Anova 3-way Interactions: Deconstructed

2008 Winter Stata Users Group

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Explaining 2-way interactions is pretty routine but 3-way interactions can be intimidating to some people. This presentation will look at three approaches to understanding a 3-way interaction:

- 1 Conceptual Approach
- 2 Anova Approach
- 3 Regression Approach
Meet the data

. use http://www.ats.ucla.edu/stat/stata/faq/threeway, clear

This is a synthetic dataset for a 2x2x3 factorial anova design with 2 observation per cell. The data were constructed to have different two-way interactions for each level of A.
Anova table

```
. anova y a b c a*b a*c b*c a*b*c

<table>
<thead>
<tr>
<th>Source</th>
<th>Partial SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>Prob &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>497.833333</td>
<td>11</td>
<td>45.2575758</td>
<td>33.94</td>
<td>0.0000</td>
</tr>
<tr>
<td>a</td>
<td>150</td>
<td>1</td>
<td>150</td>
<td>112.50</td>
<td>0.0000</td>
</tr>
<tr>
<td>b</td>
<td>.666666667</td>
<td>1</td>
<td>.666666667</td>
<td>0.50</td>
<td>0.4930</td>
</tr>
<tr>
<td>c</td>
<td>127.583333</td>
<td>2</td>
<td>63.7916667</td>
<td>47.84</td>
<td>0.0000</td>
</tr>
<tr>
<td>a*b</td>
<td>160.166667</td>
<td>1</td>
<td>160.166667</td>
<td>120.13</td>
<td>0.0000</td>
</tr>
<tr>
<td>a*c</td>
<td>18.25</td>
<td>2</td>
<td>9.125</td>
<td>6.84</td>
<td>0.0104</td>
</tr>
<tr>
<td>b*c</td>
<td>22.5833333</td>
<td>2</td>
<td>11.2916667</td>
<td>8.47</td>
<td>0.0051</td>
</tr>
<tr>
<td>a<em>b</em>c</td>
<td>18.5833333</td>
<td>2</td>
<td>9.2916667</td>
<td>6.97</td>
<td>0.0098</td>
</tr>
<tr>
<td>Residual</td>
<td></td>
<td>16</td>
<td>12</td>
<td>1.33333333</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>513.833333</td>
<td>23</td>
<td>22.3405797</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

Phil Ender
Anova 3-way Interactions: Deconstructed
b*c means plot at a1 – possible interaction
b\*c means plot at a2 – unlikely interaction
Conceptual Approach
About the conceptual approach

Basically, this approach involves running separate anovas on subsets of the original model and manually computing the correct F-ratio using the MS residual from the original 3-factor model.

You will need to save the MS residual value from the original anova model.

MSresidual = 1.333333333
Run 2-way anova at a1

. anova y b c b*c if a==1

<table>
<thead>
<tr>
<th>Source</th>
<th>Partial SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>Prob &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>70.0833333</td>
<td>1</td>
<td>70.0833333</td>
<td>56.07</td>
<td>0.0003</td>
</tr>
<tr>
<td>c</td>
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<td>12.3333333</td>
<td>9.87</td>
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<tr>
<td>b*c</td>
<td>40.6666667</td>
<td>2</td>
<td>20.3333333</td>
<td>16.27</td>
<td>0.0038</td>
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<tr>
<td>Residual</td>
<td>7.5</td>
<td>6</td>
<td>1.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>142.916667</td>
<td>11</td>
<td>12.9924242</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

F-ratio for b*c interaction does not use correct error term.
b*c at a1 (cont)

Manually compute correct F-ratio for b*c interaction.

F(a*b at a1) = \frac{20.33333333}{1.33333333} = 15.25
Repeat 2-way anova at a2

```
. anova y b c b*c if a==2
```

<table>
<thead>
<tr>
<th>Source</th>
<th>Partial SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>Prob &gt; F</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>90.75</td>
<td>64.06</td>
<td>0.0002</td>
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<tr>
<td>c</td>
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<td>2</td>
<td>60.5833333</td>
<td>42.76</td>
<td>0.0003</td>
</tr>
<tr>
<td>b*c</td>
<td>0.5</td>
<td>2</td>
<td>0.25</td>
<td>0.18</td>
<td>0.8424</td>
</tr>
<tr>
<td>Residual</td>
<td>8.5</td>
<td>6</td>
<td>1.41666667</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>220.916667</td>
<td>11</td>
<td>20.0833333</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Again, F-ratio for b*c interaction does not use correct error term.
b*c at a2 (cont)

Manually compute correct F-ratio for b*c interaction.

\[ F(a*b \text{ at } a1) = \frac{1.41666667}{1.33333333} \]

\[ = 0.1875 \]
Summary for b*c anovas

F-ratio for b*c at a1 = 15.25
F-ratio for b*c at a2 = 0.1875

It is likely that the F-ratio for b*c at a1 is statistically significant while the F-ratio at a2 is not. We will postpone the discussion of critical values until the last section.
Follow up tests of simple main effects

Since it is likely that the b*c interaction at a1 will be significant, we will need to follow up with some tests of simple main effects. In this case, we will focus on differences in the levels of c at b1 and b2 at a1.
b*c means plot at a1
### Test of simple main effects

**Oneway anova for c at b1 & a1**

```
. anova y c if b==1 & a==1
```

<table>
<thead>
<tr>
<th>Source</th>
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<th>df</th>
<th>MS</th>
<th>F</th>
<th>Prob &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>64</td>
<td>2</td>
<td>32</td>
<td>16.00</td>
<td>0.0251</td>
</tr>
<tr>
<td>Residual</td>
<td>6</td>
<td>3</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>70</td>
<td>5</td>
<td>14</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Recompute F-ratio for c using error term from original model.

\[
F(c \text{ at } b1 \& a1) = \frac{32}{1.33333333} = 24
\]
Test of simple main effects (cont)

Oneway anova for c at b2 & a1

\[ \text{anova } y \ c \ if \ b==2 \ & \ a==1 \]

<table>
<thead>
<tr>
<th>Source</th>
<th>Partial SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>Prob &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>1.33333333</td>
<td>2</td>
<td>.666666667</td>
<td>1.33</td>
<td>0.3852</td>
</tr>
<tr>
<td>Residual</td>
<td>1.5</td>
<td>3</td>
<td>.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>2.83333333</td>
<td>5</td>
<td>.566666667</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Recompute F-ratio for c using error term from original model.

\[ F(c \ at \ b1 \ & \ a1) = \frac{.666666667}{1.33333333} = 0.5 \]
Summary for tests of simple main effects

F-ratio for c at b1, a1 = 24.0
F-ratio for c at b2, a2 = 0.5

It is likely that the F-ratio for differences in c at b1 is statistically significant while F-ratio at b2 is not. We are still postponing the discussion of critical values.
Anova Approach
The anova approach involves running several anova models, creating contrast matrices and using the test command to test the effects of interest.

We could, of course, do this with the original 3-factor model but there are way too many terms to keep track of so, instead, we will do this in several steps using simpler models.
### Anova 3-way Interactions: Deconstructed

**b\*c at levels of a**

```
. anova y b c b\*c|a /* b\*c is nested in a */
```

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<tr>
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<th>df</th>
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</thead>
<tbody>
<tr>
<td>Model</td>
<td>497.8333333</td>
<td>11</td>
<td>45.2575758</td>
<td>33.94</td>
<td>0.0000</td>
</tr>
<tr>
<td>b</td>
<td>0.666666667</td>
<td>1</td>
<td>0.666666667</td>
<td>0.50</td>
<td>0.4930</td>
</tr>
<tr>
<td>c</td>
<td>127.5833333</td>
<td>2</td>
<td>63.7916667</td>
<td>47.84</td>
<td>0.0000</td>
</tr>
<tr>
<td>b*c</td>
<td>a</td>
<td>369.5833333</td>
<td>8</td>
<td>46.1979167</td>
<td>34.65</td>
</tr>
<tr>
<td>Residual</td>
<td></td>
<td>16</td>
<td>1.33333333</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>513.8333333</td>
<td>23</td>
<td>22.3405797</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
showorder

. test, showorder

Order of columns in the design matrix
  1: _cons
  2: (b==1)
  3: (b==2)
  4: (c==1)
  5: (c==2)
  6: (c==3)
  7: (b==1)*(c==1)*(a==1)
  8: (b==1)*(c==1)*(a==2)
  9: (b==1)*(c==2)*(a==1)
 10: (b==1)*(c==2)*(a==2)
 11: (b==2)*(c==1)*(a==1)
 12: (b==2)*(c==1)*(a==2)
 13: (b==2)*(c==2)*(a==1)
 14: (b==2)*(c==2)*(a==2)
 15: (b==2)*(c==3)*(a==1)
 16: (b==2)*(c==3)*(a==2)
 17: (b==2)*(c==3)*(a==1)
 18: (b==2)*(c==3)*(a==2)
create contrast matrices for b*c at levels of a

. matrix bc1=(0,0,0,0,0,0,1,0,0,0,-1,0,-1,0,0,0,1,0,  
0,0,0,0,0,0,0,1,0,-1,0,0,0,-1,0,1,0)  ///

. matrix bc2=(0,0,0,0,0,0,0,0,1,0,0,0,-1,0,-1,0,0,0,1,  
0,0,0,0,0,0,0,0,0,1,0,-1,0,0,0,-1,0,1)
test $b \times c$ at $a1$ & $b \times c$ at $a2$

/* test $b \times c$ at $a==1$ */
. test, test(bc1)

( 1) $b[1]\times c[1]\times a[1] - b[1]\times c[3]\times a[1] - b[2]\times c[1]\times a[1] + b[2]\times c[3]\times a[1] = 0$

F( 2, 12) = 15.25
Prob > F = 0.0005

/* test $b \times c$ at $a==2$ */
. test, test(bc2)


F( 2, 12) = 0.1875
Prob > F = 0.8314
c at levels of b – for tests of simple main effects

. anova y c|a*b /* c is nested in a*b */

<table>
<thead>
<tr>
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<th>F</th>
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<td>c</td>
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<td>497.833333</td>
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<td>33.94</td>
</tr>
<tr>
<td>Residual</td>
<td></td>
<td>16</td>
<td>12</td>
<td>1.33333333</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>513.833333</td>
<td>23</td>
<td></td>
<td>22.3405797</td>
<td></td>
</tr>
</tbody>
</table>

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showorder for c nested in a*b

```
. test, showorder

Order of columns in the design matrix
  1: _cons
  2: (c==1)*(a==1)*(b==1)
  3: (c==1)*(a==1)*(b==2)
  4: (c==1)*(a==2)*(b==1)
  5: (c==1)*(a==2)*(b==2)
  6: (c==2)*(a==1)*(b==1)
  7: (c==2)*(a==1)*(b==2)
  8: (c==2)*(a==2)*(b==1)
  9: (c==2)*(a==2)*(b==2)
 10: (c==3)*(a==1)*(b==1)
 11: (c==3)*(a==1)*(b==2)
 12: (c==3)*(a==2)*(b==1)
 13: (c==3)*(a==2)*(b==2)
```
create contrast matrices for c nested in a*b

```matlab
. matrix c1=(0,1,0,0,0,0,0,0,0,-1,0,0,0 \ ///
  0,0,0,0,0,1,0,0,0,-1,0,0,0)

. matrix c2=(0,0,1,0,0,0,0,0,0,-1,0,0,0 \ ///
  0,0,0,0,0,0,1,0,0,0,-1,0,0)
```
test c at b1 & c at b2

/* test c at b==1 */
. test, test(c1)

( 1) c[1]*a[1]*b[1] - c[3]*a[1]*b[1] = 0
( 2) c[2]*a[1]*b[1] - c[3]*a[1]*b[1] = 0

F( 2, 12) = 24.00
Prob > F = 0.0001

/* test c at b==2 */
. test, test(c2)


F( 2, 12) = 0.50
Prob > F = 0.6186
Regression Approach
The regression approach involves creating dummy variables for all the main effects and interactions and then testing them in the proper combinations to get the tests of simple interactions and simple main effects.
Create dummies and interactions

. recode a (1=0)(2=1)
. recode b (1=0)(2=1)
. generate c1=c==1
. generate c2=c==2
. generate ab=a*b
. generate ac1=a*c1
. generate ac2=a*c2
. generate bc1=b*c1
. generate bc2=b*c2
. generate abc1=a*bc1
. generate abc2=a*bc2
Regression model

```
.regress y a b c1 c2 ab ac1 ac2 bc1 bc2 abc1 abc2, noheader

+---------------------------------------------+-------+-------------+----------+---------------------+---------------------+
|       | Coef.   | Std. Err.   | t        | P>|t|    | [95% Conf. Interval] |
|-------+---------+-------------+----------+----------+---------------------+---------------------|
| a     | -.5     | 1.154701    | -0.43    | 0.673    | -3.015876 2.015876  |
| b     | -9.5    | 1.154701    | -8.23    | 0.000    | -12.01588 -6.984124|
| c1    | -8      | 1.154701    | -6.93    | 0.000    | -10.51588 -5.484124|
| c2    | -4      | 1.154701    | -3.46    | 0.005    | -6.515876 -1.484124|
| ab    | 15      | 1.632993    | 9.19     | 0.000    | 11.44201 18.55799  |
| ac1   | 0       | 1.632993    | 0.00     | 1.000    | -3.557986 3.557986 |
| ac2   | 1       | 1.632993    | 0.61     | 0.552    | -2.557986 4.557986 |
| bc1   | 9       | 1.632993    | 5.51     | 0.000    | 5.442014 12.55799 |
| bc2   | 5       | 1.632993    | 3.06     | 0.010    | 1.442014 8.557986 |
| abc1  | -8.5    | 2.309401    | -3.68    | 0.003    | -13.53175 -3.468247|
| abc2  | -5.5    | 2.309401    | -2.38    | 0.035    | -10.53175 -.4682473|
| _cons | 19      | .8164966    | 23.27    | 0.000    | 17.22101 20.77899 |
+---------------------------------------------+-------+-------------+----------+---------------------+---------------------+

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Anova 3-way Interactions: Deconstructed
Test of 3-way interaction

. test abc1 abc2

( 1) abc1 = 0
( 2) abc2 = 0

F(  2,    12) = 6.97
Prob > F =    0.0098
/* test b*c at a1 */
.test bc1 bc2
( 1) bc1 = 0
( 2) bc2 = 0

F( 2, 12) = 15.25
Prob > F = 0.0005

/* test b*c at a2 */
.test bc1+abc1=0
.test bc2+abc2=0, accum
( 1) bc1 + abc1 = 0
( 2) bc2 + abc2 = 0

F( 2, 12) = 0.1875
Prob > F = 0.8314
Test of c at b1 & c at b2

/* test for c at b==1 & a==1 */
.test c1 c2
(1) c1 = 0
(2) c2 = 0

F(2, 12) = 24.00
Prob > F = 0.0001

/* test for c at b==2 & a==1 */
.test c1+bc1=0
.test c2+bc2=0, accum
(1) c1 + bc1 = 0
(2) c2 + bc2 = 0

F(2, 12) = 0.50
Prob > F = 0.6186
Determining critical values
Computing critical values

There are at least four methods for computing critical values found in the literature.

- Dunn’s procedure
- Marascuilo & Levin
- Per family error rate
- Simultaneous test procedure (closely related to the Scheffé’s multiple comparison procedure)

No clear consensus as to which approach is best.
critical values for a*b at two levels of a

. smecriticalvalue, number(2) df1(2) df2(12) dfmodel(11)

number of tests: 2
   numerator df: 2
   denominator df: 12
original model df: 11

Critical value of F for alpha = .05 using ...  
---------------------------------------------

Dunn’s procedure = 6.2753765
Marascuilo & Levin = 7.1335873
per family error rate = 5.0958672
simultaneous test procedure = 10.245969
Critical value of F for alpha = .05 using ...

Dunn’s procedure = 6.2753765
Marascuilo & Levin = 7.1335873
per family error rate = 5.0958672
simultaneous test procedure = 10.245969

Using the critical values from the previous slide, both the F-ratios for for b*c at a1 (15.25) and c at b1, a1 (24.0) were statistically significant regardless of which method was used to determine the critical value.
download ado-file

Use -findit- command

. findit smecriticalvalue

Follow installation instructions.
Reference

The End