Financial System Architecture: The Role of Systemic Risk, Added Value and Liquidity

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Abstract
Risky investment projects make the coordination among small, uninformed investors hard to achieve, and generate inefficient low levels of investment. Several authors have pointed out the benefits to an economy from multiple avenues of financial intermediation. This paper explains endogenously different financial architectures and classifies them according to the capacity of financial intermediaries to reallocate risks and create added value. In some of these architectures, financial intermediaries improve coordination among agents by providing insurance over the primitive payoffs available in decentralized financial markets. This enhances efficiency and stabilizes the economy against fundamental shocks and confidence shifts. In other financial architectures financial intermediation plays a minimal role or is unfeasible.
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"What we perceived in the United States in 1998 may reflect an important general principle: Multiple alternatives to transform an economy’s savings into capital investment act as backup facilities should the primary form of intermediation fail. In 1998 in the United States, banking replaced the capital markets. Far more often it has been the other way around, as it was most recently in the United States a decade ago."
Alan Greenspan (1999)

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1 Introduction

Alan Greenspan (1999), (2000) has suggested that multiple sources of finance may help to protect economies against systemic problems affecting financial markets. Davis (2001) finds empirical evidence favorable to this hypothesis. He argues that there is a low correlation between the volumes of intermediated funds and market based finance, which helps to smooth aggregate financial flows. This feature is a long term pattern, and is also true in periods of crisis in either the direct or indirect finance markets. Our paper justifies such arguments by building up a model that mimics the above empirical results. Our explanation lies on the important role that banks have played in channeling funds and coordinating the investors’ decisions in modern economies.

Our analysis is related to a number of strands in the literature. On the game theoretical side, we build upon recent work concentrated on the mechanisms through which agents take decisions when information is imperfect and coordination is important for the final payoffs. This literature had its origin on the study of a class of games, entitled global games, by Carlsson and van Damme (1993) and was fostered by Morris and Shin (1998). Many economic problems are naturally modeled within this class of games. Global games stress the importance of coordination among agents due to strategic complementarities in their actions.

In what follows, the returns on investment are determined by the state of the fundamental variables in the economy and the mass of investors. Hence, when deciding about investing, agents take into consideration the state of the fundamentals and the actions of the other players. Coordination among potential investors is important because there is an externality loss caused by an insufficient mass of investors. We relate this negative externality to illiquid markets. Coordination is not easy to achieve because the economic fundamentals are random and information is imperfect. In the framework that we present, apart from public information known by everybody, every potential investor receives a piece of information known only to him. Private information introduces idiosyncratic uncertainty and removes common knowledge about the actions of other players. To decide whether to invest or not, agents take into account their beliefs about the state of the economy, and their guesses about what other agents will do. Under this setup we examine the interactions among systemic risk and the degree of coordination throughout the economy.

Typically the equilibrium reached in global games involves inefficiencies: often there is underinvestment because agents who receive bad private signals refrain from investing, even though the fundamentals are sound. In this context, Morris and Shin (2001a), (2001c) have highlighted that potential investors look into public information to coordinate their actions, which confers it a powerful strategic effect. For certain parameters, improving the precision of public information makes agents more confident about the investment behavior of their
peers. Therefore agents expect lower externality losses and higher expected returns. This induces more agents to invest, improving returns and efficiency.

The basic idea behind this paper is that Financial Intermediaries (FIs hereafter) may partially substitute for public information. We define direct finance as investments made directly in firms that represent the fundamentals of the economy. Households that choose direct finance face the structure of information that we have alluded before. We model an FI as an institution which offers investment opportunities different from those available in decentralized financial markets. Our approach is to introduce a new security, representing the financial intermediation sector. This security has public information different from the existing public information on the fundamentals (or, equivalently, the risk patterns of intermediated and direct finance differ). For example, an FI may pick up a risky security issued by a firm and use its own capital to create a new (composite) security with lower risk. The FI invests in the firm on behalf of uninformed investors. The FI is de facto reducing the uncertainty in payoffs, inducing effects over investment, returns and efficiency similar to an improvement in the precision of public information.

The key features that characterize financial intermediation are the capacity to create added value and the ability to perform risk transformation. These justify why we find financial architectures based either on direct or intermediated finance or both. Our contribution implies a set of empirical predictions for each possible financial architecture. In all architectures investment is procyclical and one should expect a higher volume of investment during an economic expansion. The behaviour of the intermediation sector during the business cycle depends on the particular financial architecture under consideration. On the one hand, intermediated funds mimic the behavior of overall investment in intermediation based financial systems. On the other hand, when coexistence between direct and indirect finance is possible, intermediation may act as a buffer against sudden shocks to the fundamentals and in confidence.

We are not the first to aim at building a framework that helps explaining financial architecture. Our effort adds to a small recent literature concerned with the coexistence of direct and intermediated finance. Several previous papers have modeled the choice between market and bank finance by considering an entrepreneurial moral hazard problem that can be ameliorated through (costly) bank monitoring. Holmstrom and Tirole (1997) and Repullo and Suarez (2000) examine the role of net worth of firms in the distribution of external finance. Probably the most complete model explaining the demand for finance is the one by Bolton and Freixas (2000). By assuming the existence of dilution costs and that bank debt is easier to renegotiate, they justify why firms demand bank loans, private debt and equity. Yet, most studies concentrate solely on the choice of finance by the firm, while we are mainly concerned with the supply of finance. Some authors offered an integrated view of the demand and supply of funds, namely Boot and Thakor (1997) and Gorton and Pennacchi (1990).
For Boot and Thakor, financial markets permit noncolluding informed agents to compete and convey valuable information to the firms. Banks do not have any informational advantage. Nonetheless they coordinate noninformed traders and resolve moral hazard problems. On the other hand, Gorton and Pennacchi argue that informed agents collude to exploit liquidity traders. Liquidity traders break the informed traders coalition by creating a bank. Banks mitigate informational asymmetries by splitting the cash flows of the assets in the economy. We borrow from them the security design approach and the fact that FIs may provide safer securities to their depositors. In contrast with the literature that we have mentioned so far, we impose much weaker requirements to justify the existence of FIs. We claim that informational heterogeneity is enough to justify the existence of financial intermediation since, under heterogeneous information, coordination among potential investors becomes very difficult, and FIs can improve it.

Another body of literature justifies the existence of FIs based on their role in liquidity creation, where liquid funds are those that can be immediately used for consumption\textsuperscript{1}. Here FIs help solving a coordination problem related to the best allocation of resources across technologies. These models, based on Diamond and Dybvig (1983), had difficulties in explaining coexistence between direct and indirect finance. Diamond (1997) pointed out that limited participation by some agents in some markets is a sufficient condition to guarantee coexistence. In his model, FIs emerge endogenously to solve the coordination problems generated by limited participation and we borrow this idea from him. Our argument complements Kashyap, Rajan and Stein (2002), who argue that there are synergies between the deposit taking and lending activities of a bank. This happens as long as the demands for liquidity from depositors and borrowers are not perfectly correlated. We justify endogenously this assumption because financial intermediation becomes important when direct finance dries up, and we explore its macroeconomic implications. In the same line of research, Gatev and Strahan (2003) study the commercial paper market and document that banks can provide firms with insurance against market-wide liquidity shocks because deposit inflows provide a hedge for loan demand shocks. Although our main concern is the role of FIs in financial architecture, our results carry over to a more general framework with different securities (for example, debt versus equity). The role of coordination distinguishes our work from the literature dealing with models of multi-asset securities markets under heterogeneous beliefs, as in Admati (1995).

The paper is organized as follows. We devote the next section to present the basic model and justify our main assumptions. We proceed by showing the resulting equilibrium and exhibiting the different financial architectures that may emerge. Section 4 alludes to the policy implications suggested by our model and is followed by a short conclusion. The proofs of the most important results are given in the mathematical appendix.

\textsuperscript{1}Note that we follow a different interpretation for liquidity since we relate it to the total mass of funds available in financial markets.
2 The Model

The model has three types of agents:

- a continuum of households with unit mass, indexed by \( i \in [0, 1] \), each with one unit of funds. Households must decide whether to invest their funds in any of the assets available in the economy or not invest at all.

- a continuum of identical firms, with unit mass, which have access to the same technology.

- many financial intermediaries which issue financial securities to households.

Households receive two types of information, which we will define later, about the state of the economy: public and private information. There are three dates in the economy: initial, interim and final. Public information is revealed at the initial date. At the interim date private information is given to each household, financial contracts are signed and investment decisions are made. At the final date investment returns are realized and financial claims are settled. All parties are risk neutral.

2.1 The Real Sector

Each firm has one project which requires one unit of funds and yields an uncertain payoff. Firms have no funds of their own and need to fully finance their project by resorting to either direct or intermediated finance. Hence the total demand of funds is one.

Let \( n \) be the mass of noninvestors in the economy. If \( 1 - n \) households become investors then each household, who invests directly in the firm sector, obtains an excess return \( f_{DF} = r_{DF} - n \). The risk factor \( r_{DF} \) would have been the return, had every household decided to invest. Nonetheless, non investors impose a negative externality on returns, and this effect is captured by \( n \). We call this effect externality loss and we associate it to how illiquid markets are. The factor \( r_{DF} \) is random and has a normal distribution \( N \left( \tau_{DF}, \sqrt{1/\alpha} \right) \), where \( 0 < \tau_{DF} < 1 \). These facts about the risk factor \( r_{DF} \) are common knowledge and we call them public information. We call the inverse of the variance of factor \( r_{DF}, \alpha \), the precision of public information. The uncertainty about the realization of \( r_{DF} \) may spring from two different sources.

- It may be related to the technology of the firm. Since all firms have access to the same technology, we may interpret \( r_{DF} \) as the state of technology in the economy. Being a systemic risk factor, \( r_{DF} \) could be used to assess the interaction between the financial system and the business cycle.

- Even if the fundamentals of the firm were not intrinsically uncertain, information systems are imperfect and do not reveal the true value of the
payoffs without error. These sources of uncertainty translate into public information which incorporates some risk.

The literature has presented justifications why non-investors originate negative externalities. Consult for example Morris and Shin (2001b), (2004) and Rochet and Vives (2002). We present some (informal) justifications for the existence of negative externalities. First consider the case in which our model represents the whole economy. Consider that direct finance is carried out through a mutual fund which invests in a portfolio composed of all the firms in the economy. At what we call the "initial date", participants in the fund have the choice of maintaining their investment or withdraw their funds. Hence the (open) mutual fund must liquidate some of its assets to redeem the funds from agents who decided to drop out. If we want to think of a model which encompasses the whole financial system, liquidation can only mean "physical liquidation" of real assets. If we assume that the technology used by firms is illiquid, that is with costs of premature liquidation, then some projects are, partially or totally, liquidated and the capital goods are sold at fire sale prices. Alternatively, one could also think that these goods are sold at reasonable prices but the production function displays increasing returns to scale at the aggregate level\textsuperscript{2}: reducing the global level of capital in the economy reduces (more than proportionally) future output and returns. Second, consider the case in which our model represents only part of the financial system. In this case liquidation can be financial. Assets are sold at fire sale prices to investors not participating in the mutual fund. Fire sale pricing may happen due to asymmetric information as in Rochet and Vives (2002).

2.2 The Financial System

The financial system encompasses two forms of finance: investors may decide to invest their funds directly in the firm or deposit their funds in one of the many FIs that constitute the intermediation sector. FIs offer securities with returns different from those issued directly by the firm. We postulate that all FIs issue an identical security with excess return $f_{FI} = r_{FI} - n$. FIs receive an amount $t_{FI}$ of funds from households which they invest in projects to which they have access. These projects yield a return $f_M$. Hence the revenues derived from the intermediation activity are $t_{FI}f_M$ and the payments to depositors are given by $t_{FI}f_{FI}$. Since FIs are risk neutral, and we assume no limited liability, their incentive rationality condition is given by $E\left[t_{FI} (f_M - f_{FI})\right] \geq 0$.

On their liability side we assume that factor $r_{FI}$ has a normal distribution $N(\overline{r}_{FI}, \sigma_{FI})$ with $0 < \overline{r}_{FI} < 1$ and $\text{Cov}(r_{DF}, r_{FI}) = \sigma_{DFFI}$. FIs may have two roles in our model: they can either create value or shift risks. Our formulation allows us to study the effects of these two distinct roles through the study of the three statistical components that describe $r_{FI}$:

\textsuperscript{2}This could happen due to positive externalities among firms. A production function $f(K) = K(K + r)$, where $K = 1 - n$, yields a gross rate of return equal to $1 + r - n$. 

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• the mean of the risk factor \( r_{FI} \), which depends on the ability of FIs to create value and on the margins charged to their clients.

• the standard deviation, \( \sigma_{FI} \), measures the level of risk in the securities issued by FIs.

• the correlation coefficient between the risk factors \( r_{DF} \) and \( r_{FI} \). Since agents are risk neutral, diversification is not an issue anymore (in fact most agents will hold one single asset). Yet, imperfect correlation between returns makes agents hold different securities according to their beliefs. For example, investors that are pessimistic about economic conditions may adopt more conservative investment strategies by selecting safer securities.

On their asset side, FIs invest their funds in projects that they have access to. We present two different views about the way FIs reinvest the funds that they collect from investors.

2.2.1 Security Design View versus Asset Management View

We may assume that FIs reinvest the funds received in the firms and collect their returns at the final date. This means that \( f_M = f_{DF} = r_{DF} - n \) and the profits made by FIs are given by \( 
\alpha_{FI} (r_{DF} - r_{FI}) \). Implicitly we are disregarding limited liability and FIs use their funds to amplify or offset the deviations of factor \( r_{DF} \) from its mean. The intermediation sector offers securities with returns which have a different risk and mean from the returns obtained through direct finance. When performing an insurance role, FIs are very similar to ”with-profits funds”: FIs loose money whenever \( r_{FI} > r_{DF} \), in which case they must use their own capital to make payments to depositors, and make profits whenever \( r_{FI} < r_{DF} \), in which case they retain earnings from their investments. Typically FIs smooth the return for their investors, making profits in ”good times” and loosing money in ”bad times”.

The idea is that FIs, by performing a risk transformation role, can facilitate coordination across households, increasing investment and efficiency. For example, a strategy in which FIs offer securities with returns with a standard deviation lower than the returns from direct finance, has an effect similar to an improvement in public information, whose beneficial effects over investment and efficiency have been discussed by Morris and Shin (2001a). These efficiency gains may then be shared by all agents. We mention later that, under competition with direct finance, FIs may not be able to appropriate any of these gains and direct finance may prevent financial intermediation.

\(^3\)At the initial date, we could have a coalition of agents forming an FI (not protected by limited liability). The FI invests its capital together with the depositor’s funds. At the final period, returns are realized and a realization of \( r_{FI} \) is drawn. The FI makes its payments to depositors and the members of the coalition are the residual claimants. Note that the ”no limited liability” assumption allows us to assume that FIs may start their activity, at the initial date, without any capital.
We could also think of FIs as having a more active role as far as the management of their funds is concerned. When we adopt an Asset Management perspective we are mostly interested in evaluating what happens when FIs are able to change the primitive payoffs available in the economy. Implicitly we are assuming that FIs are better at managing risks or creating value, than unsophisticated, uninformed, small investors. A possible justification is monitoring. Monitoring could either shift the \textit{ex ante} expected payoff in the projects or change the uncertainty associated to them. Under the Asset Management view we assume that $f^*_M = r_M - n$ with $r_M \neq r_{DF}$. Hence the profits from the intermediation sector are given by $\ell_{FI}(r_M - r_{FI})$.

### 2.3 Households

Households must decide between investing their funds in either direct or intermediated finance, in which case the payoffs are the respective returns, or not investing, in which case the payoff is zero. When investment decisions are taken, households know neither the values taken by the risk factors, $r_{DF}$ and $r_{FI}$, nor the mass of investors, $1 - n$. To decide whether to invest or not, they will guess the values taken by these variables, relying on the information they have. Households receive three pieces of information.

- public information on the risk factors $r_{DF}$ and $r_{FI}$, which consists of the two probability distributions that we have described earlier.

- each household receives a private signal $\omega_i$ about the true value of the risk factor $r_{DF}$, where

\[
\omega_i = r_{DF} + \varepsilon_i \quad \text{where } \varepsilon_i \sim N \left(0, \frac{1}{\beta}\right) \quad \text{iid across agents}
\]

where $\beta$ is the precision of private information. Note that agents receive no private information about the behavior of $r_{FI}$.

Given the information received, households update their (public) priors with their private information. Let $\rho_i = E[r_{DF}|\omega_i] = \frac{\alpha \sigma_{DF} + \beta \omega_i}{\alpha + \beta}$ be the updated belief of $r_{DF}$ upon observing signal $\omega_i$. Then the posterior distribution of $r_{DF}$ is the following

\[
r_{DF}|\omega_i = r_{DF}|\rho_i \sim N \left(\rho_i, \frac{1}{\alpha + \beta}\right)
\]

Conditioning on $\omega_i$ or $\rho_i$ is equivalent: when convenient we condition the random variables on $\rho_i$. Households also use their private information to update their expectation about the realization of factor $r_{FI}$.

\[
E[r_{FI}|\rho_i] = \tau_{FI} + \alpha \sigma_{DFFI} (\rho_i - \tau_{DF})
\]

Intuitively, households use their information about economic conditions to infer about the payoffs of the securities issued by FIs.
Neither firms nor FIs receive any private information about economic conditions and, in order to avoid a fully revealing equilibrium, we assume they are unable to aggregate information. In order to state our results more economically we assume that, when investing and refraining yield the same expected payoff, households prefer to invest, and, when indifferent between direct and intermediated finance, investors choose the former. Since agents are risk neutral, and we want to prevent cases in which households invest infinite amounts in one asset, we do not allow for short selling.

3 Equilibrium and Financial System Architecture

To take their decision households compare the expected net returns from investing in any portfolio composed of the available securities against the payoff of not investing. Not every household has the same perspectives about investing because there is heterogeneous information. Not only do they have different expectations about each risk factor, but also they are not sure about the actions taken by other players. This fact is central to our model because coordination of actions has important consequences over returns.

As described before, expected returns encompass two components: the expected gain derived from the risk factors, $r_{DF}$ and $r_{FI}$, and the expected loss associated with the externality loss, $n$. Households must make a guess about both components to take their decisions.

The expected gain for an investor, who formed a posterior $\rho_i$, depends on the selected portfolio. It turns out that, for each household, the portfolio with the maximum expected gain is constituted by one single security. Such result is presented in the next lemma.

**Lemma 1** Denote by $G(\rho_i)$ the expected gain from investing, for an agent with updated belief $\rho_i$. Then

$$G(\rho_i) = \max \{\rho_i, \bar{\tau}_{FI} + \alpha\sigma_{DFFI}(\rho_i - \bar{\tau}_{DF})\}$$

where $E[r_{DF}|\rho_i] = \rho_i$ and $E[r_{FI}|\rho_i] = \bar{\tau}_{FI} + \alpha\sigma_{DFFI}(\rho_i - \bar{\tau}_{DF})$. Graphically $G(\rho_i)$ is the upper envelope of these two straight lines which intersect at

$$\rho^I = \frac{\bar{\tau}_{FI} - \alpha\sigma_{DFFI}\bar{\tau}_{DF}}{1 - \alpha\sigma_{DFFI}} \quad (1)$$

**Proof.** See the mathematical appendix. ■

As for the expected loss originated by the negative externality, it is harder to compute since it depends on the actions that household $i$ believes that other households will take. To tackle this issue we need to introduce the concept of strategies. A strategy is a rule of action which determines which investment
decision to take for each updated belief \( \rho_i \) that the household \( i \) might have\(^4\). Thus it is a mapping

\[
s_i(\rho_i) : R \rightarrow \{ \text{invest in direct finance, invest in intermediated finance, not invest} \}
\]

The set of possible strategies has an infinite number of elements. We are especially interested in one type of strategy called switching strategy (hereafter SS) around \( \bar{\rho} \). This particular strategy prescribes not investing if the household receives a signal which makes its updated belief inferior to \( \bar{\rho} \). When its belief falls above the threshold \( \bar{\rho} \) then the household decides to invest in the security with highest expected return. Formally

\[
s^{\bar{\rho}}(\rho) = \begin{cases} 
\text{invest in the security with highest expected return} & \text{if } \rho \geq \bar{\rho} \\
\text{not invest} & \text{if } \rho < \bar{\rho}
\end{cases}
\]

A profile of strategies (one for each agent) is an equilibrium if, conditional on their information and the strategies followed by other households, households’ strategies maximize their conditional expected utility. At first sight this might seem a daunting task, given the amount of freedom involved, and it makes the next proposition quite surprising. Denote by \( \Phi(\cdot) \) the cumulative density function of a standard normal distribution and let \( \gamma = \frac{a^2(\alpha + \beta)}{\beta(\alpha + 2\beta)} \).

**Proposition 2** Provided that \( \gamma \leq 2\pi \) and \( \sigma_{DFI} \geq \frac{1}{\sqrt{2\pi}} \), there is a unique equilibrium. In this equilibrium every household follows a switching strategy around \( \rho^* \), where \( \rho^* \) is the unique solution to

\[
G(\rho^*) = \Phi(\sqrt{\gamma}(\rho^* - \tau_{DF}))
\]

given its updated belief \( \rho_i \), each investor invests its funds in the security with the highest expected return.

**Proof.** Our economic problem can be represented by a several stage game.
- at the first stage, nature draws a realization from the distribution of the risk factor \( r_{DF} \). This realization is unknown to everybody.
- at the second stage, nature draws a private signal for each household \( i \).
- at the third stage, each household decides whether to invest or not.
- at the final stage, those households who decided to become investors choose the best security to invest their funds.

Treating each realization of household \( i \)'s signal as a possible "type", we are solving for the Bayesian Nash Equilibrium of the game. The way we solve the game is by Backwards Induction. First, one must determine the optimal action for the possible movements at the final stage of the game. As we have alluded before, once the household has decided to invest, then the optimal action is to invest the whole amount of funds in the security with the highest expected

\(^4\)In fact, a strategy is a rule of action which determines which investment decision to take for each signal \( \omega_i \) that the agent might have. Given the equivalence between signals and updated beliefs, this formulation is correct.
return. Then we can proceed to the next-to-last decision stage and determine if each household should invest or not, given that they anticipate the action that will follow at the final stage. This last decision problem has been thoroughly discussed in the global games literature and, in particular, we apply the ideas presented by Morris and Shin (2001b). The full proof of the argument is given in the mathematical appendix. ■

The above result is extremely attractive: not only does it give a plain characterization of a unique equilibrium, but also states that the decision rule used by agents is very simple. The conditions presented amount to saying that private information is precise enough relative to public information and the amount of insurance provided by the FIs is not too big. These conditions are sufficient to guarantee uniqueness of the equilibrium. Proposition 2 is an extension of the work by Morris and Shin (2001b) in which they consider the existence of one single security. Our work extends their approach to a setup with two different securities. For the sake of completeness, we present the Morris and Shin result, adapted to our particular application, in the next remark.

**Remark 3** When only direct finance is available, provided that \( \gamma \leq 2\pi \), there is a unique equilibrium. In this equilibrium, every household refrains from investing if and only if \( \omega \leq \omega' \), where \( \omega' \) is the unique solution to

\[
\omega' = \Phi \left( \frac{1}{\sqrt{\beta}} (\omega' - \tau_{DF}) \right)
\]

Moreover \( \omega^* \leq \omega' \).

By looking at the definitions of \( \omega' \) and \( \omega^* \) it is easy to check that \( \omega^* \leq \omega' \) since \( G(x) \geq x \). Denote by \( \iota \) the level of investment in one economy in which both direct and intermediated finance are available.

**Corollary 4** The level of investment in the economy, denoted by \( \iota \), is a random variable which depends (positively) on the realization of the risk factor \( r_{DF} \)

\[
\iota(r_{DF}) = 1 - \Phi \left( \frac{\alpha + \beta}{\beta} \omega^* - \frac{\alpha}{\beta} \tau_{DF} - r_{DF} \right)
\]

**Proof.** See the mathematical appendix. ■

A consequence of the above corollary is that, the lower the threshold \( \omega^* \), the higher the level of investment in the economy. Intermediation is important because of its potential to lower the investment threshold below the level which exists in a financial system based solely on direct finance (\( \omega' \)). Introducing FIs in a financial system based on direct finance generates two different effects.

- Reallocation of funds effect: since intermediated finance does not mimic perfectly the payoffs of investing directly in the firms, there are investors who transfer their resources to FIs because they find their securities more attractive.
• Recycling of funds effect: under certain conditions, FIs lower \( \rho^* \) below the level existing in financial systems based solely on direct finance. This brings new funds, which previously remained idle, into the financial system. This diminishes the externality loss caused by non investors and increases expected returns throughout the economy, motivating further agents to invest. FIs are able to improve coordination among agents.

Under the general set of parameters that we have so far, it is difficult to give a more precise characterization of the equilibrium and, in particular, how funds are distributed among direct and intermediated finance. Hence in the next subsections we present, according to the values taken by the parameters, four possible equilibrium configurations, each of them corresponding to a specific financial architecture. We will pay some attention to the effects on the level of insurance provided by FIs, as measured by \( \sigma_{DFFI} \), and their ability to create value, as measured by \( \pi_{FI} \).

Since each investor invests his whole amount of funds in one single security, the amount of funds invested through FIs equals the amount of investors who choose intermediated finance. We denote this mass by \( \lambda_{FI} \) and it is a proxy for the size of the intermediation sector. We denote the mass of funds invested directly by \( \lambda_{DF} \). These quantities are random variables since they depend on the realization of the risk factor \( r_{DF} \).

### 3.1 Coexistence Between Direct and Intermediated Finance With Synergies

We start by presenting the equilibrium configuration which we consider to be the most interesting one.

**Corollary 5** When parameters are such that \( \alpha \sigma_{DFFI} < 1 \) and \( \rho^* < \rho^I \) there is coexistence between direct and intermediated finance. Households with an updated belief below \( \rho^* \) do not invest, households with updated beliefs belonging to the set \( \{ \rho^*, \rho^I \} \) deposit their funds in FIs and those with higher updated beliefs invest directly in the firm. Hence the amount of intermediated funds is \( \lambda_{FI} (r_{DF}) = \Phi \left( \sqrt{\beta} \left( \frac{\alpha+\beta}{\beta} \rho^I - \frac{\alpha}{\beta} \rho^* - \frac{\alpha}{\beta} \pi_{DF} - r_{DF} \right) \right) \) and the amount of direct finance is \( \lambda_{DF} (r_{DF}) = \Phi \left( \sqrt{\beta} \left( \frac{\alpha+\beta}{\beta} \rho^I - \frac{\alpha}{\beta} \pi_{DF} - r_{DF} \right) \right) \).

**Proof.** See the mathematical appendix. ■

Having \( \alpha \sigma_{DFFI} < 1 \) means that FIs offer securities either with imperfect correlation with the primitive payoffs of the economy or with lower risk. To see this, let \( \zeta \) be the correlation coefficient between both risk factors, and note that \( \alpha \sigma_{DFFI} < 1 \Leftrightarrow \zeta \sigma_{FI} < \sqrt{\frac{1}{\alpha}} \). This inequality is satisfied under two circumstances:

- \( \zeta < 1 \), in which case insurance is offered through imperfect correlation in returns.
\[ \zeta = 1 \text{ and } \sigma_{FI} < \sqrt{\frac{1}{2}}, \] in which case the securities issued by FIs have returns with lower risk than the returns obtained through direct finance.

In either case FIs will be offering insurance over the returns of the firms. Introducing financial intermediation brings new investors into the financial system, improves coordination among agents and increases returns throughout the economy, which further stimulates investment (recycling of funds effect).

One interpretation for such result is the following. Volatile economic fundamentals (which we may identify with public information which is not very precise) creates an adverse environment for investment. Uncertainty makes agents pessimistic about the investment decisions of other agents, and they become reluctant about their own investment decisions. When the fundamentals are moderately good, we see many agents rejecting potentially profitable investment projects and imposing negative externalities on other investors. The final outcome is an inefficient low level of investment compared with the Pareto optimum. Suppose now that an institution offers a new investment opportunity with returns more stable than the returns obtained through direct investment in the firms. This makes potential investors more confident about the investment decisions of other agents, spurring investment across the economy and leading to a Pareto improvement.

It is easy to check that both the mean and realization of the risk factor \( r_{DF} \) have positive effects over investment. If we interpret the fluctuations in this risk factor as being technologically driven, then positive technological shocks are associated with high levels of investment and direct finance. If we relate this result with the business cycle literature, one would expect very active financial markets during upturns and procyclical investment (eventually leading output). From the financial point of view our model predicts that, in periods of (moderate) crisis, investors tend to move their funds into safer securities. Under the equilibrium configuration described in corollary 5, the effect of the risk factor \( r_{DF} \) over the size of the intermediation sector are unclear. The intermediation sector becomes very small when the realization of \( r_{DF} \) is either very low (and almost every household decides not to invest) or is very high (where most investors choose direct finance). Nonetheless, one could argue that FIs act as a "buffer" when the economy is doing less well. For example, intermediation is important when \( r_{DF} = (\rho^* + \rho^I)/2 < \rho^I \). In this case most investors start dropping from direct finance, and FIs become more important and this is coherent with Greenspan’s view about the way the financial system works.

The relative weight of the intermediation sector in the economy depends not only on the realization of the fundamentals in the economy (as expressed by \( r_{DF} \)), but also on the variables which influence \( \rho^I - \rho^* \). Among these, it is important to stress the level of insurance provided by FIs, as measured by \( \sigma_{DFFI} \), and the mean of the risk factor \( r_{FI} \), which measures the value created
by an intermediary. Hence we may justify the different weight of intermediation, across financial systems, on the basis of the values taken by these parameters.

The following lemma is useful to characterize better the equilibrium.

**Lemma 6** When $\alpha \sigma_{DFI} < 1$ then $\frac{\partial (\rho' - \rho^*)}{\partial \sigma_{DFI}} > 0$

**Proof.** See the mathematical appendix. \[\Box\]

The above result is quite intuitive: it implies that the size of the intermediation sector increases as the mean of its returns increases. Hence margins charged by FIs have a negative effect over the size of the intermediation sector.

It is interesting to note that, under the Security Design View, the contract offered by the FIs suffers from a *winner’s curse* due to the coexistence with direct finance. Given their information, investors with $\rho_i < \rho^I$ choose indirect finance. When the realization of $r_{DF}$ is inferior to $\rho^I$ there is a large mass of agents with posteriors below $\rho^I$. In this case many investors give up direct finance and tap FIs for investment since FIs pay (on average) $r_{FI} > r_{DF}$ to their depositors. On the other hand, when the realization of $r_{DF}$ is large, FIs (on average) do not distribute the full return from their investments. But, in this case, FIs get few depositors since most investors choose direct finance. As a result, FIs must charge a (expected) margin $\tau_{DF} - \tau_{FI} > 0$ to their depositors in order to carry on with their activity. We have performed some numerical simulations and we were unable to find parameters for which this "naive" insurance activity is profitable. It seems that the winner’s curse effect is too strong\(^5\). In good states of the world (i.e. when FIs expect to be compensated with positive revenues) most investors choose direct finance and escape the insurance scheme offered by FIs. This leak, generated by competition from direct finance, makes insurance hard to offer. Under such circumstances, although FIs are socially desirable (in the sense that they enhance efficiency), financial intermediation seems to be unfeasible. Since FIs have difficulties in appropriating the social surplus generated by their activity the only feasible equilibrium involves direct finance alone.

Our model borrows from Allen and Gale (1997) the idea that there are institutions which may use capital to hedge *nondiversifiable* risks. Like them, FIs may be vulnerable to a market based system (which we identify with a system based on direct finance), unless they possess special investment opportunities. Note that a financial system based solely on financial intermediation would generate an equilibrium with lower threshold $\rho^*$, higher investment than the direct finance equilibrium and FIs making nonnegative profits.

Finally, regarding Asset Management View, we have considered the case in which $r_M = \tau_M + \sqrt{\alpha} \sigma_M (r_{DF} - \tau_{DF})$ which means that $r_M \sim N(\tau_M, \sigma_M)$. We have numerically simulated the case in which $\tau_M = \tau_{DF}$ and $\sigma_M < 1/\sqrt{\alpha}$.

\(^5\)"With-profits funds" seem to overcome this difficulty by charging exit penalties.
We interpret this assumption as if FIs have access to a monitoring technology which reduces the uncertainty in outcomes. We have found that FIs have (local) maximum profits when they offer a security with return \( r_{FI} \) such that \( r_{FI} < r_{DF} \) and \( \sigma_{DFFI} < 1 \), that is by offering insurance.\(^6\)

### 3.2 A Financial System Based on Direct Finance

For some parameters the economic environment becomes extremely unfavorable for financial intermediation.

**Corollary 7** When parameters are such that \( \sigma_{DFFI} < 1 \) and \( \rho^* \geq \rho^l \) financial intermediation is not possible. Every investor chooses direct finance.

**Proof.** See the mathematical appendix. \( \blacksquare \)

For the above set of parameters, the securities issued by FIs have unattractive expected returns and are dominated by direct finance. One possible justification for the inexistence of FIs is a low mean for the risk factor \( r_{FI} \).

**Proposition 8** When \( \sigma_{DFFI} < 1 \) then, for each constellation of parameters, there is a threshold for \( r_{FI} \) above which intermediation exists and below which intermediation is not possible.

**Proof.** See the mathematical appendix. \( \blacksquare \)

Such result is intuitively appealing: if \( r_{FI} \) is very low, when compared with the amount of insurance provided, investors find more profitable to invest directly in the firm sector.

### 3.3 Coexistence Between Direct and Intermediated Finance Without Synergies

We now study the case in which FIs, instead of offering insurance over the primitive payoffs of the economy, offer securities whose expected returns are very sensitive to the state of technology in the economy\(^7\). Specifically we mean that \( \sigma_{DFFI} > 1 \). We start by presenting an equilibrium with coexistence.

**Corollary 9** When parameters are such that \( \sigma_{DFFI} > 1 \) and \( \rho^* \leq \rho^l \) there is coexistence between direct and indirect finance. Households with an updated belief below \( \rho^* \) do not invest, households with updated beliefs belonging to the set \( [\rho^*, \rho^l] \) invest directly in the firm and those with higher updated beliefs deposit their funds in FIs. Hence the amount of intermediated funds is \( \iota_{FI} (r_{DF}) = 1 - \Phi \left( \sqrt{\sigma} \left( \frac{\alpha + \beta}{\beta} \rho^l - \frac{\alpha + \beta}{\beta} r_{DF} - r_{DF} \right) \right) \) and the amount of direct finance is

\[
\iota_{DF} (r_{DF}) = \Phi \left( \sqrt{\sigma} \left( \frac{\alpha + \beta}{\beta} \rho^l - \frac{\alpha + \beta}{\beta} r_{DF} - r_{DF} \right) \right) - \Phi \left( \sqrt{\sigma} \left( \frac{\alpha + \beta}{\beta} \rho^* - \frac{\alpha + \beta}{\beta} r_{DF} - r_{DF} \right) \right).
\]

\(^6\)We have also numerically simulated cases for which \( \rho_{M} = \sigma_{DFF} - \varepsilon \) (with \( \varepsilon \) small) and \( \sigma_{M} < 1/\sqrt{\sigma} \). This means that monitoring is costly. We obtained that FIs make positive profits for \( \sigma_{DFFI} < 1 \).

\(^7\)For example, stocks from financial institutions seem to be highly volatile with respect to the business cycle.
Proof. See the mathematical appendix. ■

Unlike the sort of coexistence presented in section 3.1, now there is no recycling of funds effect and no improvements in coordination. Under the above set of parameters, introducing FIs leads only to a reallocation of funds among the most optimistic investors and does not bring new investors into the financial system.

Again, one of the factors which determines the difference $\rho^l - \rho^*$ is the mean of the risk factor $r_{FI}$.

**Lemma 10** When $\alpha \sigma_{DFI} > 1$ then $\frac{\partial (\rho^l - \rho^*)}{\partial r_{FI}} < 0$

Proof. See the mathematical appendix. ■

The above result makes the relative weight of direct and indirect finance depend on $r_{FI}$. Such result is intuitive: for low values of the mean of the risk factor $r_{FI}$ most households prefer direct finance.

### 3.4 A Financial System Based on Intermediated Finance

We present the last possible equilibrium configuration in the next corollary.

**Corollary 11** When parameters are such that $\alpha \sigma_{DFI} > 1$ and $\rho^* > \rho^l$ every investor chooses intermediated finance.

Proof. See the mathematical appendix. ■

For the above parameters, apart from a recycling of funds effect, there is an extreme reallocation of funds: FIs absorb all funds from investors and direct finance is unfeasible. FIs perform better than direct finance, not because they offer any insurance, but rather because they offer more attractive investment opportunities. One possible justification for such equilibrium configuration is the high mean for the risk factor $r_{FI}$.

**Proposition 12** When $\alpha \sigma_{DFI} > 1$ then, for each constellation of parameters, there is a threshold for $r_{FI}$ above which only intermediation exists and below which there is coexistence.

Proof. See the mathematical appendix. ■

Such result is intuitively appealing since $r_{FI}$ is directly associated with the expected return made on the securities issued by the FIs.

The recycling of funds effect, present under the equilibria described in corollaries 5 and 11, may be associated with the justifications for intermediation exposed by Gershenkron (1962). His work suggests that countries in the early stages of their development process and with financial systems based on direct finance suffer from information and coordination difficulties which depress investment. FIs emerge due to their ability to overcome these difficulties. Many a historian\(^8\) agrees that in the early days of the industrial revolution banks played

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\(^8\)Among whom Cottrell (1980).
an important coordinating role, collecting and reinvesting idle funds from small and uninformed investors. We finish this section by presenting a classification of the different financial architectures in figure 1, according to the values taken by the two parameters $\tau_{FI}$ and $\sigma_{DFFI}$.

4 Policy and Regulatory Implications

The threshold $\rho^*$ used by households in their switching strategies is a central variable to our analysis. A lower threshold improves coordination among agents and induces two effects over the economy.

- it increases investment and returns. For positive thresholds, a lower $\rho^*$ may enhance efficiency in the economy since projects have ex ante positive net present value.
- it influences the fluctuations in the levels of investment and output in the economy. In this sense, different thresholds induce more or less stable financial markets and pronounced business cycles\(^9\). Note that efficiency and stability may become conflicting objectives.

Financial intermediation has important effects over the determination of the threshold $\rho^*$. We have seen that, unless FIs have a special investment set,

\(^9\)Note that $\text{Var}[\rho (r_{DF})] = E[\Phi^2(\sqrt{\Phi}\left(\frac{\alpha + \beta}{\Phi} \rho^* - \frac{\alpha}{\Phi} \tau_{DF} - r_{DF}\right))] - \\
\{E[\Phi(\sqrt{\Phi}\left(\frac{\alpha + \beta}{\Phi} \rho^* - \frac{\alpha}{\Phi} \tau_{DF} - r_{DF}\right))])\}^2 + \frac{\partial \text{Var}[\rho (r_{DF})]}{\partial \rho^*} = \\
\text{Cov}\left[\Phi\left(\sqrt{\Phi}\left(\frac{\alpha + \beta}{\Phi} \rho^* - \frac{\alpha}{\Phi} \tau_{DF} - r_{DF}\right)\right)\phi\left(\sqrt{\Phi}\left(\frac{\alpha + \beta}{\Phi} \rho^* - \frac{\alpha}{\Phi} \tau_{DF} - r_{DF}\right)\right)\right] \neq 0.$
financial intermediation might be difficult when facing competition from direct finance. One possible solution to such problem would be the creation of FIs at the initial date, before private information is revealed. _Ex ante_, given that a lower threshold for SS may increase welfare, and every agent recognizes the importance of FIs, private and social interests are aligned. Hence FIs could emerge since every agent is willing, _ex ante_, to finance insurance scheme at the final date.

However, often the distinction initial versus interim date is artificial: private and public information are simultaneously released and then, even if it is efficient to have intermediation, it might be extremely difficult to find a standard contract or a stable coalition that finances financial intermediaries.

Once it is recognized that FIs increase efficiency and _ex ante_ contracts are unfeasible, then there is scope for an institution to be created which helps to finance the payments made by the FIs, and helps to replace the _ex ante_ contract. In this sense a deposit insurance scheme could be seen as an instrument to overcome coordination difficulties.

## 5 Conclusion

Concluding, it seems that FIs have an important role to play in the economy. When they provide securities safer than those available in financial markets, they may improve coordination among agents and increase efficiency. Yet, such activity might be difficult to be profitably performed. Hence, in order to exist, FIs must be special in some sense, either by having access to some investment opportunities not available to other investors or having other particular abilities. Under this setup, the asset management opportunities of FIs become extremely important and centralized intervention might be desirable\(^{10}\).

We have considered that the mass of non investors, \(n\), has a common effect to both securities which implies that the externality loss does not depend on the type of investment made. One interesting extension of our model is to consider explicitly different externality losses generated by non investors for each of the securities in the economy. Arguably, FIs divert resources from markets for direct finance diminishing its liquidity. On the other hand, when markets for direct finance are incipient, financial intermediation might reveal as the most efficient mechanism to channel funds from agents into the productive activity. As a consequence, encompassing these features in our model would point out both the virtues of financial intermediaries when markets are illiquid (as in the Japanese and German types of financial architecture) and the virtues of a fully fledged security markets when externalities are limited an liquidity is high. Again, our ideas run close to the arguments by Allen and Gale (1997).

\(^{10}\)As suggested by Gershenkron (1962).
A Mathematical Appendix

A.1 Proof of Lemma 1

Note that

\[ G(\rho_i) = \max_{\lambda \in [0,1]} E[\lambda r_{DF} + (1 - \lambda) r_{FI}|\rho_i] = \max_{\lambda \in [0,1]} \lambda E[r_{DF}|\rho_i] + (1 - \lambda) E[r_{FI}|\rho_i] \]

where \( E[r_{DF}|\rho_i] = \rho_i \) and \( E[r_{FI}|\rho_i] = \tau_{FI} + \alpha\sigma_{DF}\tau_{FI} \) (\( \rho_i - \tau_{DF} \)). It is easy to see that, under our assumptions, that \( \lambda \) will be either 0 or 1. Hence the result.

A.2 Proof of Proposition 2

Let \( S \) be the set of possible strategies, \( s_i \) the strategy followed by agent \( i \) and \( s_{-i} \) the profile of strategies chosen by all other households except \( i \). Agents start by computing the maximum expected net return that they can make in the available portfolios

\[ \max_{\lambda \in [0,1]} E[\lambda (r_{DF} - n) + (1 - \lambda) (r_{FI} - n)|\rho_i, s_{-i}] \]

It is useful to study this function. Note that the above expression is equivalent to

\[ G(\rho_i) - E[n|\rho_i, s_{-i}] \]

Since households are risk neutral, investors invest their whole amount of savings in the security with the highest expected return. Given that the second component of the expected net returns is common to both securities, this implies that investors will select the security with the highest expected gain.

In the above expression we can identify both components of the expected net return. Let us study the externality loss. First note that household \( i \), given its information, may also compute the posterior for the updated belief that he believes other agents have:

\[ \rho_{-i}|\rho_i \sim N \left( \frac{\alpha\tau_{DF} + \beta\rho_i}{\alpha + \beta}, \sqrt{\frac{\beta(\alpha + 2\beta)}{(\alpha + \beta)^2}} \right) \]

Let

\[ L_1(\rho_i) = E[n|\rho_i, s_{-i}^{\rho_i}] = \text{prob}[\rho_{-i} < \rho_i|\rho_i] = \Phi[\sqrt{\tau}(\rho_i - \tau_{DF})] \]

This function represents the expected externality loss from underinvestment, for an agent who receives \( \rho_i \) and believes every other agent is using a SS around \( \rho_i \). The first equality holds because the noise in private information is independent of the true value of \( r_{DF} \), which makes the expected proportion of non investors equal to the probability that any particular household does not invest. Since
everyone follows a SS around \( \rho_i \), the probability that any particular household does not invest, is given by the probability that this household’s updated belief falls below \( \rho_i \), and this justifies the second equality. Note that \( 0 < L(\rho_i) < 1 \).

Define

\[
U_1(\rho_i) = G(\rho_i) - L_1(\rho_i)
\]

as the expected net return, given \( \rho_i \), when every other agent is using a SS around \( \rho_i \). Our assumptions imply the following lemmas.

**Lemma 13** Both functions, \( G(\rho_i) \) and \( L_1(\rho_i) \), are continuous.

**Proof.** Trivial. ■

**Lemma 14** The function \( G(\rho_i) \) is upward sloping and there is a unique value \( \rho_i \) which solves \( G(\rho_i) = 0 \).

**Proof.** Trivial. ■

**Lemma 15** If \( \gamma \leq 2\pi \) and \( \sigma_{DFI} \geq \frac{1}{\alpha} \sqrt{\frac{\pi}{2}} \), then there is a unique solution \( \rho^* \) to \( U_1(\rho^*) = 0 \).

**Proof.** The slope of \( U_1(\rho_i) \) is \( G'(\rho_i) - \sqrt{\gamma} \phi[\sqrt{\gamma} (\rho_i - \tau_{DF})] \) which is at least \( G'(\rho_i) - \sqrt{\frac{\pi}{2}} \). This is positive if \( \sigma_{DFI} > \frac{1}{\alpha} \sqrt{\frac{\pi}{2}} \) and \( \gamma < 2\pi \). Since \( U_1(\rho_i) \) takes both positive and negative values, then there is at most one solution to \( U_1(\rho_i) = 0 \). If either \( \sigma_{DFI} = \frac{1}{\alpha} \sqrt{\frac{\pi}{2}} \) or \( \gamma = 2\pi \) or both, then the point at which the slope may be zero is at \( \rho_i = \tau_{DF} \). At this point, though the second derivative of \( U_1(\rho_i) \) is zero, the third derivative is positive which implies that there is one single solution. ■

From these results follows the statement in proposition 2. The argument goes as follows. Let

\[
L_2(\rho_i, \hat{\rho}) \equiv \text{prob} [\rho_{-i} < \hat{\rho} | \rho_i] = \Phi \left[ \sqrt{\gamma} \left( \hat{\rho} - \tau_T + \frac{\beta}{\alpha} (\hat{\rho} - \rho_i) \right) \right]
\]

represent the expected externality loss from underinvestment, for an agent who receives \( \rho_i \) and believes every other agent is using a SS around \( \hat{\rho} \). Define

\[
U_2(\rho_i, \hat{\rho}) = G(\rho_i) - L_2(\rho_i)
\]

be the expected net return, conditional on posterior \( \rho_i \), when every other agent is using a SS around \( \hat{\rho} \). Note that

\[
U_2(\rho_i, \rho_i) = U_1(\rho_i) \quad (3)
\]

We start by stating the following lemmas.

**Lemma 16** \( U_2(\rho_i, \hat{\rho}) \) is increasing in its first argument and decreasing in its second.
Proof. Trivial. ■

Lemma 17 Let \( \tilde{\rho} \) be a solution to \( U_2 (\tilde{\rho}, \tilde{\rho}) = 0 \). Then \( \tilde{\rho} \) is unique and \( \rho_1 < \tilde{\rho} \).

Proof. Note that \( U_2 (\tilde{\rho}, \tilde{\rho}) = 0 \iff U_1 (\tilde{\rho}) = 0 \). Hence \( \tilde{\rho} = \rho^* \) and this is unique. Moreover, we may define \( \rho_1 \) as \( \lim_{\tilde{\rho} \to -\infty} U_2 (\rho_1, \tilde{\rho}) = 0 \). By \( U_2 (\tilde{\rho}, \tilde{\rho}) = 0 \) and lemma 16 the result follows. ■

Under our assumptions there is a single point \( \rho^* \) that satisfies \( U_1 (\rho^*) = 0 \). This point defines the unique (symmetric) equilibrium existing in this game, in which every player follows an SS around \( \rho^* \). The marginal investor who receives posterior \( \rho^* \), and believes that others use an SS around \( \rho^* \), is indifferent between investing or not. No other posterior has this property. It is easy to see, from lemma 16, that if every agent believes others use an SS around \( \rho^* \), then agents with \( \rho_i \geq \rho^* \) decide to invest while other agents decide not to invest.

Having proved the existence of a unique equilibrium in SS, we will show that there can be no other equilibrium, in the spirit of the proof by Morris and Shin [19].

If \( \rho_i \) is sufficiently unfavorable, then \( G (\rho_i) < 0 \), and not investing is the dominant action irrespective of what other agents do. Note that \( \rho_1 \) is the threshold for the posterior, below which not investing is the dominant action.

If agents believed that others were following an SS around \( \rho_1 \), then their best reply would be an SS around \( \rho_2 \), where \( \rho_2 \) solves \( U_2 (\rho_2, \rho_2) = 0 \).

The most optimistic investor believes that the proportion of non investors is higher or equal than that implied by an SS around \( \rho_1 \). Since the payoff to investing is decreasing in the mass of non investors, we rule out every strategy that prescribes investing for posteriors inferior to \( \rho_2 \) since they are dominated.

Proceeding this way, we construct an increasing sequence of thresholds \( \rho_k \) (due to lemma 16) which do not survive \( k \) iterations of deletion of dominated strategies.

\[
\rho_1 < \rho_2 < \ldots < \rho_k < \ldots
\]

Since \( \rho^* \) is the unique solution to \( U_2 (\tilde{\rho}, \tilde{\rho}) = 0 \) (because of (3)), then it is the least upper bound of the sequence of thresholds (again by lemma 16) and hence its limit. Any strategy that dictates investing for \( \rho_i < \rho^* \) does not survive iterated deletion of dominated strategies.

On the other hand, there is a symmetric argument for \( \rho_i > \rho^* \). From this argument we derive that any strategy that decides not to invest for \( \rho_i > \rho^* \) does not survive iterated deletion of dominated strategies.

Thus there is one single strategy surviving iterated elimination of dominated strategies, which is the SS around \( \rho^* \). ■

\footnote{Note that other agents might not follow an SS around \( \rho_1 \), but they deinitely follow strategies that prescribe not invest for \( \rho_i < \rho_1 \).}

\footnote{Since this is the most “optimistic” guess about the strategies followed by other players.}

\footnote{By similar strategies that decide not invest for \( \rho_i < \rho_2 \).}
A.3 Proof of Corollary 4

The mass of investors is equal to the probability that any particular household invests. Given the realization of the factor \( r_{DF} \), this is the probability that a household’s updated belief falls above \( \rho^* \). Since \( \rho_i | r_{DF} \sim N \left( \frac{\alpha r_{DF} + \beta^2 r_{DF}}{\alpha + \beta}, \frac{\sqrt{\beta}}{\alpha + \beta} \right) \) then

\[
\iota (r_{DF}) = \text{prob} [\rho_i > \rho^* | r_{DF}] = 1 - \Phi \left( \sqrt{\beta} \left( \frac{\alpha + \beta}{\beta} \rho^* - \frac{\alpha}{\beta} r_{DF} - r_{DF} \right) \right).
\]

\[\square\]

A.4 Proof of Corollary 5

Households with updated beliefs above \( \rho^* \) have positive expected returns. Their expected gain is equal to

- \( \rho_i \) if they choose direct finance
- \( \tau_{FI} + \alpha \sigma_{DFFI} (\rho_i + \tau_{DF}) \) if they choose intermediated finance

Investors with updated beliefs in the interval \([\rho^*, \rho']\) have \( \rho_i < \tau_{FI} + \alpha \sigma_{DFFI} (\rho_i + \tau_{DF}) \) and they choose intermediated finance. Agents with updated beliefs above or equal to \( \rho' \) have \( \rho_i \geq \tau_{FI} + \alpha \sigma_{DFFI} (\rho_i + \tau_{DF}) \) and choose direct finance. Hence, for a given realization of \( r_{DF} \), the mass of intermediated funds is

\[
\iota_{FI} (r_{DF}) = \text{prob} [\rho^* \leq \rho_i \leq \rho' | r_{DF}] = \Phi \left( \sqrt{\beta} \left( \frac{\alpha + \beta}{\beta} \rho' - \frac{\alpha}{\beta} r_{DF} - r_{DF} \right) \right) - \Phi \left( \sqrt{\beta} \left( \frac{\alpha + \beta}{\beta} \rho^* - \frac{\alpha}{\beta} r_{DF} - r_{DF} \right) \right)
\]

and the amount of direct finance is

\[
\iota_{DF} (r_{DF}) = \text{prob} [\rho_i \geq \rho' | r_{DF}] = 1 - \Phi \left( \sqrt{\beta} \left( \frac{\alpha + \beta}{\beta} \rho' - \frac{\alpha}{\beta} r_{DF} - r_{DF} \right) \right).
\]

\[\square\]

A.5 Proof of Lemma 6

Note that

\[
\frac{\partial (\rho' - \rho^*)}{\partial \tau_{FI}} = \begin{cases} 
\frac{1}{1 - \alpha \sigma_{DFFI}} & \text{if } G(\rho^*) = \rho^* \\
\frac{1}{1 - \alpha \sigma_{DFFI}} - \frac{1}{\alpha (\sqrt{\rho^* - \tau_{DF}}) \sqrt{\rho^* - \alpha \sigma_{DFFI}}} & \text{if } G(\rho^*) = \tau_{FI} + \alpha \sigma_{DFFI} (\rho^* + \tau_{DF}) 
\end{cases}
\]
In any of these cases \( \frac{\rho(r^*-\rho^*)}{\sigma_{F1}} > 0. \)

A.6 Proof of Corollary 7

When \( \rho^* \geq \rho^l \), the expected gain on direct finance is higher than the expected gain on intermediated finance for those households with updated beliefs above \( \rho^* \). Hence

\[
\iota_{DF}(r_{DF}) = \text{prob}[\rho_i \geq \rho^*|r_{DF}] = 1 - \Phi\left(\sqrt{\beta} \left(\frac{\alpha + \beta}{\beta} \rho^* - \frac{\alpha}{\beta} r_{DF} - r_{DF}\right)\right).
\]

A.7 Proof of Proposition 8

When \( \rho^l > \rho^* \) financial intermediation exists and when \( \rho^l \leq \rho^* \) financial intermediation is not possible. It is easy to find a sufficient high value for the mean of the risk factor \( r_{F1} \) for which intermediation necessarily exists (for example \( r_{F1} > \rho^* \)) and a sufficient low value for which intermediation is not feasible (for example \( r_{F1} < 0 \)). Given the result in lemma 6, the result follows.

A.8 Proof of Corollary 9

Investors with updated beliefs belonging to the set \([\rho^*, \rho^l]\) find direct finance more attractive since \( \rho_i \geq r_{F1} + \alpha \sigma_{DF} \) \( (\rho_i + r_{DF}) \), while more optimistic investors prefer direct finance. Hence, for a given realization of \( r_{DF} \), the expected amount of direct finance is

\[
\iota_{DF}(r_{DF}) = \text{prob}[\rho^* \leq \rho_i \leq \rho^l|r_{DF}] = \\
= \Phi\left(\sqrt{\beta} \left(\frac{\alpha + \beta}{\beta} \rho^l - \frac{\alpha}{\beta} r_{DF} - r_{DF}\right)\right) \\
- \Phi\left(\sqrt{\beta} \left(\frac{\alpha + \beta}{\beta} \rho^* - \frac{\alpha}{\beta} r_{DF} - r_{DF}\right)\right)
\]

and the expected mass of intermediated funds is

\[
\iota_{FI}(r_{DF}) = \text{prob}[\rho^l < \rho_i|r_{DF}] = 1 - \Phi\left(\sqrt{\beta} \left(\frac{\alpha + \beta}{\beta} \rho^l - \frac{\alpha}{\beta} r_{DF} - r_{DF}\right)\right).
\]
A.9 Proof of Lemma 10

We have

$$\frac{\partial (\rho^* - \rho^t)}{\partial \tau_{FI}} = \begin{cases} \frac{1}{1-\alpha \sigma_{DFFI}} & \text{if } G(\rho^*) = \rho^* \\ \frac{\phi(\sqrt{\tau(\rho^* - \tau_{DF}))})}{(1-\alpha \sigma_{DFFI})[\phi(\sqrt{\tau(\rho^* - \tau_{DF}))})]} \sqrt{\tau-1} & \text{if } G(\rho^*) = \tau_{FI} + \alpha \sigma_{DFFI} (\rho^* + \tau_{DF}) \end{cases}$$

In any of these cases $\frac{\partial (\rho^* - \rho^t)}{\partial \tau_{FI}} < 0$. ■

A.10 Proof of Corollary 11

When $\rho^* > \rho^t$ investors prefer intermediated finance. Hence

$$\nu_E(\tau_{DF}) = \text{prob}[\rho_i \geq \rho^* | \tau_{DF}] = 1 - \Phi \left( \sqrt{\beta} \left( \frac{\alpha + \beta}{\beta} \rho^* - \frac{\alpha}{\beta} \tau_{DF} - \tau_{DF} \right) \right).$$

■

A.11 Proof of Proposition 12

When $\rho^t \geq \rho^*$ there is coexistence and when $\rho^t < \rho^*$ direct finance is not possible. It is easy to find a sufficient high value for the mean of the risk factor $\tau_{FI}$ for which direct finance is not feasible and a sufficient low value for which direct finance necessarily exists. Given the result in lemma 10, the result follows. ■

References


