The dynamics of consumers’ expenditure:
the UK consumption ECM redux.

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Abstract

Simple intertemporal consumption theory implies that non-durable consumption is a random walk, but that consumption cointegrates with income and wealth. By the Granger representation theorem, there must be a (vector) error correction mechanism ((V)ECM) representation of the data; but from the theory, the equilibrating ECM cannot be in consumption. Instead, even with generalisations such as habit persistence, this equilibration should take place via income or wealth. Furthermore, unless the relative price of durables and non-durables is constant, the relative price needs to be taken into account in modelling. In this paper, the short-run dynamics and long-run relationship of and between non-durable consumption, labour income, wealth and the relative price of durable goods are examined. A cointegrating relationship is found to exist. Estimating VECMs, it is found that the adjustment towards the long-run common trend does indeed occur partly via changes in wealth, consistent with forward looking behaviour on the part of agents. The result implies that consumption will predict asset returns, and this is confirmed by a regression of excess equity returns on the lagged disequilibrium term. A decomposition of shocks hitting the system reveals that between 30 and 90% of fluctuations in non-human wealth are transitory. Even if the lower figure applies, this means a substantial part of short-term fluctuations in wealth are decoupled from permanent consumption.

Key words: Error correction, consumption, dynamics

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Summary

Since the seminal paper of Davidson, Hendry, Srba and Yeo (DHSY) that helped to popularise the error correction mechanism (ECM), innumerable consumption ECMs have been estimated. In the United Kingdom in particular, research has concentrated on the variables explaining consumption in the long- and short-runs. With single equation consumption ECMs, the implication is that deviations from the common trend in consumption, income and wealth are corrected only through consumption. This is despite the fact that in the simplest intertemporal models of household consumption, there should be no consumption ECM. Instead, equilibration should operate via the income or wealth drivers. The former result does not hold with all extensions, for example to habit persistence, but the latter does. This issue, introduced by John Campbell in the 1980s, has been revived with a number of papers on US data by Sydney Ludvigson and her co-authors. In those papers, deviations from common trends tend to be corrected via changes in wealth. In this setup, deviations from the long-run relationship appear to lead to changes in income or wealth. But the causality here is from expected future events to current consumption and saving decisions; it is not that (eg) higher consumption causes higher income growth through, say, some aggregate demand mechanism. In this paper we examine the evidence for the United Kingdom.

We pay some attention to the treatment of non-durable consumption. We construct a simple model of the consumption of both durable and non-durable goods. We construct appropriately defined data, and the short-run dynamics and long-run relationship of and between non-durable consumption, labour income, wealth and the relative price of durable goods are examined. One cointegrating relationship is found to exist. The relative price of durables to non durables may play a role in this process. Estimating vector error correction mechanisms (VECMs), we find that adjustment towards the long-run common trend does indeed occur partly via changes in wealth. This is consistent with forward looking behaviour on the part of agents. It also means consumption can predict asset returns. This result is confirmed by a regression of excess returns to equities on the disequilibrium term from the long-run relationship.

We also perform a decomposition of shocks hitting the system into temporary and permanent components. Almost all of the variation in the consumption and income process can be ascribed to permanent shocks. Depending on the treatment of the relative price of durables, we find that between 30 and 90% of fluctuations in non-human wealth are transitory. Even if the lower figure
applies, this means a substantial part of short-term fluctuations in wealth are decoupled from permanent consumption.

Our analysis implies that we can welcome the return of the UK consumption ECM, in the context of a complete VECM analysis of the system explaining the relationship between consumption and permanent income.
1 Introduction

Following the publication of Davidson et al (1978), many equations embodying consumption error correction mechanisms (ECMs) have been estimated. An alternative is the Euler equation pioneered by Hall (1978). One interpretation of the two approaches is that Euler equations test theories, while ECMs are designed to answer different questions, primarily about the role of different variables in the ‘consumption function’, and to provide forecasts. The difference is that an Euler equation embodies the predictions for the consumption path of the particular maximisation problem the investigator specifies; while, although this is not always spelt out, a ‘consumption function’ is derived by taking the intertemporal budget constraint, and substituting the Euler equation and specific assumptions about the stochastic driving processes into the expression to yield a solved-out relationship between consumption, some exogenous variables, and lagged endogenous variables (such as wealth). So in the UK research has largely centered around the variables which ‘explain’ consumption, although it became less popular in the 1980s, largely because of its vulnerability to the Lucas critique. (3)

But since Campbell (1987) it had been clear that a long-run relationship between consumption, income and wealth can be derived solely from the intertemporal budget constraint. While a consumption function may be specified if we are prepared to assume enough structure, about both preferences and the stochastic processes generating the variables, it may be convenient to think about estimated consumption ECMs in this way. A series of papers by Sydney Ludvigson and her co-authors (1999, 2000, 2001), inspired by Campbell (1987, 1993), have re-examined the information in the long-run consumption relationship, and ask to what extent consumption performs the correction when deviations from the common trend in consumption, income and wealth occur in US data. The answer is, not very much. These papers conclude that disequilibria tend to be corrected via changes in wealth, and not consumption. For those used to thinking in terms of a solved-out consumption function this conclusion may not immediately be easy to interpret, but the insight that consumption reacts to expected future events transforms our understanding of the relationship. ‘Causality’ is often associated with the notion that events in the past have an effect on subsequent outcomes. But in this case, it is what is expected to occur in the

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(1) This paper, named ‘Daisy’ after the initials (DHSY) of the authors, was influential in introducing the error correction mechanism to economics.
(2) Muellbauer and Lattimore (1995) provide a review of the theoretical and empirical literature on consumption.
(3) Church, Smith and Wallis (1994).
future that impacts on current decisions, and the ‘adjustment’ process has to be interpreted within this framework.\(^{(4)}\) In this paper we examine whether this is evident in the United Kingdom data.

We also consider the role of the relative price of durables. Non-durable, rather than total, consumption is commonly used in the empirical literature, because of its theoretical appeal. Consumers derive utility from the service flows that goods provide, and not from expenditure. Therefore the correct way to model consumption is first to calculate the service flow that goods yield, and then to use such measures to test the theory. For non-durable goods and services, expenditure equals the service flow rendered over a chosen time period. But durable expenditure cannot be a good proxy for its service flow. As a result, non-durables and services are typically used to test consumption theories.\(^{(5)}\) However, it was observed as early as Blinder and Deaton (1985) and Campbell (1987) that the share of durable goods in total consumption had been increasing, and this can be important in some exercises, including our own.\(^{(6)}\) As Chart 1 shows, for the United Kingdom such constancy in the real ratio is not observed, although it is observed in the nominal ratio (Chart 2), which would be consistent with Cobb-Douglas preferences over durable and non-durable consumption.

The plan of the paper follows. In Section 2 we briefly relate the single equation to the system VECM. In Section 3 we set out the basic relationships implied by intertemporal optimisation that motivate our work. In Section 4 we describe the data needed to estimate a theoretically sensible model. Section 5 presents our results. In Section 6 we decompose shocks into their transitory and permanent components. In Section 7 we assess the ability of our model to forecast asset returns as a cross-check on the interpretation of our results, and the final section concludes.

\(^{(4)}\) One can develop a taxonomy of empirical relationships ‘explaining’ consumption. In addition to the Euler equation estimating structural parameters, the budget constraint approach we follow that places minimal restrictions on the data and the solved-out reduced-form ‘consumption function’, there is an approach that estimates both the Euler equation and the intertemporal dynamics. An excellent example is Fuhrer (2000). Each type has different purposes and functions. Our model may be seen partly as a test of some minimal theory, and partly as an exploration of the informational content of consumption. As will become clear, our model would not be much use in a forecasting context, unless one was interested in medium to long horizon forecasts of asset returns.

\(^{(5)}\) Campbell (1987), Campbell and Deaton (1989), Flavin (1981) for early tests of the permanent income hypothesis using non-durable data for the US; Attfield, Demery and Duck (1990) for an equivalent test for the UK.

\(^{(6)}\) Campbell’s (1987) footnote 15, page 1260 states ‘Blinder and Deaton report that the share of non-durables and services in measure total consumption expenditure has displayed a secular decline over the sample period. This casts some doubt on the practice of using non-durables and services consumption as a proxy for the total; nevertheless I follow this tradition and estimate a constant scale factor.’
Chart 1: Ratio of total to non-durable consumption

![Chart 1](image1)

Chart 2: Ratio of total to non-durable consumption: nominal

![Chart 2](image2)
2 ECMs versus a VECM

In the following section we need the notion of a VECM, so we briefly review the notions of an ECM and a VECM here. Consider a vector, \( x_t \), of \( I(1) \) variables. In our case

\[
\begin{align*}
\begin{bmatrix}
C_t^n, y_t, a_t, p_t^d
\end{bmatrix}
\end{align*}
\]

where \( C^n_t \) is non-durable consumption, \( Y_t \) is labour income (or more strictly, non-asset income), \( A_t \) is the stock of assets, \( P_t^d \) is the relative price of durables to non-durables, and where lower-case letters denotes the log of a variable, so that \( z_t = \log Z_t \). Assuming a unique cointegrating relationship exists, it is possible to estimate the following VECM:

\[
\Delta x_t = \nu + \alpha^\prime \Delta x_{t-1} + \Gamma(L)\Delta x_{t-1} + C(L)z_{t-1} + e_t
\]

where \( \nu \) and \( \alpha \equiv (\alpha_c, \alpha_y, \alpha_a, \alpha_p)^\prime \) are \((4 \times 1)\) vectors, \( z_{t-1} \) is a \((n \times 1)\) vector of (weakly) exogenous \( I(0) \) variables, \( \Gamma(L) \) and \( C(L) \) are finite order distributed lag operators and \( \beta \equiv (1, -\beta_y, \beta_a, \beta_p)^\prime \) is the \((4 \times 1)\) vector of cointegrating coefficients. This yields sensible estimates as \( \beta^\prime \Delta x_{t-1} \) is \( I(0) \); \( \beta^\prime \) is a matrix of long run coefficients such that \( \beta^\prime \Delta x_{t-1} \) represents (in our specific case) a single cointegration relationship. \( \beta^\prime \Delta x_{t-1} \) may also be interpreted as the equilibrium error from the previous time period, and \( \alpha \) is a vector which determines how fast adjustment occurs to restore the equilibrium error made the previous period. For stability, at least one of the coefficients in \( \alpha \) must be different from zero, or there would be no adjustment towards the long-run, and we would be left with the estimation of a vector autoregressive process in first differences, which will have no long-run solution. It may well be of economic interest, as in our case, to determine which of the components in \( \alpha \) are different from zero, as that gives us the variables that participate in the restoration of equilibrium. So when a single error correction equation for consumption is estimated, then the assumption that \( \alpha \equiv (\alpha_c, 0, 0, 0)^\prime \) is made. The implication of this assumption is that \( \alpha_c \) represents the speed at which consumption must change to restore the equilibrium relationship between consumption, income, wealth and the relative price. If any of the terms \( \alpha_y, \alpha_a, \alpha_p \) are different from zero, then equilibration will occur not only through consumption (if \( \alpha_c \neq 0 \)) but also through other variables. As a result, \( \alpha_c \) is not a sufficient statistic to describe how quickly consumption adjusts to equilibrium. Moreover, there are also important statistical implications. If \( \alpha_t = 0 \) then the variable \( x_t \) is weakly exogenous with respect to the long-run parameters. Operationally, this is crucial. If \( x_t \) is not weakly exogenous, consistent estimation of the long-run parameters requires one row of the VECM to determine \( \Delta x_t \). In other

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(7) Lettau, Ludvingson and Barczi (2001) have a very clear exposition of when single equation ECMs are appropriate, and when they will mislead.

(8) That is, they become stationary if they are differenced once, implying that \( \Delta x_t = [\Delta c_t^n, \Delta y_t^d, \Delta a_t, \Delta p_t^d] \) is stationary.
words, only if wealth, income and the relative price are weakly exogenous can a single equation ECM be consistently estimated for consumption.

3 Consumption growth, labour income and asset returns

In this section we explain our approach. The main analysis is set out in previous related work referred to below, so our exposition is relatively brief. Most of our analysis depends only on the intertemporal budget constraint, with a little more structure to allow for durables.

3.1 Implications of the intertemporal budget constraint

We take as our starting point the accumulation equation for aggregate (human and asset) wealth $W_t$ using total consumption $C_t$:

$$W_{t+1} = (1 + r_{t+1})(W_t - C_t) \tag{2}$$

where $r_t$ is the rate of return on (broadly defined) wealth. Total consumption is given by $C_t = C^n_t + P^d_t C^d_t$, where $C^d_t$ is durable consumption. Campbell and Mankiw (1989) show that taking logs of (2), a first order Taylor expansion yields

$$w_{t+1} - w_t \approx r_{t+1} + k + (1 - 1/v) (c_t - w_t) \tag{3}$$

where lower case letters denote the log of the variable, $v = \frac{W-C}{W} < 1$ and $k = \log(v) - (1 - 1/v) \log(1 - v)$. Solving this equation forward to eliminate future wealth yields

$$c_t - w_t \approx \sum_{i=1}^{\infty} v^i (r_{t+i} - \Delta c_{t+i}) + \theta \tag{4}$$

where $\theta$ is a constant (a function of $v$ and $k$), and we require the condition

$$\lim_{t \to \infty} v^i (c_{t+i} - w_{t+i}) = 0,$$

which is easily satisfied. Substitution of the approximations

$$\log(C_t) = \log(C^n_t + P^d_t C^d_t) \approx \frac{C^n_t}{C} \log(C^n_t) + \left(\frac{P^d_t C^d_t}{C}\right) \left[\log(C^d_t) + \log(P^d_t)\right]$$

$$\approx \pi \log(C^n_t) + (1 - \pi) \left[\log(C^d_t) + \log(P^d_t)\right]$$

(9) Here and elsewhere, in common with the literature, we ignore issues of aggregation over heterogeneous agents. Attanasio and Weber (1993) present a devastating case against the aggregate Euler condition. However, in our case the results are driven almost entirely by the linearised budget constraint, which holds in aggregate in the absence of liquidity constraints.
and

\[ w_t \approx \omega a_t + (1 - \omega) h w_t \]

where \( \omega = \frac{1}{\pi} \), the share of non-labour wealth in total wealth, and \( \pi = \frac{c^\pi}{c} \), the share of non-durable consumption in the total, into (4) yields an expression in terms of durables, non-durables and the relative price of durables to non-durables,

\[
\frac{\pi}{c^\pi} + (1 - \pi) (c^d_t + p^d_t) - \omega a_t - (1 - \omega) h
\approx \sum_{i=1}^{\infty} v^i \left[ r_{i+t} - \pi \Delta c^\pi_{t+i} - (1 - \pi) (\Delta c^d_{t+i} + \Delta p^d_{t+i}) \right]
\]

where the constant is suppressed. This expression follows solely from the budget constraint. It tells us that there is a long-run relationship between consumption and wealth (the left hand side of the expression) which from the budget constraint equals a discounted sum of future returns and consumption growth. This holds \textit{ex post} in the data, but also \textit{ex ante} in expectation. Thus it is also true that

\[
\frac{\pi}{c^\pi} + (1 - \pi) (c^d_t + p^d_t) - \omega a_t - (1 - \omega) h
\approx \mathbb{E}_i \sum_{i=1}^{\infty} v^i \left[ r_{i+t} - \pi \Delta c^\pi_{t+i} - (1 - \pi) (\Delta c^d_{t+i} + \Delta p^d_{t+i}) \right]
\]

where \( \mathbb{E}_i \) is the expectation operator. After a little manipulation, and the use of some minimal theory, we can use this expression to explore the evolution of consumption.

Before doing so, we note that (5) contains an unobservable quantity, human wealth. To eliminate it, following Lettau and Ludvigson (2001), we assume labour income \( Y \) is non-stationary and human capital \( H \) can be described by

\[
h_t = \kappa + y_t + z_t
\]

where \( \kappa \) is a constant and \( z \) a stationary random variable. In particular, as in Campbell (1996), labour income \( Y \) is the ‘dividend’ on human capital \( H \), so

\[
r_{i+1}^h = (H_{t+1} + Y_{t+1})/H_t
\]
and a log-linear approximation implies that

\[ z_t = E_t \sum_{j=0}^{\infty} \psi_h (\Delta y_{t+j} - r_{t+j}) \]  

(9)

Using (7) in (5) and ignoring the constant \( \kappa \),

\[ \pi c^n_t + (1 - \pi) (c^d_t + p^d_t) - \omega a_t - (1 - \omega) y_t \]

\[ \approx E_t \sum_{i=1}^{\infty} \psi [r_{t+i} - \pi \Delta c^n_{t+i} - (1 - \pi) (\Delta c^d_{t+i} + \Delta p^d_{t+i})] + (1 - \omega) z_t. \]

(10)

3.2 Non-durables

As we have already observed, it is common to analyse non-durable consumption in structural models. This is not strictly necessary, as our analysis is based on the budget constraint, but use of non-durables helps focus the results. Although non-durable consumption is usually found not to follow a random walk, it is close: and in our case ‘durables’ encompass semi-durables.\(^{(10)}\) By contrast, as one would expect, durable expenditure has more complex dynamics. So choice of non-durable expenditure has the potential to make our results sharper. In addition, our results are directly comparable with previous work. But durable consumption has increased more than non-durable, and the relative price has fallen. To take account of this we introduce some structure beyond the intertemporal budget constraint. In the Appendix we make the simple points that intertemporal optimisation implies the ratio of marginal utilities between non-durable consumption and the durable consumption stock will be equal to an expression including the relative price, and that on the steady-state growth path non-durable consumption is proportional to the stock. To say more requires an explicit utility function. We assume there is a long-run linear relationship between durable consumption, non-durable consumption and the relative price,

\[ c^d = \phi_1 c^n - \phi_2 p^d. \]

(11)

Making this substitution for the level, using (11) to substitute for the differenced terms in durable consumption (as \( \Delta c^d = \phi_1 \Delta c^n - \phi_2 \Delta p^d \)) and again ignoring constants, we obtain the expression

\(^{(10)}\)See Attanasio (1999) for some discussion of the time series data for the UK and US.
that is the basis for the results presented below.

\[ c^n_t + \psi_1 p^d_t + \psi_2 a_t + \psi_3 y_t \]
\[ \approx E_t \sum_{i=1}^{\infty} v_1^i \left[ r_{t+i} - (\pi + \phi_1 (1 - \pi)) \Delta c^n_{t+i} - (1 - \pi)(1 - \phi_2) \Delta p^d_{t+i} \right] + \\
E_t \sum_{i=1}^{\infty} v_2^i \left[ \Delta y_{t+1+j} - r_{h,T+1+j} \right] \tag{12} \]

### 3.3 Interpretation and implications

The left-hand side of expression (12) gives the long-run relationship between non-durable consumption, assets, labour income and the relative price, the equivalent to the notion of ‘saving’ in Campbell (1987) or the consumption to broad wealth ratio in Campbell and Mankiw (1989) and Campbell (1993), and is the long-run relationship that we estimate. As we explain below, even with the minimal structure we have imposed there are important economic implications to be explored.

We have assumed, uncontroversially, that labour income is non-stationary. By definition, the expression on the left-hand side must (approximately) equal the right-hand side of the expression, driven by expected future returns, expected changes in labour income, planned consumption growth and expected changes in the relative price. To reiterate, this follows from the intertemporal budget constraint and only minimal structure from the theory explaining optimal choice of non-durables and durables. Consumption growth will be stationary (given the budget constraint, consumption cannot be an order of integration higher than income). If, again uncontroversially, \( p^d_t \) is assumed to be at most \( I(1) \) and the rates of return to be stationary, then as \( \Delta c^n_t \) and \( \Delta c^d_t \) are \( I(0) \), it follows that the right-hand side is itself a stationary object (for \( n_i < 1 \)). Thus the \( I(1) \) variables on the left-hand side form a stationary combination. Another way to express this is that \( (c^n_t, p^d_t, a_t, y_t) \) form a cointegrating relationship. The economic implication is that deviations from the long-run trend reflect, and are therefore able to predict, future returns to assets, future changes in labour income, lower (planned) future growth of non-durable consumption, or future increases in the relative price of durables. And this needs to be borne in mind when interpreting the statistical implication, namely, the existence of a VECM.
One can take (12) in several directions. If we assume particular preference structures and specify stochastic processes for the driving variables, we can in some cases obtain closed form solutions and use the Euler equation to substitute out for consumption growth. Then (12) can actually be directly estimated. An excellent example of such an exercise is Fuhrer (2000). This is demanding in terms of the identifying structure. A weaker approach is the traditional consumption ECM approach, which can be described in terms of the investigator implicitly substituting out the forward parts of the expression with a reduced form forecasting equation based on lagged information. But without imposing any further structure, (12) is telling us two things must hold. Firstly, the existence of cointegration implies a VECM exists in \( (c^a_t, p^d_t, a_t, y_t) \). It automatically follows that the long-run ‘disequilibrium’ error must ‘equilibrate’ \( \text{via} \) at least one of the four variables. But the economics underlying (12) make it clear that this equilibration follows from the forward looking nature of the problem. Furthermore, there may be stronger implications from the theory.

In basic permanent income hypothesis (PIH) models, marginal utility follows a martingale process, a result emphasised by Hall (1978). For example with a quadratic utility function and a rate of time preference equal to the rate of return on wealth, non-durable consumption and the durable stock follow random walks. Campbell’s (1987) insight was to observe that the theory offers stronger, overidentifying restrictions on the evolution of consumption and savings. In his model what he termed ‘savings’\(^{(11)}\) are the discounted (negative) sum of expected future changes in labour income. If labour income is expected to fall in the future, households will have higher savings. This is true in our model too, of course. Thus the process driving income has implications for the stochastic process driving savings, and this forms the basis for tests in Campbell (1987) and Campbell and Deaton (1989), and subsequently Quah (1990) and Falk and Lee (1990, 1998).\(^{(12)}\) The apparently odd feature of this is that although income and consumption (and in our case wealth) cointegrate, from the Euler equation we know consumption is a random walk. Thus there should be no consumption ECM. Yet if a cointegrating vector exists, then from the Granger representation theorem (Engle and Granger (1987)), we know an ECM also applies.

We can square this circle by first recalling that we are dealing with a system here, where the non-stationary series are consumption, income and wealth. Campbell (1987) observes that in the basic model the ECM relationship should move from the lagged disequilibrium term to income or wealth.

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\(^{(11)}\) Close to but not necessarily identical to consumption less resources or consumption less permanent income.

\(^{(12)}\) See Flavin 1993 for a careful look at how this approach works in a modified version of the PIH.
wealth. The interpretation of this is that if consumption deviates from the current equilibrium relationship it must be because of an expected change in a future consumption driver. Suppose households know that labour income is about to permanently fall. Then consumption will be low relative to the long-run level. This was Campbell’s insight. In his 1987 paper he emphasised the possibility that labour income may be expected to change; in Campbell and Mankiw (1989) and Campbell (1993) it is the return to wealth that varies; and we now add that the relative price of durables may also vary. Moreover, we are able to infer from the data what expectations agents are holding.\(^{(13)}\) All this follows from what are uncontroversial assumptions: that agents face an intertemporal budget constraint, and that they follow some form of PIH behaviour (essentially, intertemporal maximisation). Thus deviations from equilibrium potentially forecast income or wealth, and if so would be significant in the ECM relations.

Campbell used a model with the consumption martingale property so that there is no consumption ECM, but this is not crucial to the argument. It is easy to specify models where a consumption ECM does exist: for example, with habit persistence; rule-of-thumb satisfiers; or liquidity constrained households. With habit persistence,\(^{(14)}\) in the widely used model introduced by Abel (1990) utility is given by

\[
U_t = \frac{1}{(1 - \sigma)} \left( \frac{C_t}{X_t^\gamma} \right)^{1-\sigma}
\]

where \(X_t = \rho X_{t-1} + (1 - \rho X) C_{t-1}\) is the ‘reference consumption level’ and \(1 \geq \gamma \geq 0\) indexes the degree of habit persistence. In this model the Euler equation will include future consumption growth; a consumption ECM will exist. But the point remains that ECMs for income or wealth should also exist.

Thus to reiterate, in our empirical work we take a log approximation to the long-run relationship, standard in every respect except that we explicitly take account of the relative price of non-durables and durables. Specifically, we define a long-run relationship

\[
e_t^n + \psi_1 p_t^d + \psi_2 a_t + \psi_3 y_t = \epsilon_t.
\]

This combination of non-stationary variables equals a stationary expression, so \(\epsilon_t\) is the long-run or equilibrium error. Thus, not only do \(c_t^n, a_t, y_t, p_t^d\) cointegrate, but they are explicitly related to a stationary discounted sum of stationary future changes in themselves. This is the approach taken in this paper, and, as we shall see below, is consistent with the UK data.

\(^{(13)}\) Campbell’s (1987) superior information test: households have better information than the econometrician about future changes in labour income, so saving allows us to ‘view’ those expected movements. \(^{(14)}\) See Carroll, Overland and Weil (1997) or Fuhrer and Klein (1998) for examples.
4 Data requirements

The data produced by the Office of National Statistics (ONS) require adjustments to the consumption and labour income data to obtain series that can be used in our theoretical structure. The data requirements have been discussed at length elsewhere; Blinder and Deaton (1985) and Ludvigson and Steindel (1999) comment on the necessary adjustments for US data. Attfield et al (1990) discuss the equivalent adjustments for the UK.

The ONS divide consumption expenditure into durables, semi-durables, non-durables and services. Our definition of non-durable consumption is defined as total consumption minus consumption of durables and semi-durables; we deflate by the correspondingly defined deflator.\(^{(15)}\) We also construct an alternative definition of non-durable consumption, which we term non-durable clothing and footwear, defined as total consumption minus durable consumption minus the consumption of clothing and footwear, to aid comparison with the results obtained by Ludvigson and her co-authors in the US. Note that our preferred measure of non-durable consumption excludes all semi-durables, not just clothing and footwear.\(^{(16)}\) All data are quarterly, seasonally adjusted and real. The source of all the consumption data is the ONS (Consumer Trends). Chart 3 plots the data.\(^{(17)}\)

The ONS does not produce a direct measure of after-tax labour income: it only produces measures of after tax total income.\(^{(18)}\) Our preferred measure of labour income is given by

\[
Y = (Y^T - Y^d - T)/P
\]

where \(Y\) is our measure of post-tax labour income, \(Y^T\) is total household sector pre-tax income, \(Y^d\) is households’ non-labour income, \(T\) is defined as taxes on labour income and \(P\) is the consumers’ expenditure deflator. Taxes are defined as the share of labour income in total income times total taxes paid. Wages and salaries include self employment income and pre-tax income takes into account benefits and social contributions. Again, the resulting series are quarterly, seasonally adjusted and real. The data are shown in Chart 4.

Total wealth is defined as gross housing wealth \((W^G)\) plus net financial wealth. Net financial

\(^{(15)}\) All nominal series other than non-durables are deflated by the total consumers’ expenditure deflator.

\(^{(16)}\) Thus it differs from the measure used in the papers mentioned above.

\(^{(17)}\) In estimation we include dummies for the rise in VAT in 1979, an unexplained outlier at the end of 1980 and a change in PAYE rules affecting disposable income in 1998.

\(^{(18)}\) See Tables A38 to A40 in Economic Accounts.
wealth is obtained from Table A64 in Economic Accounts. $W^G$ is constructed as follows

$$W^G_t = \left( P^{GD}_t I^H_t \right) + W^G_{t-1} \left( \frac{P^H_t}{P^H_{t-1}} \right)$$

where $P^{GD}_t$ denotes the GDP deflator at factor cost, $I^H_t$ denote private sector dwellings investment and $P^H_t$ denotes UK house prices. $I^H$ comes from Table A8 in Economic Accounts, and $P^H$ is the Department of Transport, Local Government and the Regions (DTLR) house price index. Chart 5 plots the data. Population data come from the Monthly Digest of Statistics, Table 2.1. Population figures are mid year estimates, interpolated to obtain quarterly series. We divide non-durable consumption, post-tax labour income and wealth by the total consumers expenditure deflator to obtain real per capita series. We obtain deflators for both our non-durable and durable series (the latter defined as total consumption minus the specific measure of non-durable consumption) by dividing the real consumption measures by the corresponding nominal measures. The relative price series is then obtained as the deflator for the durable series divided by the non-durable deflator, and is plotted in Chart 6. Finally, the real interest rate is defined as the base rate less contemporaneous retail price index (RPI) annual inflation.
Chart 4: Log of labour income

Chart 5: Log of wealth
Chart 6: Relative price of durables to non-durables

Table A: Augmented (4th order) Dickey-Fuller statistics; no trend

<table>
<thead>
<tr>
<th></th>
<th>C.V. (10%)</th>
<th>( \Delta y' )</th>
<th>( y' )</th>
<th>( \Delta a )</th>
<th>( a )</th>
<th>( \Delta e^a )</th>
<th>( e^a )</th>
<th>( \Delta e^{clothing} )</th>
<th>( e^{clothing} )</th>
<th>( \Delta p'^d )</th>
<th>( p'^d )</th>
<th>( \Delta p'^{clothing} )</th>
<th>( p'^{clothing} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-2.57</td>
<td>-3.83</td>
<td>0.15</td>
<td>-5.24</td>
<td>0.42</td>
<td>-3.63</td>
<td>-0.55</td>
<td>-3.51</td>
<td>-0.28</td>
<td>-2.88</td>
<td>0.19</td>
<td>-2.68</td>
<td>0.29</td>
</tr>
</tbody>
</table>
5 Econometric results

5.1 Order of integration and cointegrating rank

In this section we report the results of estimating the system (1) over the period 1975 Q1 to 2001 Q2. Before we can do so, we must test for the order of integration of each of the variables in $x_t$, and then for the number of cointegrating relationships amongst the four variables. Table A reports the results of ADF tests\(^{(19)}\) for the order of integration of all (log) series. We cannot reject the hypotheses that all the series are $I(1)$, as one would anticipate from inspection of the series. Given these results we can test for common trends in these variables. To do so, we first use the Johansen method. The choice for the correct number of lags is important and can affect the results of the cointegration test, so we follow standard procedure by running an unrestricted VAR in the levels of the $I(1)$ variables, and impose restrictions that coefficients of successively higher order lags are zero. We employ the Schwarz criterion to test for the significance of these lags. It is also important to ensure the residuals are Gaussian (normal and white noise) as the method is maximum likelihood. According to the Schwarz criterion, one lag is sufficient. Moreover, there is no autocorrelation and the residuals are normal.\(^{(20)}\) Once the optimal lag length is chosen we test for co-integration using the Johansen procedure. We allow for trends in the data but no trend in the cointegrating space. We include the dummies described in footnote 17. The results are shown in Tables B and C for the two measures of non-durable consumption we examine.\(^{(21)}\)

The results are that for our preferred consumption measure there is evidence at the 10% significance level that a single cointegrating vector exists, whereas for the consumption measure which excludes clothing and footwear only, there is no such evidence. Application of a small-sample Reimers (1992) correction leaves our conclusions unchanged. Although the former tests are below the 5% level, they only marginally reject. Johansen (1995) advocates conservative choice of the cointegrating rank (assume the higher value for the reduced rank test holds when in doubt), as the consequences of falsely rejecting the null are more severe than maintaining it when it is false.

\(^{(19)}\) Here and in the rest of the paper, test statistics whose significance exceeds 10% are in bold.
\(^{(20)}\) The diagnostics are reported with the VECM results, in Table E. Hendry and Juselius (2000) conclude that ‘[s]imulation studies have demonstrated that statistical inference is sensitive to the validity of some of the assumptions, such as, parameter non-constancy, serially-correlated residuals and residual skewness, while moderately robust to others, such as excess kurtosis (fat-tailed distributions) and residual heteroscedasticity.’
\(^{(21)}\) To assess sensitivity, we repeated the tests with with two and three lags. We find that in each case at least one cointegrating vector exists (results not reported).
Table B: Cointegration tests for non-durables; no trend in cointegrating space

<table>
<thead>
<tr>
<th>Series: $c^n$, $y^l$, $a$, $p^d$</th>
<th>Linear deterministic trend, lags in VAR model: 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trace</td>
</tr>
<tr>
<td>Eigenvalue</td>
<td>Statistic</td>
</tr>
<tr>
<td>0.228240</td>
<td>46.58478</td>
</tr>
<tr>
<td>0.109204</td>
<td>20.41754</td>
</tr>
<tr>
<td>0.079684</td>
<td>8.737880</td>
</tr>
<tr>
<td>0.003469</td>
<td>0.350972</td>
</tr>
<tr>
<td></td>
<td>Max-Eigen</td>
</tr>
<tr>
<td>Eigenvalue</td>
<td>Statistic</td>
</tr>
<tr>
<td>0.228240</td>
<td>26.16724</td>
</tr>
<tr>
<td>0.109204</td>
<td>11.67966</td>
</tr>
<tr>
<td>0.079684</td>
<td>8.386909</td>
</tr>
<tr>
<td>0.003469</td>
<td>0.350972</td>
</tr>
</tbody>
</table>

Although the relative price is undoubtedly non-stationary in this sample, it could be argued that this cannot be true in population; there would be a positive probability that the price would become infinitesimal. When we test for cointegration excluding the relative price, we obtain similar results, as Table D shows for our preferred variable. The results are at first sight hard to reconcile with those in Table B, as both suggest a cointegrating vector exists, but Table D includes one less I(1) variable. However, as we shall see shortly, the relative price enters with a small coefficient, and the equilibrium errors are numerically close. In view of this, we present results for both cases. Ogaki and Park (1997) point out that tests of the null of no cointegration are known to have very low power against some alternatives, and often fail to reject the null with high probability even though the variables are actually cointegrated. Ogaki and Park argue that when the economic model implies cointegration, as it does for the framework in question here, it is more appropriate to test the null of cointegration than it is to test the null of no cointegration. We performed such a test using Park’s canonical cointegrating regression (CCR).\(^{(22)}\) This method may have another advantage. Our fundamental relationship, (12), shows that the long-run relationship is equated to what amounts to a complex error structure involving long overlapping leads. It may be that the Johansen method, which assumes the dynamic process can be modelled by a well-behaved VAR, is not ideal as an estimation technique. But the CCR, a fully modified estimator, may be more robust. For both the full and restricted (excluding relative price) set of variables we found that we could not not reject the null of cointegration. The test statistics were

\(^{(22)}\) We are grateful to an anonymous referee for pointing us at this technique.
Table C: Cointegration Tests for non-durables excluding clothing; no trend in cointegrating space

<table>
<thead>
<tr>
<th>Series: $c^{clothing}, y^d, a, p^{clothing}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear deterministic trend, lags in VAR model: 1</td>
</tr>
<tr>
<td>Eigenvalue</td>
</tr>
<tr>
<td>------------</td>
</tr>
<tr>
<td>0.160760</td>
</tr>
<tr>
<td>0.081079</td>
</tr>
<tr>
<td>0.076916</td>
</tr>
<tr>
<td>0.002355</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Max-Eigen Statistic</th>
<th>5 Percent Critical Value</th>
<th>1 Percent Critical Value</th>
<th>Hypothesized No. of CE(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.160760</td>
<td>17.70111</td>
<td>27.07</td>
<td>32.24</td>
<td>None</td>
</tr>
<tr>
<td>0.081079</td>
<td>8.540035</td>
<td>20.97</td>
<td>25.52</td>
<td>At most 1</td>
</tr>
<tr>
<td>0.076916</td>
<td>8.083494</td>
<td>14.07</td>
<td>18.63</td>
<td>At most 2</td>
</tr>
<tr>
<td>0.002355</td>
<td>0.238116</td>
<td>3.76</td>
<td>6.65</td>
<td>At most 3</td>
</tr>
</tbody>
</table>

$\chi_1^2 = 0.137$ and $0.079$ respectively (p-values 0.71 and 0.78). Thus both sets are potential candidates for cointegration. We report the parameter estimates below.

5.2 Evidence from VECMs

Given we have evidence for a unique cointegrating vector, we proceed to estimate VECMs. As there is some doubt about whether the relative price should be included, we report results both with and without it.

5.2.1 Including the relative price

We begin with the Johansen results reported in Table E, which gives the relationship between consumption, labour income, wealth and the relative price, including the long-run coefficients. The long-run parameters are appropriately signed. Non-durable consumption increases with wealth, labour income and the relative price of durables. As noted above, the elasticity with respect to the relative price is small, although significantly different from zero in these Johansen results. Were consumption the total, we would expect the coefficients on income and assets to sum
Table D: Cointegration Tests for non-durables; excluding relative price; no trend in cointegrating space

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Trace</th>
<th>5 Percent Critical Value</th>
<th>1 Percent Critical Value</th>
<th>Hypothesized No. of CE(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.17512</td>
<td>28.46013</td>
<td>29.68</td>
<td>35.65</td>
<td>None</td>
</tr>
<tr>
<td>0.07833</td>
<td>9.01560</td>
<td>15.41</td>
<td>20.04</td>
<td>At most 1</td>
</tr>
<tr>
<td>0.00766</td>
<td>0.77685</td>
<td>3.76</td>
<td>6.65</td>
<td>At most 2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Max-Eigen</th>
<th>5 Percent Critical Value</th>
<th>1 Percent Critical Value</th>
<th>Hypothesized No. of CE(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.17512</td>
<td>19.44453</td>
<td>20.97</td>
<td>25.52</td>
<td>None</td>
</tr>
<tr>
<td>0.07833</td>
<td>8.23875</td>
<td>14.07</td>
<td>18.63</td>
<td>At most 1</td>
</tr>
<tr>
<td>0.00766</td>
<td>0.77685</td>
<td>3.76</td>
<td>6.65</td>
<td>At most 2</td>
</tr>
</tbody>
</table>

to unity, but as we deal with only non-durables, this need not hold. Although we cannot reject the hypothesis that they do sum to unity ($\chi^2 = 1.73$), they add to 0.85. We argue below that this is a plausible number. The ‘marginal propensities to consume’ (MPC) depend upon the ratios of consumption to income or wealth. For income, the MPC is 0.58 (evaluated at the sample mean); for wealth, it is 0.050. Lettau and Ludvigson (2002) report an almost identical figure, 0.046, for the US. These figures are plausible.

In the dynamics of this system adjustment takes place in both the wealth and relative price equations, and not through consumption or post-tax labour income. The (normalised) loadings (‘speed of adjustment’ coefficients) for the wealth equation, $\alpha_d$, and for the relative price equation, $\alpha_p$, are 0.170 and 0.017 respectively and are both significant. By contrast, the loadings for consumption and labour income, $\alpha_c$, $\alpha_y$, are individually and jointly ($\chi^2 = 1.22$) insignificant. Thus the major driving shocks are via changes in wealth and the relative price. The implication is that the income process is not expected to exhibit much short run variation; and that the simple PIH model is a good approximation. From a narrowly statistical error correction perspective, all the loadings are correctly signed.

Turning to the other dynamics, we find that for consumption only lagged changes in labour income and the relative price are important. This differs from Lettau and Ludvigson (2001), who

(23) Each loading reported in the text here is normalised by the coefficient on the relevant level in the long-run relation, rather than on the coefficient on consumption, reported in the table.
find only lagged consumption growth significant. This may follow from habit persistence, be
evidence in favour of near-rational rules of thumb, or imply that consumers are liquidity
constrained. In income, apart from the 1998 dummy and the constant, nothing is significant:
income growth is a random walk. In the wealth equation, none of the lagged endogenous variables
are significant and only the lagged real interest rate term is significant.

5.2.2 Excluding the relative price

As we suggest above, there may be a question mark over the role of the long-run relative price,
and we investigate this further. Once cointegrating rank has been established, a robust alternative
this method, we find that while the coefficients on assets and income are well determined, the
coefficient on the relative price is both incorrectly signed and poorly determined. We
experimented with several lag and lead lengths: one typical result with two leads and lags is

\[ c_t^n = -0.66 + 0.467 y_t^l + 0.25 a_t - 0.02 p^d \]

There is also evidence from the CCR. For the full set of variables, we find

\[ c_t^n = -0.64 + 0.481 y_t^l + 0.241 a_t - 0.011 p^d \]

while for the restricted set

\[ c_t^n = -0.62 + 0.483 y_t^l + 0.240 a_t \]

Again, the picture that emerges is that the relative price does not have much of a role to play. In
this sample, the relative price is undoubtedly I(1), but the coefficient is effectively being set to zero
in the first two equations.

In Table F we therefore report the Johansen results excluding the relative price, while in Table G
we report DOLS results. As the DOLS parameter estimates are close to those obtained with the
CCR, we do not report detailed results for that case. The dynamics are very similar for each. The
crucial coefficients are of course the loadings, and the pattern and significance of these is similar
in each case. The main difference is that the loading on assets rises from around 0.7 to around 0.9
when the relative price is excluded.

Nevertheless, we are reluctant to dismiss the possibility that the relative price matters, for two
reasons. First, in the sample there is apparently strong evidence for non-stationarity in the price.
And the second point relates to the size of the coefficients. Were we dealing with total consumption, we would expect the coefficients on the non-human and human wealth (or income) to sum to unity. As durable expenditure is excluded, the coefficients would be expected to sum to a smaller number, the share of non-durables in total consumption. In our sample, that averages 0.90. In Table E the sum is 0.85. By contrast, when we exclude the relative price, the coefficient sums are 0.72 in the Johansen results, 0.74 in the DOLS case and 0.72 for the CCR, all substantially below the expected value. Moreover, the coefficients on income and assets should correspond to the shares of labour and profits in the economy. The implied shares, calculated by scaling the coefficients on income and wealth by the inverse sum of the coefficients, are 0.71 and 0.29 respectively in Table E, which are very plausible figures: one measure of the UK labour share is also 0.71 over this period. And the coefficient on the relative price would be expected to be small, given that the share of durables is around 0.1. This all suggests that the relative price may need to be included in the cointegrating regression.

Thus overall, these results, including or excluding the relative price in the total, suggest that deviations from the shared trend in consumption, labour income and assets are better described as transitory movements in asset wealth (and possibly the relative price) than as transitory movements in consumption or labour income. Thus when log non-durable consumption is above or below its long-run trend, it is asset wealth that is forecast to adjust, rather than consumption or labour income; forward-looking households foresee changes in the return on their future wealth.

6 Permanent and transitory effects

An extension of the analysis in the previous section would be to look explicitly at the contributions of shocks to the evolution of the variables in the system. In order to do this, we need a meaningful identification scheme for the shocks. In the framework of a cointegrated VAR there is an obvious decomposition of shocks that have permanent and transitory effects, and this sits perfectly with our economic framework. This can be achieved using the method suggested by Gonzalo and Ng (2001). Briefly, if the model is written as:

$$\Delta X_t = \Gamma(L) \Delta X_{t-1} + a'X_{t-1} + \varepsilon_t$$

(15)

(24) The measurement issue revolves round the treatment of ‘mixed’ income, formerly known as self-employed income. This is assumed to be allocated between employee compensation and profits as it is where the two categories are explicitly recorded. Data are from the National Accounts Blue Book.
Table E: VECM including the relative price: Johansen

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_t^n$</td>
<td>$\Delta c_t^n = -0.76 + \frac{0.603}{y_t^d} + \frac{0.25}{a_t} + \frac{0.09}{p_t^d}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\Delta c_t^n$</th>
<th>$\Delta y_t^d$</th>
<th>$\Delta a_t$</th>
<th>$\Delta p_t^d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta c_{t-1}$</td>
<td>$-0.119$</td>
<td>$-0.123$</td>
<td>$-0.381$</td>
<td>$-0.111$</td>
</tr>
<tr>
<td></td>
<td>$(1.23)$</td>
<td>$(0.84)$</td>
<td>$(1.09)$</td>
<td>$(1.22)$</td>
</tr>
<tr>
<td>$\Delta y_{t-1}$</td>
<td>$0.167$</td>
<td>$-0.150$</td>
<td>$0.088$</td>
<td>$0.157$</td>
</tr>
<tr>
<td></td>
<td>$(2.26)$</td>
<td>$(1.34)$</td>
<td>$(0.33)$</td>
<td>$(2.26)$</td>
</tr>
<tr>
<td>$\Delta a_{t-1}$</td>
<td>$0.051$</td>
<td>$0.079$</td>
<td>$0.161$</td>
<td>$-0.002$</td>
</tr>
<tr>
<td></td>
<td>$(1.78)$</td>
<td>$(1.83)$</td>
<td>$(1.55)$</td>
<td>$(0.07)$</td>
</tr>
<tr>
<td>$\Delta p_{t-1}^d$</td>
<td>$0.279$</td>
<td>$0.163$</td>
<td>$0.611$</td>
<td>$0.425$</td>
</tr>
<tr>
<td></td>
<td>$(2.56)$</td>
<td>$(0.99)$</td>
<td>$(1.55)$</td>
<td>$(4.15)$</td>
</tr>
<tr>
<td>$r_{t-1}$</td>
<td>$0.00002$</td>
<td>$0.0003$</td>
<td>$0.0018$</td>
<td>$0.0003$</td>
</tr>
<tr>
<td></td>
<td>$(0.12)$</td>
<td>$(1.01)$</td>
<td>$(2.25)$</td>
<td>$(1.49)$</td>
</tr>
<tr>
<td>$DUM79$</td>
<td>$0.042$</td>
<td>$0.0006$</td>
<td>$0.023$</td>
<td>$0.04$</td>
</tr>
<tr>
<td></td>
<td>$(7.27)$</td>
<td>$(0.06)$</td>
<td>$(0.99)$</td>
<td>$(6.91)$</td>
</tr>
<tr>
<td>$DUM804$</td>
<td>$-0.021$</td>
<td>$-0.010$</td>
<td>$0.008$</td>
<td>$-0.005$</td>
</tr>
<tr>
<td></td>
<td>$(-2.53)$</td>
<td>$(0.80)$</td>
<td>$(0.27)$</td>
<td>$(-0.68)$</td>
</tr>
<tr>
<td>$DUM981$</td>
<td>$0.0002$</td>
<td>$-0.035$</td>
<td>$0.059$</td>
<td>$0.0004$</td>
</tr>
<tr>
<td></td>
<td>$(0.02)$</td>
<td>$(-2.82)$</td>
<td>$(1.92)$</td>
<td>$(0.04)$</td>
</tr>
<tr>
<td>Constant</td>
<td>$0.007$</td>
<td>$0.006$</td>
<td>$0.011$</td>
<td>$-0.004$</td>
</tr>
<tr>
<td></td>
<td>$(4.29)$</td>
<td>$(2.62)$</td>
<td>$(2.10)$</td>
<td>$(-3.0)$</td>
</tr>
<tr>
<td>Loadings</td>
<td>$-0.079$</td>
<td>$0.011$</td>
<td>$0.681$</td>
<td>$0.185$</td>
</tr>
<tr>
<td></td>
<td>$(-1.17)$</td>
<td>$(0.11)$</td>
<td>$(2.78)$</td>
<td>$(2.90)$</td>
</tr>
<tr>
<td>$\overline{R}^2$</td>
<td>$0.47$</td>
<td>$0.08$</td>
<td>$0.12$</td>
<td>$0.44$</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>$\chi^2_5 = 5.20$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>$\chi^2_{48} = 60.56$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

where $\epsilon_t \sim N(0, \Omega)$, it also has a multivariate Wold representation given by:

$$\Delta X_t = C(L)\epsilon_t$$

(16)

where $C(L)$ is a lag polynomial of potentially infinite order.

Gonzalo and Ng (2001) show that if we define:

$$G = \begin{bmatrix} \alpha' \\ \beta' \end{bmatrix}$$

(17)

where $\perp$ denotes orthogonal complement then:

$$\Delta X_t = C(L)G^{-1}G\epsilon_t = D(L)\epsilon_t = \begin{bmatrix} D_{11}(L) & D_{12}(L) \\ D_{21}(L) & D_{22}(L) \end{bmatrix} \begin{bmatrix} \epsilon_t^p \\ \epsilon_t^T \end{bmatrix}$$

(18)

decomposes the shocks into those with permanent effects ($\epsilon_t^p$) and those that are only transitory ($\epsilon_t^T$). Shocks are continually hitting the system, but not all are passed through on to the long-run. By the Granger Representation Theorem, these shocks are filtered through $\alpha$ and $\beta$ in the VECM.
Orthogonality can be thought of as perfect non-association between variables. Clearly, the transitory shocks are defined to be shocks that have no impact on the long-run. What is happening in $G$ is that we are choosing a vector $\alpha_\perp$ that ensures that only the $n - r$ shocks are passed through. Speaking somewhat loosely, the orthogonal complement of $\alpha$ is defined in such a way that $G$ ‘knocks out’ the relevant non-permanent components in the VECM.$^{(25)}$ However, the shocks are not necessarily identified, so if we further define:

$$\Delta X_t = C(L)G^{-1}HH^{-1}G\epsilon_t = D(L)HH^{-1}\epsilon_t = \tilde{D}(L)\tilde{\eta}_t$$  

(19)

where $H$ is a lower triangular matrix such that $HH' = \text{cov}(G\epsilon_t)$ then the last $r$ elements in $\tilde{\eta}_t$ are the identified transitory shocks. With a single cointegrating vector, this shock is the only transitory one in the system.$^{(26)}$

It is unnecessary to identify the shocks explicitly in order to decompose the effect on the overall

$^{(25)}\alpha_\perp$ is defined by the condition $\alpha_\perp^\prime\alpha = 0$.

$^{(26)}$The variable ordering is the same as in the VECM. While the specific ordering affects the proportion of variance in the three permanent shocks, the transitory-permanent decomposition is unaffected.
Table G: VECM excluding relative price: long-run estimates, DOLS (2 leads and lags)

<table>
<thead>
<tr>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_t^n = -0.66 + 0.500 y_t^d + 0.244 a_t )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>( \Delta c_t^n )</th>
<th>( \Delta y_t^d )</th>
<th>( \Delta a_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta c_{t-1} )</td>
<td>-0.009 (1.00)</td>
<td>-0.088 (-0.63)</td>
<td>-0.207 (-0.61)</td>
</tr>
<tr>
<td>( \Delta y_{t-1} )</td>
<td>0.197 (2.72)</td>
<td>-0.120 (-1.12)</td>
<td>0.136 (0.92)</td>
</tr>
<tr>
<td>( \Delta a_{t-1} )</td>
<td>0.061 (2.10)</td>
<td>0.091 (2.12)</td>
<td>0.202 (1.95)</td>
</tr>
<tr>
<td>( r_{t-1} )</td>
<td>0.000006 (3.30)</td>
<td>0.00004 (1.33)</td>
<td>0.0015 (2.12)</td>
</tr>
<tr>
<td>( DU M79 )</td>
<td>0.042 (6.58)</td>
<td>-0.0010 (-0.11)</td>
<td>0.018 (0.82)</td>
</tr>
<tr>
<td>( DU M804 )</td>
<td>-0.027 (-3.29)</td>
<td>-0.013 (-1.02)</td>
<td>-0.003 (-0.08)</td>
</tr>
<tr>
<td>( DU M981 )</td>
<td>0.0002 (0.03)</td>
<td>-0.036 (-2.83)</td>
<td>0.054 (1.79)</td>
</tr>
<tr>
<td>( Constant )</td>
<td>0.004 (3.69)</td>
<td>0.005 (2.89)</td>
<td>0.008 (2.08)</td>
</tr>
<tr>
<td>( Loadings )</td>
<td>-0.078 (-1.05)</td>
<td>0.074 (0.67)</td>
<td>0.893 (3.32)</td>
</tr>
<tr>
<td>( \overline{R}^2 )</td>
<td>0.49</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td>( Jarque-Bera )</td>
<td>( \chi^2_6 = 5.32 )</td>
<td>( \chi^2_27 = 35.03 )</td>
<td></td>
</tr>
</tbody>
</table>

variances of the various series. In Table H we report the variance decomposition with:

\[
\alpha = [ -0.0792 \ 0.0111 \ 0.6811 \ 0.1846 ]'.
\]

In the second panel we set the first two insignificant loading coefficients to zero, i.e., as recommended by Gonzalo and Ng:

\[
\hat{\alpha} = [ 0.0000 \ 0.0000 \ 0.6811 \ 0.1846 ]'.
\]

Although this has some impact on the variance decomposition, the overall picture is unchanged.\(^{27}\)

As with Lettau and Ludvigson (2002), we find that the forecast error in consumption and income is almost entirely attributable to the impact of shocks to the stochastic trends. In contrast to their results, the forecast error in both wealth and the relative price is also dominated by the permanent component, although slightly less than 30% of the variance in each case is explained by the single shock with only transitory effects. They found over 85% of the variance of assets was explained

\(^{27}\)We also explored the implications of increasing the lag length on the dynamics. As the lag length in the VAR increases, the proportion of permanent shocks in consumption and income fall: in the case of income, the decrease is only marginal. The permanent proportions of assets and relative prices both rise. While the details change, the broad conclusions are unchanged.

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\(27\) We also explored the implications of increasing the lag length on the dynamics. As the lag length in the VAR increases, the proportion of permanent shocks in consumption and income fall: in the case of income, the decrease is only marginal. The permanent proportions of assets and relative prices both rise. While the details change, the broad conclusions are unchanged.
by transitory shocks, but a transitory component of nearly a third should not be considered small. One explanation of this might be the importance of housing in UK wealth, which has been rising, and is now (2003) about 50%. In the US it is less, although not a great deal; it is around 40%, up from around 30% in 2000 (Federal Reserve’s Flow of Funds data). If shocks to housing are disproportionately permanent compared to other wealth, as Lettau and Ludvigson find, this could help to explain some of the contrasting results. However, the main explanation seems to be the treatment of relative prices. Table I reports the results excluding the relative price. In this case consumption and income continue to be dominated by permanent shocks, but the shocks to assets are now overwhelmingly transitory, particularly for the results where the loadings are restricted. These results are numerically close to those reported in Lettau and Ludvigson. Given the empirical uncertainty about whether the relative price enters the cointegrating regression, this disparity in the pattern is unfortunate. However, in both cases the conclusion regarding the importance of permanent shocks to consumption and income is unchanged.

Table H: Variance decomposition: including relative price, VECM Table E

<table>
<thead>
<tr>
<th></th>
<th>( \Delta c_{t+h} - \Delta c_{t+h} )</th>
<th>( \Delta y_{t+h} - \Delta y_{t+h} )</th>
<th>( \Delta a_{t+h} - \Delta a_{t+h} )</th>
<th>( \Delta p_{t+h} - \Delta p_{t+h} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>h</td>
<td>P T</td>
<td>P T</td>
<td>P T</td>
<td>P T</td>
</tr>
<tr>
<td>1</td>
<td>0.949 0.051</td>
<td>1.000 0.000</td>
<td>0.712 0.288</td>
<td>0.686 0.314</td>
</tr>
<tr>
<td>2</td>
<td>0.866 0.134</td>
<td>0.973 0.027</td>
<td>0.718 0.282</td>
<td>0.736 0.264</td>
</tr>
<tr>
<td>3</td>
<td>0.861 0.139</td>
<td>0.972 0.028</td>
<td>0.719 0.281</td>
<td>0.750 0.250</td>
</tr>
<tr>
<td>4</td>
<td>0.861 0.139</td>
<td>0.970 0.030</td>
<td>0.715 0.285</td>
<td>0.749 0.251</td>
</tr>
<tr>
<td>( \infty )</td>
<td>0.859 0.141</td>
<td>0.969 0.031</td>
<td>0.710 0.290</td>
<td>0.740 0.260</td>
</tr>
<tr>
<td></td>
<td>Unrestricted loadings</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Restricted loadings</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.000 0.000</td>
<td>1.000 0.000</td>
<td>0.618 0.382</td>
<td>0.584 0.416</td>
</tr>
<tr>
<td>2</td>
<td>0.932 0.068</td>
<td>0.968 0.032</td>
<td>0.623 0.377</td>
<td>0.646 0.354</td>
</tr>
<tr>
<td>3</td>
<td>0.929 0.071</td>
<td>0.968 0.032</td>
<td>0.636 0.364</td>
<td>0.670 0.330</td>
</tr>
<tr>
<td>4</td>
<td>0.929 0.071</td>
<td>0.968 0.032</td>
<td>0.641 0.359</td>
<td>0.679 0.321</td>
</tr>
<tr>
<td>( \infty )</td>
<td>0.923 0.077</td>
<td>0.966 0.034</td>
<td>0.647 0.353</td>
<td>0.684 0.316</td>
</tr>
</tbody>
</table>

7 Forecasting stock returns

We have argued that significant loading of the long-run consumption relationship in the wealth ECM reflects agents’ expectations of future changes in wealth. The implication, as stressed by Lettau and Ludvigson (2001), is that the disequilibrium term(28) should forecast asset returns. This is a strong prediction, as it involves forecasting data not used in the generating regressions, and is therefore a good test of the model. We follow their methodology, and look at total equity returns, 

(28)Lettau and Ludvigson refer to this as detrended wealth.
in our case from the UK FT-Actuaries All-Share Total Return Index. Table J reports a regression of the quarterly excess return over the three-month T-bill rate on lagged excess returns and the lagged disequilibrium term from the Johansen estimates reported in Table E. To account for overlapping returns a Newey-West correction is employed. The disequilibrium is both significant, and of a similar size to those reported in Lettau and Ludvigson. Thus deviations from the long-run consumption relation reflect anticipated future asset returns.

We also examine the horizon over which the disequilibrium term can forecast. An indication of this is given in Table K, which where the results of cumulative \( i \)-period returns regressed on the disequilibrium term are reported. The peak forecasting power (measured by \( R^2 \)) is at 5 periods. The timing of this peak is identical to the one obtained by Lettau and Ludvigson (2001): moreover, the estimates are numerically similar. We also report results using the Johansen cointegrating vector excluding the relative price in Table L. The results are similar, although there is some evidence that the disequilibrium term can forecast consumption at short horizons.\(^{(29)}\) It remains insignificant at any conventional level for all horizons in excess of two.\(^{(30)}\)

\(^{(29)}\) The results using the DOLS method are similar to the Johansen estimates reported, not only qualitatively but also numerically.

\(^{(30)}\) Brennan and Xia (2002) have argued Lettau and Ludvigson’s result is spurious, and the residual ‘\( tay \)’ formed by regressing time on assets and income does better at forecasting returns. Lettau and Ludvigson (2003) offer a spirited defence. In our case, there is nothing to defend as \( tay \), defined with or without the relative price, has no predictive power for excess returns at any horizon.
Table J: Forecasting quarterly stock market returns

<table>
<thead>
<tr>
<th>variable</th>
<th>coefficient</th>
<th>t-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.04</td>
<td>3.55</td>
</tr>
<tr>
<td>$r_{t-1} - r_{f,t-1}$</td>
<td>0.03</td>
<td>0.38</td>
</tr>
<tr>
<td>$r_{t-2} - r_{f,t-2}$</td>
<td>-0.30</td>
<td>-3.68</td>
</tr>
<tr>
<td>$r_{t-3} - r_{f,t-3}$</td>
<td>0.01</td>
<td>0.10</td>
</tr>
<tr>
<td>$r_{t-4} - r_{f,t-4}$</td>
<td>-0.19</td>
<td>-1.92</td>
</tr>
<tr>
<td>$ecm_{t-1}$</td>
<td>1.20</td>
<td>2.14</td>
</tr>
</tbody>
</table>

$R^2$ 0.18

Table K: Regression of i-period excess returns on consumption disequilibrium: including relative price

<table>
<thead>
<tr>
<th>forecast horizon</th>
<th>R_i coefficient</th>
<th>t statistic</th>
<th>R^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.30</td>
<td>1.76</td>
<td>0.062</td>
</tr>
<tr>
<td>2</td>
<td>2.89</td>
<td>2.18</td>
<td>0.153</td>
</tr>
<tr>
<td>4</td>
<td>4.93</td>
<td>3.11</td>
<td>0.271</td>
</tr>
<tr>
<td>5</td>
<td>5.67</td>
<td>3.23</td>
<td>0.305</td>
</tr>
<tr>
<td>6</td>
<td>5.64</td>
<td>3.41</td>
<td>0.295</td>
</tr>
<tr>
<td>8</td>
<td>4.97</td>
<td>3.95</td>
<td>0.266</td>
</tr>
<tr>
<td>12</td>
<td>7.65</td>
<td>2.84</td>
<td>0.266</td>
</tr>
<tr>
<td>16</td>
<td>8.05</td>
<td>2.69</td>
<td>0.232</td>
</tr>
<tr>
<td>24</td>
<td>7.94</td>
<td>2.21</td>
<td>0.150</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>forecast horizon</th>
<th>C_i coefficient</th>
<th>t statistic</th>
<th>R^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.15</td>
<td>0.79</td>
<td>0.059</td>
</tr>
<tr>
<td>2</td>
<td>-0.13</td>
<td>1.02</td>
<td>0.024</td>
</tr>
<tr>
<td>4</td>
<td>-0.16</td>
<td>0.65</td>
<td>0.014</td>
</tr>
<tr>
<td>5</td>
<td>-0.09</td>
<td>0.33</td>
<td>0.004</td>
</tr>
<tr>
<td>6</td>
<td>-0.11</td>
<td>0.34</td>
<td>0.004</td>
</tr>
<tr>
<td>8</td>
<td>-0.09</td>
<td>0.02</td>
<td>0.000</td>
</tr>
<tr>
<td>12</td>
<td>-0.03</td>
<td>0.06</td>
<td>0.000</td>
</tr>
<tr>
<td>16</td>
<td>-0.07</td>
<td>0.16</td>
<td>0.000</td>
</tr>
<tr>
<td>24</td>
<td>-0.18</td>
<td>0.30</td>
<td>0.002</td>
</tr>
</tbody>
</table>

8 Conclusions

The PIH has profound implications for the dynamic behaviour of (non-durable) consumption. In the simplest models, it should follow a random walk, although in general that may not be true: models incorporating habit persistence, for example, predict the consumption ECM exists. It should also be true that consumption cointegrates with income and wealth. From the Granger representation theorem, we know an equilibrating ECM must exist: but the theory suggests it will not lie solely in consumption. Instead, this equilibration should also take place via income or wealth. At first sight this may appear unintuitive, but it simply reflects the forward looking aspect of consumer behaviour. Households save in response to expected future changes in income and asset returns. Thus consumption does not economically cause future income and wealth; but because current behaviour is affected by expected future events, deviations of consumption from
Table L: Regression of i-period excess returns on consumption disequilibrium: excluding relative price

<table>
<thead>
<tr>
<th>Forecast horizon</th>
<th>R&lt;sub&gt;i&lt;/sub&gt; coefficient</th>
<th>t statistic</th>
<th>R&lt;sup&gt;2&lt;/sup&gt;</th>
<th>C&lt;sub&gt;i&lt;/sub&gt; coefficient</th>
<th>t statistic</th>
<th>R&lt;sup&gt;2&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.47</td>
<td>1.48</td>
<td>0.057</td>
<td>-0.22</td>
<td>2.32</td>
<td>0.100</td>
</tr>
<tr>
<td>2</td>
<td>3.36</td>
<td>1.99</td>
<td>0.151</td>
<td>-0.24</td>
<td>1.75</td>
<td>0.062</td>
</tr>
<tr>
<td>4</td>
<td>5.69</td>
<td>2.76</td>
<td>0.271</td>
<td>-0.41</td>
<td>1.44</td>
<td>0.066</td>
</tr>
<tr>
<td>5</td>
<td>6.44</td>
<td>2.81</td>
<td>0.287</td>
<td>-0.37</td>
<td>1.10</td>
<td>0.040</td>
</tr>
<tr>
<td>6</td>
<td>6.07</td>
<td>2.78</td>
<td>0.248</td>
<td>-0.45</td>
<td>1.23</td>
<td>0.047</td>
</tr>
<tr>
<td>8</td>
<td>4.52</td>
<td>2.52</td>
<td>0.160</td>
<td>-0.47</td>
<td>1.07</td>
<td>0.035</td>
</tr>
<tr>
<td>12</td>
<td>7.77</td>
<td>2.37</td>
<td>0.197</td>
<td>-0.69</td>
<td>1.27</td>
<td>0.046</td>
</tr>
<tr>
<td>16</td>
<td>8.50</td>
<td>2.47</td>
<td>0.187</td>
<td>-0.80</td>
<td>1.44</td>
<td>0.045</td>
</tr>
<tr>
<td>24</td>
<td>10.58</td>
<td>2.55</td>
<td>0.186</td>
<td>-6.818</td>
<td>1.24</td>
<td>0.024</td>
</tr>
</tbody>
</table>

the long-run equilibrium Granger-cause wealth. One could take this as a good example of how Granger causality is not a good guide to economic causality.

To examine these issues, we first constructed a data set, being careful to define the data appropriately. Part of this was simply the construction of a post-tax labour income series, but we also excluded semi-durables as well as durables from our preferred consumption measure. Then the short-run dynamics and long-run relationship of and between non-durable consumption, labour income, wealth and the relative price of durable goods were examined. The relative price of durables is important because of our use of non-durable consumption, over a period which saw a large rise in the real share of expenditure on durables, and a large fall in the relative price. A cointegrating relationship between these series was found to exist. Estimating VECMs using the long-run estimates from the the Johansen, DOLS and CCR methods, we found that in each case adjustment towards the long-run common trend does indeed occur largely via changes in wealth. A theoretical implication is that consumption may predict asset returns, and this is confirmed by a regression of stock returns on the lagged disequilibrium consumption term, interpretable as detrended wealth. The results imply that a full understanding of consumption dynamics requires analysis of the entire system; single equation results will be misleading. Moreover, we are able to decompose the shocks hitting the wealth-consumption system into their transitory and permanent components. Almost all of the variation in the consumption and income process can be ascribed to permanent shocks. Depending on the treatment of the relative price of durables, we find that between 30 and 90% of fluctuations in non-human wealth are transitory. Even if the lower figure applies, this means a substantial part of short-term fluctuations in wealth are decoupled from
permanent consumption.

In summary, we find that wealth does most of the work of equilibration in the relation between consumption, income and wealth. There is no non-durable consumption ECM; there is, however, a VECM in which it plays a role. A full analysis requires the complete system to be estimated. The ECM ‘consumption function’ became less popular by the 1990s, largely because of its vulnerability to the Lucas critique. Our analysis implies that we can welcome its return, in the context of a complete VECM analysis of the system explaining the relationship between consumption and its determinants.
Appendix: A model of consumption with non-durables

In this section we sketch a model of durable and non-durable consumption.

Consumers face the following problem:

\[ \text{Max } E_t \sum_{i=0}^{\infty} \rho^i U \left( C^n_{t+i}, K_{t+i} \right) \]  

(A-1)

subject to

\[ A_{t+1} = (1 + r)A_t + Y_t - C^n_t - P^d_t C^d_t \]  

(A-2)

and

\[ K_t = (1 - \delta) K_{t-1} + C^d_t \]  

(A-3)

where \( E \) is the expectation operator, \( U \) denotes the utility function, \( C^n \) and \( C^d \) denote expenditure on non-durables and durables respectively, \( K \) is the stock of durables, \( A \) are assets, \( Y \) is labour income, \( P^d \) is the relative price of durable to non-durable goods, \( r \) is the (real) rate of return on assets, \( \rho \) is the preference discount factor and \( \delta \) is the depreciation rate of durables.

Solving the household’s problem yields the first-order conditions

\[ (1 + r) \rho E_i \left[ U_{C^n} \left( C^n_{t+1}, K_{t+1} \right) \right] = U_{C^n} \left( C^n_t, K_t \right) \]  

(A-4)

\[ (1 + r) \rho E_i \left[ U_K \left( C^n_{t+1}, K_{t+1} \right) \right] = U_K \left( C^n_t, K_t \right) \]  

(A-5)

where \( U_x \) denotes the partial derivative of \( U \) with respect to \( x \), and the efficiency condition

\[ U_{C^n} \left( C^n_t, K_t \right) = (1 + r) \rho \frac{1}{P^d_t} U_K \left( C^n_t, K_t \right) \]  

(A-6)

(A-6) implies a relationship between the durable stock, non-durable consumption and the relative price exists. In the steady state durable consumption is proportional to the stock through the stock accumulation identity. From (A-3) along the steady growth path

\[ K^*_t = (1 - \delta) K^*_{t-1} \left( 1 - g^K \right) + C^{d*}_t \]  

(A-7)

where \( g^K \) is the steady state growth rate of the durable stock, from which it follows that

\[ c^{d*}_t \approx (\delta + g^K) k^*_t. \]  

(A-8)

If utility were Cobb-Douglas,

\[ U \left( C^n_t, K_t \right) = \alpha c^n_t + \beta k_t \]  

(A-9)
then there would be a simple long-run relationship between $c^n$, $c^d$ and $p^d$, namely

$$c^d = c^n - p^d.$$  \hspace{1cm} (A-10)

For our purposes, we require a weaker condition, that the logs of non-durable consumption, durable consumption and the price of durables be cointegrated.
References


Lettau, M and S C Ludvigson (2003), ‘stay’s as good as stay: Reply’, unpublished, NYU.


