Evaluating the Taylor Principle Over the Distribution of the Interest Rate: Evidence from the US, UK and Japan

Paul Mizen,\textsuperscript{a,\textdagger} Tae-Hwan Kim\textsuperscript{a,b} and Alan Thanaset\textsuperscript{c}

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Abstract
Support for the Taylor principle is considerable but the focus of empirical investigation has been on estimated coefficients at the mean of the interest rate distribution. We offer a new approach that estimates the response of interest rates to inflation and the output gap at various points (quantiles) on the conditional distribution corresponding to different levels of interest rates. We find support for the Taylor principle at all but low rates in normal times for the US and the UK, but an increasingly aggressive (nonlinear) response to inflation as rates increase. This is robust to the inflation horizon, instrument choice and use of a real time output gap data. In abnormal times, described by events in Japan, we find strong support for the Taylor principle, and increasing aggression to inflation when rates increase. We confirm that increasing aggression towards inflation can be observed as interest rates approach zero. The results have implications for the modeling of economies when inflation is very low, and provides some insights into Japanese monetary policy in particular.

Keywords: Taylor Principle, policy rules, quantile regression, low inflation, Japan

JEL classifications: E42, E52

\textsuperscript{a}School of Economics, University of Nottingham, \textsuperscript{b}Department of Economics, Yonsei University, and \textsuperscript{c}Department of Economics, University of Leicester, \textsuperscript{\textdagger}Corresponding Author: Paul Mizen, University of Nottingham, University Park, Nottingham, NG7 2RD, UK. email: paul.mizen@nottingham.ac.uk. We are grateful to the Experian Centre of Economic Modelling for funding this research. We also thank Anindya Banerjee, Denise Osborn, Stefan Gerlach, Petra Gerlach-Kristen, Adam Posen, Volker Wieland, Tony Yates, James Yatman, Feng Zhu and seminar participants at the European University Institute, Florence, Hong Kong University, University of Leicester and the Swiss National Bank for comments.
1 Introduction

The Taylor principle - which suggests central banks should raise short term interest rates by more than the rate of inflation, increasing real interest rates when inflationary pressures emerge - is derived from the Taylor rule. Empirical estimates of this relationship abound for a range of countries and time periods. A selection of the literature includes Taylor (1999), Clarida et al. (1998), Bernanke and Gertler (1999) for the US, Nelson (2001) for the UK, and Kuttner and Posen (2001, 2003) for Japan, among many others. Where attempts have been made to estimate the Taylor rule, the estimation has typically been conducted using a Least Squares (LS) method or variants such as Instrumental Variable (IV) and Generalized Method of Moments (GMM) estimation that handle the potential endogeneity of the inflation rate and the output gap in the Taylor rule. These approaches have offered considerable international support for the Taylor rule, and the Taylor principle, but their focus has been exclusively on the estimated coefficients when the level of interest rate is at the mean of the distribution conditional on the inflation rate and the output gap.

The interpretation of these results has therefore assumed that the covariates— inflation and the output gap—affect only the location of the conditional distribution of the interest rate, not its scale, or any other aspect of its distributional shape, but some authors have explored the possibility of asymmetries and nonlinearities in the Taylor rule with respect to inflation and the output gap (c.f. Gerlach, 2000, and Bec et al., 2002). The evidence for or against asymmetries and nonlinearities is an empirical matter, but when nonlinearities emerge we can no longer take for granted the assumption that central banks respond with the same degree of aggression to inflation and the output gap under different conditions, such as upswings and downswings of the business cycle or deflationary conditions, for example. This issue has been central to the debate concerning monetary policy options under very low interest rates (c.f Bernanke and Reinhart, 2004; Coenen and Weiland, 2004; Eggertson and Woodford, 2003; Blinder, 2000; Reisneider and Williams, 2000; and McCallum, 2000) and not least for circumstances where nominal rates have effectively reached the zero lower bound in Japan (Ueda, 2000, 2004; Kuttner and Posen 2001, 2003).

In this paper we consider how the response to inflation and the output gap differs at various points on the interest rate distribution. The question is whether the monetary authorities of the United States, the United Kingdom and Japan react identically when their operational interest rates take values at different points on their respective interest rate distributions. To put it another way, would the response to inflation be the same if rates were equal to 10 percent compared to very low rates of one percent (say)? Given that interest rates are strongly positively correlated with inflation this is equivalent to determining the central bank reaction function when inflation is higher or lower than its mean value.

Our paper offers a new method for evaluating the Taylor Principle. We generate estimates of the response to inflation at each of the points of the interest rate distribution, labeled quantiles, so that we are able to provide an indication of the degree of response at the median (as does the conventional approach described by Clarida et al. (1998), Bernanke and Gertler (1999) and others) and at the upper and lower tails. We make use of the two-stage quantile regression (2SQR) proposed by Kim and Muller (2004a, 2004b), which is a natural extension of the regression quantile method first introduced by Koenker and Bassett (1978), and this takes into account the endogeneity problem associated with our explanatory variables\(^1\). The results we report are generated for

\(^1\)The regression quantile model has distinct advantages over the conventional approach since it can be used to characterize the entire conditional distribution of the interest rate (see Koenker and Hallock, 2001). The objective function of the quantile regression is a weighted sum of absolute deviations, and therefore the estimated coefficient vector is not sensitive to outlier observations of the dependent variables. Indeed when the error term is non-normal, quantile regression estimators can be more efficient than the least squares estimator.
monthly observations for three major economies - the US, the UK and Japan. The samples differ to reflect the monetary policy regimes. The US sample covers 1979/10, when Chairman Paul Volcker took office, to 2003/09; the UK sample begins with the start of inflation targeting in 1993/02 and runs to 2003/07; and the sample for Japan runs from 1979/04 to 2003/12 which begins with the point where the interbank lending rate became the chief operating instrument of monetary policy and all capital controls were finally abandoned. These samples include periods of high interest rates (maximum values are 19.1 percent, 7.5 percent and 12.7 percent in the US, the UK and Japan respectively) and exceptionally low rates (minimum values are 1.01 percent, 3.5 percent and 0.001 percent respectively). Interest rates map closely to the inflationary experiences of the countries concerned, since high nominal rates occurred when inflation was high by current standards, and low nominal rates have been associated with low inflation and deflation in the case of Japan.

Our findings show that the response to inflation varies across the conditional distribution, and the degree of responsiveness of the interest rate towards the inflation rate increases as inflation rises. Our findings support the Taylor Principle at all points of the distribution except low levels in normal times. The evidence for abnormal times is drawn from Japan, which has been much more responsive to inflation than the US or the UK. The aggressiveness of the Bank of Japan detected here has been found by other studies, notably Kohn (1996), Clarida et al. (1998) and Bernanke and Gertler (1999). We also find that rates become more responsive to inflation as the lower bound approaches. The response to output gap over the conditional distribution declines progressively as we move to higher quantiles. These results offer some insights into the nature of the policy rule at low levels of inflation and interest rates that might usefully be employed in studies of low inflation economies, and some specific insights into recent policy choices by the Bank of Japan.

The paper is organized as follows. In Section 2 we outline the quantile regression method used to provide estimates of the reaction to inflation at different points on the conditional distribution. Section 3 explains the data set. Our empirical findings based on IV estimation and quantile regression are discussed and interpreted in Section 4. Section 5 offers some discussion of the implications and insights from our results. The conclusions from our paper are given in the final section.

2 Quantile Regression

Consider the estimation of the relationship between $T$ observations on the interest rate, $i_t$, inflation, $\pi_t$, and the output gap, $\tilde{y}_t$, in a monetary policy reaction function. Conventional estimation methods such as OLS, IV or GMM evaluate the relationship between these observations at the mean of the conditional distribution, and estimates of the parameter values ($\alpha$, $\beta$ and $\gamma$) refer to the following conditional mean function of $i_t$ given $\pi_t$ and $\tilde{y}_t$;

$$E(i_t \mid \pi_t, \tilde{y}_t) = \alpha + \beta \pi_t + \gamma \tilde{y}_t.$$  

The Taylor Principle is evaluated by $\beta$, the marginal effect of $\pi_t$ on $E(i_t \mid \pi_t, \tilde{y}_t)$, defined as $\beta = \frac{\partial E(i_t \mid \pi_t, \tilde{y}_t)}{\partial \pi_t}$, which measures how much $i_t$ changes at the mean when the inflation rate increases by one percent. The same kind of interpretation applies to $\gamma$, the coefficient on the output gap. By ascribing the relationship to the entire distribution, it is assumed that the relationship that is uncovered holds even when observations of the dependent variable (the interest rate in our case) are taken from above or below the mean value. This may be a valid assumption, but typically it is not put to the test, and it may not be upheld empirically. This section explores ways in which this assumption could be investigated by evaluating the relationship at different points (quantiles) of the conditional distribution using quantile regression.
For a given number \( \theta \in (0,1) \), the \( \theta^{th} \) conditional quantile of \( i_t \) given \( \pi_t \) and \( \tilde{y}_t \), denoted by \( q_\theta (i_t \mid \pi_t, \tilde{y}_t) \), is defined as the solution to the following equation:

\[
\int_{-\infty}^{q_\theta (i_t \mid \pi_t, \tilde{y}_t)} f_{i_t \mid \pi_t, \tilde{y}_t} (x \mid \pi_t, \tilde{y}_t) \, dx = \theta
\]

where \( f_{i_t \mid \pi_t, \tilde{y}_t} (\cdot \mid \pi_t, \tilde{y}_t) \) is the conditional density of \( i_t \) given \( \pi_t \) and \( \tilde{y}_t \). Here, it is implicitly assumed that the two explanatory variables \( \pi_t, \tilde{y}_t \) affect not only the mean value of \( i_t \) but also other parts of the entire distribution of the interest rate. As we increase \( \theta \) continuously from 0 to 1 we effectively trace the entire conditional distribution of the interest rate, \( i_t \), conditional on the rate of inflation, \( \pi_t \), and the output gap, \( \tilde{y}_t \). Our objective in this paper is to analyze how \( \pi_t \) and \( \tilde{y}_t \) affect the interest rate over the range of the conditional distribution as it approaches the upper and lower tails.

As in the conditional mean case above, we will assume that the relationship between the interest rate and the explanatory variables is linear; i.e. the conditional quantile function is linearly specified as follows\(^2\)

\[
q_\theta (i_t \mid \pi_t, \tilde{y}_t) = \alpha (\theta) + \beta (\theta) \pi_t + \gamma (\theta) \tilde{y}_t.
\]

The parameters \( \beta (\theta) \) and \( \gamma (\theta) \) are allowed to change with \( \theta \) and measure the degree of responsiveness of the interest rate to \( \pi_t \) and \( \tilde{y}_t \) respectively when the interest rate is located at the \( \theta^{th} \) quantile of the conditional distribution. For a fixed value of \( \theta \), the parameters \( \alpha (\theta), \beta (\theta) \), and \( \gamma (\theta) \) are estimated through the following minimization:

\[
\min_{\alpha, \beta, \gamma} \sum_{t=1}^{T} \rho_{\theta} (i_t - \alpha (\theta) - \beta (\theta) \pi_t - \gamma (\theta) \tilde{y}_t)
\]

where \( \rho_{\theta} (u) \) is the so-called check function given by \( \rho_{\theta} (z) = z (\theta - 1_{[z \leq 0]} ) \). Effectively, \( \rho_{\theta} (z) \) imposes different weights on positive and negative residuals (c.f. Koenker and Hallock, 2001) and where \( \rho_{\theta} (z) \) takes the value of a half this is the median estimator.

The one-step quantile estimation in (3) assumes that there is no endogeneity problem; that is, the explanatory variables \( \pi_t \) and \( \tilde{y}_t \) are not correlated with the error term \( \varepsilon_t \). But, when a forward-looking inflation rate is used in (2) and (3), this variable is likely to be correlated with \( \varepsilon_t \). If endogeneity is present, then the quantile estimators of \( \hat{\alpha} (\theta), \hat{\beta} (\theta), \) and \( \hat{\gamma} (\theta) \) obtained from (3) will be biased as demonstrated by simulation in Kim and Muller (2004a). In order to avoid this biasness, we follow the Two-Stage Quantile Regression (2SQR) methodology proposed in Kim and Muller (2004b), in which (i) the forward-looking inflation rate is replaced by fitted values obtained by OLS regression (first stage) and (ii) the quantile regression in (3) is carried out using the OLS fitted values as explanatory variables. Kim and Muller (2004b) have proved that the slope quantile estimators \( \hat{\beta} (\theta) \) and \( \hat{\gamma} (\theta) \) are consistent, although the intercept estimator \( \hat{\alpha} (\theta) \) remains biased. In what follows we do not attempt to interpret the intercept.

To interpret the coefficients, we note that, since the \( \theta^{th} \) conditional quantile of \( i_t \) given \( \pi_t \) and \( \tilde{y}_t \) is given by (2), the marginal change in the \( \theta^{th} \) conditional quantile due to marginal change in \( \pi_t \) simply equals the partial derivative of the conditional quantile of \( i_t \) with respect to \( \pi_t \):

\[
\hat{\beta} (\theta) = \frac{\partial q_\theta (i_t \mid \pi_t, \tilde{y}_t)}{\partial \pi_t}.
\]

\(^{2}\)There are a few papers in which a possible non-linear relationship is investigated. See Kim et al. (2002) for example. However, the empirical evidence for non-linearity is not very strong so that we use the linear specification as in (2).
Here, \( \hat{\beta}(\theta) \) can be interpreted as the amount of change in the interest rate at the \( \theta^{th} \) quantile induced by one unit increase of inflation rate, giving an evaluation of the Taylor Principle at each point or quantile on the conditional distribution. Similarly, the marginal change in the \( \theta^{th} \) conditional quantile due to marginal change in \( \hat{y}_t \) equals:

\[
\hat{\gamma}(\theta) = \frac{\partial q_\theta(i_t | \pi_t, \hat{y}_t)}{\partial \hat{y}_t}
\]

which is the response of the interest rate to output gaps at each point of the conditional distribution.

## 3 Data Sources

In estimating the central bank reaction function we use monthly data. For the United States the sample period is 1979/10 to 2003/09, providing 288 observations in total, beginning with the period when Chairman Paul Volcker took over the Federal Reserve and signalled his intention to reign in inflation and extending to the present. The policy instrument is the Fed Funds rate, the inflation rate is the annualized change in the Consumer Price Index (CPI) and monthly output gap are constructed using the Industrial Production Index detrended using the HP-filter (HP)\(^3\).

The sample for the UK is the shortest sample and begins with 1993/02, when the Bank of England adopted inflation targeting, to 2003/07, giving 126 observations. The monetary policy instrument of the Bank, the repo rate is the dependent variable. Annualized changes in the Retail Price Index excluding Mortgage Interest Payments (RPIX) are used to measure the inflation rate, and, as with the US the detrended Index of Industrial Production is used to construct the output gap.\(^4\)

For Japan the sample period runs from 1979/04 to 2003/12, giving 297 observations. The starting date corresponds to the point where the interbank lending rate became the chief operating instrument of monetary policy and all capital controls were finally abandoned. The operating rate is the overnight call rate, while inflation and the output gap are defined in similar terms to the US data using the Consumer Price Index (CPI) and the Index of Industrial Production.\(^5\)

Time series plots of the Fed Funds rate, the UK Repo rate, the Japanese Call rate, and the respective inflation rates are given in Figures 1a, 2a and 3a. The histograms of the interest rates are given for each country in Figures 1b, 2b, and 3b respectively. For both the US and the UK the distributions have heavy mass in the lower tail, and for Japan the modal observation is just above the zero lower bound following the decision of the Bank of Japan to lower short term rates to near zero in 1999 as part of a zero interest rate policy (ZIRP), where rates have remained ever since. Descriptive statistics for these variables and the explanatory variables are given in Table 1. Table 2 displays various quantiles of the nominal interest rate in each country, and by examining the selected quantiles, it is apparent that the interest rate distribution for Japan is more heavily concentrated around zero than the other two countries; for example, the 30th quantile for Japan is

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\(^3\)When the output gap was constructed using an alternative detrending process based on a quadratic trend the results were comparable.

\(^4\)The measure of inflation corresponds to the inflation measure used within our sample for inflation targeting. Since November 2004 the target has been specified in terms of the consumer price index.

\(^5\)The particular difficulties associated with handling the sustained decline of economic growth in Japan in the 1990s attaches a greater degree of uncertainty to output gap measures such as the HP filter, as Kuttner and Posen (2003) illustrate, but as they also point out the alternative bottom-up strategies for measuring the deviation of output from trend have their own share of difficulties. Therefore, although we take these points seriously, we make use of top-down filtering approaches in subsequent sections in the absence of a clearly dominating alternative, but our interpretation of the reliability of the coefficient estimates is suitably cautious.
at a level of interest rates equal to just 0.43 while the corresponding quantile for the US and UK is found at levels of the interest rate that equal 5.25 for both countries. It is no surprise to find that Japan constitutes an unusual case compared to the US and the UK, and this will have a bearing on the interpretation of the results that follow.

4 Empirical Evaluation of the Taylor Principle

4.1 The Taylor Rule at the Conditional Mean

Clarida et al. (1998) estimate the Taylor-type interest rate reaction function using a function of the following type:

\[ i_t = (1 - \rho) \alpha + (1 - \rho) \beta \pi_{t+n} + (1 - \rho) \gamma \tilde{y}_t + \rho i_{t-1} + \varepsilon_t. \]  

(4)

This embeds a contemporaneous response to inflation when \( n = 0 \), and a forward-looking property when the value of \( n \) takes positive values. We can vary the value of \( n \) (\( n = 3, 6, 9, 12, 15, 18, \) and 24) to determine the effect of the horizon on the estimation of the rule. Estimation of this forward-looking Taylor rule relies upon specification of a choice set of instruments drawn from the variables in the central bank’s information set. These are often lagged variables that help forecast inflation and output gap or any other contemporaneous variables that are uncorrelated with the shock \( \varepsilon_t \).

A central part of the estimation strategy above is the inclusion of a parameter, \( \rho \), which has been used to explain smoothing of rates by central banks in the face of uncertainty (Goodhart, 1998, Sack, 2000). This is also often necessary to control for the serial correlation in the interest rate but because estimates of \( \rho \) are close to one, the parameter estimates can shrink to zero. We can confirm that this behavior is found in the estimates of the US, UK and Japanese results. To avoid this distortion, we return to Taylor’s original rule, setting \( \rho = 0 \); that is, we assume that the model is specified by

\[ i_t = \alpha + \beta \pi_{t+n} + \gamma \tilde{y}_t + \varepsilon_t. \]

(5)

The econometric effect of excluding the lagged dependent variable is to ensure that parameter estimates do not shrink, but the restriction introduces serial correlation in the residuals. In order to handle the serial correlation issue, we compute the heteroskedasticity and autocorrelation consistent (HAC) standard errors proposed by Newey and West (1987). We use a forward-looking rate of inflation and the output gap constructed using the HP filter; both variables are instrumented using a constant and lags of \( \pi_t \) and \( \tilde{y}_t \).

The IV results for the three countries are shown in Tables 3-5. First, we estimate (5) using data for the US reported in Table 3; it is obtained using a forward-looking rate of inflation twelve months ahead with the contemporaneous output gap. The estimated coefficient of inflation embodies the Taylor principle since it is greater than unity, and takes the value of 1.75 - a value within one standard deviation of the proposed coefficient of 1.50 suggested by Taylor (1993) and insignificantly different from it. This value is similar to the estimates of Bernanke and Gertler (1999) and Clarida

\footnote{All the results based on instruments and forward-looking horizons other than the reported ones in the paper are available upon request. We discuss some cases in the section on robustness later in the paper. What emerges is that the choice of \( n \) does not prove critical to the main findings of the paper.}

\footnote{The results are not reported in the paper but are available from the authors on request. The high values of the smoothing parameter are close to unity in line with other studies such as Bernanke and Gertler (1999) for example. These result in explosive estimates of the coefficients in the Taylor rule. This may be indicative of near I(1) behavior of the interest rates as Gerlach-Kristen (2003) documents.}
et al. (1998) who obtain estimates of 1.60 and 1.79 respectively.\footnote{Bernanke and Gertler (1999) estimate the Taylor rule in (4), setting $n = 12$ (12-month forward-looking), using GMM on a sample from 1960/01 to 1998/12. Clarida et al. (1998) use the sample 1979/10 to 1994/12 to produce estimates using GMM using the same specification.} The coefficient on the output gap is 0.56 and is close to the value of 0.50. Therefore, we cannot reject the hypothesis that the coefficients are equal to those suggested by Taylor (1993).

Similar properties are reported for the UK. Table 4 shows the IV estimation results setting a three-month forward-looking horizon. The estimated coefficient on inflation is close to 1.5, and is statistically significant at the 5% significance level; this result is in line with the result obtained in Nelson (2001) who also used a three-month horizon. The output gap coefficient is 0.53 and is also significant at the 5% level. Again we cannot reject the null that these coefficients differ significantly from Taylor’s proposed values.

The empirical evidence for Japan is reported for a contemporaneous output gap and a twelve-month forward-looking inflation rate in Table 5. It becomes immediately apparent that the estimated coefficient on the inflation variable at 2.04 is much greater than unity, although it upholds the Taylor principle and is not significantly different from 1.5. The output gap coefficient at 0.06 is much smaller than the estimates for the US and the UK, and is significantly different from 0.5, but not significantly different from zero. These coefficients estimates are consistent with the evidence presented by Clarida et al. (1998), who estimate the monthly forward-looking Taylor rule (4) using GMM over the sample period from 1979/04 to 1994/12 and find that the Bank of Japan responds aggressively towards inflation, but mildly towards the output gap, based on estimates of the response to inflation and to the output gap equal to 2.04 and 0.08, respectively. Bernanke and Gertler (1999) also apply the same methodology over the period from 1979/04 to 1997/12 and their estimated parameters of inflation rate and output gap are very similar at 2.21 and 0.20, respectively. Finally, Kuttner and Posen (2003) using monthly data from 1987/05 to 1999/12 find that the inflation response of Japan is approximately 2.97 - a surprisingly high value - with a negative response to the output gap of -0.19. Not surprisingly, they are skeptical about the reliability of their output gap measure.

We make use of our IV results as a benchmark to be compared with the quantile regression results that follow, where the uncovered estimates $\hat{\beta}$ and $\hat{\gamma}$ are estimated over the entire conditional distribution. When producing the quantile regression results, we use equation (5) with the same set of instruments and the same forward-looking horizon for each country as in the IV estimation method reported above.

### 4.2 The Taylor Rule at Various Quantiles

We estimate the model using two-stage quantile regression (2SQR) method developed by Kim and Muller (2004b) to avoid endogeneity problems with the forward-looking inflation rate and the output gap, which are dated later than $t$ and are expected to be correlated with the error term in (5). As proved in Kim and Muller (2004b), the 2SQR estimator based on LS predictions provides consistent estimates for the slope parameters ($\beta$ and $\gamma$) for a given value of $\theta$. This is not the case for the intercept parameter $\alpha$, which we ignore, since the intercept parameter will be biased.

The first-stage regression involves the regression of $\pi_{t+n}$ and $\tilde{y}_t$ using OLS on the instrument set and obtain the generated fitted values $\tilde{\pi}_{t+n}$ and $\tilde{y}_t$, respectively. The instrumental variables are $IV \in \{1, \pi_{t-1}, \ldots, \pi_{t-s}, \tilde{y}_{t-1}, \ldots, \tilde{y}_{t-s}\}$, where $s$ is the chosen lag of the instrument set. Subsequently, after having obtained the predicted values of the independent variables from the first-stage regression, we perform quantile regression estimation treating the fitted values from the first stage as the independent variables entering the second-stage estimation. The value of $\theta$ is chosen to range from
0.05, 0.10, 0.20, ..., 0.95. The following quantile regression model is estimated:
\[ q_\theta \left( i_t \mid \hat{\pi}_{t+n}, \hat{y}_t \right) = \alpha (\theta) + \beta (\theta) \hat{\pi}_{t+n} + \gamma (\theta) \hat{y}_t \]  
(6)

where \( q_\theta \) is the conditional quantile function of \( i_t \). The variables \( \hat{\pi}_{t+n} \) and \( \hat{y}_t \) are the fitted values of \( \pi_{t+n} \) and \( \hat{y}_t \), respectively. The linear quantile regression is estimated by solving the minimization problem:
\[ \min_{\theta} \sum_{t=1}^{T} \rho_\theta \left( i_t - \alpha (\theta) - \beta (\theta) \hat{\pi}_{t+n} - \gamma (\theta) \hat{y}_t \right) \]  
(7)

The estimated coefficients \( \beta (\theta) \) and \( \gamma (\theta) \) indicate the responsiveness of the interest rate to changes in inflation and the output gap at the \( \theta^{th} \) quantile of the interest rate, respectively.

4.2.1 United States

We estimate (6) for the United States over the sample 1979/10 to 2003/09 using 2SQR, and the estimation result is shown in Table 6 where 90% confidence interval for the estimates are also reported in parentheses. The coefficient on the inflation rate variable is significantly different from zero in all quantiles and at all points is above unity supporting the Taylor Principle. From the 5th to the 10th quantile, the response of the interest rate to inflation given by the point estimates is less than 1.50, but the mild response to inflation at low interest rates undoubtedly reflects the fact that the spectre of deflation has been a more serious consideration than inflationary pressure to the Fed which warned of this danger in public testimony.\(^9\)

Within the range from the 20th to the 95th quantile, the interest response is positive and greater than 1.5 supporting the Taylor Principle. In addition the response to inflation consistently increases from a value of 1.67 to 2.57 at the upper end of the range. With the exception of the 30th percentile there is a clear upward drift in the point estimates at successive quantiles. This steady increase in the responsiveness of the interest rate towards inflation suggests that at the relatively higher levels of interest rates (and inflation), the Fed becomes more aggressive towards inflation. The Fed increases the real interest rate by greater amounts as interest rates and inflation increase. The response becomes very aggressive at the 95th percentile where the Federal Reserve raises its Fed Funds rate by almost 2.5 percentage points in response to a one percent rise in inflation, a real increase of 1.5 percentage points. A plot of the estimated quantile regression coefficients at each percentile against the IV estimates, superimposing the confidence intervals in each case, is given in Figure 4a; it shows that there is a significant deviation of the estimated inflation coefficients taken from IV (light lines) compared to the quantile regressions (heavy lines) at the lower and upper quantiles. Since the quantile regression estimates are more reliable with non-normal distributions, we compare the IV point estimates to the estimates and confidence intervals from the quantile regressions (rather than the other way around where the confididence intervals will be wider). We find that the IV point estimates lie outside the confidence intervals for the quantile regression for quantiles below the 20th and above the 70th percentiles implying a significantly different response to inflation across the distribution.

The finding of a nonlinear response of interest rates to inflation has been found before in the work of Bec et al. (2002) but their results refer to stages of the business cycle since the estimates are drawn from a modified forward-looking Taylor rule of the type proposed by Clarida et al. (1998)

\(^9\) The correlation between the interest rate and the inflation rate is 0.80, therefore the low observations of the interest rate (when interest rates are in the lowest quantiles of the distribution) correspond to periods of low inflation and vice versa.
with a split sample to reflect upswings (positive output gaps) and downswings (negative output gaps). Using monthly data from 1982/10 to 1998/08, they found that the weight attached to the inflation objective was greater during upswings, i.e., when demand pressure was increasing. In general, they found the Fed was more aggressive regarding any inflation gap during expansions than during recessions. Our result finds greater aggression towards inflation when interest rates are high, since the coefficient on inflation increases progressively over the distribution, but this is not necessarily a business cycle phenomenon. The response to output gaps declines for higher quantiles, since the coefficient on the output gap falls from the 10th to the 80th percentile. It is quite clear from the 90% confidence intervals for \( \gamma(\theta) \) that the Fed’s response to the output is significantly different from the figure proposed by Taylor at low and high quantiles. At all points on the distribution the coefficient on the output gap is significant but its magnitude declines steadily. At the highest quantiles the response to output picks up, but this is the reaction of interest rates in the early part of the sample from 1979/10 to 1982/6. Figure 5a shows that the estimates of the output gap coefficients from the IV regression lie well outside the confidence interval for the quantile regression at the upper quantiles, which suggests that IV estimates at the mean overstate the importance attached to output gaps by central banks when there is an overriding concern with inflation.

A three dimensional graph of the response surface in Figure 5a for five different quantiles demonstrates that the interest rate response to inflation become more aggressive, for a given output gap, but the response to the output gap hardly varies at all for a given level of inflation.

### 4.2.2 United Kingdom

The quantile results for the UK provide a useful comparison with the results from US data, since the Bank of England has been an explicit inflation targeter from 1993 and has had operational control over interest rates from 1997 in order to meet its objectives, we might therefore expect it to tackle inflation as the Taylor Principle suggests. We can see from Table 7 that the estimated coefficients on inflation rate and output gap are statistically significant in the majority of the quantiles. Only at the lower quantiles (\( \theta = 0.05 \) and 0.10) and the upper quantiles (\( \theta = 0.9 \) and 0.95) does the interest rate fail to respond significantly to changes in the inflation rate.\(^\text{10}\)

For the UK the estimated coefficients on the rate of inflation are greater than one and the confidence interval for \( \beta(\theta) \) includes 1.50 in all cases except the lowest quantiles, and as with the US results the point estimates of the coefficient value generally increase as we move to successive quantiles. The pattern for \( \beta(\theta) \) implies an increasingly aggressive attitude to inflation rate as \( \theta \) increases over the range of interest rates experienced, although the point estimates do not appear to be estimated with the same precision as the US results. This is most likely due to the fact that there is a relatively smaller sample of observations compared to the US data set: there are 126 observations for the UK while we have 288 observations for the US i.e. less than half the number of observations. This may account for the insignificant coefficient below the 10th percentile. Regarding the coefficient on the output gap, it is seen that the point estimates \( \gamma(\theta) \) on the output gap decline as we move to the higher quantiles of the interest rate’s distribution. These results confirm the US results above.

Figures 4b and 5b show that the IV estimates and the quantile regression estimates for the response over the distribution to inflation and the output gap deviate far less than in the US case but the upward slope for coefficients on inflation and the downward slope for coefficients on output gap are clearly discernible. The IV estimate lies outside the confidence interval for the quantile

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\(^{10}\)This may be the result of larger standard errors of quantile estimates as \( \theta \) approaches either 0 or 1 as demonstrated by simulation in Kim and Muller (2004a, 2004b), which would be more noticeable in a smaller sample.
regression estimates over the entire range of quantiles, but the widening of confidence intervals at the extremes due to the small sample does not allow us to be as certain that there is a significant deviation across the quantiles, in contrast to the US case where the result is unambiguous.

The three dimensional graph of the response surface given in Figure 3b has a similar characteristic to the US graph. The angled planes for different quantiles show a tendency for interest rates to react more strongly to inflation but not particularly more strongly to output gaps.

4.2.3 Japan

Unlike the US or the UK, Japan’s experience has been exceptional due to the very low levels of both interest rates and inflation over most of the sample. For almost the entire sample the annualized inflation rate has been lower than the Call rate, and on occasions the inflation rate has been negative, causing real rates of interest to increase. A prolonged deflationary episode has been experienced since 1999 from which time a zero interest rate policy (ZIRP) has been initiated, and during this period the interest rate has remained almost unchanged at just above zero with other measures being implemented to ease monetary policy. It is not entirely surprising, therefore, that our IV estimates of the previous subsection showed a degree of responsiveness towards the inflation rate in Japan that was much higher at $\beta = 2.04$ than in other countries.

Kohn (1996) argues that, in order to guard against the possibility of deflation, central banks operating near price stability should be willing to act especially forcefully and quickly when they suspect downward demand shocks. Other authors such as Orphanides and Wieland (1999) also suggest that the presence of the zero bound makes policymakers inclined to be more responsive to inflation as inflation falls. Their explanation is that the forward-looking policymaker properly recognizes the costs of implementing policy under the zero bound and takes precautionary measures to reduce the probability of deflation, thereby responding aggressively at the low level of the interest rate. Yates (2003) summarizes that a central bank facing a particularly large shock to demand that threatens to push interest rates to zero will have the option of cutting rates more aggressively in response to the initial shock. Aggressive behavior on the part of the central bank would cause the expected inflation to fall by less in response to the initial shock, since the private sector will take into account the pattern of the central bank’s behavior. This may explain why very high coefficient estimates were found in the previous section, if the Bank of Japan was prepared to make aggressive interest rate cuts when a zero-bound incident threatened.

In order to investigate whether the Bank of Japan actually was more aggressive towards inflation as rates approached zero we proceed to estimate the reaction function using the quantile regression (2SQR) method. The estimation result is displayed in Table 8. The estimated coefficient on the inflation rate is higher than 1.50 across all quantiles. For all but the $95^{th}$ quantile, where the confidence interval just overlaps 1.50, and the lowest quantiles where the lower confidence interval drops quite dramatically, these are significantly different from 1.50. Our estimates suggest that the Bank of Japan reacted more aggressively to inflation over the whole range of the interest rate distribution compared with the other central banks in our study. For example, the response of the nominal interest rate to inflation at the $5^{th}$ quantile is aggressive at 2.04, since a fall in inflation by a one percentage point induces a cut in the real interest rate by the Bank of Japan of just over

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11 The exact details of the policies adopted and the movements in the call rate have been summarized in Posen (2001, 2003). These can be categorized as a policy to ensure quantitative easing and to keep interest rates at approximately zero. The zero interest rate policy as expounded in the April 1999 Bank of Japan Policy Board minutes explained that 'it was important to maintain the current decisive easy stance of monetary policy...until deflationary concerns were dispelled'. In March 2001 this was supplemented with a statement that 'The new procedure will be kept in place until the CPI registers a stable 0% or increase year on year and should affect people's expectations in order to help reduce the deflationary bias'
one percent. This compares with a reaction in nominal rates of 0.89 for the US at the 5th quantile which raises the real rate by a modest amount. Interest rates in Japan for the vast majority of the sample have been very low, and very nearly zero for over five years at the end of the sample, therefore, even the mid-ranges of the distribution represent levels of the interest rate that are quite low by comparison with the UK and the US. Rates in Japan were exceptionally low at the 5th quantile by comparison even with the US and therefore we should also compare responses in the middle quantiles. Taking the 60th quantile for Japan where rates were 4.70 percentage points, the reaction of 2.31 is much more aggressive than the US reaction of 1.53 or the UK reaction of 1.81 when interest rates in these countries were at similar levels. The three dimensional response surface in Figure 3c shows the interest rate reaction to inflation for five quantiles and illustrates a much steeper reaction than for either the US or the UK cases. This suggests that there is much greater responsiveness of interest rates to inflation at all points on the distribution in Japan. As with the US, there is increasing aggression towards inflation up to the 80th percentile and beyond that, in the period of higher and more volatile inflation more similar to the previous monetary regime, a decline. The response to output gap is fairly small and significant in the middle range of the distribution, but insignificant elsewhere.\footnote{Estimates of the response to output gaps at the mean by among others Jinushi et al. 2000, Kuttner and Posen 2001, 2003, and Alcarne et al. 2002, find the output gap is insignificant. Kuttner and Posen (2003) offer serious warnings about the construction of the output gap as we have mentioned earlier.}

Figures 4c and 5c compare the IV estimates with quantile estimates and we observe that the point estimates of the IV regressions lie within the confidence intervals except when rates falls to the near zero levels below the 20th percentile. While this does not suggest much evidence of variation over the distribution, the dominating feature of the Japanese experience has been the deflation - where a significantly different response to inflation is observed. According to Table 8, when Japan experience deflation, the ZIRP was implemented, and rates fell to exceptionally low levels of 0.5 percent and below - and here the response to inflation took a significantly different turn. As predicted by Kohn (1996), Orphanides and Williams (1998), and Yates (2003) we observe the response coefficient fall to its lowest level at the 20th percentile before increasing at the 10th and 5th percentiles. Taking the confidence intervals as our indicator, the increases were significant, since the upper confidence interval value of 2.01 for the 20th percentile was exceeded by the point estimate of 2.04 at the 5th percentile, and this value also exceeds the upper confidence interval value for the 5th percentile. Therefore our results show clear support for the views of Kohn (1996), Orphanides and Williams (1998), and Yates (2003) since the degree of aggression increases significantly as the rate approaches the two lowest quantiles, as we might expect if their prediction were true. This is a novel result showing clear evidence of nonlinearity in the response to inflation at the lowest quantiles. We discuss the effectiveness of this pre-emptive action in the discussion that follows but before we do so we consider the robustness of our results.

5 Robustness of the Results

In this section we consider the robustness of our findings, illustrating our point with reference to the US quantile regression results. We find our results are remarkably consistent over the range of instruments and inflation horizons and measures of output gaps using historical and real time data.

When we vary the instrument choice we find that the same characteristic of increasing aggression towards inflation as the quantile increases are apparent for all specifications. Our results in Tables 6 were given for a twelve-month inflation horizon where the instruments were one month lags of inflation and the historical output gap. In Table 9 we show the estimates when we vary the
instruments to include 1-6 month lags of inflation and the historical output gap. The point estimates and the quantile estimates do not differ significantly from each other. The underlying pattern of a monotonically increasing response to inflation and a decline in the response to the output gap are found for all specifications as we take higher quantiles.

When we compare historical and real time measures of the output gap in our specification we find a similar robustness. Keeping the inflation horizon and instrument set identical to Table 6 we report the results in Table 10 where the historical output gap measure is replaced with an output gap derived from the real time output series from the Federal Reserve Bank of Philadelphia real time dataset. The results show that the response to inflation and the output gap are not significantly different from the estimated responses using historical data, in fact there is a remarkable degree of similarity between the results. The same increasing response to inflation and a declining response to the output gap at successively higher quantiles is found when real time data are used.

We conclude that the findings are not unduly influenced by the choice of inflation horizon, instrument set or the use of historical data for output.

6 Discussion

The quantile regression results reported above are robust and have several interesting implications for monetary policy evaluation. First, there are obvious nonlinearities in the monetary reaction function over the range of the interest rate distribution with respect to inflation. Our analysis clearly demonstrates for a number of economies that a uniform response to inflation is not upheld - responses differ with respect to inflation at various quantiles of the distribution, and the degree of responsiveness of the interest rate towards the inflation rate increases as inflation rises within the range of interest rates experienced by central banks in the era of price stability. In general there is evidence in favor of the Taylor Principle except where the level of the interest rate is very low (essentially the lowest quantiles, which in the US corresponds to interest rates of around 1 percentage point). Since interest rates and inflation rates are positively correlated, the steady increase in the response towards inflation for higher levels of the interest rate shows that central banks apply more pressure at the higher quantiles of the distribution when inflation is generally higher. This is evidence in favor of a nonlinear response to inflation, supporting the view that central banks possess a precautionary demand for price stability\textsuperscript{13}.

The reasons for the nonlinearity in the response to inflation are not easy to establish. They may reflect the preferences of central banks over the range of interest rates, which as we have noted are positively correlated with inflation rates. In this case we can infer that as interest rates and inflation rates have taken higher values, the priorities of central banks (indicated by their desire to response to inflation relative to the desire to respond to the output gap) have altered significantly. This would suggest that the weight attached to inflation vis-a-vis output gaps in the central bank loss function is not a constant but a nonlinear relation that varies with the level of interest rates and inflation. This would greatly complicate the proposal of Svensson (2001) discussed by Bean (2001) that central banks or governments should specify such a weight. However, if it is the case then it shows that central banks adjust their behavior towards inflation according to the degree of inflationary pressure - there is not a constant response rate for all inflation rates.

Alternatively, the findings may reflect the observation that, over the period under study, the credibility for inflation fighting has been growing, the level of inflation has fallen and the size of interest rate changes required to control inflation has also fallen. This is particularly apparent in

\textsuperscript{13}In contrast to the response to inflation, the reaction to output gaps declines as we progress towards higher levels of interest rates (higher quantiles).
the figures indicating the absolute size of interest rate changes over time in the three countries reported in Figures 6a - 6c. As trust in the institutions of monetary policy has increased so the need to use instruments of policy in large steps has declined, indeed the use of rate changes greater than 25 basis points in recent years has been reserved for occasions when a signal of intent with respect to inflation was needed. Also, to a large extent, ‘open mouth operations’ have substituted for or supplemented open market operations as Thornton (2004) illustrates, so that inflation and inflation expectations have been talked down without the need for large adjustments in interest rates. If the magnitude of interest rate changes has fallen with inflation levels and interest rate levels then changes in the response to inflation over the conditional distribution of interest rates may indicate growing credibility as nominal magnitudes have fallen over time.

Whatever the underlying reasons for the nonlinearity, simulation exercises used to examine the behavior of economies under low inflation conditions need to recognize these nonlinearities in the policy rule with respect to inflation when determining the properties of economies under choices such as the optimal inflation target. Recent examples such as Fuhrer and Madigan (1997), Orphanides and Wieland (1998), Reinschneider and Williams (2000) and Coenen et al. (2003) offer laudable attempts to simulate the macroeconomic effects of low inflation on the assumption that the reaction function has constant parameters across the distribution of interest rates equal to the coefficients suggested by Taylor (1993). Our evidence suggests that because the reaction function is nonlinear in inflation and the output gap, optimal policy should be determined with a reaction function that has calibrated nonlinear responses to inflation and output gaps rather than the responses found when the reaction function is specified at the mean. Without these features the derived optimal policy will be distorted because it will overstate the monetary reaction to inflation when considering behavior of policy in the lower tail at low interest rates.

The second major finding in the paper is the unique experience of Japanese monetary policy, where the zero lower bound and the prospect of deflation have influenced the response of the central bank at the lowest points of the distribution and have resulted in sharper responses to inflation than in other countries. We find supportive evidence in favor of the more aggressive response to inflation as rates approach the zero lower bound. On the basis of this evidence we support the predictions of the literature that a central bank would attempt to pre-emptively deal with deflation by being more aggressive towards rate cutting as inflation approached the ZLB. The interpretation of this finding is complex..

Despite the evidence that rates increased in Japan as the zero lower bound approached, a prolonged deflation of five years was experienced, in spite of the apparent attempt to pre-emptively tackle it. This could be explained in two ways. We might conclude that the Bank of Japan did act aggressively but failed to take sufficiently strong measures when it was necessary, and nominal rates were cut without sufficient force at an early enough stage before rates reached the zero lower bound. Our evidence of an inflation in the response to inflation at the third lowest quantile shows action was taken, but it may have been too little, too late. An alternative conclusion might be that the Bank of Japan did not act aggressively, but changed its monetary strategy, accepting the need to reduce short term interest rates to near zero levels as inflation fell to zero and below. Discussion of its strategy by Ueda, 2000, 2004; Kuttner and Posen, 2001, 2003; Ito and Mishkin, 2004, Kuttner 2004 appears to indicate a shift of emphasis of monetary control away from variation in the short term interest rate towards the use of other instruments such as quantitative easing while committing to effectively keep the short rate at zero (thus attempting to influence longer maturity interest rates)14. Many authors, including Bernanke and Reinhart (2004), Coenen

14The Bank of Japan has made an ‘unprecedented’ commitment not to increase short term interest rates so long as the deflation continues.
and Weiland (2004), Eggertson and Woodford (2003), Blinder (2000), Reifsneider and Williams (2000), and McCallum (2000), have suggested that alternative instruments for monetary policy would be effective as interest rates approach the lower bound. It is possible that Japan provides the first example of a central bank that came to rely on alternative instruments to the short-term interest rate. If this is the case then the Bank of Japan may have actively decided not to be aggressive towards inflation as rates fell because it was not relying on the effectiveness of the short term instrument to achieve its objectives. In fact any change to interest rates, and particularly aggressive changes, could have been interpreted as a loss of confidence by the central bank in retaining monetary control: an impression which it has studiously sought to avoid by emphasizing the other controls at its disposal. If this was the case then we must explain why an inflection in the response to inflation was observed. One potential explanation draws on the fact that whatever the Bank of Japan’s intentions, and the clear statements by the policy board that rates would remain at zero, these were overshadowed by an apparent reversal of the zero interest rate policy in August 2000 when deflationary pressures were still being felt. Kuttner and Posen (2001, 2003) find that expectations, as measured by the yield spread, show evidence of ‘deflation scares’ on at least five occasions, one of which is directly associated with the policy reversal. They argue that the switch to alternative instruments of monetary policy did not tackle the prospect of future inflation because inflation expectations were not managed effectively, a charge that is supported by Ito and Mishkin (2004) who are also sharply critical of the handling of expectations by the Bank of Japan. This was a mistake that undermined credibility of the conduct of policy, and may explain why we observe an upturn in the reaction to inflation when rates were at close to zero. The consistency of both interpretations with the observed outcome suggests further research is warranted to explain the reaction to inflation by the Bank of Japan during this episode.

7 Conclusion

In this paper we have taken the Taylor rule reaction function and have allowed the responses to inflation and to output gaps to vary according to the location on the conditional distribution of interest rates. We have used the two-stage quantile regression method of Kim and Mueller (2004a, 2004b) to evaluate the responses for three countries the US, UK and Japan over 25 years. There is strong support for the Taylor Principle in the estimated reaction functions, except when interest rates are very low, but interestingly we find increasing aggression towards inflation as we move from the lower tail to the upper tail of the conditional distribution. Therefore central banks increase real interest rates by larger amounts as the level of interest rates - and the inflation rate, which is strongly positively correlated with interest rates - rises.

The implications of our results are twofold. First, the finding of nonlinearity in the response to inflation suggests that the analysis of optimal monetary policy when rates are very low should take into account the variation in the degree of aggression to inflation at different points on the interest rate distribution. This recommendation contrasts with the current practice of assuming that the calibrated responses of reaction functions to inflation and output gaps correspond to those that are found when the reaction function is evaluated at the mean. We have strong evidence to show these coefficient values do not apply at all points on the interest rate distribution.

Second, the evidence for a more aggressive response to inflation as the interest rate approaches zero in Japan leads to two conclusions: either the Bank of Japan cut rates more aggressively as the zero bound approached but insufficiently strongly and not soon enough, and therefore missed a vital opportunity to avoid deflation zero, or it willingly reduced short-term rates to zero because it was prepared to rely heavily on the alternative monetary policy measures suggested by Bernanke
and Reinhart (2004), Coenen and Weiland (2004), Eggertson and Woodford (2003), Blinder (2000), Reifschneider and Williams (2000), and McCallum (2000) among others which did not require use of the short term interest rate. Nevertheless the inconsistent manner in which this policy was pursued, which has been criticized by Kuttner and Posen (2001, 2004) and Ito and Mishkin (2004) for failing to control expectations of future inflation, has resulted in some evidence of aggression towards inflation even when rates were very low.

Both findings imply that greater attention should be given to nonlinear responses to inflation and output gaps over the range of interest rates experienced by central banks in the era of price stability. Without taking these features into account the modeling and simulation of optimal policy will be distorted.
References


### Table 1. Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>US Data (1979/10-2003/09)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$i_t$ (Fed Fund Rate)</td>
<td>6.97</td>
<td>3.72</td>
<td>1.01</td>
<td>19.10</td>
</tr>
<tr>
<td>$\pi_t$</td>
<td>3.98</td>
<td>2.69</td>
<td>1.06</td>
<td>13.76</td>
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<tr>
<td>$\tilde{y}_t$</td>
<td>−0.007</td>
<td>1.09</td>
<td>−4.09</td>
<td>3.16</td>
</tr>
<tr>
<td><strong>UK Data (1993/02-2003/07)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$i_t$ (Repo Rate)</td>
<td>5.70</td>
<td>1.03</td>
<td>3.50</td>
<td>7.50</td>
</tr>
<tr>
<td>$\pi_t$</td>
<td>2.51</td>
<td>0.40</td>
<td>1.50</td>
<td>3.43</td>
</tr>
<tr>
<td>$\tilde{y}_t$</td>
<td>0.00</td>
<td>0.72</td>
<td>−2.92</td>
<td>1.43</td>
</tr>
<tr>
<td><strong>Japan Data (1979/04-2003/12)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$i_t$ (Call Rate)</td>
<td>3.75</td>
<td>3.18</td>
<td>0.001</td>
<td>12.7</td>
</tr>
<tr>
<td>$\pi_t$</td>
<td>1.50</td>
<td>1.99</td>
<td>−1.62</td>
<td>8.43</td>
</tr>
<tr>
<td>$\tilde{y}_t$</td>
<td>−0.002</td>
<td>1.62</td>
<td>−5.17</td>
<td>5.73</td>
</tr>
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</table>

HP filter output gap

### Table 2. Sample quantiles of the US Fed Funds Rate, the UK Repo Rate and Japan Call Rate

<table>
<thead>
<tr>
<th>$\theta$th Quantile</th>
<th>Fed Funds Rate (US)</th>
<th>Repo Rate (UK)</th>
<th>Call Rate (Japan)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>1.01</td>
<td>3.50</td>
<td>0.001</td>
</tr>
<tr>
<td>10th</td>
<td>3.00</td>
<td>4.00</td>
<td>0.01</td>
</tr>
<tr>
<td>20th</td>
<td>4.14</td>
<td>5.00</td>
<td>0.23</td>
</tr>
<tr>
<td>25th</td>
<td>4.81</td>
<td>5.25</td>
<td>0.41</td>
</tr>
<tr>
<td>30th</td>
<td>5.25</td>
<td>5.25</td>
<td>0.43</td>
</tr>
<tr>
<td>40th</td>
<td>5.54</td>
<td>5.75</td>
<td>2.23</td>
</tr>
<tr>
<td>50th (Median)</td>
<td>6.05</td>
<td>6.00</td>
<td>3.79</td>
</tr>
<tr>
<td>60th</td>
<td>6.89</td>
<td>6.00</td>
<td>4.70</td>
</tr>
<tr>
<td>70th</td>
<td>8.24</td>
<td>6.00</td>
<td>6.15</td>
</tr>
<tr>
<td>75th</td>
<td>8.61</td>
<td>6.25</td>
<td>6.43</td>
</tr>
<tr>
<td>80th</td>
<td>9.29</td>
<td>6.75</td>
<td>6.69</td>
</tr>
<tr>
<td>90th</td>
<td>12.00</td>
<td>7.00</td>
<td>7.39</td>
</tr>
<tr>
<td>Max</td>
<td>19.10</td>
<td>7.50</td>
<td>12.70</td>
</tr>
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</table>
Table 3. Monthly IV estimates of the Taylor rule using US data
(HAC standard errors)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Err.</th>
<th>t-Stat</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.943</td>
<td>1.287</td>
<td>0.733</td>
<td>0.464</td>
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<tr>
<td>$\beta$</td>
<td>1.750</td>
<td>0.413</td>
<td>4.241</td>
<td>0.000</td>
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<tr>
<td>$\gamma$</td>
<td>0.561</td>
<td>0.297</td>
<td>1.888</td>
<td>0.056</td>
</tr>
</tbody>
</table>

Twelve-month forward-looking horizon
Sample (adjusted): 1979/11 - 2002/9 (275 observations)
Dependent variable: Fed Funds Rate
Output gap historical data detrended using Hodrick-Prescott filter
Instruments set $IV \in (1, \pi_{t-1}, \tilde{y}_{t-1})$.
Newey-West HAC standard errors with lag truncation = 5
$R^2 = 0.354$, Residual Sum of Squares = 2271.69

Table 4. Monthly IV estimates of the Taylor rule using UK data
(HAC standard errors)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Err.</th>
<th>t-Stat</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>2.049</td>
<td>1.070</td>
<td>1.915</td>
<td>0.058</td>
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<tr>
<td>$\beta$</td>
<td>1.504</td>
<td>0.461</td>
<td>3.264</td>
<td>0.002</td>
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<tr>
<td>$\gamma$</td>
<td>0.526</td>
<td>0.150</td>
<td>3.508</td>
<td>0.007</td>
</tr>
</tbody>
</table>

Three-month forward-looking horizon
Sample (adjusted): 1994/11 - 2003/04 (102 observations)
Dependent variable: Repo Rate
Output gap historical data detrended using Hodrick-Prescott filter
Instruments set $IV \in (1, \pi_{t-1}, \pi_{t-3}, \pi_{t-6}, \pi_{t-9}, \pi_{t-12}, \pi_{t-15}, \pi_{t-18}, \pi_{t-21},$
$\tilde{y}_{t-1}, \tilde{y}_{t-3}, \tilde{y}_{t-6}, \tilde{y}_{t-9}, \tilde{y}_{t-12}, \tilde{y}_{t-15}, \tilde{y}_{t-18}, \tilde{y}_{t-21})$
Newey-West HAC standard errors with lag truncation = 4
$R^2 = 0.118$, Residual Sum of Squares = 102.54

Table 5. Monthly IV estimates of the Taylor rule using Japan data
(HAC standard errors)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Err.</th>
<th>t-Stat</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>1.211</td>
<td>0.410</td>
<td>2.953</td>
<td>0.034</td>
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<tr>
<td>$\beta$</td>
<td>2.038</td>
<td>0.282</td>
<td>7.225</td>
<td>0.000</td>
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<tr>
<td>$\gamma$</td>
<td>0.063</td>
<td>0.110</td>
<td>0.578</td>
<td>0.564</td>
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</tbody>
</table>

Twelve-month forward-looking horizon
Sample (adjusted): 1979/06 - 2002/12 (283 observations)
Dependent variable: Call Rate
Output gap historical data detrended using Hodrick-Prescott Filter
Instruments set $IV \in (1, \pi_{t-1}, \pi_{t-2}, \tilde{y}_{t-1}, \tilde{y}_{t-2})$.
$R^2 = 0.250$, Residual Sum of Squares = 2109.00

20
Table 6. Monthly regression quantile estimates using US data (point estimates and 90% confidence intervals)

<table>
<thead>
<tr>
<th>θ</th>
<th>β_θ</th>
<th>γ_θ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>1.24</td>
<td>(1.20, 1.55)</td>
</tr>
<tr>
<td>0.1</td>
<td>1.50</td>
<td>(1.29, 1.71)</td>
</tr>
<tr>
<td>0.2</td>
<td>1.67</td>
<td>(1.44, 1.78)</td>
</tr>
<tr>
<td>0.3</td>
<td>1.62</td>
<td>(1.47, 1.86)</td>
</tr>
<tr>
<td>0.4</td>
<td>1.69</td>
<td>(1.44, 1.93)</td>
</tr>
<tr>
<td>0.5</td>
<td>1.73</td>
<td>(1.52, 2.05)</td>
</tr>
<tr>
<td>0.6</td>
<td>1.93</td>
<td>(1.66, 2.30)</td>
</tr>
<tr>
<td>0.7</td>
<td>2.09</td>
<td>(1.85, 2.41)</td>
</tr>
<tr>
<td>0.8</td>
<td>2.27</td>
<td>(1.96, 2.48)</td>
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<tr>
<td>0.9</td>
<td>2.42</td>
<td>(2.04, 2.90)</td>
</tr>
<tr>
<td>0.95</td>
<td>2.57</td>
<td>(2.06, 2.80)</td>
</tr>
</tbody>
</table>

Twelve-month forward-looking horizon

Sample (adjusted): 1979/11 - 2002/09 (275 observations)
Dependent variable: Fed Funds Rate
Output gap historical data detrended using Hodrick-Prescott filter
Instruments set IV ∈ (1, π_t−1, ŷ_t−1).

Table 7. Monthly regression quantile estimates using UK data (point estimates and 90% confidence intervals)

<table>
<thead>
<tr>
<th>θ</th>
<th>β_θ</th>
<th>γ_θ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>−0.73</td>
<td>(−1.17, 1.17)</td>
</tr>
<tr>
<td>0</td>
<td>−0.20</td>
<td>(−1.20, 1.37)</td>
</tr>
<tr>
<td>0.2</td>
<td>1.17</td>
<td>(0.03, 1.68)</td>
</tr>
<tr>
<td>0.3</td>
<td>1.41</td>
<td>(1.25, 1.78)</td>
</tr>
<tr>
<td>0.4</td>
<td>1.59</td>
<td>(1.16, 1.77)</td>
</tr>
<tr>
<td>0.5</td>
<td>1.48</td>
<td>(0.79, 2.54)</td>
</tr>
<tr>
<td>0.6</td>
<td>1.81</td>
<td>(0.73, 3.09)</td>
</tr>
<tr>
<td>0.7</td>
<td>2.43</td>
<td>(1.55, 3.18)</td>
</tr>
<tr>
<td>0.8</td>
<td>2.57</td>
<td>(0.71, 3.32)</td>
</tr>
<tr>
<td>0.9</td>
<td>1.86</td>
<td>(−0.19, 3.74)</td>
</tr>
<tr>
<td>0.95</td>
<td>1.37</td>
<td>(−0.78, 4.41)</td>
</tr>
</tbody>
</table>

Three-month forward-looking horizon

Sample (adjusted): 1994/02 - 2003/01 (108 observations)
Dependent variable: Repo Rate
Output gap detrended using Hodrick-Prescott filter
Instruments set IV ∈ (1, π_t−1, ..., π_t−3, π_t−6, π_t−9, π_t−12, π_t−15, π_t−18, π_t−21, ŷ_t−1, ..., ŷ_t−3, ŷ_t−6, ŷ_t−9, ŷ_t−12, ŷ_t−15, ŷ_t−18, ŷ_t−21).
Table 8. Monthly regression quantile estimates using Japan data (point estimates and 90% confidence intervals)

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\beta_\theta$</th>
<th>$\gamma_\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>2.04</td>
<td>(0.28, 2.08)</td>
</tr>
<tr>
<td>0.1</td>
<td>1.93</td>
<td>(0.77, 1.94)</td>
</tr>
<tr>
<td>0.2</td>
<td>1.89</td>
<td>(1.83, 2.01)</td>
</tr>
<tr>
<td>0.3</td>
<td>1.95</td>
<td>(1.85, 2.15)</td>
</tr>
<tr>
<td>0.4</td>
<td>2.09</td>
<td>(1.96, 2.27)</td>
</tr>
<tr>
<td>0.5</td>
<td>2.22</td>
<td>(2.06, 2.36)</td>
</tr>
<tr>
<td>0.6</td>
<td>2.31</td>
<td>(2.08, 2.50)</td>
</tr>
<tr>
<td>0.7</td>
<td>2.31</td>
<td>(1.93, 2.67)</td>
</tr>
<tr>
<td>0.8</td>
<td>2.14</td>
<td>(1.81, 2.56)</td>
</tr>
<tr>
<td>0.9</td>
<td>1.85</td>
<td>(1.65, 2.39)</td>
</tr>
<tr>
<td>0.95</td>
<td>1.62</td>
<td>(1.49, 2.36)</td>
</tr>
</tbody>
</table>

Twelve-month forward-looking horizon

Sample (adjusted) from 1979/10 to 2002/06 (273 observations)
Dependent variable: Call Rate
Output gap historical data detrended using Hodrick-Prescott Filter
Instruments set $IV \in (1, \pi_{t-1}, \pi_{t-2}, \bar{y}_{t-1}, \bar{y}_{t-2})$.

Table 9. Instrument robustness: monthly regression quantile estimates using US data

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\beta_\theta$</th>
<th>$\gamma_\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>1.66</td>
<td>(1.50, 2.01)</td>
</tr>
<tr>
<td>0.1</td>
<td>1.85</td>
<td>(1.54, 1.99)</td>
</tr>
<tr>
<td>0.2</td>
<td>1.75</td>
<td>(1.47, 1.96)</td>
</tr>
<tr>
<td>0.3</td>
<td>1.73</td>
<td>(1.47, 1.91)</td>
</tr>
<tr>
<td>0.4</td>
<td>1.74</td>
<td>(1.49, 1.99)</td>
</tr>
<tr>
<td>0.5</td>
<td>1.90</td>
<td>(1.62, 2.11)</td>
</tr>
<tr>
<td>0.6</td>
<td>2.04</td>
<td>(1.80, 2.16)</td>
</tr>
<tr>
<td>0.7</td>
<td>2.08</td>
<td>(1.91, 2.56)</td>
</tr>
<tr>
<td>0.8</td>
<td>2.21</td>
<td>(1.88, 2.52)</td>
</tr>
<tr>
<td>0.9</td>
<td>2.53</td>
<td>(1.92, 2.92)</td>
</tr>
<tr>
<td>0.95</td>
<td>2.74</td>
<td>(2.38, 3.19)</td>
</tr>
</tbody>
</table>

Twelve-month forward-looking horizon

Sample (adjusted): 1904/04 - 2002/09 (270 observations)
Dependent variable: Fed Funds Rate
Output gap historical data detrended using HP filter
Instruments set $IV \in (1, \pi_{t-1}...\pi_{t-3}, \pi_{t-6}, \bar{y}_{t-1}...\bar{y}_{t-3}, \bar{y}_{t-6})$. 
Table 10. Real v. Historical Data: monthly regression quantile estimates using US data (point estimates and 90% confidence intervals)

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\beta_\theta$</th>
<th>$\gamma_\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>1.33 (1.25, 1.66)</td>
<td>0.76 (0.08, 1.61)</td>
</tr>
<tr>
<td>0.1</td>
<td>1.42 (1.17, 1.75)</td>
<td>1.06 (0.22, 1.37)</td>
</tr>
<tr>
<td>0.2</td>
<td>1.60 (1.46, 1.77)</td>
<td>0.94 (0.77, 1.10)</td>
</tr>
<tr>
<td>0.3</td>
<td>1.62 (1.49, 1.89)</td>
<td>0.82 (0.70, 1.00)</td>
</tr>
<tr>
<td>0.4</td>
<td>1.71 (1.37, 1.94)</td>
<td>0.74 (0.45, 1.03)</td>
</tr>
<tr>
<td>0.5</td>
<td>1.75 (1.52, 2.06)</td>
<td>0.64 (0.25, 0.95)</td>
</tr>
<tr>
<td>0.6</td>
<td>1.86 (1.67, 2.32)</td>
<td>0.49 (0.13, 0.83)</td>
</tr>
<tr>
<td>0.7</td>
<td>2.09 (1.86, 2.42)</td>
<td>0.27 (0.09, 0.70)</td>
</tr>
<tr>
<td>0.8</td>
<td>2.25 (1.96, 2.50)</td>
<td>0.22 (0.15, 0.33)</td>
</tr>
<tr>
<td>0.9</td>
<td>2.35 (2.07, 2.90)</td>
<td>0.30 (0.21, 0.36)</td>
</tr>
<tr>
<td>0.95</td>
<td>2.58 (2.02, 2.92)</td>
<td>0.41 (0.29, 0.51)</td>
</tr>
</tbody>
</table>

Twelve-month forward-looking horizon

Sample (adjusted): 1979/11 - 2002/09 (275 observations)
Dependent variable: Fed Funds Rate
Output gap real-time data detrended using HP filter
Instruments set $IV \in (1, \pi_{t-1}, \bar{y}_{t-1})$. 

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Figure 1a: US Fed Funds rate and inflation

Series: FED
Sample 1979:10 2003:09
Observations 288

Mean  6.965243
Median  6.045000
Maximum  19.10000
Minimum  1.010000
Std. Dev.  3.722022
Skewness  1.076843
Kurtosis  4.352606
Jarque-Bera  77.61492
Probability  0.000000
Repo rate

Inflation

Sample 1993:02 2003:07
Observations 126

Mean     5.702381
Median  6.000000
Maximum  7.500000
Minimum  3.500000
Std. Dev.   1.026019
Skewness  -0.382970
Kurtosis   2.467576
Jarque-Bera  4.568240
Probability  0.101864
Series: CALL
Sample 1979:04 2003:12
Observations 297

Mean 3.747677
Median 3.790000
Maximum 12.70000
Minimum 0.001000
Std. Dev. 3.178677
Skewness 0.380524
Kurtosis 2.288908
Jarque-Bera 13.42497
Probability 0.001216
Figure 3a: Quantile Planes for the US Data

Figure 3b: Quantile Planes for the UK Data
Figure 3c: Quantile Planes for Japan Data

Figure 4a: The Responses of the Fed Funds Rate towards Inflation (OLS and QR)
Figure 4b: The Responses of the Base Rate towards Inflation (OLS and QR)

Figure 4c: The Responses of the Call Rate towards Inflation (OLS and QR)
Figure 5a: The Responses of the Fed Funds Rate towards the Output Gap (OLS and QR)

Figure 5b: The Responses of the Base Rate towards the Output Gap (OLS and QR)
Figure 5c: The Responses of the Call Rate towards the Output Gap (OLS and QR)

Figure 6a: Absolute Value of Changes in the Fed Funds Rate
Figure 6b: Absolute Value of Changes in Base Rate

Figure 6c: Absolute Value of Changes in Call Rate