Understanding Labour Market Frictions: A Tobin’s Q Approach

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(Preliminary, Please do not quote without permission)

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Abstract

Labour market friction is viewed as the Tobin’s Q of an employed worker as opposed to the position of the Beveridge curve. This Tobin’s Q is inversely proportional to the average quality of the match between employers and workers. Based on this measure, I find that the labour market friction has a procyclical trend in the US, which is indicative of the fact that firms compromise on the quality of the skill match during an expansion.

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1 Without implicating I would like to thank John Cochrane for inspiring me to undertake this project. Martin Robson is acknowledged for constructive comments on an earlier version of this paper. Thanks are also due to Mauricio Armellini and Soyeon Lee for able research assistance.
1. Introduction

The relative price of investment to consumer goods has significantly declined over time in the US. This decline is particularly noticeable in the 80s, which coincided with the great period of moderation of output volatility. A number of papers ascribe this recent decline to elimination of investment frictions (Justiniano and Primiceri, 2006, Chari, Kehoe and McGrattan, 2005). Although there is a near consensus that the degree of capital market frictions in the US has substantially decreased recently, less is known about labour market frictions.

Following the work of Pissariades (1985), by labour market friction I mean the degree of mismatch between the worker and the employer. Little is known about this job-matching variable at the aggregate level. A sizable literature focuses on the behaviour of the unemployment-vacancy relationship (known as the Beveridge curve) as a measure of this friction. There are both empirical and theoretical limitations of this Beverdige curve approach. Vacancy is usually measured by the help-wanted index which is less reliable particularly after the internet revolution when job openings are mostly available online. Valletta (2005) attempts to remedy this deficiency by creating a synthetic job vacancy ratio and argues that the Beveridge curve has shifted inward in the 80s after an outward shift in the 70s. Shimer (2005) argues that the vacancy-unemployment ratio has a remarkable volatility (almost 20 times higher than the labour productivity). This volatility makes it difficult to arrive at a definitive conclusion about the time path of the labour market frictions. ²

Theoretically, it may be misleading to use unemployment-vacancy ratio as a measure of labour market frictions even though it is measured with reasonable accuracy. During an expansionary phase of the cycle the unemployment-vacancy ratio may decline because firms may be keen on filling vacancies in a tight labour market even though the match is poor.³ Thus the skill match might progressively deteriorate as the labour market tightens. To explore this issue further I use a price-based approach to measure the quality of the skill match. A firm’s decision to fill a job vacancy is considered as an investment

² Hornstein et al. (2005) extend Shimer’s (2005) work and find additional problems in replicating the observed unemployment-vacancy fluctuations using the extant matching models.
³ Abraham and Katz (1986) argue that a downward sloping unemployment-vacancy relationship postulated by a Beveridge curve is not consistent when unemployment is driven by job separations.
problem. Just like the law of motion of the physical capital, the representative firm takes a dynamic Beveridge curve as given and then makes optimal choices about the time paths of employment as well as physical capital. The relative price of a worker with respect to capital is shown to be the Tobin’s Q of an employed worker. I show that this Tobin’s Q is inversely related to the average match quality of the worker and the employer. The Q of the worker shows endogenous fluctuations driven by the TFP shock. Parallel to investment friction, in my model, more frictions in the labour market means a higher Tobin’s Q of the existing worker. Chari, Kehoe and McGrattan (2005) define labour market friction in terms of an implicit tax on wages. My model differs from Chari et al. (2005) in an important dimension. While in their model the labour wedge in a real prototype model is equivalent to stickiness of nominal wages, in my model, this labour wedge is explicitly identified with the quality of the match between workers and the employers.

I employ a production based asset-pricing model drawing on the work of Merz and Yashiv (2005) and Cochrane (1991). Using a calibrated version of this model, I argue that there is a rising trend in the labour market friction in the US economy during the post-war period which accords well with the observed behaviour of the relative price of labour. The model predicts that the labour market friction represented by this Tobin’s Q measure shows a procyclical movement. This is indicative of the fact that firms compromise on the match quality in hiring new employees in a booming economy when the labour market is tight.

The plan of the paper is as follows. In the following section, I report some stylized facts about the time series behaviour of the relative price of labour in terms of capital. In section 3, a production-based asset-pricing model is laid out to show the precise relationship between the labour market friction and the value of a worker. Section 4 reports some calibration results. Section 5 concludes.

2. Capital and Labour Market Frictions: Some Stylized Facts

Chari et al. (2005) interpret the input market friction in terms of the relative price of the relevant input. Based on this measure, a decline in the relative price of investment
goods with respect to consumption goods means a decline in investment frictions. In Figure 1, I plot the ratio of US producer price index of finished capital goods to the consumer price index. Following the oil shock in the early 70s, there is a steady decline in this relative price of investment goods, which reconfirms the decrease in capital market frictions in the 80s.

Motivated by this price-based measure of input frictions, I calculate the relative price of labour with respect to capital for the US economy over the period 1948-2001 to arrive at a measure of labour market friction. This relative price is measured by the ratio of the annual index of compensation per worker to the producer price index of finished capital goods over the period 1948-2001 taking 1992 as the base year. Data for compensation per worker came from Hall (2001) who compiled these data from Bureau of Labour Statistics (BLS). The producer price index of finished capital goods came from the Federal Reserve St Louis database. Figure 2 plots the series. The relative price of a worker shows a steady increase except for the period of the oil shocks during 1973-74 when all producer prices increased.

Figure 3 plots the cyclical components of GDP and the relative price of worker. The cyclical component is measured by the percent deviation from the Hodrick-Prescott trend. The value of worker is procyclical. The correlation coefficient between the cyclical components is 0.50.

In the rest of the paper, I will argue that this procyclical trend in the relative value of labour with respect to capital is driven by a decrease in the average quality of the match between workers and the employers during an expansion. As the labour market becomes tighter in an expansionary phase of the cycle, firms start compromising on the quality of the match while recruiting. This makes already employed workers more valuable to the

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4 This procyclical movement in the value of the worker is consistent with the stylized fact that the real wage is procyclical in nature. The correlation coefficient between the cyclical component of the real wage (deflated by CPI) and the GDP cycle is 0.47.
firm. Based on this analysis, I will argue that the relative value of worker with respect to tangible capital is a reasonable measure of labour market friction as opposed to unemployment-vacancy ratio. To make this point transparent, in the next section, I focus on the production sector of the economy and develop a simple asset-pricing model.

3. The Model

I propose a production-based asset-pricing model, which builds on Merz and Yashiv (2005). The production sector consists of identical firms sharing the same production and investment technology facing a market wage rate, $w_t$ whose time path is exogenously specified. The timeline is as follows. At the start of date $t$, the firm observes a TFP shock $\varepsilon_t$ and produces output with the predetermined tangible capital $K_t$ and the human resources $N_t$ using the following Cobb-Douglas production function:

$$Y_t = \varepsilon_t K_t^\alpha N_t^{1-\alpha}$$  \hspace{1cm} (1)

where $\alpha$ is the capital share in output. The firm then disburses the existing employees a real wage of $w_t$. Finally it undertakes two types of investment decisions: investment in tangible capital $I_t$ and posting of new vacancy, $V_t$. The cost of posting new vacancy, $X_t$ is proportional to the number of posting as follows:

$$X_t = aV_t; \quad \text{with } a > 0$$  \hspace{1cm} (2)

Investment in tangible capital augments firm’s the physical capital following a standard linear depreciation rule:

$$K_{t+1} = (1-\delta)K_t + I_t$$  \hspace{1cm} (3)

where $\delta$ is the constant rate of depreciation of physical capital.

Regarding the latter investment, I follow Merz and Yashiv (2005), to postulate the following law of motion for the employees:

$$N_{t+1} = (1-\psi)N_t + q_tV_t$$  \hspace{1cm} (4)

where $\psi \in (0,1)$ is an exogenous job destruction rate, and $q_t$ is the average match quality between the workers and the employers. One may think of this law of motion as a

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5 Merz and Yashiv (2005) use a production based asset-pricing model of the type pioneered by Cochrane (1991). Their innovation is to show that the market value of a firm can be decomposed into the value of capital and the value of labour.
dynamic Beveridge curve in an employment-vacancy plane.\textsuperscript{6} The higher the $q_t$, the lesser the friction in the labour market which means that the increase in employment will be higher for a given number of vacancies making investment in human capital a cheaper option to the firm compared to physical capital. As we will see later that $q_t$ is endogenous in this model and determined by the firm’s valuation of a worker, which in turn depends on economic fundamentals.

The representative firm facing a constant discount factor $\rho$ solves the following problem\textsuperscript{7}:

\[
\text{Max } E_0 \left[ \sum_{t=0}^{\infty} \rho^t \{e_t K_t^\alpha N_t^{1-\alpha} - w_t N_t - V_t - I_t \} \right] \quad (P)
\]

s.t. \hspace{1cm} (1) through (4), given $K_0, N_0$.

The TFP shock $\epsilon_t$ is specified as a geometric random walk as follows:\textsuperscript{8}

\[
\ln e_{t+1} = \ln e_t + \xi_{t+1} \quad (5)
\]

where $\xi_{t+1} \sim N(0, \sigma^2)$

The first order conditions with respect to $I$ and $X$ are as follows:

\[
I: \quad 1 = \rho E_t \left[ e_{t+1} k_{t+1}^{\epsilon-1} + 1 - \delta \right] \quad (6)
\]

\[
V: \quad aq_t^{-1} = \rho E_t \left[ e_{t+1} (1 - \alpha) k_{t+1}^\alpha - w_{t+1} + (1 - \psi) aq_{t+1}^{-1} \right] \quad (7)
\]

\textsuperscript{6} To see it clearly, normalize the labour force at unity (ignore population growth). Then (3) can be rewritten in an unemployment-vacancy plane as: $U_{t+1} = \psi + (1 - \psi) U_t - q_t V_t$ where $U_t$ defined as 1-$N_t$ is the rate of unemployment and $V_t$ is the vacancy rate. This is a familiar dynamic Beveridge curve used in the literature (see for example, Nickell et al, 2001).

\textsuperscript{7} I ignore any convex adjustment cost in this benchmark model. There is, however, some built in adjustment cost of shifting resources from tangible to intangible capital. The firm incurs a relative price of $1/q_t$ to switch from tangible to intangible investment.

\textsuperscript{8} According to Prescott (1986) US TFP is a near random walk process while I assume that it is an exact random walk. Benaerjee and Magnus (2001) show that the forecast sensitivity due to difference stationary specification when the process is truly trend stationary is of first order insignificance. See also Banerjee and Basu (2001) for a related paper. Moreover, I also performed a unit root test for the logarithm of the TFP series used in the following section. One cannot reject the null of a unit root.
where $k_t$ is the capital/employment ratio at date $t$. Given the random walk nature of the TFP shock, it is straightforward to verify that the capital-employment ratio is:

$$k_{t+1} = \left[ \frac{\alpha \mu_t \varepsilon_{t+1}}{1 - \rho (1 - \delta)} \right]^{1-\alpha}$$  \hspace{1cm} (8)

where

$$\mu_t = \exp\left( \frac{\sigma^2}{2} \right)$$  \hspace{1cm} (9)

The first order conditions (6) and (7) can be rewritten in the following valuation equation form:

$I: K_{t+1} = \rho E_t \left[ CF_t^k + K_{t+2} \right]$  \hspace{1cm} (10)

$X: \frac{aN_{t+1}}{q_t} = \rho \left[ CF_t^n + \frac{aN_{t+2}}{q_{t+1}} \right]$  \hspace{1cm} (11)

where $CF_t^k = \varepsilon_t \alpha k_t^{\alpha-1} K_t - I_t$ and $CF_t^n = \varepsilon_t (1 - \alpha) k_t^{\alpha} N_t - w_t N_t - V_t N_t$.

Using (10) and (11) one can have the following value decomposition for the firm:

$$V_t = V_t^K + V_t^N$$  \hspace{1cm} (12)

where

$$V_t^K = K_{t+1}$$  \hspace{1cm} (13)

$$V_t^N = \frac{N_{t+1}}{q_t}$$  \hspace{1cm} (14)

The Tobin’s $Q$ of capital is unity while the Tobin’s $Q$ of a worker is the inverse of the average match quality $q_t$. This match quality variable $q_t$ drives a wedge between the Tobin’s $Q$ of capital and the Tobin’s $Q$ of labour. The relative value of a worker is defined as the Tobin’s $Q$ of a worker to the Tobin’s $Q$ of tangible capital. This relative
value is the inverse of the match quality $q_t$. A higher relative value of a worker thus reflects a lower match quality or a greater degree of labour market friction.$^9$

Define $q_t^* = a q_{t}^{-1}$. Using (4) and (7), one can write the following valuation equation for a worker:

$$q_t^* - 1 = \rho (1 - \alpha) e_t^{1 - \alpha} \left[ \frac{\alpha \rho}{1 - \rho (1 - \delta)} \right]^{\alpha} - E_t w_{t+1} \alpha + (1 - \psi) E_t q_{t+1}^* \] (15)$$

This valuation equation is just like a standard asset pricing equation. The worker is valued as an intangible asset to the firm. The Tobin’s Q of an installed worker is typically the expected present value of cash flows or surplus arising from his/her continued employment. This cash flow is the difference between worker’s productivity the real wage.

**Specification of the Process for Wages**

There are two alternative views of the real wage story: (i) sticky wage version, (ii) flexible wage version. Hall (2005) provides a comprehensive survey of this debate and arrives at a synthesis. As far as the US labour market is concerned, the punchline of this debate boils down to the link between real wage and productivity. To nest these alternative views of the real wage formation, I posit the following process for real wage:

$$w_t = \Omega [MPL_t]^{\theta} \] (16)$$

where the parameter $\theta \in (0,1)$ captures the elasticity of real wage with respect to the contemporaneous marginal product of labour and $\Omega$ is a scale parameter. A zero value of $\theta$ means that the real wage is unresponsive to change in labour productivity.

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$^9$ To see why the relative price of physical capital is $q_t$, note from (2) and (3) that the firm has to invest $1/q_t$ to augment the number of employees by one unit.
Solution for the Tobin’s Q of Worker

The key equation is (15) which involves the Tobin’s Q of the worker. Using the method of undetermined coefficient, one arrives at the following solution for the worker’s Tobin’s Q:

\[ q_t = \frac{A}{1 - \rho(1 - \psi)\mu_3} e^{\frac{1}{2(1 - \alpha)}} - \frac{B}{1 - \rho(1 - \psi)\mu_2} e^{\frac{\theta}{2(1 - \alpha)}} \]  \hspace{1cm} (17)

where

\[ A = \rho(1 - \alpha)\mu_1^{1/(1 - \alpha)} \left[ \frac{\alpha \rho}{1 - \rho(1 - \delta)} \right]^{1/(1 - \alpha)} \]  \hspace{1cm} (18)

\[ B = \rho \Omega(1 - \alpha) \left[ \frac{\alpha \rho \mu_1}{1 - \rho(1 - \delta)} \right]^{\alpha \theta / (1 - \alpha)} \mu_2 \]  \hspace{1cm} (19)

\[ \mu_2 = \exp \left( \frac{\sigma^2 \theta^2}{2(1 - \alpha)^2} \right) \]  \hspace{1cm} (20)

\[ \mu_3 = \exp \left( \frac{\sigma^2}{2(1 - \alpha)^2} \right) \]  \hspace{1cm} (21)

The appendix outlines the derivation of (17). The Tobin’s Q of a worker is basically driven by the TFP. Whether a positive TFP shock increases or decreases the Tobin’s Q depends on the how the TFP impacts the revenue and cost of the firm. If revenue increases more than the cost, the currently employed worker will be valued more by the firm. Another way to look at this is that a higher valuation attached to the currently employed worker means a higher demand for labour in a tighter labour market. The equilibrium match quality \( q_t \) must be lower in a tighter labour market to make the employed worker more worthwhile.
4. Calibration

Parameter Values

There are five parameters: $\alpha$, $\delta$, $\rho$, $\psi$, $\sigma^2$. Following Prescott (1986) I set the benchmark values, $\alpha = 0.36$, and $\delta = 0.1$ (annual data), $\rho = 0.96$ and $\sigma^2$ is fixed at 0.00763. There is no published estimate of the parameter $\psi$. The closest one is the average job separation rate of 3% in the US economy over the period 1946-2001 found in Hall (2001). The remaining parameters are $\theta$ and $\Omega$ in (16). These parameters were identified at values equal to .62 and 1 respectively by running a loglinear regression of real wage index on a moving average of the TFP indices.\(^{10}\)

Data

I use the annual manufacturing multifactor productivity index as a proxy for the overall TFP of the US economy. The data came from the Bureau of Labour Statistics. The real wage series was constructed by deflating the compensation per worker by the CPI. The data series again came from Hall (2001). The sample period ranges from 1949 to 2001.

Trend and Cyclicl Components of the Labour Market Frictions

Using the baseline parameter values and the observed series for the TFP, I next compute the series for the Tobin’s $Q$ of worker based on equation (17). Figure 3 plots the model and actual Tobin’s $Q$ of a worker over the entire sample period. The actual $Q$ is the same series reported in Figure 1. The model series is normalized at unity for the base year 1992 to make it comparable to the actual relative price of labour. The model performs really well in tracking down the trend in the Tobin’s $Q$ of the worker.

\(^{10}\)Using the Cobb-Douglas production function (1) and the TFP process (5), verify that (16) reduces to:

$$
\ln w_t = \kappa + \theta \left[ \ln \epsilon_t + \frac{\alpha}{1-\alpha} \ln \epsilon_{t-1} \right]
$$

where $\kappa = \ln \Omega + (\alpha / (1-\alpha) \ln \left[ \frac{\alpha \rho \mu_1}{(1- \rho)(1-\delta)} \right]$. Setting $\alpha = 0.36$, I obtain an estimate of $\theta$ equal to .62, which was significant at 1% level. The constant coefficient was found statistically insignificant. Given that the structural parameters $\alpha$, $\rho$, and $\delta$ cannot be zero, I take the insignificant $\kappa$ as an evidence that $\Omega$ is close to unity. The $R^2$ for this real wage regression was .96. This real wage regression simply reconfirms the procyclical behaviour of the US real wage of worker.
Figure 5 plots the cyclical components of the model’s Tobin’s Q of a worker and the cyclical component of GDP. The model’s Tobin’s Q show procyclical fluctuations. The correlation coefficient is 0.82. This is reconfirmed in Table 1 which presents the cross correlation between the cyclical components of GDP, model Tobin’s Q and actual Tobin’s Q.

This quantitative exercise based on the model’s Tobin’s Q equation suggests that the average quality of the match deteriorates during an expansion when unemployment is lower. This match quality is determined in equilibrium by firms’ valuation of the installed worker, which is the Tobin’s Q of the worker. The intuition for a higher Tobin’s Q of a worker during an expansionary phase goes as follows. A positive TFP shock at date \( t \) triggers an increase in capital-employment ratio \( (k_{t+1}) \) in the following period (see equation 8). Due to the constant returns to scale property of the production function, a higher \( k_{t+1} \) lowers the marginal product of capital at date \( t+1 \), and raises the marginal product of a worker. Thus a higher TFP realization today basically signals a higher prospective relative return to human capital with respect to physical capital. In response to this, firms switch gear from physical investment to investment in human capital, which means posting more vacancy (higher \( V_t \)). This increased demand for workers raises the value of the worker meaning lower match quality \( q_t \). Thus in equilibrium a lower unemployment coexists with a lower match quality. Basically firms compromise on the quality of the match during a boom when the labour market is tight.

An Estimate of the Match Probability

In this section, I estimate the match probability \( q_t \) based on the model’s Tobin’s Q of the worker. I normalize this probability by setting a value of the job posting cost parameter \( a \) in equation (2) such that maximum value of \( q_t \) equals unity. This means \( a \) equals .53.

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11 Chari, Kehoe and McGrattan (2005) measure labour frictions, which they call labour wedge, in terms of an implicit tax on wages. Their labour wedge also covaries positively with output although for reasons fundamentally different from my model. The labour wedge in their model is equivalent to stickiness of wages while in my model the labour wedge is equivalent to a matching friction.
Since this matching probability is a stationary variable, I detrend the model Tobin’s $Q$ by taking out a log-linear trend component out of it. Figure 6 plots this matching probability and the detrended TFP series.\textsuperscript{12}

One may note that the matching probability declined during the 70s and then it revived in the 80s while TFP shows the opposing pattern. The matching probability increased during the 80s when there was productivity slowdown. These results reinforce my hypothesis that the quality of the match shows a countercyclical pattern.

Note that $q_t$ determines the shift of the Beveridge curve (see footnote 5). Our results thus also accord well with Valletta (2005) who finds that the US Beveridge curve shifted out during the 70s and then shifted back in during the 80s. In the present setting, the slope of the Beveridge curve is endogenously driven by the TFP. My framework shows the direct link between the TFP and the matching probability which is inversely related to labour market frictions. The reversal of the match probability labour market frictions is basically driven by the reversal in the TFP movements in the US economy in the 80s.

*General equilibrium*

In this paper, I have posed the issue of labour market friction and the related Tobin’s $Q$ of worker from a partial equilibrium angle. I only look at the firm’s side of the problem. In a general equilibrium, the average quality of the match (the inverse of the Tobin’s $Q$ of the worker) is determined by the interaction between firm’s search for the right employee and the household’s search for the right match. In the appendix, I outline a general equilibrium version of the model following Merz (1995) and argue that the procyclical behavior of the labour market friction is theoretically robust. The search friction is modeled as a social planning problem where the planner internalizes both advertisement cost and search cost. A positive TFP shock triggers a wealth effect, which means more vacancy posting by the firms and more search efforts by the households. Due to convexity of the search cost function, this means a lower match quality between workers and the employees.

\textsuperscript{12} The TFP series is also normalized at unity taking 1992 as the base year.
5. Conclusion

There is no consensus whether the labour market friction has increased or decreased in the US economy over the last few decades. The traditional literature identifies labour market friction in terms of an upward shift of the Beveridge curve. In this paper, I question this interpretation of the labour market friction. I take an asset pricing approach to understand the friction. Higher friction means a lower match quality, which implies a higher relative value of a worker with respect to capital. Viewed from this perspective, I find that the labour market friction has a procyclical pattern. The increased friction is reflected by a lower match quality during an expansion. This basically indicates that firms find it difficult to have the right match in an expansionary economy with a tighter labour market. Contrary to conventional wisdom, there is no conflict between a higher labour market friction and lower unemployment.
Appendix A

Derivation of equation 15

Conjecture a solution

\[ q_t^{-1} = \lambda_1 e_t^{1/(1-\alpha)} - \lambda_2 \theta_t^{1/(1-\alpha)} \quad (A.1) \]

Upon substitution in (4) and using the geometric lognormal random property of the TFP process \( \{\varepsilon_t\} \) one obtains:

\[ \lambda_1 e_t^{1/(1-\alpha)} - \lambda_2 \theta_t^{1/(1-\alpha)} = A e_t^{1/(1-\alpha)} - B \theta_t^{1/(1-\alpha)} + \rho(1-\psi)\lambda_1 \mu_3 e_t^{1/(1-\alpha)} - \rho(1-\psi)\mu_2 \lambda_2 \theta_t^{1/(1-\alpha)} \quad (A.2) \]

Using the method of undetermined coefficients it immediately follows that

\[ \lambda_1 = \frac{A}{1 - \rho \mu_3 (1-\psi)} \]

and

\[ \lambda_2 = \frac{B}{1 - \rho \mu_2 (1-\psi)} \]

which proves (15). //

Appendix B

Tobin’s Q of a Worker in General Equilibrium

I consider a social planning problem based on Merz (1995) as follows. The social planner chooses consumption \( (C_t) \), employment \( (N_t) \), unemployment \( (1-N_t) \), search intensity \( (S_t) \) and job vacancies \( (V_t) \) posted per firm to solve the following maximization problem:

\[ E_0 \sum_{t=0}^{\infty} \rho^t [U(C_t) - W(N_t)] \]

s.t.

\[ C_t + I_t + c(S_t)(1-N_t) + aV_t = Y_t : \text{Resource constraint} \quad (A.1) \]
\[
Y_t = \varepsilon_t F(K_t, N_t) \quad \text{: Production function} \quad \text{(A.2)}
\]

\[
K_{t+1} = (1 - \delta)K_t + I_t \quad \text{: Law of motion of physical capital} \quad \text{(A.3)}
\]

\[
N_{t+1} = (1 - \psi)N_t + M_t \quad \text{: Law of Motion of Employment} \quad \text{(A.4)}
\]

\[
M_t = V_t^{1-\lambda}[S_t(1 - N_t)]^{\lambda}, \quad 0 < \lambda < 1 \quad \text{: Matching Function} \quad \text{(A.5)}
\]

\[
K_0, N_0 = \text{given} \quad \text{(A.6)}
\]

All the notations are the same as before except \(S_t\) and \(M_t\) which stand for household’s search intensity and the extent of matching between workers and firms. The cost of worker’s search is represented by the function \(c(S_t)\) which satisfies the properties that \(c'(S_t) > 0\) and \(c''(S_t) > 0\). The social planner internalizes both these costs which explains the resource constraint (A.1) facing the planner. Equation (A.5) represents a standard Pissarides (1985) type matching technology, which means that the quality of the match between employers and the workers depends on the interaction between search intensities of firms and workers. The social planner instantaneous felicity function represents household’s utility function of consumption, \(U(C_t)\), and disutility function of work, \(V(N_t)\).

Our central concern here is about the Tobin’s Q of the worker which is the inverse of the search quality \(q_t\). At the optimum, it can be rewritten as:

\[
\frac{1}{q_t} = \frac{V_t}{M_t} \quad \text{(A.7)}
\]

It is straightforward to verify that a key first order condition must hold equating the ratio of marginal products of search and advertisements to the ratio of the corresponding marginal costs. In other words, at the optimum we must have:

\[
\frac{\partial M_t}{\partial S_t} / \frac{\partial S_t}{\partial V_t} = \frac{c'(S_t)(1 - N_t)}{a} \quad \text{(A.8)}
\]
using (A.5), (A.6) and (A.7), it is straightforward to verify that

\[
\frac{1}{q_t} = \left[ \frac{(1-\lambda)c'(S_t)}{\lambda a} \right]^\lambda
\]

(A.9)

Given the convexity of the search cost function, the Tobin’s Q of the worker positively correlates with worker’s search intensity. Following Merz (1995), one can argue that a positive technology shock via a positive resource wealth effect creates congestion by raising the search intensity \((S_t)\) of workers. This raises the Tobin’s Q of worker in a general equilibrium.
Figure 1: Relative Price of Investment Goods with Respect to Consumption Goods
Figure 2: Relative Price of a Worker in terms of Capital Goods

Figure 3: Plot of the Cyclical Components of the Relative Price of Worker and the GDP
Figure 4: Value of Worker: Model vs Actual

Figure 5: Cyclical Components of Model Tobin's Q of Worker and GDP
Table 1: Correlation between the Cyclical Components of GDP, Model’s Value of Worker, Actual Value of Worker

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<th>Actual q(^{t})</th>
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