Anova 3-way Interactions: Deconstructed

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Three approaches

Explaining 2-way interactions is pretty routine but 3-way interactions can be intimidating to some people. This presentation will look at three approaches to understanding a 3-way interaction:

- 1 Conceptual Approach
- 2 Anova Approach
- 3 Regression Approach

Meet the data

```
. use http://www.ats.ucla.edu/stat/stata/faq/threeway, clear
```

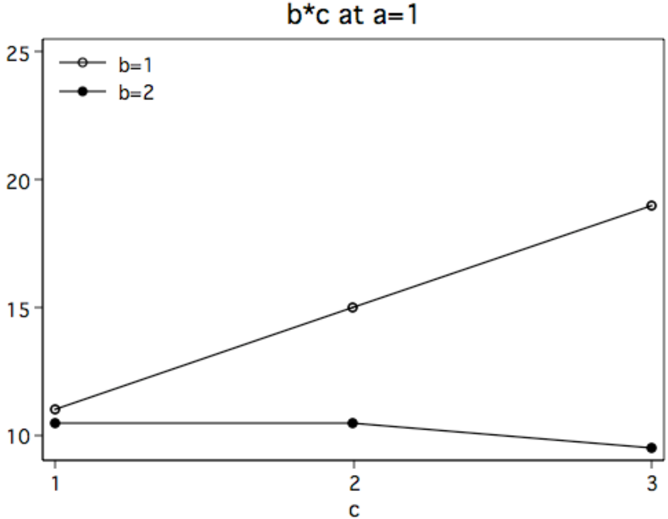
This is a synthetic dataset for a 2x2x3 factorial anova design with 2 observation per cell. The data were constructed to have different two-way interactions for each level of A.

Anova table

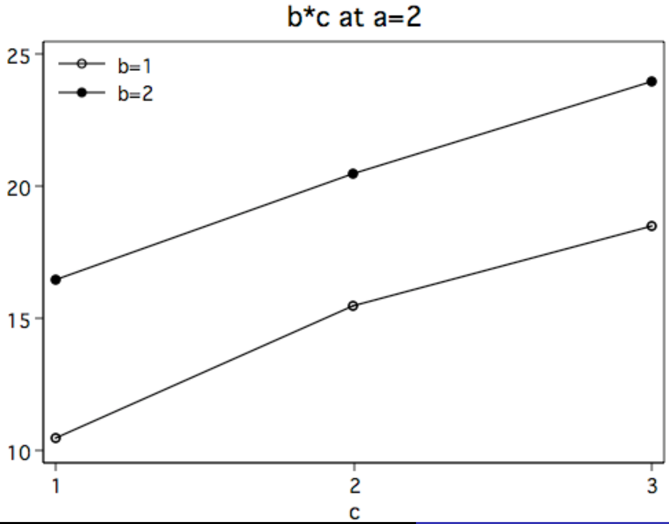
```
. anova y a b c a*b a*c b*c a*b*c

    Source | Partial SS    df       MS           F     Prob > F
-----------+----------------------------------------------------
     Model |  497.833333    11  45.2575758      33.94     0.0000
         a |         150     1         150     112.50     0.0000
         b |  .666666667     1  .666666667       0.50     0.4930
         c |  127.583333     2  63.7916667      47.84     0.0000
       a*b |  160.166667     1  160.166667     120.13     0.0000
       a*c |       18.25     2       9.125       6.84     0.0104
       b*c |  22.5833333     2  11.2916667       8.47     0.0051
     a*b*c |  18.5833333     2  9.29166667       6.97     0.0098
           |
  Residual |          16    12  1.33333333
-----------+----------------------------------------------------
     Total |  513.833333    23  22.3405797
```

b*c means plot at a1 – possible interaction



b*c means plot at a2 – unlikely interaction



Conceptual Approach

About the conceptual approach

Basically, this approach involves running separate anovas on subsets of the original model and manually computing the correct F-ratio using the MS residual from the original 3-factor model.

You will need to save the MS residual value from the original anova model.

```
MSresidual = 1.333333333
```

b*c at a1

Run 2-way anova at a1

```
. anova y b c b*c if a==1

      Source |  Partial SS    df       MS           F     Prob > F
  -----------+----------------------------------------------------
           b |  70.0833333     1  70.0833333      56.07     0.0003
           c |  24.6666667     2  12.3333333       9.87     0.0127
         b*c |  40.6666667     2  20.3333333      16.27     0.0038
             |
    Residual |         7.5     6        1.25
  -----------+----------------------------------------------------
       Total |  142.916667    11  12.9924242
```

F-ratio for b*c interaction does not use correct error term.

b*c at a1 (cont)

Manually compute correct F-ratio for b*c interaction.

```
F(a*b at a1) = 20.33333333/1.33333333

             = 15.25
```

b*c at a2

Repeat 2-way anova at a2

```
. anova y b c b*c if a==2

      Source |  Partial SS     df        MS           F     Prob > F
  -----------+------------------------------------------------------
           b |       90.75      1       90.75      64.06     0.0002
           c |  121.166667      2  60.5833333      42.76     0.0003
         b*c |          .5      2         .25       0.18     0.8424
             |
    Residual |         8.5      6  1.41666667
  -----------+------------------------------------------------------
       Total |  220.916667     11  20.0833333
```

Again, F-ratio for b*c interaction does not use correct error term.

b*c at a2 (cont)

Manually compute correct F-ratio for b*c interaction.

```
F(a*b at a1) = 1.41666667/1.33333333

             = 0.1875
```

Summary for b*c anovas

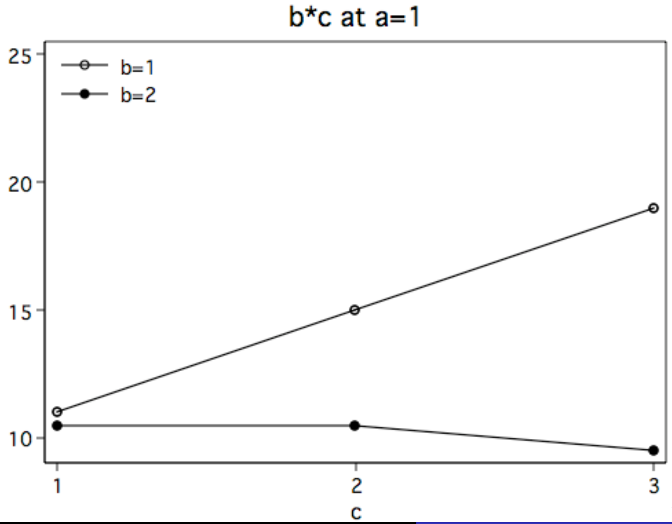
F-ratio for b*c at a1 = 15.25
F-ratio for b*c at a2 = 0.1875

It is likely that the F-ratio for b*c at a1 is statistically significant while the F-ratio at a2 is not. We will postpone the discussion of critical values until the last section.

Follow up tests of simple main effects

Since it is likely that the b*c interaction at a1 will be significant, we will need to follow up with some tests of simple main effects. In this case, we will focus on differences in the levels of c at b1 and b2 at a1.

b*c means plot at a1



Test of simple main effects

Oneway anova for c at b1 & a1

```
. anova y c if b==1 & a==1

    Source |  Partial SS    df       MS           F     Prob > F
-----------+----------------------------------------------------
         c |          64     2          32      16.00     0.0251
  Residual |           6     3           2
-----------+----------------------------------------------------
     Total |          70     5          14
```

Recompute F-ratio for c using error term from original model.

```
F(c at b1 & a1) = 32/1.33333333 = 24
```

Test of simple main effects (cont)

Oneway anova for c at b2 & a1

```
. anova y c if b==2 & a==1

      Source |  Partial SS    df       MS           F     Prob > F
  -----------+----------------------------------------------------
           c |  1.33333333     2  .666666667       1.33     0.3852
    Residual |         1.5     3          .5
  -----------+----------------------------------------------------
       Total |  2.83333333     5  .566666667
```

Recompute F-ratio for c using error term from original model.

```
F(c at b1 & a1) = .666666667/1.33333333 = 0.5
```

Summary for tests of simple main effects

F-ratio for c at b1, a1 = 24.0
F-ratio for c at b2, a2 = 0.5

It is likely that the F-ratio for differences in c at b1 is statistically significant while F-ratio at b2 is not. We are still postponing the discussion of critical values.

Anova Approach

About the anova approach

The anova approach involves running several anova models, creating contrast matrices and using the test command to test the effects of interest.

We could, of course, do this with the original 3-factor model but there are way too many terms to keep track of so, instead, we will do this in several steps using simpler models.

b*c at levels of a

```
. anova y b c b*c|a  /* b*c is nested in a */

      Source |  Partial SS    df       MS           F     Prob > F
  -----------+----------------------------------------------------
       Model |  497.833333    11  45.2575758      33.94     0.0000
             |
           b |  .666666667     1  .666666667       0.50     0.4930
           c |  127.583333     2  63.7916667      47.84     0.0000
       b*c|a |  369.583333     8  46.1979167      34.65     0.0000
             |
    Residual |          16    12  1.33333333
  -----------+----------------------------------------------------
       Total |  513.833333    23  22.3405797
```

showorder

```
. test, showorder

 Order of columns in the design matrix
      1: _cons
      2: (b==1)
      3: (b==2)
      4: (c==1)
      5: (c==2)
      6: (c==3)
      7: (b==1)*(c==1)*(a==1)
      8: (b==1)*(c==1)*(a==2)
      9: (b==1)*(c==2)*(a==1)
     10: (b==1)*(c==2)*(a==2)
     11: (b==1)*(c==3)*(a==1)
     12: (b==1)*(c==3)*(a==2)
     13: (b==2)*(c==1)*(a==1)
     14: (b==2)*(c==1)*(a==2)
     15: (b==2)*(c==2)*(a==1)
     16: (b==2)*(c==2)*(a==2)
     17: (b==2)*(c==3)*(a==1)
     18: (b==2)*(c==3)*(a==2)
```

create contrast matrices for b*c at levels of a

```
. matrix bc1=(0,0,0,0,0,0,1,0,0,0,-1,0,-1,0,0,0,1,0\ ///
             0,0,0,0,0,0,0,0,1,0,-1,0,0,0,-1,0,1,0)

. matrix bc2=(0,0,0,0,0,0,0,1,0,0,0,-1,0,-1,0,0,0,1\ ///
             0,0,0,0,0,0,0,0,0,1,0,-1,0,0,0,-1,0,1)
```

test b*c at a1 & b*c at a2

```
/* test b*c at a==1 */
. test, test(bc1)

 ( 1)  b[1]*c[1]*a[1] - b[1]*c[3]*a[1] - b[2]*c[1]*a[1] + b[2]*c[3]*a[1] = 0
 ( 2)  b[1]*c[2]*a[1] - b[1]*c[3]*a[1] - b[2]*c[2]*a[1] + b[2]*c[3]*a[1] = 0

       F(  2,    12) =   15.25
            Prob > F =    0.0005

/* test b*c at a==2 */
. test, test(bc2)

 ( 1)  b[1]*c[1]*a[2] - b[1]*c[3]*a[2] - b[2]*c[1]*a[2] + b[2]*c[3]*a[2] = 0
 ( 2)  b[1]*c[2]*a[2] - b[1]*c[3]*a[2] - b[2]*c[2]*a[2] + b[2]*c[3]*a[2] = 0

       F(  2,    12) =    0.1875
            Prob > F =    0.8314
```

c at levels of b – for tests of simple main effects

```
. anova y c|a*b  /* c is nested in a*b */

     Source |  Partial SS    df       MS           F     Prob > F
 -----------+----------------------------------------------------
      Model |  497.833333    11  45.2575758      33.94     0.0000
            |
      c|a*b |  497.833333    11  45.2575758      33.94     0.0000
            |
   Residual |          16    12  1.33333333
 -----------+----------------------------------------------------
      Total |  513.833333    23  22.3405797
```

showorder for c nested in a*b

```
. test, showorder

 Order of columns in the design matrix
      1: _cons
      2: (c==1)*(a==1)*(b==1)
      3: (c==1)*(a==1)*(b==2)
      4: (c==1)*(a==2)*(b==1)
      5: (c==1)*(a==2)*(b==2)
      6: (c==2)*(a==1)*(b==1)
      7: (c==2)*(a==1)*(b==2)
      8: (c==2)*(a==2)*(b==1)
      9: (c==2)*(a==2)*(b==2)
     10: (c==3)*(a==1)*(b==1)
     11: (c==3)*(a==1)*(b==2)
     12: (c==3)*(a==2)*(b==1)
     13: (c==3)*(a==2)*(b==2)
```

create contrast matrices for c nested in a*b

```
. matrix c1=(0,1,0,0,0,0,0,0,0,-1,0,0,0\ ///
             0,0,0,0,0,1,0,0,0,-1,0,0,0)

. matrix c2=(0,0,1,0,0,0,0,0,0,0,-1,0,0\ ///
             0,0,0,0,0,0,1,0,0,0,-1,0,0)
```

test c at b1 & c at b2

```
/* test c at b==1 */
. test, test(c1)

 ( 1)  c[1]*a[1]*b[1] - c[3]*a[1]*b[1] = 0
 ( 2)  c[2]*a[1]*b[1] - c[3]*a[1]*b[1] = 0

       F(  2,    12) =   24.00
            Prob > F =    0.0001

/* test c at b==2 */
. test, test(c2)

 ( 1)  c[1]*a[1]*b[2] - c[3]*a[1]*b[2] = 0
 ( 2)  c[2]*a[1]*b[2] - c[3]*a[1]*b[2] = 0

       F(  2,    12) =    0.50
            Prob > F =    0.6186
```

Regression Approach

About the anova approach

The regression approach involves creating dummy variables for all the main effects and interactions and then testing them in the proper combinations to get the tests of simple interactions and simple main effects.

Create dummies and interactions

```
. recode a (1=0)(2=1)
. recode b (1=0)(2=1)
. generate c1=c==1
. generate c2=c==2
. generate ab=a*b
. generate ac1=a*c1
. generate ac2=a*c2
. generate bc1=b*c1
. generate bc2=b*c2
. generate abc1=a*bc1
. generate abc2=a*bc2
```

Regression model

```
. regress y a b c1 c2 ab ac1 ac2 bc1 bc2 abc1 abc2, noheader

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           a |        -.5   1.154701    -0.43   0.673    -3.015876    2.015876
           b |       -9.5   1.154701    -8.23   0.000    -12.01588   -6.984124
          c1 |         -8   1.154701    -6.93   0.000    -10.51588   -5.484124
          c2 |         -4   1.154701    -3.46   0.005    -6.515876   -1.484124
          ab |         15   1.632993     9.19   0.000     11.44201    18.55799
         ac1 |          0   1.632993     0.00   1.000    -3.557986    3.557986
         ac2 |          1   1.632993     0.61   0.552    -2.557986    4.557986
         bc1 |          9   1.632993     5.51   0.000     5.442014    12.55799
         bc2 |          5   1.632993     3.06   0.010     1.442014    8.557986
        abc1 |       -8.5   2.309401    -3.68   0.003    -13.53175   -3.468247
        abc2 |       -5.5   2.309401    -2.38   0.035    -10.53175   -.4682473
       _cons |         19   .8164966    23.27   0.000     17.22101    20.77899
------------------------------------------------------------------------------
```

Test of 3-way interaction

```
. test abc1 abc2

 ( 1)  abc1 = 0
 ( 2)  abc2 = 0

       F(  2,    12) =    6.97
            Prob > F =    0.0098
```

Test of b*c at a1 & b*c at a2

```
/* test b*c at a1 */
. test bc1 bc2
 ( 1)  bc1 = 0
 ( 2)  bc2 = 0

       F(  2,    12) =   15.25
            Prob > F =    0.0005

 /* test b*c at a2 */
. test bc1+abc1=0
. test bc2+abc2=0, accum
 ( 1)  bc1 + abc1 = 0
 ( 2)  bc2 + abc2 = 0

       F(  2,    12) =    0.1875
            Prob > F =    0.8314
```

Test of c at b1 & c at b2

```
 /* test for c at b==1 & a==1 */
. test c1 c2
 ( 1)  c1 = 0
 ( 2)  c2 = 0

      F(  2,    12) =   24.00
           Prob > F =    0.0001

/* test for c at b==2 & a==1 */
. test c1+bc1=0
. test c2+bc2=0, accum
 ( 1)  c1 + bc1 = 0
 ( 2)  c2 + bc2 = 0

      F(  2,    12) =    0.50
           Prob > F =    0.6186
```

Determining critical values

Computing critical values

There are at least four methods for computing critical values found in the literature.

- Dunn's procedure
- Marascuilo & Levin
- Per family error rate
- Simultaneous test procedure (closely related to the Scheffé's multiple comparison procedure)

No clear consensus as to which approach is best.

critical values for a*b at two levels of a

```
. smecriticalvalue, number(2) df1(2) df2(12) dfmodel(11)

  number of tests: 2
     numerator df: 2
   denominator df: 12
original model df: 11

Critical value of F for alpha = .05 using ...
------------------------------------------------
Dunn's procedure              = 6.2753765
Marascuilo & Levin            = 7.1335873
per family error rate         = 5.0958672
simultaneous test procedure   = 10.245969
```

critical values for a*b at two levels of a (cont)

```
Critical value of F for alpha = .05 using ...
-----------------------------------------------
Dunn's procedure                = 6.2753765
Marascuilo & Levin              = 7.1335873
per family error rate           = 5.0958672
simultaneous test procedure     = 10.245969
```

Using the critical values from the previous slide, both the F-ratios for for b*c at a1 (15.25) and c at b1, a1 (24.0) were statistically significant regardless of which method was used to determine the critical value.

download ado-file

Use -findit- command

```
. findit smecriticalvalue
```

Follow installation instructions.

Reference

(1995) Kirk, R.E. Experimental design: Procedures for the behavioral sciences (3rd ed). Pacific Grove, CA: Brooks/Cole

The End