On the Relevance of Open Market Operations for Short-run Effects of Monetary Policy\textsuperscript{1}

Andreas Schabert\textsuperscript{2}
University of Cologne

This version: April 16, 2003

Abstract
This paper reexamines the role of open market operations for monetary transmission and equilibrium determinacy. Money demand is due to a cash constraint, while the central bank supplies money exclusively via repurchase agreements. Though, the duration of holding money is finite, settlement of repurchase agreements avoids the Hahn-paradox. We consider a legal restriction for open market operations by which only government bonds are accepted in exchange for money. In this case, agents care about open market operations when private debt earns a higher interest than government bonds. The model is then it able to generate liquidity effects of monetary injections regardless whether prices are flexible or set in a staggered way. Money is neutral in the former case, whereas a monetary injection leads to a real and a nominal expansion in the latter case. A nominal interest rate peg is associated with a uniquely determined price level for flexible prices and equilibrium determinacy for sticky prices, and is equivalent to a constant money growth policy.

JEL classification: E52, E32.
Keywords: Repurchase agreements, cash-in-advance constraint, liquidity effects, staggered price setting, policy equivalence, determinacy.

\textsuperscript{1}The author would like to thank Klaus Adam, Matthias Bruckner, Matthew Canzoneri, Peter Funk, Campbell Leith, Ludger Linnemann and Maurizio Zanardi as well as seminar participants at the University of Cologne, Glasgow, and Wuerzburg where the author received useful comments and suggestions.

\textsuperscript{2}University of Cologne, Department of Economics, D-50923 Koeln, Germany, email: schabert@wiso.uni-koeln.de, fax: +49/221/470-5077, tel: +49/221/470-4532.
1 Introduction

Central banks in most industrial countries exert control over money via open market operations. Herein, money is supplied in exchange for risk-free securities discounted with a short-run nominal interest rate. Hence, the costs of cash acquisition depend on the current discount rate and the availability of collateral. Monetary theory, however, has not reached a consensus on the effects of open market operations and even claimed open market operations to be irrelevant (see, e.g., Wallace, 1981, or, Sargent and Smith, 1987). In accordance with the latter view, recent contributions to the monetary policy literature commonly disregard open market operations assuming that money is injected via lump-sum transfers (see Walsh, 1997). In this paper, we depart from this approach and introduce open market operations similar to Drèze and Polemarchakis (2000) and Bloise et al. (2002). In particular, we assume that money is supplied via repurchase agreements such that households temporarily acquire money in exchange for securities discounted by a short-run nominal interest rate. Money is demanded by households due to a Clower (1967)-constraint and does not serve as a store of value. Hence, money is flow and, thus, serves as a true medium of exchange, rather than treated as an asset ‘demanded to be held at the end of the period’ (see Hellwig, 1993). Though, the duration of households’ money holdings does not exceed the length of one period, settlement of repurchase agreements avoids Hahn’s (1965) paradox, i.e., the puzzle about how to guarantee that money is held over a finite horizon.

We develop a business cycle model where aggregate prices are allowed to be set by monopolistically competitive retail firms in a staggered way. Households’ financial wealth comprises contingent claims on other households and government bonds. In each period they can acquire money from the central bank via repurchase agreements. We impose a legal restriction for open market operations, by which government bonds are exclusively accepted as collateral. The central assumption is that households internalize this particular money market restriction when they decide on their optimal plan. When private debt earns a higher interest than public debt, open market operations matter and households’ demand for government bonds is determined by their transaction demand for cash and the prevailing repo rate. As a consequence, public debt policy needs to be specified in an explicit way given that Ricardian equivalence now does not hold. When, however, the expected returns from both assets are identical, open market operations are irrelevant and the model can be reduced to a set of equilibrium conditions isomorphic to the consensus monetary business cycle model, as, e.g., applied by Clarida et al. (1999) or Woodford (2002). Given that we are primarily interested

---

3More recently, Dupor (2001) has shown that open market operations are not irrelevant when they imply the fiscal policy regime to be non-Ricardian.
in the former scenario, we show that there exists a well-behaved steady state with a positive spread and derive the local dynamics and monetary policy effects in its neighborhood.

Starting with the case where the central bank sets the growth rate of money, it is shown that a monetary injection raises prices and reduces the nominal interest rate regardless whether prices are flexible or sticky. This effect, which is repeatedly found in the data and also known as the liquidity effect (see Christiano et al., 1999),\textsuperscript{4} can hardly be generated in conventional sticky price models (see Galí, 2001, or, Andrés et al., 2002). When prices are sticky the model further predicts a real expansion. In the case where the central bank sets the nominal interest rate on government bonds, the model with relevant open market operations lacks some unpleasant features of conventional models. A nominal interest rate peg is associated with a uniquely determined price level even for flexible prices and without relying on fiscal policy to be specified in a non-Ricardian way, as, e.g., in Woodford (1994) or Benhabib et al. (2001). It is further shown that a central bank can switch between these instruments without changing the equilibrium sequences of the sticky price version, whereas an analogous exercise is impossible when open market operations are irrelevant. Equilibrium determinacy in the sticky price case does not require the fulfillment of the so-called Taylor-principle (see Woodford, 2001) when the central bank sets the nominal interest rate contingent on inflation rates. On the other hand, the central bank should refrain from setting the interest rate in a highly reactive way to avoid the economy to be destabilized in cases where debt interest payments are not completely tax financed. Macroeconomic stability, thus, demands monetary policy to be coordinated with fiscal policy.\textsuperscript{5}

The remainder is organized as follows. Section 2 develops the model. In section 3 we present results for the flexible and sticky price version for money growth policy. The case of an interest rate policy is examined in section 4. Section 5 concludes.

2 The model

The outline of the model  Household-firm units are endowed with government bonds and claims on other households carried over from the previous period. They produce a wholesale good employing labor from all households. Aggregate uncertainty is due to monetary policy shocks, which are realized at the beginning of the period. Then goods are produced and asset markets open, where households can freely trade in government bonds and in contingent claims. Money demand is induced by assuming that purchases of consumption goods are

\textsuperscript{4}Limited participation and segmentation of asset markets are commonly recommended as solutions for the so-called liquidity puzzle (see Christiano et al., 1997, and, Alvarez et al., 2002)

\textsuperscript{5}The destabilizing effect of aggressive interest rate policy due to 'debt-interest spirals' is also examined by Leith and Wien-Lewis (2000) in an overlapping generations framework.
restricted by a liquidity constraint. The central bank supplies money exclusively via open
market operations, i.e., via repurchase agreements, where money is traded in exchange for
securities. Hence, households can acquire money to an amount equal to the discounted
asset value.\textsuperscript{6} After goods are traded, households repurchase securities from the central bank.
Hence, there is no accumulation of money. In appendix 6.1 we provide a sufficient condition
for households not willing to carry over money from one period to the other.

In order to allow for a nominal rigidity, which is commonly perceived as the main source for
short run non-neutrality of money, we introduce monopolistically competitive retail firms, who
differentiate the wholesale goods. To minimize distortions induced by liquidity constraints,
we assume that households first buy coupons for the differentiated consumption goods from
the retail firms, which enables the latter to purchase the wholesale good from the household-
firm units. Given these assumption, the log-linearized approximation of the model nests the
prototype New Keynesian model.

**Households** Lower (upper) case letters denote real (nominal) variables. There is an in-
finite number of time periods \( t \) (\( t = 0, 1, 2, ... \)). Let \( s^t = (s_0, ...., s_t) \) denote the history of events up
to date \( t \) and \( g(s^t|s^{t-1}) \) denote probability of state \( s_t \) and, thus, of the history \( s^t \) conditional
on the history \( s^{t-1} \) at date \( t - 1 \). The initial state, \( s^0 \), is given so that \( g(s^0) = 1 \). There is a
continuum of perfectly competitive household-firm units distributed uniformly over \((0, 1)\).
In each period \( t \) a household \( j \in (0, 1) \) consumes a composite goods \( c(j, s^t) \) and supplies working
time \( l(j, s^t) = \int_0^1 l^k(j, s^t) dk \) to household-firm units, where \( l^k(j, s^t) \) denotes the working time
of households \( j \) in firm \( k \). Further, household \( j \) produces a wholesale good \( x(j, s^t) \) using the
technology: \( x(j, s^t) = \int_0^1 \bar{p}(k, s^t) dk \), and sells the wholesale good to retail firms charging a
price \( P^w(s^t) \) per unit. The objective of household \( j \) is given by
\[
\sum_{t=0}^{\infty} \sum_{s^t} \beta^t g(s^t) \ u(c(j, s^t), l(j, s^t)), \text{ with } 0 < \beta < 1, \tag{1}
\]
where \( \beta \) denotes the subjective discount factor.

**Assumption 1** The utility function \( u \) is assumed to be strictly increasing in consumption \( c \),
strictly decreasing in labor \( l \), strictly concave, twice continuously differentiable with respect to
both arguments, satisfies the usual Inada conditions, and is additively separable.

We separate the household problem into a temporal and an intertemporal part. In the
temporal part they make their optimal decisions on production and on the composition of

\textsuperscript{6}One can equally assume that financial intermediaries engage in open market operations on the behalf of
households.
consumption. Profit maximizing demand for labor $v^j(k, s^t)$ implies

$$P(s^t)w(s^t) = P^u(j, s^t),$$

where $P(s^t)$ denotes the aggregate price level and $w(s^t)$ the real wage rate. Let $c(j, s^t)$ be consumption of a composite good which is defined as a CES aggregate of the differentiated goods and a differentiated good $y^j(i, s^t)$ is bought from a retailer $i \in (0, 1)$

$$c(j, s^t) = \left[ \int_0^1 y^j(i, s^t)^{\frac{1}{\epsilon}} di \right]^{\frac{\epsilon}{1-\epsilon}}, \quad \epsilon > 1,$$

where $\epsilon$ is the constant elasticity of substitution between any two retail goods. Let $P(i, s^t)$ denote the price of the retail good $y^j(i, s^t)$, then the price of the composite good $P(s^t)$ is given by

$$P(s^t) = \left[ \int_0^1 P(i, s^t)^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}}.$$

Minimizing costs for purchasing a unit of the composite good leads to the following optimal demand for the retail good $y^j(i, s^t)$

$$y^j(i, s^t) = \left( \frac{P(i, s^t)}{P(s^t)} \right)^{-\epsilon} c(j, s^t).$$

We now turn to the intertemporal part. At the beginning of period $t$ households are endowed with financial wealth $A(j, s^{t-1})$ which comprises government bonds holdings $B(j, s^{t-1})$ and state contingent claims on other households $D(j, s^{t-1}) : A(j, s^{t-1}) = B(j, s^{t-1}) + D(j, s^{t-1})$. Both assets exhibits payoffs contingent on the aggregate state $s^t$. Let $z(s^{t+1}, s^t)$ be the price in state $s^t$ of a claim that pays off one unit of currency if and only if state $s^{t+1}$ occurs.

Before agents enter the asset market, the aggregate shocks arrive, goods are produced, and wages are credited on checkable accounts. Then households enter the assets market, where their assets holdings pay off $D(j, s^{t-1})$ and $R(s^t)B(j, s^{t-1})$ and the new portfolio of contingent claims costs $\sum_{s^{t+1} \mid s^t} z(s^{t+1}, s^t)g(s^{t+1}|s^t)D(j, s^t)$. To acquire cash, households participate in repurchase agreements, where they can exchange interest bearing assets $B^c(j, s^t)$ for cash $M(j, s^t)$. The amount $M(j, s^t)$ supplied by the central bank equals the discounted value $B^c(j, s^t)/R(s^t)$, implying that the exchange rate equals the gross nominal interest rate on government bonds:

$$M(j, s^t) = \frac{B^c(j, s^t)}{R(j, s^t)}, \quad \text{with } B^c(j, s^t) \geq 0.$$

Then the goods market opens. We assume that the purchase of consumption goods is subject
to the following liquidity constraint:

\[ P(s')c(j, s') \leq M(j, s') + \left[ P(s')w(s')l(j, s') - P(s')w(s') \int_0^1 l(k, s') dk \right]. \tag{6} \]

The modification of the Clower (1967) constraint, i.e., the term in the square brackets, is introduced to avoid the cash-credit good distortion between consumption and leisure, which would unnecessarily make the model structure more complicated (see, Jeanne, 1998). The household receives a labor income \( P(s')w(s')l(j, s') \) and have to pay \( P(s')w(s') \int_0^1 l(k, s') dk \) for the wage outlays for its own firm. Assuming that wages are credited at demand deposits and that checks drawn on these accounts are accepted as a means of payment, we end up with (6). On the other hand, the household receives cash by selling its product \( x(j, s') \) and in form of profits of retail firms \( P(s') \int_0^1 \omega^j(i, s') di \) such that they leave the goods market with the amount \( \overline{M}(j, s') \)

\[
\overline{M}(j, s') = P^w(s')x(j, s') + P(s') \int_0^1 \omega^j(i, s') di - P(s')c(j, s')
+ M(j, s') + P(s')w(s')l(j, s') - P(s')w(s') \int_0^1 l(k, s') dk.
\]

Thus, the cash constraint can be combined to \( \overline{M}(j, s') \geq P^w(s')x(j, s') + P(s') \int_0^1 \omega^j(i, s') di. \) Then repurchase agreements are completed, i.e., \( M(j, s') \) is bought back by the central bank and the households receive \( B^c(j, s')/R(s') \). To allow open market operations to play a non-negligible role, we impose the legal restriction that only government bonds are accepted by the central bank such that

\[
B^c(j, s') \leq B(j, s'). \tag{7}
\]

Given that the opportunity costs of money accumulation is equal to the expected return from contingent claims, the utility of household \( j \) will not be higher for a plan where they carry over cash from one period to the other. Hence, we disregard accumulation of money for simplicity,\(^7\) such that household \( j \)'s asset market constraint can be written as

\[
\sum_{s^{t+1}|s^t} z(s^{t+1}, s^t)g(s^{t+1}|s^t)D(j, s^t) + B(j, s^t) + (R(s') - 1) M(j, s^t)\]

\[
\leq R(s^t)B(j, s^{t-1}) + D(j, s^{t-1}) - P(s')c(j, s^t) - P(s^t)\tau(s^t)
+ P(s^t)w(s^t)l(j, s^t) - P(s')w(s^t) \int_0^1 l(k, s') dk + P^w(s^t)x(j, s^t) + P(s^t) \int_0^1 \omega^j(i, s') di,
\]

where \( \tau \) denotes a lump-sum tax. We further assume that households are aware of the fact

\(^7\)Appendix 6.1 provides sufficient conditions for households not to carry over money from one period to the other.
that their access to cash is restricted by their holdings of government bonds. This restricting is obviously irrelevant when they can issue private debt earning an interest rate not higher than the interest rate on government bonds. However, as the monetary authority sets the latter, a positive spread $R^d(s^t) - R(s^t)$ cannot generally be ruled out such that they internalize the constraint, which can be reduced to

$$M(j,s^t) \leq \frac{B(j,s^t)}{R(s^t)},$$

(9)

when they derive their optimal decisions. Maximizing (1) subject to the asset market constraint (8) a no-Ponzi-game condition

$$\lim_{t \to \infty} \sum_{s^{t+i}} g(s^{t+i})A(j,s^{t+i}) \prod_{v=1}^{i} z(s^{t+v}, s^t) \geq 0,$$

(10)

the goods market constraint (6) and the open market constraint (9) for a given initial value of total nominal wealth $A(j,s^0)$ leads to the following conditions for consumption, leisure, contingent claims, government bonds and money

$$u_c(j,s^t) = \lambda(j,s^t) + \psi(j,s^t),$$

(11)

$$u_c(j,s^t) = -u_l(j,s^t)/w(s^t),$$

(12)

$$\frac{\lambda(j,s^t)}{P(s^t)} = \beta \sum_{s^{t+1}|s^t} g(s^{t+1}|s^t) \frac{\lambda(j,s^{t+1})}{P(s^{t+1})} \frac{1}{z(s^{t+1}, s^t)},$$

(13)

$$\eta(j,s^t) = \beta \sum_{s^{t+1}|s^t} g(s^{t+1}, s^t) \frac{\lambda(j,s^{t+1})}{P(s^{t+1})} \left( \frac{1}{z(s^{t+1}, s^t)} - R(s^{t+1}) \right),$$

(14)

$$\psi(j,s^t) = (R(s^t) - 1) \lambda(j,s^t) + R(s^t) \eta(j,s^t),$$

(15)

$$\eta(j,s^t) \geq 0, \quad \psi_t \left[ m(j,s^t) + w(s^t)l(j,s^t) - w(s^t) \int_0^1 \psi(k,s^t) dk - c(j,s^t) \right] = 0,$$

(16)

$$\eta(j,s^t) \geq 0, \quad \mu_t \left[ b(j,s^t) - R(s^t)m(j,s^t) \right] = 0,$$

(17)

and (6) and (9), where $\lambda$ denotes the shadow price of wealth, $\psi$ the Lagrange multiplier on the cash-in-advance constraint, and $\eta$ the Lagrange multiplier on the open market constraint (9). The cash-in-advance constraint holds with equality when $\psi$ is positive. In the optimum (10) further holds with equality serving as the transversality condition. We assume that private debt is issues in form of one period bonds that costs one unit of currency in $t$ and pays $R^d(s^{t+1})$ in $s^{t+1}$, implying

$$R^d(s^{t+1}) = z(s^{t+1}, s^t)^{-1}.$$
Retailer There is a monopolistically competitive retail sector with a continuum of retail firms indexed on \( i \in (0, 1) \). Each retail firm, owned by the households, buys a quantity \( x^i(j, s^t) \) of the wholesale good produced by household \( j \) at price \( P_w(s^t) \). We assume that a retailer is able to differentiate the wholesale good without further costs. The differentiated retail good \( y(i, s^t) = \int_0^1 x^i(j, s^t) dj \) is then sold at a price \( P(i, s^t) \). We assume that retailers set their prices according to Calvo’s (1983) staggered price setting model. The retailer changes its price when he receives a signal, which arrives with in a given period with probability \( (1 - \phi) \), where \( 0 \leq \phi < 1 \). A retailer who does not receive a signal adjusts its price by the steady state aggregate inflation rate \( \pi \), such that \( P(i, s^t) = \pi P(i, s^{t-1}) \). On the other hand, a retailer that receives a price change signal in period \( t \) chooses an optimal price \( \tilde{P}(i, s^t) \) to maximize the expected sum of future discounted profit streams given by

\[
\sum_{v=0}^{\infty} \sum_{s^{t+v}} (\beta \phi)^v q(s^{t+v}, s^t) \tilde{\omega}(i, s^{t+v}, s^t),
\]

where \( q(s^{t+1}, s^t) \equiv z(s^{t+1}, s^t)g(s^{t+1}|s^t) \) is the stochastic discount factor and \( \tilde{\omega}(i, s^{t+v}, s^t) \) denotes his real profits in period \( t + v \) for own prices not being adjusted after period \( t \) :

\[
P(s^t)\tilde{\omega}(i, s^{t+v}, s^t) = \tilde{P}(i, s^t) y(i, s^{t+v}) - P_w(s^{t+v}) \int_0^1 x^i(j, s^{t+v}) dj.
\]

Maximizing (18) subject to the demand function (4), taking the price \( P_w(s^t) \) of the wholesale good and the aggregate final goods price index \( P(s^t) \) and the initial price level \( P(s^0) \) as given, yields the first-order condition for \( \tilde{P}(i, s^t) \)

\[
\tilde{P}(i, s^t) = \frac{\epsilon}{\epsilon - 1} \sum_{v=0}^{\infty} \sum_{s^{t+v}} (\beta \phi)^v q(s^{t+v}, s^t) x(s^{t+v}) P(s^{t+v}) \exp(-\epsilon P_w(s^{t+v})).
\]

Public sector The public sector consists of a fiscal and a monetary authority. The monetary authority supplies money in open market operations in exchange for government bonds and transfers the seignorage to the fiscal authority. The budget constraint of the central bank is given by

\[
B^c(s^t) - M(s^t) = (R(s^t) - 1)M(s^t) = P(s^t)\tau^c(s^t),
\]

where \( \tau^c \) denotes transfers to the fiscal authority. The latter issues risk free one period bonds earning a gross nominal interest rate \( R(s^t) \), collects lump-sum taxes \( \tau \) from the households and receives the transfer from the monetary authority \( \tau^c \)

\[
R(s^t)B(s^{t-1}) = B(s^t) + P(s^t)\tau^c(s^t) + P(s^t)\tau(s^t).
\]

We assume that the central bank sets the growth rate of money \( \mu(s^t) = M(s^t)/M(s^{t-1}) \) according to: \( \mu(s^t) = \mu^{1-\rho} \mu(s^{t-1})^\rho \exp(\varepsilon_t) \), with \( 0 \leq \rho < 1 \) and \( 1 \geq \mu \). The innovation \( \varepsilon_t \) has
an expected value zero and is serially uncorrelated. The fiscal policy regime is characterized by the following simple rule which relates debt obligations to tax receipts and transfers from the central bank:

$$\vartheta(R(s^t) - 1)(s^t)B(s^{t-1}) = P(s^t)\left(\tau(s^t) + \tau_c(s^t)\right), \quad \text{with } 0 < \vartheta \leq 1. \quad (21)$$

The fiscal policy parameter $\vartheta$ governs the portion of government expenditures covered by tax receipts. Using the fiscal policy rule (21) to eliminate the transfers in the budget constraint (20) leads to the following consolidated budget constraint of the public sector

$$B(s^t) = [(1 - \vartheta)(R(s^t) - 1) + 1] B(s^{t-1}). \quad (22)$$

Hence, a higher value for the fiscal policy parameter $\vartheta$ reduces the growth rate of government bonds. Given that $\vartheta > 0$ it can immediately be seen from (22) that solvency of the public sector is guaranteed as

$$\lim_{i \to \infty} \sum_{s^{t+i}} g(s^{t+i})B(s^{t+i}) \prod_{v=1}^{i} R(s^{t+v})^{-1} = 0 \quad (23)$$

is always satisfied. In other words, our specification of public policy is Ricardian.\(^8\)

**Symmetry and market clearing** Given that households are symmetric we know that

$$c(j, s^t) = c(s^t), \quad l(j, s^t) = l(s^t), \quad l(k, s^t) = l^d(k, s^t) = l^d(s^t), \quad m(j, s^t) = m(s^t), \quad \overline{m}(j, s^t) = \overline{m}(s^t), \quad b(j, s^t) = b(s^t), \quad \overline{b}(j, s^t) = b^c(s^t), \quad x(j, s^t) = x(s^t), \quad \lambda(j, s^t) = \lambda(s^t), \quad \eta(j, s^t) = \eta(s^t), \quad \psi(j, s^t) = \psi(s^t), \quad d(j, s^t) = d(s^t),$$

where real values for the assets are denoted by lower case letters. Aggregation of retailer profits yields $P(s^t)\omega(s^t) = P(s^t) - P^u(s^t)y(s^t)$. Market clearing further implies

$$l(s^t) = l^d(s^t), \quad y(s^t) = x(s^t), \quad d(s^t) = 0, \quad \overline{m}(s^t) = m(s^t), \quad a(s^t) = b(s^t), \quad y(s^t) = c(s^t).$$

In what follows we restrict our attention on the cases where the cash constraint is binding ($c(s^t) = m(s^t)$). For this, it is sufficient that the nominal interest rate on government bonds $R_t - 1$ is strictly positive (see 15) such that $\psi_t > 0$.

\(^8\)An analysis of open market operations in an environment with a non-Ricardian fiscal policy regime can be found in Dupor (2001).
3 Money growth policy and liquidity effects

In this section we first derive the conditions for open market operations to be relevant. Then we examine the effects of a money growth shock ($\varepsilon_f > 0$ and $\varepsilon_t = 0 \forall t : t \neq f$), when prices are flexible and sticky. To lighten the notion, the reference to the state is suppressed.

3.1 Open market operations

The model features two fundamentally different versions depending on the relevance of open market operations, i.e., if the open market constraint (9) enters the set of equilibrium conditions as an equality. When open market operations are not legally restricted by (7), which demands that only government bonds are accepted as collateral, open market operations are obviously irrelevant as money can be acquired in exchange for securities issued by the households themselves. However, even if open market operations are legally restricted by (7), open market operations are irrelevant as long as households’ government bonds holdings are sufficiently large such that $B_t \geq B^g_t$ always holds. Given the timing of events and markets in our model, households can easily afford the latter when government bonds earn the same interest as private bonds ($R_t = R^d_t$). In this case, households can freely issue private debt to invest costlessly in government bonds to any amount. In contrast, when the interest rate on government bonds is smaller than the interest rate on private bonds, this strategy becomes costly such that households are willing to minimize on the interest rate loss. In this case, they will only hold government bonds equal to desired amount of money times the actual discount rate, $B_t = B^g_t$ and the open market constraint (9) is binding. This result is summarized in the following proposition.

**Proposition 1** Open market operations matter ($\eta_t > 0 \Rightarrow B_t = B^g_t$) iff $E_t[R^d_{t+1} - R_{t+1}] > 0$.

**Proof.** The claim made in the proposition immediately follows from the first order condition (14), which reads $\eta_t = \beta E_t[\frac{\lambda_{t+1}}{\pi_{t+1}}(R^d_{t+1} - R_{t+1})]$ and from (16). □

Whether the open market constraint is binding or not has an important consequence for the relation between government bonds, money, and, thus, consumption. Suppose that the expected spread between the interest rate on private debt and the interest rate on government bonds is positive. According to the result in proposition 1 the open market constraint then demands that money and, thus, consumption is linked to real government bonds $c_t = b^g_t R_t = b_t / R_t$. Otherwise, the amount of securities traded in open market operations $b^g_t$ is not directly linked to public debt and to the remainder of the model. We proceed with the case of flexible prices.
3.2 Flexible prices

In what follows we reduce the set of endogenous variables and restrict our attention on the response of inflation, consumption, the interest rate on government bonds, and households assets to changes in the money growth rate. When prices are flexible ($\phi = 0$) the state equals the realization of the shock to the money growth rule ($s_t = \varepsilon_t$) such that the model exhibits no sluggishness. Further using that $c_t = l_t$ and $a_t = b_t$ and introducing the inflation rate $\pi_t \equiv P_t/P_{t-1}$, we can define the equilibrium in five endogenous variables. It should be noted that the shadow prices $\lambda_t$ and $\eta_t$ can be recursively determined by (13) and (14). Though, the shadow price for the open market constraint matters is clearly relevant in equilibrium, we make use of the fact that it is actually the sign of $\eta_t$ that governs the evolution of the variables of interest. Hence, we are able to characterize the equilibrium contingent on the sign of $\eta_t$.

**Definition 1** A rational expectations equilibrium of the flexible price model with $\psi_t > 0$ is a set of sequences $\{c_t, R_t, m_t, a_t, \pi_t\}_{t=0}^{\infty}$ satisfying

\[
\begin{align*}
uc(c_t) &= -uc(c_t) \frac{\epsilon}{\epsilon - 1}, \quad (24) \\
c_t &= m_t, \quad (25) \\
R_t &= \begin{cases} a_t/c_t & \text{if } \eta_t > 0 \\ u_{c}(c_{t+1})E_t \left[u_{c}(c_{t+1})^{-1} \pi_{t+1}\right] / \beta & \text{if } \eta_t = 0 \end{cases}, \quad (26) \\
a_t &= [(1 - \vartheta)(R_t - 1) + 1] a_{t-1} \pi_t^{-1}, \quad (27) \\
m_t \pi_t / m_{t-1} &= \mu_t, \quad \text{with } \mu_t = \mu^{1-\rho} \mu_{t-1}^{\rho} \exp(\varepsilon_t), \quad (28)
\end{align*}
\]

and the transversality condition for a given initial value $A_0$.

In the flexible price environment the mark-up of the retail price over the wholesale price is constant and given by $\frac{\epsilon}{\epsilon - 1}$. Hence, the real wage is, by (2), also constant: $w_t = w = \frac{\epsilon - 1}{\epsilon}$. Given that the real wage is constant, consumption is uniquely pinned down by (24): $c_t = c$.

This property, which simplifies the analysis, is actually a virtue of avoiding the cash-credit good distortion between consumption and leisure by applying a modified cash-constraint.

The behavior of the inflation rate and the nominal interest rate on government bonds can now directly be deduced from the equilibrium conditions (24)-(28). We start with the case where open market operations matter. Given that consumption is fixed, a money growth policy $\mu_t = m_t \pi_t / m_{t-1}$ implies by $M_t = P_t c$ that the inflation rate equals the money growth rate: $\mu_t = \pi_t$. Turning to the interest rate on government bonds, we use (27) and $a_t = cR_t$ to obtain

\[
\mu_t = \left[1 + (1 - \vartheta)(R_t - 1)\right] \frac{R_{t-1}}{R_t}.
\]
Equation (29) indicates that the nominal interest rate $R_t$ declines for a rise in the money growth rate given that we assumed the fiscal authority to satisfy $\vartheta > 0$. However, this reaction is opposed to the one for the case where open market operations are irrelevant ($\eta_t = 0$). In this case, the model exhibits no liquidity effect and the nominal interest rate rises due to the so-called expected inflation effect (see Christiano et al., 1997). The following proposition summarizes these results.

**Proposition 2 (Liquidity effect, $\phi = 0$)** Suppose that prices are flexible. Then an expansionary money growth shock is accompanied by a rise in inflation and

1. a declining nominal interest rate if the open market constraint is binding ($\eta_t > 0$), and
2. a non-declining nominal interest rate if the open market constraint is not binding ($\eta_t = 0$).

**Proof.** The first part of the proposition immediately follows from the former discussion and from (29). In the case where $\eta_t = 0$, the nominal interest rate $R$, which is equal to $R^d$ in this case, is determined by $u_c(c_t) = R_t\beta E_t\left[\frac{u(c_{t+1})}{\pi_{t+1}}\right]$. Given that $c_t = c$ holds, the nominal interest rate rises with the expected future inflation $R_t = \beta^{-1}E_t\pi_{t+1}$, with $\pi_t = \mu_t$. ■

When $\mu_t$ is not serially correlated ($\rho = 0$), the nominal interest rate is constant for $\eta_t = 0$. However, when we allow that the money growth rate exhibits a positive autocorrelation ($\rho > 0$), the model predicts, by (13), a rise in the nominal interest rate. This equilibrium for this version is, thus, uniquely pinned down. Using the equation (29) we can further examine the conditions for the model with $\eta_t > 0$ to exhibit a unique rational expectations equilibrium path which converges to the steady state. Actually, it remains to check if the differential equation (29) is unstable such that the forward looking variable $R_t$ is uniquely determined. The following proposition summarizes the result.

**Proposition 3 (Determinacy, $\eta_t > 0$)** Suppose that prices are flexible and the open market constraint is binding. Then the model exhibits a unique rational expectations equilibrium path converging to the steady state.

**Proof.** To establish the claim made in the proposition, we log-linearize the deterministic version of equation (29) at the steady state: $\mu \hat{R}_t = (1 - \vartheta)R\hat{R}_t + [(1 - \vartheta)R + \vartheta]\hat{R}_{t-1}$, where $\hat{R}_t$ is defined as $\hat{R}_t = \log(R_t/R)$. Using the steady state restrictions $\mu = (1 - \vartheta)R + \vartheta$, we end up with $\hat{R} = \frac{\mu}{\vartheta}R_{t-1}$. Hence, the eigenvalue is unstable (including the unit root) and, thus, the model is uniquely determined given that $\mu \geq \vartheta$ is ensured by assumption. ■

---

9 The complete set of steady state conditions can be found in the appendix 6.2.
3.3 Staggered price setting

In this section we consider the case where prices are not completely flexible \((\phi > 0)\). In order to analyze the effects of a monetary injection, we take a linear approximation of the model at the steady state. The steady state of the model, which consistent with the 'monetary facts' of McCandless and Weber (2001) regardless whether open market operations matter or not, is presented in appendix 6.2. The model now additional features an aggregate supply constraint stemming from the partial price adjustment of retailers. It can be derived by log-linearizing the first order condition (19) and using the assumption on non-price-adjusting retailer. As shown in appendix 6.3, the evolution of the inflation rate can then be summarized by the following aggregate supply constraint:

\[
\hat{\pi}_t = \chi \hat{m}_t \hat{c}_t + \beta E_t \hat{\pi}_{t+1}, \quad \chi \equiv (1 - \phi)(1 - \beta \phi)\phi^{-1} > 0,
\]

where \(\hat{\pi}_t\) denotes percent deviations of \(\pi_t\) from the steady state value \(\pi\) and \(mc_t = P_t^w/P_t(= w_t)\) denotes the retailer’s real marginal costs. We further log-linearize and reduce the remaining equilibrium conditions of the model at the steady state to obtain an analytically tractable representation. The equilibrium of the linearized model with a staggered price setting is defined as follows.

**Definition 2** A rational expectations equilibrium of the log-linear approximation to the sticky price model at the steady state with \(\bar{R} > 1\) and \(\mu < \bar{\mu}\) is a set of sequences \(\{\hat{m}_t, \hat{c}_t, \hat{\pi}_t, \hat{R}_t, \hat{\alpha}_t\}_{t=0}^{\infty}\) satisfying

\[
\hat{c}_t = \begin{cases} 
\hat{a}_t - \hat{R}_t, & \text{if } \eta_t > 0 \\
E_t \hat{c}_{t+1} - (\hat{R}_t - E_t \hat{\pi}_{t+1})/\sigma & \text{if } \eta_t = 0,
\end{cases}
\]

(31)

\[
\hat{m}_t = \hat{c}_t,
\]

(32)

\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \gamma_1 \hat{c}_t,
\]

(33)

\[
\hat{a}_t = \hat{a}_{t-1} + \gamma_2 \hat{R}_t - \hat{\pi}_t,
\]

(34)

\[
\hat{m}_t = \hat{m}_{t-1} - \hat{\pi}_t + \hat{\mu}_t, \quad \text{with} \quad \hat{\mu}_t = \rho \hat{\mu}_{t-1} + \varepsilon_t,
\]

(35)

\[\text{with } \gamma_1 \equiv \chi (\sigma + v) > 0, \quad \gamma_2 \equiv \frac{\mu - \vartheta}{\mu} \geq 0, \quad \sigma \equiv -\frac{\pi_c}{\pi_c \pi} > 0, \quad v \equiv \frac{\pi_l}{\pi_{ll}} > 0,\]

and the transversality condition for given initial values \(A_0\) and \(P_0\).

A closer look at the equilibrium conditions reveals that real financial wealth and, thus, the real value of government debt outstanding only affects consumption and inflation in the case where open market operations matter \((\eta_t > 0)\). Otherwise, the equilibrium sequences of consumption, inflation, real balances, and the nominal interest rate are completely unaffected by real wealth, given that they are already determined by the conditions (31), (32), (33) and
the money growth rule (35). Real wealth and, thus, real government debt can recursively be
determined by (34). Furthermore, the public financing decision, which is represented by the
parameter \( \vartheta \) governing the ratio of tax to debt financing, is also irrelevant, except for the
sequence of real debt. This property, which our model with \( \eta_t = 0 \) shares with the majority
of monetary business cycle models, is known as debt neutrality or Ricardian equivalence. The
more interesting property for the novel version is summarized in the following proposition.

Remark 1 When prices are sticky and the open market constraint is binding, Ricardian
equivalence does not hold.

Ricardian non-equivalence immediately follows from the fact that \( \vartheta \) affects the evolution of
real wealth by (34), while the latter alters the access to money and, thus, the ability to
consume as revealed by the condition in the upper row of (31). To discriminate between the
two cases, we derive a particular steady state condition for the central bank which ensures
that the open market constraint binds (\( \eta > 0 \)). The result is summarized in the following
proposition.

Proposition 4 (Steady state with \( \eta > 0 \)) Suppose that the fiscal policy is sufficiently re-
active such that \( \vartheta > 1 - \beta \). Then there exists a steady state where the open market constraint
is binding (\( \eta > 0 \)) if the central bank sets its target money growth rate \( \mu \) such that \( \mu < \overline{\mu} \),
with \( \overline{\mu} = \vartheta / (1 - (1 - \vartheta) / \beta) \).

Proof. The steady state demands \( \frac{\xi}{\beta} = R^d \), \( R = (\mu - \vartheta) / (1 - \vartheta) \) (see appendix 6.2). Thus,
\( \mu < \overline{\mu} \) ensures that \( R < \frac{\xi}{\beta} = R^d \) implying \( \eta > 0 \). Further, existence of a particular \( \mu \), which
requires that \( 1 < \overline{\mu} \), is guaranteed by \( 1 < \beta \). ■

Hence, we can identify cases where the open market constraint is binding by referring to
a restriction for the long-run money growth rate (\( \mu < \overline{\mu} \)). This strategy further requires
that the support for the innovations \( \varepsilon \) is small enough such that \( \eta_t > 0 \) also holds in the
neighborhood of the steady state.

Before we turn to the model’s solution, we examine the local determinacy properties
of the model with a binding open market constraint. In particular, we want to derive the
conditions for the model to exhibit a unique and stable equilibrium path. Given that there
is one endogenous state variable (\( \hat{a}_{t-1} \)), this requires exactly one eigenvalue of the model in
(36) to lie inside the unit circle. The following proposition summarizes the local determinacy
result.

Proposition 5 (Equilibrium determinacy, \( \eta_t > 0 \)) Suppose that prices are rigid and the
open market constraint is binding. Then the model exhibits a unique rational expectations
equilibrium path converging to the steady state.
**Proof.** The model with \( \eta_t > 0 \) given in definition 2 can easily be reduced to a \( 2 \times 2 \) system in real wealth and inflation which reads

\[
M_0 \begin{pmatrix} \hat{a}_t \\ E_t \hat{\pi}_{t+1} \end{pmatrix} = M_1 \begin{pmatrix} \hat{a}_{t-1} \\ \hat{\pi}_t \end{pmatrix} + M_\mu \hat{\mu}_t
\]

with

\[
M_0 = \begin{pmatrix} \gamma_1 & \beta \\ 1 & 0 \end{pmatrix}, \quad M_1 = \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}, \quad M_\mu = \begin{pmatrix} -\gamma_1 \pi / \vartheta \\ 1 - \pi / \vartheta \end{pmatrix},
\]

where we used that the constant money growth rule implies the following solution for the nominal interest rate: \( \hat{R}_t = -\pi \vartheta^{-1} \hat{\mu}_t \). The characteristic polynomial of \( M_0^{-1} M_1 \) is \( f(X) = X^2 - \frac{\beta + \gamma_1 + 1}{\beta} X + \frac{1}{\beta} \). Given that \( f(0) \) is equal to \( 1 / \beta \) and, therefore, strictly positive and \( f(1) \) is negative \( f(1) = -\gamma_1 / \beta < 0 \), the model exhibits one eigenvalue lying between zero and one and one unstable eigenvalue. □

Once we know that the model is locally determined, we can easily derive the impact effects of a monetary injection on the endogenous variables of the model. For this we use that the state space now features an endogenous state, \( s_t = (a_{t-1}, \hat{\mu}_t) \), such that the general solution form of (36) is given by

\[
\begin{align*}
\hat{a}_t &= \delta_a \hat{a}_{t-1} + \delta_a e \hat{\mu}_t, \\
\hat{\pi}_t &= \delta_{\pi a} \hat{a}_{t-1} + \delta_{\pi \varepsilon} \hat{\mu}_t, \\
\hat{m}_t &= \hat{c}_t = \delta_{ca} \hat{a}_{t-1} + \delta_{c \varepsilon} \hat{\mu}_t, \\
\hat{R}_t &= \delta_{ra} \hat{a}_{t-1} + \delta_{r \varepsilon} \hat{\mu}_t.
\end{align*}
\]

Using that \( \delta_a \in (0, 1) \) and applying the method of undetermined coefficients, we are able to identify the signs of the impact multiplier which are summarized in the following proposition.

**Proposition 6 (Monetary policy effects, \( \eta_t > 0 \))** Suppose that prices are rigid and the open market constraint is binding. Then an expansionary money growth shock leads to

1. a rise in consumption and real balances \( (\partial C_t / \partial \varepsilon_t = \partial \hat{m}_t / \partial \varepsilon_t > 0) \),
2. a decline in real wealth \( (\partial a_t / \partial \varepsilon_t < 0) \) and the nominal interest rate \( (\partial \hat{R}_t / \partial \varepsilon_t < 0) \), and
3. a rise in the inflation rate \( (\partial \hat{\pi}_t / \partial \varepsilon_t > 0) \) if \( \vartheta > \check{\vartheta} \), with \( \check{\vartheta} = 1 - \beta \gamma_1 / (1 - \delta_a + \gamma_1) < 1 \).

**Proof.** See appendix 6.4.

Hence, the solution of the model reveals that a monetary injection leads to qualitative impact effects which are in accordance with common priors about monetary policy effects on real activity and prices. However, for the response of the latter to be positive, the degree of fiscal responsiveness should be sufficiently large \( (\vartheta > \check{\vartheta}) \). The model further predicts that real wealth declines in response to a monetary injection, which is mainly caused by the surge in

---

15
inflation. Most importantly, proposition 6 states that the model always generates a liquidity effect.

As in the case where prices were flexible, the latter result (a liquidity effect) can hardly be found in conventional sticky price models (see, e.g., Christiano et al., 1997, Galí, 2001, or, Andrès et al., 2002). In the version of our model where open market operations are irrelevant we find that a liquidity effects can only occur when the inverse of the elasticity of substitution, $\sigma \equiv -\frac{\sigma_{cc}}{\sigma_{u}}$, exceeds one given that the autocorrelation of the money growth process is sufficiently small.

**Proposition 7 (Monetary policy effects, $\eta_t = 0$)** Suppose that prices are rigid and that the open market constraint is not binding. Then the equilibrium is uniquely determined and a monetary expansion leads to

1. a rise in consumption, real balances, and inflation ($\partial c_t/\partial \varepsilon_t$, $\partial m_t/\partial \varepsilon_t$, $\partial R_t/\partial \varepsilon_t > 0$),
2. a decline in the nominal interest rate ($\partial R_t/\partial \varepsilon_t < 0$) if $\rho < 1 - \delta_m$ and $\sigma > \sigma(\rho)$, with $\sigma(\rho) \geq 1$, and $\sigma'(\rho) > 0$.

**Proof.** See appendix 6.5.

4 Interest rate policy, determinacy, and policy equivalence

Consider that the central bank sets the nominal interest rate $R$. Identifying the monetary policy instrument in this way is actually more realistic and in line with the recent monetary policy literature (see, Clarida, et al., 1999). However, several studies have shown that an interest rate policy can lead to price level indeterminacy and can easily destabilize the economy by allowing for multiple equilibrium paths (see, e.g., Benhabib et al., 2001, or, Carlstrom and Fuerst, 2001). In contrast, it can easily be shown that our model is not associated with nominal indeterminacy for an interest rate peg.\textsuperscript{10} This result is summarized in the following proposition.

**Proposition 8 (Price level determinacy, $\phi = 0$)** Suppose that prices are flexible. Then an interest rate peg $R_t = R$ is associated with

1. price level indeterminacy if the open market constraint not binding ($\eta_t = 0$), and
2. a uniquely determined price level if the open market constraint is binding ($\eta_t > 0$).

**Proof.** Recall that the equilibrium is defined for a given initial value for financial wealth $A_0$ and that financial wealth evolves according to $A_t = \alpha A_{t-1} \Rightarrow A_t = \alpha^t A_0$, with $\alpha \equiv$

\textsuperscript{10}This result is closely related to the finding in Canzoneri and Diba (2000) showing that prices level indeterminacy can be resolved when government bonds serve as a means of payment.
$(1 - \theta)R + \vartheta > 0$ (see definition 1). Hence, price level determinacy requires a uniquely determined value for real financial wealth \(a_t = A_t/P_t\). For \(\eta_t = 0\), an interest rate peg fixes the inflation rate by \(\pi = R\beta\). While the growth rate of real financial wealth is given by \(a_t/a_{t-1} = \alpha/(R\beta)\), its level cannot be determined. For \(\eta_t > 0\), real financial wealth is uniquely determined by \(a_t = a = Rc\), allowing for the determination of the price level by \(P_t = A_t/a_t = \alpha^t A_0/(cR)\).\\

Regarding the consensus monetary business cycle framework with sticky prices, which is represented by the version with \(\eta_t = 0\), local determinacy requires the central bank to set the nominal interest rate in an active way (see Woodford, 2001). On the other hand, an interest rate peg leads to local indeterminacy given that fiscal policy is Ricardian (\(\vartheta > 0\)). In contrast, the version of our model where the open market constraint is binding does not allow for multiple equilibrium paths in the case of an peg. This feature immediately follows from the property that a constant interest rate is equivalent to a constant money growth policy and from proposition 5.

**Proposition 9 (Policy equivalence, \(\phi > 0, \eta_t > 0\))** Suppose that prices are rigid and the open market constraint is binding. Then an interest rate peg \(R_t = R\) is associated with a uniquely determined equilibrium and leads to the identical fundamental solution as a constant money growth policy \(\eta_t = \mu\).

**Proof.** The claim made in the proposition immediately follows from the solution for the nominal interest rate \(\tilde{R}_t = -\pi \vartheta^{-1} \varepsilon_t^\mu\) implying \(R_t = R\) for a deterministic money growth rule \(\varepsilon_t = 0\) and from the local determinacy property derived in proposition 5.\\

The fact that the central bank can switch between its instruments for \(\eta_t > 0\) without altering the allocation and, thus, the determinacy properties of the model is not self-evident. It is one the one hand based on the fact that money acquisition is costly for households when \(R_t^d - R_t > 0\). On the other hand, the change in the policy instrument is not associated with a change in the state space dimension of the model given that real wealth remains the single predetermined state variable of the fundamental solution. On the contrary, the state space dimension changes with an instrument switch in the case where open market operations are irrelevant (\(\eta_t = 0\)). The reason for this is the validity of Ricardian equivalence which implies that the real wealth is now an irrelevant state variable. Hence, money growth policy introduces a new state variable (real balances), while interest rate policy leaves the set of endogenous states empty. Consequently, a central bank cannot replace an exogenous money growth policy by an exogenous interest rate policy without changing the allocation. This result is summarized in the following proposition.
Proposition 10 (Equilibrium indeterminacy, $\phi > 0$, $\eta_t = 0$) Suppose that prices are rigid and the open market constraint is not binding. Then an interest rate peg is non-equivalent with a constant money growth policy and leads to multiple equilibrium paths converging to the steady state.

Proof. The equilibrium conditions (31) and (32) imply that an interest rate peg implements a sequence of money growth rates satisfying $\mu_t = (1 - \sigma^{-1}) \hat{\pi}_t$, such that $\mu_t = \mu$ only holds if $\sigma = 1$. While this property only refers to a structural identity, we want to show that both regimes lead to non-equivalent solutions. This result follows from the fact that real balances $m_{t-1} = M_{t-1}/P_{t-1}$ is a relevant predetermined variable when the central bank sets the money growth rate in a sticky price environment, whereas the economy is entirely forward-looking in the case of an interest rate peg such that the fundamental solution exhibits no predetermined variable. Recalling that the coefficients on the endogenous state variable are non-zero in the case of the money growth policy, the state space representations are obviously non-equivalent for both regimes. The second claim made in the proposition directly follow from the determinacy condition presented in section 5 in Carlstrom and Fuerst (2001) for a model featuring the equilibrium conditions (31) and (33) for $\eta_t = 0$. ■

Turning back to the case where $\eta_t > 0$, an interest rate peg was shown to lead to real determinacy, while the stability of equilibrium paths is not yet guaranteed for the case where the nominal interest rate is set contingent on changes in inflation, such as in the often recommended Taylor-rule (see, Woodford, 2001). Consider the simple rule $\bar{R}_t = \rho_\pi \hat{\pi}_t$ and that the central bank chooses a high inflation elasticity $\rho_\pi$. When inflation rises then the fiscal authority can be forced to issue new debt – for $\vartheta$ smaller than one – if the associated rise in $R_t$ is sufficiently high such that debt obligations rise. Hence, a highly aggressive interest rate policy might lead to debt spirals when the fiscal authority is less responsible (small $\vartheta$).\textsuperscript{11} This results is summarized in the following proposition.

Proposition 11 (Equilibrium determinacy, $\phi > 0$, $\eta_t > 0$) Suppose that prices are rigid and the open market constraint is binding. Then an interest rate policy satisfying $\bar{R}_t = \rho_\pi \hat{\pi}_t$ is associated with a unique rational expectations equilibrium path converging to the steady state iff

$$\rho_\pi < \overline{\rho}_\pi,$$

with $\overline{\rho}_\pi = \frac{(1 - \vartheta)R + \vartheta}{(1 - \vartheta)\bar{R}} > 1$.

\textsuperscript{11}A similar outcome can occur in a sticky price model with overlapping generations (see Leith and Wren-Lewis, 2000).
Proof. In the case of the interest rate policy, the matrices in the general form of the $2 \times 2$ model in (36) takes the form

$$M_0 = \begin{pmatrix} \gamma_1 & \beta \\ 1 & 0 \end{pmatrix}, \quad M_1 = \begin{pmatrix} 0 & 1 + \gamma_1 \rho_\pi \\ 1 & \gamma_2 \rho_\pi - 1 \end{pmatrix}, \quad M_\xi = \begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix}.$$

The characteristic polynomial of $M_0^{-1}M_1$ which is given by $f(\delta_a) = \delta_a^2 - \{ (\gamma_1[(1 - \gamma_2) \rho_\pi + 1] + 1 + \beta)\delta_a - (1 + \gamma_1 \rho_\pi) \}/\beta$ is positive at $\delta_a = 0 : f(0) = \frac{1}{\beta} (1 + \gamma_1 \rho_\pi) > 0$ and is negative at $\delta_a = 0$ for $\rho_\pi < 1/\gamma_2 : f(1) = \frac{\gamma_1}{\beta} (\gamma_2 \rho_\pi - 1)$. Hence, the model exhibits one stable and one unstable eigenvalue if $\rho_\pi < \pi/(\pi - \vartheta) = [(1 - \vartheta)R + \vartheta]/[(1 - \vartheta)R]$. □

The stability condition presented in proposition 11 reveals that the model is stable regardless of $\rho_\pi$ when the fiscal authority runs a balanced budget policy ($\vartheta = 1$). On the contrary, a non-Ricardian regime ($\vartheta = 0$) would require a passive interest rate policy to escape explosiveness.

5 Conclusion

We developed a business cycle model with repurchase agreements, where only government bonds are accepted as collateral for money. When private debt earns a higher interest than government bonds, agents care about open market operations and the monetary stance depends on the nominal interest rate and the amount of government bonds outstanding. The model is able to generate a real and nominal expansion in response to monetary injections and solves the so-called liquidity puzzle regardless of prices being sticky or flexible. An interest rate peg is not associated with indeterminacy of the price level or the equilibrium. However, given that public debt holdings affects households’ access to money, debt policy interacts with monetary policy in that a low tax to debt ratio can destabilize the economy when interest rate setting is highly reactive to the state.
6 Appendix

6.1 Allowing for money accumulation

In this appendix, we examine when households are not willing to carry over money from one period to another. For this, we allow for households to accumulate cash $M^h(j, s^t) \geq 0$ starting with $M^h(j, s_0) = 0$. Further, the open market constraint changes to $B(j, s^t) \geq (M(j, s^t) + M^h(j, s^t) - M^h(j, s^{t-1})) R(s^t)$. The Lagrangian is given by

$$
\max L = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t g(s^t) \{ u(c(j, s^t), l(j, s^t))
$$

$$
+ \frac{\lambda(j, s^t)}{P(s^t)} \left[ \begin{array}{l}
R(s^t)B(j, s^t-1) + D(j, s^t-1) + P(s^t)w(s^t)l(j, s^t) \\
\int_{s^t}^{s_{t+1}} g(s|s^t)B(j, s|s^t) + D(j, s|s^t) + P(s|s^t)w(s|s^t)\int_{s^t}^{s_{t+1}} \omega(i, s^t)di - P(s^t)\tau(s^t), \\
- \sum_{s_{t+1}} z(s^t+1, s^t)g(s|s^t+1|s^t)D(j, s^t) - B(j, s^t) - P(s^t)c(j, s^t) \\
- (R(s^t) - 1) M(j, s^t) - R(s^t) (M^h(j, s^t) - M^h(j, s^{t-1})))
\end{array} \right]
$$

$$
+ \frac{\psi(j, s^t)}{P(s^t)} \left[ \begin{array}{l}
M(j, s^t) + M^h(j, s^t) + P(s^t)w(s^t)l(j, s^t) \\
\int_{s^t}^{s_{t+1}} g(s|s^t)B(j, s|s^t) + D(j, s|s^t) + P(s|s^t)w(s|s^t)\int_{s^t}^{s_{t+1}} \omega(i, s^t)di - P(s^t)\tau(s^t), \\
- (R(s^t) - 1) M(j, s^t) - R(s^t) (M^h(j, s^t) - M^h(j, s^{t-1}))
\end{array} \right]
$$

$$
+ \frac{\eta(j, s^t)}{P(s^t)} \left[ B(j, s^t) - \left( M(j, s^t) + M^h(j, s^t) - M^h(j, s^{t-1}) \right) R(s^t) \right]
$$

$$
+ \frac{\xi(j, s^t)}{P(s^t)} \left[ M^h(j, s^t) - M^h(j, s^{t-1}) \right]
$$

where $\xi$ measures the opportunity costs of cash holdings. Maximization then leads to the first order conditions (11)-(17) and the following condition for money holdings

$$
\xi(j, s^t) - \beta \sum_{s^t+1|s^t} g(s^t+1, s^t) \frac{\xi(j, s^t+1)}{\pi(s^{t+1})} = R(s^t) \left( \lambda(j, s^t) + \eta(j, s^t) \right) - \psi(j, s^t)
$$

$$
- \beta \sum_{s^t+1|s^t} g(s^t+1, s^t) \frac{R(s^t+1)}{\pi(s^{t+1})} \left( \lambda(j, s^t+1) + \eta(j, s^{t+1}) \right),
$$

$$
\xi(j, s^t) \left( M^h(j, s^t) - M^h(j, s^{t-1}) \right) = 0, \quad \xi(j, s^t) \geq 0, \quad M^h(j, s^t) - M^h(j, s^{t-1}) \geq 0,
$$

where a positive value $\xi(j, s^t)$ implies that households are not willing to accumulate cash $(M^h(j, s^t) - M^h(j, s^{t-1}) = 0)$. Using (11), (13)-(15) and considering $R^t(s^{t+1}) = \left[ z(s^{t+1}, s^t) \right]^{-1}$ we obtain:

$$
\xi(j, s^t) - \beta E_t \left[ \frac{\xi(j, s^t+1)}{\pi(s^{t+1})} \right] = \eta(j, s^t) - \beta E_t \left[ \frac{R(s^t+1)}{\pi(s^{t+1})} \eta(j, s^{t+1}) \right].
$$

(38)

Given that $\eta(j, s^t)$ (and $\xi(j, s^t)$) cannot be negative, the multiplier $\xi(j, s^t)$ can only be non-positive if either expected value of $\eta(j, s^t)$ times the real interest rate on government bonds
exactly offsets $\eta(j, s^t)/\beta$ or if $\eta(j, s^t) = E_t \eta(j, s^{t+1}) = 0$. Before we derive a sufficient condition which ensures that money is not accumulated, we will first demonstrate that households are indifferent in this regard when the latter holds.

Suppose that $\eta(j, s^t) = E_t \eta(j, s^{t+1}) = 0$ holds implying that open market operations are irrelevant and that $R(s^t) = R^d(s^t)$. Then the (38) reads $\xi(j, s^t) - \beta E_t [\xi(j, s^{t+1})/\pi(s^{t+1})] = 0$. Consider the case where prices are flexible such that wages are constant $w(s^t) = \bar{w}$ such that the households conditions (11) and (12) imply that consumption is constant $c(s^t) = c$. Then $R(s^t) = E_t \pi(s^{t+1})/\beta$ holds by (13) and (38) can be written as $\xi(j, s^t) [1 - E_t \xi(j, s^{t+1})/(R(s^t)\xi(j, s^t))] = 0$. When $\rho = 0$, the term in the square brackets is strictly positive implying that the multiplier $\xi(j, s^t)$ must be equal to zero.\textsuperscript{12}

To ensure that households are not willing to carry over cash from one period to the other it is sufficient for our purpose, i.e., analysis of the local dynamic properties, to derive a particular condition for an equilibrium with a time invariant state $\bar{\pi}$. For $s^t = s^{t+1} = \bar{\pi}$, the condition (38) can be written as

$$
\xi(j, \bar{\pi}) = \lambda(j, \bar{\pi}) \left[1 - \frac{R(\bar{\pi})}{R^d(\bar{\pi})}\right]^{\frac{2}{1 - 1/R^d(\bar{\pi})}},
$$

which implies that household $j$ is not willing to accumulate money if $R^d(\bar{\pi}) - R(\bar{\pi}) > 0$ given that a binding budget constraint (8) implies $\lambda(j, \bar{\pi}) > 0$. Hence, in a steady state where the open market constraint (9) holds with equality ($\eta > 0$, see proposition 4), households will voluntarily acquire money exclusively via repurchase agreements such that money is not carried over to the next period.

6.2 Steady state

The steady state of the model is assumed be characterized by stationary values for $\lambda$, $c$, $\pi$, $a$, $m$, $R^d$, $R$, and $\eta$ satisfying following conditions:

$$
\frac{u_c(\bar{\pi})}{u_l(\bar{\pi})} = \frac{\epsilon}{\epsilon - 1}, \quad c = m, \quad \frac{\pi}{\beta} = R^d, \quad \pi = (1 - \bar{\vartheta})R + \bar{\vartheta}, \quad \mu = \pi,
$$

$$
\frac{\eta}{\lambda} = \frac{R^d - R}{R^d}, \quad \lambda = \begin{cases} 
\frac{u_c(\bar{\pi}) [R (1 + \mu/\lambda)]^{-1}}{u_l(\bar{\pi})} & \text{if } \eta > 0, \\
u_c(\bar{\pi}) R^{-1} & \text{if } \eta = 0,
\end{cases} \quad m = \begin{cases} 
a/R & \text{if } \eta > 0 \\
c & \text{if } \eta = 0.
\end{cases}
$$

Further, $\mu$ and $\vartheta$ are set by the central bank and the fiscal authority, respectively.

\textsuperscript{12}Note that we assumed $R(s^t) > 1$ to hold in equilibrium (see definition 1).
6.3 Derivation of the aggregate supply constraint

Using a simple price rule for the fraction \( \phi \) of the firms (\( P_{lt} = \pi P_{l,t-1} \)), the price index for the final good \( P_t \) evolves recursively over time. In a symmetric equilibrium we obtain the following condition for the evolution of the price level: \( P_t^{1-\epsilon} = \phi (\pi P_{t-1})^{1-\epsilon} + (1 - \phi) \tilde{P}_t^{1-\epsilon} \), which can be written in stationary variables as:

\[
1 = \left( \phi (\pi_t)^{1-\epsilon} + (1 - \phi) \tilde{P}_{qt}^{1-\epsilon} \right) \frac{1}{1 - (1 - \phi) \int_0^1 \frac{1 - \phi}{1 - \phi} \pi_t \, dt}, \text{ with } \tilde{P}_{qt} = \frac{\tilde{P}_t}{P_t}, \text{ and } \pi_t = \frac{P_t}{P_{t-1}},
\]

(39)

where \( \pi \) denotes the percent deviation of \( x \) from its steady state value \( \pi^* \). Linearization of (39) at the steady state leads to: \( \frac{\tilde{\pi}_t}{1 - \phi} \pi_t = \tilde{P}_{qt} \). Further, we transform the first order condition for the firm’s optimal price \( \tilde{P}_{lt} \) (19) into:

\[
\tilde{P}_{qt} \frac{\epsilon - 1}{\epsilon} \sum_{s=0}^{\infty} (\beta \phi)^s E_t \left[ z_{t,t+s} y_{t+s} \pi_t^{1-\epsilon} + \gamma \pi_t^{1-\epsilon} \right] = \sum_{s=0}^{\infty} (\beta \phi)^s E_t \left[ z_{t,t+s} y_{t+s} \pi_t^{1-\epsilon} + \gamma \pi_t^{1-\epsilon} \right] m_{c,t+s} \pi_t^{1-\epsilon} \equiv s,
\]

(40)

where \( m_{c,t} = P_t^{1-\epsilon} / P_t \) denotes the retailer’s real marginal costs and \( \pi_{t,t+s} \) denotes a cumulative inflation rate: \( \pi_{t,t+s} = \frac{P_{t+s}}{P_t} = \prod_{k=1}^{s} \pi_{t+k} \). Linearizing equation (40) at the steady state we obtain:

\[
\sum_{s=0}^{\infty} (\beta \phi)^s \frac{\epsilon - 1}{\epsilon} \pi_t^{1-\epsilon} \gamma \pi_t^{1-\epsilon} E_t \left[ z_{t,t+s} y_{t+s} \pi_t^{1-\epsilon} + \gamma \pi_t^{1-\epsilon} \right] m_{c,t+s} \pi_t^{1-\epsilon} \equiv s
\]

(41)

Using \( \frac{\epsilon - 1}{\epsilon} \pi_t^{1-\epsilon} = \pi c(t) \) and substituting \( \pi q \) out with \( \frac{\phi}{1 - \phi} \pi_t = \tilde{P}_{qt} \), equation (41) can be simplified to:

\[
\frac{\phi}{1 - \phi} \pi_t = (1 - \beta \phi) \sum_{s=0}^{\infty} (\beta \phi)^s E_t \left[ \pi_{t,t+s} + m_{c,t+s} \right].
\]

(42)

Taking the period \( t + 1 \) version of (42) times \( \beta \phi \) and subtracting from (42), gives:

\[
\frac{\phi}{1 - \phi} (\pi_t - \beta \phi E_t [\pi_{t+1}]) = (1 - \beta \phi) \left( m_{c,t} - \beta \phi \sum_{s=0}^{\infty} (\beta \phi)^s E_t [-\pi_{t+1}] \right).
\]

(43)

Rewriting equation (43) leads to the ’New Keynesian Phillips Curve’ (30).

6.4 Proof of proposition 6

Using the general solution form in (37) to replace the endogenous variables in the equilibrium equations (31)-(35), we obtain the following conditions for the undetermined coefficients \( \delta_n \).
\[ \delta_{\pi a}, \delta_{ae}, \text{and } \delta_{ae} \text{ effects} \]

\[ \gamma_1 \delta_a + \beta \delta_{\pi a} \delta_a - \delta_{\pi a} = 0, \quad \gamma_1 \delta_{ae} + \beta \delta_{\pi a} \delta_{ae} + \gamma_1 \frac{\pi}{\vartheta} - \delta_{\pi e} = 0, \]

\[ \delta_a - 1 + \delta_{\pi a} = 0, \quad \delta_{ae} + \delta_{\pi e} + \frac{\pi}{\vartheta} = 0. \]

Manipulating these conditions, we obtain the following impact multiplier on inflation and real wealth

\[ \delta_{\pi e} = \frac{1}{\vartheta} \gamma_1 + (\vartheta - \pi)(1 - \delta_a) \beta, \quad \delta_{ae} = -\frac{1}{\vartheta} \gamma_1 + (1 - \beta \rho)(\pi - \vartheta). \]

Hence, \( \delta_{ae} \) is strictly positive when \( \vartheta \gamma_1 + (\vartheta - \pi)(1 - \delta_a) \beta > 0 \). Using that \( \mu \) is assumed to be strictly smaller than \( \varpi \equiv \vartheta/[1 - (1 - \vartheta)/\beta] \), we can conclude that \( \delta_{ae} \) is strictly positive if \( \vartheta > 1 - \beta \gamma_1/(1 - \delta_a + \gamma_1) \). The coefficient \( \delta_{ae} \) is further used together with the solution for \( \hat{c}_t, \hat{c}_t = \delta_a \hat{a}_{t-1} + (\delta_{ae} + \frac{\varrho}{\vartheta}) \varepsilon_t \), to derive the impact multiplier on consumption and real balances \( \delta_{ce} \), which reads

\[ \delta_{ce} = \frac{1}{\vartheta} \gamma_1 + \pi \frac{\beta (1 - \delta_a)}{1 - \vartheta \beta} > 0. \]

Combining the conditions (34) and (35) further gives \( \hat{R}_t = -(\mu/\vartheta) \varepsilon_t \) and, thus, \( \delta_{ce} = -\mu/\vartheta < 0 \), which completes the proof of proposition 6.

### 6.5 Proof of proposition 7

When open market operations are not binding the model given in definition 2 can be reduced to the following 2 \times 2 system in real balances and inflation,

\[ \gamma_1 \tilde{m}_t + \beta \mathbb{E}_t \tilde{\pi}_{t+1} = \tilde{\pi}_t \]

\[ \tilde{m}_t = \tilde{m}_{t-1} - \tilde{\pi}_t + \tilde{\mu}_t, \quad \text{with} \quad \tilde{\mu}_t = \tilde{\rho} \tilde{m}_{t-1} + \varepsilon_t. \]

Given that real balances \( \tilde{m}_{t-1} \) are predetermined, real determinacy requires the existence of exactly one stable eigenvalue. To establish the latter, the model is rewritten in matrix form

\[ M_0^f \left( \begin{array}{c} \hat{m}_t \\ E_t \hat{\pi}_{t+1} \end{array} \right) = M_1^f \left( \begin{array}{c} \hat{m}_{t-1} \\ \hat{\pi}_t \end{array} \right) + \left( \begin{array}{c} 0 \\ 1 \end{array} \right) \hat{\mu}_t, \quad \text{with} \quad M_0^f \equiv \left( \begin{array}{cc} \gamma_1 & \beta \\ 1 & 0 \end{array} \right), \quad M_1^f \equiv \left( \begin{array}{cc} 0 & 1 \\ 1 & -1 \end{array} \right). \]

The characteristic polynomial of \((M_0^f)^{-1} M_1^f \), thus, reads \( f(X) = X^2 + \frac{\beta - \gamma_1}{\beta} X + \frac{1}{\beta} \). As \( f(0) \) is equal to \( 1/\beta \) and, therefore, strictly positive and \( f(1) \) is negative \( f(1) = -\gamma_1/\beta < 0 \), the model exhibits exactly one stable eigenvalue lying between zero and one. In order to establish the claims made in part 1 and 2 of the proposition, we use the following general
The solution form (46) is then used to replace the endogenous variables in (44) and (45), yielding the following conditions for the undetermined coefficients $\delta_m$, $\delta_{m\mu}$, $\delta_{\pi m}$, and $\delta_{\pi \mu}$

$$
\begin{align*}
\gamma_1 \delta_m + \beta \delta_{\pi m} \delta_m - \delta_{\pi m} &= 0, \\
\delta_{\pi \mu} - \gamma_1 \delta_{m\mu} - \beta \delta_{\pi m} \delta_{m\mu} - \beta \delta_{\pi \mu} \rho &= 0, \\
\delta_m + \delta_{\pi m} - 1 &= 0, \\
\delta_{m\mu} + \delta_{\pi \mu} - 1 &= 0.
\end{align*}
$$

Using that $0 < \delta_m < 1$ is already established, the claims made in part 1 follow from the following expressions for the coefficients $\delta_{m\mu}$, $\delta_{\pi m}$, and $\delta_{\pi \mu}$

$$
\delta_{\pi \mu} = \left[\gamma_1 + \beta (1 - \delta_m)\right]/\gamma_3 > 0, \quad 0 < \delta_{m\mu} = (1 - \beta \rho)/\gamma_3 < 1, \quad 0 < \delta_{\pi m} = 1 - \delta_m < 1,
$$

where $\gamma_3 \equiv 1 - \beta \rho + \gamma_1 + \beta (1 - \delta_m) > 0$. Turning to part 2 of the proposition, we want to derive the sign of the interest rate response $\partial \hat{R}_t/\partial \hat{\mu}_t$. For this, we apply the following structural equation which governs the nominal interest rate response for $\eta_t = 0$ (see definition 2): 

$$
\hat{R}_t = \sigma (\hat{c}_{t+1} - \hat{c}_t) + E_t \hat{\pi}_{t+1}.
$$

Using the cash-constraint and the constant money growth rule, we obtain the condition \( \hat{R}_t = \sigma \rho \hat{\mu}_t + (1 - \sigma) E_t \hat{\pi}_{t+1} \), which determines the nominal interest rate for given sequences of inflation, real balances, and the money growth rate. Applying the solution form (46) the fundamental solution, thus, reads

$$
\hat{R}_t = \delta_{rm} \hat{m}_{t-1} + \delta_{r\mu} \hat{\mu}_t, \quad \text{with} \quad \delta_{rm} \equiv (1 - \sigma) (1 - \beta \rho) [(1 - \delta_m) - \rho] / \gamma_3 + \rho, \quad \delta_{r\mu} \equiv (1 - \sigma) \delta_{\pi m} \delta_m.
$$

Given that we are interested in the case, where $\sigma$ takes reasonable values ($\sigma \geq 1$), it can immediately be seen that the impact response $\partial \hat{R}_t/\partial \hat{\mu}_t$ is $\delta_{rm}$ cannot be negative if $\rho \geq 1 - \delta_m$. Hence, we consider the case where $(1 - \delta_m) > \rho$. Then, the partial derivative of the coefficient $\delta_{rm}$ with respect to $\rho$, which reads,

$$
\frac{\partial \delta_{rm}}{\partial \rho} = (\sigma - 1) \left( \frac{\gamma_1 \beta - \beta [1 - \delta_m] - \rho + (1 - \beta \rho) \rho + (1 - \beta \rho)^2}{\{1 - \beta \rho + \gamma_1 \beta (1 - \delta_m) \}^2} \right) + 1 > 0,
$$

is positive such that $\delta_{rm}$ rises with the autocorrelation of the money growth rate for this case. However, the coefficient $\delta_{rm}$ can take negative values for a sufficiently large values for $\sigma : \sigma > \sigma$. The lower bound $\sigma$ can be expressed as a function of $\rho$, $\delta_m$, $\beta$ and $\gamma_1$

$$
\sigma(\rho) = \frac{\rho \gamma_1 + (1 - \delta_m)}{(1 - \beta \rho) [(1 - \delta_m) - \rho]}, \quad \text{with} \quad \sigma(\rho) \geq 1, \quad \sigma'(\rho) > 0 \ \forall \rho \in (0, 1 - \delta_m).
$$

This completes the proof of the claims made in the proposition. ■
7 References


