Interactions of monetary and fiscal policy in a business cycle model with open market operations

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Abstract

Consensus monetary business cycle theory is hardly able to rationalize why fiscal policy is repeatedly found to stimulate private consumption and why monetary policy should care about Ricardian fiscal policy. In this paper we demonstrate that this changes when government bonds provide liquidity services. We develop a simple business cycle, which can be solved analytically, where money is supplied via open market operations. When only government bonds are accepted as a collateral for money and private debt earns a higher interest, real public debt eases households' access to money and Ricardian equivalence does not hold. Interest rate policy is not restricted by requirements for equilibrium determinacy and its effects are consistent with common priors. Shocks are propagated via changes in financial wealth and persistence is altered by the stance of fiscal and monetary policy. Government expenditures, which are not completely tax financed, can raise private consumption when monetary policy is not too reactive. Similarly, a moderate interest rate policy allows a deficit financed tax cut to stimulate real activity.

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1 Introduction

Consensus monetary business cycle theory, i.e., the New Keynesian theory or the New Neoclassical Synthesis, has evidently proven its usefulness for monetary policy analysis (see, e.g., Clarida et al., 1999, or, Woodford, 2002). However, it leaves some open questions, or, puzzles, in particular with regard to the effects of fiscal policy measures and interactions thereof with monetary policy. Empirical evidence, for example, indicate that government spending is able to cause a surge in private consumption (see, Fatas and Mihov, 2001, and Blanchard and Perotti, 2002), whereas the consensus model can at most generate a rise in total aggregate demand (see Canzoneri et al., 2002a, or, Linnemann and Schabert, 2003). It also lacks any mechanism by which a deficit financed tax cut can lead to a real expansion as, e.g., found by Mountford and Uhlig (2002). Furthermore, a Ricardian fiscal policy regime is irrelevant for price stability and equilibrium determinacy (see Woodford, 2002), such that the consensus framework cannot provide a rationale for high fiscal responsiveness as, for example, demanded by the stability and growth pact.

The main source for these shortcomings is the validity of Ricardian equivalence, which hardly allows for substantial fiscal policy effects. As a promising alternative Leith and Wren-Lewis (2000) and Chadha and Nolan (2002) develop a business cycle framework with overlapping generations. Given that Ricardian equivalence does not hold, they show that the interaction of monetary and fiscal policy can be particularly relevant for macroeconomic stability and for determinacy. In this paper we develop a novel approach based on the role of government bonds for money supply, i.e., in open market operations. In particular, we identify cases where open market operations are relevant and demonstrate that allowing for government bonds to provide liquidity services can help solving the aforementioned puzzles.[2] For this we develop a model where the central bank supplies money via open market operations, or, to be more precisely, via repurchase agreements, while money demand is induced by a cash-in-advance constraint. Households do not carry over money from one period to the other and the so-called Hahn (1965) paradox, i.e., the puzzle about how to guarantee that money has a positive value over a finite horizon, is resolved by settlement of repurchase agreements similar to the approach of Drèze and Polemarchakis (2000) and Bloise et al. (2002). Households' financial wealth comprises private debt and government bonds; only the latter are accepted in exchange for money in repurchase agreements. When government bonds earn a lower interest than private debt, agents care about open market operations and government bonds provide liquidity services. To facilitate comparisons with related studies, we assume that the central bank sets the discount (repo) rate, which equals the interest rate on treasury bills, contingent on current inflation, and that firms set prices in a staggered way.

The model exhibits two fundamentally different versions, depending on whether open

[2] Recently, Canzoneri and Diba (2000) have shown that the price level indeterminacy problem can be solved by allowing for liquidity services of bonds.

market operations matter or not.[3] In the latter case the model is isomorphic to the consensus New Keynesian model. Herein, money supply is de facto unrestricted and monetary policy has a bearing on prices and on real activity by shifting the *real* interest rate such that households are willing to intertemporally substitute consumption and leisure. Ricardian equivalence holds such that government financing is irrelevant for the allocation and monetary policy is responsible for macroeconomic stability (see, e.g., Clarida et al., 1999, or, Woodford, 2001). In the case where agents care about open market operations the monetary stance depends on the nominal interest rate and on the stock of government bonds outstanding. Changes of the former immediately affects money supply and, thus, private consumption, while the consumption Euler equation residually determines the interest rate on private debt.[4] Fiscal policy alters the monetary stance as real public debt eases households' access to cash via open market operations. In contrast to the former version, the equilibrium sequence of real financial wealth, which serves as an endogenous state variable, affects private consumption and inflation, and Ricardian equivalence does not hold. Furthermore, real wealth exerts an stabilizing impact on the economy such that interest rate policy must not be restricted to guarantee a stable and uniquely determined equilibrium path.

The simple structure of the model allows to analytically derive the local determinacy properties and the state space representation facilitating a straightforward analysis of interest rate, government spending, and tax cut shocks. When open market operations are relevant such that government bonds provide transaction services, fiscal policy interacts with monetary policy and our model is able to overcome the shortcomings of the consensus model mentioned above. Innovations to the discount rate alters money supply, inflation, and, by rigid prices, real activity qualitatively consistent with, e.g., Clarida et al.'s (1999) New Keynesian model. In contrast to models of the latter type, real financial wealth is a relevant state variable and can induce impulse responses not to die out immediately after a shock disappears. While a more reactive interest rate policy lowers the persistence, the latter is shown to rise for higher degrees of fiscal responsiveness. Turning to the effects of fiscal policy measures, it is further shown that the sign of the responses, in particular for private consumption and output, crucially rely on the structural part of fiscal and monetary policy. Regarding the effects of an unexpected rise in public consumption, the model generates essentially neoclassical effects in the case where government expenditures are predominantly tax financed. When, on the other hand, the portion of public consumption financed by debt is sufficiently large and interest rate policy is moderate, private consumption can actually rise as, e.g., found by Blanchard and Perotti (2002). Moreover, a deficit financed tax cut raises the inflation rate as well as real wealth, and can stimulate real activity, as reported by Mountford and Uhlig (2002), when interest rate policy is not too aggressive. Otherwise, interest rate adjustments

[3] This question has already been discussed by Sargent and Wallace (1982) and Smith (1988).

[4] The latter feature is in fact consistent with Canzoneri et al.'s (2002b) empirical findings, which cast servere doubts on the central role of the consumption Euler equation for the transmission of interest rate policy.

triggered by higher inflation can lead to a contractionary monetary stance which prevails over expansionary fiscal policy measures.

The remainder is organized as follows. In section 2 we develop a sticky price model with repurchase agreements. Section 3 provides the linear approximation to the model for two versions differing in whether open market operation are relevant or not. In section 4 we derive the solution for the former version and examine monetary and fiscal policy effects herein. Section 5 concludes.

2 The model

Timing of events At the beginning of a period, households are endowed with government bonds and claims on other households carried over from the previous period. There are three sources of aggregate uncertainty: a monetary policy shock, a government spending shock and a tax cut shock. After these shocks arrived, goods are produced, and factor remunerations are credited at a financial intermediary.[5] Then asset markets open, where households can adjust bond holdings and borrow (lend) from (to) other households. Given that purchases of consumption goods are restricted by a liquidity constraint, households are willing to acquire cash. The central bank is assumed to supply money exclusively via open market operations, i.e., via repurchase agreements. Households carry over a certain amount of interest bearing assets B^c to the financial intermediary, which engages in repurchase agreements on the behalf of the households. The central bank supplies money M to an amount equal to the value of households' securities discounted by the nominal interest rate $i : M = B^c/(1+i)$. Then the goods market opens, where households purchase consumption goods from firms. Their cash earnings are then paid to the owners (households), transferred to the intermediary, and repurchased by the central bank.

Households Lower (upper) case letters denote real (nominal) variables. The time index is dropped to denote steady state values. There is a continuum of identical and infinitely lived households of mass one. At the beginning of period t, the representative household's financial wealth A_{t-1} comprises government bonds B_{t-1} and private debt D_{t-1} holdings, $A_{t-1} = B_{t-1} + D_{t-1}$, carried over from the previous period. Before the goods market opens, the household enters the asset market, where beginning-of-period assets holdings earn $(1+i_t)B_{t-1}$ and $(1+i_t^d)D_{t-1}$ and the household can adjust its portfolio such that assets holdings are now equal to B_t and D_t. To acquire money, they carry over securities B_t^c to a financial intermediary, which participates in repurchase agreements with the central bank. The amount of money M_t supplied by the latter equals the discounted value of the securities B_t^c:

$$M_t = \frac{B_t^c}{1+i_t}, \qquad \text{with } B_t^c \geq 0. \tag{1}$$

[5] The households' accounts at the financial intermediary are further charged by the wage outlays of the final goods producing firms, which are owned by the households.

The discount (repo) rate, which equals the gross interest rate on government bonds $1+i_t$, is set by the central bank. The household then enters the goods market. When goods trading has ended and taxes and profits are transferred, cash is repurchased by the central bank in exchange for the securities B_t^c less the seignorage $\frac{i_t}{1+i_t}B_t^c = i_t M_t$. It should be noted that the opportunity costs of holding cash from one period to the other would not be lower than the costs of repurchase agreements as long as the nominal interest rate on government bonds is not higher than the one on private debt. As we will focus on this case in the remainder of this paper, we can disregard the possibility of money accumulation by the households. In order to introduce a meaningful role of open market operations, we introduce a legal restriction on open market operations and assume that only securities issued by the government can be used in repurchase agreements:

$$B_t^c \leq B_t. \tag{2}$$

When private debt is also accepted as collateral in open market operations, money supply would obviously become irrelevant. Households receive income from labor supply l_t, interest earnings from assets, and firms' profits ω_t. The budget constraint, which is actually the constraint for the asset market, is

$$A_t + P_t c_t + P_t \tau_t \leq (1+i_t^d)A_{t-1} - (i_t^d - i_t)B_{t-1} - i_t M_t + P_t w_t l_t + P_t \omega_t, \tag{3}$$

where P_t denotes the aggregate price level, c_t the consumption good, w_t the real wage, and τ_t a lump-sum tax. The representative household holds an checkable account at the financial intermediary. After goods are produced its labor income is credited on this account, while it is charged for wage outlays of firms which are owned by the households. Entering the goods market, consumption expenditures are restricted by the following liquidity constraint:

$$P_t c_t \leq M_t + \left(P_t w_t l_t - P_t w_t \int_0^1 l_{it} di \right), \tag{4}$$

The conventional cash-in-advance constraint is augmented by allowing for checkable accounts to be accepted as a means of payment (see the term in round brackets in 4). Hence, an individual labor income, which exceeds the average wage payments of final goods producing firms indexed with $i \in (0,1)$, leads to an relaxation of the cash constraint (4). This assumption, which is adopted from Jeanne (1998), is introduced to avoid the cash-credit good distortion between consumption and leisure and, thus, to simplify the analysis and to facilitate comparisons with related studies. Obviously, we obtain a standard cash-in-advance specification ($P_t c_t \leq M_t$) in equilibrium. The objective of a representative household is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \text{ with } 0 < \beta < 1, \tag{5}$$

where β denotes the subjective discount factor and E_0 the expectation operator conditional on the information in period 0. Regarding the instantaneous utility function $u(c_t, l_t)$, we state the following assumptions.

Assumption 1 *The utility function $u(c_t, l_t)$, with $u : R^2_{++} \rightarrow R$, is assumed to be strictly increasing in consumption c, strictly decreasing in labor l, strictly concave, twice continuously differentiable with respect to both arguments, satisfies the usual Inada conditions, and is additively separable.*

We assume that the household internalizes that its access to money is restricted by their holdings of government bonds.[6] Hence, the household considers the following constraint for the money market

$$M_t \leq \frac{B_t}{1+i_t}, \tag{6}$$

when it decides on its optimal plan. Maximizing (5) subject to the budget constraint (3), a no-Ponzi-game condition, $\lim_{i\to\infty} E_t A_{t+i} \prod_{v=1}^{i}(1+i^d_{t+v})^{-1} \geq 0$, the liquidity constraint (4) and the open market constraint (6) for a given initial value A_0, leads to the following first order conditions for consumption, leisure, private debt, government bonds and money:

$$u_c(t) = \lambda_t + \psi_t, \tag{7}$$

$$u_l(t) = -w_t u_c(t), \tag{8}$$

$$\frac{1}{\beta}\lambda_t = E_t\left[\frac{\lambda_{t+1}}{\pi_{t+1}}\left(1+i^d_{t+1}\right)\right], \qquad \text{with } \pi_t \equiv P_t/P_{t-1}, \tag{9}$$

$$\eta_t\frac{1}{\beta} = E_t\left[\frac{\lambda_{t+1}}{\pi_{t+1}}(i^d_{t+1} - i_{t+1})\right], \tag{10}$$

$$\psi_t = i_t\lambda_t + (1+i_t)\eta_t, \tag{11}$$

$$\psi_t(m_t + w_t(l_t - L_t) - c_t) = 0, \quad \psi_t \geq 0, \quad m_t + w_t(l_t - L_t) - c_t \geq 0, \tag{12}$$

$$\eta_t(b_t - m_t(1+i_t)) = 0, \quad \eta_t \geq 0, \quad b_t - m_t(1+i_t) \geq 0, \tag{13}$$

where λ_t, ψ_t, and η_t denotes the Lagrange multiplier on the asset market constraint (3), on the goods market constraint (4), and on the money market constraint (6), respectively. From (10) and (13) it can immediately be seen that a positive value for the spread $i^d_{t+1} - i_{t+1}$ leads to a binding open market constraint constraint. The conditions (11) and (12) further imply that the cash-in-advance constraint is binding if the nominal interest rate exceeds zero ($i_t > 0$). When open market operations are not legally restricted by (2), the liquidity value for government bonds is zero ($\eta_t = 0 \Rightarrow i_t = i^d_t$) and the first order conditions change to

$$u_c(t) = \lambda_t R_t, \qquad \psi_t = i_t\lambda_t, \qquad \frac{1}{\beta}\frac{u_c(t)}{R_t} = E_t\frac{u_c(t+1)}{\pi_{t+1}}. \tag{14}$$

and (8) and (12). In the optimum the budget constraint (3) must hold with equality and the no-Ponzi-game condition turns into a transversality condition providing a terminal condition for the household's intertemporal problem:

$$\lim_{i\to\infty} E_t\lambda_{t+i}\beta^{t+i}\frac{A_{t+i}}{P_{t+i}} = 0; \tag{15}$$

[6] This restriction becomes irrelevant when private debt is also accepted as a collateral for money.

Production sector The final consumption good is an aggregate of differentiated goods produced by monopolistically competitive firms indexed with $i \in (0,1)$. The aggregator of differentiated goods is defined as follows: $y_t^{\frac{\epsilon-1}{\epsilon}} = \int_0^1 y_{it}^{\frac{\epsilon-1}{\epsilon}} di$, with $\epsilon > 1$, where y_t is the number of units of the final good, y_{i_t} the amount produced by firm i, and ϵ the constant elasticity of substitution between these differentiated goods. Let P_{it} and P_t denote the price of good i set by firm i and the price index for the final good. The demand for each differentiated good is $y_{it} = (P_{it}/P_t)^{-\epsilon} y_t$, with $P_t^{1-\epsilon} = \int_0^1 P_{it}^{1-\epsilon} di$. A firm i produces good y_i employing a technology which is linear in labor: $y_{it} = l_{it}$. We introduce a nominal stickiness in form of staggered price setting as developed by Calvo (1983). Each period firms may reset their prices with the probability $1-\phi$ independent of the time elapsed since the last price setting. The fraction ϕ of firms are assumed to adjust their previous period's prices according to the following simple rule: $P_{it} = \pi P_{it-1}$. The log-linearized version of this rule and of the optimal condition for the price $\widetilde{P}_{it}$ of firms, who are allowed to reset their prices, can be shown to lead to the following aggregate supply constraint (see, e.g., Yun, 1996)

$$\widehat{\pi}_t = \chi \widehat{mc}_t + \beta E_t \widehat{\pi}_{t+1}, \qquad \text{with } \chi \equiv (1-\phi)(1-\beta\phi)\phi^{-1}, \tag{16}$$

which is most commonly applied in the recent business cycle literature. Note that $\widehat{x}$ denotes the percent deviation from the steady state value $x : \widehat{x} = \log(x_t) - \log(x)$ and mc_t the real marginal costs. Labor demand in the symmetric equilibrium satisfies

$$w_t = mc_t. \tag{17}$$

Public sector The public sector consists of a fiscal and a monetary authority. The monetary authority supplies money M_t in open market operations in exchange for bonds $B_t^c = (1+i_t)M_t$. It sets the exchange rate, i.e., the gross nominal interest rate $R_t \equiv 1 + i_t$, according to the following state contingent interest rate rule

$$R_t = \rho(\pi_t, \varepsilon_t^r), \qquad \text{with } \rho(\pi_t, \varepsilon_t^r) \geq 1 \text{ and } \partial\rho/\partial\pi_t \geq 0. \tag{18}$$

where the innovation ε_t^r has an expected value of zero and is serially uncorrelated. We assume that the steady state condition on the nominal interest rate has a solution for $\rho(\pi, 0) > 1$.[7] Furthermore, the realizations of ε^r are restricted to be small enough such that the gross interest rate always exceeds one in the neighborhood of the steady state.

Open market operations are conducted in form of repurchase agreements, i.e., swaps of the ownership over securities B_t^c at the rate i_t. Hence, the central bank earns $B_t^c - M_t = i_t M_t$ from repurchase agreements such that its budget is given by $i_t M_t = P_t \tau_t^c$, where τ_t^c denotes the transfer to the fiscal authority. The latter issues risk free one period bonds earning the nominal interest rate i_t, collects lump-sum taxes τ_t from the households, receives transfers

[7] An example for the interest rate rule is: $\rho(\pi_t, \varepsilon_t^r) = \overline{\rho}\pi_t^{\rho_\pi} \exp(\varepsilon_t^r)$, where the inflation elasticity ρ_π governs the reactiveness of the interest rate policy and $\overline{\rho}$ guarantees $\overline{\rho}\pi^{\rho_\pi} > 1$.

τ^c from the monetary authority and purchases the amount g of the final good

$$P_t g_t + (1 + i_t) B_{t-1} = B_t + P_t \tau_t^c + P_t \tau_t. \tag{19}$$

The fiscal policy regime is characterized by the following simple rule which relates expenditures g and debt obligations to their tax receipts and transfers from the central bank:

$$\kappa_t P_t g_t + i_t B_{t-1} = P_t \left(\tau_t + \tau_t^c\right). \tag{20}$$

The policy variable κ_t decides on the portion of government expenditures not covered by public debt. By setting κ_t equal to one the fiscal authority, for example, chooses a balanced budget policy. A lower value for the 'fiscal stance' κ_t raises the debt financed fraction of government expenditures such that κ_t can be interpreted as a measure of fiscal responsiveness. To facilitate an analysis of unexpected changes in the fiscal stance, we assume that it follows the stochastic process

$$\log \kappa_t = \log \kappa + \varepsilon_t^\kappa, \qquad \text{with } \kappa \in (0, 1), \tag{21}$$

where ε_t^κ has an expected value of zero and is serially uncorrelated.[8] Similarly, government expenditures are assumed to be exogenous and to follow a first order autoregressive process,

$$\log g_t = \log g + \varepsilon_t^g, \qquad \text{with } g \in (0, c), \tag{22}$$

where the shock ε_t^g has an expected value of zero and is serially uncorrelated. It should be noted that public consumption is assumed not to exceed private consumption in the long-run (see 22). Using the fiscal policy rule (20), and the budget constraint (19) leads to the following consolidated public sector budget constraint

$$B_t = (1 - \kappa_t) P_t g_t + B_{t-1}. \tag{23}$$

Given that the sequences of κ_t and g_t are, by (21) and (22), stationary, it can immediately be seen from (23) that our specification of the public sector implies that public debt B grows asymptotically with a rate smaller than the gross nominal interest rate ($1 < R_t$). The reason is that our fiscal policy rule (20) demands that all debt interest payments are financed by taxes or transfers. As a consequence, solvency of the public sector is guaranteed, i.e., $\lim_{i \to \infty} E_t B_{t+i} \prod_{v=1}^{i} (1 + i_{t+v})^{-1} = 0$ is always satisfied, such that our specification of public policy is Ricardian (see Woodford 2002) or 'well posed' (see Buiter, 2002).

Rational expectations equilibrium Given that the initial price level P_0 as well as the initial stock of nominal financial wealth A_0 is given, real financial wealth $a_0 = A_0/P_0$ is a predetermined variable. In equilibrium private debt is equal to zero $d_t = 0$ such that households' financial wealth solely consists of government bonds $a_t = b_t$. As we will disregard

[8] Note that κ_t is not restricted to lie between zero and one.

the case where the cash constraint is not binding ($R_t = 1$), we define the equilibrium of our model only for the case where $R_t > 1$.

Definition 1 *A rational expectations equilibrium of the model with $R_t > 1$ is a set of sequences $\{\lambda_t,\ \eta_t,\ c_t,\ l_t,\ y_t,\ \pi_t,\ mc_t,\ w_t,\ a_t,\ m_t,\ b_t^c,\ b_t,\ R_t^d \equiv 1 + i_t^d,\ R_t \equiv 1 + i_t,\ g_t,\ \kappa_t\}_{t=0}^{\infty}$ satisfying the households' first order conditions (7), (8), (9), and (10) and*

$$c_t = m_t, \tag{24}$$

$$m_t = b_t^c / R_t, \tag{25}$$

$$b_t^c = \begin{cases} b_t & \text{if} \quad \eta_t > 0 \\ c_t R_t & \text{if} \quad \eta_t = 0 \end{cases}, \tag{26}$$

the aggregate production function, $y_t = l_t$, and the firms' first order conditions (16) and (17), the consolidated public sector budget constraint (23), the monetary policy rule (18), and the law of motion for the fiscal stance κ_t (21) and for government expenditures (22), market clearance, such that the aggregate resource constraint ($y_t = c_t + g_t$) and $a_t = b_t$ holds, for a given initial condition a_0 and the terminal condition (15).

3 Fundamental properties

In this section we present fundamental properties of the model, which will be used for the analysis of cyclical effects of monetary and fiscal policy in the subsequent section. We derive a linear version of the model by log-linearizing the equilibrium conditions listed in definition 1 at the steady state. The complete set of steady state conditions can be found in appendix 6.1. For the model 's local dynamics and for the interaction between monetary and fiscal policy is it crucial whether the open market constraint is binding or not. However, this distinction is irrelevant for the long-run relation between the inflation, and the growth rate of money and real output, as both versions are consistent with McCandless and Weber's (1995) 'monetary facts'. The steady state conditions (see appendix 6.1), particularly imply that, in contrast to overlapping generation models (see, e.g., Leith and Wren-Lewis, 2000), real government liabilities and, thus, the rate of inflation do not affect private consumption in the long run equilibrium.

When open market operations are irrelevant Rather than to apply a comprehensive approach allowing for a joint analysis of both versions, we treat them separately for simplicity. We are primarily interested in the dynamic behavior of the log-linear approximation to the model in a small neighborhood of a steady state where the open market constraint is binding. However, before we turn to the latter case, we start with a brief characterization of the log-linearized version where assets traded in open market operations are not restricted by (2). In this case, open market operations are irrelevant for households' optimal decisions ($\eta_t = 0$), as can be seen from (14). As a consequence, the nominal interest rates on government bonds and private debt are identical and the amount of securities B_t^c can be recursively determined by $b_t^c = c_t R_t$ (see 24-26). Given that the policy variable κ_t, which governs the ratio of tax to debt financing, exclusively enters the government budget constraint (23) and real wealth does not affect the remaining variables,[9] it can immediately be concluded that κ_t is irrelevant for the equilibrium sequences of the remaining variables. Hence, Ricardian equivalence holds in this version, as in the majority of general equilibrium business cycle models with lump-sum taxes. This result and a simplified representation of the rational expectations equilibrium is presented in the following proposition.

Proposition 1 *Suppose that the open market operations are irrelevant ($\eta_t = 0$). Then Ricardian equivalence holds and the rational expectations equilibrium of the log-linear approximation to the model at the steady state can be reduced to a set of sequences $\{\widehat{c}_t, \widehat{\pi}_t, \widehat{R}_t, \widehat{a}_t\}_{t=0}^{\infty}$ satisfying*

$$\sigma\widehat{c}_t = \sigma E_t\widehat{c}_{t+1} - \widehat{R}_t + E_t\widehat{\pi}_{t+1}, \tag{27}$$

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \gamma_1\widehat{c}_t + \gamma_2\varepsilon_t^g, \tag{28}$$

$$\widehat{R}_t = \rho_\pi\widehat{\pi}_t + \varepsilon_t^r, \tag{29}$$

$$\widehat{a}_t = \pi^{-1}\widehat{a}_{t-1} - \pi^{-1}\widehat{\pi}_t + (1-\kappa)\frac{g}{a}\varepsilon_t^g - \kappa\frac{g}{a}\varepsilon_t^\kappa, \tag{30}$$

$$\gamma_1 \equiv \chi[\sigma + \vartheta\frac{c}{c+g}] > 0, \quad \gamma_2 \equiv \chi\vartheta\frac{g}{c+g} \geq 0, \quad \sigma \equiv -\frac{\overline{u}_c}{\overline{u}_{cc}\overline{c}} > 0, \quad \vartheta \equiv \frac{\overline{u}_l}{\overline{u}_{ll}\overline{l}} > 0,$$

together with the transversality condition and a given initial value for real wealth a_0.

Proof. Log-linearizing the equilibrium conditions given in definition 1 for $\eta_t = 0$ and simplifying gives (27)-(30). The equilibrium sequences for $\widehat{c}_t$, $\widehat{\pi}_t$, and $\widehat{R}_t$ are determined by (27)-(29), while the equilibrium sequence for $\widehat{a}_t$ results from (30). Ricardian equivalence follows immediately from the fact that the ratio of deficit financed expenditures (κ and ε_t^κ) does not enter (27)-(29). *Q.E.D.*

The three equations (27)-(29) solely determine the equilibrium sequences of consumption, inflation, the nominal interest rate, and government expenditures. Real financial wealth a_t, which is a predetermined variable, does not affect these variables and can be recursively determined by (30) for given equilibrium sequences of inflation, government expenditures, and shocks ε_t^κ. The model consisting of the subset (27)-(29) is isomorphic with the consensus monetary business cycle model, the so-called New Keynesian model (see, e.g., Clarida et al., 1999, or Woodford, 2001), with government expenditures. Given that the fiscal stance κ_t only matters for the evolution of real financial wealth such that public financing is irrelevant for the effects of monetary and government spending shocks, and that the transmission of the latter can be found in the literature (see Canzoneri et al., 2002a, and Linnemann and Schabert, 2003), we abstain from a further investigation of this version.

When open market operations matter Now we turn to the version where households internalize that their access to money is restricted by the open market constraint (6). The corresponding Lagrange multiplier is larger than zero whenever the spread between the nominal interest rates on private debt and on government bonds is expected to be positive (see 10). When the spread is equal to zero, the model reduces to the one presented in proposition 1.

[9] This property relates to the 'real bond indeterminacy' in standard models (see Canzoneri and Diba, 2000).

Given that we are interested in the model's local dynamics, it is sufficient for our purpose to examine the case where the steady state exhibits a positive spread. The following proposition presents the particular steady state restriction.

Proposition 2 *The open market constraint is binding in the steady state ($\eta > 0$) if and only if the central bank sets the nominal interest rate on government bonds such that*

$$R < \overline{R}, \qquad \text{with} \quad \overline{R} \equiv \beta^{-1} + (1-\kappa)g/c. \tag{31}$$

Proof. Suppose that $R \geq \overline{R}$ and that the open market constraint is binding ($\eta > 0$) such that $a = Rc$. Using the steady state conditions on R^d and on π (see appendix 6.1), $R \geq \overline{R}$ can then be rewritten as $0 \geq \left(R^d - R\right)[1 - (1-\kappa)g/(Rc)]$. Given that the term in the square brackets is strictly positive and that R cannot exceed R^d, the spread must be equal to zero ($R^d = R$). The steady state condition $\eta R^d/\lambda = R^d - R$ then demands $\eta = 0$, which contradicts the initial assumption implying $R < \overline{R} \Leftrightarrow \eta > 0$. *Q.E.D.*

In what follows we assume that the support of ε_t^g, ε_t^r, and ε_t^κ is sufficiently small such that, first, the log-linearization at the steady state with $R < \overline{R}$ is a suitable approximation of the non-linear model and that, second, the nominal interest rate R_t^d always exceeds R_t. Hence, open market operations matter ($\eta_t > 0$) and private consumption is determined by $c_t = a_t/R_t$ (see 24-26). The consumption Euler equation, $\sigma \widehat{c}_t = \sigma E_t \widehat{c}_{t+1} - \widehat{R}_t^d + E_t \widehat{\pi}_{t+1}$, is now irrelevant for the determination of the equilibrium sequences of consumption, inflation, and real wealth. It only serves as an equilibrium condition which can recursively be applied to determine the equilibrium sequence for the interest rate on private debt. The following proposition presents the log-linearized model.

Proposition 3 *Suppose that the open market constraint is binding ($\eta_t > 0$). Then the rational expectations equilibrium of the log-linear approximation to the model at the steady state with $R < \overline{R}$ is a set of sequences $\{\widehat{c}_t, \widehat{\pi}_t, \widehat{R}_t, \widehat{a}_t\}_{t=0}^\infty$ satisfying (29) and*

$$\widehat{c}_t = \widehat{a}_t - \widehat{R}_t \tag{32}$$

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \gamma_1 \widehat{a}_t - \gamma_1 \widehat{R}_t + \gamma_2 \varepsilon_t^g \tag{33}$$

$$\widehat{a}_t = \pi^{-1} \widehat{a}_{t-1} - \pi^{-1} \widehat{\pi}_t + (1-\kappa)\frac{g}{a}\varepsilon_t^g - \kappa \frac{g}{a}\varepsilon_t^\kappa \tag{34}$$

together with the transversality condition and a given initial value for real wealth a_0.

Proof. Given that $R < \overline{R}$, the spread is positive such that $\eta > 0$ and, thus, $\eta_t > 0$ holds at the steady state. Hence, in addition to the conditions (28)-(30), the equality $\widehat{c}_t = \widehat{a}_t - \widehat{R}_t$, which results from linearizing and simplifying (24)-(26) and $b_t = a_t$, must be satisfied in equilibrium. Eliminating $\widehat{c}_t$ in (28) then gives (33). *Q.E.D.*

As can be seen from proposition 3, real wealth $\widehat{a}_{t-1}$ constitutes a relevant endogenous state variable of the model, while the rate of inflation and the nominal interest rate can jump. The equilibrium path of the model is therefore stable and unique (saddlepoint stable) if there is exactly one eigenvalue lying inside the unit circle. It turns out that the assumptions made in the previous section ensure real determinacy. This result is summarized in the following proposition.

Proposition 4 *Suppose that the open market constraint is binding ($\eta_t > 0$). Then the model exhibits a unique rational expectations equilibrium path converging to the steady state.*

Proof. The linear model presented in proposition 3 reads in deterministic form:

$$M_1 \begin{pmatrix} \widehat{a}_t \\ \widehat{\pi}_{t+1} \end{pmatrix} = M_0 \begin{pmatrix} \widehat{a}_{t-1} \\ \widehat{\pi}_t \end{pmatrix}, \qquad \text{with } M_1 \equiv \begin{pmatrix} 1 & 0 \\ \gamma_1 & \beta \end{pmatrix} \text{ and } M_0 \equiv \begin{pmatrix} \pi^{-1} & -\pi^{-1} \\ 0 & 1+\gamma_1\rho_\pi \end{pmatrix}.$$

For the analysis of the stability conditions we examine the characteristic polynomial of $M_1^{-1}M_0$ given by $F(X) = X^2 - \frac{\gamma_1+\pi+\pi\gamma_1\rho_\pi+\beta}{\pi\beta}X + \frac{1+\gamma_1\rho_\pi}{\pi\beta}$. We want to show that exactly one root of $F(X)$ lies between zero and one. For this we examine $F(0)$, which is strictly positive: $F(0) = \frac{1+\gamma_1\rho_\pi}{\beta\pi} > 0$. For $X = 1$, we obtain $F(1) = -\frac{1}{\beta\pi}(\gamma_1 + (\pi-1)(1-\beta+\gamma_1\rho_\pi)) < 0$ given that $\pi \geq 1$. Hence, one root of $F(X)$ lies between zero and one, whereas the other root exceeds one. *Q.E.D.*

Hence, stability and uniqueness of the rational expectations equilibrium is guaranteed. It should, however, be noted that the specific choice of the fiscal policy rule (20) is responsible for this results. In particular, when debt obligations are not completely tax financed, as implied by the fiscal policy rule (20), debt-interest spirals, as discussed in Leith and Wren-Lewis (2000), can occur for an aggressive interest rate setting. Remarkably, the Taylor principle ($\rho_\pi > 1$) which is necessary in standard New Keynesian models for real determinacy (see Clarida et al., 1999, or Woodford 2001), is irrelevant in this model.

4 Monetary and fiscal policy interaction

In this section we examine the responses to monetary and fiscal policy shocks for the version with a binding open market constraint ($\eta_t > 0$) described in proposition 3. Given the state space solution of the model, which is presented in the first part of this section, we investigate the impulse responses and changes thereof with respect to variations of policy parameter. In particular, we are interested in how fiscal policy affects the model's responses to a monetary policy shock and vice versa. We abstain from presenting the policy effects in the model's version where open market operations are irrelevant, as it lacks any interaction of monetary policy and public financing (see proposition 1), while the effects of fiscal policy shocks herein can be found in the literature (see, e.g., Linnemann and Schabert, 2003).

Fundamental solution Given that the model consists of one endogenous ($\widehat{a}_{t-1}$) and three exogenous states ($\varepsilon_t^g, \varepsilon_t^r, \varepsilon_t^g$), the state space representation of the model in a, π, and g reads

$$\widehat{a}_t = \delta_a \widehat{a}_{t-1} + \delta_{ag}\varepsilon_t^g + \delta_{ar}\varepsilon_t^r + \delta_{a\kappa}\varepsilon_t^\kappa, \tag{35}$$

$$\widehat{\pi}_t = \delta_{\pi a} \widehat{a}_{t-1} + \delta_{\pi g}\varepsilon_t^g + \delta_{\pi r}\varepsilon_t^r + \delta_{\pi\kappa}\varepsilon_t^\kappa, \tag{36}$$

As shown in proposition 4, the coefficient δ_a is the single stable eigenvalue of the model and lies between zero and one ($0 < \delta_a < 1$). The remaining coefficients in (35)-(36), which are derived by applying the method of undetermined coefficients (see, e.g., Uhlig, 1999), are presented in the following lemma.

Lemma 1 *The state space representation (35)-(36) of the model with $\eta_t > 0$ is characterized by coefficients δ_i with $i \in \{a,\, ag,\, ar,\, a\kappa,\, \pi a,\, \pi g,\, \pi r,\, \pi\kappa\}$ satisfying*

1. $0 < \delta_a = \frac{1}{2\beta\pi}(\alpha_1 - \alpha_2^{1/2}) < 1/\pi$ *and* $0 < \delta_{\pi a} = 1 - \delta_a \pi < 1$,
2. $\delta_{ag} = [\gamma_3 \pi (1-\kappa)\frac{g}{a} - \gamma_2]/\gamma_4 \lesseqgtr 0$ *and* $\delta_{\pi g} = \pi[(\beta\,(1-\delta_a\pi) + \gamma_1)\,(1-\kappa)\frac{g}{a} + \gamma_2]/\gamma_4 > 0$,
3. $\delta_{ar} = \gamma_1/\gamma_4 > 0$ *and* $\delta_{\pi r} = -\gamma_1 \pi/\gamma_4 < 0$,
4. $\delta_{a\kappa} = -\kappa\pi\frac{g}{a}\gamma_3/\gamma_4 < 0$ *and* $\delta_{\pi\kappa} = -\kappa\pi\frac{g}{a}[\gamma_1 + \beta(1-\delta_a\pi)]/\gamma_4 < 0$,

with $\alpha_1 \equiv \pi\gamma_3 + \beta + \gamma_1 > 1$, $\alpha_2 \equiv \alpha_1^2 - 4\beta\pi\gamma_3 > 0$, $\gamma_3 \equiv 1 + \gamma_1\rho_\pi > 1$, *and* $\gamma_4 \equiv \gamma_3\pi + \beta\,(1-\delta_a\pi) + \gamma_1 > 1$.

Proof. See appendix 6.2.

Once the coefficients are identified, we can easily examine the effects of monetary and fiscal policy shocks. Rather than restricting the attention on the cyclical behavior of inflation and real wealth, we will also examine the responses of consumption and output. The corresponding coefficients are derived from $\widehat{c}_t = \widehat{a}_t - \widehat{R}_t = \widehat{a}_t - \rho_\pi\widehat{\pi}_t - \varepsilon_t^r$ and $\widehat{y}_t = \frac{c}{c+g}\widehat{c}_t + \frac{g}{c+g}\varepsilon_t^g$.

Interest rate shocks We start our policy analysis with the case where a nominal interest rate shock hits the economy in period s, while the realizations of the remaining stochastic variables are equal to zero ($\varepsilon_t^g = \varepsilon_t^\kappa = 0\ \forall t$). In particular, we assume that the sequence of interest rate rule innovations ε^r satisfies $\varepsilon_s^r > 0$ and $\varepsilon_t^r = 0\ \forall t : t \neq s$. The following proposition summarizes the results.

Proposition 5 *A positive innovation to the interest rate rule in period s leads to a decline in inflation ($\partial\widehat{\pi}_s/\partial\varepsilon_s^r < 0$), a rise in real wealth ($\partial\widehat{a}_s/\partial\varepsilon_s^r = \delta_{ar} > 0$), and to a decline in consumption and output ($\partial\widehat{c}_s/\partial\varepsilon_s^r$, $\partial\widehat{y}_s/\partial\varepsilon_s^r < 0$).*

Proof. The first two claims made in the proposition immediately follow from $\partial\widehat{\pi}_s/\partial\varepsilon_s^r = \delta_{\pi r} < 0$ and $\partial\widehat{a}_s/\partial\varepsilon_s^r = \delta_{ar} < 0$ (see in part 3 of lemma 1). As $\widehat{c}_t = \widehat{a}_t - \rho_\pi\widehat{\pi}_t - \varepsilon_t^r$ holds in equilibrium, the impact effect on consumption is given by $\partial\widehat{c}_s/\partial\varepsilon_s^r = \delta_{ar} - \rho_\pi\delta_{\pi r} - 1$. Applying the results in part 3 of lemma 1, we obtain $\partial\widehat{c}_s/\partial\varepsilon_s^r = (\gamma_1 + \rho_\pi\gamma_1\pi - \gamma_4)/\gamma_4 = -(\pi + \beta\,(1-\delta_a\pi))/\gamma_4 < 0$ and, thus, $\partial\widehat{y}_s/\partial\varepsilon_s^r = \frac{c}{c+g}\partial\widehat{c}_s/\partial\varepsilon_s^r < 0$. *Q.E.D.*

As summarized in proposition 5, an unanticipated rise in the nominal interest rate causes a decline in inflation, consumption, and, therefore, in output. These responses qualitatively accord to the predictions of the standard New Keynesian model and are consistent with common priors about monetary policy effects. The model further predicts that real wealth rises due to a decline in the price level and eases, ceteris paribus, households' access to money. Subsequent to the impact period s, real wealth, which evolves sluggishly according to (34), is temporarily higher than in the long-run equilibrium which is responsible for a smooth recovery of consumption and inflation. In contrast to the standard New Keynesian model, which exhibits no endogenous state variable and, thus, lacks any propagation mechanism, our model predicts that the endogenous variables do not immediately return to the steady state when a transitory monetary policy shock disappears. The persistence of monetary policy effects is governed by the stable eigenvalue δ_a, which depends – inter alia – on the parameter κ and ρ_π describing the reactiveness of fiscal and monetary policy. While a rise in the fiscal stance κ raises the eigenvalue δ_a, a higher inflation elasticity ρ_π leads to the opposite effect. These findings are summarized in the following proposition.

Proposition 6 *The eigenvalue δ_a and, therefore, the persistence of impulse responses*

1. *increases with the fiscal responsiveness κ ($\partial\delta_a/\partial\kappa > 0$) and*
2. *decreases with the inflation elasticity ρ_π ($\partial\delta_a/\partial\rho_\pi < 0$).*

Proof. To establish the first claim, we use that κ affects the solution only via its negative effect on the steady state inflation rate ($\partial\pi/\partial\kappa < 0$) and that the stable eigenvalue is given by $\delta_a = \frac{1}{2\beta\pi}(\alpha_1 - \alpha_2^{1/2})$ (see part 1 of lemma 1). Applying the definitions for α_1 and α_2, the partial derivative of the eigenvalue with respect to inflation is $\partial\delta_a/\partial\pi = [\pi\gamma_3\alpha_3 - (\alpha_1 - \alpha_2^{1/2})]/(2\beta\pi^2)$, with $\alpha_3 \equiv 1 - \alpha_2^{-1/2}(\alpha_1 - 2\beta) < 0$. Given that $(\alpha_1 - \alpha_2^{1/2}) > 0$ and that α_3 can easily be shown to be negative, we can conclude that $\partial\delta_a/\partial\pi < 0$ and, thus, $\partial\delta_a/\partial\kappa > 0$. The partial derivative of δ_a with respect to the inflation elasticity ρ_π is further given by $\delta_a/\partial\rho_\pi = \gamma_1\alpha_3/(2\beta) < 0$, which establishes the second claim. *Q.E.D.*

While the propagation of monetary policy shocks is often discussed in the literature, it is obvious that changes in the eigenvalue δ_a also affect the persistence of other shocks effects. The reason for the fiscal stance κ to increase the persistence is due to the fact that a lower mean value for the fiscal stance κ directly raises the steady state inflation rate. Inspecting the linearized equilibrium condition (34) reveals that the inverse of the latter weights the backward dependence of real wealth. Hence, whenever real wealth deviates from its steady state value its recovery is, ceteris paribus, more pronounced for higher steady state inflation rates. The effect of the inflation elasticity ρ_π on persistence, which does not operate via on changes in steady state values, can easily be understood for the current experiment of a positive interest rate shock. For a higher inflation elasticity ρ_π the decline in inflation triggered by the contractionary monetary policy shock causes the central bank to adjust the nominal interest rate more strongly mitigating the initial interest rate hike. Hence, a higher reactiveness of interest rate policy leads to a faster recovery to the long-run equilibrium in the subsequent periods.

Government expenditure shocks We now turn to the experiment where a government expenditure shock hits the economy in period s ($\varepsilon_s^g > 0$, $\varepsilon_t^g = 0$ $\forall t : t \neq s$, and $\varepsilon_t^g = \varepsilon_t^\kappa = 0$ $\forall t$). The solutions for the coefficients presented in lemma 1 reveal that inflation always rises in response to a positive innovation to government expenditures, while the response of output depends on the reactiveness of the interest rate rule. The result is summarized in the following proposition.

Proposition 7 *A positive government expenditure shock in period s leads to a rise in*

1. *inflation* ($\partial\widehat{\pi}_s/\partial\varepsilon_s^g > 0$), *and in*
2. *output* ($\partial\widehat{y}_s/\partial\varepsilon_s^g > 0$) *if* $\rho_\pi < \overline{\rho_{\pi 1}}$, *with* $\overline{\rho_{\pi 1}} \equiv [\beta(1-\delta_a\pi)]^{-1} > 1$.

Proof. The first claim immediately follows from the sign restriction for $\delta_{\pi g} = \partial\widehat{\pi}_s/\partial\varepsilon_s^g$ given in part 2 of lemma 1. The impact multiplier on output is given by $\partial\widehat{y}_s/\partial\varepsilon_s^g = \delta_{yg} = \frac{c}{c+g}\delta_{cg} + \frac{g}{c+g}$. Using $\delta_{cg} = \delta_{ag} - \rho_\pi\delta_{\pi g}$ and the definitions for γ_1, γ_2, γ_3, and γ_4, we obtain $\delta_{yg} = \frac{g}{c+g}\frac{1}{c\gamma_4}\left[\pi(1-\kappa)\frac{1}{R}\left[1-\rho_\pi\beta\left(1-\delta_a\pi\right)\right] + \pi + \chi\sigma\left(1+\rho_\pi\pi\right) + \beta\left(1-\delta_a\pi\right)\right]$. Hence, for a rise in $\widehat{y}_t$ a moderate inflation elasticity $\rho_\pi < [\beta(1-\delta_a\pi)]^{-1}$ is sufficient, where the upper bound is strictly larger than one given that $\beta < 1$ and $\delta_a\pi < 1$. *Q.E.D.*

A comparison with Linnemann and Schabert (2003) reveals that both results presented in proposition 7, are consistent with the findings on fiscal policy effects in a standard New Keynesian model, i.e., in the version presented in proposition 1 with irrelevant open market operations. The rise in government consumption leads to a price pressure by (33) and tends to raise aggregate demand. However, the rise in the price level has an adverse effect on aggregate demand when the central bank endogenously responds to higher inflation by raising the nominal interest rate. Hence, a highly aggressive monetary policy reaction ($\rho_\pi > \overline{\rho_{\pi 1}}$) can cause a decline in aggregate demand in response to a government expenditure shock.

This result immediately leads to the probably most interesting effect of government spending shocks, i.e., the response of private consumption. Given that consumption rises with real financial wealth and declines with the nominal interest rate (see 32), a positive consumption response obviously requires that prices are not too flexible. Hence, the sign of the impact multiplier on consumption does not only to depend on the monetary stance ρ_π, but also hinges on the degree of price rigidity χ and on the fiscal reactiveness κ. When government expenditures are highly debt financed and prices are not too flexible, real wealth can rise such that households can afford to consume more. Hence, a rise in real wealth requires prices to be sufficiently rigid (low χ) and a fiscal stance not to be too reactive (low κ), while a rise in private consumption further demands a moderately reactive interest rate policy (low ρ_π). The following proposition summarizes the results.

Proposition 8 *Suppose that prices are sufficiently rigid* $\chi \leq \overline{\chi}$, *with* $\overline{\chi} \equiv \beta(c+g)/(\vartheta c)$. *Then a positive innovation to government expenditures in period s leads to a rise in*

1. *real wealth* ($\partial\widehat{a}_s/\partial\varepsilon_s^g > 0$) *if* $\kappa < \overline{\kappa}_1$, *with* $\overline{\kappa}_1 \in (0,1)$, *and in*
2. *consumption* ($\partial\widehat{c}_s/\partial\varepsilon_s^g > 0$) *if* $\kappa < \overline{\kappa}_2$, *with* $\overline{\kappa}_2 \in (0,\overline{\kappa}_1)$ *and* $\rho_\pi < \overline{\rho_{\pi 2}}$

with $\overline{\kappa}_1 \equiv 1 - \frac{\gamma_2 c/(g\beta)}{1+\gamma_1\rho_\pi}$, $\overline{\kappa}_2 \equiv 1 - \frac{\gamma_2 c(1+\rho_\pi)/(g\beta)}{1-\rho_\pi[\beta(1-\delta_a\pi)+\gamma_2]}$, *and* $\overline{\rho_{\pi 2}} \equiv \frac{1-c\gamma_2/(g\beta)}{\beta(1-\delta_a\pi)+\gamma_2[1+c/(g\beta)]} > 0$.

Proof. To establish the first claim, we consider that $\partial \widehat{a}_s / \partial \varepsilon_s^g = \delta_{ag} = [\frac{\gamma_3(1-\kappa)}{Rc/g-(1-\kappa)} - \gamma_2]/\gamma_4$. Using that R is assumed to exceed $\overline{R}$ (see proposition 2 and 3), we obtain $\kappa < \overline{\kappa}_1 \leq 1$ as a sufficient condition for $\delta_{ag} > 0$. To establish the second claim, suppose that $\rho_\pi < \widetilde{\rho_\pi}$, with $\widetilde{\rho_\pi} \equiv [\beta(1-\delta_a \pi) + \gamma_2]^{-1}$. Using the upper bound $\overline{R}$ and $\pi^{-1} = 1 - (1-\kappa)\frac{g}{cR}$, it can easily be shown that $\partial \widehat{c}_s / \partial \varepsilon_s^g = \delta_{cg} = \left[\pi(1-\kappa)\frac{g}{a}\left[1 - \rho_\pi \beta\left(1 - \delta_a \pi\right)\right] - \left(1 + \rho_\pi \pi\right)\gamma_2\right]/\gamma_4$ is positive in this case if but not only if $\kappa < \overline{\kappa}_2$. Further, $\rho_\pi < \overline{\rho_\pi}_2$ $(< \widetilde{\rho_\pi})$ ensures that $0 \leq \overline{\kappa}_2 \leq \overline{\kappa}_1$, while $\chi \leq \beta(c+g)/(\vartheta c)$ guarantees that $\overline{\kappa}_1$ and $\overline{\rho_\pi}_2$ are non-negative. *Q.E.D.*

The result presented in the second part of proposition 8 is, in particular, remarkable as several empirical studies (see, e.g., Fatas and Mihov, 2001, Blanchard and Perotti, 2002) find that private consumption rises in response an fiscal expansion. While standard business cycle theory fails to reproduce the rise in private consumption (see, e.g., Canzoneri et al., 2002a, or, Linnemann and Schabert, 2003). As shown by Baxter and King (1993), in a neoclassical-style business cycle model government expenditures induce households to reduce leisure (to raise labor supply) by intertemporal substitution, which is accompanied by an increased willingness to postpone private consumption. In contrast, the novel wealth effect in our model has in fact the potential to solve this 'puzzle' when prices are sufficiently rigid.[10] The fundamental difference is that the response of private consumption is determined by the response of real wealth and of the nominal interest rate (see 32), whereas the consumption Euler equation, which governs the consumption response in a standard model, just residually determines the nominal interest rate on private debt R_t^d. The results presented in proposition 7 and 8 further provide a rationale for responses to government expenditure shocks hardly being robust for different countries and over time (see, e.g., Perotti, 2002).

Temporary fiscal consolidation In this subsection we check for the effects of shock to the parameter κ which governs the fiscal stance. In particular, we consider a positive κ_s shock ($\varepsilon_s^\kappa > 0$, $\varepsilon_t^\kappa = 0$ $\forall t : t \neq s$, and $\varepsilon_t^g = \varepsilon_t^r = 0$ $\forall t$), which can be interpreted as a temporary fiscal consolidation. Obviously, this policy experiment emphasizes the primary difference to a conventional business cycle models. Given that the latter are regularly characterized by the validity of Ricardian equivalence (see proposition 1), a switch from deficit to tax financing has no further impact on the allocation in these models. This result is clearly at odds with recent empirical evidence of Mountford and Uhlig (2002), showing that the most significant fiscal stimulus is brought about a deficit financed tax cuts. Our model, in fact, allows to reproduce this result ($\kappa_t \downarrow \Rightarrow y_t \uparrow$) when open market operations are relevant. The following proposition summarizes the effects a temporary fiscal consolidation.

Proposition 9 *An unanticipated and temporary fiscal consolidation in period s leads to a decline in*

1. *inflation ($\partial \widehat{\pi}_s / \partial \varepsilon_s^\kappa < 0$) and real financial wealth ($\partial \widehat{a}_s / \partial \varepsilon_s^\kappa < 0$), and in*

[10] Note that the constraint $\chi \leq \overline{\chi}$ is hardly restrictive for a conventional set of parameter values (e.g., $\sigma = \vartheta = 2$, $\beta = 0.99$, $g/y = 0.2$, $\phi = 0.8$, $\Rightarrow \chi = 0.052$ and $\overline{\chi} = 0.61875$).

2. *consumption and output* ($\partial\widehat{c}_s/\partial\varepsilon_s^\kappa$, $\partial\widehat{y}_s/\partial\varepsilon_s^\kappa < 0$) *if and only if* $\rho_\pi < \overline{\rho_{\pi 1}}$.

Proof. The claims made in the first part immediately follow from $\partial\widehat{\pi}_s/\partial\varepsilon_s^\kappa = \delta_{\pi\kappa} < 0$ and $\partial\widehat{a}_s/\partial\varepsilon_s^\kappa = \delta_{a\kappa} < 0$ (see part 4 of lemma 1). For the second part, we use that $\partial\widehat{c}_s/\partial\varepsilon_s^\kappa = \frac{c+g}{c}\partial\widehat{y}_s/\partial\varepsilon_s^\kappa = \delta_{c\kappa} = \delta_{a\kappa} - \rho_\pi\delta_{\pi\kappa}$. Applying the solutions for $\delta_{a\kappa}$ and $\delta_{\pi\kappa}$ yields $\delta_{c\kappa} = -\frac{\kappa}{\pi}\frac{g}{Rc}[1-\rho_\pi\beta(1-\delta_a\pi)]/\gamma_4$, which is strictly positive for $\rho_\pi < [\beta(1-\delta_a\pi)]^{-1} = \overline{\rho_{\pi 1}}$, with $\overline{\rho_{\pi 1}} > 1$ (see proposition 7). *Q.E.D.*

A rise in fiscal responsiveness κ_t is, by (23), associated with a decline in nominal government bonds outstanding, accompanied by a decline in inflation and real government debt given that prices are sticky. Thus, consumption and output tends, by (32), to decline as long as interest rate policy is not too reactive.. For high inflation elasticities ($\rho_\pi > \overline{\rho_{\pi 1}}$), the decline in inflation can cause the central bank to lower the nominal interest rate in an extreme way such that households might be willing to increase consumption expenditures, even though real wealth declines. In other words, a deficit financed tax cut can stimulate real activity when the reactiveness of monetary policy is moderate ($\rho_\pi < \overline{\rho_{\pi 1}}$).

5 Conclusion

We developed a simple business cycle model where the central bank supplies money via repurchase agreements and sets the repo rate. We allow for a non-negligible role of open market operations by assuming that private debt is not accepted as a collateral for money. When households internalize this restriction, demand for government bonds depends on the monetary policy stance if private debt earn a higher interest. Otherwise, the model is isomorphic to the conventional New Keynesian model. While the latter has proven is usefulness for monetary policy analysis, it is hardly able to explain why government expenditures or deficit financed tax cut are found to be able to stimulate private consumption, or why public debt matters for monetary policy. Obviously, these shortcomings are primarily founded in the validity of Ricardian equivalence.

When agents care about open market operations, Ricardian equivalence breaks down as government bonds provide liquidity services. Real financial wealth serves as a relevant predetermined state variable, which stabilizes the model's dynamics such that interest rate policy is not restricted by the requirements for real determinacy. Further, the structural part of monetary and fiscal policy is shown to affect the stable eigenvalue and, thus, the persistence of impulse responses. Interest rate shock effects are consistent with common priors, while government expenditure and tax cut shock are able to stimulate output as well as private consumption when the central bank is not too reactive. Otherwise, interest rate adjustments triggered by higher inflation can lead to a monetary tightening which prevails over expansionary fiscal policy impulses. Hence, our novel mechanism for fiscal and monetary policy to interact via liquidity services of government bonds, is able to generate policy effects qualitatively consistent with recent empirical findings.

6 Appendix

6.1 Steady state

The model's steady state consists of stationary values λ, c, π, a, m, R^d, and η satisfying:

$$\frac{u_c(c)}{-u_l(c)} = \frac{\epsilon}{\epsilon-1}; \quad c = m; \quad R^d = \frac{\pi}{\beta}; \quad \pi = [1-(1-\kappa)\frac{g}{cR}]^{-1} \geq 1;$$

$$\frac{\eta}{\lambda} = \frac{R^d - R}{R^d}, \text{ with } \eta \geq 0; \quad \lambda = \begin{cases} u_c(c)\,[R\,(1+\eta/\lambda)]^{-1} & \text{if } \eta > 0 \\ u_c(c)R^{-1} & \text{if } \eta = 0 \end{cases}; \quad m = \begin{cases} a/R & \text{if } \eta > 0 \\ c & \text{if } \eta = 0 \end{cases},$$

while g and κ are exogenously determined by fiscal policy (see 21 and 22), and R is determined by (18) if $\eta > 0$, or by $R = R^d$ if $\eta = 0$.

6.2 Proof of lemma 1

The equilibrium conditions of the model with a binding open market constraint given in proposition 3 can be reduced to

$$\widehat{a}_t = \pi^{-1}\widehat{a}_{t-1} - \pi^{-1}\widehat{\pi}_t + (1-\kappa)\frac{g}{a}\varepsilon_t^g - \kappa\frac{g}{a}\varepsilon_t^\kappa \tag{37}$$

$$\beta E_t\widehat{\pi}_{t+1} + \gamma_1\widehat{a}_t = \gamma_3\widehat{\pi}_t - \gamma_2\varepsilon_t^g + \gamma_1\varepsilon_t^r, \qquad \text{with } \gamma_3 \equiv 1 + \gamma_1\rho_\pi > 1 \tag{38}$$

Replacing the endogenous variables in (37) and (38) by using the general solution form given by (35)-(36), leads to the following conditions for the undetermined coefficients

$$\delta_a - \pi^{-1} + \pi^{-1}\delta_{\pi a} = 0, \qquad \gamma_3\delta_{\pi a} - \beta\delta_{\pi a}\delta_a - \gamma_1\delta_a = 0, \tag{39}$$

$$\delta_{ar} + \pi^{-1}\delta_{\pi r} = 0, \qquad \beta\delta_{\pi a}\delta_{ar} + \gamma_1\delta_{ar} - \gamma_1 - \gamma_3\delta_{\pi r} = 0, \tag{40}$$

$$(1-\kappa)g/a - \pi^{-1}\delta_{\pi g} - \delta_{ag} = 0, \qquad \beta\delta_{\pi a}\delta_{ag} - \gamma_3\delta_{\pi g} + \gamma_1\delta_{ag} + \gamma_2 = 0, \tag{41}$$

$$-\delta_{a\kappa} - \pi^{-1}\gamma_{\pi\kappa} - \kappa g/a = 0, \qquad \gamma_1\delta_{a\kappa} + \beta\delta_{\pi a}\delta_{a\kappa} - \gamma_3\delta_{\pi\kappa} = 0. \tag{42}$$

Eliminating $\delta_{\pi a}$ in the two conditions given in (39), leads to the following quadratic equation in eigenvalue $\delta_a : \beta\delta_a^2\pi - (\pi\gamma_3 + \beta + \gamma_1)\,\delta_a + \gamma_3 = 0$. Exactly one root of this equation lies between zero and one (see proposition 4), which is given by

$$0 < \delta_a = \frac{1}{2\beta\pi}\left(\alpha_1 - \sqrt{\left(\alpha_1^2 - 4\beta\pi\gamma_3\right)}\right) < 1, \qquad \text{with } \alpha_1 \equiv \pi\gamma_3 + \beta + \gamma_1 > 1.$$

We can, further, conclude from (39) that the coefficient $\delta_{\pi a}$ is strictly positive, $\delta_{\pi a} = 1 - \pi\delta_a > 0$, given that $\pi\delta_a$ can easily be shown to be smaller than one. Turning to the impact responses to a *monetary policy shock*, the conditions in (40) immediately lead to the solutions for $\delta_{\pi r}$ and δ_{ar} given in part 3 of lemma 1. Rearranging the conditions in (41), gives the solution for the coefficients $\delta_{\pi g}$ and δ_{ag} presented in part 2 of lemma 1, which govern the effects of *government spending shocks*. The solution for remaining coefficients $\delta_{\pi\kappa}$ and $\delta_{a\kappa}$ in part 4 of lemma 1, which refer to the responses to a *fiscal consolidation shock*, immediately follow from the conditions in (42) and $\delta_{\pi a} = 1 - \delta_a\pi$. *Q.E.D.*

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