Inflation-Target Expectations and Optimal Monetary Policy

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Abstract

Under credible inflation targeting, we suggest that, instead of forming a rational expectation, some firms ("*inflation-targeters*") might simply expect future inflation to always equal its target. This paper analyses the implications of this for optimal monetary policy in a standard new-Keynesian model. If shocks have any persistence and inflation-targeters are present, the optimal policy frontier is improved under discretion. Under commitment, although the gains from commitment are diminished (stabilisation bias is reduced), overall loss is still reduced relative to the rational expectations benchmark for plausible parameter values. These results formally show how policies which encourage expectations anchoring may be beneficial.

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1 Overview

Over the past fifteen years, several countries have adopted inflation targeting. The anchoring of inflation expectations which this frequently brings about is often argued to be beneficial for the economy. In particular, it is often claimed that inflation targeting reduces the impact of shocks, with inflation and output both being more stable as a result. It is also generally accepted that inflation targeting central banks should communicate extensively with the public about the plans and objectives of monetary policy.[1]

This paper makes several contributions which both help to provide formal theoretical support for these propositions, and attempt to analyse the potential theoretical implications of inflation targeting on expectations formation and hence some well-known results in the monetary policy literature. Specifically, we propose a new expectations formation mechanism ("*inflation-target expectations*") which differs from both rational and adaptive expectations, but which may be relevant in countries operating under credible inflation targeting regimes. This mechanism is designed to capture potential anchoring of inflation expectations onto the inflation target. We assume that inflation-target expectations are adopted by a fraction of firms, while the remaining firms use rational expectations. We then use these assumptions to illustrate precisely how anchored expectations may be beneficial for the economy in a formal theoretical model. In particular, we provide a concrete theoretical explanation of why the impact of shocks is likely to be muted under inflation targeting, with both inflation and output being more stable as a result. Finally, we also show that stabilisation bias will be reduced if some firms adopt inflation-target expectations, thus implying that the gains from commitment may not be as great as the current literature suggests.

In terms of policy, our results suggest that if some firms use inflation-target expectations, central banks should respond less aggressively than they otherwise would to cost-push shocks which exhibit any persistence. They also have the clear implication that inflation targeting is likely to be more successful if the central bank can persuade firms to believe that the inflation target will always be hit, thus anchoring expectations. Therefore, our model provides concrete theoretical support for the notions that central banks should publicise their targets widely, signal that they are fully committed to them, and regularly explain how they are trying to meet them.

2 Introduction

Although the first countries to adopt formal inflation targeting only did so in the early 1990s, it has rapidly become one of the main frameworks for conducting monetary policy. It is already used in a wide range of countries and there are continuing discussions

[1]For example, see Bernanke, Laubach, Mishkin and Posen (1999) and Bernanke (2003), who argue (informally) in favour of all of these propositions.

about whether the United States Federal Reserve should also adopt a formal inflation target (for a recent example, see Goodfriend, 2005).

The empirical evidence tends to suggest that once the regime is established and has gained credibility, inflation targeting helps to lower inflation expectations, effectively by inducing them to be anchored onto the inflation target. (In almost all countries which have adopted inflation targeting, inflation expectations were initially above the specified inflation target.) For example, using both survey data on expectations and evidence from interest-rate differentials for New Zealand, Canada, the United Kingdom and Sweden, Bernanke, Laubach, Mishkin and Posen (1999) find that once credibility had been established, inflation targeting helped to reduce private-sector inflation expectations. They conclude by noting (p. 297) that: "In all the cases we have studied, countries using inflation targets significantly reduced both the rate of inflation and the public's inflation expectations relative to their previous experience and, probably, relative to what they would have been in the absence of inflation targets". Similar conclusions are also reached by Johnson (2003). Slightly more direct empirical evidence that inflation expectations may be anchored under inflation targeting is presented by Sheridan (2001, cited in Ball and Sheridan, 2005), who finds that the regime dampens movements in expected inflation. Moreover, Gavin (2004) and Gurkaynak, Sack and Swanson (2003) present evidence suggesting that inflation expectations are better anchored in inflation targeting countries than in non-inflation targeting countries. Finally, Castelnuovo, Nicoletti-Altimari and Rodriguez-Palenzuela (2003) find that it is indeed the inflation target (or, where a symmetric target range is used, the midpoint of that range) onto which expectations are anchored in inflation targeting countries.

In terms of expectations, the experience of the United Kingdom is particularly striking. As King (2005, p. 13) observes, "whether you measure them by bond yields, index-linked versus conventional yields, or surveys, inflation expectations in Britain are now pretty well anchored on the [inflation] target". Moreover, considering this set of data over the past few years, Bank of England (2004, p. 10) argues that there is "reasonably strong evidence that inflation expectations have been close to the target since 1997, despite the shocks that have occurred".[2] Although data from the financial markets tend to reflect inflation expectations at fairly long time horizons, surveys usually reflect expectations at much shorter time horizons (typically one year ahead). Since short-term inflation expectations are more likely to be the relevant variable for firms' pricing decisions, the evidence from surveys is particularly interesting. As illustrated by King (2002, Chart 7) and Bank of England (2004, Chart A), if we exclude the general public, UK inflation expectations one year ahead as measured by the Barclays Basix quarterly survey have been very close to the official inflation target ever since the Bank of England was given sole responsibility for setting monetary policy in 1997. Moreover,

[2] King (2002) discusses some of this evidence in more detail.

inflation expectations were consistently higher before 1997 (falling dramatically almost immediately after the Bank of England's independence was announced) and even higher (and also much more volatile) prior to the initial adoption of inflation targeting in 1992.

In terms of whether the adoption of inflation targeting has reduced the impact of shocks and led to improved macroeconomic performance, Mishkin (1999) claims that it has. So do Bernanke, Laubach, Mishkin and Posen (1999): at the end of their detailed discussion which considers both cross-country empirical evidence and case studies, they conclude (p. 298) that "there does seem to be evidence that the effects of inflationary shocks are somewhat muted under [inflation targeting]". Moreover, in their more recent cross-country study, Neumann and von Hagen (2002) find that the adoption of inflation targeting has helped to curb the volatility of inflation and interest rates. Finally, for the specific case of the United Kingdom, Benati (2004) finds that both inflation and output have been much more stable since the adoption of inflation targeting. As argued by King (2002), this is despite the fact that shocks to the economy have been at least as large as in previous decades.

Although the discussion above strongly suggests that inflation targeting helps to anchor expectations and reduce the impact of shocks, the evidence, particularly that relating to supposed superior macroeconomic performance as a consequence of inflation targeting, has been challenged by Ball and Sheridan (2005). They argue that countries which introduced inflation targeting had high and volatile inflation initially and that their improved performance is simply a consequence of generic regression to the mean. When they control for this in their empirical analysis of 20 OECD countries, they find that there is no evidence that the inflation targeting countries performed better. However, Gertler (2005) is somewhat sceptical of their results. He notes that the decision to adopt inflation targeting is likely to be endogenous and it may have been the case that the countries which adopted the regime did so because they initially had high and unstable inflation. As a result, the (significant) coefficients on the initial condition terms in Ball and Sheridan's regressions may reflect the embedded impact of inflation targeting rather than reversion to the mean. Moreover, in their analysis, Bernanke, Laubach, Mishkin and Posen (1999) dealt with the issue of regression to the mean to a certain extent by comparing the inflation targeting countries with "control" countries which they regarded as having very similar characteristics and initial conditions (e.g. Italy, for comparisons with the United Kingdom and Sweden; and Australia, which was not inflation targeting for most of the period they were considering, for comparisons with Canada and New Zealand). They generally found that the performance of the control countries was inferior. Therefore, despite the evidence of Ball and Sheridan (2005), it still seems likely that inflation targeting does help to improve macroeconomic performance to a certain extent, and many authors retain this view.

Supposing that it is indeed the case that inflation targeting has muted the impact of shocks and resulted in more stable inflation and output, this then raises the question of what aspect(s) of inflation targeting may have caused this. As noted by Benati (2004), this question becomes particularly intriguing when we realise that although reduced inflation volatility could potentially be attributed to a more aggressive monetary policy (as might be associated with inflation targeting), most existing new-Keynesian models predict that output volatility should actually *increase* under this explanation. Therefore, to try to explain the facts, many authors (e.g. Bank of England, 2004; Bernanke, 2003; King, 2005) have instead directly cited the beneficial effects deriving from the anchoring of inflation expectations often associated with inflation targeting. However, in the existing literature, theoretical discussion of this potential link is extremely limited.

The one major exception is Orphanides and Williams (2005). They adopt a learning model in which agents are always trying to forecast inflation rationally but are hindered in their ability to do so by imperfect knowledge of both the model's structural parameters and the central bank's preferences. One of their key insights is that if the central bank's inflation target becomes known (perhaps because it is formalised or better publicised), agents will no longer have to estimate it, meaning that their forecasting problem is simplified and their inflation expectations will be more stable and better anchored. As a result, the optimal policy frontier is improved. However, in their model, inflation and output under a known inflation target are still more volatile than under the perfect knowledge benchmark, in which agents are assumed to either know the model's parameters or be able to estimate them perfectly. By contrast, we feel that expectations anchoring may yield much more significant benefits, in the sense that outcomes could be improved relative to the (perfect knowledge) rational expectations benchmark. We view this as stemming from the fact that the introduction of inflation targeting may change the way in which some agents actually form their expectations: this is in stark contrast to Orphanides and Williams (2005), who assume that all agents try to form expectations in the same way, regardless of whether or not there is an inflation target.

Specifically, in light of the evidence on expectations presented above, we feel that under inflation targeting, some firms may simply expect future inflation to always equal its target rather than ever attempting to form a rational expectation.[3] The introduction of this expectations formation mechanism (which we describe as "*inflation-target expectations*" in what follows) is the key novelty in our model. We justify it more fully in the main body of the paper, once we have specified the precise assumptions it entails. However, we note here that forming inflation-target expectations may be viewed as near-rational behaviour. This is because the gains from using rational rather than inflation-target expectations may be very small if the target is credible and the central bank is usually fairly successful in meeting it. As a result, for some firms, these gains

[3] In this sense, our work may be viewed as being loosely related to the recent literature which considers non-standard ways of modelling inflation expectations (e.g. Ball, 2000; Carroll, 2003; Mankiw and Reis, 2002; Reis, 2004).

may be more than outweighed by the thinking or calculation costs which are likely to be associated with forming a rational expectation.[4]

We introduce the assumption of inflation-target expectations into the standard new-Keynesian Phillips curve and, more broadly, into the new-Keynesian framework for analysing monetary policy. Although there are some well-known problems with the new-Keynesian Phillips curve (Mankiw, 2001), we use it as our baseline since it still represents the benchmark model in the literature and there is no clear consensus on which (if any) of the proposed alternatives is superior. However, we should note that our main idea is of broader relevance since, conceptually, inflation-target expectations could easily be introduced into most alternative Phillips curve specifications.

Having specified a policy objective, we analyse the implications of some firms adopting inflation-target expectations by solving our model for optimal monetary policy in response to a cost-push shock. We consider both the discretionary case, in which it is assumed that the central bank is unable to make commitments over the path of future policy, and the full commitment case, in which it is assumed that it can.[5] Under discretion, we find that if the shock has any persistence, the central bank should respond less aggressively to it as the proportion of firms adopting inflation-target expectations increases, due to the beneficial anchoring effects that these firms provide. Therefore, the effect of the shock is muted: the optimal policy frontier is improved; inflation and output are more stable; and loss is reduced. The results are slightly more complex under commitment. As more firms adopt inflation-target expectations, the optimal solution under commitment increasingly resembles the optimal solution under discretion (or, put another way, stabilisation bias is reduced). Since the gains from commitment are diminished, loss relative to the rational expectations benchmark will actually increase in this case if there is no persistence in the shock. However, for plausible parameter values and mild persistence in the shock, overall loss will be reduced because the beneficial anchoring effects of near-rational firms will more than outweigh the minor losses stemming from the reduced effectiveness of commitment. Taking all of this together, we therefore argue that the presence of firms adopting inflation-target expectations will generally be beneficial for the economy, regardless of whether or not the central bank can make commitments over the path of future policy. Our model therefore provides a

[4]For an early microeconomic illustration of why near-rational behaviour may be plausible in some contexts, see Cochrane (1989). For an early discussion of possible near-rational behaviour towards inflation, see Turnovsky (1970, p. 1453); for a more extensive discussion, see Akerlof, Dickens and Perry (2000). To a certain extent, our paper draws on their work.

[5]There is an extensive literature discussing whether or not central banks can make credible commitments over future policy (see Walsh, 2003, chapter 8, for a recent survey). Although this is clearly an important issue, we leave it aside for two reasons. Firstly, we wish to identify the potential gains from commitment in our model. Therefore, it makes sense to consider the two polar cases: discretion, considered in section 4, and full commitment, considered in section 5. Secondly, our interest is in the impact of firms adopting inflation-target expectations on optimal monetary policy and it therefore seems sensible to consider their impact in both of the benchmark cases commonly discussed in the literature.

concrete theoretical explanation of precisely how the anchoring of expectations which may be associated with inflation targeting can help to reduce the impact of shocks and lower the volatility of *both* inflation and output.[6]

The proportion of firms adopting inflation-target expectations is likely to depend on how well-known and credible the inflation target is. Therefore, in terms of policy, our model is consistent with the standard recommendations that central banks should publicise their inflation targets widely and attempt to raise awareness about monetary policy decisions. However, in contrast to much of the existing literature (with the exception of Orphanides and Williams, 2005), it provides a clear theoretical reason for precisely why these policies may be beneficial in terms of superior macroeconomic outcomes.

The remainder of this paper is structured as follows. Section 3 introduces the basic model, characterises the notion of inflation-target expectations more precisely, and provides further motivation for the assumption. Section 4 solves our model for optimal monetary policy under discretion, while section 5 solves it for the commitment case. Finally, section 6 concludes and discusses the policy implications of our results.

3 Introducing the Model

We work within the new-Keynesian framework for analysing monetary policy. Specifically, we adopt a simple closed economy model similar to those used in Clarida, Gali and Gertler (1999) and Walsh (2003, chapter 11). It consists of three components: a new-Keynesian Phillips curve modified to incorporate inflation-target expectations; a standard expectational IS curve; and a standard central bank objective function. To simplify and clarify the analysis, we mainly focus on the key aggregate relationships in what follows: for an extensive discussion of the model's microfoundations, see Woodford (2003).

3.1 The Inflation-Target Expectations Phillips Curve

3.1.1 Basic Assumptions

We assume that the economy is populated by small, identical, monopolistically competitive firms, each of whom faces an isoleastic demand curve for its differentiated product. However, firms are constrained in their ability to reset prices. Specifically, following Calvo's (1983) model of staggered price adjustment, we assume that in each period, firms must keep their prices fixed with probability λ. Moreover, we assume that each

[6] In the context of the United Kingdom, it could also help to explain why the Bank of England has never missed its inflation target by more than one percentage point (and therefore never been required to write an Open Letter to the Chancellor of the Exchequer), even though early predictions (e.g. Bean, 1998) suggested that Open Letters would be triggered fairly frequently.

firm has the same ex ante probability of being able to adjust its price in any given period - this means that the firm's reset probability is independent of its history. This formulation implies that, for any given firm, the expected number of periods between price adjustments is $1/(1-\lambda)$: from this, we can see how the parameter λ may be viewed as a measure of the degree of price rigidity in the economy.

With all variables expressed in logs (as they will be throughout this paper), it can then be shown that the aggregate price index at time t, p_t, evolves according to:

$$p_t = \lambda p_{t-1} + (1-\lambda)\,\overline{p}_t \tag{1}$$

where $\overline{p}_t$ is an index reflecting the reset price of firms that are able to change their price in period t. It can also be shown that the optimal reset price for fully rational firms, p_t^r, is given by:

$$p_t^r = (1-\beta\lambda)\sum_{k=0}^{\infty}(\beta\lambda)^k E_t\left\{mc_{t+k}^n\right\} \tag{2}$$

where mc_t^n is the firm's nominal marginal cost (as a percentage deviation from its steady-state level) and β is a discount factor (constrained to lie between zero and one).[7] Note that (2) may be re-expressed as:

$$p_t^r - p_t = (1-\beta\lambda)\sum_{k=0}^{\infty}(\beta\lambda)^k E_t\left\{mc_{t+k}\right\} + \sum_{k=1}^{\infty}(\beta\lambda)^k E_t\left\{\pi_{t+k}\right\} \tag{3}$$

where mc_t is the firm's real marginal cost (as a percentage deviation from its steady-state level) and $\pi_t = p_t - p_{t-1}$ is inflation at time t.

Equations (1) and (2) both have intuitive interpretations. From (1), we can see that if no firm is able to change its price ($\lambda = 1$), the aggregate price index will be unchanged from the previous period; if all firms can change their price ($\lambda = 0$), the aggregate price index will equal the index reflecting the reset price of firms in each period; while if $0 < \lambda < 1$, then, as we might expect, the aggregate price index will be a weighted average of these two extreme cases. Equation (2) shows that when choosing a new price, fully rational firms will take account of current and expected future nominal marginal costs. However, expected nominal marginal costs further into the future are given less

[7] The derivations of (1) and (2) assume that inflation is zero in steady-state: with a positive inflation target (which will be allowed for in this paper), steady-state inflation will be positive and, as shown by various authors (e.g. Rotemberg, 2002; Bakhshi, Burriel-Llombart, Khan and Rudolf, 2003; Ascari, 2004), there will be additional terms in (1) and (2). However, when allowing for positive inflation targets, we have very low targets in mind (similar in magnitude to the targets currently in place in developed inflation targeting countries). As a result, any steady-state inflation will be minimal and the additional terms in (1) and (2) will be of minor significance (for further discussion of this point, see Woodford, 2003, chapter 3, footnote 32). Therefore, for simplicity and clarity, it seems sensible to omit them. Moreover, note that we always have the option of setting the inflation target to be zero in our model, in which case there are no additional terms in (1) and (2). We therefore proceed by adopting (1) and (2) as stated, even though we will be allowing for a positive inflation target.

weight. This is both because of discounting, and because, the further away the date, the greater the probability that the firm will be able to change its price again before that date is reached, thus making the current expectation irrelevant.

We now introduce near-rational firms into the model in a comparable way to Gali and Gertler (1999), Gali, Gertler and Lopez-Salido (2001) and Steinsson (2003). We assume that only a fraction $1-\omega$ of firms are fully rational and reset their prices according to (3). The remaining fraction ω of firms are assumed to be near-rational. As will be discussed below, these near-rational firms form their expectations in a different way to fully rational firms and will therefore choose a different reset price, p_t^{nr}.[8]

The introduction of near-rational firms means that the index of newly reset prices is given by:

$$\overline{p}_t = (1-\omega)\, p_t^r + \omega p_t^{nr} \tag{4}$$

We can then use (1) and (4) to derive the following equation for inflation:

$$\pi_t = \left(\frac{1-\lambda}{\lambda}\right) \left[(1-\omega)\left(p_t^r - p_t\right) + \omega\left(p_t^{nr} - p_t\right)\right] \tag{5}$$

Thus far, our model is similar to the those contained in Gali and Gertler (1999) and Gali, Gertler and Lopez-Salido (2001). However, these papers proceed by assuming that near-rational firms adopt a simple backward-looking rule of thumb, choosing their reset price to be the previous period average reset price, corrected for inflation using the lagged inflation rate. We do not adopt this assumption. Instead, we assume that firms are operating under a regime of inflation targeting and that all firms are aware of the (constant) inflation target, π^T. (Consistent with the targets currently in place in developed inflation targeting countries, we have a very small, positive target in mind here.) Moreover, we assume that near-rational firms form expectations according to:

$$E_t\left\{\pi_{t+k}\right\} = \pi^T \quad \forall k \geq 1 \tag{6}$$

$$E_t\left\{mc_t\right\} = mc_t \tag{7}$$

$$E_t\left\{mc_{t+k}\right\} = 0 \qquad \forall k \geq 1 \tag{8}$$

[8]Note that this way of introducing rule of thumb behaviour into the benchmark new-Keynesian model differs from the approach of Yun (1996) and Christiano, Eichenbaum and Evans (2005). In these papers, it is assumed that all firms sometimes set their prices optimally, but *between* (optimal) price adjustments they adopt a rule of thumb indexing rule which updates their prices in every period in line with steady-state inflation (Yun) or previous period inflation (CEE). This only makes sense if we assume that the impediments to setting optimal prices more frequently are solely thinking or calculation costs rather than menu costs. However, as discussed by Taylor (1999), microeconomic evidence strongly suggests that most firms do not change their prices as frequently as every quarter, presumably because menu costs do not make it worthwhile for them in low inflation environments. Therefore, we feel that the real root of near-rational behaviour stems from the fact that, due to excessive thinking or calculation costs, some firms may never set their prices in a fully optimal way, while menu costs reduce the frequency with which they change prices (whether or not they are behaving in a fully optimal way when they change their price).

In other words, near-rational firms observe and take into account current marginal cost shocks but expect future inflation to always equal its target and future marginal costs to always be at their steady-state level. In what follows, if firms adopt (6)-(8), we will describe them as "*inflation-targeters*" who are forming "*inflation-target expectations*". Note that by substituting (6)-(8) into (3), we can see that the reset price of near-rational firms in our model is given by:

$$p_t^{nr} - p_t = (1 - \beta\lambda)\, mc_t + \frac{\beta\lambda}{1 - \beta\lambda}\pi^T \tag{9}$$

3.1.2 Motivating Inflation-Target Expectations

The introduction of inflation-target expectations represents the key innovation in our model and it drives all of our new results. Therefore, it is important to motivate the set of assumptions it encapsulates. The first thing to note is that if firms have assumed (6), the only consistent expectation regarding future marginal costs is for them to assume that they are zero. This is because forming a rational expectation of future marginal costs would incur the costs of being rational, meaning that the firm may as well form a rational expectation of future inflation as well. Moreover, adopting a rule of thumb implying non-zero future marginal costs would only be consistent with a rule of thumb implying that future inflation would be different from its target (note that if firms expect future marginal costs to be positive (negative), they should expect future inflation to be above (below) its target). Hence (8) must follow as a direct consequence of (6). As for (7), it seems plausible to suggest that near-rational firms are aware of their current costs when setting prices. As a result, it seems reasonable to assume that they can observe and take account of current marginal cost shocks.

Therefore, the key component of inflation-target expectations is captured by (6). Some empirical motivation for this assumption has already been provided in section 2; here we assess it in a more theoretical sense. The first thing to note is that if there are no shocks, the expectation will be correct because the central bank will always hit the inflation target in this case. In other words, inflation-target expectations are correct in steady-state. Perhaps more interestingly, and in stark contrast to adaptive expectations and other backward-looking rules of thumb, they will also be correct if all shocks are white noise. This is because when shocks have a mean of zero and are completely unpredictable, the best (i.e. rational) forecast of future inflation is simply the inflation target. By contrast, firms using backward-looking rules will make systematic mistakes if shocks are white noise. To see this, consider the case of a positive, non-persistent shock in period t which raises inflation in that period. The rational forecast of inflation in period $t+1$ is still π^T. However, firms using backward-looking rules will extrapolate in some way based on period t inflation and will therefore (incorrectly) expect inflation in period $t+1$ to be above π^T.

Inflation-target expectations also have a second major advantage over backward-looking rules of thumb: firms using them do still take account of the expected time until their next price change (compare, for example, with the rule of thumb proposed by Gali and Gertler, 1999, for which this is not the case). Therefore, inflation-target expectations may be regarded as being more forward-looking than other rules of thumb in two separate senses: firms using them both consider the future inflation target, and realise that they may not be able to change their prices again for several periods.

Nevertheless, if shocks are persistent, inflation-target expectations will be incorrect. However, there are three things to note. Firstly, although rational expectations will outperform inflation-target expectations in this case, there may be significant costs to being fully rational. These could be viewed as thinking or calculation costs. Critically, if inflation is low and stable, the benefits from being fully rational rather than adopting inflation-target expectations are likely to be minimal and may therefore be outweighed by these costs, at least for some firms. Secondly, if a high proportion of firms are using inflation-target expectations, then, collectively, near-rational firms will not be getting too much wrong because, as we will see below, expectations will be so well anchored that inflation will always be fairly near to the target, even in the face of large, persistent shocks. Finally, even if shocks are persistent, inflation-target expectations could still be more correct (i.e. closer to the rational forecast) than backward-looking rules of thumb (though the precise relative performance of inflation-target expectations against other rules will depend on the exact nature of the shock).

Taking all of this together, we feel that under a regime of inflation targeting, inflation-target expectations are just as plausible (and probably more plausible) than other rules of thumb which have previously been proposed. Therefore, near-rational firms who find it too costly to form a rational expectation may well choose to adopt inflation-target expectations as a quick and easy rule for forming expectations.

3.1.3 The New-Keynesian Phillips Curve under Inflation-Target Expectations

Having motivated inflation-target expectations, we are now able to derive our modified version of the new-Keynesian Phillips curve. In Appendix A, we show that provided that $\omega \neq 1$, the marginal cost version of this is given by:

$$\pi_t = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}\left[mc_t - \omega\beta\lambda E_t\left\{mc_{t+1}\right\}\right] + \beta\left[1-\omega(1-\lambda)\right]E_t\left\{\pi_{t+1}\right\} + \omega\beta(1-\lambda)\pi^T \tag{10}$$

Our model of the Phillips curve is now almost complete. However, before being able to use it for policy analysis, we must firstly relate the marginal cost terms to some measure of output, and, secondly, introduce a potential source of shocks into it. We deal with both of these issues by assuming that:

$$mc_t = \delta \left(y_t - y^* \right) + e_t = \delta x_t + e_t \tag{11}$$

where y_t is output at time t, y^* is the natural rate of output, x_t is the output gap at time t, and e_t is a cost-push shock. The cost-push shock is assumed to evolve according to:

$$e_t = \rho e_{t-1} + \varepsilon_t \tag{12}$$

where $\rho \in [0, 1]$ is a parameter measuring the degree of persistence in the shock and ε_t is an independent and identically distributed random variable with zero mean and variance σ_ε^2 (i.e. an unforecastable error term).

Provided that firms face constant returns to scale, it can be shown that this framework generates the proportional relationship between marginal cost and output contained in (11) (see, for example, Rotemberg and Woodford, 1997, or Woodford, 2003). Clarida, Gali and Gertler (1999, footnote 15) then argue that the cost-push shock contained in new-Keynesian models may be interpreted as a deviation from this equilibrium condition. They also give some examples of potential shocks which may mean that marginal cost does not change proportionately with output. More specifically, Clarida, Gali and Gertler (2001) formally relate the cost-push shock in (11) to wage mark-up shocks. Therefore, despite the lack of a clear consensus on how best to introduce shocks into the new-Keynesian Phillips curve, we feel that the approach adopted above is the one which is most clearly consistent with microfoundations.

We can now use (11) and (12) to rewrite (10). This gives:

$$\pi_t = \kappa \left[x_t - \omega\beta\lambda E_t \left\{ x_{t+1} \right\} \right] + \beta \left[1 - \omega \left(1 - \lambda \right) \right] E_t \left\{ \pi_{t+1} \right\} + \omega\beta \left(1 - \lambda \right) \pi^T + \eta e_t \tag{13}$$

where $\kappa = \delta \left(1 - \lambda\right) \left(1 - \beta\lambda\right) / \lambda$ and $\eta = \left(1 - \lambda\right) \left(1 - \beta\lambda\right) \left(1 - \omega\beta\lambda\rho\right) / \lambda$. Equation (13) is our modified version of the standard new-Keynesian Phillips curve. In what follows, we describe it as the "*inflation-target expectations Phillips curve (ITEPC)*".

We may use (13) to derive the steady-state output gap, $x_t = \overline{x}$, which is consistent with the inflation target being hit. To do this, note that in steady-state, $\pi_t = E_t \left\{ \pi_{t+1} \right\} = \pi^T$, $x_t = E_t \left\{ x_{t+1} \right\} = \overline{x}$, and $e_t = 0$. Substituting these expressions into (13) and rearranging gives:

$$\overline{x} = \frac{(1 - \beta)}{\kappa \left(1 - \omega\beta\lambda\right)} \pi^T \tag{14}$$

From (14), we can see that, as in other similar new-Keynesian models (e.g. Woodford, 2003, p. 246), $\overline{x}$ is increasing in the inflation target and there is a small long-run output-inflation trade-off. Note, however, that if $\beta = 1$ (i.e. there is no discounting), then $\overline{x} = 0$ and there is no long-run trade-off, while for plausible values of β (i.e. close to one), $\overline{x}$ will be close to zero and will only increase very slowly in the inflation target.

Having derived (14), we can use it to rewrite our Phillips curve, (13), in terms of inflation and output gaps. This gives:

$$\pi_t - \pi^T = \kappa\left[(x_t - \overline{x}) - \omega\beta\lambda\left(E_t\{x_{t+1}\} - \overline{x}\right)\right] + \beta\left[1 - \omega(1-\lambda)\right]\left(E_t\{\pi_{t+1}\} - \pi^T\right) + \eta e_t \tag{15}$$

3.1.4 Comments on the Inflation-Target Expectations Phillips Curve

Equation (15) represents a Phillips curve which is entirely new in the literature. It derives solely from the assumption that some firms adopt inflation-target expectations. As we would expect, when $\omega = 0$ (i.e. when there are no inflation-targeters), (15) reduces to the conventional new-Keynesian Phillips curve which is extremely widespread in the literature. Therefore, we can see that our model encompasses the standard framework.

It is also interesting to consider what happens to our Phillips curve in the limit as $\omega \to 1$ (recall that the derivation in Appendix A is invalid when $\omega = 1$).[9] In this limit, if we ignore the shock, (15) becomes:

$$\pi_t - \pi^T = \kappa\left(x_t - \overline{x}\right) - \kappa\beta\lambda\left(E_t\{x_{t+1}\} - \overline{x}\right) + \beta\lambda\left(E_t\{\pi_{t+1}\} - \pi^T\right) \tag{16}$$

Since the coefficient on the expected inflation term does not go to zero in this case, it may at first seem as if the ITEPC retains a forward-looking component in this limit even though no firms are forward-looking. However, this is an illusion since the unique bounded solution of (16) in steady-state is:

$$\pi_t - \pi^T = \kappa\left(x_t - \overline{x}\right) \tag{17}$$

which clearly has no forward-looking component. Moreover, (17) is identical to the solution we obtain if we solve our model directly for the case where $\omega = 1$.[10]

For values of ω between zero and one, the ITEPC is quite intuitive. If expected inflation is above the target (i.e. $E_t\{\pi_{t+1}\} > \pi^T$), it is clear from (15) that near-rational firms act as a counterbalance, providing a stabilising force for inflation and reducing it to a lower level than it would otherwise be. This gives a preliminary indication of how the anchoring of expectations associated with inflation-target expectations is likely to stabilise inflation following a shock. The $-\omega\beta\lambda\left(E_t\{x_{t+1}\} - \overline{x}\right)$ term in (15) is also interesting. It implies that if the output gap is expected to be above its steady-state level in the next period, then, if near-rational firms are present (i.e. $\omega > 0$), inflation will not be as high as it would otherwise be. So, in effect, it may be viewed as another damping term. It arises because near-rational firms assume that future marginal costs (and hence

[9] This discussion is motivated by Steinsson (2003, p.1435).

[10] This can be shown by firstly setting $\omega = 1$ in (5) and then substituting (9) into the resultant expression. Using (11) (with $e_t = 0$) and (14) (with $\omega = 1$), we can then obtain (17).

future output gaps) will always be at their steady-state levels. Therefore, if the output gap is expected to be above its steady-state level in the future (as calculated by rational firms), near-rational firms who can reset their prices will not raise them by as much as fully rational firms. Current inflation will therefore be lower than it would otherwise be: again this is quite intuitive.[11]

Inflation Persistence

The ITEPC does not include a term in lagged inflation. This may seem to be at odds with the empirical evidence which suggests that inflation does exhibit some persistence. However, if cost-push shocks are serially correlated, then (15) is able to generate the kind of inflation persistence necessary to match the observed data. Indeed, Jensen (2002) finds that persistence in US inflation over the period 1960-1998 is better explained empirically by highly correlated shocks than by the introduction of lagged inflation terms into the Phillips curve.

Moreover, there is no clear consensus on how to introduce a lagged inflation term into the Phillips curve in a way which is consistent with microfoundations, especially when the inflation target is constant and known by all firms, which is what we are assuming in this paper.[12] For example, as argued by Taylor (1999), the frequently cited approach of Fuhrer and Moore (1995) is inconsistent with optimising behaviour. Meanwhile, although the recent approaches of Gali and Gertler (1999) and Christiano, Eichenbaum and Evans (2005) are more consistent with microfoundations, both approaches still incorporate an ad hoc rule of thumb. Moreover, as shown by Mash (2005), if the coefficients on the rules of thumb assumed in these papers are chosen optimally by firms, then these models are no longer able to explain inflation persistence.

Finally, it seems unlikely that including a term in lagged inflation will change the qualitative theoretical results of this paper. Therefore, since this paper is intended to analyse the theoretical implications for monetary policy of a specific change in inflation expectations formation rather than attempting to match the data, it seems sensible not to include a lagged inflation term, both for simplicity and clarity. However, note that conceptually, it is a simple extension to incorporate such a term into our model. For example, we could assume the existence of three types of firms in our framework: fully rational firms, inflation-targeters, and backward-looking firms who adopt the rule of thumb proposed by Gali and Gertler (1999). We regard this as a fruitful area for future research, especially if some consensus can be reached on how best to motivate the presence of a lagged inflation term in the standard new-Keynesian Phillips curve.

[11] Note that $E_t\{\pi_{t+1}\} = E_t\{p_{t+1}\} - p_t$ and $E_t\{p_{t+1}\}$ is increasing in $E_t\{x_{t+1}\}$, so it is not the case that a rise in $E_t\{x_{t+1}\}$ actually lowers inflation.

[12] This assumption rules out the explanation, proposed by Erceg and Levin (2003), that inflation persistence may arise when agents do not perfectly observe the inflation target and are therefore unable to disentangle permanent changes in it from transitory shifts in monetary policy caused by shocks.

3.2 The IS Curve

We adopt a standard expectational IS curve which may be derived by log-linearising the representative household's Euler equation for optimal consumption. This yields:

$$x_t = E_t\{x_{t+1}\} - \left(\frac{1}{\sigma}\right)(i_t - E_t\{\pi_{t+1}\}) \tag{18}$$

where i_t is the nominal interest rate. For simplicity, we assume that the interest rate may be varied freely and costlessly, and that the central bank has no desire to smooth interest rates. This, together with the fact that the goods market adjusts instantaneously (as implied by the form of the IS curve), means that (18) imposes no real constraint on the central bank.[13] As a result, we can view the central bank as choosing x_{t+i} directly (rather than i_{t+i}) in all of what follows.

3.3 Policy Objective

We assume that the central bank attempts to minimise the loss function:

$$L_t = \frac{1}{2}E_t\left\{\sum_{i=0}^{\infty}\beta^i\left[\left(\pi_{t+i} - \pi^T\right)^2 + \phi\left(x_{t+i} - \overline{x}\right)^2\right]\right\} \tag{19}$$

where ϕ represents the relative weight placed by the central bank on stabilising the output gap. As is common in the recent literature, we have assumed that the central bank does not have an overly ambitious output target (recall that $\overline{x}$ is the steady-state output gap consistent with the inflation target being hit). Therefore, our model will not generate an average inflation bias.

Although we have not derived (19) explicitly from a utility-based welfare function for our specific model, loss functions of this type are used widely in the literature. Moreover, Svensson (1999) argues that, in practice, central banks generally regard the loss function to be of the form described by (19), and Bean (2003) appears to confirm this for the case of the United Kingdom. Therefore, for ease of comparison with the existing literature, practical relevance, and simplicity, we proceed by adopting (19).

[13] Instantaneous adjustment in the goods market may be viewed as unrealistic. However, since this paper is intended to analyse the theoretical implications of a change in the new-Keynesian Phillips curve, we wish to keep the remaining components of the model as simple as possible. Nevertheless, modifying the IS curve to include sluggish adjustment in the goods market is clearly a potential extension.

4 Optimal Monetary Policy under Discretion

4.1 Solving the Model

We now proceed to solve our model for optimal monetary policy in response to a cost-push shock. In other words, we derive the optimal path for inflation and the output gap that minimises the central bank's loss function following a shock.

We start by considering the discretionary case, in which it is assumed that the central bank is unable to make commitments over the path of future policy. Therefore, it is assumed that the central bank simply chooses inflation and the output gap in each period to minimise the loss function (19) subject to the ITEPC (15). Since the actions of the central bank at time t do not bind it at any future dates under discretion, it cannot influence firms' expectations about future inflation. Therefore the central bank's optimisation problem reduces to the single-period problem of:

$$\min_{\pi_t, x_t} \frac{1}{2}\left[\left(\pi_t - \pi^T\right)^2 + \phi\left(x_t - \overline{x}\right)^2\right]$$

subject to:

$$\pi_t - \pi^T = \kappa\left[\left(x_t - \overline{x}\right) - \omega\beta\lambda\left(E_t\left\{x_{t+1}\right\} - \overline{x}\right)\right] + \beta\left[1 - \omega\left(1 - \lambda\right)\right]\left(E_t\left\{\pi_{t+1}\right\} - \pi^T\right) + \eta e_t$$

The full details of the solution to this problem are contained in Appendix B. Combining the first order conditions for inflation and the output gap, we obtain the following condition which relates the two variables together:

$$\pi_t - \pi^T = \frac{-\phi\left(x_t - \overline{x}\right)}{\kappa} \tag{20}$$

This condition is standard in the literature (especially when $\pi^T = 0 \Rightarrow \overline{x} = 0$). By leading (20) by one period and then substituting both the resultant and original expressions into the Phillips curve (15), we can obtain an expectational difference equation in terms of x_t. Solving this equation using the method of undetermined coefficients, we find that the solution for the output gap under discretion is given by:

$$x_t - \overline{x} = c_1 e_t \tag{21}$$

where:

$$c_1 = \frac{-\delta\left(1-\lambda\right)^2\left(1-\beta\lambda\right)^2\left(1-\omega\beta\lambda\rho\right)}{\lambda^2\phi\left(1-\beta\rho\right) + \beta\rho\phi\omega\lambda^2\left(1-\lambda\right) + \delta^2\left(1-\lambda\right)^2\left(1-\beta\lambda\right)^2\left(1-\omega\beta\lambda\rho\right)} \tag{22}$$

Finally, substituting (21) into (20) gives the outcome for equilibrium inflation:

$$\pi_t - \pi^T = -\frac{\phi}{\kappa} c_1 e_t \tag{23}$$

4.2 Discussion of the Solution

As we would expect, when there are no near-rational firms in the economy ($\omega = 0$), these results reduce to the standard case in the literature. Following a positive cost-push shock, we can see that output will contract while inflation will rise above its target. More interesting is what happens when $\omega > 0$. From (22), $\frac{dc_1}{d\omega} > 0$ if $\rho > 0$ (if $\rho = 0$, $\frac{dc_1}{d\omega} = 0$). Since c_1 is always negative, this means that as ω increases, the magnitude of c_1 decreases. Therefore, in the presence of near-rational firms, it is optimal for the central bank to respond less aggressively to cost-push shocks which exhibit any persistence. (If there is no persistence in the shock, then as noted in section 3.1.2, near-rational firms will actually be acting rationally and, as a result, will not make any difference to the solution.) As a result, provided that the central bank behaves optimally (and is aware of the presence of near-rational firms), the impact of (persistent) shocks on inflation and the output gap will be reduced as the proportion of near-rational firms increases, with the implication that both inflation and output will be more stable as a result.

This is an intuitive result. By assuming that future inflation will equal the target and by ignoring any persistence which may be present in cost-push shocks, near-rational firms are acting as an (inflation) anchor: they serve to dampen down the effects of shocks and permit the central bank to be less aggressive when responding to them. This is beneficial for the economy as a whole: the next subsection shows this by deriving an expression for loss in our model; the following subsection illustrates it by showing how the optimal policy frontier changes as the proportion of near-rational firms increases.

4.2.1 Loss under Discretion

We are able to use our solutions for inflation and the output gap to derive an exact expression for the magnitude of the loss under discretion following a one-off shock in period t. First note that from (21) and (23) respectively:

$$E_t \{x_{t+i} - \overline{x}\} = c_1 E_t \{e_{t+i}\} = c_1 \rho^i e_t \tag{24}$$

$$E_t \left\{\pi_{t+i} - \pi^T\right\} = -\frac{\phi}{\kappa} c_1 E_t \{e_{t+i}\} = -\frac{\phi}{\kappa} c_1 \rho^i e_t \tag{25}$$

where we have used the fact that $E_t \{e_{t+i}\} = \rho^i e_t$, which follows from (12). Substituting (24) and (25) into the loss function (19) and simplifying, we can obtain an expression for the loss associated with a one-off shock:

$$L_t = \frac{1}{2}\sum_{i=0}^{\infty}\beta^i\left[\left(-\frac{\phi}{\kappa}c_1\rho^i e_t\right)^2 + \phi\left(c_1\rho^i e_t\right)^2\right] = \frac{\phi\left(\phi+\kappa^2\right)c_1^2 e_t^2}{2\kappa^2\left(1-\beta\rho^2\right)} \tag{26}$$

Since the magnitude of c_1 is decreasing in ω when $\rho > 0$, we can immediately see from (26) that the loss from a (persistent) shock decreases as the proportion of near-rational firms increases. We can also illustrate this graphically by plotting loss against ω. To do this, we first need to calibrate the parameters in the model. We interpret the time interval as one quarter and therefore set λ to be 0.75 (implying that, consistent with the discussion in Taylor, 1999, firms reset their price once a year on average) and the discount factor, β, to be 0.99. We assume that ϕ is 0.0625. This implies that in *annual* terms, the central bank places equal weight on output and inflation deviations – a standard benchmark case in the literature. Estimates of κ (and hence the implied value of δ) vary widely. We set δ to be 0.6, implying that κ is approximately 0.05, which is the value chosen by Walsh (2003, p. 527). Finally, we assume that the shock, e_t, is of magnitude 0.1 (note that this is an innocuous assumption since e_t only has a scale effect in (26)).

Given these parameters, we plot loss (multiplied by 10^5 for greater clarity) against ω for various values of ρ, as depicted in Figure 1. From the diagram, we can clearly see that provided $\rho > 0$, loss decreases as ω increases. Moreover, the gains are particularly large when ρ is high.

4.2.2 The Optimal Policy Frontier under Discretion

We are also able to plot the optimal policy frontier for different values of ω in our model. The optimal policy frontier illustrates the trade-off between inflation and output volatility that the central bank faces following a cost-push shock. From (21) and (23) respectively, we can see that:

$$Var\left(x_t - \overline{x}\right) = c_1^2 Var\left(e_t\right) = \frac{c_1^2\sigma_\varepsilon^2}{1-\rho^2} \tag{27}$$

$$Var\left(\pi_t - \pi^T\right) = \frac{\phi^2 c_1^2}{\kappa^2}Var\left(e_t\right) = \frac{\phi^2 c_1^2\sigma_\varepsilon^2}{\kappa^2\left(1-\rho^2\right)} \tag{28}$$

where we have used the fact that $Var\left(e_t\right) = \frac{\sigma_\varepsilon^2}{1-\rho^2}$, which follows from (12).

In Figure 2, we trace out optimal policy frontiers by taking different values of ω and then plotting (27) against (28) over the range $\phi \in [0.01, 0.25]$.[14] As can be seen,

[14]For this exercise, we use the calibrated parameter values from the previous section and additionally assume that σ_ε^2 is 0.01 (another innocuous assumption which only has a scale effect) and that ρ is 0.5. We also multiply $Var\left(\pi_t - \pi^T\right)$ by a factor of 16 so that it can be interpreted as the variance of annual inflation, and then scale both axes by a factor of 100 for greater clarity.

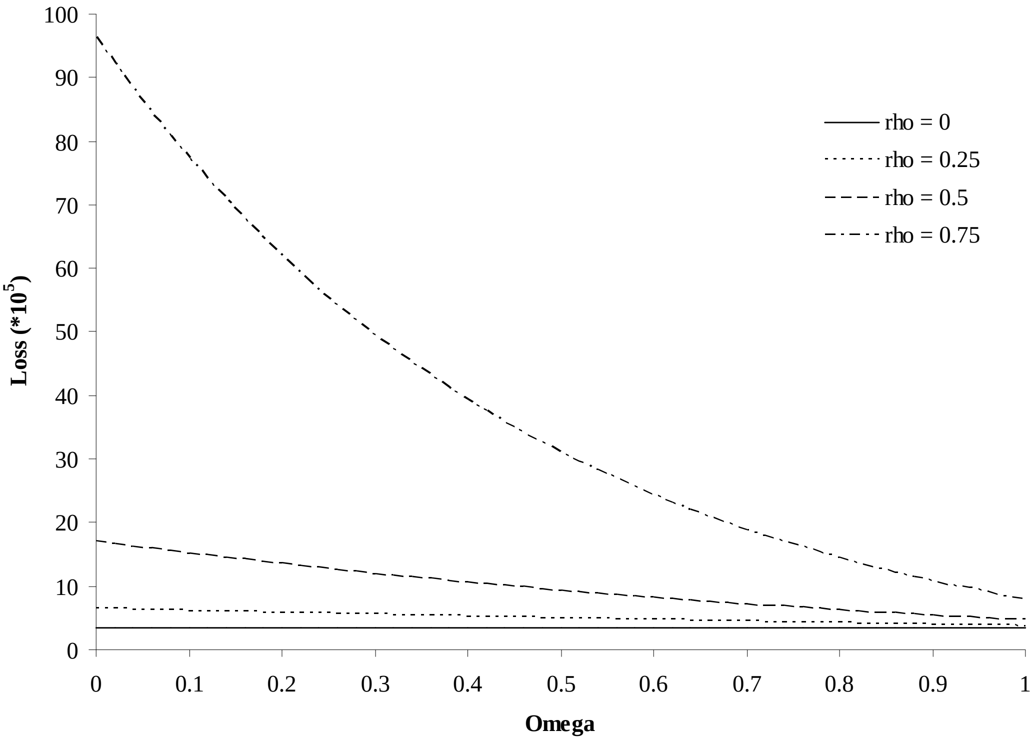

Figure 1: Loss under Discretion

each policy frontier illustrates the level of volatility in the output gap which must be tolerated to achieve a given volatility in inflation. Considering a fixed policy frontier (e.g. the $\omega = 0$ frontier), it is clear that if the central bank wishes to reduce inflation volatility, it must accept greater output volatility. From this, we can therefore see why most existing new-Keynesian models find it difficult to explain a simultaneous reduction in both inflation and output volatility.

However, it is also evident from the diagram that as ω increases, the optimal policy frontier improves as signified by its inward shift. Therefore, we can clearly see that in our model, it is possible for both inflation and output volatility to fall at the same time.

4.3 Summary

In this section, we have shown that if some firms adopt inflation-target expectations, the central bank should respond less aggressively to cost-push shocks which exhibit any persistence. Moreover, under the optimal policy, the effect of (persistent) shocks on the economy will be muted, with both inflation and output being less volatile. As a result, losses will not be as severe. This provides a concrete theoretical explanation of exactly how and why the anchoring of expectations in an inflation targeting regime may

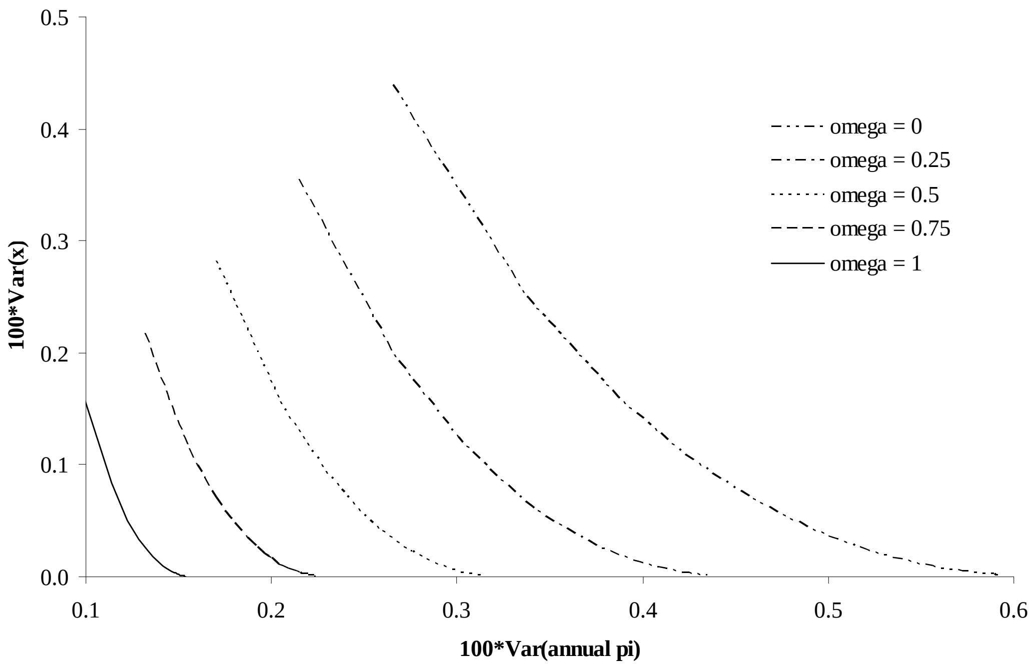

Figure 2: Optimal Policy Frontier under Discretion

be beneficial for the economy.

5 Optimal Monetary Policy under Commitment

5.1 Solving the Model

We now consider how our results extend when the central bank is able to make commitments into the future. In this case, the central bank specifies a path for inflation and the output gap in all future periods at time t and is assumed to be able to commit to this plan even if it would be in its best interest to choose a different policy in some subsequent period. As a result, it is able to influence private sector inflation expectations. Therefore, in period t, subject to the ITEPC (15) holding in every period $t+i$ (where $i \geq 0$), it attempts to minimise the loss function (19) over its entire future rather than period by period as in the discretionary case. This implies that its minimisation problem is given by:

$$\min_{\pi_{t+i}, x_{t+i}} \frac{1}{2} E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[\left(\pi_{t+i} - \pi^T \right)^2 + \phi \left(x_{t+i} - \overline{x} \right)^2 \right] \right\}$$

subject to (for all $i \geq 0$):

$$\begin{aligned} E_t \{\pi_{t+i}\} - \pi^T = {} & \kappa \left[\left(E_t \{x_{t+i}\} - \overline{x} \right) - \omega \beta \lambda \left(E_t \{x_{t+i+1}\} - \overline{x} \right) \right] \\ & + \beta \left[1 - \omega (1 - \lambda) \right] \left(E_t \{\pi_{t+i+1}\} - \pi^T \right) + \eta e_{t+i} \end{aligned}$$

The full details of the solution to this problem are contained in Appendix C. Letting μ_{t+i} be the period $t+i$ Lagrange multiplier, the first order conditions for π_t, x_t, π_{t+i} for $i \geq 1$, and x_{t+i} for $i \geq 1$ are respectively given by:

$$\left(\pi_t - \pi^T\right) - \mu_t = 0 \tag{29}$$

$$\phi\left(x_t - \overline{x}\right) + \mu_t \kappa = 0 \tag{30}$$

$$E_t\left\{\left(\pi_{t+i} - \pi^T\right) - \mu_{t+i} + \left[1 - \omega\left(1 - \lambda\right)\right]\mu_{t+i-1}\right\} = 0 \qquad (i \geq 1) \tag{31}$$

$$E_t\left\{\phi\left(x_{t+i} - \overline{x}\right) + \kappa\mu_{t+i} - \kappa\omega\lambda\mu_{t+i-1}\right\} = 0 \qquad (i \geq 1) \tag{32}$$

Comparing (29) and (30) with (31) and (32), we can immediately see the time inconsistency present in the optimal policy. For example, comparing the inflation equations, we can see that it is optimal to adopt (29) in the present period but to promise to use (31) in future periods. However, when the next period arrives, it will then be optimal for the central bank to adopt an updated version (29) in that period rather than abiding by its earlier promise. In what follows, we assume that this problem is either not an issue or has been resolved in some way (possibly by institutional design).

Having obtained the first order conditions, we solve the model by adopting Woodford's (1999) timeless perspective approach which assumes that the central bank ignores (29) and (30), and instead implements (31) and (32) in every period, including the present period. McCallum and Nelson (2004) argue that this is the most plausible approach for solving for equilibrium: in particular, they stress that the approach makes sense if we think of the decision to implement (31) and (32) as having been made at some point in the distant past.[15]

Solving (31) and (32) simultaneously, we obtain an expression for expected inflation as a function of output gap terms. We then use this to eliminate the inflation terms from the Phillips curve constraint to obtain a difference equation in terms of x_t. Solving this equation using the method of undetermined coefficients, we obtain the solution for the output gap, which can then be used to derive the outcome for equilibrium inflation. These solutions may be summarised by the following set of equations.

The solution for the output gap may be written in two equivalent forms:

$$x_t - \overline{x} = \gamma_0 e_t + \sum_{j=1}^{\infty} \gamma_j \left(x_{t-j} - \overline{x}\right) \tag{33}$$

$$x_t - \overline{x} = \left(\gamma_1 + \omega\lambda\right)\left(x_{t-1} - \overline{x}\right) + \gamma_0 e_t - \omega\lambda\gamma_0 e_{t-1} \tag{34}$$

[15] Note, however, that since the timeless perspective policy ignores the first-order conditions associated with the very first period, it is not quite fully optimal. Indeed, as shown by Blake (2001) and Jensen and McCallum (2002) in the standard case (where there are no near-rational firms), it is not even optimal within the class of time-invariant policy rules. However, as these authors acknowledge, calibration exercises for plausible parameter values suggest that the relative loss associated with the timeless perspective policy is likely to be very small.

while equilibrium inflation is given by:

$$\pi_t - \pi^T = \omega\lambda\left(\pi_{t-1} - \pi^T\right) + \frac{\phi}{\kappa}\left(1 - \gamma_1 - \omega\right)\left(x_{t-1} - \overline{x}\right) - \frac{\phi}{\kappa}\gamma_0 e_t + \frac{\phi}{\kappa}\omega\lambda\gamma_0 e_{t-1} \quad (35)$$

where γ_1 is given by the solution less than one of the quadratic equation:

$$h_3\gamma_1^2 - \left(h_2 - \omega\lambda h_3\right)\gamma_1 + h_4 = 0 \quad (36)$$

and:

$$\gamma_0 = \frac{-h_5}{h_2 - h_3\rho - h_3\gamma_1} \quad (37)$$

$$\gamma_j = (\omega\lambda)^{j-1}\gamma_1 \quad \forall j \geq 2 \quad (38)$$

$$h_2 = 1 + \frac{\kappa^2}{\phi} + \beta\left[1 - \omega\left(1 - \lambda\right)\right]\left(1 - \omega\right)$$

$$h_3 = \frac{\kappa^2\omega\beta\lambda}{\phi} + \beta\left[1 - \omega\left(1 - \lambda\right)\right]$$

$$h_4 = \left(1 - \omega\right)\left\{1 - \omega\lambda\beta\left[1 - \omega\left(1 - \lambda\right)\right]\right\}$$

$$h_5 = \frac{\kappa\eta}{\phi}$$

5.2 Discussion of the Solution

We first consider what happens to our solution when $\omega = 0$ and $\omega = 1$. As we would expect, when there are no near-rational firms, the results reduce to the standard case in the literature. To see this, note that when $\omega = 0$, (34) and (35) simplify to:

$$x_t - \overline{x} = \gamma_1\left(x_{t-1} - \overline{x}\right) + \gamma_0 e_t$$

$$\pi_t - \pi^T = \frac{\phi}{\kappa}\left(1 - \gamma_1\right)\left(x_{t-1} - \overline{x}\right) - \frac{\phi}{\kappa}\gamma_0 e_t$$

while the equations for γ_1 and γ_0 (i.e. (36) and (37)) reduce to:

$$\beta\gamma_1^2 - \left(1 + \frac{\kappa^2}{\phi} + \beta\right)\gamma_1 + 1 = 0$$

$$\gamma_0 = \frac{-\kappa\eta}{\phi\left[1 + \beta\left(1 - \rho - \gamma_1\right)\right] + \kappa^2}$$

These expressions constitute the standard solution for optimal monetary policy under commitment. In this solution (and, indeed, also in our model's solution for all $\omega < 1$), the central bank commits to make future output gaps dependent on the current output gap following a shock, even if there is no persistence in that shock. In other words, the optimal policy introduces inertia into the output gap (and hence inflation).

The rationale for this can be explained intuitively by fixing ω and considering the case of a (non-persistent) positive cost-push shock at time t. Assuming that the economy was at steady-state before the shock, this will cause an immediate contraction in output and rise in inflation. However, by (credibly) committing to contract output in future periods (when e_t will be zero), the central bank is able to reduce period t expectations of future inflation. Since inflation in period t is partly determined by expectations of future inflation, this means that period t inflation will be lower than would otherwise be the case. As a result, the impact of the shock on inflation is reduced and, for a fixed ω, loss is lower compared to the discretion case in which the central bank is unable to make credible commitments.[16] Therefore, as highlighted by Currie and Levine (1993) and discussed more recently by Clarida, Gali and Gertler (1999), there may be gains from commitment even if the policy maker does not have an over-optimistic output target. Since output is stabilised more in future periods under discretion than under commitment, policy under discretion is often referred to as suffering from a "stabilisation bias".

When $\omega = 1$, the optimal solution under commitment reduces so that it is identical to the optimal solution under discretion.[17] The intuition for this result is straightforward. By its very nature, the commitment solution relies on some firms looking forward in period t and taking into account the future path of policy when setting prices. However, when all firms are adopting inflation-target expectations, none of them takes future policy into account. Therefore, there is no point in the central bank making commitments over the path of future policy: only current policy can influence inflation and the output gap. As a result, when $\omega = 1$, the central bank's optimal solutions for inflation and the output gap are determined period by period, as happens under discretion. This has implications both for the extent of stabilisation bias, and for loss values.

5.2.1 Stabilisation Bias

We have already discussed the presence of stabilisation bias when $0 \leq \omega < 1$. However, since the optimal solutions under commitment and discretion coincide when $\omega = 1$, there is no stabilisation bias in this case. Therefore, if all firms are near-rational, there are no gains from commitment. To analyse whether stabilisation bias decreases monotonically in ω, we set $\rho = 0$. This eliminates all beneficial anchoring effects

[16] We have already noted above that the timeless perspective policy is not fully optimal since it ignores the first-order conditions in the first period. Therefore, as demonstrated by Blake (2001), it is actually possible for losses under optimal discretionary policy to be lower than under the timeless perspective policy. However, this relies on both κ and β being fairly low relative to what we might normally expect: for plausible ranges of the parameter values, McCallum and Nelson (2004) show that the timeless perspective policy is always superior. Therefore, when comparing the discretion and commitment solutions in what follows, our discussion assumes that the timeless perspective policy under commitment always dominates the optimal policy under discretion.

[17] To see this formally, first note that the solution less than one to (36) when $\omega = 1$ is $\gamma_1 = 0$. Therefore, from (38), $\gamma_j = 0 \ \forall j \geq 1$ and, as a result, from (33), the solution for the output gap is given by $x_t - \overline{x} = \gamma_0 e_t$. Straightforward algebra then shows that γ_0, as given by (37), is equal to c_1, as given by (22), when $\omega = 1$. This then proves that the solutions under commitment and discretion coincide when $\omega = 1$.

provided by near-rational firms (recall that their presence has no impact on loss under discretion when $\rho = 0$) and therefore allows us to isolate the impact of near-rational firms on the difference between the commitment and discretion solutions, and hence on stabilisation bias. In Appendix D, we show that the magnitude of γ_0 is monotonically increasing in ω. In other words, as the proportion of near-rational firms increases, provided that $\rho = 0$, it is optimal under commitment to respond more strongly in the current period to shocks. This is what we would expect intuitively because, as more firms adopt inflation-target expectations, the ability to control inflation by making commitments about future output gaps is diminished. Therefore, as ω increases, the optimal solution under commitment monotonically converges onto the optimal solution under discretion, and (by implication) stabilisation bias decreases monotonically.

5.2.2 Loss Index under Commitment

From the discussion above, it is clear that the gains from commitment are reduced as the proportion of near-rational firms increases because fewer firms consider the future policy of the central bank. Therefore, if $\rho = 0$ (meaning that near-rational firms have no beneficial anchoring effects), subject to footnote 16, it must be the case that loss under commitment is increasing in ω.

However, if $\rho > 0$, the presence of near-rational firms results in a trade-off between the beneficial anchoring effects that they provide and the costs associated with the fact that commitment is less effective. Unfortunately, in the commitment case, it is not possible to analytically derive an exact expression for loss as in section 4.2.1 (unless $\rho = 0$). However, Blake (2001) and Woodford (2003, pp. 496-497) both show that loss for the objective function (19) is asymptotically proportional to the index:[18]

$$LI = \frac{1}{2}\left[Var\left(\pi_t\right) + \phi Var\left(x_t\right)\right] \tag{39}$$

Using (34) and (35), we can compute expressions for $Var\left(\pi_t\right)$ and $Var\left(x_t\right)$.[19] We then use the calibrated parameter values assumed in section 4.2.1 to plot the loss index (again multiplied by 10^5 for greater clarity) against ω for various values of ρ. Figure 3 restricts attention to low values of ρ; Figure 4 considers higher values of ρ as well.

From Figure 3, we can see that, as we already know, loss is increasing in ω when $\rho = 0$. However, even a small rise in ρ (to about 0.15) overturns this result: loss rises slightly for small values of ω (due to the reduced effectiveness of commitment) but, as ω increases, this is more than offset by the gains from greater anchoring, so that when

[18] Technically, these authors do not include the 1/2 term in (19) and hence it is not present in their version of (39). In addition, they assume that the inflation target is zero. However, assuming a positive inflation target makes no difference to the variance of inflation since the target is a constant.

[19] Though conceptually straightforward, the calculations are very long and are therefore omitted from the paper. However, they are available on request from the author.

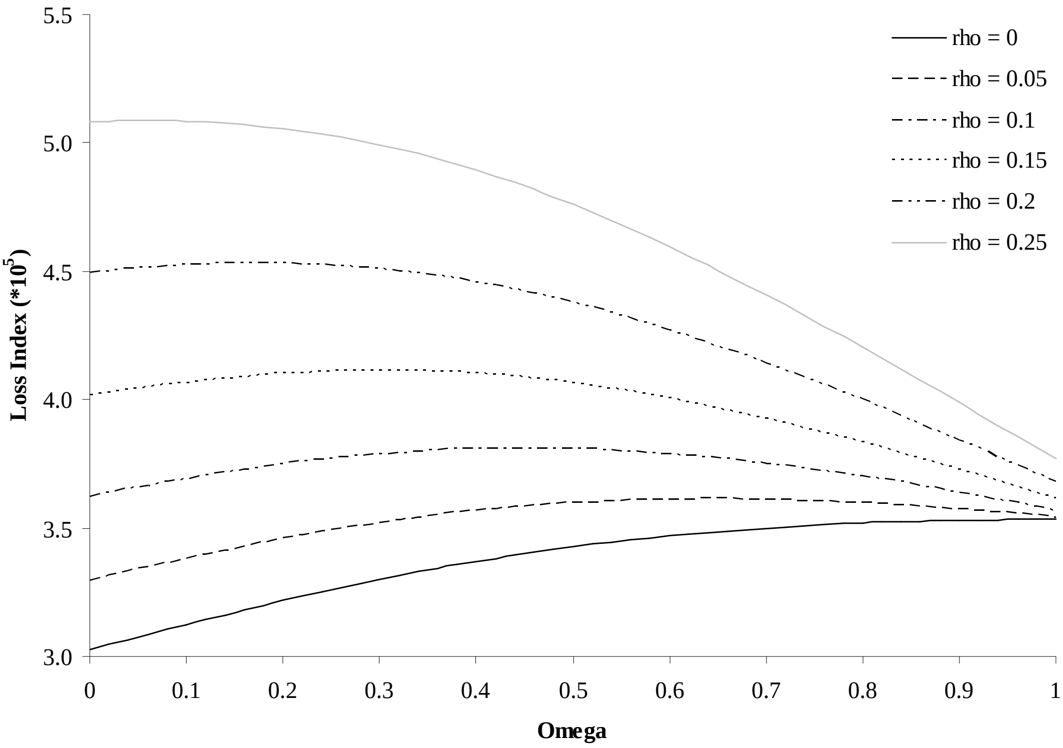

Figure 3: Loss Index under Commitment (for low values of ρ)

$\omega = 1$, loss is lower than when $\omega = 0$. As is clear from Figure 4, for higher values of ρ, loss falls monotonically in ω – the large gains deriving from the anchoring of expectations are completely dominant.

It is possible that these graphs are affected by the choice of calibrated parameters. We therefore perform robustness checks by varying the parameters over a range of plausible values.[20] The results suggest that the broad picture in Figures 3 and 4 is not sensitive to the particular parameter values used. However, the exact critical value of ρ above which near-rational firms help to reduce loss is clearly dependent on the choice of parameters. Therefore, we change the parameters in such a way that increases the critical ρ: specifically, we reduce ϕ to 0.01 (approximately the lower end of its plausible range) and raise the implied κ to 0.1 (approximately the upper bound for estimates of κ). Nevertheless, even in this extreme case, the results are not radically different: we find that as long as ρ is greater than about 0.35, loss is lower when $\omega = 1$ than when $\omega = 0$, and as long as ρ is greater than about 0.65, loss falls monotonically in ω.

Given the assumption that the time interval is one quarter, and citing the evidence of Jensen (2002) discussed in section 3.1.4, we would argue that cost-push shocks are very likely to be highly correlated. In particular, we would argue that it is certainly plausible for ρ to be greater than 0.65. Moreover, note that this is an extreme critical value in

[20]Full results from this analysis are not presented here. However, the spreadsheet which allows robustness checks to be performed is available on request from the author.

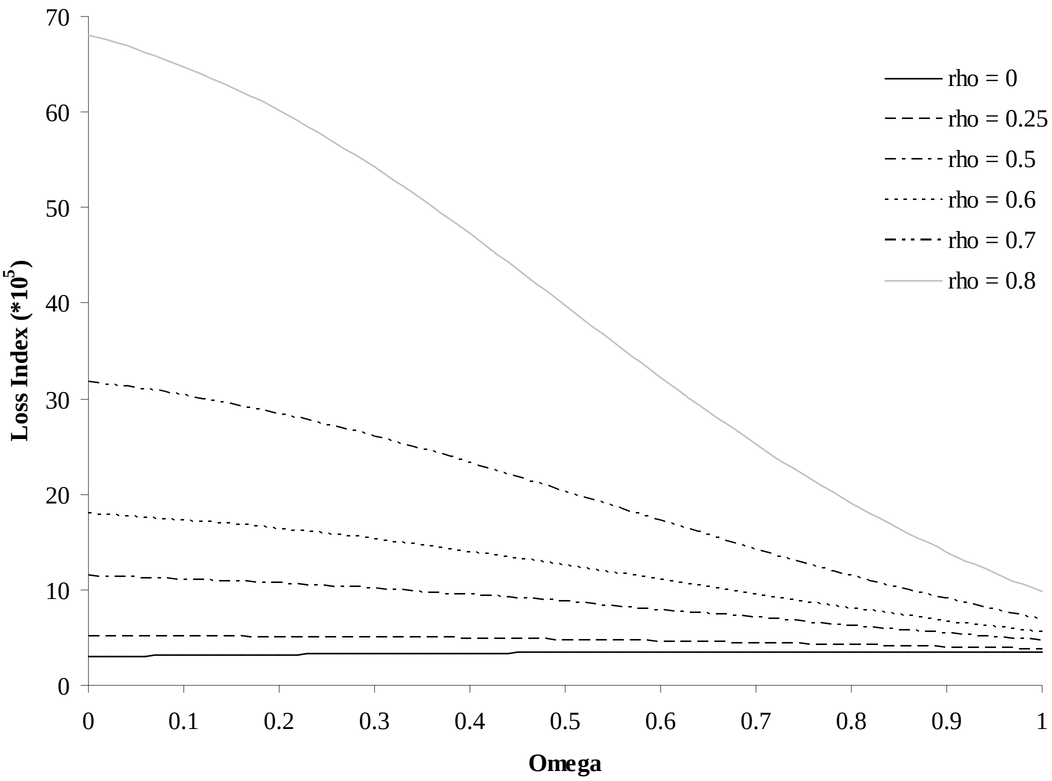

Figure 4: Loss Index under Commitment (for high values of ρ)

two separate senses: firstly, it is obtained by using highly unfavourable values for the parameters; secondly, even in this extreme case, slightly lower values of ρ only result in small increases in loss when ω is very low, but can still yield fairly large gains (relative to the $\omega = 0$ case) when ω is high. Finally, even if $\rho = 0$, we can see that the relative losses caused by firms adopting inflation-target expectations are extremely small. By contrast, the potential gains are very large when ρ is high, Therefore, even if ρ varies according to the shock and is sometimes (but not always) zero, near-rational firms are still, on average, very likely to be beneficial for the economy.

Based on this discussion, we would argue that, for plausible parameter values, it is highly likely that loss (or at least average loss) under commitment is reduced as the proportion of near-rational firms increases. So, even in the commitment case, the presence of near-rational firms is very likely to reduce the variance of both inflation and the output gap (recall that the loss index (39) is dependent on these two variances) and be beneficial for the economy.

5.3 Summary

In this section, we have shown that, as the proportion of near-rational firms increases, the gains from commitment fall, with the optimal solution under commitment increasingly resembling the optimal solution under discretion. Therefore, given that some firms may well adopt inflation-target expectations, our results indicate that stabilisation bias may not be as important an issue as the current literature suggests.

We have also shown that for plausible parameter values and mild persistence in the shock, overall loss is reduced as the proportion of near-rational firms increases, despite the fact that commitment becomes less effective. Therefore, the analysis in this section does not overturn the broad results derived when considering the optimal policy under discretion in section 4. In particular, it is still the case that the adoption of inflation-target expectations by some firms is very likely to lower both inflation and output volatility, and be beneficial for the economy.

6 Conclusion

6.1 Summary of the Paper and its Main Results

This paper was motivated by the idea that in countries which have adopted credible inflation targeting, some firms might simply expect future inflation to always be equal to its target. We formalised this notion in the concept of inflation-target expectations. We then introduced inflation-target expectations into the benchmark new-Keynesian model. Solving our model for optimal monetary policy under discretion, we found that as the proportion of near-rational firms using inflation-target expectations increased, the impact of (persistent) cost-push shocks on the economy was reduced, with losses being less severe, and inflation and output both being less volatile. In the commitment case, we found that stabilisation bias was reduced when some firms were adopting inflation-target expectations, suggesting that it may be less of an issue than the literature currently suggests. We also found that loss was reduced for plausible parameter values and mild persistence in the shock. Taken together, these results may be viewed as providing a precise illustration of exactly how the anchoring of expectations often associated with inflation targeting may help to reduce the impact of shocks on the economy and make both inflation and output more stable as a result.

6.2 Possible Extensions

Some possible extensions to this paper have already been proposed. For example, the IS curve could be adjusted to incorporate sluggish adjustment in the goods market (perhaps due to habit persistence), while a term in lagged inflation could be added to the Phillips curve. The model could also clearly be extended to the open economy context.

Another potentially interesting extension would be to endogenise the parameter ω. It seems plausible that the proportion of inflation-targeters could depend on how successfully the central bank has met their target in recent periods. Therefore, ω could be made a decreasing function of some metric representing recent deviations of inflation from its target. Such a system would probably be non-linear but could be interesting to simulate. In particular, the system would probably be fairly stable within certain bands (possibly reflecting the target ranges which are associated with many inflation targeting frameworks) but could be quite unstable outside those bands. Indeed, movement outside those bands and the associated loss of expectations anchoring could reflect a potential situation which might create severe problems for the central bank and, more generally, the inflation targeting framework. Moreover, endogenising ω could also enable analysis of the transition dynamics which might be present as an inflation target is being introduced (when it still may not have gained full credibility).

Inflation-target expectations could also be incorporated into a greater range of inflation models to investigate how the predictions of these models might change. Moreover, it would be interesting to see whether the introduction of inflation-target expectations into empirical models could help these models to match the data on inflation and output-inflation dynamics better.

Finally, the broad principle behind inflation-target expectations could apply in other contexts. In particular, if a target for a particular variable is announced and that target is credible, then it may generally be the case that some agents will adopt the target itself as their expectation of that variable rather than using rational (or adaptive) expectations. Therefore, inflation-target expectations could perhaps be viewed as a subset of a more general expectations formation mechanism which may potentially have implications for models in other areas of economics.

6.3 Policy Implications

The policy implications of our results are clear. Firstly, in almost all circumstances, if some firms are using inflation-target expectations, central banks should react less aggressively than they otherwise would to cost-push shocks. Secondly, since it is generally beneficial for the economy if firms adopt inflation-target expectations, the central bank should take any steps it can which may encourage firms to switch to this mode of expectations formation. For example, central banks could publicise their inflation targets more widely, signal that they are fully committed to meeting them, and attempt to raise general awareness about monetary policy decisions. In this regard, it may be useful for inflation targeting central banks to introduce schemes similar to the Bank of England's "Target Two Point Zero" competition. More generally, greater transparency regarding the conduct of monetary policy is likely to be beneficial. In addition, as suggested by Bean (2003) and Svensson (2001), point targets may help to anchor expectations more successfully than target bands: if this is the case, our results strongly suggest that certain central banks (such as the European Central Bank) may wish to adopt a more precise, symmetric inflation target. Finally, although this paper has not compared inflation targeting with other frameworks for conducting monetary policy, its arguments could be taken as bolstering the theoretical case for formal inflation targeting if it is believed that expectations are more likely to be anchored under this regime than under alternative monetary policy regimes. In this regard, the paper sheds interesting light on the current debate about whether the United States Federal Reserve should adopt a formal inflation target.

Appendix A

In this appendix, we derive the marginal cost version of the new-Keynesian Phillips curve under inflation-target expectations. To do this, we first substitute (3) and (9) into (5). This gives:

$$\pi_t = \frac{(1-\lambda)(1-\omega)(1-\beta\lambda)}{\lambda}\sum_{k=0}^{\infty}(\beta\lambda)^k E_t\{mc_{t+k}\} + \frac{(1-\lambda)(1-\omega)}{\lambda}\sum_{k=1}^{\infty}(\beta\lambda)^k E_t\{\pi_{t+k}\} + \frac{\omega(1-\lambda)(1-\beta\lambda)}{\lambda}mc_t + \frac{\omega\beta(1-\lambda)}{1-\beta\lambda}\pi^T$$

which may be re-expressed as:

$$\pi_t = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}mc_t + \frac{(1-\lambda)(1-\omega)(1-\beta\lambda)}{\lambda}\sum_{k=1}^{\infty}(\beta\lambda)^k E_t\{mc_{t+k}\} + \frac{(1-\lambda)(1-\omega)}{\lambda}\sum_{k=1}^{\infty}(\beta\lambda)^k E_t\{\pi_{t+k}\} + \frac{\omega\beta(1-\lambda)}{1-\beta\lambda}\pi^T \quad (40)$$

Note that we may also rewrite (40) as:

$$\pi_t = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}mc_t + (1-\lambda)(1-\omega)(1-\beta\lambda)\beta E_t\{mc_{t+1}\} + \frac{(1-\lambda)(1-\omega)(1-\beta\lambda)}{\lambda}\sum_{k=2}^{\infty}(\beta\lambda)^k E_t\{mc_{t+k}\} + (1-\lambda)(1-\omega)\beta E_t\{\pi_{t+1}\} + \frac{(1-\lambda)(1-\omega)}{\lambda}\sum_{k=2}^{\infty}(\beta\lambda)^k E_t\{\pi_{t+k}\} + \frac{\omega\beta(1-\lambda)}{1-\beta\lambda}\pi^T \quad (41)$$

Now note that if we update (40) by one period and rearrange, we can obtain:

$$\frac{(1-\lambda)(1-\omega)(1-\beta\lambda)}{\lambda}\sum_{k=2}^{\infty}(\beta\lambda)^k E_t\{mc_{t+k}\} + \frac{(1-\lambda)(1-\omega)}{\lambda}\sum_{k=2}^{\infty}(\beta\lambda)^k E_t\{\pi_{t+k}\} = \beta\lambda\left[E_t\{\pi_{t+1}\} - \frac{\omega\beta(1-\lambda)}{1-\beta\lambda}\pi^T - \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}E_t\{mc_{t+1}\}\right] \quad (42)$$

Finally, provided that $\omega \neq 1$ (in which case the left-hand side of (42) is zero), we can substitute (42) into (41). Doing this, collecting terms and simplifying gives:

$$\pi_t = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}\left[mc_t - \omega\beta\lambda E_t\{mc_{t+1}\}\right] + \beta\left[1-\omega(1-\lambda)\right]E_t\{\pi_{t+1}\} + \omega\beta(1-\lambda)\pi^T$$

which is equation (10) in the main text.

Appendix B

In this appendix, we provide the details of the model's solution for optimal monetary policy under discretion. The central bank's minimisation problem is given by:

$$\min_{\pi_t, x_t} \frac{1}{2}\left[\left(\pi_t - \pi^T\right)^2 + \phi\left(x_t - \overline{x}\right)^2\right]$$

subject to:

$$\pi_t - \pi^T = \kappa\left[(x_t - \overline{x}) - \omega\beta\lambda\left(E_t\{x_{t+1}\} - \overline{x}\right)\right] + \beta\left[1-\omega(1-\lambda)\right]\left(E_t\{\pi_{t+1}\} - \pi^T\right) + \eta e_t \tag{15}$$

The Lagrangian is:

$$\begin{aligned} M &= \frac{1}{2}\left[\left(\pi_t - \pi^T\right)^2 + \phi\left(x_t - \overline{x}\right)^2\right] \\ &-\mu_t\left\{\left(\pi_t - \pi^T\right) - \kappa\left[(x_t - \overline{x}) - \omega\beta\lambda\left(E_t\{x_{t+1}\} - \overline{x}\right)\right] - \beta\left[1-\omega(1-\lambda)\right]\left(E_t\{\pi_{t+1}\} - \pi^T\right) - \eta e_t\right\} \end{aligned}$$

where μ_t is the Lagrange multiplier. The first order conditions for π_t and x_t are respectively given by:

$$\left(\pi_t - \pi^T\right) - \mu_t = 0 \tag{43}$$

$$\phi\left(x_t - \overline{x}\right) + \mu_t\kappa = 0 \tag{44}$$

Combining (43) and (44) to eliminate μ_t gives:

$$\pi_t - \pi^T = -\frac{\phi\left(x_t - \overline{x}\right)}{\kappa} \tag{20}$$

which is contained in the main text. If we now update (20) by one period, we obtain:

$$E_t\left\{\pi_{t+1} - \pi^T\right\} = -\frac{\phi\left(E_t\{x_{t+1}\} - \overline{x}\right)}{\kappa} \tag{45}$$

Substituting (20) and (45) back into the constraint (15) gives:

$$-\frac{\phi\left(x_t - \overline{x}\right)}{\kappa} = \kappa\left[(x_t - \overline{x}) - \omega\beta\lambda\left(E_t\{x_{t+1}\} - \overline{x}\right)\right] + \beta\left[1-\omega(1-\lambda)\right]\left(-\frac{\phi\left(E_t\{x_{t+1}\} - \overline{x}\right)}{\kappa}\right) + \eta e_t$$

This expression simplifies to:

$$\left(1+\frac{\kappa^2}{\phi}\right)(x_t-\overline{x})=\beta\left[1+\frac{\omega\lambda\kappa^2}{\phi}-\omega(1-\lambda)\right](E_t\{x_{t+1}\}-\overline{x})-\frac{\eta\kappa}{\phi}e_t \tag{46}$$

To solve this expectational difference equation for x_t, we guess a solution of the form:

$$x_t-\overline{x}=c_1e_t \tag{47}$$

Given this guess, it must be the case that:

$$E_t\{x_{t+1}-\overline{x}\}=c_1E_t\{e_{t+1}\}=c_1\rho e_t \tag{48}$$

where we have used the fact that $E_t\{e_{t+1}\}=\rho e_t$, which follows from (12). We then substitute (47) and (48) into (46) to determine c_1. Doing this gives:

$$\left(1+\frac{\kappa^2}{\phi}\right)c_1e_t=\beta\left[1+\frac{\omega\lambda\kappa^2}{\phi}-\omega(1-\lambda)\right]c_1\rho e_t-\frac{\eta\kappa}{\phi}e_t$$

Therefore:

$$c_1=\frac{-\eta\kappa}{\phi(1-\beta\rho)+\kappa^2(1-\omega\beta\lambda\rho)+\beta\rho\phi\omega(1-\lambda)}$$

Finally, substituting back in for η and κ gives the expression in the main text:

$$c_1=\frac{-\delta(1-\lambda)^2(1-\beta\lambda)^2(1-\omega\beta\lambda\rho)}{\lambda^2\phi(1-\beta\rho)+\beta\rho\phi\omega\lambda^2(1-\lambda)+\delta^2(1-\lambda)^2(1-\beta\lambda)^2(1-\omega\beta\lambda\rho)} \tag{22}$$

Appendix C

In this appendix, we provide the details of the model's solution for optimal monetary policy under commitment. The central bank's minimisation problem is given by:

$$\min_{\pi_{t+i},x_{t+i}}\frac{1}{2}E_t\left\{\sum_{i=0}^{\infty}\beta^i\left[\left(\pi_{t+i}-\pi^T\right)^2+\phi(x_{t+i}-\overline{x})^2\right]\right\}$$

subject to (for all $i\geq 0$):

$$\begin{aligned}E_t\{\pi_{t+i}\}-\pi^T=\kappa\left[(E_t\{x_{t+i}\}-\overline{x})-\omega\beta\lambda(E_t\{x_{t+i+1}\}-\overline{x})\right]&\\+\beta\left[1-\omega(1-\lambda)\right]\left(E_t\{\pi_{t+i+1}\}-\pi^T\right)+\eta e_{t+i}&\end{aligned}$$

Letting μ_{t+i} be the period $t+i$ Lagrange multiplier, the Lagrangian may therefore be written as:

$$M = E_t \sum_{i=0}^{\infty} \beta^i \{(1/2) \left[\left(\pi_{t+i} - \pi^T \right)^2 + \phi \left(x_{t+i} - \overline{x} \right)^2 \right] -$$
$$\mu_{t+i} \left[\left(\pi_{t+i} - \pi^T \right) - \kappa \left[\left(x_{t+i} - \overline{x} \right) - \omega \beta \lambda \left(x_{t+i+1} - \overline{x} \right) \right] - \beta \left[1 - \omega \left(1 - \lambda \right) \right] \left(\pi_{t+i+1} - \pi^T \right) - \eta e_{t+i} \right] \}$$

The first order conditions for π_t, x_t, π_{t+i} for $i \geq 1$, and x_{t+i} for $i \geq 1$ are respectively given by:

$$\left(\pi_t - \pi^T \right) - \mu_t = 0 \tag{29}$$

$$\phi \left(x_t - \overline{x} \right) + \mu_t \kappa = 0 \tag{30}$$

$$E_t \left\{ \left(\pi_{t+i} - \pi^T \right) - \mu_{t+i} + \left[1 - \omega \left(1 - \lambda \right) \right] \mu_{t+i-1} \right\} = 0 \qquad (i \geq 1) \tag{31}$$

$$E_t \left\{ \phi \left(x_{t+i} - \overline{x} \right) + \kappa \mu_{t+i} - \kappa \omega \lambda \mu_{t+i-1} \right\} = 0 \qquad (i \geq 1) \tag{32}$$

To obtain the timeless perspective solution, we first need to solve (31) and (32) simultaneously to eliminate the Lagrange multiplier terms. We may rearrange (32) as:

$$\kappa \left[1 - \omega \lambda L \right] E_t \left\{ \mu_{t+i} \right\} = -\phi \left(E_t \left\{ x_{t+i} \right\} - \overline{x} \right) \tag{49}$$

where L denotes the lag operator. Therefore, from (49):

$$E_t \left\{ \mu_{t+i} \right\} = -\frac{\phi}{\kappa} \left[1 - \omega \lambda L \right]^{-1} \left(E_t \left\{ x_{t+i} \right\} - \overline{x} \right) \tag{50}$$

$$E_t \left\{ \mu_{t+i-1} \right\} = -\frac{\phi}{\kappa} \left[1 - \omega \lambda L \right]^{-1} \left(E_t \left\{ x_{t+i-1} \right\} - \overline{x} \right) \tag{51}$$

Substituting (50) and (51) into (31) gives:

$$E_t \left\{ \pi_{t+i} \right\} - \pi^T = -\frac{\phi}{\kappa} \left[1 - \omega \lambda L \right]^{-1} \left(E_t \left\{ x_{t+i} \right\} - \overline{x} \right)$$
$$+ \frac{\phi}{\kappa} \left[1 - \omega \left(1 - \lambda \right) \right] \left[1 - \omega \lambda L \right]^{-1} \left(E_t \left\{ x_{t+i-1} \right\} - \overline{x} \right) \tag{52}$$

Therefore:

$$E_t \left\{ \pi_{t+i} \right\} - \pi^T = -\frac{\phi}{\kappa} \left[1 + \left(\omega \lambda \right) L + \left(\omega \lambda \right)^2 L^2 + ... \right] \left(E_t \left\{ x_{t+i} \right\} - \overline{x} \right)$$
$$+ \frac{\phi}{\kappa} \left[1 - \omega + \omega \lambda \right] \left[1 + \left(\omega \lambda \right) L + \left(\omega \lambda \right)^2 L^2 + ... \right] \left(E_t \left\{ x_{t+i-1} \right\} - \overline{x} \right)$$

Cancelling and then collecting terms, we obtain the following expression for expected inflation as a function of output gap terms:

$$E_t\{\pi_{t+i}\}-\pi^T = -\frac{\phi}{\kappa}\left\{(E_t\{x_{t+i}\}-\overline{x})-(1-\omega)\left[\sum_{j=1}^{\infty}(\omega\lambda)^{j-1}(E_t\{x_{t+i-j}\}-\overline{x})\right]\right\}$$

For $i=0$ and $i=1$ respectively, this general expression reduces to:

$$\pi-\pi^T = -\frac{\phi}{\kappa}\left\{(x_t-\overline{x})-(1-\omega)\left[\sum_{j=1}^{\infty}(\omega\lambda)^{j-1}(x_{t-j}-\overline{x})\right]\right\} \tag{53}$$

$$E_t\{\pi_{t+1}\}-\pi^T = -\frac{\phi}{\kappa}\left\{(E_t\{x_{t+1}\}-\overline{x})-(1-\omega)\left[\sum_{j=1}^{\infty}(\omega\lambda)^{j-1}(x_{t+1-j}-\overline{x})\right]\right\} \tag{54}$$

Substituting (53) and (54) into the $i=0$ constraint (15), multiplying through by $-\frac{\kappa}{\phi}$, and simplifying, we obtain the following difference equation in terms of x_t:

$$h_2(x_t-\overline{x}) = h_3(E_t\{x_{t+1}\}-\overline{x})+h_4\sum_{j=1}^{\infty}(\omega\lambda)^{j-1}(x_{t-j}-\overline{x})-h_5e_t \tag{55}$$

where:

$$\begin{aligned}
h_2 &= 1+\frac{\kappa^2}{\phi}+\beta[1-\omega(1-\lambda)](1-\omega)\\
h_3 &= \frac{\kappa^2\omega\beta\lambda}{\phi}+\beta[1-\omega(1-\lambda)]\\
h_4 &= (1-\omega)\{1-\omega\lambda\beta[1-\omega(1-\lambda)]\}\\
h_5 &= \frac{\kappa\eta}{\phi}
\end{aligned}$$

To solve this difference equation for x_t, we guess a solution of the form:

$$(x_t-\overline{x}) = \gamma_0e_t+\sum_{j=1}^{\infty}\gamma_j(x_{t-j}-\overline{x}) \tag{56}$$

Given this guess, and using the fact that $E_t\{e_{t+1}\} = \rho e_t$ (which follows from (12)), it must be the case that:

$$
\begin{aligned}
E_t\{x_{t+1}\} - \overline{x} &= \gamma_0 E_t\{e_{t+1}\} + \sum_{j=1}^{\infty} \gamma_j \left(x_{t-j+1} - \overline{x}\right) \\
&= \gamma_0 \rho e_t + \gamma_1 \left(x_t - \overline{x}\right) + \sum_{j=1}^{\infty} \gamma_{j+1} \left(x_{t-j} - \overline{x}\right) \\
&= \gamma_0 \rho e_t + \gamma_0 \gamma_1 e_t + \gamma_1 \sum_{j=1}^{\infty} \gamma_j \left(x_{t-j} - \overline{x}\right) + \sum_{j=1}^{\infty} \gamma_{j+1} \left(x_{t-j} - \overline{x}\right) \quad (57)
\end{aligned}
$$

We then substitute (56) and (57) into (55) to enable us to determine the γ_i. Doing this gives:

$$
\begin{aligned}
h_2 \left[\gamma_0 e_t + \sum_{j=1}^{\infty} \gamma_j \left(x_{t-j} - \overline{x}\right)\right] = h_3 &\left[\gamma_0 \rho e_t + \gamma_0 \gamma_1 e_t + \gamma_1 \sum_{j=1}^{\infty} \gamma_j \left(x_{t-j} - \overline{x}\right) + \sum_{j=1}^{\infty} \gamma_{j+1} \left(x_{t-j} - \overline{x}\right)\right] \\
&+ h_4 \sum_{j=1}^{\infty} (\omega\lambda)^{j-1} \left(x_{t-j} - \overline{x}\right) - h_5 e_t \quad (58)
\end{aligned}
$$

Equating terms in e_t in (58) and solving for γ_0, we obtain:

$$
\gamma_0 = \frac{-h_5}{h_2 - h_3 \rho - h_3 \gamma_1} \quad (37)
$$

as in the main text. Equating terms in x_{t-j} $\forall j \geq 1$ and solving for γ_j, we find that:

$$
\gamma_j = \frac{h_3 \gamma_{j+1} + h_4 (\omega\lambda)^{j-1}}{h_2 - h_3 \gamma_1} \quad (59)
$$

Solving this recurrence relation forwards and noting that for convergence, it must be the case that $\gamma_n \to 0$ as $n \to \infty$, we can obtain the following expressions for γ_j and hence γ_1:

$$
\begin{aligned}
\gamma_j &= \frac{h_4 (\omega\lambda)^{j-1}}{h_2 - h_3 \gamma_1} \left[1 + \frac{h_3 \omega\lambda}{h_2 - h_3 \gamma_1} + \frac{h_3^2 (\omega\lambda)^2}{(h_2 - h_3 \gamma_1)^2} + \frac{h_3^3 (\omega\lambda)^3}{(h_2 - h_3 \gamma_1)^3} + ...\right] \\
&= \frac{h_4 (\omega\lambda)^{j-1}}{h_2 - h_3 \gamma_1} \left[\frac{h_2 - h_3 \gamma_1}{h_2 - h_3 \gamma_1 - h_3 \omega\lambda}\right] \\
\gamma_j &= \frac{h_4 (\omega\lambda)^{j-1}}{h_2 - h_3 \gamma_1 - h_3 \omega\lambda} \quad (60) \\
\gamma_1 &= \frac{h_4}{h_2 - h_3 \gamma_1 - h_3 \omega\lambda} \quad (61)
\end{aligned}
$$

Rearranging (61) gives the quadratic equation for γ_1 contained in the main text:

$$h_3\gamma_1^2 - (h_2 - \omega\lambda h_3)\,\gamma_1 + h_4 = 0 \tag{36}$$

Moreover, for the solution to be stable, it must be the case that γ_1 is given by the solution less than one of (36). Finally, from (60) and (61):

$$\gamma_j = (\omega\lambda)^{j-1}\,\gamma_1 \quad \forall j \geq 2 \tag{38}$$

Taking all of this together, we can therefore see that, as in the main text, the solution for the output gap is given by:

$$x_t - \overline{x} = \gamma_0 e_t + \sum_{j=1}^{\infty} \gamma_j \left(x_{t-j} - \overline{x}\right) \tag{33}$$

where γ_1 is given by the solution less than one of the quadratic equation:

$$h_3\gamma_1^2 - (h_2 - \omega\lambda h_3)\,\gamma_1 + h_4 = 0 \tag{36}$$

and:

$$\gamma_0 = \frac{-h_5}{h_2 - h_3\rho - h_3\gamma_1} \tag{37}$$

$$\gamma_j = (\omega\lambda)^{j-1}\,\gamma_1 \quad \forall j \geq 2 \tag{38}$$

We now demonstrate that (33) may be rewritten in the alternative form (34). To show this, first note that (33) may be rewritten as:

$$\begin{aligned} x_t - \overline{x} &= \gamma_0 e_t + \gamma_1 \left(x_{t-1} - \overline{x}\right) + \sum_{j=1}^{\infty} \gamma_{j+1} \left(x_{t-j-1} - \overline{x}\right) \\ &= \gamma_0 e_t + \gamma_1 \left(x_{t-1} - \overline{x}\right) + \gamma_1 \sum_{j=1}^{\infty} (\omega\lambda)^j \left(x_{t-j-1} - \overline{x}\right) \end{aligned} \tag{62}$$

where we have used the fact that $\gamma_{j+1} = (\omega\lambda)^j\,\gamma_1$, which follows from (38). Meanwhile, lagging (33) gives:

$$\begin{aligned} x_{t-1} - \overline{x} &= \gamma_0 e_{t-1} + \sum_{j=1}^{\infty} \gamma_j \left(x_{t-j-1} - \overline{x}\right) \\ &= \gamma_0 e_{t-1} + \gamma_1 \sum_{j=1}^{\infty} (\omega\lambda)^{j-1} \left(x_{t-j-1} - \overline{x}\right) \end{aligned} \tag{63}$$

using (38). Multiplying through (63) by $\omega\lambda$ and rearranging gives:

$$\gamma_1 \sum_{j=1}^{\infty} (\omega\lambda)^j \left(x_{t-j-1} - \overline{x}\right) = \omega\lambda \left(x_{t-1} - \overline{x}\right) - \omega\lambda\gamma_0 e_{t-1} \tag{64}$$

Substituting (64) into (62) (note that this step is only valid if $\gamma_1 \neq 0 \Rightarrow \omega \neq 1$), we therefore obtain the alternative solution form:

$$x_t - \overline{x} = (\gamma_1 + \omega\lambda)\left(x_{t-1} - \overline{x}\right) + \gamma_0 e_t - \omega\lambda\gamma_0 e_{t-1} \tag{34}$$

Finally, to derive the outcome for equilibrium inflation, note that the $i = 0$ version of (52) can be expressed as:

$$[1 - \omega\lambda L]\left(\pi_t - \pi^T\right) = -\frac{\phi}{\kappa}\left(x_t - \overline{x}\right) + \frac{\phi}{\kappa}\left[1 - \omega\left(1 - \lambda\right)\right]\left(x_{t-1} - \overline{x}\right) \tag{65}$$

Substituting (34) into (65) and simplifying gives the solution in the main text:

$$\pi_t - \pi^T = \omega\lambda\left(\pi_{t-1} - \pi^T\right) + \frac{\phi}{\kappa}\left(1 - \gamma_1 - \omega\right)\left(x_{t-1} - \overline{x}\right) - \frac{\phi}{\kappa}\gamma_0 e_t + \frac{\phi}{\kappa}\omega\lambda\gamma_0 e_{t-1} \tag{35}$$

Appendix D

We wish to show that when $\rho = 0$, the magnitude of γ_0 is monotonically increasing in ω over the range $\omega \in [0, 1)$. Since h_5 does not depend on ω, and since both $h_5 > 0$ and $h_2 - h_3\gamma_1 > 0$, we can see from (37) (with $\rho = 0$) that this is equivalent to showing that:

$$\frac{d\left(h_2 - h_3\gamma_1\right)}{d\omega} < 0 \quad \forall\omega \in [0, 1)$$

Unfortunately, there is no analytical way of showing this directly. However, using *Maple 9*, we can show that the only value of ω for which $\frac{d(h_2 - h_3\gamma_1)}{d\omega} = 0$ is $\omega = 1$. This implies that the function has no stationary points in the range $\omega \in [0, 1)$. Since the function is continuous, this means that in the range $\omega \in [0, 1)$, it must be the case that $(h_2 - h_3\gamma_1)$ is either everywhere increasing in ω or everywhere decreasing in ω.

For a particular choice of parameter values, it can be shown on a spreadsheet that $(h_2 - h_3\gamma_1)$ is decreasing. (Alternatively, after a very lengthy calculation, it can be shown that $\frac{d(h_2 - h_3\gamma_1)}{d\omega}$ is always negative when $\omega = 0$ for all values of the other parameters.) This implies that the function cannot be everywhere increasing in the range $\omega \in [0, 1)$. Therefore, by contradiction, it must be everywhere decreasing and we have proved the result.

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