Linking process to outcome

The `seqlogit` package

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Introduction

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- Outcome: The final outcome of the process, e.g. highest achieved level of education.

Outline

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Process and Outcome

Empirical example

The seqlogit package

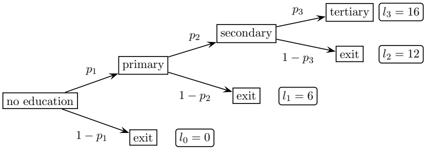
Sequential logit model

- This model is know under a variety of other names:
  - sequential response model (maddala 1983),
  - continuation ratio logit (Agresti 2002),
  - model for nested dichotomies (fox 1997), and
  - the Mare model (shavit and blossfeld93) (after (Mare 1981))

Sequential logit model

- Model each choice separately using a `(m)logit` on the sub-sample that is 'at risk'

Figure: Hypothetical educational system



sequential logit to end result

$$\hat{p}_{ki} = \frac{\exp(\alpha_k + \lambda_k SES_i)}{1 + \exp(\alpha_k + \lambda_k SES_i)} \quad \text{if} \quad y_{k-1\,i} = 1$$

$$E(ed) = (1-\hat{p}_{1i})l_0 + \hat{p}_{1i}(1-\hat{p}_{2i})l_1 + \hat{p}_{1i}\hat{p}_{2i}(1-\hat{p}_{3i})l_2 + \hat{p}_{1i}\hat{p}_{2i}\hat{p}_{3i}l_3$$

Effect of explanatory variable on outcome

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$$
\begin{aligned}
&\frac{\partial E(ed)}{\partial SES} = \\
&\{1 \times \hat{p}_{1i}(1-\hat{p}_{1i}) \times [(1-\hat{p}_2)l_1 + \hat{p}_2(1-\hat{p}_3)l_2 + \hat{p}_2\hat{p}_3 l_3 - l_0]\}\lambda_1 + \\
&\{\hat{p}_{1i} \times \hat{p}_{2i}(1-\hat{p}_{2i}) \times [(1-\hat{p}_3)l_2 + \hat{p}_3 l_3 - l_1]\}\lambda_2 + \\
&\{\hat{p}_{1i}\hat{p}_{2i} \times \hat{p}_{3i}(1-\hat{p}_{3i}) \times [l_3 - l_2]\}\lambda_3
\end{aligned}
$$

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Effect of explanatory variable on outcome

$$\frac{\partial E(ed)}{\partial SES} =$$
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Effect of explanatory variable on outcome

proportion at risk

$$\frac{\partial E(ed)}{\partial SES} =$$
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Effect of explanatory variable on outcome

variance of the variable indicating whether one passes or not

$$\frac{\partial E(ed)}{\partial SES} =$$
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Effect of explanatory variable on outcome

expected increase in the level of education after passing

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Effect of explanatory variable on outcome

expected level of education for those that pass

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Effect of explanatory variable on outcome

minus the expected level of education for those that fail

$$\frac{\partial E(ed)}{\partial SES} =$$
$$\{1 \times \hat{p}_{1i}(1-\hat{p}_{1i}) \times [(1-\hat{p}_2)l_1 + \hat{p}_2(1-\hat{p}_3)l_2 + \hat{p}_2\hat{p}_3 l_3 - l_0]\}\lambda_1 +$$
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In words:

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- weights = at risk $\times$ variance $\times$ gain

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Empirical example

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Data

- General Social Survey (GSS).
- 20 surveys held between 1977 and 2004 with information on cohorts 1913-1978.
- 13,400 men aged between 27 and 65 with complete information.

Variables

- Father's highest achieved level of education measured in (pseudo) years.

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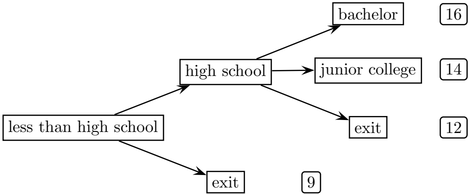
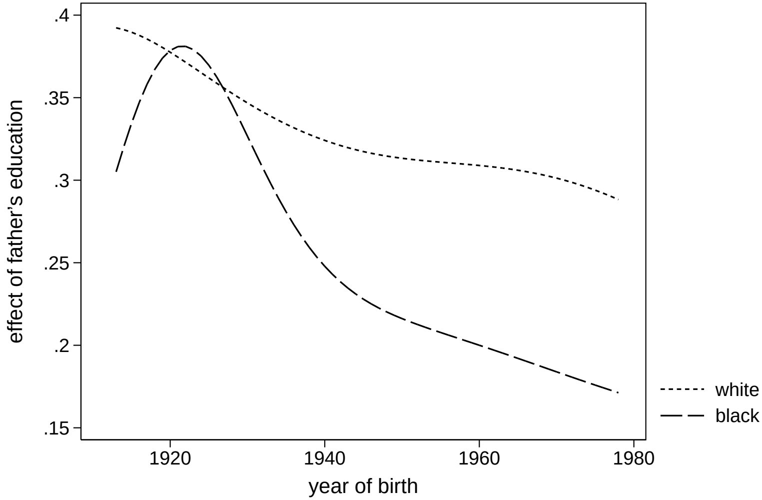
- Respondent's highest achieved Level of education in (pseudo) years
- Time measured as a restricted cubic spline with one knot in 1946.

Simplified model of the US educational system



Change in effect on outcome over cohorts



Decomposition of effect on outcome

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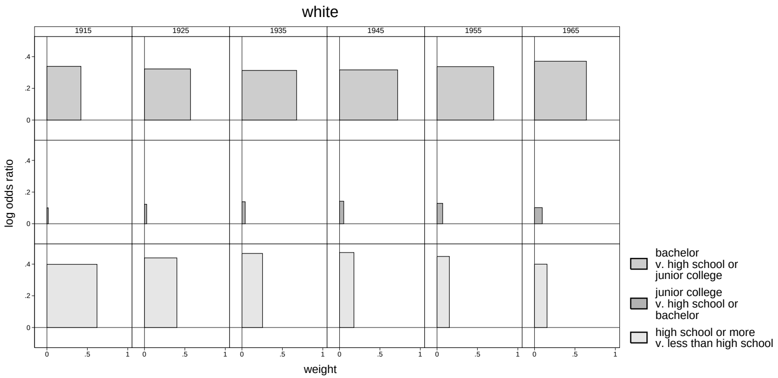
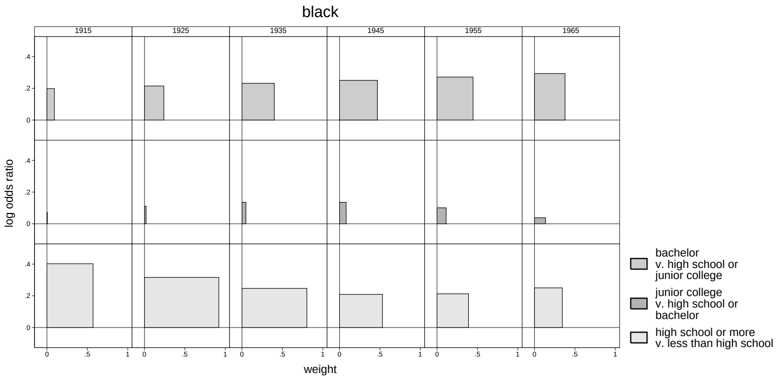
- The contribution of the first transition is: $w_1\ \lambda_1$
- This can be visualized as the area of a rectangle with width w_1 and height λ_1.
- The effect on the outcome is the sum of the areas of these rectangles

Decomposition of effect on outcome for white men



Decomposition of effect on outcome for black men



Decomposition of weights

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- These three elements are all a function of the proportions that pass the transitions

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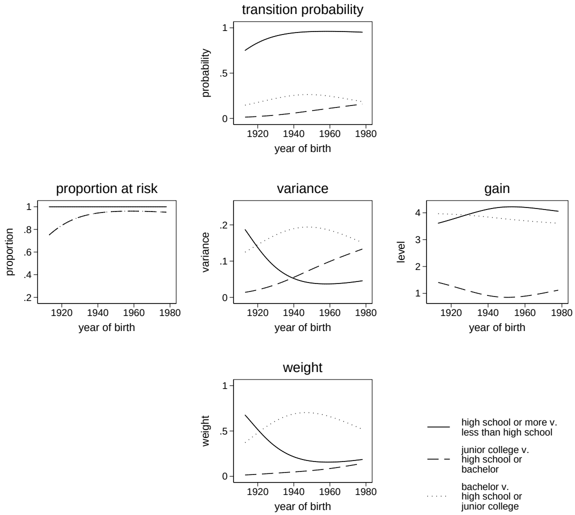

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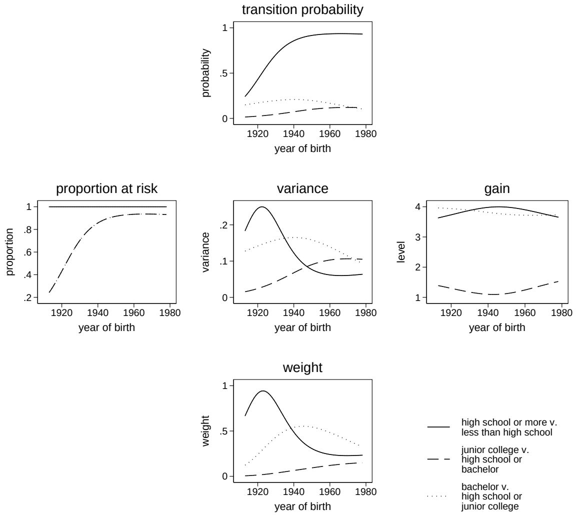

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- `seqlogit` will estimate a sequential logit model
- It can `predict` the weights and its components
- `seqlogitdecomp` shows the decomposition of the effect on the outcome into effects on the transitions and their weights.

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example:

```
seqlogit degree south padeg coh padegXcoh, /*
*/ tree(0:1 2 3 , 1:2:3 )
```

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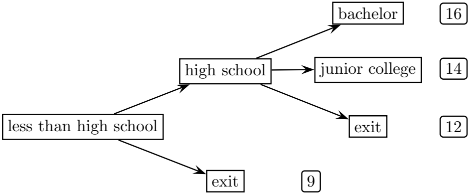

`predict`

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- trvar variance of the indicator variable indicating whether or not the respondent passed the transition
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- pr probability that an outcome is the highest achieved outcome.
- y† expected highest achieved level

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example:

```
predict weib*, trweight /*
*/ levels(0=9, 1=12, 2=14, 3=16)
```

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- Differences in the effect on the outcome may be due to:
  - differences in weights, or
  - differences in effects on transitions (log odds ratios)
- Both need to be specified
  - The weights can be specified by fixing all values of all explanatory variables
  - The effects on transitions can be specified directly

Specify the weights

- Model:
  ```
  seqlogit degree south padeg coh padegXcoh, /*
  */ tree(0:1 2 3 , 1:2:3 )
  ```
- We want to compare cohorts 1920 1940 1960

```
seqlogitdecomp,
overat( coh 1920 padegXcoh `mean20' ,
coh 1940 padegXcoh `mean40' ,
coh 1960 padegXcoh `mean60' )
overlodds( _b[padeg] + 1920*_b[padegXcoh] ,
_b[padeg] + 1940*_b[padegXcoh] ,
_b[padeg] + 1960*_b[padegXcoh] )
at(south 0)
```

Locals `` `mean20' ``, `` `mean40' ``, and `` `mean60' `` contain the mean of `padeg` times 1920, 1940, 1960 respectively.

Specify the odds ratios

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at(south 0)
```

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- The effect on the outcome depends in an understandable way on the effects on the process.
- The effect on the outcome is a weighted sum of the effects on the transition probabilities, and the weights increase if:
  - the proportion at risk increases,
  - the proportion that passes is closer to .50,
  - the expected increase in the outcome increases

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- This relationship can be used to:
  - to relate the process to the outcome.
  - identify important and less important transitions,
  - to explain differences in effect on outcome with well documented phenomena like educational expansion or racial differences in educational attainment.