Do Domestic Macroeconomic Factors Play a Role in Determining Long-Term Nominal Interest Rates?
Application in the Case of a Small Open-Economy

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Abstract

The determinants of long-term nominal interest rates have not yet been fully explained by either economic theory or empirical studies. Since long-term nominal interest rates are the sum of long-term real interest rates and inflation expectations, any macroeconomic factor that impacts expected inflation, real rates or both should affect long-term nominal interest rates. The objective of this paper is to examine how the dynamics of nominal bond yields is related to domestic macroeconomic fundamentals. We consider a structural VECM where identification is achieved by imposing long-run restrictions. A technical innovation of the paper is the identification of structural stochastic trends in a VECM including exogenous variables, which enables us to address the special features of a small-open economy like Canada. We then assess the impact of various shocks - monetary, fiscal and supply shocks - on nominal bond yields. Our analysis supports the view that domestic macroeconomic policies play a determining role in the long-run dynamics of nominal bond yields. First, an unexpected permanent fiscal deterioration results in large increases in long-term nominal interest rates. Second, a permanent shock to inflation results in higher nominal long rates. Supply shocks, however, have no significant long-run impact on long-term nominal rates.

JEL Classification: E43
Keywords: long-term nominal interest rates, structural cointegrated model

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1. Introduction

Although many theoretical and empirical studies have been devoted to understanding the determinants of long-term nominal interest rates, controversies still exist regarding the role of economic fundamentals in interest rate dynamics. The long-term nominal interest rate is the sum of the long-term real interest rate and inflation expectations, thus any factor that impacts expected inflation, the real rate or both should affect long-term nominal interest rates. While economic theory suggests that real long-term interest rates are influenced by potential GDP, households’ time preference and the rate of return on investment, inflation expectations are strongly influenced by monetary policy, which depends itself on the various macroeconomic variables that enter the central bank’s reaction function. Macroeconomic shocks should, therefore, have a role to play in explaining long-term nominal interest rates. Existing literature indicates that monetary policy is widely viewed as an important determinant of long-term nominal interest rates, while the impact of fiscal policy and supply shocks on long-term yields remains an open issue with no clear-cut conclusion.

The objective of this paper is to examine how the dynamics of nominal bond yields is related to domestic macroeconomic fundamentals. To that end, we specify a structural vector-error-correction model following the methodology of King, Plosser, Stock, and Watson (1991), where identification is achieved by imposing long-run restrictions. By using long-run restrictions, this methodology is similar to the one proposed by Blanchard and Quah (1989) except that it incorporates the information contained in the cointegrating vector. We first formally test for the presence of cointegration. Our results effectively support the existence of an equilibrium relationship between interest rates and the fundamentals we consider. We next use this relationship to specify a structural VAR in error correction form.

A technical innovation of the paper is the identification of structural stochastic trends in a VECM including exogenous variables, which addresses the special features of a small-open economy. This methodology allows us to assess the importance of various disturbances—defined in terms of monetary, fiscal and supply shocks—as sources of movements in nominal bond yields. Moreover, it provides a convenient way to assess the level of nominal interest rates consistent with the funda-
mentals. The focus on the long-run impact has the advantage of filtering out temporary responses of public policies to business-cycle movements. As a result, it is easier to make the distinction between genuine (fiscal and monetary) policy shocks and systematic (business cycle-related) reactions to stabilize economic activity in the short run. The methodology is applied in the Canadian context over the 1962-2003 sample.

Three main results emerge from our empirical analysis. First, the fiscal position has a sizeable effect on interest rates. More specifically, an unexpected permanent fiscal deterioration - defined as a one percentage point increase in the primary deficit-to-GDP ratio - results in a 250 basis points increase in long-term nominal interest rates. This impact - higher than what is generally found in existing studies - can be explained by the methodology used here to assess the impact of fiscal policy on interest rates. More precisely, within a VAR framework, the results provide an estimate of the impact of unexpected movements—basically the structural shocks—and not an estimate of the systematic component of the variables in the model. Furthermore, the structural shocks are defined in term of permanent shifts. Consequently, such long-lasting movements in fundamentals have a stronger impact on interest rates than temporary movements.

Second, we provide additional evidence regarding the importance of monetary shocks in the dynamics of nominal variables, thereby confirming the impact of monetary policy on the inflationary component of nominal interest rates. A one per cent permanent unexpected rise in inflation increases the long-term nominal interest by around 0.6 percent in the long-run.

Finally, in the long-run, we find that supply shocks have no significant impact on long-term nominal interest rates.

The remainder of the paper is organized as follows. Section 2 reviews existing literature regarding the potential determinants of long-term nominal interest rates. Section 3 explains the methodology we use to achieve identification in a structural VECM in the case of a small-open economy. Section 4 presents the main results. Section 5 concludes.
2. Literature survey

The determinants of long-term nominal interest rates remain a debated issue, both theoretically and empirically.

According to the widely accepted Fisher relationship, the long-term nominal interest rate is equal to the sum of the long-term real interest rate and inflation expectations:

\[ i_{nt} = r r_{nt} + p^e_t \]  \hspace{1cm} (2.1)

where \( i_{nt} \) is the long-term nominal interest rate, \( r r_{nt} \) the real long-term interest rate and \( p^e_t \) the long-run expected inflation.

Any macroeconomic variable able to impact expected inflation, the real rate or both should thus affect long-term nominal interest rate. Economic theory effectively suggests that real interest rates are influenced by several macroeconomic factors - potential GDP, the rate of return on investment, households’ time preference and investors’ behavior towards risk (Orr, Edey and Kennedy 1995; Evans and Marshall 2001; Laubach and Williams 2003 among others). Fiscal policy is another potential factor influencing real interest rates, but the relationship between fiscal policy and interest rates remains a vigorously debated issue with no clear-cut conclusion to date. On the other hand, inflation expectations are mainly influenced by monetary policy, which depends itself on the various macroeconomic variables that enter the central bank’s reaction function. Consequently, it seems highly possible that macroeconomic factors have a key role to play in explaining long-term nominal interest rates.

Existing literature on the determinants of long-term nominal interest rates generally considers individually one of those potential factors.\(^2\) The purpose of this section is to survey this literature. After a brief presentation of the two “competing/complementary” theories of the long-term interest

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\(^2\) The main exception is Evans and Marshall (2001), who consider a wide set of macroeconomic shocks - including a monetary policy shock, a fiscal policy shock, supply and demand shocks- and study the impact of those shocks on the yield curve.
rate, we consider the impact of macroeconomic shocks on nominal interest rates before focusing on the impact of monetary and fiscal policy on interest rates.

2.1 Two “competing/complementary” theories of the determinants of long-term interest rates

At the theoretical level, two theories coexist to explain long-term interest rates: the first one is based on the expectation hypothesis, while the second one relies on the loanable funds model.

According to the expectation hypothesis, the long-term interest rate can approximate long-run expectations about the future value of short-term rates (plus maturity premium):

\[
(1 + i_n)^n = (1 + i_1)(1 + i_{e,1})(1 + i_{e,2})\cdots(1 + i_{e,n-1})(1 + \text{premium}_n)
\]  

(2.2)

where \( i_n \) is the long-term interest rate (n years), \( i_1 \) the short-term interest rate (one year), \( i_{e,k} \) the short-term interest rate expected to prevail k years ahead (k=1 to n-1) and \( \text{premium}_n \) is the maturity premium (increases with the maturity).

This approach gives current and expected future monetary policy an important role in explaining long-term interest rates since it is widely accepted that monetary policy affects short-term market interest rates (see 2.3).

In the loanable funds model, the long-term real interest rate is the equilibrium price resulting from the demand and the supply of loanable funds in the economy. Therefore, the long-term real interest rate can be influenced by the various factors affecting the demand and supply of funds in the economy. The supply of loanable funds comes from domestic saving - private and public - and, because of the integration of international capital markets, from foreign saving, while the borrowing needs come from the private and the public sectors. If one of those elements is modified, everything else unchanged, the long-term interest rate should be affected (see Box 1 for the case of a fiscal deterioration as an illustration).
2.2 Macroeconomic shocks and long-term interest rates

As explained before, both components of long-term nominal interest rates are potentially affected by macroeconomic variables. Existing literature regarding the impact of macroeconomic shocks on long-term interest rates is, however, rather limited. The main contributions are Evans and Marshall (2001), Ang and Piazzesi (2003) and Wu (2003). With similar US data, however, their conclusions are not the same.

Using a structural VAR approach with different identification strategies, Evans and Marshall (2001) consider the impact of both demand (preference) and supply (technology) shocks on the US yield curve, on the 1959:1-2000:12 period. Whatever the identification strategy, they show that aggregate demand shocks induce the largest, most significant and most persistent responses in nominal yields, because demand shocks move the real interest rates and inflation in the same direction. Regarding the impact of aggregate supply (technology) shocks, on the contrary, they do not obtain robust conclusions. In the context of a structural VAR based on Gali’s identification strategy, the responses of nominal yields to a supply shock are not statistically significant, reflecting the opposite moves in real interest rate and inflation following a supply shock. In the case of a structural VAR identified from model-based shocks, the impact of the technology shock on nominal yields is sensitive to the features of the VAR system (over- versus exactly-identified) and to the ordering of shocks (in exactly-identified systems): technology shocks induce a significant response of the nominal yield level in over-identified systems and in exactly-identified systems where technology shock enters the system after the demand shock, but have no impact if the demand shock precedes the technology shock. The response of nominal yields following a supply shock is therefore particularly sensitive to the identification strategy, which conducts Evans and Marshall (2001) to conclude that this remains an open question.

Ang and Piazzesi (2003) use a Vector Autoregression model where identifying restrictions are

3. They first use the approach proposed by Gali (1999) where identification is achieved with strong a priori restrictions on the covariance structure of the VAR innovations. They next identify a structural VAR by using model-based shock measures. In that case, the identifying restrictions are closely tied to specific economic theories and few prior restrictions are placed on the covariance structure of the VAR innovations.
4. Their results regarding monetary and fiscal policy shocks are presented in 2.3 and 2.4 respectively.
based on the absence of arbitrage to investigate how macroeconomic variables (inflation and real activity) as well as unobservable factors affect the dynamics of the US yield curve with data covering the 1952:6-2000:12 period. Variance decompositions show that macroeconomic factors explain movements at the short and middle ends of the yield curve, while unobservable factors still account for most of the movement at the long end of the yield curve. Therefore, Ang and Piazzesi’s conclusions do not support the idea that macroeconomic variables affect long-term nominal interest rates.

In the context of a structural VAR framework where shocks are identified using a recursive strategy with US data covering the 1967:1-1998:12 sample, Wu (2003) shows that a positive shock to real output raises all the interest rates with a similar magnitude along the yield curve. Moreover, this effect on the level of the yield curve is more persistent than the effect created by a monetary policy shock. Wu’s results support thus the idea that a supply shock impacts interest rates.

To date, there is hence no firm conclusion in the literature regarding the effect of macroeconomic shocks on long-term nominal interest rates.5

2.3 Monetary policy and long-term nominal interest rate

Both the Fisher relation and the expectations hypothesis give monetary policy an \textit{a priori} role in determining long-term nominal interest rates.

Since inflation is ultimately a monetary phenomenon (see Bullard, 1999, for a detailed survey on long-run monetary neutrality), long-run inflation expectations are largely set by monetary policy, thereby making monetary policy a relevant candidate as a determinant of nominal interest rates. Several empirical studies (conducted with US data) effectively support the view that long-term nominal interest rates are affected by monetary policy through its impact on inflation.6

\footnotesize{5. Moreover, the longest-term interest rate used in those studies is the 60-month zero coupon bond yields, which can be viewed as a medium-term rather than long-term interest rate. In our empirical study, we use the 10-year bond yield as the long-term interest rate.  
6. Figure 1 illustrates the broadly similar movements in nominal bond yields and inflation over time in Canada.}
Using monthly data on the 1952:1-1987:2 period, Campbell and Ammer (1993) show that bond returns are largely driven by news about future inflation, while real rates have little impact. They find however a small difference in the variance decomposition of bond returns according to the sample period: while the variation in bond returns is essentially explained by news about future inflation over the 1952-1979 period, the news about future excess bond returns also contributes to the overall variance of bond returns in sample periods that include the 1980s. Using cointegration and error-correction methodology in a multivariate framework, Mehra (1996) finds a long-run equilibrium relationship between the US bond rate and the inflation rate that can be interpreted as a Fisher relation in which the (trend) rate of inflation determines the bond rate. The long-run effect of monetary policy on bond yields occurs therefore primarily through the inflation channel.\(^7\) Finally, on the basis of the Lucas’s generalization of the Fisherian theory,\(^8\) Ireland (1996) shows that movements in nominal bond yields primarily reflect changes in long-run inflationary expectations.

Moreover, as noted before, the expectation hypothesis gives current and expected future monetary policy an important role in explaining long-term interest rates.

Empirical studies generally find a weak relationship between monetary actions and long-term interest rates (see Roley and Sellon 1995 for a detailed survey of those empirical studies in the US case), hence questioning monetary authority’s ability to influence longer-term interest rates and, eventually, aggregate demand. In the context of structural VAR models, Evans and Marshall (2001) show that monetary policy shocks have a significant impact on the slope of the yield curve, but no effect on the level of the yield curve. Wu (2003), with a similar approach, confirms that monetary policy shocks have a large and significant but short-lived effect on short-term interest rates with a dissipating effect on longer-term interest rates.

This weak impact of monetary policy on long-term nominal interest rates can be explained by the fact that previous studies only consider current monetary policy, while the expectations theory re-

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7. In the short-run, however, Mehra (1996) finds that monetary policy also affects the real rate component. This is, nevertheless, out of the scope of our study which focuses on the long-run.
8. Lucas (1978) generalizes the Fisher relation by identifying a risk premium as the third determinant of the nominal bond yields. This risk premium compensates investors for holding dollar-denominated bonds in a context characterized by inflation uncertainty.
lates long-term interest rates not only to the current short rate but also to market expectations of future short-term rates. Any change in the view of market participants about future monetary policy can consequently affect the long-term interest rate. By explicitly including market expectations of future monetary actions, Roley and Sellon (1995) find a larger response of long-term interest rates to monetary policy than traditionnally: they show that the magnitude of the response of long-term interest rates to monetary actions depends on the expected persistence of those actions. A change in the current short-term interest rate can therefore influence longer yields only if market participants view this change as permanent or as the first of a serie of actions. The effect of monetary policy on long-term nominal interest rates is thus linked with the persistence of monetary decisions.

In summary, existing literature shows that, in the long run, monetary policy is able to impact long-term nominal interest rates through its inflation component, that is in affecting inflation expectations.

2.4 Fiscal policy and long-term interest rates

The relationship between fiscal policy and long-term interest rates is a vigorously debated issue, both theoretically and empirically. Moreover, it is a politically sensitive issue for which there is no widely-accepted conclusion. There are both elements that indicate that fiscal policy should not influence long-term interest rates, and others that suggest an impact of the fiscal position on long-term interest rates.

According to the Ricardian equivalence (Barro 1974), economic agents understand that any increase in current fiscal deficits will conduct to tax raises in the future. To smooth their consumption over time, economic agents therefore increase their present saving in face of the higher fiscal deficits. This parallel increase in both private saving and public borrowing needs results then in unchanged long-term interest rates. As a result, fiscal policy does not influence interest rates.

Moreover, since international asset markets are increasingly integrated, the relationship between domestic saving and borrowing needs has necessarily weakened, therefore reducing the potential impact of domestic fiscal policy on interest rates.

Nevertheless, the evolution of fiscal positions, savings rate and long-term interest rates over the 1980s and the 1990s in the main industrialized countries has questioned the relevance of the Ricardian equivalence.

The following box illustrates how a deteriorating fiscal position can impact on long-term real interest rate through its impact on the supply of loanable funds (in face of unchanged borrowing needs).
Situation [1] is relatively implausible because empirical studies have shown that only 20 to 50 per cent of a decrease in public saving is offset by a rise in private saving.\(^\text{10}\) Situation [A] is also implausible because empirical evidence suggests that changes in net foreign investment flows account only for 25 to 40 percent of changes in national saving (Feldstein and Horioka 1980; Obstfeld and Rogoff 2000). Consequently, situation [B] can be viewed as the most realistic one given existing empirical evidence about the behavior of private and foreign saving in face of a deteriorating...

\(^{10}\) See Gale and Orzag (2003), pages 6-7, for a detailed survey of those empirical studies.
fiscal position. Consequently, according to the loanable funds approach, fiscal policy should impact long-term interest rates. 11

To date, there is no definitive conclusion about which of the previous arguments is correct, the Ricardian view or the approach based on the loanable funds model. However, several large-scale macroeconometric models have detected an economically significant link between changes in fiscal position and long-term interest rates: a one per cent increase in the fiscal deficit-to-GDP ratio would raise long-term interest rates by about 100 basis points after 10 years (see Gale and Orzag 2003, Table 1, page 18), thus providing an additional argument in favor of a link between fiscal policy and long-term interest rates. 12

The debate is also far from being close at the empirical level because existing empirical studies of the relationship between fiscal policy and long-term interest rates have produced mixed results. 13

In the Canadian case, existing empirical studies give mixed results too. Siklos (1988), with spectral analysis and time series (with annual and quaterly data), finds no evidence to support the view that fiscal deficits influence interest rates (real and nominal). Nunes-Correia and Stemitsiotis (1993), on the contrary, find that, in Canada, a one percentage point increase in the deficit ratio creates a 53 basis points increase in the long-term interest rate. Furthermore, they show that the average fiscal deficit ratio has induced a 236 basis points increase in the long-term interest rate over the 1980-90 period, concluding therefore that fiscal deficits have been an important determinant of long-term interest rates in the 1970s and 1980s. Finally, using a VECM approach on the 1972-1994 period, Fillion (1996) finds a strong cointegration relationship between real long-term interest rates in Canada and the United-States and the Canadian public debt. He next shows that a simulated public debt shock (one percentage point increase in the public debt ratio) induces a 3.1 basis points increase in long-term real interest rates and concludes that the public debt increase in Canada from

11. Moreover, fiscal policy can impact on interest rates because an increasing public debt creates an eviction effect on capital, rising thus the capital returns and, consequently, the returns on other assets, including Government bonds.
12. Moreover, if there is a risk of monetization of increasing fiscal deficits, inflation expectations could be affected upwards following a fiscal deterioration. In that case, fiscal policy would influence nominal interest rates not only through its impact on real interest rates as described before, but also through its impact on (expected) inflation.
13. See Brook (2003) and Gale and Orzag (2003) for very detailed surveys of those empirical studies.
1990 has induced a 85 to 135 basis point increase in real interest rates.

It is more and more widely accepted that the temporal dimension of the relationship between fiscal policy and interest rates must be taken into account in studying this relationship (Feldstein 1986; Brook 2003; Gale and Orzag 2003). Because of the forward-looking nature of financial markets, long-term interest rates respond to expectations of future fiscal policy, rather than to the current policy stance. Studies using projected fiscal deficits effectively find a positive and significant impact of expected fiscal deficits on expected future interest rates.¹⁴

Using the Congressional Budget Office (CBO) budget balance projections, Canzoneri, Cumby and Diba (2002) find that a one per cent of current GDP increase in the projected future deficits raises the spread between long and short term interest rates by 53 to 60 basis points for the five-year projections, and by 41 to 45 basis points for the ten-year projections.

Laubach (2003) measures the impact of both the CBO projections and the Office of Management and Business (OMB) projections on the real five-years-ahead ten-year treasury yield, while controlling for other variables viewed as influencing the long-term interest rate (potential GDP growth and equity premium). He shows that a one per cent increase in the projected deficit-to-GDP ratio induces a 28 to 40 basis points rise in the long-term interest rate in the future and that a one per cent increase in the projected debt-to-GDP ratio raises the interest rate by 5.2 basis points, which is consistent with economic theory (neoclassical model of growth).

Finally, following Feldstein (1986), it is more and more widely accepted that the potential impact of fiscal policy on long-term interest rates depends on the “nature” of the fiscal situation: the fiscal position can affect long-term interest rates only if it is viewed as permanent or structural, while a temporary fiscal deterioration - aiming to fight an economic slowdown - has no impact on interest rates.

¹⁴ These studies are based on US data only because of the data availability of fiscal projections.
2.5 Does the U.S. long-term nominal interest rate influence the Canadian one?

Our objective is to assess whether Canadian domestic macroeconomic factors have a role to play in explaining Canadian long-term nominal interest rates. It may be argued, however, that, due to the greater integration of international capital markets - and more precisely the stronger linkages among major bond markets - the behavior of national bond yields should be explained not only by domestic developments, but also by movements in foreign bond yields. Several studies (Brook 2003 and Laopodis 2004 for the most recent ones) effectively conclude in favour of more synchronized movements of long-term yields across developed countries following the increasing global financial integration. Moreover, Brook (2003) concludes in favor of a predominantly unilateral nature of the international transmission going from the U.S. long rates to other countries' rates. This may be particularly true for Canada. As a small open-economy, Canada should have a negligible impact on how interest rates are set on the world market. Moreover, Canadian domestic long-term interest rates should be a priori influenced by the U.S. long rates because of the influence of these latters on the “world” rate and the close economic relationship between the two countries. As shown by Figure 2, Canadian and U.S. bond yields have effectively evolved in a broadly similar manner over the past four decades. Such a similar evolution, however, is not sufficient to conclude that Canadian bond yields are determined by the U.S. ones, thereby implying that Canadian domestic factors may have no - or a very small - effect on the determination of Canadian long-term nominal interest rates.

For countries with a fixed exchange-rate arrangement, policymakers’ ability to pursue independent policies is limited, and market integration results in a greater “systematic” synchronization in the movements of their interest rates. Under flexible exchange rates and independent macroeconomic policies, however, long-term nominal interest rates should be influenced, at least partly, by domestic developments. This argument can be used in the Canadian case for the past thirty years at least. If Canada would have failed to implement sound monetary and fiscal policies over the 1980s and the 1990s - that is if Canadian monetary and fiscal authorities would have been unable to produce low and stable inflation and to improve fiscal position respectively - Canadian bond yields would certainly have been higher due to higher premia (inflation premium and risk premium associated with...)

associated with the fiscal position) and would have evolved differently from the U.S. yields. The choice of implementing those macroeconomic policies is purely Canadian, in that there was no constraint for Canadian policymakers to follow U.S. policies. Canadian and U.S. macroeconomic policies, however, have largely converged over time: (i) accommodating monetary policies associated with expansive fiscal policies during the 1960s and the 1970s, (ii) monetary restriction to reduce inflation associated with attempts to improve fiscal positions in the 1980s,\textsuperscript{16} and (iii) credible monetary policies producing low and stable inflation and a stronger commitment to fiscal improvement in the 1990s. This convergence of independently chosen macroeconomic policies may reflect successive consensus of opinion among policymakers regarding the desirable policies to implement to favour economic growth. The broadly similar developments of Canadian and U.S. bond yields over the past four decades can therefore be viewed as reflecting the impact of similar - but independent - domestic policies. If this argument is valid, we should therefore find in our empirical study that Canadian domestic factors have been fundamental determinants of bond yields in Canada. If, on the contrary, Canadian bond yields were to be determined by U.S. bond yields only, we would find that domestic factors play no role in explaining long-term nominal interest rates in Canada. As shown later in this paper, we effectively conclude that domestic factors (monetary policy, supply shocks and fiscal policy) have been long-run determinants of Canadian bond yields over the past four decades.\textsuperscript{17}

This result, however, does not mean that U.S. bond yields have no influence at all on Canadian bond yields, but only that, in the long-run, Canadian long-term nominal interest rates have been (largely) explained by domestic developments. The short-run fluctuations of Canadian bond yields (high-frequency data), however, may be strongly influenced by short-run movements in the U.S. bond yields. The focus of our paper, however, is not on explaining high-frequency movements in Canadian bond yields, but how the long-run dynamics of nominal bond yields is related to domestic macroeconomic fundamentals.

\textsuperscript{16} This is true for other industrialized countries too. In the 1980s, OECD countries have effectively reached a consensus regarding the elements which should provide the basis for sustained economic growth (sound monetary policy, sound fiscal policy, trade liberalization and structural reforms). This consensus has resulted, notably, in the gradual implementation of “sound” monetary and fiscal policies in industrialized countries, which can explain the increasingly synchronized movements of their bond yields from the 1980s.

\textsuperscript{17} It would be interesting to confort the view that the aforementioned domestic developments have played a similar role in explaining long-term nominal interest rates in the U.S. case. This will be the topic of future research using a similar approach with U.S. data.
3. Statistical Framework

As mentioned in the introduction, the objective of this paper is to examine how the dynamics of Canadian nominal bond yields is related to several domestic macroeconomic factors. In most of macroeconomic models, including recent dynamic general equilibrium models, cycles are usually driven by some combination of monetary, fiscal, and technological innovations. We can therefore reasonably assume that nominal interest rates respond to the news coming from these macroeconomic impulses. In that section, we propose an econometric approach that allows us to assess the importance of various disturbances as sources of movements in nominal bond yields.

We specify the following vector-error-correction model (VECM):

\[
\begin{bmatrix}
\Delta \text{INF}_t \\
\Delta \text{GDP}_t \\
\Delta \text{DEF}_t \\
\Delta r_t
\end{bmatrix}
= \sum_{i=1}^{p-1} \Gamma_i
\begin{bmatrix}
\Delta p\text{com}_{t-i} \\
\Delta \text{INF}_{t-i} \\
\Delta \text{GDP}_{t-i} \\
\Delta \text{DEF}_{t-i} \\
\Delta r_{t-i}
\end{bmatrix}
+ \alpha \beta'
\begin{bmatrix}
p\text{com}_{t-i} \\
\text{INF}_{t-i} \\
\text{GDP}_{t-i} \\
\text{DEF}_{t-i} \\
r_{t-i}
\end{bmatrix}
+ \mu + e_t.
\] (1)

The endogenous variables are the following quarterly Canadian variables: the Consumer Price Index (CPI) year-over-year inflation rate, the real gross domestic product (GDP), the government primary balance - that is the government net lending excluding the interest debt service - expressed as a share of GDP (DEF), and the 10-year government bond rate (r). Given that the set of lagged variables is assumed to be a good proxy for the information set available to economic agents, we also include commodity prices (pcom) as an exogenous variable since Canada is a small open economy which exports mainly primary products and is, thereby, highly sensitive to commodity price developments.

We estimate this VECM specification over the 1962-2003 period using a 10 lags structure, \(^\text{18}\) which is consistent with the usual information criteria (Hannan-Quinn and Schwartz) and large enough to remove the residual autocorrelation.

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18. Our results are qualitatively robust to specifications with 9 and 11 lags.
Unit root tests suggest that all the variables are integrated of order one; the variables are therefore specified in first difference in our model.\textsuperscript{19}

The results obtained from cointegration tests corrected for the presence of one exogenous variable, as proposed by Pesaran, Shin and Smith (2000), are presented in Table 1. Both the L-max and Trace tests indicate the presence of one cointegration vector.

### Table 1: Cointegration Tests\textsuperscript{1}

<table>
<thead>
<tr>
<th>L-max</th>
<th>Trace</th>
<th>H0:r=</th>
<th>L-max (.10)</th>
<th>Trace (.10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>39.46</td>
<td>57.86</td>
<td>0</td>
<td>28.2</td>
<td>54.84</td>
</tr>
<tr>
<td>14.32</td>
<td>18.39</td>
<td>1</td>
<td>22.1</td>
<td>35.8</td>
</tr>
<tr>
<td>3.98</td>
<td>4.08</td>
<td>2</td>
<td>15.9</td>
<td>20.7</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>3</td>
<td>9.5</td>
<td>9.5</td>
</tr>
</tbody>
</table>

\textsuperscript{1} The critical values corrected for the presence of one exogenous variable are taken from Table T.3 in Pesaran et al. (2000).

### Table 2: Testing restrictions on the cointegrating vector\textsuperscript{a}

<table>
<thead>
<tr>
<th>INF</th>
<th>GDP</th>
<th>DEF</th>
<th>LR</th>
<th>pcom</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.36</td>
<td>-0.077</td>
<td>-2.408</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>(.37)</td>
<td>(.026)</td>
<td>(.38)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The LR test, $\chi^2(1) = 2.3$, p-value = 0.13

\textsuperscript{a} Standard errors in brackets.

The coefficients of the cointegrating relationship cannot be interpreted as elasticities even if the variables are in logarithm form, because a shock to one variable usually induces a shock to all the

\textsuperscript{19} There exists some evidence that inflation may have become stationary since the adoption of an inflation targeting regime in 1991 [St-Amant and Tessier (2000)]. However, over a longer period - as the one we use here - formal unit-root tests tend to support the nonstationarity hypothesis. This issue remains widely debated. In the U.S. case, for instance, Cogley and Sargent (2001) argue that there has been a downward shift in the degree of persistence in inflation, while others (see Stock, 2001) consider that the statistical evidence in favour of such a break is weak.
variables in the long run. Hence, the coefficients do not generally allow for a *ceteris paribus* interpretation [see Lütkepohl (1994)]. More formally, Wickens (1996) shows that reduced-form cointegration vectors should not be interpreted without further structural assumptions.\(^2\)

King et al. (1991) [KPSW hereafter] propose an identification methodology based on long-run restrictions that allow for a structural interpretation of a cointegrated VAR. In order to address the special features of a small-open economy like Canada, this paper proposes a technical innovation that allows the identification of structural stochastic trends in a VECM including exogenous variables.\(^2\) By structural interpretation, we mean that it is possible to identify different shocks related to macroeconomic fundamentals, and to derive meaningful impulse response functions. In a structural VAR with long-run restrictions, the identification is achieved by positing a lower-triangular structure for the matrix of long-run impact, which requires thus to impose various long-run neutrality conditions. Since economic theory provides long-run relationships between variables, imposing long-run neutrality conditions is far more reliable than adopting contemporaneous restrictions. The focus on the long-run impact has the further advantage of filtering out temporary responses of public policies to business cycle movements. As a result, it is easier to make the distinction between genuine (fiscal and monetary) policy shocks and systematic (business cycle-related) reactions to stabilize economic activity in the short run.

Given the set of variables included in our empirical framework, we now explain how nominal bond yields can be decomposed into monetary, fiscal and supply (or productivity) shocks following the aforementioned methodology. In structural VARs, the ordering of variables matters. More precisely, in the context of long-run restrictions, the variables are put in decreasing order of long-run exogeneity. In the present case, with four endogenous I(1) variables and one cointegration relationship, we need to impose three restrictions. The first set of restrictions comes from our definition of a monetary shock. As suggested by Roberts (1993), we adopt the view that inflation is ultimately a monetary phenomenon and, accordingly, we define a monetary policy shock as a permanent shock to inflation. By using this monetarist approach, we suppose that the trend of inflation is fully under the control of the central bank. Consequently, any permanent movement in inflation results from changes in inflation that the central bank is inclined to tolerate.

\(^2\) Interpreting those coefficients on a *ceteris paribus* basis would tell that a permanent increase in inflation is associated with a permanent decrease in nominal interest rates. We will illustrate latter how this interpretation is at odd with the impact of structural shocks.

\(^2\) In Appendix A, we show that the KPSW methodology can be generalized in the context of a VECM with exogenous variables provided the exogenous variables are not cointegrated with the endogenous variables. This assumption is accepted with a p-value of 0.2. We have used MATLAB to implement the identification procedure.
Inflation is thus the most exogenous variable in the long-run, which explains the first two restrictions (the two zeros in the first row of Table 3).

The remaining restriction comes from our definition of a supply shock. It is effectively widely accepted that disturbances with a permanent impact on output can be thought as aggregate supply shocks (often referred as technology or productivity shocks). Consequently, only supply shocks—beyond the monetary shocks\(^22\)—can have a permanent effect on output and thus explain the third restriction (the zero in the second row of Table 3). Finally, the fiscal policy shock is an exogenous, permanent disturbance to the primary fiscal balance (expressed as a percentage of GDP).\(^23\) Using the primary fiscal deficit as the measure of fiscal position has three main advantages. First, it is representative of the government’s financial needs and, thereby, is more likely to have an impact on bond yields than a fiscal measure based on taxes or government expenditures only. Second, by defining a fiscal shock in terms of its long-run impact, we implicitly assume that the fiscal shock is purged of any business cycle movements. Third, since the shocks we consider are permanent, our approach is consistent with the more and more widely accepted view that fiscal policy can impact interest rates only if it is considered permanent or structural, as opposed to a temporary fiscal change.

4. Shock Analysis

The long-run impact of the three structural shocks is displayed in Table 3.

<table>
<thead>
<tr>
<th></th>
<th>(\eta^\text{in})</th>
<th>(\eta^y)</th>
<th>(\eta^f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>inf</td>
<td>0.66</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>gdp</td>
<td>1.05</td>
<td>1.21</td>
<td>0</td>
</tr>
<tr>
<td>def</td>
<td>0.5</td>
<td>-0.03</td>
<td>0.17</td>
</tr>
<tr>
<td>r</td>
<td>0.41</td>
<td>0.02</td>
<td>0.42</td>
</tr>
</tbody>
</table>

\(^22\) This identification structure allows for the possibility that monetary superneutrality would not hold in the long run. Such a scenario is consistent with the view that inflation could have distortionary effects on real output.

\(^23\) Assessing the potential impact of fiscal policy on long-term interest rates effectively requires to remove the component of fiscal policy that is explained by interest rates: the interest paid on the public debt.
4.1 Monetary Policy Shock

The typical nominal (inflation) shock increases inflation by around 0.7 percent in the long-run while the real long-term interest rate decreases both in the short and the long run, thus violating the Fisher effect (see Figure 5). To date, there is no widely accepted conclusion in the literature regarding the impact of a permanent change in inflation on long-term interest rates, that is whether a permanent change in inflation could permanently affect the real long-term interest rate. The permanent decrease in the real interest rate we find here is consistent with King and Watson (1997) who show that, in a large set of identification schemes, nominal interest rates do not fully adjust to permanent inflation shocks. This is also in line with Rapach (1999), who finds that, in 14 industrialized countries, the real interest rate decreases following a permanent increase in inflation, and with Gauthier and Li (2004), in the context of a larger model for Canada. This result, however, is at odd with Mehra (1996) who concludes in favor of a cointegration relationship between inflation and the long rates in the U.S., thus implying that long-term interest rates are explained solely by inflation in the long-run.

A permanent increase in inflation affects the level of real output (upwards), both in the short and the long-run. While the short-run increase in output following the inflation shock is consistent with the accommodative lower real interest rate, the long-run positive effect of inflation on real output may be considered puzzling. Existing empirical evidence regarding the long-run impact of inflation on real output is mixed. For example, King and Watson (1997) conclude that superneutrality with respect to output can be rejected for some identification schemes that they consider reasonable. They also find that the effect can be either way. Bullard and Keating (1995) also document that permanent inflation shocks permanently increase the level of output for certain low inflation countries. The positive impact of inflation on output in the long-run found in the present study, however, should not be interpreted as illustrating a beneficial effect of inflation on real output. Effectively, this positive effect can be explained by some features of the sample period covered by our empirical study and the methodology we use. First, in the early 1970s, the strong rise in inflation has been accompanied by a noticeable increase in the growth rate of real GDP. Next, in both the early 1980s and the early 1990s, restrictive monetary policy actions have resulted in the expected decrease in inflation, but have also been accompanied by long-lasting negative effects on real output. The methodology we use, however, captures the average effect of an inflation shock to other variables only. Since the three episodes described above are

24. Notice how the impact of the (structural) nominal shock is at odd with the (false) interpretation of the cointegrating vector coefficients (see footnote 20).
25. There exist theories that are consistent with both possibilities.
episodes of particularly large magnitude, the average effect is therefore strongly affected, which explains why our methodology associates a strongly persistent rise in inflation with a benefit in terms of real output.

4.2 Supply Shock

The typical supply shock (see Figure 6) increases output by around 1.2 per cent in the long-run. In conformity with most theoretical models, an unexpected permanent rise in output increases excess supply, thereby pushing inflation down in the short-run.

Since higher income results in higher fiscal revenues and lower transfers, the fiscal situation improves (decrease in the deficit) in the short term following the supply shock.

The short-run drop in the real interest rate that accompanies the supply shock suggests that the monetary authority has historically accommodated supply shocks. In the long-run, however, the real interest rate is unaffected by supply (technology) shocks. This is consistent with the model of Ramsey in which the interest rate is determined by the rate of time preferences while the level of capital is set by technology so that the marginal product of capital is equal to the interest rate. These results are in line with Gauthier and Li (2004). As explained before, no consensus has been reached yet in the existing literature regarding the impact of supply shocks on interest rates, thus providing very few benchmark to assess the relevance of our results.

4.3 Fiscal Policy Shock

The impact of a permanent unexpeced increase in the government primary deficit (deterioration of fiscal position) is reported in Figure 7. As predicted by standard macroeconomic models, a positive fiscal shock (coming from either an increase in spending or a decrease in taxes) stimulates the economy and slightly raises inflation in the short-run as excess demand builds up. Our results suggest that along with deficit increases in Canada, important risk premiums were incorporated in the long-term interest rate. More precisely, a permanent *unexpected* deficit increases of around 26. The real cost of those disinflation policies - measured by the sacrifice ratio - in Canada has been documented by several studies, including Ball 1994, Jordan 1997, Johnson and Gerlich 2002.

27. See Figure 3 (shaded areas represent the aforementioned three “atypical” episodes).
0.20 percent is associated with a 40 basis points rise in the long-term interest rate. The influence of fiscal policy on bond yields in Canada has been documented in previous empirical studies. Comparing our results with those provided by this literature, however, is delicate given the differences in both variables and methodologies. For example, Fillion (1996) suggests that the increase in the public debt ratio from 1990 to 1994 accounts for an increase of 85 to 135 basis points in real interest rates. Not only does Fillion examine the impact of an increase in the debt ratio while we focus on a deficit measure, but his results are based on the long-run coefficients of the cointegrating relationship, while our results are based on the impact of structural shocks, which may differ considerably from the reduced form coefficients (see footnotes 20 and 24). Moreover, our data span 40 years, twice longer than Fillion’s sample (1975-1994). Nevertheless, both papers conclude that fiscal policy has been an important long-run determinant of long-term interest rates in Canada.

4.4 Historical Decomposition

The historical decomposition of long-term nominal interest rate (Figure 8) gives a broader view of the respective contributions of these three domestic macroeconomic factors.

The accommodating monetary policy conducted in the 1970s - as illustrated by the strongly persistent increase in inflation - has contributed to the rise in bond yields over the 1973-1980 period (for about 250 basis points). On the contrary, the sustained decrease in inflation in the 1990s - with the adoption of inflation targets (1991) - is responsible for about 200 basis points of the decrease in bond yields. The impact of supply shocks is more mitigated. The investment boom of the 1990s, however, has contributed to a 200 basis points decrease in nominal long-term interest rates. As shown by Figure 8, fiscal policy has played a sizeable role in explaining the dynamics of bond yields in Canada over the past 30 years. The huge fiscal deterioration observed from the mid-1970s to the mid-1980s is responsible for a large part - about 700 basis points - of the rise in bond yields. Symmetrically, the gradual and sustained improvement of the fiscal position from the mid-1980s has largely contributed to the decrease in bond yields.

28. Fillion estimates one model including both the debt ratio and the deficit ratio, but this model is mis-specified if, as appears to be the case, both variables are treated as being integrated of the same order.
Thus, in terms of magnitude, fiscal policy is the most important domestic factor explaining the dynamics of long-term nominal interest rates in Canada, responsible for about 700 basis points of the rise in the 1970s and 650 basis points of the decrease in the 1980s-1990s. Monetary policy (inflation component) has also been an important determinant of the dynamics of bond yields, explaining about 250 basis points of the rise in the 1970s and about 200 basis points of the decrease in the 1990s.

5. Conclusion

The aim of this paper was to detect whether, in the context of increasingly integrated financial markets, domestic factors still have a role to play in the dynamics of nominal bond yields. This question is relevant for national policymakers since, ultimately, their ability to influence economic growth depends on their ability to impact long-term interest rates (through monetary and fiscal policy mainly). Thus, the present paper proposed to relate the dynamics of nominal bond yields to various domestic potential macroeconomic drivers. Based on existing literature, monetary policy, fiscal policy and supply shocks should be relevant candidates to determining long-term nominal interest rate. Using a structural VECM that includes one exogenous variable, we studied the dynamics of Canadian long-term nominal bond yields on the 1962-2003 period.

Our empirical study supports the view that domestic developments have been key determinants of nominal bond yields in Canada. Caporale and Williams (2002), by investigating the information content of domestic macroeconomic developments for the determination of nominal long-term interest rates in the G7, also conclude in favour of such a significant impact of fiscal and monetary developments on long-term interest rates.

We confirm the impact of monetary policy on nominal bond yields, through their inflationary component: a one per cent permanent unexpected rise in inflation increases the long-term nominal interest by around 0.6 percent in the long-run. Supply shocks, on the contrary, have played a mitigate role. One particularly interesting result of this paper is the strong impact fiscal policy is found to have on nominal bond yields: a permanent unexpected deficit increases of around 0.20 per cent produces a 40 basis points rise in the long-term nominal interest rate. In the current vigorous debate regarding the relationship between fiscal policy and long-term interest rates, our results support then the view that fiscal policy has a role to play in explaining the long-run dynamics of long-term interest rates.
In future research, we plan to extend our empirical study to the U.S. case, to confirm our view that the comovements of U.S. and Canadian nominal bond yields over time may illustrate the convergence of their respective domestic policies, rather than only an unilateral international transmission going from the U.S. long rates to the Canadian ones. We do not reject, however, that high-frequency movements in U.S. bond yields may strongly impact the short-run dynamics of Canadian bond yields, but this is not the scope of the present paper which is based on a long-run perspective.
REFERENCES


Figure 1. Inflation rate and Government of Canada 10-year Bond Yields

Figure 2. 10-year Bond Yields, Canada Versus U.S.
Figure 3. Real GDP Growth Rate

![Real GDP Growth Rate graph](image)

Figure 4. The Data

![Inflation graph](image)
![Output graph](image)
![Deficit graph](image)
![Interest rate graph](image)
Figure 5. Impact of a typical permanent inflation shock

- Inflation shock on inflation
- Inflation shock on output
- Inflation shock on deficit
- Inflation shock on interest rate
Figure 6. Impact of a typical supply shock
Figure 7. Impact of a typical deficit shock
Figure 8. Historical components of the long-term interest rate

- Permanent inflation shock contribution on interest rate
- Supply shock contribution on interest rate
- Permanent deficit shock contribution on interest rate
Appendix A. Identification of shocks in a VECM with exogenous variables

In a non-cointegrated VAR model, the structural shocks’ identification procedure (Blanchard and Quah [1989] for example) is clearly invariant to the presence or not of exogenous variables in the model. However, in presence of cointegration, this is not obvious as the common stochastic trends must be consistent with the cointegrating relations which possibly include exogenous variables. Wickens and Motto (2001) has shown how to identify the shocks when the following restrictions are made: the variables can be classified as endogenous or exogenous, there are as many cointegrating relations as endogenous variables, the cointegrated vectors are identified and they contain at least one exogenous variable. In Wickens and Motto (2001) the complete model need to be estimated. In this paper, we show how King et al. (1991)’s identification procedure can be applied to a VECM with either weakly exogenous I(1) variables restricted not to be in the cointegrating relations, or strongly exogenous variables. A simple way to invert a VECM with exogenous variables is also suggested.

A.1 Efficient estimation of a VECM with weakly exogenous variables

Economic systems often have so many potentially useful variables that the system gets extremely large. Johansen (1992) has shown, however, that a partial model can be efficiently estimated when some of the variables are weakly exogenous. Consider an m-dimensional VAR(p) process \( \{ z_t \}_{t=1}^{\infty} \) expressed as the vector error correction model (VECM):

\[
\Delta z_t = a + \sum_{i=1}^{p-1} \Gamma_i \Delta z_{t-i} + \Pi z_{t-1} + e_t, \quad t=1,2,\ldots
\]  (A.1)

where \( \Delta = 1 - L \) with \( L \) being the lag-operator, the long-run multiplier \( \Pi \) and the short-run response matrices \( \Gamma_i \) are \( m \times m \) constant coefficient matrices, \( a \) is a constant vector, and the \( m \)-dimensional disturbance \( e_t \sim IN(0, \Omega) \).

We now partition the \( m \)-vector of random variables \( z_t \) into the \( n \)-vector \( y_t \) and the \( k \)-vector \( x_t \), where \( k = m - n \); that is \( z_t = (y'_t, x'_t)' \), \( t = 1, 2, \ldots \). By partitioning the error term \( e_t \) conformably with \( z_t = (y'_t, x'_t)' \) as \( e_t = (e'_{yt}, e'_{xt})' \) and its variance matrix as
we are able to express $e_{yt}$ conditionally in terms of $e_{xt}$ as

$$e_{yt} = \Omega_{yx}^{-1}\Omega_{xx}^{-1}e_{xt} + u_t,$$

where $u_t \sim IN(0, \Omega_{uu})$, $\Omega_{uu} \equiv \Omega_{yy} - \Omega_{yx}\Omega_{xx}^{-1}\Omega_{xy}$ and $u_t$ is independent of $e_{xt}$. We also use a similar partitioning of the parameter vectors and matrices $a = (a'_y, a'_x)'$, $\Pi = (\Pi'_y, \Pi'_x)'$ and $\Gamma_i = (\Gamma_{yi}, \Gamma_{xi})'$, $i = 1, ..., p - 1$. Following Johansen (1992) and Boswijk (1992, Chapter 3), we make the following assumption:

Assumption 2.1. $\Pi_x = 0$.

Under Assumption 2.1, i.e. the process $\{x_t\}_{t=1}^{\infty}$ is weakly exogenous with respect to the matrix of long-run multiplier $\Pi$, the following conditional model in terms of $z_{t-1}$, $\Delta x_t$, $\Delta z_{t-1}$, $\Delta z_{t-2}$, ... is efficiently estimated by maximum likelihood without using the equations for $\{x_t\}_{t=1}^{\infty}$:

$$\Delta y_t = c + \Lambda \Delta x_t + \sum_{i=1}^{p-1} \psi_i \Delta z_{t-i} + \Pi_y z_{t-1} + u_t, \quad t = 1, 2, ...$$

(A.3)

where $c \equiv a_y - \Omega_{yx}\Omega_{xx}^{-1}a_x$, $\Lambda \equiv \Omega_{yy}\Omega_{xx}^{-1}$, and $\psi_i \equiv \Gamma_{yi} - \Omega_{yx}\Omega_{xx}^{-1}\Gamma_{xi}$, $i = 1, ..., p - 1$.

A.2. Identification of the permanent shocks

The identifying procedure documented in King et al. (1991) is based on the infinite moving average (MA) form obtained by inverting the estimated VECM. This inversion cannot be directly made because of the presence of cointegration. An easier way to invert a VECM than those commonly suggested in the literature (see Yang [1998] for example) is proposed in Appendix B. The inverted reduced form model obtained is:

$$\Delta y_t = \mu + C_x(L)\Delta x_t + C(L)u_t$$

(A.4)

where all the parameters are defined in Appendix A. Notice that, since $u_t$ is independent of $e_{xt}$, $u_t$ is independent of $\Delta x_t$.

Consider a structural model of the form:

$$\Delta y_t = \mu + C_x(L)\Delta x_t + \Gamma(L)\eta_t$$

(A.5)
where \( \eta_t \sim IN(0, \Omega_\eta) \) is a \( n \times 1 \) vector of serially uncorrelated disturbances independent of \( \Delta x_t \) (being a linear combination of \( u_t \)), and where the endogenous variables’ response to a change in the exogenous variables is given by \( C_x(L) \).

The identifying problem consist in identifying the individual components in \( \eta_t \) from the estimated reduced form model given by (4) and can be described as follows. There are \( s = n - r \) identifiable common stochastic trends driving the \( n \times 1 \) vector \( y_t \) where \( r = \text{Rank}[\Pi_y] \).\(^{29}\) We express \( \Pi_y = \alpha_y \beta' \) where the \( n \times r \) loading matrix \( \alpha_y \), and the \( m \times r \) matrix of cointegrating vector \( \beta \) are each full column rank and identified up to an arbitrary \( r \times r \) non-singular matrix.\(^{30}\)

Partition \( \beta \) conformably with \( z_t \) as \( \beta' = (\beta_y', \beta_x')' \) where \( \beta_y \) and \( \beta_x \) are respectively \( n \times r \) and \( k \times r \), and partition the vector of structural disturbances \( \eta_t \) into two components, \( (\eta^1_t, \eta^2_t)' \), where \( \eta^1_t \) contains the \( s \) disturbances that have permanent effects on the components of \( y_t \) and where \( \eta^2_t \) contains \( n - s \) elements that have only temporary effects.

Partition the matrix of long-run multipliers, \( \Gamma(1) \), conformably with \( \eta_t \) as \( \Gamma(1) = [\Theta, 0] \), where \( \Theta \) is the \( n \times s \) matrix of long-run multipliers for \( \eta^1_t \) and 0 is a \( n \times (n - s) \) matrix of zeros corresponding to the long-run multipliers of \( \eta^2_t \).

**Assumption 3.1** \( \beta_x' = 0 \)

Under Assumption 3.1, \( \beta'z_t \) being stationery implies that \( \beta'y_t \) is stationery, which implies \( \beta'_y \Gamma(1) = 0 \). Hence the matrix of long-run multipliers is determined by the condition that its columns are orthogonal to \( \beta'_y \), and \( \Theta \eta^1_t \) represents the innovations in the long-run components of \( y_t \). While the cointegration restrictions identify the permanent innovations \( \Theta \eta^1_t \), they fail to identify \( \eta^1_t \) because \( \Theta \eta^1_t = (\Theta P)(P^{-1} \eta^1_t) \) for any non-singular matrix \( P \). To identify the individual elements of \( \eta^1_t \), we need the following identifying restrictions:

**Assumption 3.2.** \( u_t = \Gamma_0 \eta_t \) where \( \Gamma_0^{-1} \) exists.

\(^{29}\)We implicitly make the assumption that \( s \) is strictly positive. Wickens (1996) has shown that if \( \text{rank}(\Pi) = n \), then the full model has to be estimated and the common stochastic trends can be equated with the non-stationary component of the exogenous variables.

\(^{30}\)That is, \( (\alpha, K^{-1})(K\beta') = (\alpha, \beta') \) for any \( (r, r) \) non-singular matrix \( K \).
Under assumption 3.2, the structural disturbances are in the space spanned by current and lagged values of $z_t$ and that there are no singularities in the structural model.

**Assumption 3.3.** $\Theta$ is assumed triangular which permits writing $\Gamma(1) = \begin{bmatrix} \tilde{\Theta} \Pi, 0 \end{bmatrix}$ where $\tilde{\Theta}$ is a $n \times s$ matrix with no unknown parameters whose columns are orthogonal to $\beta_y'$, and $\Pi$ is a $s \times s$ lower triangular matrix with full rank and 1’s on the diagonal.\textsuperscript{31}

The covariance matrix of the structural disturbances is partitioned conformably with $\eta_t = (\eta^1_t, \eta^2_t)'$ and is assumed to be

**Assumption 3.4.** $\Omega_{\eta} = \begin{bmatrix} \Omega_{\eta^1} & 0 \\ 0 & \Omega_{\eta^2} \end{bmatrix}$ where $\Omega_{\eta^1}$ is diagonal.

That is, the permanent shocks, $\eta^1_t$, are assumed to be uncorrelated with the transitory shocks, $\eta^2_t$, and the permanent shocks are assumed to be mutually uncorrelated.

The permanent innovations, $\eta^1_t$, can be determined from the reduced form (7) as follows. From equations (7) and (8) and Assumption 3.2, $C(L) = \Gamma(L) \Gamma_0^{-1}$ and $C(1) = \Gamma(1) \Gamma_0^{-1}$. Let $D$ be any solution of $C(1) = \tilde{\Theta} D$. Thus, $\tilde{\Theta} D u_t = \tilde{\Theta} \Pi \eta^1_t$ and $D \Omega_u D' = \Pi \Omega_{\eta^1} \Pi'$. Let $\Pi = \text{chol}(D \Omega_u D') = \Pi \Omega_{\eta^1}^{1/2}$. Since $\Pi$ is a triangular matrix, and $\Omega_{\eta^1}$ is diagonal, there is a unique solution for $\Pi$ and $\Omega_{\eta^1}$. We can thus identify the permanent shocks $\eta^1_t = \Pi^{-1} D u_t$. Defining $G = \Pi^{-1} D$, it is then easy to show that the dynamic multipliers associated with $\eta^1_t$ are $C(L) \Omega_u G \Omega_{\eta^1}^{-1}$.

---

\textsuperscript{31}The diagonal elements of $\Pi$ are normalised to unity without loss of generality, since the variances of $\eta^1_t$ in Assumption 4.3 are unrestricted.
Appendix B. A simple way to invert a VECM with exogenous variables.

The identifying procedure documented in King et al. (1991) is based on the infinite moving average (MA) form obtained by inverting the estimated VECM. This inversion cannot be directly made because of the presence of cointegration. In this section, we propose an easier way to invert a VECM than those commonly suggested in the literature (see Yang [1998] for example).

By partitioning \( \Pi_y \) and \( \Psi_i \) conformably with \( z_t = (y_t', x_t')' \) as \( \Pi_y = (\Pi^y_y, \Pi^x_y)' \) and \( \Psi_i = (\psi^y_i, \psi^x_i)' \), where \( \Pi^y_y \) and \( \Psi^y_i \) are \( n \times n \) and \( \Pi^x_y \) and \( \Psi^x_i \) are \( n \times k \) constant coefficient matrices, we can rewrite (A.3) as:

\[
y_t = c + B_0 x_t + \sum_{i=1}^{p} A_i y_{t-i} + \sum_{i=1}^{p} B_i x_{t-i} + u_t \tag{B.1}
\]

where \( B_0 = \Lambda \), \( B_1 = -(\Lambda - \Pi^x_y - \Psi^y_1) \), \( B_i = (\psi^y_i - \psi^x_{i-1}) \) for \( i = 2, \ldots, p-1 \), \( B_p = -\psi^x_{p-1} \), \( A_1 = (\psi^y_1 + \Pi^y_y + I_n) \), \( A_i = (\psi^y_i - \psi^y_{i-1}) \) for \( i = 2, \ldots, p-1 \) and \( A_p = -\psi^y_{p-1} \).

We then write (A.4) as the following VARX(1):

\[
y_t = C + A y_{t-1} + B x_t + U_t \tag{B.2}
\]

where \( y_t = (y_t', y_{t-1}', \ldots, y_{t-p+1}', x_t', x_{t-1}', \ldots, x_{t-p+1}')' \), \( U_t = (u_t', 0, 0, \ldots, 0)' \) and \( c = (c', 0, 0, \ldots, 0)' \) are \( mp \times 1 \) matrices. Matrices \( A \) and \( B \), respectively of dimension \( mp \times mp \) and \( mp \times k \), are defined accordingly to \( \dot{Y} \) and \( x \) following Lutkepohl (p.335). Assuming that the process starts at a finite time \( t = 0 \), it is straightforward to obtain the inverted form:\footnote{In this unstable system, a one time impulse may have a permanent effect in the sense that it shifts the system to a new equilibrium, but the impulse responses may be calculated just as in the stable case. See Lutkepohl, Reimers (1992) for further details on this point.}

\[
y_t = A^T y_0 + \sum_{i=0}^{t-1} A^i c + \sum_{i=0}^{t-1} A^i B x_{t-i} + \sum_{i=0}^{t-1} A^i U_{t-i} \tag{B.3}
\]
Taking the first difference of (B.3), assuming for simplicity that \( U_0 = x_0 = y_0 = 0 \), and extracting the endogenous variables with the appropriate \( nm \times p \) matrix \( J = [I_n, 0, \ldots, 0] \), we get:

\[
\Delta y_t = \mu + C_x(L)\Delta x_t + C(L)u_t \tag{B.4}
\]

where \( \mu = J A^{t-1} c \), \( C_x(L) = \sum_{i=0}^{t-1} J A^i B L^i \), \( C(L) = \sum_{i=0}^{t-1} C_i L^i \),

\[
C_i = J(A^i - A^{i-1})J' L^i \quad \text{for} \quad i = 1, \ldots, t-1 \quad \text{and} \quad C_0 = I_n.
\]