Optimal Monetary Policy and Asset Price Misalignments

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\textbf{Abstract}

This paper analyses the relationship between monetary policy and asset prices in the context of optimal policy rules. The transmission mechanism is represented by a linearized rational expectations model augmented for the effect of asset prices on aggregate demand. Stabilization objectives are represented by a discounted quadratic loss function penalizing inflation and output gap volatility. Asset prices are allowed to deviate from their intrinsic value since they may be positively affected by past price changes. We find that in the presence of wealth effects and inefficient markets, asset price misalignments from their fundamentals should be included in the optimal interest rate reaction function.

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1. Introduction

Over the last twenty years, significant changes have occurred in the institutional and macroeconomic framework that central banks operate. In particular, there has been a widespread move towards financial liberalization, both within and across national borders, especially after the 1980s, while inflation rates have become lower and less variable. The disinflation process of the 1990s has been a global phenomenon since it is observed both in countries where formal inflation targets are in use, and in non-targeting countries\(^1\). The decline in inflation has gone hand in hand with a similar decline in interest rates. In many countries, both short term and long term interest rates are close to, or even bellow, post-war lows. As Bean (2003) argues, price stability has not been achieved at the expense of the real economy, as unemployment has been decreasing in a number of countries, while growth has also been relatively stable. Despite the good macroeconomic record of the past decade, there has been a growing concern among academics and policymakers that the achievement of price stability may be associated with an increased risk of financial instability.

Some commentators claim that the lower cost of capital along with exuberant growth projections have boosted the late 1990s stock market bubble. For instance, Borio and Lowe (2002) argue that booms and busts in asset prices should be considered as part of a broader set of symptoms that typically also include a build-up of debt and high rate of capital accumulation. Rising asset prices and debt accumulation lead to stretched household and corporate balance sheets, vulnerable to sharp corrections of the type witnessed recently in global equity markets. In a series of articles, Goodhart and Hofmann (2000, 2003) establish empirically the link between output growth, credit aggregates, and asset price movements in a number of major economies. Kiyotaki and Moore (1997) develop a theoretical model that exhibits a crucial interaction between collateral values, asset prices, credit and economic activity. During the period of boom,
balance sheets may look healthy as the increase in asset prices, and consequently the value of the collateral, offsets the build-up of debt. However, when optimism about further increases in asset values turns to pessimism, leading to a decrease in the net worth of households and firms, then financial distress may be the result of financial imbalances unwinding. It has been argued that the widespread financial deregulation of asset markets may have contributed to an increase in the frequency of such boom-bust episodes (IMF, 2003).

An important issue related to the above concerns is the establishment of the appropriate monetary policy response to asset price movements. Should the central bank care about the financial instability associated with large asset price fluctuations? Nowadays, everyone recognizes price level stability as the primary objective of monetary policy. Indeed, as Issing (2003) emphasizes, price stability and financial stability tend to mutually reinforce each other in the long run. However, as the examples of the US in the 1920s and 1990s and Japan in the late 1980s demonstrate, financial imbalances may build up even in an environment of stable prices (Borio and Lowe, 2002). Exponents of the ‘new environment’ hypothesis argue that low and stable rates of inflation may even foster asset price bubbles, due e.g. to excessively optimistic expectations about future economic development. Thus, price stability is not a sufficient condition for financial stability. Among the exponents of the new environment hypothesis, Crocket (2003) claims that: “…if the monetary policy reaction function does not incorporate financial imbalances, the monetary anchor may fail to deliver financial stability”. The current consensus however, stresses that monetary policy should be directed exclusively at achieving price stability, and its role in promoting financial stability should be restricted to minimising the negative effects from bubbles bursting and financial imbalances unwinding.²

¹ See e.g. Johnson (2002) for international evidence.
² For instance, Alan Greenspan (2002) argues that: “The notion that a well-timed incremental tightening could have been calibrated to prevent the late 1990s bubble is almost surely an illusion. Instead, we...need to focus on policies to mitigate the fallout when it occurs and, hopefully, ease the transition to the next expansion.”
A number of studies have tried to provide an answer to the question of whether monetary policy should respond to asset prices, by simulating macroeconomic models where aggregate demand is affected by consumption wealth effects and/or investment balance sheet effects. The simulation evidence of Bernanke and Gertler (1999, 2001) opts for a *reactive* monetary policy response since they show that a central bank dedicated to price stability should pay no attention to asset prices per se, except insofar as they signal changes to expected inflation (see also Gilchrist and Leahy, 2002). On the other hand, Cecchetti, Genberg, Lipsky and Wadhwani (2000), and Kontonikas and Ioannidis (2003) find that, in line with the new environment *proactive* view, overall macroeconomic volatility can be reduced with a (mild) reaction of interest rates to asset price misalignments from fundamentals. Also, recent econometric evidence by Kontonikas and Montagnoli (2003) for the UK, and Chadha, Sarno and Valente (2003) for UK, US and Japan, suggests that monetary policymakers may use asset prices not only as part of their information set for setting interest rates, but also as elements in their reaction function.

All the aforementioned papers use the assumption that monetary policy is characterised by an augmented Taylor rule, where the nominal interest rate responds positively to inflation, demand pressures, and asset prices. Following the seminal work by Taylor (1993), feedback rules conditioning the interest rate instrument on current or expected inflation and the output gap have been extensively analysed in both theoretical and empirical literature. Svensson (1997), Clark, Goodhart and Huang (1999) among others, show that such a feedback rule is optimal in that it derives from the first order condition for the optimisation of the central bank’s objectives\(^3\). In this paper, we try to shed some more light in the relationship between monetary policy and asset prices in the context of optimal policy rules. In essence, we will examine whether there is any

\[^3\] One should keep in mind though, that simple instrument rules like the Taylor rule and its variants may not correspond to fully optimal policy in the context of a particular economic model (see e.g. Woodford, 2001). Also, as Svensson, (2003) argues, no central bank has so far made a commitment to a simple instrument rule like the Taylor rule or variants thereof. In addition, neither has any central bank announced a particular instrument rule as a guideline.
underlying theoretical motivation for the increasingly frequent assumption of an augmented (for asset prices) Taylor rule. To do so, we start from a backward-looking structural macro model where asset prices affect future inflation indirectly, through direct wealth effects on aggregate demand. In our model, market inefficiency implies that asset prices may deviate from their fundamental value due to ‘momentum’ effects from past asset price changes.

The optimality conditions suggest that monetary policy should respond to asset price misalignments from their fundamental value, with the aggressiveness of the response being a positive function of the impact of asset prices on aggregate demand. This result has important implications for the conduct of monetary policy and contributes crucially to the existing literature, as previous work on optimal rules considering asset prices, either fails to find a role for asset prices (Bean, 2003), or obtains complex, non linear rules (Bordo and Jeanne, 2002).

The remainder of the paper is structured as follows. Section 2 describes the theoretical model that will be employed, while section 2.1 focuses on the asset price block of the model and provides econometric evidence to support the chosen specification. In section 2.2 the model is solved, and in section 3 we calculate the optimal interest rate rule based upon dynamic optimization of the central bank’s objectives. In section 3.1 we analyze the results with a special interest on the interaction between the magnitude of wealth effects and the interest rate reaction coefficients. Section 4 concludes.

2. The model

We use a structural backward-looking model of a (closed) economy that allows for the effect of asset prices on aggregate demand. The model augments the standard macroeconomic system (aggregate demand, aggregate supply) by taking into account asset prices, which themselves are assumed to stochastically evolve influenced by both fundamentals and momentum. The model is given by the following equations:
\[ \pi_{t+1} = \pi_t + ay_{t+1} + \epsilon_{t+1} \quad (1) \]
\[ y_{t+1} = \beta_1 y_t - \beta_2 (i_t - E_t[\pi_{t+1}]) + \beta_3 q_t + \eta_{t+1} \quad (2) \]
\[ q_t = q^*_t + b\Delta q_{t-1} \quad (3) \]
\[ q^*_t = -\delta_1 (i_t - E_t[\pi_{t+1}]) + \delta_2 E_t[y_{t+1}] + u_t \quad (4) \]

where \( y_t \) is the deviation of (log) output from its steady-state level (output gap), \( \pi_t = p_t - p_{t-1} \) is the inflation rate (strictly, the deviation from target), \( p_t \) is (log) price level, \( i_t \) is the monetary policy instrument (one-period nominal interest rate), \( q_t \) denotes (log) real asset prices and \( q^*_t \) the fundamentals. Different interpretations of \( q_t \) are possible (e.g. house prices, stock prices or the value of a portfolio containing both housing and equity investment), in what follows though we mainly treat it as an equity index. \( \eta_t, \epsilon_t, u_t \) represent exogenous random shocks to aggregate demand, inflation, and asset price fundamentals. For simplicity, we assume that they are mutually uncorrelated i.i.d. processes with zero means and constant variances. The structural parameters can be interpreted as partial elasticities with the following properties: \( 0 < \beta_1 < 1; \ a, \ \beta_2, \ \delta_1, \ \delta_2, > 0; \ \beta_3 \geq 0, 0 \leq b < 1. \)

Eq. (1) is an accelerationist (or backward-looking NAIRU type) Phillips Curve where the change in inflation is a positive function of the current output gap and the inflation shock. The presence of inflation inertia in the inflation equation implies that disinflations will be costly in terms of output losses, thus there is a short-run trade-off between inflation and the output. However, since lagged inflation enters Eq. (1) with unity coefficient, this specification implies a vertical long-run Phillips curve. Eq. (1) posits no role for expected future inflation in the inflation adjustment equation\(^4\). Fuhrer (1997) employed US inflation data and argued that forward looking

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\( ^4 \) Similar specification has been used by Svensson (1997) and Rudebusch and Svensson (1999), with the difference that their model exhibits a two-periods control lag for inflation. Rudebusch (2002) considers the hybrid Phillips
expectations are unimportant empirically. The parameter $\alpha$ is a positive constant which measures the sensitivity of inflation to excess demand$^5$.

The demand side, as given by Eq. (2), is consistent with the specification employed by Walsh (1998), Ball (1997), and Svensson (1997), with one important difference: aggregate demand depends positively on the past level of asset prices via consumption wealth effects and investment balance sheet effects. For example, a persistent increase in the level of asset prices decreases the perceived level of households’ financial distress causing a boost in consumption spending. The balance sheet channel implies a positive relationship between the firms’ ability to borrow and their net worth which in turn depends on asset valuations. There is a vast amount of empirical evidence indicating that stock and house price movements are strongly correlated with aggregate demand in most major economies$^6$. Parameter $\beta_3$ in the aggregate demand is of crucial interest since it indicates the magnitude of wealth effects. If there are no wealth effects then $\beta_3 = 0$ and Eq. (2) resembles a traditional dynamic IS curve. In our model, the central bank takes into account the effect of wealth on aggregate demand, that is, it is fully aware of the effect of $q_t$ on $y_{t+1}$ and its magnitude.

2.1 Asset price dynamics

Apart from augmenting aggregate demand to account for the effect of asset prices, our own contribution is to append Eqs. (3) and (4) to the standard model, representing the dynamic evolution of asset prices, $q_t$, and their underlying fundamentals, $q_t^*$, respectively. In order to depict actual financial market behavior, Eq. (3) indicates that observed asset prices are not always equal to their fundamental value. The Efficient Markets Hypothesis (EMH) postulates that all

\[
\pi_{t+1} = \mu_\pi \pi_t + (1-\mu_\pi)E_t[\pi_{t+1}] + a_{y_{t+1}} + \epsilon_{t+1},
\]

and points out that the accelerationist Phillips curve ($\mu_\pi \approx 1$) can be derived from well-known models of price-setting behavior (see e.g. Roberts, 1995).

$^5$ As Clark, Goodhart, and Huang (1999) point out, there are good reasons to believe that $\alpha$ is not constant. However, the assumption of linearity in the Phillips curve helps to obtain a closed-form solution for the optimal feedback rule.
information required to determine the intrinsic asset value will, by actions of rational profit-maximizing agents, be reflected in the actual market price; hence $b = 0$ and $q_t = q_t^*$. In the context of the EMH, the asset price changes if and only if the market receives new information about the asset’s underlying economic fundamentals, and the actions of speculators are stabilising, in that they drive the actual asset price towards its fundamental value rather than away from it (e.g. by buying underpriced assets and selling overpriced ones).

However, the central tenets of the EMH, that future prices are not affected by past movements in the asset price and that speculation can only have a stabilizing effect have never been quite accepted by market participants. As Kortian (1995) argues, there are several aspects of modern asset markets trading, which are clearly contrary to the sort of behavior implied by the EMH. For instance, the widespread use of technical analysis, that tries to use past asset price movements to predict future prices. Also, the frequent employment of stop-loss orders (selling orders which are activated once the asset price has fallen by a particular pre-determined amount), and the development of dynamic hedging strategies, such as portfolio insurance, according to which, investors buy in a rising market and sell into a falling one. All the aforementioned strategies, base investment decisions upon past price movements and agree with the view that investors from time-to-time act in a destabilizing manner. Economic history also provides plenty examples of destabilizing investor behavior with significant implications for asset prices and aggregate economic activity beginning as early as the seventeenth century\(^7\).

Incorporating these arguments in our analysis, Eq. (3) indicates that, if asset prices have increased in the past ($\Delta q_{t-1} > 0$) there is a positive ‘momentum’ effect on their current level ($b > 0$). In essence, investors bid up the demand for asset holdings in expectation that past capital gains will persist in the future. The higher the value of $b$, the stronger the effect from past asset

\(^7\) See among others, Kontonikas and Montagnoli (2003) for relevant empirical evidence considering the UK economy.
price changes and therefore \( q_t \) can diverge significantly from its fundamental value, \( q^*_t \), albeit not permanently\(^8\). But once asset prices revert, at an unknown future date, the downward effect on aggregate demand could be large. Eq. (3) is essentially a backward-looking version of the Frenkel and Mussa (1985) asset price equation\(^9\). Stability of the asset price path requires that the parameter \( b \) satisfies: \( 0 < b < 1 \). Eq. (4) describes fundamental asset prices in line with the standard dividend model of asset pricing. There is a positive effect from expected future dividends (assumed to depend on expected output) and a negative effect from real interest rates. This is supported by the majority of empirical studies examining the effect of macroeconomic variables on the stock market\(^10\). We also allow for uncertainty in the fundamentals’ process by including the random disturbance term, \( u_t \).

In order to gain some further insight on the suitability and empirical validity of the asset price block of our model, we substituted Eq. (4) in Eq. (3) and took 1\(^{st}\) differences to obtain an econometrically estimatable expression:

\[
\Delta q_t = -\delta_1 \Delta (i_t - E_t[\pi_{t+1}]) + \delta_2 \Delta E_t[y_{t+1}] + b \Delta^2 q_{t-1} + \xi_t
\]

where \( \xi_t = \Delta u_t \)

Eq. (3)\(^\prime\) implies that real asset returns are negatively related to changes in the real interest rate, and positively affected by (upward) revisions in output expectations, and past asset returns. Eq. (3)\(^\prime\) will be estimated using quarterly data for the United Kingdom and the United States over

\(^7\) See Garber (2000) for a discussion on the tulip mania in the early seventeenth century as well as other famous bubbles.

\(^8\) We do not regard the divergence of \( q_t \) from \( q^*_t \) as an explicit bubble because we do not assign any probabilistic structure to its evolution.

\(^9\) Frenkel and Mussa (1985) argue that a wide range of structural models for exchange rate determination can be subsumed under the reduced form asset price expression: \( q_t = q^*_t + b E_t[\Delta q_{t+1}] \).

the period 1966-2002, 1958-2002, respectively\textsuperscript{11}. To do so, expected future inflation and output are replaced with their ex post actual values, leading to Eq. (3.1)\textsuperscript{'}:

\[ \Delta q_t = -\delta_1 \Delta (i_t - \pi_{t+1}) + \delta_2 \Delta y_{t+1} + b \Delta^2 q_{t-1} + \xi_t \]  

(3.1)\textsuperscript{'}

The set of orthogonality conditions implied by Eq. (3.1)\textsuperscript{'} is:

\[ E_t \left[ \Delta q_t + \delta_1 \Delta (i_t - \pi_{t+1}) - \delta_2 \Delta y_{t+1} - b \Delta^2 q_{t-1} \right] = 0 \]

where \( Z_t \) is a vector of instruments, that is, lagged variables that help to forecast the change in inflation and output, and contemporaneous variables that are uncorrelated with the exogenous asset returns shock, \( \xi_t \).

The Generalised Method of Moments (GMM) estimation results in Table 1 used as instruments: a constant and six lags of the change in: nominal short-term interest rates, inflation, output gap, and real stock prices. Since the number of instruments is greater than the number of elements of the parameter vector \( [\delta_1, \delta_2, b] \), we test for the validity of the over-identifying restrictions using Hansen’s \( J \)-statistic.

[Table 1 about here]

The results indicate that, contrary to the EMH, real returns are not only affected by economic fundamentals, but also from their past history since the \( b \) coefficient is positive and statistically significant at the 1% level in both UK and USA. The \( b \) coefficient obtains values in the range (0,1) ensuring that the dynamic stability criterion in Eq. (3) is satisfied. There is also a negative effect from a monetary policy tightening \(( -\delta_1 = -0.024, -0.016 \text{ in UK, USA})\) and a strong positive effect from higher output \(( \delta_2 = 1.39, 1.54 \text{ in UK, USA})\). Finally, the \( J \)-statistics indicate that the over-identifying restrictions are not rejected.

\textsuperscript{11} Inflation was calculated as the log change of the GDP deflator; output gap was proxied by the deviation of real GDP from a Hodrick-Prescott trend; UK, USA real stock returns were calculated as the log change of the FTSE All Shares and Dow Jones, respectively; nominal short term interest rate was proxied by the 3 month Treasury Bill rate.
2.2 Solution of the model

The structure of our model implies that monetary policy affects the stock market contemporaneously, and inflation and output with one period lag. At time $t$, the central bank chooses $i_t$ which affects concurrent real asset prices and next period’s inflation and output gap, while contemporaneous inflation and output gap are predetermined by previous decisions and current exogenous shocks.

Substituting for fundamentals, $q^*_t$, in Eq. (3) we get an alternative expression for real asset prices:

$$q_t = b \Delta q_{t-1} - \delta_i (i_t - E_t[\pi_{t+1}]) + \delta_2 E_t[y_{t+1}] + u_t$$

We then use Eq. (5) and the expectational version of Eq. (1) to eliminate $q_t$, $E_t[\pi_{t+1}]$ from the aggregate demand Eq. (2):

$$y_{t+1} = \beta_1 y_t - (\beta_2 \delta_1 + \beta_2)(i_t - \pi_t) + \left[a(\beta_2 \delta_2 + \delta_2) + \beta_3 \delta_2 \right] E_t[y_{t+1}] + \beta_3 b \Delta q_{t-1} + v_{t+1}$$

where $v_{t+1} = \beta_3 u_t + \eta_{t+1}$

Taking expectations on both sides of the above expression, conditional upon time $t$ information, yields the following expression for $E_t[y_{t+1}]$:

$$E_t[y_{t+1}] = \lambda_1 y_t - \lambda_2 (i_t - \pi_t) + \lambda_3 b \Delta q_{t-1}$$

where

$$\lambda_1 = \frac{\beta_1}{1 - [a(\beta_2 \delta_1 + \beta_2) + \beta_3 \delta_2]}, \quad \lambda_2 = \frac{\beta_2 \delta_2 + \beta_2}{1 - [a(\beta_2 \delta_1 + \beta_2) + \beta_3 \delta_2]}, \quad \lambda_3 = \frac{\beta_3}{1 - [a(\beta_2 \delta_1 + \beta_2) + \beta_3 \delta_2]}$$

Using Eq. (7) to eliminate $E_t[y_{t+1}]$ from Eq. (6) and rearranging, gives:

$$y_{t+1} = \lambda_1 y_t - \lambda_2 (i_t - \pi_t) + \lambda_3 b \Delta q_{t-1} + v_{t+1}$$

We define $\varphi_t$ as the control variable of the central bank), since $\pi_t$, $y_t$ are predetermined when $i_t$ is chosen.

$$\varphi_t = \lambda_1 y_t - \lambda_2 (i_t - \pi_t) + \lambda_3 b \Delta q_{t-1}$$
Thus, the original system of equations (1-4) can be written compactly in terms of $\phi_i$ as:

$$\pi_{t+1} = \pi_t + a\phi_t + \omega_{t+1}$$  \hspace{1cm} (1)'

$$y_{t+1} = \phi_t + v_{t+1}$$  \hspace{1cm} (2)'

where $\omega_{t+1} = \varepsilon_{t+1} + av_{t+1}$

3. Optimal interest rate rule

The central bank objective is to solve the following stochastic control problem: choose an infinite sequence of controls, $\phi_i$, to minimise the expected discounted value of the intertemporal quadratic loss function that penalizes both inflation and output gap volatility:

$$\min_{\{\phi_i\}_{i=0}^\infty} \frac{1}{2} E_i \sum_{i=1}^{\infty} \beta^i \left[ \pi_{t+i}^2 + \mu y_{t+i}^2 \right]$$  \hspace{1cm} (10)

subject to the transition Eqs. (1)' and (2)'

where $\mu \geq 0$ is the relative weight attached by the central bank on output stabilisation. $\beta$ is the discount factor, $0 < \beta < 1$. In the absence of discounting, the postulated loss function is a weighted average of conditional volatility of inflation and output. It is evident from (2)' that at time $t$, when the interest rate (and consequently $\phi_t$) is chosen the only state variable is $\pi_t$. Therefore, the value function is defined in terms of $\pi_t$ only, $V(\pi_t)$. Applying Bellman’s dynamic programming principle, and substituting for the two constraints (1)' and (2)' in the value function, we obtain:

$$V(\pi_t) = \min_{\phi_t} E_i \left\{ \frac{1}{2} \left[ (\pi_t + a\phi_t + \omega_{t+1})^2 + \mu(\phi_t + v_{t+1})^2 \right] + \beta V(\pi_t + a\phi_t + \omega_{t+1}) \right\}$$  \hspace{1cm} (11)

The first order condition with respect to $\phi_t$ and the envelope theorem allow to derive an expression for the optimal path of the control variable $^{12}$:

$$\phi_t = -\left( \frac{a}{a^2 + \mu} \right) \pi_t + \left( \frac{\mu\beta}{a^2 + \mu} \right) E_i[\phi_{t+1}]$$  \hspace{1cm} (12)

$^{12}$ See Appendix A.1 for more details.
Since we have a linear-quadratic structure in the stochastic control problem the solution will be of the form:

\[ \varphi_t = c \pi_t \]  

(13)

Thus the optimal control will be linear function of the state variable (see Walsh, 1998). Updating one period ahead and taking expectations at time \( t \) of Eq. (13) yields:

\[ E_t[\varphi_{t+1}] = (1 + ca)\varphi_t \]  

(14)

Substitution of Eqs. (13), (14) in Eq. (12) yields the following quadratic equation, whose solution gives the optimal \( c \) value:

\[(\mu \beta a)c^2 + (\mu \beta - a^2 - \mu)c - a = 0\]  

(15)

The solution that we accept should satisfy the inflation process stability criterion. This condition implies that only the negative \( c \)-root is accepted\(^{13}\). Finally, we manage to obtain the optimal path for the interest rate using Eqs. (9) and (13), substituting for \( \lambda_1, \lambda_2, \lambda_3 \), and re-arranging.

\[ i_t = \left(1 - \frac{c[1 - A]}{\beta_1 \delta_1 + \beta_2} \right) \pi_t + \left( \frac{\beta_1}{\beta_1 \delta_1 + \beta_2} \right) y_t + \left( \frac{\beta_2 \beta}{\beta_3 \delta_1 + \beta_2} \right) \Delta q_{t-1} \]  

(16)

where \( A = a(\beta_1 \delta_1 + \beta_2) + \beta_3 \delta_2 \)

Hence, the nominal interest rate reacts to the current output gap, current consumer price inflation, and past asset price inflation. Eq. (16) can be transformed into a more intuitive expression by recalling that, according to Eq. (3), the deviations from fundamentals are a positive function of past asset price changes:

\[ q_t - q_t^* = b\Delta q_{t-1} \]  

(17)

Hence, via Eq. (17), the final expression for the optimal interest rate rule is:

\[ i_t = f_x \pi_t + f_y y_t + f_{q-q^*} \left[ q_t - q_t^* \right] \]  

(18)

\(^{13}\) See Appendix A.2 for more details.
where \( f_x = 1 - \frac{c[1-A]}{\beta_3 \delta_1 + \beta_2} \), \( f_y = \frac{\beta_1}{\beta_3 \delta_1 + \beta_2} > 0 \), \( f_{q-q} = \frac{\beta_3}{\beta_3 \delta_1 + \beta_2} \geq 0 \), are the respective interest rate weights on inflation, output and asset price misalignments from fundamentals. The ‘Taylor principle’ implies that the inflation coefficient, \( f_x \), should exceed the value of one, to ensure a real interest rate response that will lead to lower inflation.\(^{14}\)

### 3.1 Analysis of the results

The rule for adjusting nominal interest rates shown in Eqs. (16) and (18), signifies a fundamental new result in the interest rates rules literature, since we show that the central bank should not only take into consideration inflation and output when setting interest rates, but should also react to asset price misalignments. Bean (2003) also assumes a wealth effects augmented demand curve in his analysis, but the results that he obtains for optimal policy differ significantly from the ones presented in this section. In particular, Bean finds no role for asset prices in the commitment and discretionary equilibrium. Bean’s optimality conditions contain neither the policy instrument, nor anything to do with the demand side of the economy.

In our results, however, the aggressiveness of the reaction to asset price misalignments depends upon the impact of wealth effects in aggregate demand. If there are significant wealth effects, \( \beta_3 > 0 \), then the central bank should raise interest rates in response to increasing asset price misalignments (\( f_{q-q} > 0 \)). Kontonikas and Ioannidis (2003) simulate a forward-looking variant of the macroeconomic model presented here, and find that a mild response to misalignments (\( f_{q-q} = 0.1 \)) promotes overall macroeconomic stability. Such a pro-active response has also been advocated by Cecchetti et al (2000) using the Bernake and Gertler (1999) new keynesian sticky wages – financial accelerator model.

\(^{14}\) As we show in Appendix A.3, this condition is consistent with \( A < 1 \).
A common feature in the aforementioned studies is that they assume, rather than derive, a rule for interest rate setting and then examine the effects on macroeconomic volatility from reacting or not reacting to asset prices. Our main focus however, was to show that in the context of optimal central bank behavior, asset price misalignments should be an element in the monetary authority’s feedback rule. Hence, this paper extends the literature that obtains analytical expressions for interest rates based upon optimization of the central banks’ objectives. The augmented Taylor rule depicted by Eq. (18) points out explicitly that the financial and real instability associated with growing financial imbalances should not be tolerated by the central bank.

It is easy to show that the standard Taylor rule (Taylor, 1993) can be obtained as a special case of our augmented rule in two cases. First, in the absence of a link between aggregate demand and asset prices, i.e. $\beta_3 = 0$, there is no scope for monetary policy to react to asset prices ($f_{q-q^*} = 0$), and the feedback rule which implements the optimal policy takes the form of a Taylor rule with interest rates being an increasing function of inflation and the output gap\textsuperscript{15}.

\[ i_t = f_x^* \pi_t + f_y^* y_t \]  

(19)

where the inflation and output gap weights are given by:  

\[ f_x^* = 1 - \frac{c}{\beta_2} + ca, \quad f_y^* = \frac{\beta_1}{\beta_2}. \]

Second, if markets are efficient and actual asset prices are always equal to their intrinsic value, i.e. $b = 0$, there is no direct monetary policy reaction to asset prices. In this case, monetary policy takes into account asset prices, indirectly and with a lag, via their demand wealth effects. Considering, however, the empirical evidence in Section 2.1, EMH does not appear to hold since $b > 0$. This implies that a positive weight should be applied to asset price misalignments.

\textsuperscript{15} The policy rule in Eq. (19) is an \textit{direct explicit instrument rule} since it provides a formula for the setting of policy instrument that specifies feedback only from predetermined target variables $(\pi_t, y_t)$, without involving any ‘intermediate target’ variables.
In order to further examine the impact of asset prices on the interest rate setting behavior of the central bank, we calculate the elasticity of the reaction coefficients in Eq. (18) with respect to the magnitude of wealth effects, $\beta_3$. The results, presented in Table 2 below, lead to Propositions 1 to 3.

[Table 2 about here]

**Proposition 1:** *The stronger the wealth effect, $\beta_3$, the smaller is the optimal interest rate weight on inflation.*

**Proof:** Since $\delta, (\beta_3\delta + \beta_2)^2 > 0$, $c < 0$, $A < 1$, it is implied that: $\partial f_\pi / \partial \beta_3 < 0$.

**Proposition 2:** *The stronger the wealth effect, $\beta_3$, the smaller is the optimal interest rate weight on output gap.*

**Proof:** Since $\beta_1, \delta, (\beta_3\delta + \beta_2)^2 > 0$, it is implied that: $\partial f_y / \partial \beta_3 < 0$.

Thus, when the role of capital markets as creator of wealth and collateral is taken into account, the magnitude of the inflation related-interest rate adjustment should be smaller. This does not imply that the Central Bank intervenes less frequently. In fact, if the true data generation process for aggregate demand is given by the augmented IS, Eq. (2), then monetary policy may have to be more frequently adjusted. Proposition 1 suggests that as wealth effects build up, a too aggressive interest rate response to inflation will lead to recession and will threaten the price stability objective. In addition, Proposition 2 calls for a less pronounced response to the output gap in the presence of significant correlation between asset prices and aggregate demand.
**Proposition 3:** The stronger the wealth effect, $\beta_3$, the larger is the optimal interest rate weight on asset price misalignments from fundamentals.

**Proof:** Since $\beta_2, (\beta_3 \delta_1 + \beta_3)^2 > 0$, it is implied that: $\frac{\partial f_{q_q}}{\partial \beta_3} > 0$.

The intuition and policy implications of Propositions 1 and 2 become clearer when considered in combination with Proposition 3. In essence, if aggregate demand is affected by the evolution of asset prices then monetary authorities should include asset price misalignments in their optimal feedback rule and there should be a change in the distribution of the relevant interest rate weights. Particularly, the interest rate weight on inflation and output decreases while the weight attached to asset price misalignments increases. This allows asset prices to be considered as an element of the authorities’ reaction function without necessarily implying overall tighter, than before, policy since the response to inflation and output will be less aggressive. In other words, our optimal analysis results imply that first, asset price misalignments should have an independent role and not only be considered as instruments to help forecast output and inflation; and second, there should be a shift in the magnitude of reaction, away from the traditional variables, i.e. inflation and the output gap, and towards a direct response to financial imbalances.

4. **Conclusions**

Although there is still no widespread agreement among economists on whether central banks should explicitly target asset price inflation, in addition to conventional consumer price targets, a vast consensus that emerges states that the financial-market channel plays an important
role in the transmission of the monetary policy. Our aim in this paper is to examine how the conduct of monetary policy is affected by the dynamic evolution of asset prices. Starting from these considerations, we build a backward-looking structural macro model where asset price fluctuations have an impact on aggregate demand and consequently on inflation. A crucial property of our model is that the asset market is not necessarily efficient, thereby generating deviations between actual asset prices and their fundamental value. In order to construct the optimal interest rate rule, we assume that the central bank solves a stochastic control problem to minimise intertemporally the variance of the output gap and inflation.

The derived optimal policy rule conditions the monetary policy instrument not only on inflation and demand pressures, as standard in the Taylor rule literature, but also on financial imbalances, as represented by asset price misalignments from fundamentals. The magnitude of the interest rate reaction depends, among other factors, on the relative importance of wealth effects for aggregate demand. The response to deviations from fundamentals becomes more aggressive as wealth effects build up, while the reaction to inflation and the output gap becomes less pronounced. The derived augmented Taylor rule, nests the standard Taylor rule as a special case. When there is no difference between actual and intrinsic asset value (Efficient Market Hypothesis holds) and/or when there are no aggregate demand wealth effects, then the interest rate should respond to inflation and demand pressures only.

Thus, our main contribution is to extend the optimal monetary policy literature towards recognizing that, in the presence of wealth effects and inefficient capital markets, monetary authorities should grant an independent role to asset price misalignments and not only regard them as instruments to forecast inflation and output. Future work should consider an open economy model, where the firms’ financing and the households’ capital gains derive not only from domestic but also from foreign capital markets.
References

Rudebusch, G., 2002, Term Structure Evidence on Interest Rate Smoothing and Monetary Policy Inertia,. Journal of Monetary Economics, 49 (3), 1161-1187

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### Table 1: GMM Estimates of Eq. (3)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_1$</td>
<td>0.024 **</td>
<td>0.016 *</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>1.39 ***</td>
<td>1.54 **</td>
</tr>
<tr>
<td>$b$</td>
<td>0.45 ***</td>
<td>0.57 ***</td>
</tr>
<tr>
<td>S.E. Regression</td>
<td>0.069</td>
<td>0.057</td>
</tr>
<tr>
<td>J-Stat.</td>
<td>0.06</td>
<td>0.09</td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.48</td>
<td>0.50</td>
</tr>
</tbody>
</table>

**Note:**

1. Estimates are obtained by GMM estimation with correction for MA(4) autocorrelation. Two-stage least squares estimation is employed to obtain the initial estimates of the optimal weighting matrix.
2. The instruments used are a constant and lags 1 to 6 of the change in: nominal short term interest rates, inflation, output gap, and real stock prices.
3. $J$-stat denotes the test statistic for overidentifying restrictions.
4. *, **, *** indicate level of significance of 10%, 5%, and 1% respectively.

### Table 2: Partial derivatives of interest rate reaction coefficients with respect to wealth effect parameter, $\beta_3$

<table>
<thead>
<tr>
<th>$f$</th>
<th>$\partial f / \partial \beta_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_\pi$</td>
<td>$\frac{c(1-A)\delta_1}{(\beta_3\delta_1 + \beta_2)^2} &lt; 0$</td>
</tr>
<tr>
<td>$f_y$</td>
<td>$-\frac{\beta_3\delta_1}{(\beta_3\delta_1 + \beta_2)^2} &lt; 0$</td>
</tr>
<tr>
<td>$f_{q-q}$</td>
<td>$\frac{\beta_2}{(\beta_3\delta_1 + \beta_2)^2} &gt; 0$</td>
</tr>
</tbody>
</table>
APPENDIX

Appendix A1

The first order condition that yields the optimal response is:

\[
\frac{\partial}{\partial \varphi_i} V(\pi_i) = 0 \iff (a^2 + \mu) \varphi_i + a \pi_i + \beta a E_i V'(\pi_{i+1}) = 0 \quad (A1.1)
\]

We employ the envelope theorem in order to derive an expression for \( E_i V'(\pi_{i+1}) \):

\[
dV(\pi_i) = E_i \frac{\partial}{\partial \pi_i} \left[ \frac{1}{2} (\pi_{i+1}^2 + \mu y_{i+1}^2) + \beta V(\pi_{i+1}) \right] d\pi_i \iff \\
V'(\pi_i) d\pi_i = E_i \left[ \pi_{i+1} \frac{\partial \pi_{i+1}}{\partial \pi_i} + \mu y_{i+1} \frac{\partial y_{i+1}}{\partial \pi_i} + \beta V'(\pi_{i+1}) \frac{\partial \pi_{i+1}}{\partial \pi_i} \right] d\pi_i \iff \\
V'(\pi_i) = E_i \left[ \pi_{i+1} + \beta V'(\pi_{i+1}) \right]
\]

Using (2) we obtain:

\[
V'(\pi_i) = \pi_i + a \varphi_i + \beta E_i V'(\pi_{i+1}) \quad (A1.2)
\]

Multiplying (A1.2) by \( \alpha \) and adding it to (A1.1) we get:

\[
aV'(\pi_i) = -\mu \varphi_i
\]

If we multiply this expression by \( \beta \) lead it by one period and take expectations based in information at time \( t \) we get: \( \beta a E_i V'(\pi_{i+1}) = -\mu \beta E_i[\varphi_{i+1}] \).

Thus, (A1.1) can be re-written as:

\[
(a^2 + \mu) \varphi_i + a \pi_i - \mu \beta E_i[\varphi_{i+1}] = 0 \iff \\
\varphi_i = -\left( \frac{a}{a^2 + \mu} \right) \pi_i + \left( \frac{\mu \beta}{a^2 + \mu} \right) E_i[\varphi_{i+1}] \quad (A1.3)
\]
Appendix A2

The quadratic equation whose solution gives the optimal $c$ value is:

$$(\mu \beta \alpha)c^2 + (\mu \beta - a^2 - \mu)c - a = 0 \quad (A2.1)$$

The two roots of (A2.1) are given by

$$c = \frac{-\mu \beta + a^2 + \mu \pm \sqrt{\mu^2 \beta^2 + 2\mu \beta \alpha^2 - 2\mu^2 \beta + a^4 + 2a^2 \mu + \mu^2}}{2\mu \beta \alpha} \quad (A2.2)$$

Recalling that according to Eq. (2)’ inflation is given by:

$$\pi_{t+1} = \pi_t + a\varphi_t + \omega_{t+1} = \pi_t + a(c\pi_t) + \omega_{t+1} \iff \pi_{t+1} = (1 + a_t c)\pi_t + \omega_{t+1}$$

Therefore, stability of the inflation process requires that

$$|1 + ac| < 1 \iff -1 < 1 + ac < 1 \iff \frac{-2}{a} < c < 0 \quad (A2.3)$$

Since $a > 0$ it implies that only the negative $c$-root is accepted.

Appendix A3

The inflation parameter in the interest rate reaction function, $f_\pi$, has to be greater than one in order to satisfy the stability condition that real rates increase in response to inflation, with higher values implying a more aggressive response:

$$f_\pi = 1 - \frac{c[1-A]}{\beta_1 \delta_1 + \beta_2} > 1 \quad (A3.1)$$

This condition can be re-expressed as:

$$\frac{c[1-A]}{\beta_1 \delta_1 + \beta_2} < 0 \quad (A3.2)$$

As we showed in Appendix A2, only negative values of parameter $c$ are accepted. Since $\beta_1 \delta_1 + \beta_2 > 0$, it is implied that:

$$1 - A > 0 \iff A < 1 \quad (A3.3)$$