment of methods to solve the households problem.

Our first attempts to solve the problem using tensor product splines and a variety of optimization routines were unsuccessful. The splines were too poorly behaved to allow the optimizer to find anything approximating the global solution. Our next approach involved linear approximation of the grid. The linear approximation required the use of slower optimization routines and proved infeasible. Our final successful approach to solving the agent's problem uses discrete state dynamic programming. This method is extraordinarily robust and converges linearly at approximately rate β (the time preference parameter). In order to decrease computational time, we use a discrete approximation to the golden rule search method presented in Press et. al. (1992). The search must be done in each direction of the decision problem. In other words, we must first take the choice of durable state as given and then find the optimal asset choice given next period's durable state. This gives us the value corresponding to that choice of durable stock. We then search over choices of durable stock in order to find the optimal solution. Because we are not interpolating between the grid points the algorithm is very fast and the number of points which are evaluated are successively smaller percent of the grid space as that grid size increases. Hence, moving from a grid of 10 to 1000 in the asset direction requires, on average, less than 50 extra function evaluations at each state. The trade-off from not using a smooth approximation to the value function comes in the form of the necessity of computing the value function at many more grid points than would be necessary if we were using a successful interpolation technique.

With the solution method for the agent's problem in hand, we must compute the equilibrium distribution and hence the equilibrium prices for the model. The technique is essentially that used by Aiyagari (1994) or Rios-Rull (1998), guessing an initial capital stock and hence the initial wages and interest rate, then solving the agent's problem taking the prices as given. Compute the distribution implied by the solution to agent's problem. If the capital stock matches the stock which is guessed the problem has converged and the computation is complete. Otherwise, update the capital stock and iterate to convergence.

This class of models is very sensitive to the interest rate. Slight changes in the interest rate can lead to large changes in the distribution. As a result, the above algorithm is very sensitive to how the interest rate is updated between iterations. We use the following procedure. Fist, find the lower and upper bounds for the interest rate. The interest rate is bounded above by $1/\beta$ and is bounded below zero. Choose a point in between the upper and lower bounds and evaluate the agents problem. If the new interest rate is above the interest rate chosen, choose the new lower bound to be the old interest rate and update the upper bound if the new interest is below the initial rate. This method successively bounds the interest rate. The convergence criteria should be computed from the distance between the new and the old interest rate. Failure to converge almost invariably implies that the problem was solved on too coarse a grid for the desired accuracy.

The program is written in Fortran. The code is compiled using Compaq Visual Fortran Professional Edition We are happy to provide the code upon request.

9 Precautionary Motive with the Durable Good

The change in the precautionary motive in the presence of the durable good is not trivial. Whether the precautionary motive increases or decreases depends on the way in which the durable enters into the period utility function, the discount factor, the coefficient of relative risk aversion, the size of the transaction costs, the difference between the stationary transition matrix and the one step transition matrix as well as the rate of convergence.

The effect of the durable good on the degree of precautionary savings is driven by three offsetting factors. First, as the durable good becomes more and more important to the agent the agent ties up a larger portion of his wealth in the illiquid asset. Since this wealth is no longer available for smoothing of nondurable consumption, the variation of nondurable consumption increases, committed expenditure risk. As the agent is allowed to borrow against his stock of the durable good, this affect is less strong than a model with tighter borrowing constraints; but, since the user cost is strictly positive so is this effect. Second, the durable good is updated relatively infrequently. As a result, period-by-period aggregate consumption is smoothed within the inaction region, this effect is increasing in the weight of the durable good in the utility function. Third, while the durable good is updated infrequently, movements in the durable stock are large at the boundaries of the inaction region. Hence, even though the adjustments are infrequent, they contribute significantly to lifetime consumption volatility. The relative importance of this factor is determined by the rate of time preference and the volatility of the income process over long versus short time horizons. The more persistent is the income process, the greater this difference will be.

To understand how the various affects are working in our model, we will rewrite the problem and then examine the problem between stopping times (a stopping time is defined as the state-dependent date when the agent updates their capital stock). We can always write the agent's problem as the following sequence problem:

$$\max_{\{A_{t+1},c_t,h_{t+1},\}_{t=0}^{\infty},\{\tau\}} \sum_{t=0}^{\infty} \beta^t EU\left(c_t, h_{t+1}\right)$$

$$s.t.\ c_t + A_{t+1} \leq r_t A_t + we_t \quad \text{ if } t \notin \tau$$

$$c_t + \left(1+\lambda_b\right) h_{t+1} + A_{t+1} \leq r_t A_t + we_t + \left(1-\lambda_s\right) h_t \quad \text{if } t \in \tau$$

where τ is the set of stopping times for the problem and all other variables are as described in the body of the paper. We can from this formulation always rewrite the above summation as the maximum over stopping times of the interval between every stopping time. In other words, we can turn the maximization problem into a two step maximization by first choosing sequences of A, c, and h conditional on a sequence of stopping times and then maximizing over stopping time. Conditional on the sequence of stopping times, each subproblem is written in the following manner:

$$\max_{c_\tau,h_{\tau+1},A_{\tau+1}} U\left(c_\tau, h_{\tau+1}\right) + \max_{c_t,A_{t+1}} E \sum_{t=1}^{\tau'-1} \beta^t U\left(c_t, h_{t+1}\right) + \max_{c_{\tau'},h_{\tau'+1},A_{\tau'+1}} E\beta^{\tau'} U\left(c_{\tau'}, h_{\tau'+1}\right)$$

the fact that the adjustment in durable good is separated by time makes all the difference in this model. Obviously, decreases in the rate of time preference will decrease the importance of adjustment of the durable good and will reduce the precautionary motive. The greater the transaction costs the greater the relative distance between τ and τ' and the more effect small changes in β are likely to have. In addition to the weight given by the discount factor, fluctuations in h at the next stopping time are driven by the expectation in τ'. The properties of this term are then, in turn, driven by the n-step transition matrix, where n is equal to the distant $\tau' - \tau$.

An additional source of variation, since it also affects the marginal utility of nondurable consumption is the fact that τ' is also a function of the parameters of the problem. The expectation must fall outside the summation since τ' is a stopping time adapted to the filtration generated by $\{e_t\}$. Most significant is the fact that the precautionary motive is a function of the holding period for the home. Changes in the variability of the income stream affect not only c and h but also the distance between τ and τ'. Changes in the underlying structure which increase the length between transaction (increases in transaction costs, higher proportion of durable in the utility function, increased income variability) tend to raise the value of housing while changes that reduce the time lower the value. At the same time, changes in the parameters of the problem which reduce the value of the durable stock tend to reduce the user value. Hence, increases in the transaction costs also have a component of decreasing the cost.

10 Data Appendix

We will calibrate and test the model to U.S. survey data. Estimates of transaction costs are derived from the Survey of Consumer Expenditures. In that survey, households are asked to estimate the total financial costs of moving. While the households report a wide variety of associated relocation expenses, we use their estimates of taxes and agency fees. We find transactions costs account for approximately 7.5 percent of the value of the purchased home. We split these costs symmetrically between buying and selling. Our income process is estimated from the Panel Survey of Income Dynamics. We are able to estimate both the relative variance in the income process and the transition matrix directly from this data. Using the panel frees us from having to estimate the transition probabilities. Notice, the transition probabilities given from the data are quite different from those derived from estimating the transition probabilities from an AR(1) process. Finally, we test the model by comparing the distribution statistics in several dimensions with the Survey of Consumer Finances. We compare the Gini indices for total wealth, housing wealth, and financial wealth. We also match the relative dispersion and median holdings of financial wealth and housing wealth.

The model is compared to wealth statistics taken from the 2001 Survey of Consumer Finances (SCF). The SCF comprises detailed wealth and asset holding information for 4442 respondents. Our model is calibrated to a sub-sample of the total survey. We only consider households with a household head between the ages of 25 and 55, in order remove non-working age households, as well as those nearing retirement, from the sample. Our model does not allow for a retirement state, so that the proper sample excludes retired households as well as those households near enough to retirement that their saving and consumption behavior is likely to be impacted. We also eliminate households with business wealth and other property wealth. The model allows two assets, financial wealth and a consumption-yielding durable good, we do not allow entrepreneurial wealth. Additionally, our income process is derived from the PSID excluding households reporting business income. Thus, consistency demands that we remove these households from our wealth statistics. Finally, households are required to own housing in our model, so that we only consider households reporting a positive stock of housing. The sub-sample includes 970 households.

Table 2 compares our sub-sample to the entire 2001 SCF sample. As expected the sub-sample is poorer in the mean but richer in the median than the entire sample, as we have removed a number of extremely wealthy entrepreneurial households with substantial business wealth. Sub-sample households own more housing, though less financial assets on net, as they are more apt to carry mortgages. The sub-sample is younger but has roughly the same median and mean income as the total sample, both as a result of our exclusion of retirement households.

The distribution of assets across quintiles of the sub-sample is depicted in Table 3. Wealth is considerably more evenly distributed in the sub-sample, both because the high end of the distribution has been curtailed by the elimination of owners of business wealth, and the low end shortened by the consideration of home-owners. Housing is much more evenly distributed in the total sample than is total wealth, largely because of compression in the high end of the distribution. Table 2 also reports the percentage of total wealth held as housing equity for each quintile of the total wealth distribution, considering only those households with positive total wealth. In the total sample the importance of housing equity peaks in the third quintile, while in our home-owning sub-sample the importance of housing equity is uniformly decreasing. In the complete sub-sample, including households with non-positive total wealth, the median ratio of total wealth to house value is .95, implying that the median household holds a negative financial asset position equal to 5 percent of their house value.

Table 1: Model Parameters

β	σ	δ_K	δ_D	θ	α
.95	4	.1	0	.36	0.7

Income States

[.3198	.7144	1.0885	1.7689]

Transition Matrix

.6500	.2227	.0753	.0520
.2250	.5460	.1828	.0462
.0857	.1739	.5596	.1808
.0500	.0382	.1477	.7641

Table 2: Sample Statistics 2001 SCF

		Total Wealth	House Value	Net Financial Assets	Income	Business Wealth
Total Sample	Mean	398,710	122,670	137,730	69,152	75,947
	Median	86,620	75,000	586	40,089	0
Sub-Sample[a]	Mean	203,810	142,400	35,612	69,844	
	Median	99,800	110,000	-20,180	56,741	

		Age	Other Property Wealth
Total Sample	Mean	49	21,358
	Median	47	0
Sub-Sample[a]	Mean	41	
	Median	42	
Sample Size			
Total Sample			
Sub-Sample			

[a]Sub-sample excludes households reporting business and other property wealth as well as households with a head of household younger than 25 or older than 55.

Table 3 – Wealth and Asset Distributions 2001 SCF

	Gini	Quintile First	Second	Third	Fourth	Fifth	Top 90-95%	95-99%	99-100%
Total Wealth									
Total Sample	.81	-.11	1.25	4.48	11.79	82.60	12.26	25.28	32.09
Sub-Sample[a]	.63	1.19	4.67	9.83	19.02	65.30	13.38	18.96	16.04
Housing Value									
Total Sample	.64	0	1.52	12.24	22.56	63.69	13.89	18.44	12.68
Sub-Sample[a]	.41	4.93	10.79	15.26	22.12	46.91	11.36	12.94	6.41
Housing Equity / Total Wealth[b]	Mean								
Total Sample	37.6	22.9	39.5	47.4	40.0	16.3	21.9	16.4	6.9
Sub-Sample[a]	54.4	85.3	58.6	51.7	46.0	32.6	29.3	26.5	22.4
House Value / Total Wealth	Median	Mean							
Sub-Sample[a]	1.05	.69							

[a]Sub-sample excludes households reporting business and other property wealth as well as households with a head of household younger than 25 or older than 55.
[b] Percentage of total wealth held as housing equity for each quintile of the total wealth distribution for households reporting positive total wealth.

Table 4 – Model Simulated Wealth and Asset Distributions – a=0

	Gini	Quintile First	Second	Third	Fourth	Fifth	Top 90-95%	95-99%	99-100%
Housing Value									
λ_b=.01 λ_s=.01	.2356	.0800	.1493	.2102	.2497	.3108	.0794	.0684	.0184
λ_b=.03 λ_s=.03	.2461	.0765	.1469	.2284	.2304	.3178	.0817	.0722	.0193
λ_b=.05 λ_s=.05	.2529	.0764	.1428	.2059	.2538	.3214	.0827	.0724	.0200
Financial Wealth									
λ_b=.01 λ_s=.01	.5963	.0052	.0343	.1070	.2536	.5999	.1595	.1687	.0481
λ_b=.03 λ_s=.03	.5903	.0071	.0407	.0399	.3091	.6032	.1615	.1733	.0471
λ_b=.05 λ_s=.05	.5998	.0070	.0409	.0994	.2301	.6226	.1676	.1730	.0443

Aggregate Stocks	K	H	H/TW	r	Gini TW[b]	Prec Sav[a].
λ_b=.01 λ_s=.01	4.12	10.52	.71	1.045	.3300	**7.8%**
λ_b=.03 λ_s=.03	4.21	10.25	.71	1.044	.3316	**10.2%**
λ_b=.05 λ_s=.05	4.49	10.19	.70	1.042	.3399	**17.5%**

a. Precautionary Savings refers to the increase in capital stock over the economy without income risk.
b. Gini index for total wealth.

Table 5 – Model Simulated Wealth and Asset Distributions – a=natural limit

	Gini	Quintile First	Second	Third	Fourth	Fifth	Top 90-95%	95-99%	99-100%
Housing Value									
λ_b=.01 λ_s=.01	.2422	.0731	.1498	.2142	.2528	.3102	.0789	.0691	.0190
λ_b=.03 λ_s=.03	.2622	.0664	.1454	.2094	.2546	.3240	.0830	.0737	.0198
λ_b=.05 λ_s=.05	.2730	.0625	.1424	.2091	.2547	.3313	.0854	.0763	.0203
Financial Wealth									
λ_b=.01 λ_s=.01	.7250	-.0359	-.0101	.0703	.3055	.6702	.1762	.1790	.0477
λ_b=.03 λ_s=.03	.7137	-.0329	-.0011	.0751	.2867	.6722	.1791	.1820	.0470
λ_b=.05 λ_s=.05	.7159	-.0410	.0114	.1288	.2669	.6835	.0854	.2007	.0378

Aggregate Stocks	K	H	H/TW	r	Gini TW[b]	Prec Sav[a].
λ_b=.01 λ_s=.01	4.213	10.189	.71	1.0448	.3777	**10.2%**
λ_b=.03 λ_s=.03	4.234	9.946	.70	1.0470	.3823	**10.7%**
λ_b=.05 λ_s=.05	4.324	9.746	.69	1.0402	.3861	**13%**

a. Precautionary Savings refers to the increase in capital stock over the economy without income risk.
b. Gini index for total wealth.