Financial Fragility, Heterogeneous Firms and the Cross Section of the Business Cycle.

Sean Holly
University of Cambridge

Emiliano Santoro
University of Cambridge

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Abstract

There is growing empirical evidence that the cross section of the growth rate of firms is subject to systematic distortions at business cycle frequencies. In this paper we briefly review this evidence and then offer a theoretical model that incorporates nonlinearities in the way in which firms respond to aggregate and ideosyncratic shocks. We are able to replicate the most commonly found regularity - skewness in the cross section is counter-cyclical - and show that the strength of this relationship varies with the extent of financial fragility.

1 Introduction

The representative agent model has long been an important workhorse for economics and in the last 3 decades it has become the dominant macroeconomic approach. Today's representative agent models are characterized by an explicitly stated optimization problem of the representative agent, which can be either a consumer or a producer. The derived individual demand or supply curves are then in turn used for the corresponding aggregate demand or supply curves. The moments of the aggregates in these models are then compared with time series observations of the macroeconomy. Implicit in this approach is that any underlying heterogeneity across agents or firms averages out and does not have any implications for the behaviour of the aggregate economy. An aggregate shock whether to technology or to nominal demand generates a spread-preserving mean shift in the economy. However, Haltiwanger (1997) has argued that statistical agencies should report the higher moments of economic activity; for example, the distribution of aggregate output across sectors and firms. Moreover, recent research into the cross sectional distribution of behaviour at business cycle frequencies has raised some questions about the usefulness of the representative agent model for explaining certain regularities. In particular the shape of the cross section is sensitive to business cycle shocks[1]. Macroeconomic shocks do not have a spread-preserving effect on the behaviour of firms. Evidence for the US (Higson et al (2003)), for the UK (Higson et al, 2004), Germany (Dopke et al, 2005) and Italy (Santoro, 2005) suggests a systematic tendency for the cross sectional distribution in firm growth rates to vary with the business cycle.

These stylised facts of the business cycle need some explanation. In this paper we consider a model of heterogeneous firms functioning in imperfectly competitive markets who may be in different financial states[2]. Bernanke and Gertler (1989), Greenwald and Stiglitz (1993), Kiyotaki and Moore (1997), Bernanke et al. (1996, 1999) show that in the presence of asymmetric information, financing constraints can be important for both investment and production decisions.

The more recent literature, however, is essentially concerned with the emergence of financial fragility in a perfect competition setting on the real side of the economy. The model of Greenwald and Stiglitz assumes that each firm faces an infinitely elastic demand function subject to a random idiosyncratic shock, which captures uncertainty regarding relative prices. Uncertainty arises because firms are price takers and there is a one-period lag between when firms borrow on the credit market, hire workers and production takes place, and when they sell their output. Firms are unable to raise external finance on the stock market because of equity rationing (Greenwald et al. (1984) and Myers and Majluf

[1] It should be noted that these cross section findings use unweighted growth rates of firms, so it is possible that while the interaction of aggregate shocks and firm behaviour generates interesting cross sectional dynamics this does not carry over significantly to the aggregate economy which is of course the weighted sum of firm outputs.

[2] For a recent review of heterogeneous models see Hommes (2005). Of course, there may be other models that can also explain the stylised facts of the business cycle seen from the cross section, but we do not consider them here.

(1984)). Therefore, they rely on internally generated funds as the primary source of funding and resort to bank credit if internal funds are insufficient to finance the wage bill. As a consequence, firms face an explicit risk of bankruptcy.

By construction the probability of bankruptcy is a decreasing function of the net worth of the firm. The higher net worth (the lower financial fragility) relative to the wage bill, the lower the probability of bankruptcy will be. Therefore, if bankruptcy is costly, production is increasing in net worth. By assumption, Greenwald and Stiglitz rule out any strategic interaction.

A different approach is taken in the theoretical framework put forward by Delli Gatti et al. (2004, 2005) in which firms are heterogeneous in terms of size and the degree of financial fragility and interact through the credit market. Their model shows how a parsimonious non-linear framework can generate a rich set of stylized facts both from a cross-sectional and from a dynamic point of view. However the agent-based model introduced by Delli Gatti et al. (2003, 2004) only focuses on the indirect interaction on the credit market through the presence of a commercial banking sector, ruling out any interaction on the goods market. More recently, Bischi et al. (2004), incorporate imperfect competition and strategic interactions among firms.

In this paper we seek to extend this model by incorporating it into a wider macroeconomic framework. We consider a monopolistically competitive market populated by heterogeneous firms, which differ because of their financial structure. Financial conditions affect both the firm-specific level of output and the competitors' output through the conventional demand function of monopolistic competition à la Blanchard and Kiyotaki (1987). Furthermore, financial fragility affects exit and entry of firms because of an explicit risk of bankruptcy. Shocks to the economy, both of an aggregate and idiosyncratic nature, have a different impact depending on the financial condition of the firm. Depending on their financial robustness, monetary policy affects firms in different ways. At the same time, commercial banks determine the contractual interest rate on loans following a mark-up pricing rule over the interbank interest rate: the mark-up is determined as a weighted average of the firm-specific probability of bankruptcy as well as of an index of performance of the overall credit market.

Firms set their prices simultaneously given their private information, but price decisions interact both directly and indirectly, given that the demand function faced by each of them depends upon aggregate income available for consumption and on the number of firms (equal to the different varieties of goods) which operate at any given time. Hence, interaction is direct and strategic through Bertrand competition on prices and indirect and non strategic through the effect of aggregate income. Particular attention must be paid to this last term, as it is likely to determine both an aggregate income effect and an effect due to the entry-exit process. While in standard models of monopolistic competition the solution to the problem is represented by a symmetric Bertrand-Nash equilibrium, obtained by assuming that firms are homogeneous and have common knowledge, with asymmetric agents competitors prices have to be somehow forecasted. Heterogeneity implies that agents lack sufficient information on the strategies adopted by competitors, forcing them to rely on a simple static rule to form expectations on the general price level.

The remainder of the paper reads as follows: in section 2 we briefly review the empirical evidence on the dynamics of the cross section over the business cycle, in section 3 we describe the theoretical model, in section 3 we report on a number of numerical solutions of the model and establish the extent to which it can replicate the cross section dynamics in the data, finally in section 4 we conclude.

2 Some Stylised Features of the Business Cycle from the Cross Section

In this section we briefly consider some of the cross sectional features of the business cycle revealed by the analysis of longitudinal data on firms for the US, the UK, Germany and Italy. In Higson et al (2003, 2005) the methodology used measures of dispersion to examine the relationship between the business cycle and the higher moments of the cross sectional distribution calculated from firm growth rates. Here we use an alternative approach which fits a particular function (the Subbotin or Exponential Power Distribution) to the cross section in each year and then observes the extent to which the parameters of this distribution varies over the business cycle. We show that there are distinctive changes in the shape of the cross sectional distributions associated with business cycle swings in the macroeconomy.

Table (2.1) summarises the results for four countries, Italy, Germany, the UK and the US. It provides a regression of real GDP growth on the cross sectional moments (mean and skewness). All are at the annual frequency. The sample periods are given below each country column. Each of the moments is regressed on lags of itself and current and lagged GDP growth. In all cases there is a significant positive correlation between aggregate GDP growth and the mean growth rate in the cross section of firms. There is also a significant negative correlation between the aggregate growth rate and skewness in the cross section.

2.1 Subbutin Distributions

There is a large empirical literature on the dynamics of firms and industries that has established many stylised facts concerning the distribution of firms' characteristics. However, due to the limited availability of longitudinal establishment data, the literature on industrial demography [3] has focused mainly on the "static" properties of the distribution of variables such as size and growth, neglecting their possible variation over time. Following Sutton (1997), a variety of lines of research have been followed by the conventional industrial organization literature. One, based on Gibrat's law (1931), assumes that the growth rates of firms are independent of firms' size.

[3] See Steindl (1968), Geroski (1995), Sutton (1997) and Caves (1998) for comprehensive reviews of the literature.

Table 2.1: Regression of Mean and Skewness on GDP.

	Germany		**Italy**		**UK**		**US**	
momemt (μ)	Mean	Skewness	Mean	Skewness	Mean	Skewness	Mean	Skewness
constant	-0.010	0.103	0.002	-0.020	-1.5753	0.083	-0.5207	-0.0344
	(0.008)	(0.002)			-3.04	1.86	-1.12	-0.82
μ_{t-1}	0.103	0.127	0.323	0.033	0.2039	0.2169	0.4301	0.2772
	(0.124)	(0.129)			1.18	1.22	3.29	2.04
μ_{t-2}	-2.240	-0.193			-0.1668	-0.2585		
	(0.015)	(0.069)			-1.62	-2.36		
$\Delta ln(gdp_t)$	1.053	-8.077	1.426	-10.345	1.054	-0.0798	0.9443	-0.0596
	(0.000)	(0.000)			7.8	-6.54	11.33	-7.82
$\Delta ln(gdp_{t-1})$	(-)	(-)	-0.719	1.959	0.4929	-0.0309	-0.1450	0.0022
					1.97	-1.53	-1.06	0.21
Adjusted R^2	0.856	0.810	0.848	0.754	0.832	0.774	0.743	0.755
LM(2)	0.491	0.547	0.482	0.720	3.85	2.591	0.026	0.680
Sample	1971	-1998			1968	-1997	19?	2001
Company years								

Gilbrat's Law of Proportionate Effect implies that if the rates of growth of firms are identically and independently distributed, the distribution of the firms' size tends asymptotically to a lognormal. It follows from this that the distribution of firms' rates of growth is Gaussian. Nevertheless, more recent empirical studies in industrial demography have detected two empirical regularities which are so widespread across countries and persistent over time to be characterized as universal laws (a) the distribution of the size of firms is right skewed and can be fitted by a Power Law (or Zipf) probability density function[4]; (b) the growth rates of firms' output follow a Laplace distribution.

Zipf's law is the discrete counterpart to the Pareto continuous distribution (power law). It links the probability of observing the dimension of a phenomenon with rank greater than, say, z_i, with the cumulative frequency. Roughly speaking, a discrete random variable Z is said to follow a Power Law (also known as Rank-Size, or Pareto-Levy) distribution, if its cumulative distribution function takes the form

$$\Pr(Z \geq z_i) = \left(\frac{z_i}{z_0}\right)^{\alpha} \tag{2.1}$$

with $z_i \geq z_0$, $\alpha > 0$, where z_0 is the minimum efficient size and α is the scaling exponent or shape parameter.

Moreover, Stanley et al. (1996), Amaral et al. (1997) and Bottazzi and Secchi (2003) have found that the growth rate of the output of firm y_i follows,

[4] See for instance Axtell (2001) and Gaffeo et al. (2003).

instead of a normal distribution, a Laplace distribution:

$$L(y_i, b) = \frac{b}{2} \exp(-by_i) \tag{2.2}$$

where $b > 0$ is the scale parameter. In order to explain these findings, the literature has pursued two lines of research. The first focuses only on the statistical properties of the link between the distribution of the size of firms and their rates of growth. For instance, Reed (2001) shows that independent rates of change do not generate a lognormal distribution of the size of firms if the time of observation of firms is not deterministic but if it itself is a random variable following approximately an exponential distribution. In this case, even if Gibrat's law holds true at the individual level, firms will converge to a double Pareto distribution.

The second line of research stresses the importance of non-price interactions among firms hit by multiplicative shocks, hence building on the framework put forward by Herbert Simon and his co-authors during the 1950s and 60s. For example, Bottazzi and Secchi (2003) obtain a Laplace distribution of firms growth rates within Simon's model, just by relaxing the assumption of independence of firms' growth rates[5]. In the present analysis, following Marsili et al. (2004), we will test the stability of the cross sectional distribution of firms' rate of growth by fitting an asymmetric Subbotin density, whose symmetric counterpart encompasses the Laplace and the Gaussian densities as particular cases.

2.1.1 The Asymmetric Subbotin Distribution

The fuctional form of the symmetric[6] Subbotin distribution is characterised by three parameters: a position parameter m (which is at the same time the mean, the median and the mode of the density), a scale parameter a (describing the spread or width of the density) and a shape parameter b (which is inversely related to the fatness of the tails) and is described by

$$f(x; a, b, m) = \frac{\exp(-\frac{1}{b} \left| \frac{x-m}{a} \right|^b)}{2ab^{1/b}\Gamma(1 + 1/b)} \tag{2.3}$$

[5] In principle, these results can lead to reject the strong version of Gibrat's law. This law claims that the distribution of the levels (firms size measured in output or capital units) is lognormal while the empirical analysis points to Zipf's law - and the distribution of growth rates is normal while it seems to be a Laplace. As a matter of fact, things are not that simple. The idea according to which Gibrats law has to be fully discarded is wrong, given that in the recent literature a weak version seems to hold, in which growth rates seem to be independent at least in mean. In fact, Lee et al. (1998) show that the variance of growth rates depends negatively on firms size. The implications of the strong version of Gibrats law are not necessarily true in the weak version. Fujiwara et al. (2003) have shown, in fact, that if the distribution is characterized by time-reversal symmetry, i.e. the joint probability distribution of two consecutive years is symmetric in its arguments $P_{12}(x_1, x_2) = P_{12}(x_2, x_1)$, the weak version of Gibrats law can yield a power law of firms size. Hence power law and Gibrat's law (under its weak version) are not necessarily inconsistent.

[6] This distribution was introduced originally by Subbotin (1923) and popularized by Box and Tiao (1962, 1964, 1973), who used it in robustness studies (see also Tiao and Lund (1970), Swamy and Mehta (1977), West (1984), and more recently Osiewalski and Steel (1993)).

Where Γ is the Gamma distribution. The symmetric Subbotin distribution encompasses the Gaussian and the Laplace (or double exponential) distributions as special cases: for $b = 2$ it boils down to the Gaussian and for $b = 1$ to a Laplace, while for $b \longrightarrow \infty$ the distribution tends to a Uniform. The lower b, the fatter the tails: hence the distribution is platikurtic for $b > 2$ while it is leptokurtic for $b < 2$. This symmetric version of the Subbotin density has all central moments of odd order equal to zero. Following Bottazzi and Secchi (2003), the central moment of order $2l$ is:

$$M_{2l} = \left[ab^{1/b}\right]^{2l} \frac{\Gamma((2l+1)/b)}{\Gamma(1/b)}. \tag{2.4}$$

Particular interest will be attached in the subsequent analysis to the excess Kurtosis exhibited by the fitted distribution: in the symmetric case the index reads as follows

$$\gamma_k = \frac{\Gamma(1/b)\Gamma(5/b)}{[\Gamma(3/b)]^2} \tag{2.5}$$

It is relatively straightforward to check that $\partial\gamma_k/\partial b < 0$ for $b > 0$: this aspect will turn out to be particularly important for our analysis on the dynamic pattern of higher moments of the distribution.

The asymmetric Subbotin density extends the family described above by allowing the parameters a and b in the two halves of the density to take different values. Its functional form depends on five parameters: a positioning parameter m, two scale parameters a_l and a_r respectively for the values below or above m, and two shape parameters b_l and b_r characterizing, respectively, the lower and upper tail of the density. The following factorisation has been introduced by Bottazzi and Secchi (2003)

$$P(X) = \begin{cases} \frac{\exp(-(x-m)/a)^{b_l} \frac{1}{b_l}}{A} & x < m \\ \frac{\exp(-(x-m)/a)^{b_r} \frac{1}{br}}{A} & x > m \end{cases}$$

where

$$A = a_l b_l^{\frac{1}{b_l}} \Gamma\left(1 + \frac{1}{b_l}\right) + a_r b_r^{\frac{1}{br}} \Gamma\left(1 + \frac{1}{br}\right)$$

2.1.2 Some Subbutin distributions for the US.

In this section we report on some estimated Subbutin distributions for the US using the same data that was used for the US results shown in Table (2.1), but highlighting a subset of years associated with particularly extreme swings of the business cycle. These are shown below[7]. In Figure (2.1) we plot the estimated Subbutin distributions for 1972 to 1975, and in figure (2.2) for 1989 to 1993. These scan two significant business cycles in the US. They all show a shift in the distribution from one skewed to the left during major upswings to skewness to

[7] for a fuller analysis of this see Santoro (2006).

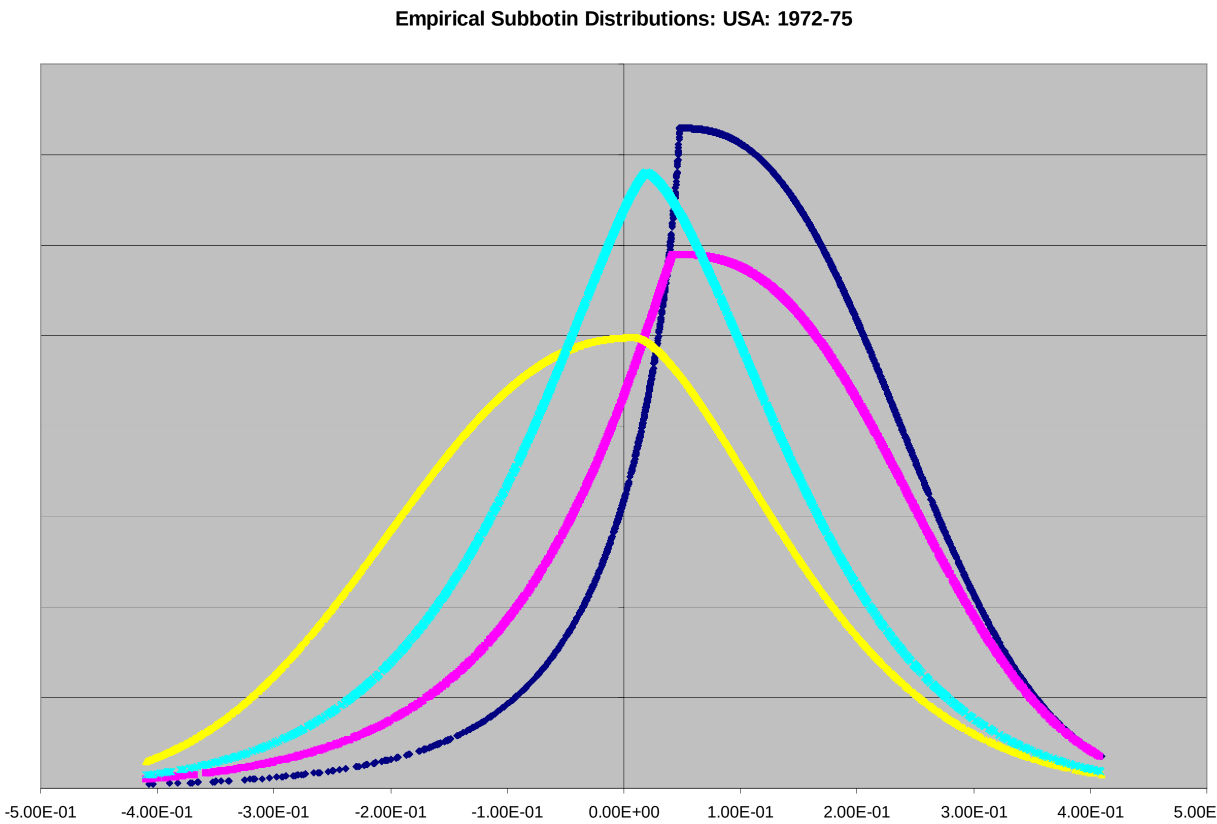

Figure 2.1:

the right in recession periods, and then a return to a distribution that is closer to a Laplace distribution.

The aim of the remainder of this paper is to describe a model that draws on the literature that emphasises the credit channel and financial accelerator aspects of the monetary transmission process, to allow for explicit forms of cross sectional heterogeneity, and then to see whether a simulated version of such a model can capture some of the movements in the moments of the cross sctional distributions at business cycle frequencies.

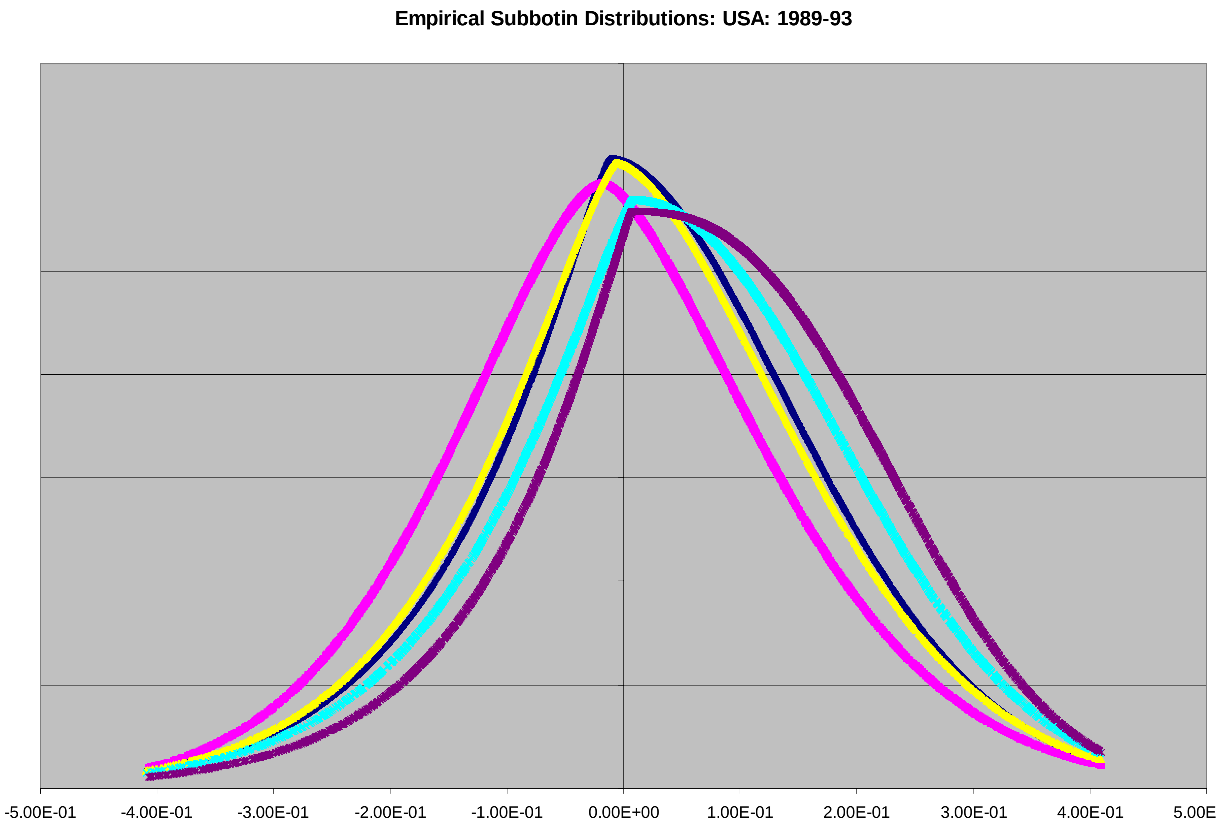
Empirical Subbotin Distributions: USA: 1989-93
-5.00E-01
-4.00E-01
-3.00E-01
-2.00E-01
-1.00E-01
0.00E+00
1.00E-01
2.00E-01
3.00E-01
4.00E-01
5.00E

Figure 2.2:

3 The Model Setup

In this section we turn to a model that we evaluate to establish whether it is capable of generating some of the stylised features of the cross sectional business cycle. The model contains heterogeneous firms in a monopolistically competitive framework.

The economy consists of two markets: goods and credit. The goods market is monopolistically competitive along the lines of Blanchard and Kiyotaki (1987). The population of firms in the economy produces a single good in n_t varieties in each period. The demand side of the economy is not explicitly modelled. It is assumed that there is a representative consumer endowed with a CES utility function, who in each period demands a bundle C_t of differentiated goods:

$$C_t = n_t^{\frac{1}{1-\eta}} \left(\sum_{i=1}^{n_t} C_{it}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

In each period the i^{th} firm in the economy faces the following demand function for its good:

$$Y_{it}^d = \left(\frac{p_{it}}{p_t} \right)^{-\eta} \frac{Y_t}{n_t}, \qquad \eta > 1 \tag{3.1}$$

where p_{it} denotes the price charged by the i^{th} firm, Y_t is income, η is the elasticity of substitution between pairs of goods, while p_t is the aggregate price level:

$$p_t = \left(\frac{1}{n_t} \sum_{i=1}^{n_t} {p_{it}}^{1-\eta} \right)^{\frac{1}{1-\eta}} \tag{3.2}$$

We assume that each firm in the economy produces a differentiated good by means of a decreasing returns to scale technology in which capital is the only input

$$Y_{it} = \phi_t {K_{it}}^{\alpha}, \qquad \alpha < 1 \tag{3.3}$$

where ϕ_t represents the productivity parameter common to all firms, which is assumed to follow an stationary AR(1) process

$$\phi_t = \nu \phi_{t-1} + \varepsilon_t \tag{3.4}$$

where $0 < \nu < 1$ and ε_t is an *iid* shock to ϕ_t with constant standard deviation σ_ε. Assuming no depreciation, the law of motion of the capital stock is expressed by:

$$K_{it} = K_{it-1} + I_{it} \tag{3.5}$$

where I_{it} denotes the investment undertaken by the i^{th} firm at time t.

Firms finance their production costs at least partially by means of internally generated funds inherited from the previous period, as net worth, denoted by A_{it}. The end of any period balance sheet implies that:

$$K_{it} = A_{it} + L_{it} \tag{3.6}$$

where L_i are bank loans. The ratio of net worth to total capital - the equity ratio - provides an index for the financial robustness of the firm. Firms accumulate net worth by means of retained profits, according to the following law of motion

$$A_{it+1} = \pi_{it} + A_{it} \tag{3.7}$$

where π_{it} denotes the retained profits for the i^{th} firm at time t.

It is assumed that firms are completely rationed on the equity market (Greenwald and Stiglitz , 1993). If internally generated funds are not enough to finance investment, the firm relies on the credit market[8].

The production cycle starts at the beginning of time t,and takes one period. At the beginning of period t, n_t firms adopt Bertrand strategies, determining their optimal price (hence their output and capital, through the demand function and the production function respectively), given income available for consumption, and the aggregate price index. The latter is an indicator of the average strategies undertaken by competitors. Goods are sold at the end of the period. Furthermore, firms must finance new capital before goods are sold. Production in period t will depend, therefore, on financial conditions inherited from the period, $t-1$. . Given this setup, each firm incurs a financing cost FC_{it} equal to the service on debt and payment of dividends (Delli Gatti et al., 2000):

$$FC_{it} = r_{it} L^d_{it} + r^A_{it} A_{it} \tag{3.8}$$

where r_{it} is the *contractual* real interest rate on loans paid by the i^{th} firm, while r^A_{it} is the return on net worth, paid out in the form of dividends. Debt commitments are determined by the firm specific interest rate on loans, set in every period by the commercial bank.

Real profits are the difference between real revenues and real costs. It is assumed that each firm's total revenues are subject to a multiplicative idiosyncratic shock ω_{it}, which is uniformly and independently (over time and across firms) distributed over the positive support $[0, \overline{\omega}]$ and has an expected value $E(\omega_{it}) = 1$. For simplicity, we assume that the contractual interest rate equals the return on net worth. Hence, real profits for the *ith* firm are:

[8] Although firms could in principle issue new equities, this option is *a priori* ruled out, due to the possibility that equity issues would be subject to adverse selection (Myers and Majluf (1984), Greenwald et al. (1984)), and would be too costly to firms. Relying on Greenwald and Stiglitz (1993), new equity issues contribute little to corporate finance: at a given share price, only overvalued firms are willing to sell their shares; potential shareholders anticipate this fact, and no trade occurs on the equity market .

$$\pi_{it} = \omega_{it} \left(\frac{p_{it}}{p_t}\right)^{1-\eta} \frac{Y_t}{n_t} - r_{it} \left(\left(\frac{p_{it}}{p_t}\right)^{-\eta} \frac{Y_t}{\phi_t n_t}\right)^{\frac{1}{\alpha}} \tag{3.9}$$

In this uncertain environment, firms risk bankruptcy if their net worth at the end of the period is negative. In order to simplify matters, we assume that the probability, ex ante, of bankruptcy PB_{it} at the beginning of the period is proportionate to the ratio of debt to total capital inherited from the previous period:

$$PB_{it} = \frac{L_{it-1}}{K_{it-1}} = 1 - \frac{A_{it-1}}{K_{it-1}} \tag{3.10}$$

The probability of bankruptcy is decreasing in net worth. Defining the *leverage* l_i as the ratio of debt to net worth, we get:

$$l_i = \frac{L_i}{A_i} = \frac{PB_i}{1 - PB_i} \tag{3.11}$$

and

$$PB_i = \frac{l_i}{1 + l_i} \tag{3.12}$$

The probability of bankruptcy is an increasing and concave function of leverage and so:

$$\lim_{l_i \to \infty} PB_i = 1.$$

When debt increases relative to net worth, the financial condition of the firm worsens and the probability of bankruptcy increases. In the case of a particularly unbalanced financial structure, even a small exogenous shock (aggregate or idiosyncratic) can trigger bankruptcy.

Bankruptcy is costly, so we assume that these costs are proportional to total sales

$$BC_i = c \left(\frac{p_{it}}{p_t}\right)^{1-\eta} \frac{Y_t}{n_t}, \quad c > 0. \tag{3.13}$$

The problem facing the firm is to maximise expected profits (with respect to the relative price $\frac{p_{it}}{p_t^e}$) net of bankruptcy costs:

$$Max\ E[\pi_{it}] = \left(\frac{p_{it}}{p_t^e}\right)^{1-\eta} \frac{Y_t}{n_t} - r_{it} \left(\left(\frac{p_{it}}{p_t^e}\right)^{-\eta} \frac{Y_t}{\phi_t n_t}\right)^{\frac{1}{\alpha}} - [BC_{it} * PB_{it}] \tag{3.14}$$

Notice that the i^{th} firm knows neither the actual price charged by competitors, nor the aggregate price level, information about which becomes available only at the end of the period. Firms set their prices simultaneously given their private information, but price decisions interact both directly and indirectly, given that the demand function faced by each firm depends upon the aggregate income available for consumption and on the number of firms and the variety of goods in the economy at time t, n_t. Hence interaction is direct and strategic through Bertrand competition on prices and indirect and non strategic through the term $\frac{Y_t}{n_t}$, which is likely to provide both an aggregate income effect (captured by Y_t) and a second round effect due to the entry-exit process (captured by n_t). While in standard models of monopolistic competition the outcome of competition is represented by a symmetric Bertrand-Nash equilibrium - obtained by assuming that firms are homogeneous and have common knowledge - with heterogeneous agents competitors' prices have to be forecasted. In the present context, since firms are subject to an idiosyncratic shock, whose effect will depend upon the financial condition of the firm, and since an entry process of new firms will be explicitly modeled, after the first period (in which a symmetric Nash equilibrium is assumed) the economy will be populated by heterogeneous agents. This feature of the model enables us to keep track of the evolution over time of the distributions of the size of firms and their rates of growth. Simulations of the model will rely on a simple rule, assuming that firms forecast their competitors' prices taking static expectations, $p_t^e = p_{t-1}$, or they rely on $AR(2)$ predictor. From the first order condition we end up with the following price decision rule for the i^{th} firm:

$$\frac{p_{it}}{p_t^e} = \left[\frac{r_{it}\eta a K_{it-1}}{(\eta - 1)[K_{it-1} - c(K_{it-1} - A_{it-1})]\phi_t^a} \left(\frac{Y_t}{n_t} \right)^{a-1} \right]^{\frac{1}{\epsilon}} \tag{3.15}$$

where $a = \frac{1}{\alpha}$ and $\epsilon = 1 + \eta(a-1)$.

The optimal price set by the i^{th} firm is an increasing function both of the interest rate on loans r_{it} and of the marginal cost of bankruptcy. Output associated with this optimal price is:

$$Y_{it}^s = \left[\frac{r_{it}\eta a K_{it-1}}{(\eta - 1)[K_{it-1} - c(K_{it-1} - A_{it-1})]\phi_t^a} \right]^{-\frac{\eta}{\epsilon}} \left(\frac{Y_t}{n_t} \right)^{\varsigma} \tag{3.16}$$

where $\varsigma = 1 - \frac{\eta}{\epsilon}(a-1)$.

The first order condition for the demand for capital goods is then:

$$K_{it}^d = \phi_t^{a(\frac{\eta}{\epsilon}-1)} \left[\frac{r_{it}\eta a K_{it-1}}{(\eta - 1)[K_{it-1} - c(K_{it-1} - A_{it-1})]\phi_t^a} \right]^{-\frac{\eta a}{\epsilon}} \left(\frac{Y_t}{n_t} \right)^{\varsigma a} \tag{3.17}$$

As one would expect, the optimal capital stock is a non-linear, negative function of the interest rate and a non-linear, increasing function of net worth, A_{it-1}. A deterioration in the financial position of a firm has a negative effect on output. The production of each firm, because it is linked to aggregate income through the negatively sloped demand function, will in turn depend on the general financial state of the economy and on the number of bankruptcies that occurred in the previous period. Therefore, relaxing the assumption of Greenwald and Stiglitz (1993) of a perfectly competitive goods market and assuming heterogeneous monopolistically competitive firms, creates interdependence among firms. This *mean field interaction* is determined, among other things, by the general financial state of the economy.

3.1 The Credit Market

Investment at time t is the difference between the desired capital stock and the capital stock inherited from the past

$$I_{it} = K_{it} - K_{it-1} \tag{3.18}$$

To finance investment, each firm uses retained profits and, if necessary, resorts to the credit market. Given the balance sheet identity, the demand for credit at the beginning of period t will be equal to:

$$L_{it} = K^d_{it} - A_{it} \tag{3.19}$$

After substituting for the demand for capital, we end up with the following relation:

$$L^d_{it} = \phi_t^{a(\frac{\eta}{\epsilon}-1)} \left[\frac{r_{it}\eta a K_{it-1}}{(\eta-1)[K_{it-1} - c(K_{it-1} - A_{it-1})]\phi^a_t} \right]^{-\frac{\eta a}{\epsilon}} \left(\frac{Y_t}{n_t} \right)^{\varsigma a} - \pi_{it-1} + A_{it-1} \tag{3.20}$$

For tractibility, we assume there is one bank in the model functioning as a vertically integrated banking sector. If all firms repay debt, the bank would be certain that its liquidity constraint is satisfied. Depending on the financial status and on the overall credit market performance of each firm, the banking sector, in each period, renegotiates the conditions on loans extended to each firm. Each contractual interest rate, negotiated at time t, embodies both a firm specific risk of default and a macroeconomic index of credit performance, which describes the rate of default in the economy as a whole. The contractual interest rate is set as a mark-up on the interbank interest rate, i_t:

$$r_{it} = [1 + \Phi_{it}(PB_{it}, d_{t-1})]i_t \tag{3.21}$$

where $\Phi_{it}(PB_{it}, d_{t-1})$ is the mark-up function, which depends upon the probability of bankruptcy for the i^{th} firm and on the rate of default at the previous period, denoted by d_{t-1}.

We compute the rate of default as the volume of performing loans allotted by the commercial banking sector in the previous period, relative to the total debt extended:

$$d_t = \frac{L^B_t}{L^T_t} \tag{3.22}$$

where L^T_t is total credit extended by the commercial banking sector in period t and L^B_t is the total "bad debt" in period t, defined as the sum of the debt of firms going bust at the end of the period.

Alternatively we can write the markup function $\Phi_{it}(PB_{it}, d_{t-1})$ in linear form, as a weighted average of its arguments:

$$\Phi_{it}(PB_{it}, d_{t-1}) = \tau PB_{it} + (1-\tau)d_{t-1}, \qquad \tau > 0.5 \tag{3.23}$$

The commercial banking sector attaches greater importance to the firm specific risk of default. Given these features, the contractual rate of interest will can be expressed as:

$$r_{it} = [1 + \tau PB_{it} + (1 - \tau)d_{t-1}]i_t \tag{3.24}$$

This produces an infinitely elastic credit supply function that shifts on the orthant $\{r_{it}, L_{it}\}$ depending on the firm specific probability of bankruptcy, on an overall index of credit default risk and on the interbank rate. Thus, the central bank can at least partially control the interest rate applied by the banking sector to each firm. Notice that, as the probability of bankruptcy goes to zero, the interest rate on a firm specific loan will equal the interest rate set by the central bank only if the systemic default rate is also equal to zero.

3.2 Industrial Demography

The process of entry and exit by firms will have an effect on the dynamics of the cross sectional distribution of the rates of growth and this will also depend on the degree of financial vulnerability of the entire economy. The demographic process influences the demand function faced by each incumbent through the interplay of the aggregate income available for consumption, that in turn depends upon the financial robustness of the economy as a whole, and through the number of incumbents, which is determined by the turnover between firms going bust and new entrants. This "direct"[9] mean interaction effect is likely to drive the dynamics of the aggregate output and to have serious distributive implications.

Davis et al.(1996) have shown that the turnover of firms entering and exiting markets contributes almost as much to employment and output fluctuations as incumbent firms. This suggets that we should pay particular attention to the way entry and exit of firms is modeled. As we already know exits are traced back to financially weak firms, whose leverage is so high that an adverse shock makes net worth become negative. In the literature, the entry process has been modelled as a purely stochastic process (Winter et al. (1997), or as an endogenous process (Hopenhayn (1992)), depending on expected profit opportunities.

[9] Delli Gatti et al. (2005) develop a perfectly competitive framework where mean field interaction arises as a result of a bank effect (Hubbard et al., 2002). In their model, if a firm goes bankrupt, not only does aggregate output but also bank's equity is directly affected. As a consequence, credit extended goes down, pushing up the interest rate charged to each firm, which spreads financial fragility and increases the risk of bankruptcy for the whole population of firms. Some of the firms which are particularly financially fragile will default and leave the market, while surviving firms' output and investment will shrink. An analogous *domino* effect is at work in our model, but in the present context the propagation mechanism arising from the demographic turnover acts through two channels. First, it propagates directly through the market for goods, *via* demand function, and second, through the credit market, given that banks set the firm-specific contractual interest rate by adding a default risk premium to the rate of interest set by the monetary authority. Furthermore, we can envisage a sort of interdependence between these two channels, since the cental bank, as we will see, follows a Taylor rule, which partially determines the interest rate by considering the deviation of the total output from its natural level.

However, according to Caves (1998), we can rely on the observation that entrants are generally unsure about the probability of prospective success, and that entries do not occur at a unique optimal size.

Here, as a first approximation we will model the entry process in an adaptive way. Each exiting firm is replaced by a new entrant. Furthermore, we assume that new firms enter at a scale of production equal to the average of the incumbent population.

3.3 The Central Bank

In our framework the dynamics of the economy are driven by the credit cycle and by the propagation mechanism of idiosyncratic and aggregate shocks, whose effect depends upon the degree of financial heterogeneity. We assume that the Central Bank follows a Taylor rule with interest rate smoothing which implies a short run trade off between output growth and the deviation of the rate of inflation from its target value:

$$i_t = \rho i_{t-1} + (1-\rho)[\gamma_1 \Delta \log y_t + \gamma_2(\pi_t - \pi^*)], \qquad 0 < \rho < 1 \tag{3.25}$$

where ρ is the interest rate smoothing parameter and π_t and π^* are the average rate of inflation and the target rate of inflation respectively and y_t is aggregate output. As usual parameters γ_1 and γ_2 describe the relative importance that the central bank attributes to output growth and to the excess of inflation over its target.

3.4 Model Simulation

The model described in the previous section does not have a closed form analytical solution. The basic properties of the framework will be analysed, therefore, through simulation. We can draw some implications from the expression describing the reponse of firm specific production to total real income. First, notice that the impact of macroeconomic fluctuations are non-linear. Second, the financial structure plays a crucial role in the response. Recall the demand function faced by each producer:

$$Y_{it} = \left(\frac{p_{it}}{p_t}\right)^{-\eta} \bar{Y}_t, \qquad \eta > 1$$

where we have replaced the term $\frac{Y_t}{n_t}$ with $\bar{Y}_t$. Notice that the latter is equal to the market share of the i^{th} firm along a symmetric equilibrium, where $p_{it} = p_t$. This situation represents a useful benchmark, since it represents the outcome of standard models of monopolistic competition, in which the solution to the problem is represented by a symmetric Bertrand-Nash equilibrium, obtained by assuming that firms are homogeneous.

The relative competitive position of the firm is then given by the ratio of the effective demand to the potential market share in the representative agent case:

$$S_{it} = \frac{Y_{it}}{\bar{Y}_t} = \left(\frac{p_{it}}{p_t}\right)^{-\eta} \qquad i = 1, .., n. \tag{3.26}$$

The latter can be seen as an index of competitiveness of the i^{th} firm, while the monopolistic competition setting can be regarded as a competition over market shares. As it is clear from expression above, S_{it} is a negative function of the firm specific price. At this stage it is useful to recall the firm's optimal reaction function in this framework, in which we assume $c = 1$ (without loss of generality) and denote the equity ratio $\frac{A_i}{K_i}$ with a_i :

$$p_{it} = p_t^e \left[\frac{r_{it}\eta a}{(\eta - 1)a_{it-1}\phi_t^a}\right]^{\frac{1}{\epsilon}} \bar{Y}_t^{\frac{a-1}{\epsilon}} \tag{3.27}$$

The price set by the i^{th} firm is a negative function of financial fitness, captured by the equity ratio at the beginning of the period[10]. A better financial structure allows the firm to be more competitive and to supply more[11].

Given these general features of the model, we need a competitive mechanism, which is strictly related to the level of financial vulnerability/strength of the firm in order to determine firm specific and aggregate prices. The mechanism we are going to use here can be formalised along the lines suggested by the literature on evolutionary dynamics. In the following we will abstract from considerations regarding changes in the level of productivity and the demographic process. There will be a fixed number of firms. Given that firms are heterogeneous from the financial point of view, the relative price changes over time. We linearise and transform the system of $n + 1$ equations described by equation (3.27) from discrete to continuous time in order to express the time evolution of the relative market share of the i^{th} firm. Denoting by $s_i(t)$ the logarithm of $S_i(t)$

$$\overset{\bullet}{s_i}(t) = -\eta \left(\frac{\log \overset{\bullet}{p_i}(t)}{\log p_i(t)} - \frac{\log \overset{\bullet}{p}(t)}{\log p(t)}\right) s_i(t) \tag{3.28}$$

If we think of $s_i(t)$, the (log)relative competitive position of the i^{th} firm, as a *fitness function*, the expression above can be thought as a *replicator function*[12]: the dynamic system describes the evolution of the relative market shares of each firm in the system.The firm will be able to survive in a competitive environment as long as its price increases less than the average price level[13]: this is possible

[10] Recall also that the real contractual interest rate is a negative function of the equity ratio: this feature amplifies the effect of the financial structure through the credit market.

[11] However, the relevance of the equity ratio for the monopolistic pricing is limited above by $K_{it-1}(a_{it-1} = 1)$.

[12] The concept of fitness is a metaphor for the ability of a firm to survive in a competitive environment.

[13] The fitness function and the replicator function describing the evolution of the population composition are sign compatible, that is they have the same sign, whenever $S_i(t) > 1$. Under this condition, we can follow the arguments reported in Joosten (1996) and Gaffeo (1999) to prove that the deterministic system (no demographic growth) admits at least one fixed point. Furthermore, every stable fixed point is a saturated equilibrium, that is an equilibrium at

if the financial structure, captured by the equity ratio, allows the firm to be competitive and to lower the price without frictions, in order to supply more and to gain market share. In such an environment, the margin of manovrability on prices, which is positively related to the equity ratio, becomes crucial for survival and growth. Hence financial structure has a direct influence on the rise of frictions in the price setting system[14]. During an expansion, the relative market share S_{it} could decrease, if Y_{it} increases less rapidly than $\bar{Y}_t$. Thus, if a firm is not able to lower its price, it will lose market shares. Given that the main determinant of the price setting is the level of financial fitness, a firm in conditions of financial distress will have a competitive disadvantage, which translates into a limited margin for manovrability of the price, namely a downward stickiness.

We simulate a model with 500 firms over 300 periods. All simulations refer to a benchmark parameter setup. Homogeneity of initial conditions was assumed in order not to bias the micro and macro dynamics. At time 0 firms experience no idiosyncratic shocks have the same financial structure, while the relative price is equal to 1. The baseline parameterisation is:

n_0	i_0	c	ν	α	$\overline{\omega}$	η	τ	ρ	γ_1	γ_2	π^*	a_0
400	0.05	1	0.98	0.95	2	1.5	0.8	0.95	0.5	1.5	0.02	0.6

In order to establish whether aggregate variables match empirical dynamic regularities and whether cross sectional moments of the firms' rate of growth distribution are in line with recent empirical work discussed above, we analyse the simulated data both from a time series and from a cross sectional perspective. Only the last 220 periods are used. In particular we are interested in whether the model can replicate the negative correlation between the aggregate business cycle and cross sectional skewness. The results of a series of regressions of the cross sectional mean and skewness on the aggregate growth rate generated by the model simulation are reported in Table (4.1). The results are shown for three values of the price elasticity of demand. As we move closer to a perfectly competitive environment, ($\eta = 10$) we note that correlation of the cross section mean with the aggregate growth rate increases and the negative correlation with skewness falls, although it is still significant. For this model with imperfectly competitive markets and heterogeneity in the financial state of each firm the higher moments of the distribution of the rates of growth are correlated with the cycle, replicating some of the characteristic patterns described in Higson et al. (2002, 2004). For the sake of convenience, the reported evidence refers to the last 50 periods of the simulation (at a cut-off of +/- 100%, but results prove to be quite robust for larger or narrower ranges of growth). What is really

which each survived firm has highest fitness. As the dynamic process goes on, firms whose financial position prevents them from having a mergin of manovrability on prices eventually go bankrupt, and the number of operating firms shrinks. Eventually, only the fittest firms will survive.

[14] Furthermore, even if not described in our framework, it is worth pointing out how the financial structure has an indirect influence on the price setting system, through the effect of R&D activities. An higher degree of financial strength allows a firm to innovate and to improve its productivity, and hence to be more competitive by decreasing prices.

striking is the counter-cyclical behaviour of skewness.

Table 3.1: Regression of Mean and Skewness on GDP weith Simulated Data.

	$\eta = 3$		$\eta = 5$		$\eta = 10$	
	Mean	Skewness	Mean	Skewness	Mean	Skewness
constant	-0.0660	0.1605	-0.0589	0.1605	-1.5753	0.083
	(5.29)	(2.24)	(4.34)	(2.24)	-3.04	1.86
moment$_{t-1}$	-0.552	-3.9358	-0.4180	-0.6270	0.2039	0.2169
	(3.53)	(7.33)	(4.447)	(7.81)	1.18	1.22
$\Delta 1n(gdp_t)$	0.4895	-3.9360	0.4187	-3.9358	1.054	-0.0798
	(6.09)	(7.33)	(6.20)	(7.33)	7.8	-6.54
$\Delta 1n(gdp_{t-1})$	0.2878	-2.2951	0.3048	-2.2951	0.4929	-0.0309
	(2.56)	(3.95)	(4.32)		1.97	-1.53
Adjusted R^2	0.662	0.552	0.473	0.754	0.832	0.774

Conclusions

This paper has attempted to construct a model of heterogeneous firms differentiated by the state of their finances. Simulations of the model suggest that we can replicate some of the cross sectional features that have been detected recently in the literature using longitudinal data on firms in the US, UK, Germany and Italy. We use a model with a monopolistically competitive market Financial conditions affect both the firm-specific level of output and competitors' output through the conventional demand function of monopolistic competition à la Blanchard and Kiyotaki (1987). The resulting economy is characterised by the presence of both aggregate and idiosyncratic shocks, that determine an explicit risk of bankruptcy, which, depending on their financial structure, is different across firms. Over the business cycle, in our simulation model we observe systematic shifts in the cross section, with in particular, a negative correlation between skewness and the aggregate business cycle.

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