Sustaining Collusion with Asymmetric Costs

David R. Collie

Cardiff Business School, Cardiff University, Aberconway Building, Colum Drive, Cardiff, CF10 3EU, United Kingdom. (E-mail: Collie@cardiff.ac.uk)

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Abstract

In contrast to conventional wisdom, as formalised by Rothschild (1999), it is shown that cost asymmetries do not make sustaining collusion more difficult. Unlike Rothschild (1999), where the cartel is assumed to maximise total-industry profits, in this paper the cartel can allocate output quotas to ensure that collusion is viable and sustainable in the face of cost asymmetries. Using the concept of balanced temptation from Friedman (1971), where output quotas are allocated so that all firms have the same incentive to defect from the cartel, this paper analyses how cost asymmetries affect the critical discount factor of the cartel.

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1. Introduction

The conventional wisdom, according to many Industrial Organization textbooks, is that cost asymmetries make it more difficult to sustain collusion; for example, in Cabral (2000, p. 138) it is stated prominently that 'Collusion is normally easier to maintain among few and similar firms'.[1] However, this assertion does not really have a strong theoretical basis, although Rothschild (1999) has recently tried to formalise this argument in an infinitely repeated Cournot oligopoly game where firms have different quadratic cost functions. When the firms use trigger strategies as in Friedman (1971) and the cartel allocates output quotas to maximise total-industry profits as in Patinkin (1947), Rothschild (1999) argues that cost asymmetries will make it more difficult for the cartel to sustain collusion. However, this argument ignores the criticism of Patinkin (1947) by Bain (1948). Patinkin (1947) argued that a cartel would maximise total-industry profits and would therefore allocate output quotas as if it was a multi-plant monopolist allocating production between plants. The cartel would allocate output quotas so that the marginal cost of production was the same for all firms, which implies that the cartel minimises the total cost of its output. In the absence of side-payments between firms, Bain (1948) argued that such an allocation might not be viable as inefficient firms may have lower profits in the cartel than in the non-cooperative outcome.[2] Therefore, a cartel would be unlikely to allocate output quotas in the manner suggested by Patinkin (1947) but would take account of the viability of collusion and the bargaining power of the firms.

Of course, Bain (1948) made his comments before the modelling of cartels was formalised in an infinitely repeated game setting by Friedman (1971). He showed that collusion, where all firms could obtain higher profits than in the static Nash equilibrium, could be sustained as a subgame-perfect equilibrium using Nash-reversion trigger strategies provided the discount factor was sufficiently large. To deal with the multiplicity of possible collusive equilibria, Friedman (1971) suggested that firms would choose the 'balanced temptation' equilibrium: an equilibrium where all firms are equally tempted to defect from the cartel and that is on the Pareto-frontier of the feasible profits set. Although balanced temptation has been criticised as a way of selecting a collusive equilibrium by Harrington (1991), it does provide a method to find the lowest discount factor that will sustain collusion on the Pareto-frontier as a subgame-perfect equilibrium. This paper will use balanced temptation to find the critical discount factor of the cartel, which is defined as the lowest discount factor that will sustain collusion on the Pareto-frontier of the feasible profits set, and to analyse how cost asymmetries affect the sustainability of collusion.

In the literature, the sustainability of collusion with asymmetries has been addressed in a number of different oligopoly models. However, the main concern has usually been the selection of a collusive equilibrium given the discount factor rather than finding the lowest discount factor that will sustain collusion, which is the issue addressed in this paper. Bae (1987) and Harrington (1991) consider a homogeneous product Bertrand duopoly model with asymmetric constant marginal costs. While Bae (1987) uses balanced temptation and the maximisation of total-industry profits to select a collusive equilibrium, Harrington (1991) uses Nash bargaining to select from the collusive equilibria that can be sustained (implemented) as subgame-perfect equilibria. Lambson (1994, 1995) and Compte, Jenny and Rey (2002) consider a homogeneous product Bertrand oligopoly model with asymmetric capacity constraints. Compte, Jenny and Rey (2002) show that asymmetric capacities make collusion more difficult when aggregate capacity is limited but make it easier when aggregate capacity is much larger than the size of the market. Verboven (1997) considers a homogeneous product Cournot duopoly with asymmetric constant marginal costs. He considers partial collusion where the profits of the firm are in the interior of the feasible profits set, and employs an equilibrium concept where both firms are equally tempted to defect.[3] In a different setting, Collie (1993) considers a trade policy game between two governments that give export subsidies to Cournot duopolists with asymmetric constant marginal costs. He shows that cost asymmetries make it more difficult for the two governments to sustain free trade (zero export subsidies) using Nash-reversion trigger strategies.[4]

Section two presents the basic Cournot duopoly supergame then clarifies the analysis of Rothschild (1999) by presenting the critical discount factors in a diagram similar to that used by Collie (1993), and making it clear that the inefficient firm has the greatest incentive to defect from the cartel. Section three employs the idea of balanced temptation to find the critical discount factor of the cartel, and to show that cost asymmetries do not make it harder to sustain collusion. Some concluding comments and suggestions for future research are given in section four.

2. The Model

Consider an infinitely repeated Cournot duopoly game with a linear demand function and quadratic cost functions. In the stage game of this supergame, both firms simultaneously and independently choose outputs. The market price of the homogeneous product is given by the inverse demand function: $P = \alpha - \beta\left(q_1 + q_2\right)$, where q_1 is the output of firm one, q_2 is the output of firm two, and the demand parameters are $\alpha, \beta > 0$. The (total) cost function of *i*th firm is $c_i\left(q_i\right) = \gamma_i q_i^2 / 2$ where the cost parameter $\gamma_i > 0$ and $i = 1,2$. Therefore, the marginal cost of the *i*th firm is a linear function of its output and equal to $\gamma_i q_i$, and the *i*th firm is said to be the efficient (inefficient) firm if its cost parameter is smaller (larger) than that of the other firm, $\gamma_i < (>) \gamma_j$.

In the supergame, this stage game is infinitely repeated and firms discount profits in future stages using the discount factor δ, where $0 < \delta \leq 1$. This discount factor is assumed to be the same for both firms and to be the same in all stages, and the stage game is assumed to be identical in all stages. Importantly, as is usual in this literature, it is assumed that side-payments between the two firms are not possible.[5] In such a supergame, Friedman (1971) has shown that collusion can be sustained as a subgame-perfect equilibrium using Nash-reversion trigger strategies, where firms revert to the Cournot equilibrium if any firm defects from the cartel, provided that the discount factor is sufficiently large.[6]

Firstly, consider the non-cooperative Nash equilibrium of the stage game, which is the usual Cournot equilibrium in a static duopoly game. In the stage game, given the linear demand and quadratic cost functions, the profits of the *i*th firm are:

$$\pi_i = Pq_i - c_i(q_i) = \left[\alpha - \beta\left(q_i + q_j\right)\right]q_i - \gamma_i q_i^2/2 \qquad i \neq j \quad i,j = 1,2 \tag{1}$$

In the Cournot equilibrium of the stage game, each firm is setting its output to maximise its profits given the output of its competitor. Therefore, the first-order conditions for a Cournot equilibrium are:

$$\frac{\partial \pi_i}{\partial q_i} = \alpha - (2\beta + \gamma_i)q_i - \beta q_j = 0 \qquad i \neq j \quad i,j = 1,2 \tag{2}$$

Given the assumptions of linear demand and quadratic costs (linear marginal costs), a unique Cournot equilibrium will exist where the output of both firms is positive so there is no need to be concerned about boundary solutions whatever the relative efficiencies of the two firms. Solving for the Cournot equilibrium outputs, market price, and profits yields:

$$q_i^N = \frac{\alpha}{A}\left(\beta + \gamma_j\right) \qquad p^N = \frac{\alpha}{A}(\beta + \gamma_1)(\beta + \gamma_2) \qquad \pi_i^N = \frac{\alpha^2}{2A^2}(2\beta + \gamma_i)\left(\beta + \gamma_j\right) \tag{3}$$

where $A \equiv 3\beta^2 + 2\beta(\gamma_1 + \gamma_2) + \gamma_1\gamma_2 > 0$.[7] This describes the Cournot equilibrium of the stage game, and both firms setting the Cournot equilibrium output in every stage is also a subgame-perfect equilibrium of the supergame. However, the folk theorem implies that collusion allows both firms to achieve higher profits than in the Cournot equilibrium.

Now consider the possibility of sustaining collusion in the supergame using Nash-reversion trigger strategies. If the output quota gives each firm higher profits in the cartel than in the Cournot equilibrium then collusion can be sustained provided that the discount factor is sufficiently large. The equilibrium strategies involve each firm choosing its output quota in the first stage and in subsequent stages if both firms have chosen their output quota in all previous stages of the supergame. If any firm has defected from the cartel and chosen an output other than its output quota in any previous stage of the supergame then each firm will choose its Cournot equilibrium output in all subsequent stages. Obviously, the threat of reversion to the Cournot equilibrium outputs is credible as these outputs are a Nash equilibrium of the stage game. Therefore, using the threat of reversion to the Cournot equilibrium to sustain collusion is a subgame-perfect equilibrium provided the discount factor is sufficiently large.

One important issue for the cartel, especially with cost asymmetries, is the allocation of the output quotas between the two firms. Rothschild (1999), following Patinkin (1947), assumes that the firms form a cartel that allocates output between firms so as to maximise total-industry profits, $\Pi = \pi_1 + \pi_2$. Then, the first-order conditions for maximisation of total-industry profits are:

$$\frac{\partial \Pi}{\partial q_i} = \alpha - (2\beta + \gamma_i) q_i - 2\beta q_j = 0 \qquad i \neq j \quad i, j = 1,2 \tag{4}$$

Given the quadratic cost function this implies a unique allocation of output quotas between the firms such that marginal cost of production is equalised for both firms. Solving (4) for the output quotas, price and profits of the cartel yields:

$$q_i^T = \frac{\alpha\gamma_j}{B} \qquad P^T = \frac{\alpha}{B}\left[\beta(\gamma_1+\gamma_2)+\gamma_1\gamma_2\right] \qquad \pi_i^T = \frac{\alpha^2\gamma_j}{2B} \tag{5}$$

where $B \equiv 2\beta(\gamma_1+\gamma_2)+\gamma_1\gamma_2 = A-3\beta^2 > 0$. The second-order conditions for the maximisation of total-industry profits will always be satisfied, as they require that $B > 0$. When the cartel maximises total-industry profits, the marginal costs of both firms are equal to $\alpha\gamma_i\gamma_j / B$ so the cartel allocates output quotas between the two firms to ensure production efficiency.

In a similar manner to Rothschild (1999), the total output, price, and total profits of the cartel can be rewritten as:

$$q_1^T + q_2^T = \frac{2\alpha}{4\beta+H} \qquad P^T = \alpha\frac{2\beta+H}{4\beta+H} \qquad \Pi^T = \frac{\alpha^2}{4\beta+H} \tag{6}$$

where H is the harmonic mean of the cost parameters of the two firms.[8] This is defined as:

$$H \equiv \left[\frac{1}{2}\left(\frac{1}{\gamma_1}+\frac{1}{\gamma_2}\right)\right]^{-1} \tag{7}$$

Hence, the total output, price, and total-industry profits of the cartel are independent of the distribution of the cost parameters, but depend only upon the harmonic mean of the cost parameters and the demand parameters. Therefore, an increase in cost asymmetry with the harmonic mean of the cost parameters held constant would not affect the total output, price or profits of the cartel.[9] This allows cost asymmetries to be analysed in an analogous way to capacity asymmetries in Compte, Jenny and Rey (2002), where total capacity in the industry is held constant.

Obviously, the cartel price is higher than the price in the Cournot equilibrium, $P^T > P^N$. Also, the market share of the *i*th firm in the cartel, $\gamma_j / (\gamma_i + \gamma_j)$, is larger than its market share in the Cournot equilibrium, $(\beta + \gamma_j)/(2\beta + \gamma_i + \gamma_j)$, if it is the efficient firm, $\gamma_i < \gamma_j$. This can be seen by noting that both firms have the same marginal cost and price-cost margin in the cartel, but the efficient firm has a larger output and hence a larger price-cost margin in the Cournot equilibrium. Hence, the marginal cost of the efficient firm is lower than the marginal cost of the inefficient firm in the Cournot equilibrium. Therefore, since marginal cost is linear, it can be deduced that the market share of the efficient firm is lower in the Cournot equilibrium than in the cartel. Thus, the efficient firm gains relatively more from collusion whereas the inefficient firm loses relatively less from reversion to the Cournot equilibrium following a defection from the cartel.

While total industry profits must be higher in the cartel than in the Cournot equilibrium, the profits of the inefficient firm may actually be lower. Comparing (3) and (5), it can be shown that the Cournot equilibrium profits are higher than the cartel profits for the *i*th firm, $\pi_i^N > \pi_i^T$, if:

$$\gamma_i > \frac{1}{2\beta + \gamma_j}\left[\left(-2\beta^2 + \gamma_j^2\right) + \left(\beta + \gamma_j\right)\sqrt{4\beta^2 + 2\beta\gamma_j + \gamma_j^2}\right] \tag{8}$$

If this condition holds then the inefficient firm will have no incentive to join the cartel, as its profits in the cartel are lower than its profits in the Cournot equilibrium. The cartel will not be viable as collusion will not be sustainable for any discount factor if the cost parameter of the inefficient firm is sufficiently high. Even if the profits of the inefficient firm are higher in the cartel than in the Cournot equilibrium, the differences in efficiency may still make it difficult to sustain collusion.

Suppose that the *i*th firm defects from the cartel in any stage, then it will maximise its one-period profits in that stage given the output of the other firm, q_j^T. Thus, the first-order condition for profit maximisation by the defecting firm is:

$$\frac{\partial \pi_i}{\partial q_i} = \alpha - \left(2\beta + \gamma_i\right) q_i - \beta q_j^T = 0 \tag{9}$$

Solving for the output, price and profits of the defecting firm yields:

$$q_i^D = \frac{\alpha\left(B - \beta\gamma_i\right)}{\left(2\beta + \gamma_i\right)B} \qquad P^D = \frac{\alpha\left(\beta + \gamma_i\right)\left(B - \beta\gamma_i\right)}{\left(2\beta + \gamma_i\right)B} \qquad \pi_i^D = \frac{\alpha^2\left(B - \beta\gamma_i\right)^2}{2\left(2\beta + \gamma_i\right)B^2} \tag{10}$$

Obviously, the profits when the firm defects are higher than the profits when the firm is part of the cartel, $\pi_i^D > \pi_i^T$. There is a one-period gain in profits for the defecting firm followed by a loss of profits forever thereafter as both firms revert to the Cournot equilibrium.

Collusion is sustainable if, for both firms, the present discounted value of profits from being part of the cartel exceeds the present discounted value of the profits from defecting from the cartel for one stage followed by the Cournot equilibrium profits in all subsequent stages. Thus, collusion is sustainable for the *i*th firm if:

$$\frac{1}{1-\delta}\pi_i^T \geq \pi_i^D + \frac{\delta}{1-\delta}\pi_i^N \qquad \Rightarrow \qquad \delta \geq \frac{\pi_i^D - \pi_i^T}{\pi_i^D - \pi_i^N} \tag{11}$$

Substituting profits from (3), (5), and (10) into (11) yields the critical discount factor for the *i*th firm:

$$\delta_i = \frac{\beta^2 \gamma_i^2 A^2}{\left(B - \beta\gamma_i\right)^2 A^2 - \left(2\beta + \gamma_i\right)^2 \left(\beta + \gamma_j\right) B^2} \tag{12}$$

This is a rather messy function of the demand parameter β (but not α) and the cost parameters γ_1 and γ_2 so it is not possible to obtain general results about the effects of the cost parameters on the critical discount factors. However, to gain some insight into how the critical discount factors depend upon the cost parameters, differentiate (12) with respect to the cost parameters, and evaluate for the symmetric case where $\gamma_1 = \gamma_2 = \gamma$. This yields the following results:

$$\begin{aligned}
\left.\frac{\partial \delta_i}{\partial \gamma_i}\right|_{\gamma_1=\gamma_2=\gamma} &= \frac{2(3\beta+\gamma)\left(66\beta^4+98\beta^3\gamma+52\beta^2\gamma^2+12\beta\gamma^3+\gamma^4\right)}{\gamma(\beta+\gamma)\left(17\beta^2+12\beta\gamma+2\gamma^2\right)^2} > 0 \\
\left.\frac{\partial \delta_i}{\partial \gamma_j}\right|_{\gamma_1=\gamma_2=\gamma} &= \frac{-2(2\beta+\gamma)(3\beta+\gamma)\left(33\beta^3+33\beta^2\gamma+10\beta\gamma^2+\gamma^3\right)}{\gamma(\beta+\gamma)\left(17\beta^2+12\beta\gamma+2\gamma^2\right)^2} < 0
\end{aligned} \tag{13}$$

An increase in the inefficiency of a firm increases its critical discount factor and decreases the critical discount factor of the other firm. Now consider the effect on the critical discount factors of the two firms of an increase cost asymmetry while keeping the harmonic mean of the cost parameters constant.[10] This involves an increase in the cost parameter of one firm with a corresponding decrease in the cost parameter of the other firm, such that $d\gamma_j / d\gamma_i = -\gamma_j^2 / \gamma_i^2$. Hence, using (13), the effect of an increase in cost asymmetry on the critical discount factors evaluated in the symmetric case is:

$$\begin{aligned}
\frac{d\delta_i}{d\gamma_i} = \left.\left(\frac{\partial \delta_i}{\partial \gamma_i} - \frac{\partial \delta_i}{\partial \gamma_j}\frac{d\gamma_j}{d\gamma_i}\right)\right|_{\gamma_1=\gamma_2=\gamma} &= \frac{2(3\beta+\gamma)^2(4\beta+\gamma)\left(11\beta^2+10\beta\gamma+2\gamma^2\right)}{\gamma(\beta+\gamma)\left(17\beta^2+12\beta\gamma+2\gamma^2\right)^2} > 0 \\
\frac{d\delta_i}{d\gamma_j} = \left.\left(\frac{\partial \delta_i}{\partial \gamma_j} - \frac{\partial \delta_i}{\partial \gamma_i}\frac{d\gamma_i}{d\gamma_j}\right)\right|_{\gamma_1=\gamma_2=\gamma} &= \frac{-2(3\beta+\gamma)^2(4\beta+\gamma)\left(11\beta^2+10\beta\gamma+2\gamma^2\right)}{\gamma(\beta+\gamma)\left(17\beta^2+12\beta\gamma+2\gamma^2\right)^2} < 0
\end{aligned} \tag{14}$$

An increase in the cost parameter of a firm leads to an increase in its critical discount factor and a decrease in the critical discount factor of the other firm. As the firm becomes less efficient, as the cost parameter increases, the gains from the initial defection from the cartel may increase or decrease but the losses suffered thereafter always decrease, and this effect dominates so that the critical discount factor becomes larger. This is because the inefficient firm has a smaller market share in the cartel (when production is allocated efficiently) than in the Cournot equilibrium. Therefore, it suffers less when the firms revert to the Cournot equilibrium outputs following a defection from the cartel than the efficient firm.

Evaluating the critical discount factors of the two firms for the symmetric case where $\gamma_1 = \gamma_2 = \gamma$, yields:

$$\delta_1 = \delta_2 = \bar{\delta} = \frac{(3\beta + \gamma)^2}{17\beta^2 + 12\beta\gamma + 2\gamma^2} \tag{15}$$

Note that the critical discount factor in the symmetric case is greater than one-half but less than one, and it is decreasing in the cost parameter γ but increasing in the demand parameter β .

Figure 1 shows the critical discount factors of the two firms plotted as a function of the cost parameter of firm one for specific parameter values ($\alpha = 25$, $\beta = 5$, $H = 10$), but seemingly without any loss of generality.[11] As the cost parameter of firm one increases the cost parameter of firm two decreases to keep the harmonic mean of the cost parameters constant. As expected, the critical discount factors of the two firms intersect and are equal when the cost parameters of the two firms are equal at $\gamma_1 = 10$. The critical discount factor of firm one is increasing in the cost parameter of firm one while the critical discount factor of firm two is decreasing in the cost parameter of firm one, as suggested by (14). For a sufficiently large value of the cost parameter, the profits of firm one in the Cournot equilibrium are larger than its profits in the cartel and the critical discount factor of firm one exceeds unity, as shown in (8). Similarly, for a sufficiently small value of the cost parameter of firm one, the profits of firm two in the Cournot equilibrium are larger than its profits in the cartel and the critical discount factor of firm two exceeds unity. Collusion can be sustained by the cartel if the discount factor is larger than the critical discount factor of both firms, $\delta \geq \text{Max}\{\delta_1, \delta_2\}$, so in region above both the curves in figure 1. Clearly, as Rothschild (1999) has shown, any cost asymmetry will make it more difficult for the cartel to sustain collusion. This leads to the following proposition:

Proposition 1*: Cost asymmetries make it more difficult to sustain collusion in a cartel that allocates output quotas to maximise total-industry profits as the inefficient firm has a greater incentive to defect than the efficient firm.*

The explanation is that the inefficient firm has more incentive to defect from the cartel as reversion to the Cournot equilibrium outputs following a defection from the cartel has less effect on its profits than on the profits of the efficient firm. This is because the market share of the inefficient firm in the Cournot equilibrium is larger than its market share in the cartel.

3. Sustaining Collusion with Asymmetric Costs

When the cartel maximises total-industry profits, as in Rothschild (1999), it has been shown that the inefficient firm has more incentive to defect from the cartel than the efficient firm, and may have no incentive to join the cartel in the first place. The latter case is shown in figure two, which shows the feasible profits set for the two firms where the inefficient firm (firm two) has higher profits in the Cournot equilibrium (N) than in the cartel (T) so obviously collusion is not viable.[12] However, there are points on the Pareto-frontier of the feasible profits set for the two firms that yield both firms higher profits than at the Cournot equilibrium. Therefore, in the absence of side payments between the two firms, the only way to make the cartel viable is to change the allocation of output quotas between the two firms so that the cartel colludes at a point such as Z rather than T. This can be done by assuming that the cartel maximises a weighted sum of the profits of the two firms with relatively less weight being given to the profits of the efficient firm (firm one) as shown in figure two. Hence, collusion can always be made to be viable for the two firms despite any differences in efficiency between the two firms.

The next question is the sustainability of collusion and the critical discount factor of the cartel. This critical discount factor of the cartel is the lowest discount factor that will sustain collusion between the two firms given their cost functions, and it can be derived using the concept of 'balanced temptation' as proposed by Friedman (1971). The cartel adjusts the output quotas of the two firms until the critical discount factor is the same for both firms and they both have the same incentive to defect from the cartel. Obviously, if one firm had a higher critical discount factor then it would have a greater incentive to defect from the cartel. However, if its output quota was increased and that of the other firm decreased then its incentive to defect from the cartel would be reduced. This adjustment of output quotas by the cartel can be modelled as a change in the relative weight that the cartel attaches to the profits of the two firms. Thus, to find the critical discount factor of the cartel that will sustain collusion between the two firms, one has to find the relative weighting of profits such that the critical discount factors of both firms are equal. Therefore, suppose that the cartel maximises the following weighted sum of the two firms profits:

$$Z = \lambda \pi_1 + \pi_2 \tag{16}$$

where $\lambda > 0$ is the weight on the profits of firm one.[13] Since it is only the relative weight on the profits of the two firms that matters, the weight on the profits of firm two has been normalised to unity. Although this weighting will affect the output quotas and cartel profits of the two firms, it will not affect the Cournot equilibrium outputs and profits (3) as each firm maximises its own profits in the Cournot equilibrium.

If the cartel allocates output quotas to the two firms to maximise the weighted-profits then the first-order conditions are:

$$\begin{aligned} \frac{\partial Z}{\partial q_1} &= \lambda\alpha - \lambda(2\beta+\gamma_1)q_1 - (\lambda+1)\beta q_2 = 0 \\ \frac{\partial Z}{\partial q_2} &= \alpha - (\lambda+1)\beta q_1 - (2\beta+\gamma_2)q_2 = 0 \end{aligned} \tag{17}$$

Unlike the equivalent equations in the previous section, these equations are not symmetric as the weight on the profits of firm one has introduced an asymmetry so in this section the expressions will have to be presented separately for each firm. These first-order conditions can be solved for the output quotas of the two firms as functions of the cost and demand parameters and the weight on the profits of firm one, λ, which yields:

$$q_1^Z(\lambda) = \frac{\alpha}{E}\left[\lambda(\beta+\gamma_2)-\beta\right] \qquad q_2^Z(\lambda) = \frac{\alpha\lambda}{E}\left[(\beta+\gamma_1)-\lambda\beta\right] \tag{18}$$

where $E \equiv \lambda B - (\lambda-1)^2\beta^2$. The second-order conditions for the maximisation of the weighted-sum of profits imply that $E > 0$, and this will be the case if the output quotas of both firms are positive, which implies that $\beta/(\beta+\gamma_2) < \lambda < (\beta+\gamma_2)/\beta$.

Substituting these output quotas into (1) yields the cartel profits of the two firms as functions of the weight on the profits of firm one:

$$\begin{aligned} \pi_1^Z(\lambda) &= \frac{\alpha^2}{2E^2}\left[\lambda(\beta+\gamma_2)-\beta\right]\left[\lambda B-(\lambda-1)\beta\gamma_1\right] \\ \pi_2^Z(\lambda) &= \frac{\alpha^2\lambda^2}{2E^2}\left[(\beta+\gamma_1)-\lambda\beta\right]\left[B+(\lambda-1)\beta\gamma_2\right] \end{aligned} \tag{19}$$

These cartel profits are on the Pareto-frontier of the feasible profits set shown in figure two, and an increase in the weight on the profits of firm one will move the cartel clockwise round the Pareto-frontier. If firm one defects from the cartel and maximises its profit for one period, given the output quota of firm two, then its first-order condition for profit maximisation is:

$$\frac{\partial \pi_1}{\partial q_1} = \alpha - \left(2\beta + \gamma_1\right) q_1 - \beta q_2^Z = 0 \tag{20}$$

Solving for the output of firm one if it defects from the cartel yields:

$$q_1^{DZ}\left(\lambda\right) = \frac{\alpha}{\left(2\beta + \gamma_1\right) E}\left[\lambda\left(B - \beta\gamma_1\right) + \left(\lambda - 1\right)\beta^2\right] \tag{21}$$

Thus, the profits of firm one if it defects from the cartel are:

$$\pi_1^{DZ}\left(\lambda\right) = \frac{\alpha^2}{2\left(2\beta + \gamma_1\right) E^2}\left[\lambda\left(B - \beta\gamma_1\right) + \left(\lambda - 1\right)\beta^2\right]^2 \tag{22}$$

Similarly, if a firm two defects from the cartel and maximises its profit for one period, given the output quota of firm one, then its first-order condition for profit maximisation is:

$$\frac{\partial \pi_2}{\partial q_2} = \alpha - \left(2\beta + \gamma_2\right) q_2 - \beta q_1^Z = 0 \tag{23}$$

Solving for the output of firm one if it defects from the cartel yields:

$$q_2^{DZ}\left(\lambda\right) = \frac{\alpha\lambda}{\left(2\beta + \gamma_2\right) E}\left[\left(B - \beta\gamma_2\right) - \left(\lambda - 1\right)\beta^2\right] \tag{24}$$

Thus, the profits of firm two if it defects from the cartel are:

$$\pi_2^{DZ}\left(\lambda\right) = \frac{\alpha^2\lambda^2}{2\left(2\beta + \gamma_2\right) E^2}\left[\left(B - \beta\gamma_2\right) - \left(\lambda - 1\right)\beta^2\right]^2 \tag{25}$$

Having derived the profits of the firms from collusion in the cartel, from defection from the cartel, and in the Cournot equilibrium, the critical discount factors of the two firms can be derived as in the previous section:

$$\delta_i(\lambda) = \frac{\pi_i^{DZ}(\lambda) - \pi_i^Z(\lambda)}{\pi_i^{DZ}(\lambda) - \pi_i^N} \tag{26}$$

The critical discount factors of the two firms are clearly functions of the weight that the cartel attaches to the profits of firm one. To gain some insight into the effect of a change in this weight, differentiate the critical discount factors with respect to the weight λ then evaluate the derivatives for the symmetric case where both firms have the same cost parameter, $\gamma_1 = \gamma_2 = \gamma$, and let $\lambda = 1$, which yields:

$$\begin{aligned} \left.\frac{\partial \delta_1}{\partial \lambda}\right|_{\gamma_1=\gamma_2=\gamma} &= \frac{-2(2\beta+\gamma)^2(3\beta+\gamma)^2(11\beta+3\gamma)}{\gamma(17\beta^2+12\beta\gamma+2\gamma^2)} < 0 \\ \left.\frac{\partial \delta_2}{\partial \lambda}\right|_{\gamma_1=\gamma_2=\gamma} &= \frac{2(2\beta+\gamma)^2(3\beta+\gamma)^2(11\beta+3\gamma)}{\gamma(17\beta^2+12\beta\gamma+2\gamma^2)} > 0 \end{aligned} \tag{27}$$

In the symmetric case, an increase in the weight the cartel attaches to the profits of firm one will decrease the critical discount factor of firm one and increase the critical discount factor of firm two. This is because an increase in the weight increases the output quota and cartel profits of firm one, and therefore reduces its incentive to defect from the cartel. Similarly, it decreases the output quota and cartel profits of firm two and increases its incentive to defect from the cartel.

To minimise the discount factor required to sustain collusion by the cartel, balanced temptation implies that the critical discount factors of the two firms should be equated so that both firms have the same incentive to defect from the cartel, and this can be done by altering the relative weight that the cartel attaches to the profits of firm one. Thus, balanced temptation implies choosing the weight $\lambda = \lambda^*$ so that critical discount factors of the two firms are equal, $\delta_1(\lambda^*) = \delta_2(\lambda^*)$, and this gives the critical discount factor of the cartel:

$$\delta^* \equiv \delta_1(\lambda^*) \equiv \delta_2(\lambda^*) \tag{28}$$

To find out the effect of an increase in the cost asymmetry (an increase in the cost parameter of firm one while holding the harmonic mean of the cost parameters constant) on the critical discount factor of the cartel, δ^*, it is first necessary to derive the effect of an increase in the cost asymmetry on the weight that the cartel attaches to the profits of firm one, λ^*. This can be obtained by totally differentiating (28), which yields:

$$\frac{\partial \delta_1}{\partial \lambda}\frac{d\lambda^*}{d\gamma_1}+\frac{d\delta_1}{d\gamma_1}=\frac{\partial \delta_2}{\partial \lambda}\frac{d\lambda^*}{d\gamma_1}+\frac{d\delta_2}{d\gamma_1} \tag{29}$$

This can be evaluated for the symmetric case where the firms are equally efficient, $\gamma_1=\gamma_2=\gamma$, which implies that $\lambda^*=1$, using the derivatives from (14) together with the derivatives from (27). Note that the derivatives in (14) were calculated for the case when the cartel maximises total-industry profits, but this is equivalent to the symmetric case considered here when $\lambda=1$. Rearranging yields the effect of an increase in cost asymmetry on the weight that the cartel attaches to the profits of firm one in the symmetric case:

$$\left.\frac{d\lambda^*}{d\gamma_1}\right|_{\gamma_1=\gamma_2=\gamma}=\frac{(4\beta+\gamma)(11\beta^2+10\beta\gamma+2\gamma^2)}{(\beta+\gamma)(2\beta+\gamma)^2(11\beta+3\gamma)}>0 \tag{30}$$

An increase in the cost parameter of firm one (holding the harmonic mean of the cost parameters constant) leads to an increase in the weight that the cartel attaches to the profits of firm one that will increase its output quota and cartel profits. Thus, the effect on the critical discount factor of the cartel is:

$$\frac{d\delta^*}{d\gamma_1}=\frac{\partial \delta_1}{\partial \lambda}\frac{d\lambda^*}{d\gamma_1}+\frac{d\delta_1}{d\gamma_1}=\frac{\partial \delta_2}{\partial \lambda}\frac{d\lambda^*}{d\gamma_1}+\frac{d\delta_2}{d\gamma_1}=0 \tag{31}$$

An increase in cost parameter of firm one (holding the harmonic mean of the cost parameters constant) leads to no change in the critical discount factor of the cartel. The direct effect of an increase in the cost parameter of firm one is to increase the critical discount factor of firm one, but this is offset by the increase in the weight on the profits of firm one that reduces its critical discount factor. While the direct effect reduces the critical discount factor of firm two, but this is offset by the increase in the weight on the profits of firm one that increases its critical discount factor. Therefore, an increase in cost asymmetry (an increase in the cost parameter of firm one holding the harmonic mean of the cost parameters constant) has no effect on the critical discount factor when evaluated in the symmetric case.

Given the complex nature of the problem, further analytical results are not possible but the model can be solved numerically using *Mathematica*, see Wolfrom (1999), and the results obtained seem to be clear and unambiguous. In fact, they are probably more useful than any analytical results that could be obtained with calculus as they allow the effect of cost asymmetry on the critical discount factors of the cartel to be seen very clearly. Figure 3 uses the same parameters as in the previous section ($\alpha = 25$, $\beta = 5$, $H = 10$) and shows the critical discount factors plotted against the cost parameter of firm one (holding the harmonic mean of the cost parameters constant). The critical discount factors of the two firms are shown for the case when the cartel maximises total-industry profits, $\lambda = 1$. If the cost parameter of firm one is $\hat{\gamma}_1$ then the critical discount factor of firm one is greater than the critical discount factor of firm two, but increasing the weight on the profits of firm one to $\lambda = 1 \cdot 1$ equalises the critical discount factors of the two firms and gives the critical discount factor of the cartel at $\hat{\gamma}_1$. While the critical discount factor of the cartel δ^* appears to be as 'flat as a pancake' in figure 3a, figure 3b clearly shows that it has a maximum in the symmetric case ($\gamma_1 = \gamma_2 = 10$) and that an increase in cost asymmetry results in a decrease in the critical discount factor! This together with (31) leads to the following proposition:

Proposition 2: *An increase in cost asymmetry reduces the critical discount factor of the cartel making it easier to sustain collusion.*

This result is in stark contrast to conventional wisdom and, particularly, the results of Rothschild (1999). The reason is that Rothschild (1999) assumes that the cartel maximises total-industry profits and does not alter the allocation of output quotas when faced with cost asymmetry. However, a cartel faced with cost asymmetries can always allocate output quotas to ensure that the cartel is viable and sustainable. Then, cost asymmetries do not make it more difficult to sustain collusion.

4. Conclusions

This paper has challenged the conventional wisdom in Industrial Organisation as formalised by Rothschild (1999) that cost asymmetries make collusion more difficult. In a model similar to Rothschild (1999) it has been shown that cost asymmetries do not make it more difficult to sustain collusion. The explanation for this result is that in Rothschild (1999) it was assumed that the cartel maximised total-industry profits as in Patinkin (1947), but as Bain (1948) has pointed out this ignores the viability of the cartel. In the face of cost asymmetry, a cartel will allocate output quotas to ensure the viability and sustainability of collusion. Using the concept of balanced temptation from Friedman (1971), where output quotas are allocated so that all firms have the same incentive to defect from the cartel, this paper has analysed the critical discount of the cartel. Thus, counter-intuitively, it was shown that cost asymmetries make it (slightly) easier to sustain collusion.

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Endnotes

[1] Also see the discussion of this issue in Vives (1999, p. 308).

[2] As Bain (1948, p. 618) noted, the maximisation of total-industry profits may require some inefficient firms to cease production and as he stated: 'The fact is that *with earnings following output*, the presumptive rationale of output *allocation* by the cartel will not be one of maximization of industry profit at all'. Bain's criticism of Patinkin has been noted by Osborne and Pitchik (1983), Schmalensee (1987) and Harrington (1991).

[3] Verboven (1997) basically uses the idea of balanced temptation without the requirement to be on the Pareto-frontier of the feasible profits set whereas in this paper the concern will be solely with collusion that yields profits on the Pareto-frontier.

[4] It should be noted that free trade (zero export subsidies) is not on the Pareto-frontier of the feasible welfare set for the two governments, as a Pareto optimal outcome would involve export taxes.

[5] Obviously, side-payments between firms would make it easy for competition authorities to detect collusion by the cartel as there would be clear evidence of explicit collusion whereas tacit collusion without side-payments is more difficult to detect.

[6] The optimal penal code to sustain collusion has been characterised by Abreu (1986) for a symmetric Cournot oligopoly game. However, it is extremely difficult to characterise the optimal penal code in a Cournot oligopoly where firms have asymmetric costs. Therefore, as in Rothschild (1999), attention will be restricted to the use of Nash-reversion trigger strategies by the cartel.

[7] The N superscript is used to denote the Nash (Cournot) equilibrium of the stage game. Using a C superscript has been avoided to prevent confusion as in a cartel model it could stand for Cournot, cheating, collusion, cartel, or cooperative.

[8] Rothschild (1999) does not use the harmonic mean of the cost parameters, H, but his variable S is obviously related to H and it can be shown that $S=2/H$.

[9] As Rothschild (1999) notes the horizontal summation of the marginal costs of the two firms gives the 'aggregate' marginal cost of the cartel, which has a slope equal to $1/S$ or $H/2$. When the cartel maximises total-industry profits it equates its 'aggregate' marginal cost with its marginal revenue.

[10] By keeping the harmonic mean of the cost parameters constant, there will be no change in the total output, price and total profits of the cartel.

[11] A similar diagram is used by Collie (1993) in an analysis of the sustainability of free trade in an infinitely-repeated version of a profit-shifting export subsidy game in which Cournot duopolists have different marginal costs.

[12] With quadratic costs, the Pareto-frontier of the feasible profits set is strictly concave whereas Schmalensee (1987) showed that the Pareto-frontier was strictly convex with constant, but different, marginal costs.

[13] The cartel is assumed to maximise weighted-profits in order to find the balanced temptation equilibrium on the Pareto frontier of the feasible profits set that minimises the critical discount factor of the cartel. It is not an assumption about how the cartel will actually allocate output quotas if the discount factor exceeds the critical discount factor of the cartel. Then, the cartel may use Nash bargaining to allocate output quotas as in Harrington (1991).

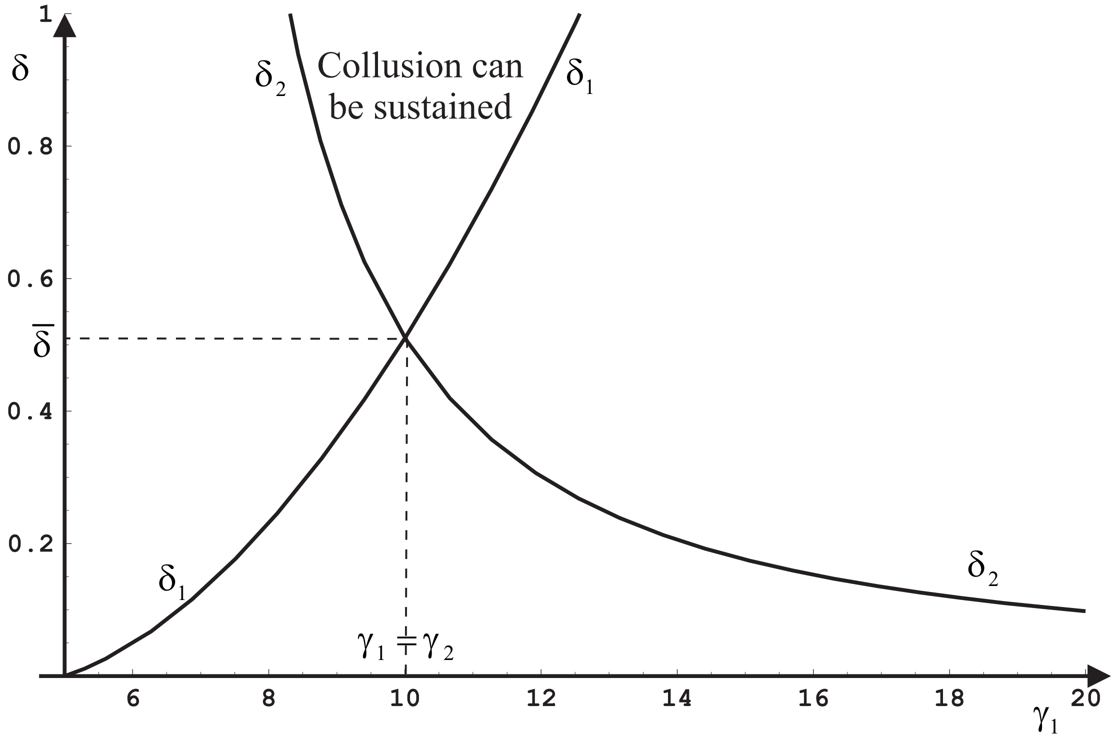

Figure 1: Cartel maximises total industry profits *H*=10

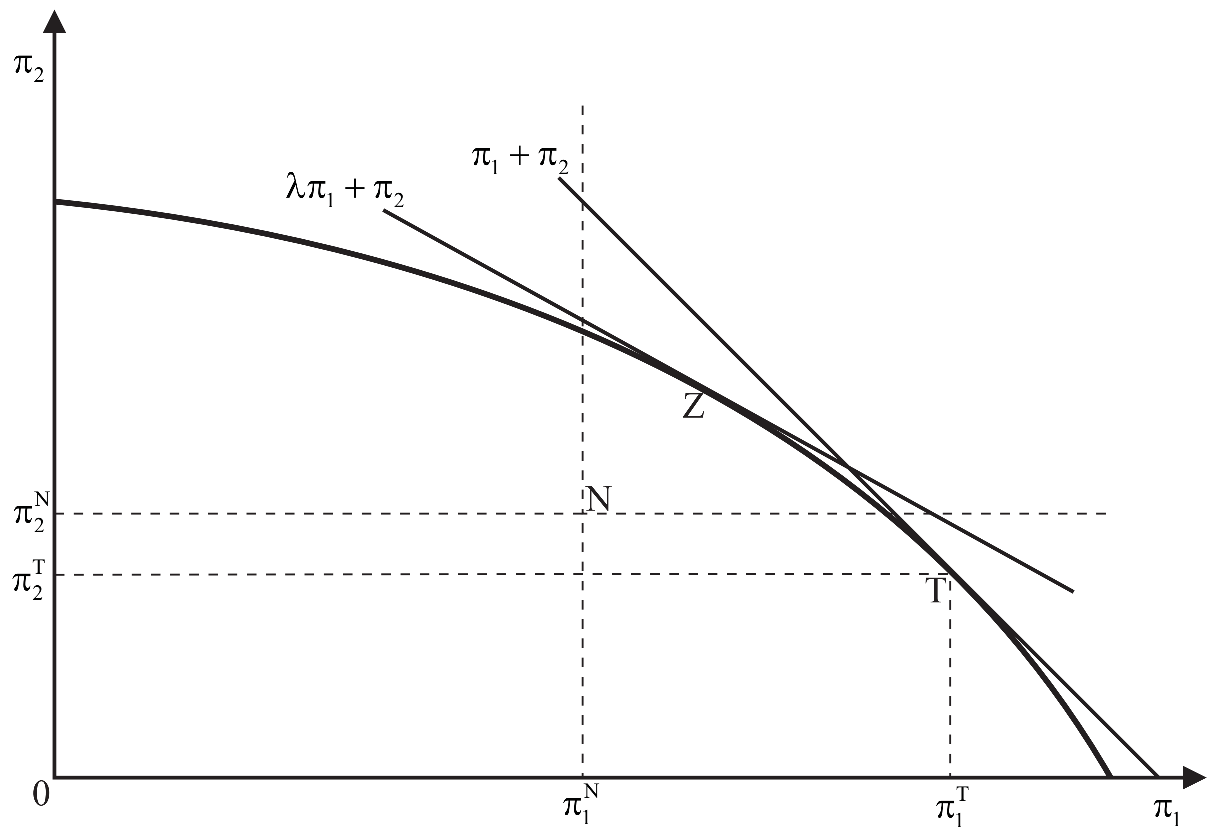

Figure 2: Pareto-frontier of feasible profits set

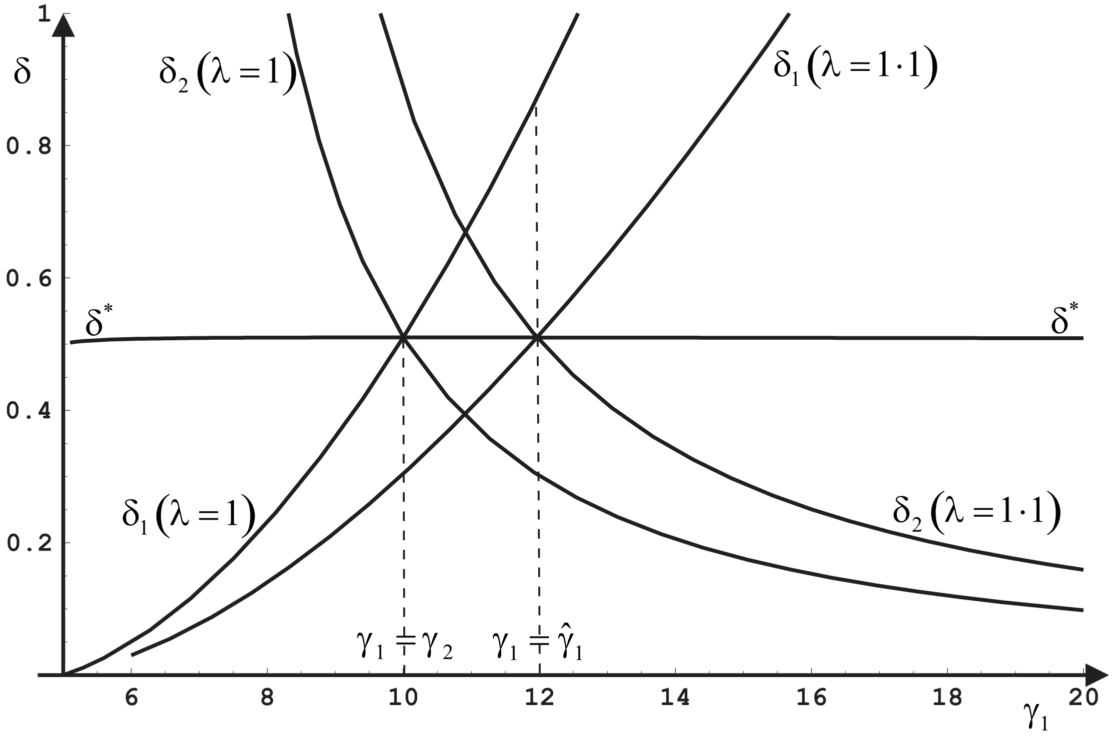

Figure 3a

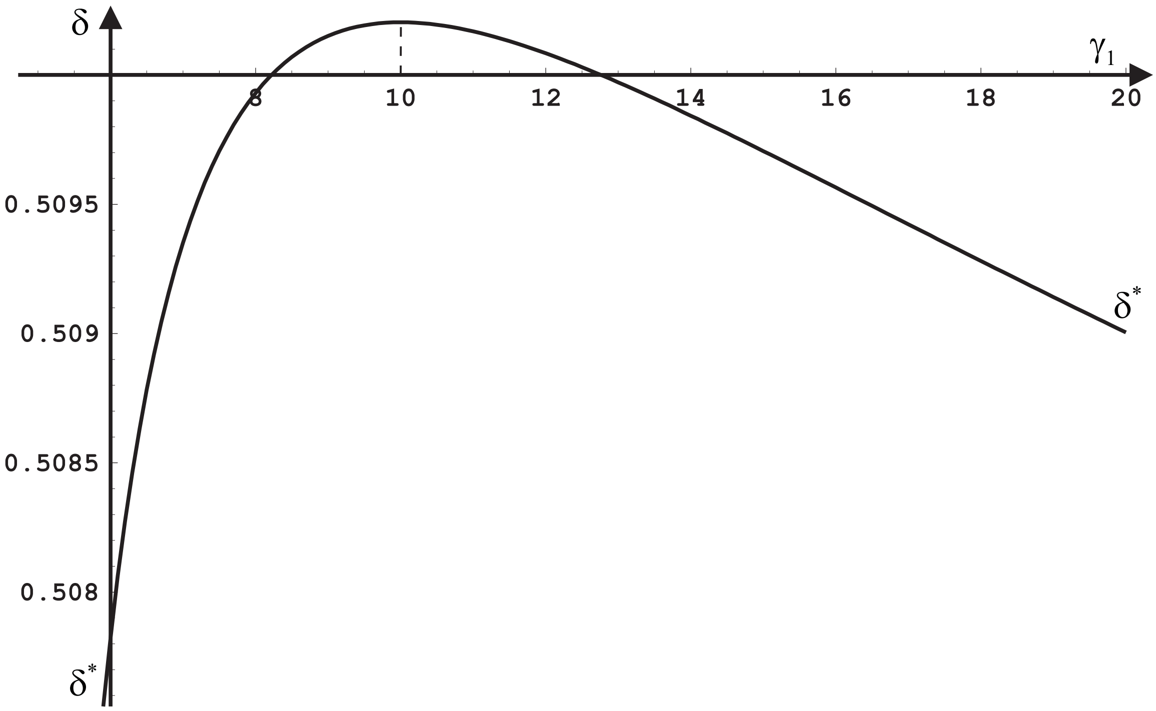

Figure 3b

Figure 3: Cost asymmetry and the critical discount factor of the cartel