Money, Wealth and Overlapping Generations

David Llewelyn Aikman*
Department of Economics, University of Warwick

January 15, 2003

Abstract

In this paper, we use Weil's [1989] overlapping infinitely-lived dynasties framework to analyse a microfounded version of the real balance effect envisaged by Pigou. This effect appears to imply that temporary monetary expansions in the limit have an unboundedly large impact on current aggregate demand, thereby eliminating Krugman's liquidity trap. The circumstances under which such an effect is operative, however, imply a condition that rules out temporary monetary expansions of this magnitude. For the set of feasible temporary monetary expansions, rather than eliminating the possibility of liquidity traps occurring, the real balance effect generated by this model makes a trap perhaps more likely due to the heightened constraints it imposes on the monetary authority.

KEYWORDS: Liquidity trap; real balance effect; Pigou effect; monetary policy; Japan.

JEL Classification System: E0; E3; E4; E5.

*Please address all correspondence to: D.Aikman, Department of Economics, University of Warwick, Coventry, CV4 7AL. e-mail: D.L.Aikman@warwick.ac.uk

1 Introduction

The theme of this paper is an old one: how can the zero lower bound on nominal interest rates represent a plausible threat to stabilisation policy when increases in real balances – *regardless of the level of the nominal interest rate* – raise household wealth and therefore consumption, as described by Pigou's [1943] argument for a "real balance effect"? The traditional critique of this particular real balance effect, due to Kalecki [1944], concerns its likely (lack of) empirical significance. As the largest part of private holdings of monetary assets, including bank deposits and so on, have direct or indirect counterparts in private debt, the net base for this effect will be a small fraction of the economy's overall stock of wealth. Nevertheless, from a purely theoretical perspective, the existence of this simple wealth effect of monetary policy represents an important challenge to the possibility of liquidity traps occurring.

Indeed, this challenge appears to have strongly influenced Paul Krugman's decision to employ a dynamic general equilibrium framework in his initial papers on the topic (Krugman [1998a], [1998b]). The following is a quote from Krugman [1999]:

> Here's how my initial argument – not that different from the debates between Keynes and Pigou – went. In the IS-LM model ... to say that increases in M were ineffective beyond some point was ... equivalent to saying that reductions in P were ineffective in raising demand – that the aggregate demand curve (was) downward-sloping over some range but vertical thereafter.
>
> But as Pigou pointed out, that simply cannot be right. If nothing else, a fall in the overall price level increases the real value of the public's holdings of money, and this wealth effect will increase consumption. If the IS-LM model seems to suggest that no full employment equilibrium exists, it is only because that model does not really get the budget constraints right.
>
> To demonstrate the truth of that supposed truism, all that was needed was to write down a model that got the budget con-

> straints right, that did not fudge the individual's decision problem. So I set out to write down the simplest such model I could. And it ended up saying something quite different.

To Krugman's evident surprise, his representative agent, cash-in-advance framework did not generate the real balance effect he anticipated. This result is not, we should note, a consequence of erroneous budget constraints or an ad hoc decision problem. The crucial equilibrium condition in his framework – the Euler equation for the optimal timing of private expenditure – does not in any way contradict the contribution of financial wealth to the intertemporal budget constraint. Rather, money affects aggregate demand in this set-up only through its effects upon real interest rates and the incentives that they provide for intertemporal subsitution of expenditure.

This strong conclusion about how the transmission mechanism operates, we should note, is not a general property shared by all optimising models. As is well known, both the money-in-the-utility function (MIUF) and shopping time approaches to modelling money demand can plausibly generate a type of real balance effect operating through the cross-partial derivative of the utility function u_{cm}. If utility is obtained from holding money, this must be because money balances facilitate transactions and it is hardly sensible that the benefits of such balances should be independent of the real volume of transactions that a household undertakes. In particular, it is plausible that the marginal benefit of additional real balances should be higher when real transactions are greater, implying $u_{cm} > 0$. This suggests the existence of a real balance effect as temporary increases in the current supply of money will directly raise current aggregate demand through the cross partial derivative.

Woodford [2002] and McCallum [2000] discuss this possibility and argue that reasonable parameterisations of the utility function lead to very small coefficients on money in the IS equation. To be exact, this last remark applies only to the money coefficient in a log-linearised version of the

IS curve. Regardless of the quantitative importance of this effect near the steady state, Woodford [2002] persuasively argues that this real balance effect would disappear completely in a liquidity trap. When there is satiation in real balances at some finite level, it would seem perverse to argue that further increases in real balances beyond the satiation point make transactions more convenient.

Real balance effects can also emerge in MIUF economies with alternative timing specifications for the money balances entering the utility function. Aikman [2002] demonstrates this for Carlstrom and Fuerst's recently suggested specification (Carlstrom and Fuerst [2001]), although intriguingly, the effect works in exactly the opposite direction from that described above: a temporary increase in the current money supply lowers the marginal utility of liquidity and this in turn *lowers* aggregate demand for a given real interest rate. Neither effect, therefore, corresponds closely to the pure wealth effect envisaged by Pigou.

In this paper, we extend Krugman's sticky-price, cash-in-advance model to a framework due to Weil [1989], [1991] hybrid of the representative agent Sidrauski and overlapping generations (OLG) approaches, in which new dynasties of infinitely-lived agents enter the economy each period. With a growing population, the model generates a real balance effect that closely resembles the one advanced by Pigou: a temporary increase in the current stock of money raises aggregate demand independently of its effect on the equilibrium real interest rate. Furthermore, this effect operates through net wealth. Following the work of Sachs [1980], Cohen [1985] and Weil [1991], we show that the economy's *net* per capita stock of monetary wealth is positive only in the case where new, financially disconnected cohorts are entering the population. In Krugman's representative agent framework, therefore, money is not actually net wealth.

The paper most closely related to our own is Ireland [2001]. Using the same Weil [1989] overlapping generations framework, Ireland finds that the real balance effect completely eliminates the possibility of liquidity traps occurring. His version of the liquidity trap, however, differs quite substantially from Krugman's. In particular, rather than examining the constraints on monetary policy imposed by the zero lower bound when the economy experiences a large adverse shock, Ireland confines his analysis to the steady state of an economy in which the central bank is following the Friedman rule. With steady state zero nominal interest rates, the cash-in-advance constraint ceases to bind and there exists, in the representative agent model, multiple equilibrium values of real balances (and hence also multiple equilibrium time paths for the price level) that are consistent with the steady state conditions. Hence, the central bank's choice of an initial value for the nominal money supply plus a steady state growth rate is not sufficient to determine a unique equilibrium time path for the price level. Introducing population growth eliminates this multiplicity by making the steady state real interest rate depend on the level of real balances. Our paper, we argue, is closer to being a direct extension of Krugman's work to an OLG economy.

The rest of this paper is organised as follows. In section 2 will lay out the basic model, which is nothing but a discrete-time, sticky-price version of Weil [1991] and in section 3, we characterise the current interest rate-output conjuncture and describe how the real balance effect operates. The main part of the analysis is contained in section 4, which examines the possibility of liquidity traps occurring. To preview the conclusions briefly: with population growth, temporary monetary expansions can have an unboundedly large impact on demand; expansions of such magnitude, however, are ruled out as the lump-sum tax required to contract the money supply next period will be so high that future generations will be unable to pay the tax and consume. Within the set of feasible temporary monetary expansions, liquidity traps are possible and, under our simple calibration of the model, more likely with an operative real balance effect!

2 The Model

We begin by presenting a sticky-price, discrete time version of Weil's [1991] monetary, overlapping infinitely-loved dynasties framework. The model builds on the Blanchard [1985] uncertain-horizons framework, which itself builds on Yaari [1965], and it provides a convenient vehicle for nesting the representative agent and OLG approaches.

2.1 Demographic Structure

We assume that the economy is composed of distinct, infinitely-lived dynasties that come into being on different dates. Households born in a particular period $v \geq 0$ belong to cohort v and the arrival of new cohorts causes the total number of households to grow at the constant rate $n \geq 0$. Define N_s as the population of households during period s. Then, given a positive population (normalised to one) at the origin of time, $s = 0$, we have:

$$N_{s+1} = (1+n)\, N_s \tag{1}$$

$\forall s \geq 0$. As we discuss, the population growth rate n serves as a measure of financial disconnectedness and heterogeneity in the population. In the special case of $n = 0$, the model collapses to the more familiar infinitely-lived representative agent model.

Households in a particular cohort are identical, so it is possible to examine the behaviour of a representative agent within each cohort.

2.2 Household υ's Maximisation Problem

An individual born on date υ lives forever and maximises:

$$U_{\upsilon}^{\upsilon} = \sum_{s=\upsilon}^{\infty} \beta^{s-\upsilon} u\left(c_s^{\upsilon}, \frac{M_s^{\upsilon}}{P_s}\right) \tag{2}$$

subject to the following real flow budget constraint:

$$\frac{B_{s+1}^{\upsilon}}{P_s} + \frac{M_s^{\upsilon}}{P_s} = (1+i_s)\frac{B_s^{\upsilon}}{P_s} + \frac{M_{s-1}^{\upsilon}}{P_s} + y_s + \tau_s - c_s^{\upsilon} \tag{3}$$

This utility function is assumed to be increasing in both arguments, strictly concave and continuously differentiable. The stock of real balances enters as a direct argument in the agent's utility function because of the liquidity services facilitating transaction making that money provides. This shortcut for incorporating money into general equilibrium models is widespread and is adopted for example by two recent graduate texts in this field (Obstfeld and Rogoff [1996] and Walsh [1998]).

There is one consumption good at each date with price P, c_s therefore being the individual's real consumption of this good on date s. There are two assets in the economy: fiat money M and bonds B offering a one-period nominal return i ($i_s \geq 0\ \forall s$), and each period the government makes a lump-sum real transfer of τ to the agent. Notice that both y and τ are taken to be the same for all agents alive. A note on the timing convention adopted: B_{s+1}^{υ} denotes individual υ's nominal bond holdings at the beginning of period $s+1$, i.e. prior to that period's interest payment; M_s^{υ} though represents the same individual's nominal cash holdings at the beginning of period $s+1$.

Define household υ's real *cum dividend* financial wealth available at the beginning of period $s+1$, as a_{s+1}^{υ}:

$$a_{s+1}^{\upsilon} \equiv \frac{(1+i_{s+1})\,B_{s+1}^{\upsilon}}{P_{s+1}} + \frac{M_s^{\upsilon}}{P_{s+1}} \tag{4}$$

A key assumption of the model is that new cohorts are not linked to pre-existing cohorts through operative gifts, as captured by the initial condition $a_s^s = 0 \; \forall s > 0$ (with $a_0^0 > 0$).

Using (4), the flow budget constraint (3) can be rewritten in terms of a:

$$\frac{a_{s+1}^v}{1 + r_{s+1}} = a_s^v + y_s + \tau_s - \widehat{c}_s^v \tag{5}$$

where $\widehat{c}_s^v \equiv c_s^v + \left(\frac{i_{s+1}}{1+i_{s+1}}\right) \frac{M_s^v}{P_s}$ represents "full consumption" – the sum of consumption on goods plus the opportunity cost of holding financial wealth in monetary form (see Sachs [1980]). Given this set-up, we also require a condition prohibiting the household from engaging in Ponzi schemes:

$$\lim_{T \to \infty} \frac{a_{s+T}^v}{\prod_{j=s+1}^{T} (1 + r_j)} \geq 0 \tag{6}$$

This standard problem in dynamic optimisation yields the following well-known first order conditions:

$$u_c\left(c_s^v, \frac{M_s^v}{P_s}\right) = \beta (1 + r_{s+1}) \, u_c\left(c_{s+1}^v, \frac{M_{s+1}^v}{P_{s+1}}\right) \tag{7}$$

$$\frac{u_m\left(c_s^v, \frac{M_s^v}{P_s}\right)}{u_c\left(c_s^v, \frac{M_s^v}{P_s}\right)} = \frac{i_{s+1}}{1 + i_{s+1}} \tag{8}$$

$$(1 + r_{s+1}) = (1 + i_{s+1}) \frac{P_s}{P_{s+1}} \tag{9}$$

$$\lim_{T \to \infty} \frac{a_{s+T}^v}{\prod_{j=s+1}^{T} (1 + r_j)} = 0 \tag{10}$$

Given $\{y_s, \tau_s\}_{s=v}^{\infty}$ and the initial condition of zero financial wealth at birth, conditions (7), (8), (9), (5) and (10) fully characterise this household's optimal program.

2.2.1 Household υ's Consumption Function

In order to derive a closed form expression for this household's consumption function we must first specify preferences. We opt for the rather restrictive additively separable, logarithmic[1] form:

$$u\left(c_s^{\upsilon}, \frac{M_s^{\upsilon}}{P_s}\right) = \log c_s^{\upsilon} + \chi \log \frac{M_s^{\upsilon}}{P_s} \tag{11}$$

Additive separability allows us to abstract from an alternative real balance effect working through the cross derivative term, u_{cm}, as discussed in the introduction, while logarithmic utility facilitates the algebra involved in solving for the household's consumption function.

Combining (11) with (7) and (8) gives:

$$c_{s+1}^{\upsilon} = \beta\left(1 + r_{s+1}\right) c_s^{\upsilon} \tag{12}$$

$$\frac{M_s^{\upsilon}}{P_s} = \chi\left(\frac{1 + i_{s+1}}{i_{s+1}}\right) c_s^{\upsilon} \tag{13}$$

Notice that under this set of preferences, full consumption will simply be proportional to goods consumption in equilibrium:

$$\widehat{c}_s^{\upsilon} = (1 + \chi)\, c_s^{\upsilon} \tag{14}$$

and as a result its dynamics will also be governed by (12).

Equation (5) can be recursively solved forward from date $s \geq \upsilon$ to obtain the household's *lifetime* budget constraint:

$$\sum_{i=s}^{\infty} \frac{1}{\prod_{j=s+1}^{i}\left(1 + r_j\right)} \widehat{c}_i^{\upsilon} = a_s^{\upsilon} + \sum_{i=s}^{\infty} \frac{1}{\prod_{j=s+1}^{i}\left(1 + r_j\right)} \left(y_i + \tau_i\right) \tag{15}$$

[1] The aggregation procedure we adopt later on restricts us to using homothetic preferences in any case.

where $\prod_{j=s+1}^{s}(1+r_j)^{-1}$ is interpreted as 1, $\prod_{j=s+1}^{s+1}(1+r_j)^{-1}$ as $(1+r_{s+1})^{-1}$ and so on.

Combining (15) with (12) and (14) leads to household υ's optimal full consumption at date s:

$$\widehat{c}_s^{\upsilon} = (1-\beta)\left(a_s^{\upsilon} + h_t\right) \tag{16}$$

where $h_s \equiv \sum_{i=s}^{\infty} \frac{1}{\prod_{j=s+1}^{i}(1+r_j)}(y_i + \tau_i)$ is the human wealth of cohort υ as of date $s \geq \upsilon$. Given (14), it is a straightforward matter to solve for the optimal goods consumption function linking consumption to total household wealth:

$$c_s^{\upsilon} = \kappa\left(a_s^{\upsilon} + h_s\right) \tag{17}$$

where $\kappa \equiv \frac{1-\beta}{1+\chi}$ is the marginal propensity to consume out of wealth. Full consumption is thus simply proportional to the sum of human and nonhuman wealth, following the permanent income hypothesis.

2.3 Per Capita Relationships

For any variable, x_s^{υ}, the corresponding per-capita variable at date s, denoted x_s, is simply given by[2]:

$$x_s = \frac{x_s^0 + \sum_{\upsilon=1}^{s} n(1+n)^{\upsilon-1} x_s^{\upsilon}}{(1+n)^s} \tag{18}$$

Applying this linear aggregation procedure to expressions (16), (13), (4), (5) and to the relationship between consumption and full consumption (14), one derives the following:

$$\widehat{c}_s = (1-\beta)\left[a_s + h_s\right] \tag{19}$$

[2] Vintage $\upsilon = 0$ has $N_0 = 1$ members. Total population next period is N_1; of this total, $N_1 - N_0 = (1+n) - 1$ are of vintage $\upsilon = 1$. Similarly, vintage $\upsilon = 2$ contains $N_2 - N_1 = (1+n)^2 - (1+n) = n(1+n)$ members, and so on. Total population at date t equals $N_t = (1+n)^t$.

$$\frac{M_s}{P_s} = \chi \left(\frac{1 + i_{s+1}}{i_{s+1}} \right) c_s \tag{20}$$

$$a_{s+1} \equiv \frac{(1 + i_{s+1}) B_{s+1}}{P_{s+1}} + \frac{M_s}{P_{s+1}} \tag{21}$$

$$a_{t+1} = \left(\frac{1 + r_{t+1}}{1 + n} \right) (a_t + y_t + \tau_t - \widehat{c}_t) \tag{22}$$

$$\widehat{c}_s \equiv c_s + \frac{i_{s+1}}{1 + i_{s+1}} \frac{M_s}{P_s} = (1 + \chi) c_s \tag{23}$$

While (19), (20) and (21) are straightforward analogs of (16), (13) and (4), a comparison of (22) to (5) reveals that per capita financial wealth grows at a slower rate than each individual household's financial wealth, as newly born households start their lives without money and bonds by assumption. This property (also present in Blanchard [1985]) plays a crucial role in our results.

2.3.1 The Per Capita Consumption Euler Equation

When we attempt to convert the household consumption Euler equation (12) into per capita terms using our aggregation method, however, the consumption of the newly born cohort does not drop out:

$$\beta (1 + r_{s+1}) c_s = c_{s+1} + n \left(c_{s+1} - c_{s+1}^{s+1} \right) \tag{24}$$

This equation has a simple interpretation. Notice, first of all, that for $n = 0$, (24) simply reduces to the standard consumption Euler equation for representative agent models. Such a result would also occur, were it the case that average consumption in the population, c_s, were equal to consumption of the newborn, c_{s+1}^{s+1}. This cannot be the case, however, as the new cohort are born with no nonhuman wealth by assumption and as a consequence, their consumption must be less than the average level in the population. This "disconnectedness" – to use Weil's term – depresses the growth rate of aggregate consumption and gives rise to non-Ricardian results.

We derive the per capita consumption Euler equation in terms of only per capita variables in two steps: we inially derive the Euler equation for full consumption and then relate this expression directly to c. Using the per capita full consumption function (19) to solve out for a in (22) yields the following equation:

$$\left[\frac{\widehat{c}_s}{1-\beta} - h_t - (\widehat{c}_s - y_s - \tau_s)\right]\frac{1+r_{s+1}}{1+n} = \frac{\widehat{c}_{s+1}}{1-\beta} - h_{s+1} \tag{25}$$

Upon further simplification, this equation can be transformed into a relationship linking the optimal time path of full consumption to the stock of human wealth:

$$\frac{\beta}{1-\beta}\left(\frac{1+r_{s+1}}{1+n}\right)\widehat{c}_s = \frac{\widehat{c}_{s+1}}{1-\beta} - \frac{n}{1+n}h_{s+1} \tag{26}$$

For our purposes in this paper, it is more useful to work with this relationship in terms of *nonhuman wealth,* however. Substituting out for h_{t+1} using (19) yields the per capita Euler equation for full consumption:

$$\beta\left(1+r_{s+1}\right)\widehat{c}_s = \widehat{c}_{s+1} + n\left(1-\beta\right)a_{s+1} \tag{27}$$

The per capita Euler equation for goods consumption then follows directly from (23):

$$\beta\left(1+r_{s+1}\right)c_s = c_{s+1} + \frac{n\left(1-\beta\right)}{1+\chi}a_{s+1} \tag{28}$$

This is the central equation in the paper. Comparing (28) with (24), we see that the difference between average consumption and consumption of the newly born is simply the marginal propensity to consume times aggregate nonhuman wealth. The higher the level of nonhuman wealth, the greater the difference c_s and c_{s+1}^{s+1} and the lower the optimal growth rate of aggregate consumption.

2.4 Pricing Assumptions

We model the supply side in an extremely simplistic fashion. Denote the current period as $s = t$. The current price level is assumed to be fixed exogenously, $P_t = \overline{P}_t$, whereas the sequence of future price levels, $\{P_s\}_{s=t+1}^{\infty}$, are assumed to be perfectly flexible. In addition to this, there exists each period an exogenous, *constant*, "full capacity" level of output, $\overline{y}$, which the economy needs to reach to prevent a recession. One may thus imagine the current supply curve as a horizontal line at $\overline{P}_t$, with full capacity chosen exogenously; clearly, nothing prevents current output deviating from this level and so y_t, in period t is endogenous. For all future periods, however, the supply curve will be vertical at the full capacity level (hence the output gap will be zero) and the price level rather than the output gap is endogenous.

2.5 Policy Regime

In order to close the model, it remains to specify the government's fiscal and monetary policy. Govenment activity here is restricted to the printing of (or destruction of) money; both government consumption and the stock of public debt are assumed to be zero at every point in time[3]. More specifically, monetary policy is specified by a positive, exogenous sequence of per capita money supplies $\{M_s\}_{s=t}^{\infty}$. Fiscal policy in turn is specified so that each

[3] We make these assumptions in order to focus on the monetary aspects of the model. As the Ricardian debt neutrality proposition does not hold in this overlapping generations set-up, however, the zero public debt assumption is not an entirely innocuous one. In particular, the equivalence between transfers or Friedman-style helicopter-drops and open market operations breaks down here as we discuss in more detail later on.

element of the sequence $\{B_s\}_{s=t}^{\infty}$ equals zero. The government's flow budget constraint, in nominal per capita terms, is therefore given by:

$$P_s \tau_s = M_s - \frac{M_{s-1}}{1+n} \tag{29}$$

$\forall s \geq t$.

3 Macroeconomic Equilibrium

3.1 Aggregate Consistency Conditions

In equilibrium, the goods and asset markets must clear:

$$c_s = y_s \tag{30}$$

$$a_s = \frac{M_{s-1}}{P_s} \tag{31}$$

$\forall s \geq t$. As, by assumption, output is not storable and the government does not issue debt, the latter condition states that all aggregate nonhuman wealth must be held in the form of real balances. Through (31), we therefore abstract from the usual critique of the Pigou effect, originally due to Kalecki [1944], that, being dwarfed in relative magnitude by other outside assets, increases real balances of all but extremely large magnitudes will fail to have a sizeable impact on aggregate financial wealth. Our framework would thus seem to be heavily biased towards finding in favour of the Pigovian real balance effect.

Combining (28) and (20) with the market clearing conditions (30), (31) and our assumptions on pricing and technology gives:

$$\beta \left(1 + i_{t+1}\right) \frac{\overline{P}_t}{P_{t+1}} y_t = \overline{y} + n\kappa \frac{M_t}{P_{t+1}} \tag{32}$$

$$\frac{M_t}{\overline{P}_t} = \chi \left(\frac{1 + i_{t+1}}{i_{t+1}} \right) y_t \tag{33}$$

3.2 Perfect Foresight Equilibrium

A *perfect foresight equilibrium* is a set of solutions y_t, $\{P_s, i_s\}_{s=t+1}^{\infty}$ satisfying (32), (33), non-negativity constraints on real balances, $\frac{M_s}{P_s} \geq 0 \; \forall s \geq t$, and each cohort's consumption at birth, $c_s^s \geq 0 \; \forall s \geq t$, and the exogenous set of sequences $\{M_s, \overline{y}\}_{s=t}^{\infty}$.

The solution method we employ is to first of all solve for the sequence of flexible prices, $\{P_s\}_{s=t+1}^{\infty}$, and future nominal interest rates, $\{i_s\}_{s=t+2}^{\infty}$, and then use the solution for P_{t+1} to find current values of output and the nominal interest rate.

3.2.1 Solving for the Path of Future Prices and Interest Rates

Combining (32) with (33) and a little algebra (see part 1 of the appendix for details), we arrive at the basic law of motion governing the dynamics of real balances, $m \equiv \frac{M}{P}$ (or equivalently the price level) in periods $t+1$, $t+2$ and so on:

$$m_{t+1} = \chi \overline{y} + \beta m_{t+2} \overline{y} \left[\left(\frac{\mu_{t+2}}{1+n} \right) \overline{y} + n \kappa m_{t+2} \right]^{-1} \equiv \theta \left(m_{t+2}, \mu_{t+2} \right) \tag{34}$$

where $\mu_s \equiv (1+n) M_s / M_{s-1}$ denotes the gross growth rate of the *aggregate* nominal money stock between periods $s-1$ and s. As m_{t+1}, like P_{t+1}, is a nonpredetermined, "jump" variable, (34) must be solved in a forward-looking manner, in which m_{t+1} is determined as the value that causes the resulting sequence $\{m_s\}_{s=t+2}^{\infty}$ to be non-divergent. For this sequence to also be unique, we require the usual saddlepath condition that the derivative of the right-hand side of (34) with respect to m_{t+2} at the steady state is strictly less than one in absolute value. In general, this solution will make

m_{t+1} a function of the entire future sequence of money growth rates. If we make the simplifying assumption that these growth rates are constant from period $t+1$ onwards at $\overline{\mu}$, however, then the unique non-explosive solution to (34) will be the steady state value, m^*.

The steady state solution to (34) solves the following quadratic polynomial:

$$\Psi(m^*) \equiv \frac{n\kappa}{\overline{\mu}}(m^*)^2 + \overline{y}\left(1 - \frac{\chi n\kappa}{\overline{\mu}} - \frac{\beta(1+n)}{\overline{\mu}}\right)m^* - \chi(\overline{y})^2 = 0 \qquad (35)$$

The equation has two distinct real roots, but only its largest root satisfies the requirement that real balances be non-negative, and this is given by:

$$m^* = \phi(n, \overline{\mu})\,\overline{y} \qquad (36)$$

where $\phi(n,\overline{\mu}) \equiv \frac{1}{2n\kappa}\left\{-[\overline{\mu} - \chi n\kappa - \beta(1+n)] + \sqrt{[\overline{\mu} - \chi n\kappa - \beta(1+n)]^2 + 4\chi nk\overline{\mu}}\right\}$. As in the representative agent model with logarithmic preferences, the steady state here has a unit income elasticity of money demand.

The price level in period s, for $s \geq t+1$, is therefore given by:

$$P_s = (\overline{\mu})^{s-(t+1)}\frac{M_{t+1}}{\phi(n,\overline{\mu})\,\overline{y}} \qquad (37)$$

Solving for the sequence of future nominal interest rates is even more simple. Rearranging (33), we see that the nominal interest rate in all future periods $s \geq t+1$ will be constant at:

$$i_s = \frac{\chi}{\phi(n,\overline{\mu}) - \chi} \qquad (38)$$

3.2.2 The IS-LM Equilibrium

We are now ready to solve for the current level of output and the nominal interest rate. For this purpose, we combine (32) and (33) with (37) to form a system of two equations in two unknowns:

$$y_t = \beta^{-1}\left(1+i_{t+1}\right)^{-1}\left[\frac{M_{t+1}}{\phi\left(n,\overline{\mu}\right)\overline{P}_t} + n\kappa\frac{M_t}{\overline{P}_t}\right] \tag{39}$$

$$\frac{M_t}{\overline{P}_t} = \chi\left(\frac{1+i_{t+1}}{i_{t+1}}\right)y_t \tag{40}$$

As is common practice in monetary theory papers nowadays, we will label (39) and (40) *microfounded* IS and LM curves, in reference to the famous workhorse model of undergraduate macro textbooks. Whilst the resemblance of (40) to a Hicksian LM curve is clear enough, the connection between (39) and the Hicksian IS curve breaks down in one crucial aspect: the presence of period t real balances on the right hand side when $n > 0$. As an implication, this framework has the interesting property that there exists a positive relationship between current real balances and current output, *holding constant the real interest rate.* My claim is that, of all the microfounded real balance effects discussed in this paper, this particular mechanism comes closest to the one advocated by Pigou [1943] in his rebuttal to Keynes' challenge to classical theory. But first, let us discuss how this feature alters the determination of income and the nominal rate.

Rearranging (40) to substitute out for i_{t+1} in (39) yields the following aggregate demand curve:

$$\begin{aligned} y_t &= \frac{M_t}{\overline{P}_t}\left\{\beta\frac{M_t}{\overline{P}_t}\left[\frac{M_{t+1}}{\phi\left(n,\overline{\mu}\right)\overline{P}_t} + n\kappa\frac{M_t}{\overline{P}_t}\right]^{-1} + \chi\right\}^{-1} \\ &\equiv y\left(M_t, M_{t+1}, n\right) \end{aligned} \tag{41}$$

The transmission mechanism works as follows. Holding constant M_{t+1}, when the monetary authority increases M_t, this has two effects on incentives for current spending: (i) a standard *liquidity effect* leading to a fall in i_{t+1} and the real interest rate, r_{t+1}; and (ii) a *real balance effect*, whereby the policy change leads to an increase in aggregate nonhuman wealth at the beginning of period $t+1$. This, in turn, decreases the optimal growth rate of average consumption and as future consumption is fixed at $\overline{y}$, that means higher consumption and output today. We provide further intuition for this latter effect below. When the monetary authority increases M_{t+1}, there is an *expected inflation effect* which raises P_{t+1} and lowers real interest rates. The first and second arguments in the function y therefore have positive partial derivatives.

4 The Liquidity Trap

4.1 Money, Wealth and the Pigou Effect

Our purpose in this paper is to analyse this transmission mechanism in extremis when the economy is caught in a liquidity trap, defined as in Aikman [2000]:

Definition 1 *A liquidity trap will be said to occur whenever temporary changes in the current nominal stock of money – that is, changes in M_t which leave the sequence $\{M_s\}_{s=t+1}^{\infty}$ unchanged – are incapable of matching aggregate demand with full capacity output.*

We initially characterise the $n=0$ case[4]. In order to establish that liquidity traps are possible in representative agent models, it suffices to show that $\lim_{M_t \longrightarrow \infty} y_t < \overline{y}$ for some feasible shock. Setting $n=0$ in (41) implies:

$$y_t = \frac{M_{t+1}\left(1-\beta\left(\overline{\mu}\right)^{-1}\right)}{\chi \overline{P}_t\left(\beta + \frac{M_{t+1}}{M_t}\left(1-\beta\left(\overline{\mu}\right)^{-1}\right)\right)} \tag{42}$$

[4] The reader may wish to consult Aikman [2002] for a more complete description of this case.

Taking the limit as the nominal money stock is expanded without bound gives:

$$\lim_{M_t \longrightarrow \infty} y_t = \frac{M_{t+1}\left(1 - \beta\left(\overline{\mu}\right)^{-1}\right)}{\beta\chi\overline{P}_t} \tag{43}$$

Graphically, this represents the level of demand where the IS curve crosses the horizontal axis in (y_t, i_{t+1}) space. It is perfectly feasible that this level of demand lies below $\overline{y}$. Starting from $y_t = \overline{y}$, a permanent time preference shock[5], for instance, which raised β sufficiently high could cause such an outcome. In IS-LM terms, this shock would shift the IS curve (drawn for a given level of expected inflation) leftwards and if the shock was of sufficient magnitude, this could cause the horizontal intercept of the curve to lie below $\overline{y}$. The condition for this to occur is:

$$\beta > \frac{M_{t+1}}{M_{t+1}\left(\overline{\mu}\right)^{-1} + \overline{y}\chi\overline{P}_t} \tag{44}$$

$$\Longleftrightarrow \quad \beta > \frac{P_{t+1}}{\overline{P}_t} \tag{45}$$

Such a shock is certainly feasible, provided expected inflation as of period t is negative.

We now turn our attention to the more interesting $n > 0$ case. Taking the limit of (41), we find that as $M_t \to \infty$, (41)$\to \frac{M_t}{P_t}\left(\frac{\beta}{n\kappa} + \chi\right)^{-1} \to \infty$. As the supply of money is expanded to infinity, therefore, aggregate demand increases without bound. This fact, therefore, may lead us to conclude that the real balance effect generated by this framework completely eliminates the possibility of a liquidity trap as defined above.

[5] Following Krugman [1998], Aikman [2002] considered a shock that permanently reduced the full capacity level of output from period $t+1$ onwards. Our use of logarithmic preferences, combined with the assumption that policy sets exogenous monetary targets implies that such a shock would not cause a trap here (see Aikman [2002] for fuller discussion).

4.1.1 Monetary Wealth Effects

Why is it that a real balance effect operating through wealth only occurs when $n > 0$? To gain a general insight into the mechanism at work, let us begin by noting that (22) can be integrated to yield the per capita lifetime budget constraint:

$$\sum_{i=t}^{\infty} \lambda_i \left[c_i + \left(\frac{i_{i+1}}{1 + i_{i+1}} \right) \frac{M_i}{P_i} \right] = \frac{M_{t-1}}{P_t} + \sum_{i=t}^{\infty} \lambda_i \left(y_i + \tau_i \right) \tag{46}$$

where $\lambda_i \equiv \frac{(1+n)^{i-t}}{\prod_{j=t+1}^{i}(1+r_j)}$ is the discount factor. As we show in part 2 of the appendix, (46) can be combined with (28) and (31) to equivalently express the per capita consumption function as:

$$c_t = \Delta \left(\Omega_t^m + \sum_{i=t}^{\infty} \lambda_i y_i \right) \tag{47}$$

where $\Delta \equiv 1 - \beta (1 + n)$ and Ω_t^m is defined as the economy's *real net per capita monetary wealth:*

$$\Omega_t^m \equiv \frac{M_{t-1}}{P_t} + \sum_{i=t}^{\infty} \lambda_i \tau_i + \frac{n\kappa}{\Delta} \sum_{i=t+1}^{\infty} \lambda_i \frac{M_{i-1}}{P_i} - \sum_{i=t}^{\infty} \lambda_i \left(\frac{i_{i+1}}{1 + i_{i+1}} \right) \frac{M_i}{P_i} \tag{48}$$

We initially analyse Ω_t^m for the $n = 0$, representative agent case. Here, the third term on the right-hand side of (48) drops out, and net per capita wealth will be the sum of nonhuman monetary wealth ($\frac{M_{t-1}}{P_t}$) and human monetary wealth (the discounted value of the stream of real transfers), less the present discounted value of the sequence of opportunity costs incurred when wealth is held in monetary form.

As we show in part 2 of the appendix, $\Omega_t^m \equiv 0$ when $n = 0$, for all paths of real and nominal interest rates, . This result, originally due to Sachs [1980] and Cohen [1985] – who both prove it for the continuous time case

– explains why the representative agent model analysed by Krugman does not display a real balance effect. With Ω_t^m equal to zero, (47) implies that monetary policy will only influence demand to the extent that it influences the equilibrium path of real interest rates in $\sum_{i=t}^{\infty} \lambda_i y_i$.

Intuitively, without population growth, the households owning the current money stock are exactly the same households who receive all the transfers and incur all the opportunity costs involved in carrying the money stock. These three components of wealth always exactly cancel when $n = 0$. As Weil [1991] discusses, this result is a modality of the Ricardian debt neutrality proposition, and as we now go on to show for the $n > 0$ case, the conditions that cause Ricardian equivalence to break down are exactly the same conditions required for money to enter the model as net wealth.

When $n > 0$, as we show in the appendix, Ω_t^m can be computed as:

$$\Omega_t^m = \left(\frac{n}{1+n}\right)\frac{M_{t-1}}{P_t} + \frac{n\kappa}{\Delta}\sum_{i=t+1}^{\infty} \lambda_i \frac{M_{i-1}}{P_i} > 0 \tag{49}$$

In this case, the real value of current monetary wealth exceeds the present discounted value of the future opportunity costs of holding money incurred by the currently alive consumers. This is because future opportunity costs fall partly upon future cohorts whose consumption is not valued by agents currently alive. Money is therefore net wealth in the sense that the stock of money enters the consumption function in a nontrivial way.

4.2 Intergenerational Effects and a Very Keynesian Conclusion

When $n = 0$, the monetary authority can, in principle, engineer temporary changes in the current stock of money of any size. This is because the representative household in that case can always use its current stock of real balances to finance the lump-sum taxes required to contract the money supply in future. When $n > 0$, however, this is not the case; households born in period $t+1$ have zero initial financial wealth and the growth rate of the money stock between periods t and $t+1$ must be sufficiently high that these households can afford to pay the taxes and still consume. Part 3 of the appendix shows that the nonnegativity constraint on c_{t+1}^{t+1} holds if and only if:

$$M_t \leq \widehat{M_t} \equiv \frac{M_{t+1}}{\phi(n, \overline{\mu}) \kappa} \tag{50}$$

Intuitively, if M_t becomes too large relative to M_{t+1}, the lump-sum tax required to contract the money supply in period $t+1$, τ_{t+1}, will be so high that human wealth as of period $t+1$ will be negative. As the newly born cohort in $t+1$ have no other assets to fall back on, such a path for the money supply would imply negative consumption for this cohort. The nonnegativity constraint on c_{t+1}^{t+1} will, of course, also have implications for the set of feasible long run rates of growth of the money stock, and we also characterise this set in part three of the appendix.

This brings us directly back to our analysis of the possibility of liquidity traps occurring when $n > 0$. We showed above that aggregate demand in this case expands without bound in the limit as the money supply is increased to infinity and put forward the claim that this ruled out the possibility of a liquidity trap occurring under this paper's definition. It is now clear, however, that such reasoning was incorrect, as temporary increases in the stock of money of this size would violate the nonnegativity constraint on the consumption of the newly born in $t+1$, (50).

The correct question, therefore, is whether $y\left(\widehat{M_t}, M_{t+1}, n\right)$ can possibly lie below the full capacity level of output. Evaluating this expression, we find that the maximum feasible level of demand when $n > 0$ is:

$$\lim_{M_t \longrightarrow \widehat{M_t}} y_t = \frac{M_{t+1}(1+n)}{\overline{P}_t \phi(n, \overline{\mu}) [\beta + \chi\kappa(1+n)]} \tag{51}$$

The maximum level of demand is therefore proportional to $\frac{M_{t+1}}{\overline{P}}$, with the coefficient of proportionality simply being a collection of parameters that are independent of $\overline{y}$. Without a priori restrictions on technology and tastes, therefore, it is clear that nothing prevents $y\left(\widehat{M}_t, M_{t+1}, n\right)$ lying below $\overline{y}$ and we can conclude that, in principle, liquidity traps are certainly possible when $n > 0$.

More practical readers may still be wondering, however, whether such an outcome represents anything other than a remote possibility? As a preliminary step in shedding some light on the problem, multiply both numerator and denominator by P_{t+1} and use the steady state solution for real balances (36); we find that aggregate demand will lie below $\overline{y}$ if and only if:

$$\frac{P_{t+1}}{\overline{P}_t} < \frac{\beta}{1+n} + \chi\kappa \tag{52}$$

Given that n, χ and κ are all likely to be small numbers, the expected deflation required to cause a liquidity trap when $n > 0$ would not appear to be that different from the $n = 0$ case, suggesting that the incorporation of the real balance effect does not offer substantially greater protection from a liquidity trap. We now sharpen this reasoning with a simple calibration of the model's key parameters.

If we take a period to equal one year, then a value of 0.957 for the subjective time discount factor, β, would correspond to Cooley and Prescott's [1995] calibration. For the steady state annual growth rate of the nominal money stock, we choose 5 percent, implying $\overline{\mu} = 1.05$. Following Ireland [2001], we calibrate the annual population growth rate, n, at 1 percent (a value also considered by Weil [1991]) and finally χ is set at 0.0127, a value that implies that the ratio of real money balances to real output in the steady state is 0.16. Walsh [1998] argues that such a value roughly corresponds to the real value of M1 relative to GDP in the U.S. in the early 1990s.

These parameter values imply that the deflation rates $\left(-\left(P_{t+1}/\overline{P}_t - 1\right)\right)$

required to cause a liquidity trap in the $n = 0$ and $n > 0$ cases are 4.3 percent and 5.2 percent respectively. Furthermore, we can apply these values directly to (43) and (51) to obtain expressions for the maximum feasible levels of demand (as a function of M_{t+1} and $\overline{P}_t$) resulting from temporary changes in M_t. When $n = 0$, $\lim_{M_t \longrightarrow \infty} y_t = 7.29 \frac{M_{t+1}}{\overline{P}_t}$; when $n > 0$, $\lim_{M_t \longrightarrow \widehat{M}_t} y_t = 6.31 \frac{M_{t+1}}{\overline{P}_t}$. The maximum feasible level of demand when the real balance effect is operative therefore lies to the *left* of the level without the real balance effect! This outcome is sketched in figure 1 below:

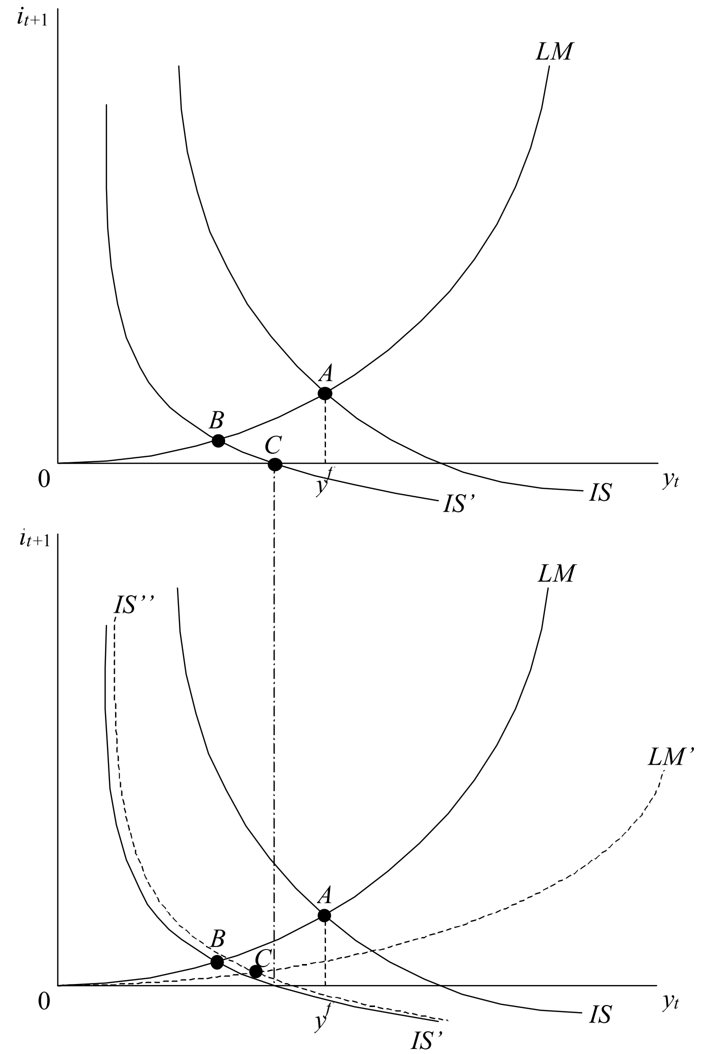

Figure 1: *A Liquidity Trap With and Without the Real Balance Effect*

In each case, we begin at point A, where demand equals the full capacity level of output. Following the increase in β, the IS curve shifts leftwards to *IS'*, implying a new equilibrium at point B. The central bank, wishing to restore the full capacity equilibrium, then raises M_t and this shifts the LM curve rightwards as in the textbook IS-LM model, but also the IS curve rightwards when $n > 0$. The movement from B to C represents the largest feasible effect on demand with and without the real balance effect. In the upper diagram, which corresponds to the $n = 0$ case, the current money supply is expanded to infinity and this brings nominal interest rates down to zero; nevertheless, the horizontal intercept of IS is the highest level of demand feasible. When $n > 0$, however, the upper bound to M_t is $\widehat{M_t}$ rather than ∞, and as the lower diagram illustrates, under our back-of-the-envelope calibration, the highest feasible level of demand is actually less than in the former case.

What then is the intuition for this perhaps at first sight surprising degree of policy ineffectiveness in the model incorporating the real balance effect? The result becomes even more surprising when we apply our calibration to the money growth constraint (50): it permits the central bank to expand the current money stock to 147 times its $t + 1$ value! So what is driving the ineffectiveness result? The answer becomes clear once we examine the magnitude of the real balance effect under our chosen calibration. The size of this effect is effectively determined by the product $n\kappa$ and under our parameter values, this is extremely small at 0.0004. Therefore, while the constraint (50) allows for truly enormous gyrations in the money stock between periods t and $t+1$, the real balance effect is so weak that even these movements have little impact on demand.

Our results, therefore, contrast sharply to those of Ireland [2001]. Rather than eliminating the possibility of liquidity traps occurring, the real balance effect generated by this model makes a trap more likely as it heightens the constraints on the monetary authority.

5 Conclusion

In this paper, we have used Weil's [1991] monetary, overlapping infinitely-lived dynasties framework to analyse a microfounded version of the real balance effect envisaged by Pigou. This effect appears to imply that temporary monetary expansions can have an unboundedly large impact on current aggregate demand, thereby eliminating Krugman's liquidity trap. The circumstances under which such an effect is operative, however, imply a condition that rules out temporary monetary expansions of this magnitude. We then showed that for the set of feasible monetary expansions, liquidity traps are possible and, under our simple calibration of the model, more likely. This result contrasts sharply with Ireland [2001].

We conclude with some thoughts for further research using this framework. The first topic would be to extend this paper to analyse the optimal monetary response to shocks that cause a liquidity trap. Notwithstanding the obvious difficulties in defining a social welfare function with heterogeneous agents, such a study would certainly be interesting as an OLG framework is likely to yield very different results from the representative agent case. The clearest such difference would surely revolve around the relative merits of temporary versus permanent monetary expansions. As discussed in this paper, temporary expansions imply an increase in future lump-sum taxes and this will have proportionally the greatest impact on future cohorts as they have no other assets to fall back on. As permanent monetary expansions also have distributional consequences, we could analyse the optimality of Krugman's suggestion of credibly committing to being irresponsible.

A second avenue of research that might prove fruitful would be to combine this framework with the expectational liquidity trap put forward by Benhabib, Schmitt-Grohe and Uribe [2001], [2002]. Conceptually and observationally quite distinct from Krugman's liquidity trap, these authors view the trap as a failure to coordinate expectations on the stable inflation equilibrium. Self-fulfilling deflations are usually ruled out as possible perfect foresight equilibrium paths by appealing to the transversality condition. Benhabib et al., however, specify a policy regime in which the transversality condition necessarily holds, regardless of the evolution of the endogenous variables, and so this argument cannot apply. Their policy regime has a particular implication for the dynamic behaviour of financial wealth (money plus bonds outstanding) and in our framework, this would influence, and be influenced by, demand via the per capita consumption Euler equation.

6 Appendix

6.1 Deriving the Law of Motion Governing Real Balances for Periods $t+1$ Onwards

Using (33) to substitute out i_{t+1} in (32), and forwarding by one period, we get the following difference equation in real balances:

$$\beta\left(1+\frac{\chi\overline{y}}{m_t-\chi\overline{y}}\right)\frac{m_{t+1}}{m_t}\left(\frac{1+n}{\mu_{t+1}}\right)\overline{y}=\overline{y}+n\kappa\left(\frac{1+n}{\mu_{t+1}}\right)m_{t+1}$$

Mutliplying out the brackets gives:

$$\beta\frac{m_{t+1}}{m_t}\left(\frac{1+n}{\mu_{t+1}}\right)\overline{y}+\beta\left(\frac{\chi\overline{y}}{m_t-\chi\overline{y}}\right)\frac{m_{t+1}}{m_t}\left(\frac{1+n}{\mu_{t+1}}\right)\overline{y}=\overline{y}+n\kappa\left(\frac{1+n}{\mu_{t+1}}\right)m_{t+1}$$

We then use the following trick:

$$\beta\frac{m_{t+1}}{m_t}\left(\frac{1+n}{\mu_{t+1}}\right)\overline{y}+\frac{\chi\overline{y}}{m_t}\left[\underbrace{\beta\left(\frac{m_t}{m_t-\chi\overline{y}}\right)\frac{m_{t+1}}{m_t}\left(\frac{1+n}{\mu_{t+1}}\right)\overline{y}}_{=\overline{y}+n\kappa\left(\frac{1+n}{\mu_{t+1}}\right)m_{t+1}}\right]=\overline{y}+n\kappa\left(\frac{1+n}{\mu_{t+1}}\right)m_{t+1}$$

Combining terms gives:

$$\beta\frac{m_{t+1}}{m_t}\left(\frac{1+n}{\mu_{t+1}}\right)\overline{y}=\left(1-\frac{\chi\overline{y}}{m_t}\right)\left(\overline{y}+n\kappa\left(\frac{1+n}{\mu_{t+1}}\right)m_{t+1}\right)$$

Finally, multiplying through by $m_t\left(\overline{y}+n\kappa\left(\frac{1+n}{\mu_{t+1}}\right)m_{t+1}\right)^{-1}$ gives the equation in the text.

6.2 Net Monetary Wealth

6.2.1 Deriving the Per Capita Consumption Function (47)

We begin by noting that by (28) after a little algebra yields:

$$\begin{aligned}
& c_t+\left(\frac{1+n}{1+r_{t+1}}\right)c_{t+1}+\frac{(1+n)^2}{(1+r_{t+1})(1+r_{t+1})}+\cdots \\
= \; & c_t+(1+n)\left(\beta c_t-\frac{n\kappa a_{t+1}}{(1+r_{t+1})}\right)+(1+n)^2\left(\beta^2 c_t-\frac{\beta n\kappa a_{t+1}}{(1+r_{t+1})}-\frac{n\kappa a_{t+2}}{(1+r_{t+1})(1+r_{t+2})}\right)+\cdots
\end{aligned}$$

Upon further simplification, it is clear that the infinite sum can be written as:

$$\begin{aligned}
\sum_{i=t}^{\infty}\lambda_i c_i &= \frac{c_t}{\Delta}-\frac{n\kappa(1+n)a_{t+1}}{(1+r_{t+1})\Delta}-\frac{n\kappa(1+n)^2 a_{t+2}}{(1+r_{t+1})(1+r_{t+2})\Delta}-\cdots \\
&= \frac{c_t}{\Delta}-\frac{n\kappa}{\Delta}\sum_{i=t}^{\infty}\lambda_i a_i
\end{aligned}$$

where $\Delta\equiv 1-\beta(1+n)$ and $\lambda_i\equiv\frac{(1+n)^{i-t}}{\prod_{j=t+1}^{i}(1+r_j)}$. Combining this with the *per capita* lifetime constraint and the asset market clearing condition (31) then yields equation (47) in the text.

6.2.2 Characterising Ω_t^m

We begin with the $n = 0$ case. The economy's net per capita monetary wealth will therefore be:

$$\Omega_t^m = \frac{M_{t-1}}{P_t} + \sum_{i=t}^{\infty} \frac{1}{\prod_{j=t+1}^{i}(1+r_j)}\tau_i - \sum_{i=t}^{\infty} \frac{1}{\prod_{j=t+1}^{i}(1+r_j)} \left(\frac{i_{i+1}}{1+i_{i+1}}\right) \frac{M_i}{P_i}$$

Using the government budget constraint $\tau_i = \frac{M_i - M_{i-1}}{P_i}$, we can expand the first two parts of this expression as follows:

$$\begin{aligned} & \frac{M_{t-1}}{P_t} + \left(\frac{M_t - M_{t-1}}{P_t}\right) + \frac{1}{1+r_{t+1}}\left(\frac{M_{t+1} - M_t}{P_{t+1}}\right) + \frac{1}{(1+r_{t+1})(1+r_{t+2})}\left(\frac{M_{t+2} - M_{t+1}}{P_{t+2}}\right) + \cdots \\ = & \frac{M_t}{P_t}\left(1 - \frac{1}{1+r_{t+1}}\frac{P_t}{P_{t+1}}\right) + \frac{1}{1+r_{t+1}}\frac{M_{t+1}}{P_{t+1}}\left(1 - \frac{1}{1+r_{t+2}}\frac{P_{t+1}}{P_{t+2}}\right) + \cdots \\ = & \left(\frac{i_{t+1}}{1+i_{t+1}}\right)\frac{M_t}{P_t} + \frac{1}{1+r_{t+1}}\left(\frac{i_{t+2}}{1+i_{t+2}}\right)\frac{M_{t+1}}{P_{t+1}} + \cdots \end{aligned}$$

where the latter equality comes from imposing the Fisher equation, linking real and nominal interest rates. Clearly the first two terms in Ω_t^m are identically equal to the third term and $\Omega_t^m \equiv 0$.

For the case where $n > 0$, we continue to follow the same tack. Here, net per capita monetary wealth will be:

$$\Omega_t^m \equiv \frac{M_{t-1}}{P_t} + \sum_{i=t}^{\infty} \lambda_i \tau_i + \frac{n\kappa}{\Delta} \sum_{i=t+1}^{\infty} \lambda_i \frac{M_{i-1}}{P_i} - \sum_{i=t}^{\infty} \lambda_i \left(\frac{i_{i+1}}{1+i_{i+1}}\right) \frac{M_i}{P_i}$$

Once again using the government's flow constraint (29), we can expand terms one and two in this expression:

$$\frac{M_{t-1}}{P_t} + \left(\frac{M_t}{P_t} - \frac{M_{t-1}}{(1+n)\,P_t}\right) + \frac{1+n}{1+r_{t+1}}\left(\frac{M_{t+1}}{P_{t+1}} - \frac{M_t}{(1+n)\,P_{t+1}}\right)$$
$$+\frac{(1+n)^2}{(1+r_{t+1})\,(1+r_{t+2})}\left(\frac{M_{t+2}}{P_{t+2}} - \frac{M_{t+1}}{(1+n)\,P_{t+2}}\right) + \cdots$$
$$= \left(\frac{n}{1+n}\right)\frac{M_{t-1}}{P_t} + \frac{M_t}{P_t}\left(1 - \frac{1}{1+r_{t+1}}\frac{P_t}{P_{t+1}}\right) + \left(\frac{1+n}{1+r_{t+1}}\right)\frac{M_{t+1}}{P_{t+1}}\left(1 - \frac{1}{1+r_{t+2}}\frac{P_{t+1}}{P_{t+2}}\right) + \cdots$$
$$= \left(\frac{n}{1+n}\right)\frac{M_{t-1}}{P_t} + \left(\frac{i_{t+1}}{1+i_{t+1}}\right)\frac{M_t}{P_t} + \frac{1+n}{1+r_{t+1}}\left(\frac{i_{t+2}}{1+i_{t+2}}\right)\frac{M_{t+1}}{P_{t+1}} + \cdots$$

Adding this to terms three and four in Ω_t^m then gives the expression in the text.

6.3 Restrictions on Admissible Monetary Policies

6.3.1 Ensuring that Consumption at Birth is Nonnegative

In equilibrium, we require that consumption at birth c_v^v be nonnegative – a necessary and sufficient condition guaranteeing that $c_s^v \geq 0$ for all $s > v \geq t$. Since dynasties are born with zero financial wealth, this amounts – when $n > 0$ – to imposing the restriction that human wealth be nonnegative (see (17)). From (19) and (23), with the prior imposition of the market clearing conditions (31), (30), this restriction is equivalent to the following condition:

$$\frac{M_{s-1}}{P_s} \leq \frac{y_s}{\kappa}$$

$\forall s \geq t$. For $s = t+1$ and using (37), this inequality implies a lower bound for the feasible set of growth rates for the nominal money stock between periods t and $t+1$:

$$M_t \leq \widehat{M}_t \equiv \frac{M_{t+1}}{\phi\,(n, \overline{\mu})\,\kappa} \tag{A1}$$

For $s = t+2$, $t+3$ and onwards, it implies an upper bound for the steady state level of real balances:

$$m^* \leq \frac{\overline{\mu y}}{\kappa}$$

From the definition of the quadratic polynominal $\Psi(m^*)$, an equivalent way of stating the above inequality is $\Psi\left(\frac{\overline{\mu y}}{\kappa}\right) \geq 0$. Simplifying, this implies a lower bound to the feasible set of *steady state* monetary growth rates, $\overline{\mu}$:

$$\overline{\mu} \geq \chi\kappa + \beta \tag{A2}$$

6.3.2 The Saddlepath Condition for $m_{t+1} = \theta\left(m_{t+2}, \mu_{t+2}\right)$

In addition to the above, we also require a restriction on the steady state money growth rate to ensure that (34) has a saddlepath solution. As usual, this amounts to requiring that the derivative of $\theta\left(m_{t+2}, \mu_{t+2}\right)$ with respect to its first argument be less than unity at the steady state, or equivalently:

$$\beta\overline{\mu} < (1+n)\left[\frac{\overline{\mu}}{1+n} + n\kappa\phi\left(\overline{\mu}, n\right)\right]^2 \tag{A3}$$

This condition implicitly places a lower bound on the admissible rate of money growth in the steady state, $\overline{\mu}$. Notice that, when $n = 0$, (A3) reduces to the standard requirement that the per capita rate of money growth exceed the discount factor, β. Depending upon the particular calibration chosen, one of the conditions (A2) and (A3) will in general be redundant.

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