

Career Progression and Formal versus on the Job Training

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February 14, 2003

VERY PRELIMINARY AND INCOMPLETE

Abstract

This paper evaluates the return to formal education over the life-cycle and compare it to informal, on the job training. More specifically, we assess the apprenticeship system in Germany by comparing the long run value of education choices and subsequent labor market outcomes for apprentices and non-apprentices. We develop a structural model of career progression and educational choice, allowing for unobserved ability, endogenous job to job transition, specific firm-worker matches, specific returns to tenure and to general experience. We estimate this model on a large panel data set which describes the career progression of young Germans. We find that formal education is more important than informal training, even when taking into account for the possible selection into education. We use the estimated model to evaluate the long-run impact of labor market policies on educational choices and career progression. We find that policies such as the Earned Income Tax Credit which subsidize low wage have a detrimental effect on the probability of further education and on job mobility.

1 Introduction

This paper evaluates the return to formal education over the life-cycle and compare it to informal, on the job training. More specifically, we assess the apprenticeship system in Germany by comparing the long run value of education choices

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2 The Data Set

2.1 The Data Set

We use a 1% extract of the German social security records. The data set follows a large number of young individual from 1975 to 1995. For each individual in the sample, we get the exact employment date (starting date, end date) for each job. The data set also reports the daily wage each year if the individual stays an entire year, or for the part of the year the individual works for the firm. We aggregate the data to obtain information on a quarterly basis.

The data set also reports the periods of apprenticeship training. For the purpose of this study, we select our sample to consist only of West-German males, with only post-secondary education and who start either work or an apprenticeship after school. In total, we follow 27525 individuals through time, quarter after quarter up to 1995.

2.2 Descriptive Data

Figure 1 displays the log wage profile as a function of years of labor market experience for apprentices and non apprentices. Individuals with an apprenticeship training have on average higher wages, but a flatter wage profile. In contrast, non apprentices start at a low wage and experience a rapid wage growth, but the wage gap never closes.

Apprentices are also more likely to work as shown in Figure 2. This is especially true in the first years in the labor market. Figure 3 shows the proportion of workers exiting from the labor market as a function of labor market experience and education. Similarly, apprentices are more likely to re-enter the labor market, conditional on not working. After ten years of experience, both education groups appear to have the same re-entry behavior.

Figure 1: Log Wage and Labor Market Experience

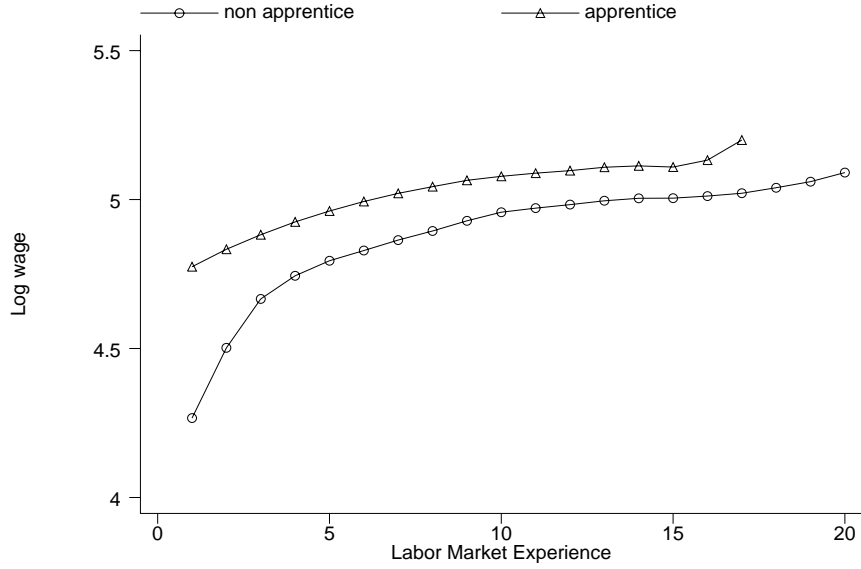


Figure 2: Proportion not Working and Labor Market Experience

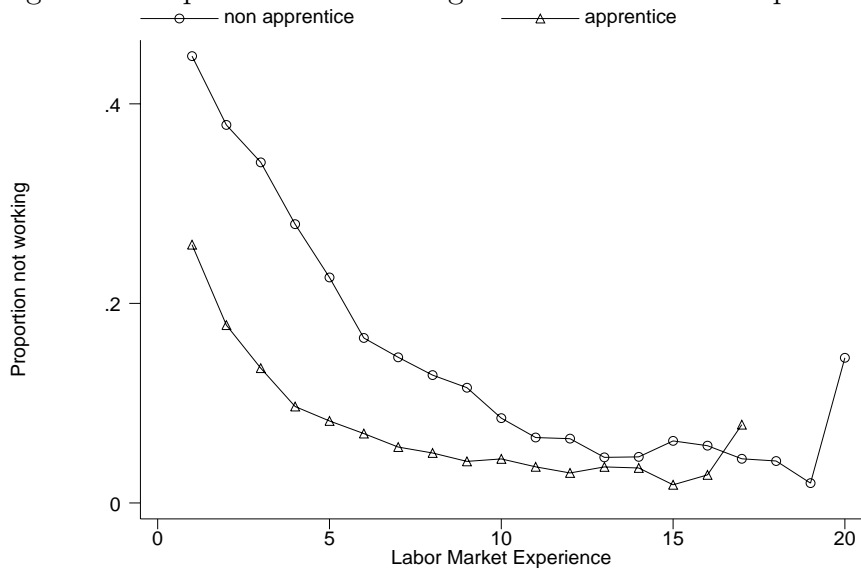


Figure 3: Probability of Exit from Labor Market

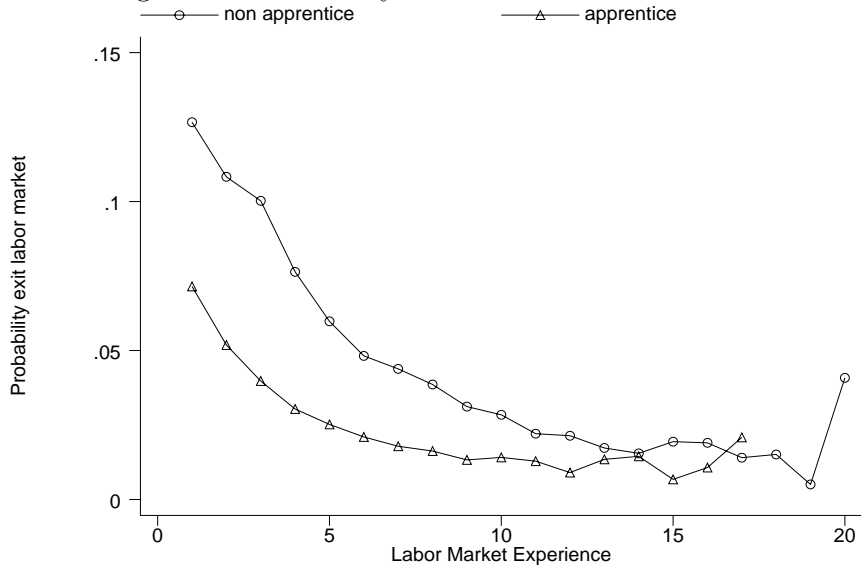
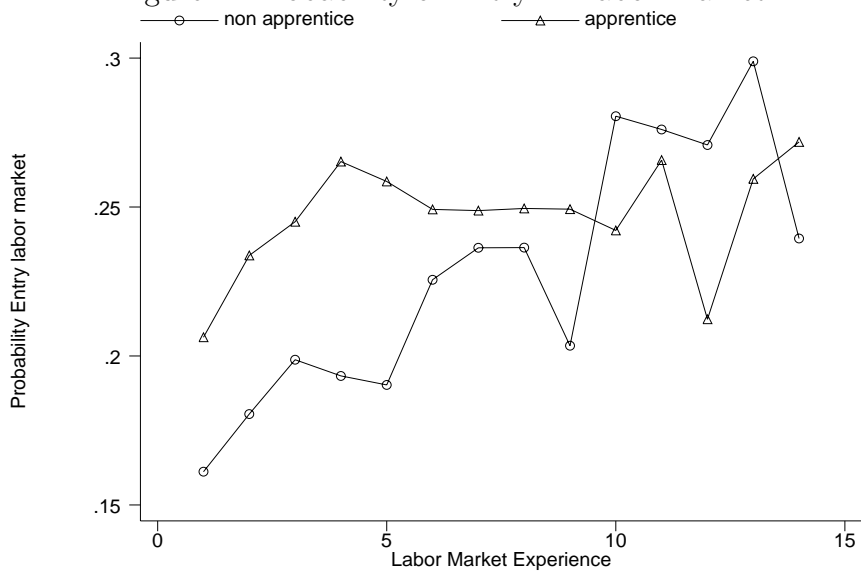
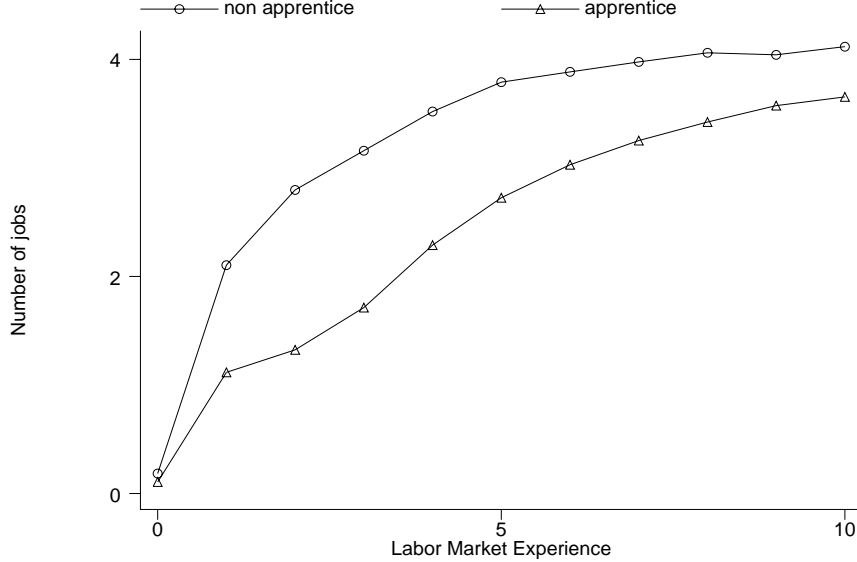


Figure 4: Probability of Entry in Labor Market



The two groups also differs by the number of jobs hold over time. Non apprentices are much more mobile, going from firm to firm, especially in the first years (Figure 5).

Figure 5: Cumulative Number of Jobs and Labor Market Experience



Next, we try to decompose the wage growth into different components. Figure 6 displays the changes in the log wage for individuals who change jobs. In the first years in the labor market, the wage growth can be substantial, at about 30% for non apprentices and 10% to 20% for apprentices. The gain in wages reduces over time, decreasing towards zero.

Figure 7 displays the wage growth conditional on staying with the same firm for two consecutive periods. The wage growth is of an order of 1 to 2% and is higher in the first 4 years for non apprentices.

Hence, most of the wage growth is due to job to job transition and very little to gains in experience or tenure. It appears that the rapid wage growth of non apprentices is mostly due to better matches and job search in the early years. However, the results in both Figures are potentially biased, because mobility may be endogenous. Our model will be able to disentangle the selection effect from the determinant of wage growth.

Figure 6: Changes in Log Wage (Between) and Labor Market Experience

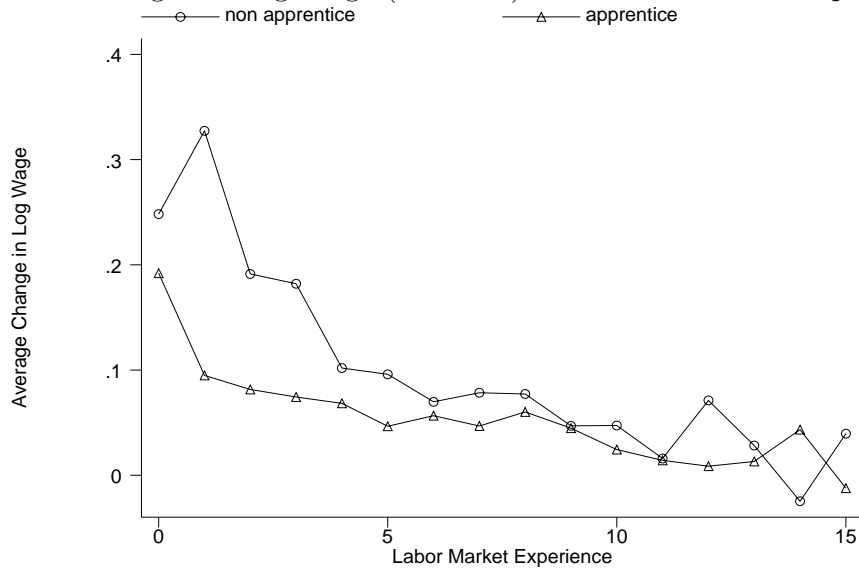


Figure 7: Changes in Log Wage (Within) and Labor Market Experience

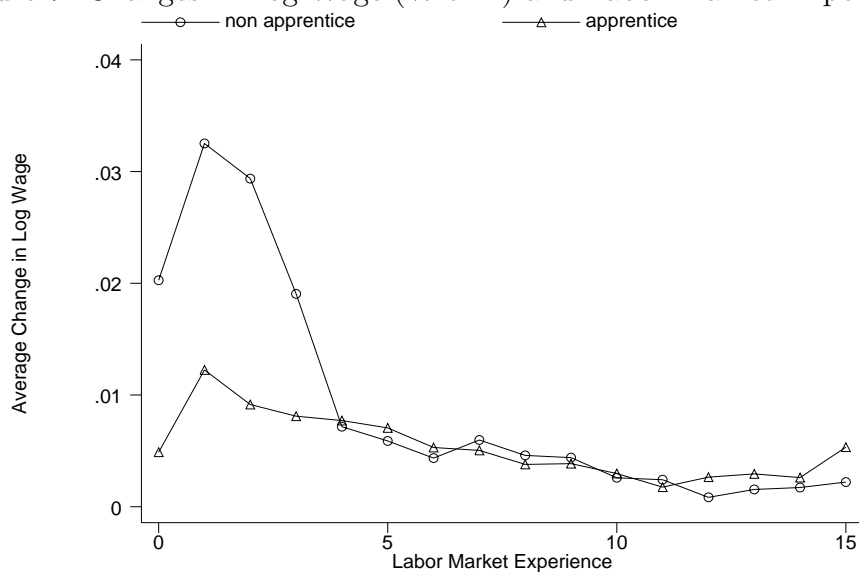
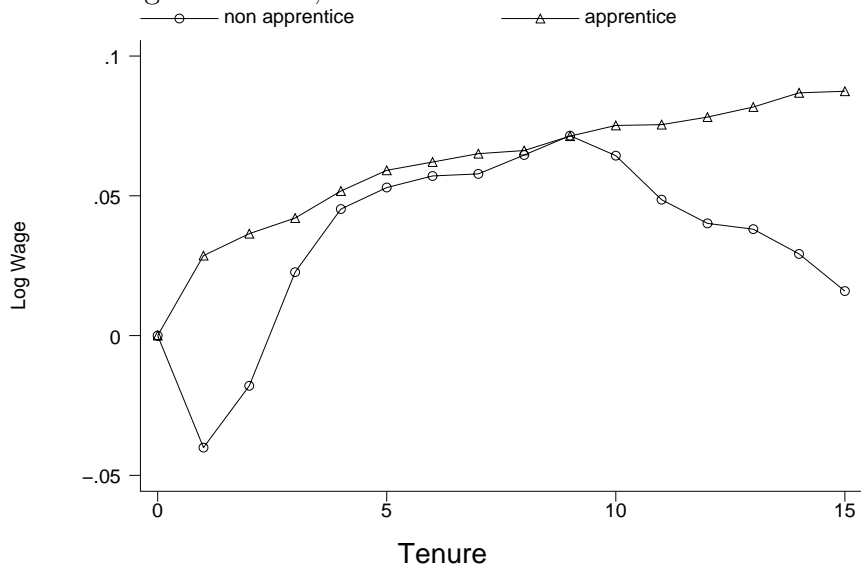


Figure 8: OLS, Estimation of Return to Experience



Figure 9: OLS, Estimation of Return to Tenure



3 The Model

We model jointly the education decision and the labor market progression for a given individual. The decision to go into apprenticeship or not depends on the future returns to apprenticeship, so we will first detail the model of career progression conditional on education choices.

3.1 Labor Market

Consider an agent with education level Ed , equal to either one (apprenticeship training) or zero (only post secondary education). While working, the agent accumulates general labor market experience, which is a function of the number of years in the labor market, X , and firm specific experience, which is a function of the number of years spent in a particular firm (tenure or seniority), T . Each employed individual experience a firm-worker match specific effect, denoted $\kappa(T)$. We allow this effect to be time varying and correlated through time. We model it as a random walk:

$$\kappa(T) = \kappa(T - 1) + u \quad V(u) = \sigma_u^2 \quad V(\kappa(0)) = \sigma_0^2$$

The match specific effect is indexed by the tenure as its variance is non stationary. The wage is a function of labor market experience, tenure, education, ability, denoted ε , the match specific effect and of an aggregate business cycle effect, denoted G :

$$w = w(X, T, Ed, G, \varepsilon, \kappa(T))$$

Two alternative job offers differs because of the match specific effect.

In any given period, the agent can be either employed or not employed. If unemployed, the agent gets unemployment benefits and receive with a probability λ_U a job offer. The agent rejects the offer if the proposed wage is below his reservation wage, waiting for a better offer (match specific effect) to come.

If employed, the agent derives a utility from the wage and keeps his job next period with a probability $1 - \delta$. The agent may also receive a job offer, while on the job, with a probability λ_W . The agent would move to a new firm if the wage is higher than the reservation wage.

We denote the value of working, i.e. the intertemporal flow of discounted utility as W_t . Similarly, we denote U_t the value of unemployment. The value of working is defined recursively as:

$$\begin{aligned} W_t(X, T, G, Ed, \varepsilon, \kappa(T)) = & w_t(X, T, Ed, G, \varepsilon, \kappa(T)) \\ & + \beta \delta E_{G'} U_{t+1}(X + 1, T + 1, G', Ed, \varepsilon, \kappa(T)) \\ & + \beta(1 - \lambda_W)(1 - \delta) E_{G', \kappa'} W_{t+1}(X + 1, T + 1, G', Ed, \varepsilon, \kappa(T + 1)) \\ & + \beta(1 - \delta) \lambda_W E_{G', \kappa', \tilde{\kappa}'} \max[W_{t+1}(X + 1, T + 1, G', Ed, \varepsilon, \kappa'(T + 1)), \\ & W_{t+1}(X + 1, 0, G', Ed, \varepsilon, \tilde{\kappa}'(0))] \end{aligned} \tag{1}$$

The value is the sum of the wage plus the discounted value of the future. With a probability δ the agent goes into unemployment, at the end of the period. He then starts next period with one additional unit of experience and of tenure, faces a new state of aggregate business cycle, and stay with the same level of the match specific effect.¹ The agent has rational expectation over the future aggregate state of the economy G' . With a probability λ_W , the agent receives an alternative offer, characterized by the match specific effect $\tilde{\kappa}'(0)$. The agent compares the value of staying in the current firm with the value of moving. If the agent moves to the new firm, he loses all tenure and starts at zero.

The value of unemployment is defined recursively as:

$$\begin{aligned}
U_t(X, T, G, Ed, \varepsilon, \kappa(T)) &= \alpha w_t(X, T, Ed, G, \varepsilon, \kappa(T)) & (2) \\
&+ \beta(1 - \lambda_U) E_{G'} U_{t+1}(X, T, G', Ed, \varepsilon, \kappa(T)) \\
&+ \beta \lambda_U E_{G', \tilde{\kappa}'} \max[U_{t+1}(X, T, G', Ed, \varepsilon, \kappa(T)), \\
&W_{t+1}(X, 0, G', Ed, \varepsilon, \tilde{\kappa}'(0))]
\end{aligned}$$

α is the fraction of the last wage the individual receives in unemployment benefits. With a probability $1 - \lambda_U$, the individual does not receive a job offer and stays in unemployment next period and faces a potential different business cycle G' . If a job offer is received, consisting in a match specific effect $\tilde{\kappa}'(0)$, the agent opts for the choice which maximizes his intertemporal utility.

The job arrival and destruction rates are heterogenous across workers and depends on the time spent in the labor market, the tenure, the ability of the agent, the business cycle and the match specific effect:

$$\begin{cases}
\delta = \delta(X, T, G, Ed, \varepsilon, \kappa) \\
\lambda_W = \lambda_W(X, T, G, Ed, \varepsilon) \\
\lambda_U = \lambda_U(X, G, Ed, \varepsilon)
\end{cases}$$

We assume that the business cycle can be described by a first order process:

$$G_t = \mu_G + \rho_G G_{t-1} + u_{Gt} \quad G_t = \ln(\text{Detrended GDP}_t)$$

3.2 Structure of education choices

Denote the value of apprenticeship as: $A(G, R, \varepsilon, T^A)$, where G is an indicator of the current business cycle, R is the region of living, ε is a measure of ability and T^A is the length of apprenticeship. We model the value of going into apprenticeship as:

$$\begin{aligned}
A(G, R, \varepsilon, T^A) &= c_A(G, R, \varepsilon) + \eta^A + \delta \beta^{T^A} E_{G'} U(T^A, T^A, G', 1, \varepsilon, \kappa(T^A)) \\
&+ (1 - \delta) \beta^{T^A} E_{G'} W_t(T^A, T^A, G', 1, \varepsilon, \kappa(T^A))
\end{aligned}$$

¹The value of unemployment keeps track of this match specific effect as well as the tenure in order to calculate the unemployment benefits, defined as a fraction of the last wage.

An apprentice incurs a cost which depends on the region of living, the business cycle and unobserved heterogeneity. In addition, the agent receive a taste shock η^A , assumed to follow an extreme value distribution. After T^A periods, the agent is a trained apprentice with T^A periods of experience and tenure. The agent can either keep the job within the same firm or be fired.

The value of not going into apprenticeship is expressed as:

$$NA(G, \varepsilon) = \eta^{NA} + U(0, 0, G, 0, \varepsilon, \underline{k}(0))$$

i.e., the agent looks for a job having zero tenure and experience. Here again, the agent experiences an extreme value distributed taste shock. Taking advantage of the property of the extreme value distribution, the probability of going into apprenticeship is expressed as:

$$P(\text{Apprentice}|G, R, \varepsilon) = \frac{\exp(A(G, R, \varepsilon, T^A))}{\exp(NA(G, \varepsilon)) + \exp(A(G, R, \varepsilon, T^A))}$$

Hence, the education choices depend on the return to apprenticeship, which differs by ability, across time through the business cycle effect and across regions. We rely on regional differences which shifts the cost of training to better identify the parameters of the model.

The model is estimated by Maximum Likelihood. The likelihood is detailed in the appendix. We follow each individual from the schooling decision and during his career for up to 20 years. For a given individual, we compute the likelihood of the education choice, the likelihood of any transition in or out the labor market, the likelihood of job to job transitions and the likelihood of observing a particular wage. We allow for fixed unobserved heterogeneity (ability) by assuming a discrete mass point distribution with two points of support.

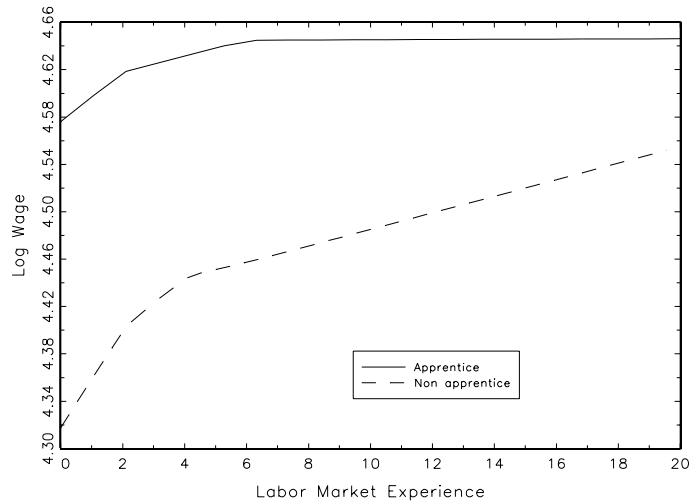
4 Results

In total we have over 100 parameters. The results are best seen through a series of graphs.

Figure 10 displays the effect of labor market experience on log wages, conditional on ability and on staying forever in the same firm (with a zero match specific effect), for apprentices and non apprentices. The increase in log wage is due to the combined effect of the return to experience and tenure. Non apprentices starts at a lower wage but catch up rapidly in the first two years. However, even after 20 years, there is still a gap between education groups suggesting that the value of training is positive. The wage growth for apprentices is very low, approximately 0.3% per year.

Figure 11 displays the effect of unobserved ability on wages. High ability types earn up to 30% more. Next we break down the increase in the log wage

Figure 10: Effect of Labor Market Experience on Wages, Conditional on Ability



between the return to experience and to tenure. For apprentices, the return to tenure is about 0.75% per year, with a non linear pattern. The individual learns more in the first years. The return to experience is extremely low. In contrast, the return to tenure for non apprentices is higher at about 1.7% per year and the return to experience is at about 1% per year. Here again, most of the effect is in the early years.

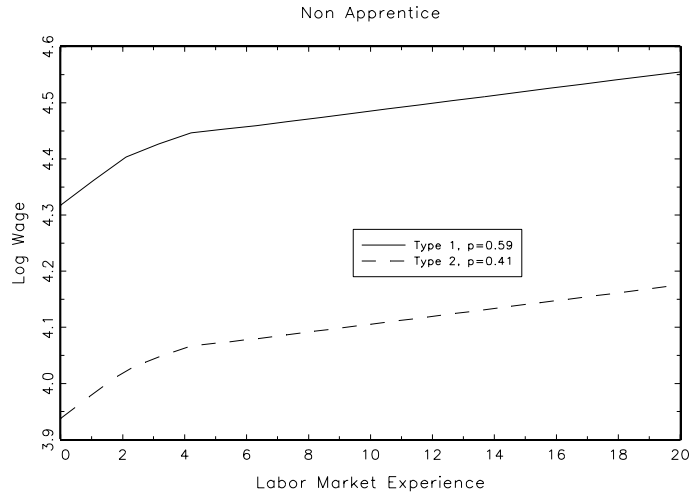
Figure 13 displays the probability of apprenticeship training by region and by ability. High ability individuals are more likely to go through apprenticeship. This explains partly the differences in wages, but not entirely as shown in Figure 10. The probabilities also vary by region, reflecting different costs.

5 Policy Evaluations

In this section, we evaluate the effect of labor market policies on career progression and education choices. In particular, we evaluate the effect of in-work benefits on human capital accumulation and acquisition of skills. These policies offer subsidies to employed individuals with a low wage. Examples of such policies are the Earn Income Tax Credit (EITC) in the US and the Working Family Tax Credit (WFTC) in the UK. These policies are in place to encourage labor market participation.

We simulate a reform similar to the EITC, where low wage individuals get a subsidy. This subsidy starts at 0 for a zero wage, increases with the wage up to

Figure 11: Effect of Ability on Wages, Non Apprentices



a first limit, stays constant over a range of income and finally declines to zero. Hence, two categories of individuals do not receive a subsidy: individuals not working and individuals with a high enough wage.

In general, these in-work benefit policies have an effect on labor market participation. However, these policies could also have detrimental long-term effects on education choices and skill acquisition. As lower wages are subsidized, individuals are less likely to obtain higher education levels as the wage gap between education groups might decrease. Second, due to the non linearity of the benefits, the policy might discourage job-to-job mobility. This would reduce the mobility of workers across jobs and slow down or prevent the best matches between firms and workers to form, decreasing over-all productivity.

Figures 14 to 16 show the results of the policies simulated from the estimated model. The policy has a positive impact on labor participation but a negative one on overall productivity.

6 Conclusion [to be written]

Figure 12: Return to Experience and Return to Tenure

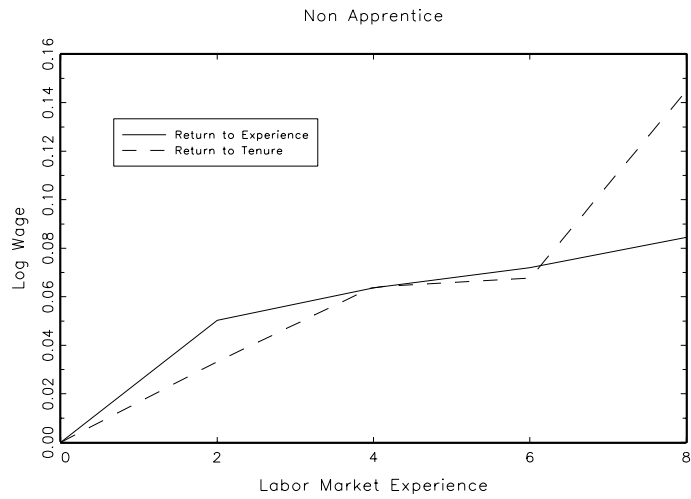
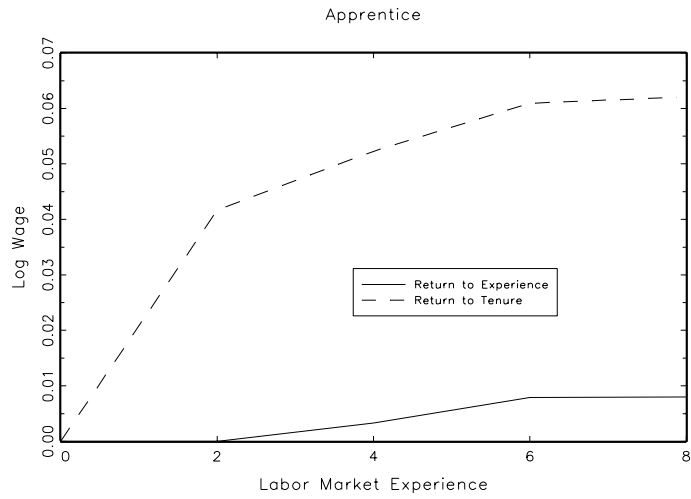
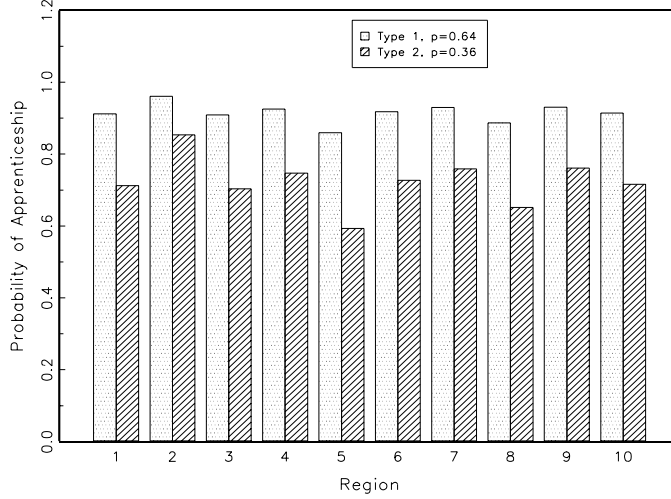


Figure 13: Probability of Apprenticeship Training by Region and Ability



Appendix

A Likelihood

The likelihood is constructed by multiplying the sequence of transition probabilities conditional on the person-specific effect and integrating it out.

A.1 Likelihood of Transitions

The data consists in a succession of transitions from one state to the other and in a succession of wages. The possible states are $S = U$, unemployed, $S = E$, employed or $S = \tilde{E}$, employed in a different firm from previous period. In the first period, the agent is by assumption unemployed, so that $S_1 = U$ and $w_1 = .$

$$L(\varepsilon) = P(S_T, S_{T-1}, \dots, S_1, w_T, \dots, w_1 | \varepsilon)$$

with the convention that $w_t = .$ if unemployed. Given the first order correlation in the match specific shock, the likelihood can be written as:

$$L(\varepsilon) = P(S_T, w_T | S_{T-1}, w_{T-1}, \varepsilon) \dots P(S_2, w_2 | S_1, w_1, \varepsilon) P(S_1, w_1 | \varepsilon)$$

with $P(S_1, w_1) = 1$. We next characterize the conditional probability $P(S_t, w_t | S_{t-1}, w_{t-1}, \varepsilon)$. There are five possible sorts of transitions:

Figure 14: Effect of In-Work Benefits on Labor Market Participation

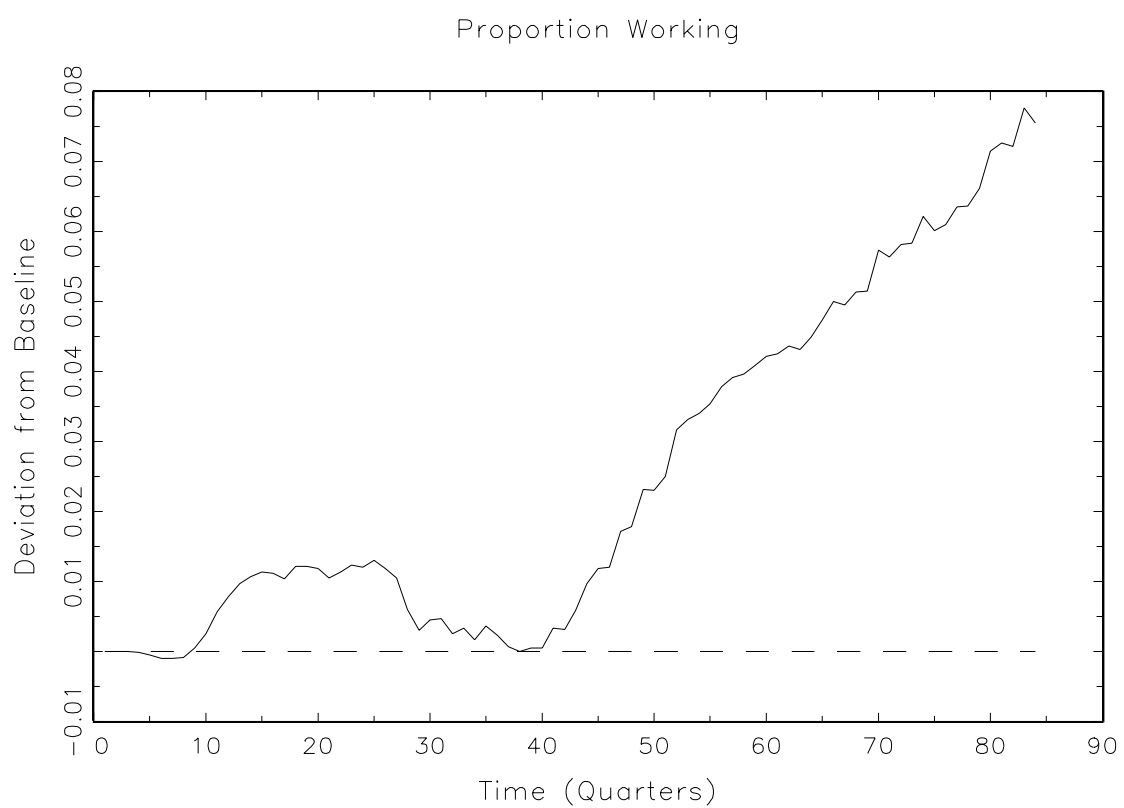


Figure 15: Effect of In-Work Benefits on Firm-Worker Match

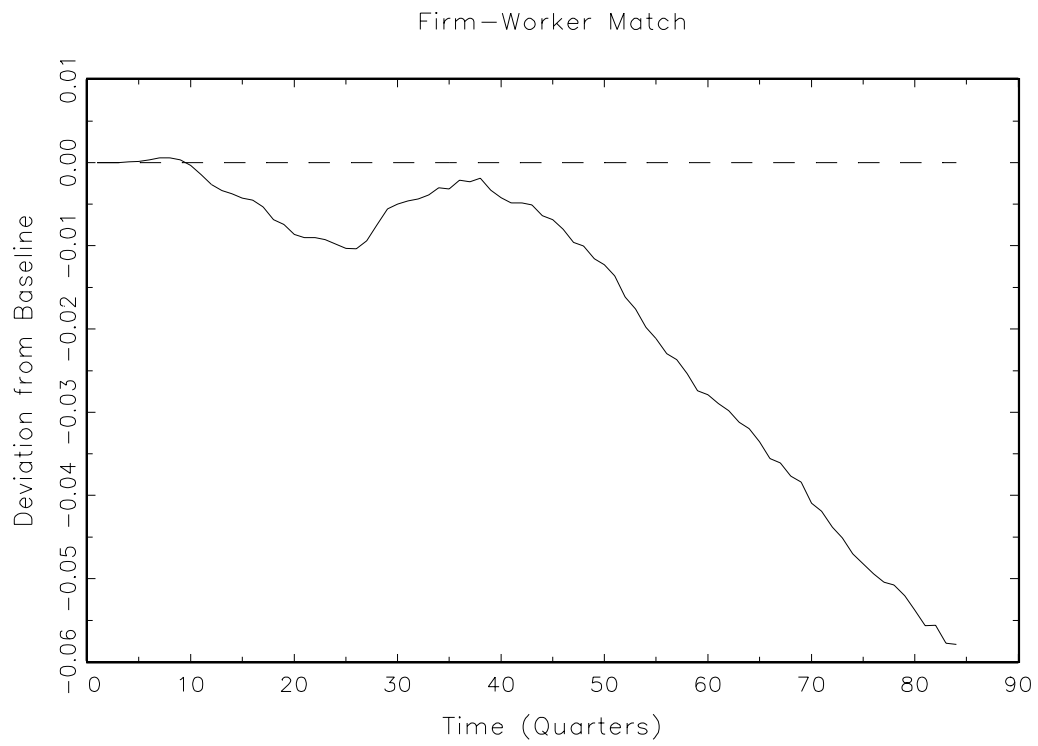
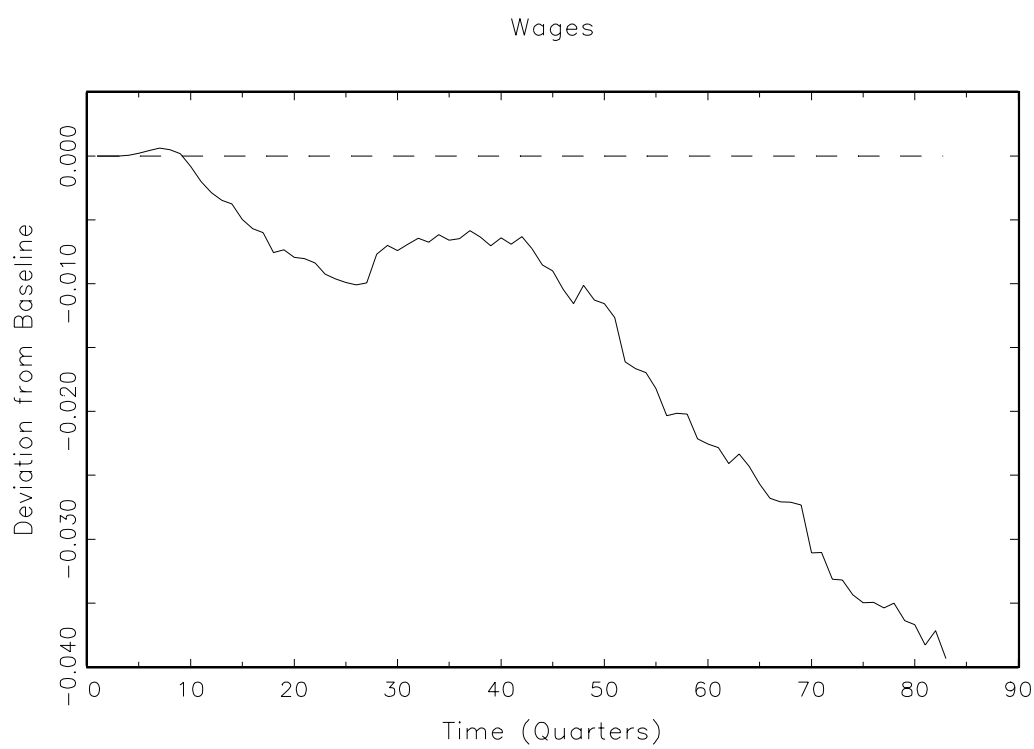


Figure 16: Effect of In-Work Benefits on Wages



1. **Transition E to U:** $S_t = U, S_{t-1} = E$:

$$\begin{aligned} P_{EU} &= P(S_t, w_t | S_{t-1}, w_{t-1}, \varepsilon) \\ &= \delta(X, T, G, Ed, \varepsilon) \end{aligned}$$

which is the probability of being fired.

2. **Transition U to U:** $S_t = U, S_{t-1} = U$:

$$\begin{aligned} P_{UU} &= P(S_t, w_t | S_{t-1}, w_{t-1}, \varepsilon) \\ &= (1 - \lambda_U) + \lambda_U P_{\tilde{\kappa}} [U(X, T, G, Ed, \varepsilon, \kappa(T)) > W_t(X, 0, G, Ed, \varepsilon, \tilde{\kappa}(0))] \end{aligned}$$

3. **Transition U to E:** $S_t = E, S_{t-1} = U$:

$$\begin{aligned} P_{UE} &= P(S_t, w_t | S_{t-1}, w_{t-1}, \varepsilon) \\ &= \lambda_U I[U(X, T, G, Ed, \kappa_t(T)) < W(X, 0, G, Ed, \kappa_t(0))] \\ &\quad P(w_t | U(X, T, G, Ed, \kappa_t(T)) < W(X, 0, G, Ed, \kappa_t(0))) \\ &= \lambda_U I[U(X, T, G, Ed, \varepsilon, w_{t'} - \gamma(\varepsilon)Z_{t'}) < W(X, 0, G, Ed, \varepsilon, w_t - \gamma(\varepsilon)Z_t)] \\ &\quad P(w_t | U(X, T, G, Ed, \varepsilon, w_{t'} - \gamma(\varepsilon)Z_{t'}) < W(X, 0, G, Ed, \varepsilon, w_t - \gamma(\varepsilon)Z_t)) \end{aligned}$$

where I is an index function and t' is the period the individual was last seen working. Define $\kappa^*(X, T, G, Ed, \varepsilon, \kappa_{t'}(T))$ as the match specific threshold for which the agent is indifferent between working and staying in unemployment.

$$\begin{aligned} P(w_t \mid U(X, T, G, Ed, \varepsilon, \kappa(T)) < W(X, 0, G, Ed, \varepsilon, w_t - \gamma(\varepsilon)Z_t)) \\ &= P(w_t | \tilde{\kappa}_t(0) > \kappa^*(X, T, G, Ed, \varepsilon, \kappa(T))) \\ &= \frac{1}{\sigma_K(0)(1 - \Phi(\frac{\kappa^*(X, T, G, Ed, \varepsilon, \kappa(T))}{\sigma_\kappa(0)})} \phi(\frac{w_t - \gamma(\varepsilon)Z_t}{\sigma_K(0)}) \end{aligned}$$

where ϕ is the density of the normal distribution with mean 0 and variance 1.

4. **Transition E to same E:** $S_t = E, S_{t-1} = E$:

We denote \tilde{w}_t the outside offer.

$$\begin{aligned} P_{EE} &= P(S_t, w_t | S_{t-1}, w_{t-1}, \varepsilon) \\ &= (1 - \delta)(1 - \lambda_W)P(w_t | w_{t-1}, \varepsilon) \\ &\quad + (1 - \delta)\lambda_W P(w_t | w_{t-1}, W(X_t, 0, G_t, Ed, \varepsilon, \tilde{\kappa}_t(0)) < W(X_t, T_t, G_t, Ed, \varepsilon, \kappa_t(T_t))) \end{aligned}$$

The first part is when no outside offer is received. The second part is when an outside offer is received but declined. Given the notation, the wages can be written as:

$$\begin{aligned}\tilde{w}_t &= \gamma(\varepsilon)\tilde{Z}_t + \tilde{\kappa}_t \\ w_t &= \gamma(\varepsilon)Z_t + \kappa_t = \gamma(\varepsilon)Z_t + \kappa_{t-1} + u_{\kappa,t} = \gamma(\varepsilon)Z_t + (w_{t-1} - \gamma(\varepsilon)Z_{t-1}) + u_{\kappa,t}\end{aligned}$$

so that

$$P(w_t|w_{t-1}, \varepsilon) = \frac{1}{\sigma_u} \phi\left(\frac{w_t - \gamma(\varepsilon)Z_t - (w_{t-1} - \gamma(\varepsilon)Z_{t-1})}{\sigma_u}\right)$$

and

$$\begin{aligned}P(w_t \mid w_{t-1}, W(X_t, T_t, G_t, Ed, \varepsilon, \kappa_t(T)) > W(X_t, 0, G_t, Ed, \varepsilon, \tilde{\kappa}_t(0))) \\ &= P(w_t|w_{t-1}, \kappa_t(T) > \kappa_{EE}^*(\tilde{\kappa}_t(0), Z_t, \varepsilon)) \\ &= P(w_t|w_{t-1}, u_{\kappa,t} > \kappa_{EE}^*(\tilde{\kappa}_t(0), Z_t, \varepsilon) - (w_{t-1} - \gamma(\varepsilon)Z_{t-1})) \\ &= P(w_t|w_{t-1}, u_{\kappa,t} > x^*(\tilde{\kappa}_t(0), Z_t, \varepsilon, w_{t-1}, Z_{t-1})) \\ &\quad - \frac{(w_t - \gamma(\varepsilon)Z_t - (w_{t-1} - \gamma(\varepsilon)Z_{t-1}))^2}{2\sigma_u^2} \\ &= \frac{e}{\sqrt{2\pi}\sigma_u} \int_{-\infty}^{\kappa^{*-1}(\kappa_t)} \frac{\phi_{\sigma_{\kappa}(0)}(\tilde{\kappa})}{1 - \Phi\left(\frac{x^*(\tilde{\kappa}_t, Z_t, \varepsilon, w_{t-1}, Z_{t-1})}{\sigma_u}\right)} d\tilde{\kappa}\end{aligned}$$

where $\kappa_{EE}^*(\tilde{\kappa}(0), Z_t, \varepsilon)$ is a threshold, beyond which the agent prefers to stay in the same job. For simplicity, we denote $x^*(\tilde{\kappa}_t(0), Z_t, \varepsilon, w_{t-1}, Z_{t-1}) = \kappa_{EE}^*(\tilde{\kappa}(0), Z_t, \varepsilon) - (w_{t-1} - \gamma(\varepsilon)Z_{t-1})$. If we discretize the process κ_t and approximate it by a Markov process, the expression simplifies to:

$$\begin{aligned}P(w_t \mid w_{t-1}, W(X_t, T_t, G_t, Ed, \varepsilon, \kappa_t(T)) > W(X_t, 0, G_t, Ed, \varepsilon, \tilde{\kappa}_t(0))) \\ &\quad - \frac{(w_t - \gamma(\varepsilon)Z_t - (w_{t-1} - \gamma(\varepsilon)Z_{t-1}))^2}{2\sigma_u^2} \\ &= \frac{e}{\sqrt{2\pi}\sigma_u} \sum_i \frac{\pi_i}{1 - \Phi\left(\frac{x^*(\tilde{\kappa}_i(0), Z_t, \varepsilon, w_{t-1}, Z_{t-1})}{\sigma_u}\right)}\end{aligned}$$

where π_i is a weight associated with $\tilde{\kappa}_i(0)$.

5. Transition E to new E: $S_t = \tilde{E}$, $S_{t-1} = E$:

$$\begin{aligned}P_{E\tilde{E}} &= P(S_t, w_t|S_{t-1}, w_{t-1}, \varepsilon) \\ &= (1 - \delta)\lambda_W P(w_t|w_{t-1}, W(X_t, 0, G_t, Ed, \varepsilon, \tilde{\kappa}_t(0)) > W(X_t, T_t, G_t, Ed, \varepsilon, \kappa_t(T)))\end{aligned}$$

Similarly,

$$\begin{aligned}
P_{E\tilde{E}} &= (1 - \delta)\lambda_W P(w_t | w_{t-1}, \tilde{\kappa}_t(0) > \kappa_{E\tilde{E}}^*(\kappa_t(T), Z_t, \varepsilon)) \\
&= (1 - \delta)\lambda_W P(w_t | \tilde{\kappa}_t(0) > \kappa_{E\tilde{E}}^*(u_{\kappa,t} + (w_{t-1} - \gamma(\varepsilon)Z_{t-1}), Z_t, \varepsilon)) \\
&= (1 - \delta)\lambda_W \frac{e^{-\frac{(w_t - \gamma(\varepsilon)Z_t)^2}{2\sigma_\kappa^2(0)}}}{\sqrt{2\pi}\sigma_\kappa(0)} \\
&\quad \int_{-\infty}^{\kappa^{*-1}(\tilde{\kappa}_0, Z_t, \varepsilon)} \frac{\phi_{\sigma_u}(u_\kappa)}{1 - \Phi\left(\frac{\kappa_{E\tilde{E}}^*(u_\kappa + (w_{t-1} - \gamma(\varepsilon)Z_{t-1}), Z_t, \varepsilon)}{\sigma_\kappa(0)}\right)} du_\kappa
\end{aligned}$$