Endogenous Business Cycle with Search in the Labour Market

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Abstract

We develop an endogenous business cycle (EBC) model with search in the labour market and equilibrium unemployment. Because of the thin market externalities the equilibrium path is indeterminate, and there is a room for sunspot equilibria (extrinsinc uncertainty). Our model generates such business cycles patterns as relative volatility, persistence and comovement in fluctuations of macroeconomic variables, with extrinsic uncertainty being the source of fluctuations. The model is able to predict high autocorrelation in output growth and hump-shaped impulse response of output - important business cycle features that existing EBC models, as well as real business cycle models, failed to explain.

Keywords: Business Cycle; Sunspots; Equilibrium Unemployment
JEL classification: E32; J64

1 Introduction

In this paper we explore the implications of labour-market search in the context of business cycles. A wide range of real business cycle (RBC) models, in which the source of economic fluctuations are random shocks to the production function (as in [14], [18]), fail to generate high autocorrelation in macroeconomic variables, unless the underlying shocks to the economy are highly persistent. Because of a weak internal propagation mechanism, in RBC models the dynamics of output replicates the dynamics of inputs (see, for example, [9], [8]). An alternative approach are the endogenous business cycle (EBC) models, in which the fluctuations are driven by *extrinsic* uncertainty, as opposed to *intrinsic* uncertainty in the RBC models. In the latter stochastic shocks affect economic fundamentals, while in the former the fundamentals do not change. The RBC models typically exhibit saddle-type stability (unique equilibrium path) and, therefore, need exogenous shocks to generate fluctuations. In the EBC models

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the equilibrium is characterized by a sink-type stability, and, hence, a continuum of equilibrium paths and a possibility of existence of a sunspot equilibrium. The source of fluctuations are self-fulfilling beliefs, or "animal spirits" of economic agents, or sunspots ([22]). Weder in [24] gives a detailed overview of various EBC models with indeterminacy. In the existing literature the models displaying this kind of dynamics are based on increasing returns [2], [10], market imperfections, such as monopolistic markups [11], [19], or distortionary taxation [21]. However, these models do not perform very well in terms of calibration to macroeconomic data: to generate indeterminacy and basic patterns of business cycles they require the degree of increasing returns or imperfections significantly higher than empirically plausible. Multi-sector models with sector-specific externalities, such as in [5], [17], [3], [4], or [20], allow for better calibration. Still, in all these models extrinsic uncertainty alone fails to explain the autocorrelation function of output growth and the hump-shaped response function of output to demand shocks. Schmitt-Grohé analyzes this issue in [20] and concludes that the EBC models of this type do not provide the propagation mechanism that RBC models are lacking, because "... for mild, and hence empirically realistic, deviations from constant returns and marginal cost pricing, these models all imply fairly similar employment and output dynamics". This suggests, in particular, that indeterminacy models with essentially different employment dynamics may be more successful in explaining business cycle phenomena. An approach we use in this paper is the slow response of employment to external shocks, due to search frictions in the labour market.

Among the first attempts to incorporate labour market search into dynamic macroeconomic model are the papers by Andolfatto [1], Merz [16], Shi and Wen [23], and Burda and Weder [7]. In these models, as in the models described above, business cycle fluctuations are generated by random shocks to the technology, *i.e.* intrinsic uncertainty. A similar approach was used by den Haan, Ramey and Watson in [12]. In their model persistence of output effects of intrinsic shocks is generated by cyclical fluctuations in the endogenous job destruction rate (and costly capital adjustment). Frictions in the labour market search create the mechanism of propagation of real shocks. In this paper we focus on the effect of purely extrinsic uncertainty. We develop a new model of endogenous business cycle with two important features. Firstly, the model exhibits a propagation mechanism, that is generated by slow adjustment of employment to the disturbances in the economy. Job matching requires that (1) the firms allocate part of their resources to creating and maintaining job vacancies, and (2) the agents allocate part of their resources to seeking a job. Hence, employment at the time of production is pre-determined and responds slowly to the shocks. The market equilibrium in this economy is characterized by a non-zero unemployment level. Secondly, because of the existence of thin market externalities in an inefficient search equilibrium, there is a possibility of indeterminacy of the equilibrium - a continuum of equilibrium paths converging to the stationary equilibrium characterized by sink-type stability. The source of fluctuations are self-fulfilling beliefs of the agents.

We analyze local dynamics of a one-sector constant-return-to-scale economy with labour market search and equilibrium unemployment. We show that, in the presence of inefficiency, and, hence, externalities, in search equilibrium, the dynamic equilibrium is indeterminate, and business cycle fluctuations can be caused by stochastic shocks to agents' beliefs (sunspots). Calibration to the data on U.S. aggregate macroeconomic variables shows that, in terms of relative volatility, serial correlation and cross-correlation of the time series of economic variables, our model, with constant returns in production and perfect competition in the markets for consumption good and capital, performs at least as good as real business cycle models and endogenous business cycle models with increasing returns and monopolistic competition. More importantly, because of the rich internal propagation structure, with i.i.d. stochastic shocks to the sunspot variable (in this paper - search efforts of the workers) alone, the model generates other features of business cycles - significant autocorrelation of output growth and hump-shaped impulse response function of output, which the existing endogenous business cycle models, as well as real business cycle models, failed to explain.

2 Model

Consider an economy with a large number of identical households and a large number of identical firms. Firms produce a single good using capital and labour inputs; this good can be used both as a consumption good and as capital input. Households own capital and labour. As capital owners, they rent capital to firms at market interest rate. As workers, they supply labour to the labour market. In order to create a job in the future, both parties, the workers and the firms, need to exert costly search efforts in the present time. Wage is determined by Nash bargaining: the surplus from creating a job is split between the firm and the worker according to their bargaining powers, which we assume to be exogenous constants. Firms are owned by households, who claim the profits.

2.1 Households

A representative infinitely lived household maximizes lifetime discounted utility $\sum_{0}^{\infty} \beta^t U(c_t, l_t)$ from the stream of consumption, $\{c_t\}$, and leisure, $\{l_t\}$. We assume that instantaneous utility function $U(\cdot, \cdot)$ is strictly jointly concave and twice differentiable. In the beginning of each period t a household chooses optimally its consumption, job search efforts (search time u_t and search intensity s_t), supply of capital k_{t+1}, and supply of labour n_{t+1}, subject to the following constraints. One unit of time available at t is allocated between employment n_t, leisure l_t, and job search u_t:

$$n_t + u_t + l_t = 1.$$

Household's income at t, consisting of capital income, wage income, and profits from production, is allocated among consumption c_t, investment i_t, and cost of intensity of job search efforts $C(s_t)$:

$$c_t + i_t + \mathcal{C}(s_t) = r_t k_t + w_t n_t + \pi_t.$$

(investment, as usually, is defined as supply of capital in the next period less non-depreciated part of existing capital, $i_t = k_{t+1} - (1-\delta)k_t$). Employment in the next period, n_{t+1}, depends on the worker's search efforts at t; in the end of period t he is separated from job at an exogenous constant rate θ, and the new job is created at (exogenous to the worker) rate m_t, with per worker number of jobs proportional to the worker's search time and search intensity:

$$n_{t+1} = n_t + m_t u_t s_t - \theta n_t$$

The Lagrangean of the household's optimization problem is:

$$\begin{aligned}\mathcal{L}^H &= \sum \beta^t \left[U(c_t, 1 - n_t - s_t) + \Omega_t^H \left(n_t \left(1-\theta\right) + m_t u_t s_t - n_{t+1}\right)\right. \\ &\quad \left. + \Psi_t \left(r_t k_t + w_t n_t + \pi_t - \mathcal{C}\left(s_t\right) - c_t - k_{t+1} + \left(1-\delta\right) k_t\right)\right]\end{aligned}$$

Here, Ω_t^H is the shadow price, in terms of utility, of employment to the household, and Ψ_t is the shadow price, also in terms of utility, of income to the household.

Under the concavity of the objective function first-order conditions hold with equality and are both necessary and sufficient. After some rearrangement we get:

$$U_t^{(1)} = \Psi_t \tag{1}$$

$$U_t^{(2)} = m_t s_t \Omega_t^H \tag{2}$$

$$\mathcal{C}'\left(s_t\right) = \frac{\Omega_t^H}{\Psi_t} m_t u_t \tag{3}$$

$$\Psi_t = \beta \Psi_{t+1} \left(r_{t+1} + 1 - \delta\right) \tag{4}$$

$$\Omega_t^H = \beta \left(\Omega_{t+1}^H \left(1-\theta\right) + \Psi_{t+1} w_{t+1} - U_{t+1}^{(2)}\right) \tag{5}$$

In the optimum, the household equates marginal utility of consumption to the shadow price of income (1), marginal utility of leisure to the shadow price of employment (2), and marginal cost of job search intensity to the price, in real terms, of additional jobs created per unit of search intensity (3). Shadow prices evolve according to the law of motions (4) and (5). The latter, together with (1) and (2), implies that wage must exceed marginal rate of substitution between consumption and leisure to ensure nonzero labour supply in the steady-state equilibrium. Therefore, $MRS_{c,l} = U_t^{(2)}/U_t^{(1)}$ is interpreted as worker's reservation wage, and $w_t - U_t^{(2)}/U_t^{(1)}$ is the worker's surplus from being employed at time t.

2.2 Firms

A representative firm uses capital and labour inputs to produce a single capital/ consumption good according to some production function, $y_t = F(k_t, n_t)$. In the end of period t employees are separated at an exogenous constant rate θ. In order to hire labour in the next period, $t+1$, the firm at time t must exert search efforts, measured by the number of vacancies v_t to be created and/or advertised at cost $\mathcal{V}(v_t)$. Therefore, firm's profit π_t at time t is the output of production less cost of inputs and cost of maintaining the vacancies.

Firm maximizes discounted stream of profits

$$\sum_{t=0}^{\infty} \frac{1}{\Pi_{\tau=0}^{t} R_\tau} \left[F\left(k_t, n_t\right) - r_t k_t - w_t n_t - \mathcal{V}\left(v_t\right)\right]$$

subject to the job flow constraint:

$$n_{t+1} = n_t + \mu_t v_t - \theta n_t,$$

where μ_t is the (exogenous to the firm) rate of filling a vacancy, by choosing optimally at time t capital k_{t+1}, labour n_{t+1}, and number of vacancies v_t. If the objective of the firm is to maximize its value (present value of cash flows) to the owner (household), then the discount factor at time t is $R_t = 1 + r_t - \delta$ (the opportunity cost of reinvesting the profit).

The Lagrangean of the firm's optimization problem is:

$$\begin{aligned} \mathcal{L}^F &= \sum_{t=0}^{\infty} \frac{1}{\Pi_{\tau=0}^{t}\left(1 + r_\tau - \delta\right)} \left[F\left(k_t, n_t\right) - r_t k_t - w_t n_t - \mathcal{V}\left(v_t\right)\right. \\ &\quad \left. + \Omega_t^F \left(n_t\left(1-\theta\right) + \mu_t v_t - n_{t+1}\right)\right] \end{aligned}$$

Here, Ω_t^F is shadow price, in real terms, of employment to the firm.

Under the concavity of the objective function, first-order conditions hold with equality and are both necessary and sufficient. After some rearrangement we get:

$$r_{t+1} = F_{t+1}^{(1)} \tag{6}$$

$$\mathcal{V}'\left(v_t\right) = \mu_t \Omega_t^F \tag{7}$$

$$\Omega_t^F = \frac{1}{1 + r_{t+1} - \delta}\left(\Omega_{t+1}^F\left(1-\theta\right) - w_{t+1} + F_{t+1}^{(2)}\right) \tag{8}$$

In the optimum, firm equates marginal cost of capital input to marginal increase in output from additional unit of capital (6), and marginal cost of search efforts to the price of additional jobs created per unit of search efforts measured by the number of vacancies (7). Shadow price of employment evolves according to the law of motion (8), which together with (7) implies that marginal productivity of labour must exceed wage to ensure nonzero employment in the steady-state equilibrium. The difference between MPL and wage, $F_t^{(2)} - w_t$, is the firm's surplus from employment of an additional unit of labour employed at time t.

2.3 Equilibrium

We define the equilibrium in the economy described above as the set of sequences of variables $\{c_t\}$, $\{k_t\}$, $\{n_t\}$, $\{u_t\}$, $\{s_t\}$, $\{v_t\}$, job matching rates $\{m_t\}$, $\{\mu_t\}$, and prices $\{r_t\}$, $\{w_t\}$, such that

- given sequence of job matching rate $\{m_t\}$ and sequences of capital and labour prices $\{r_t\}$, $\{w_t\}$, variables $\{c_t\}$, $\{k_t\}$, $\{n_t\}$, $\{u_t\}$, $\{s_t\}$ solve optimization problem of a representative household subject to the time, budget, and job flow constraints;
- given sequence of job matching rate $\{\mu_t\}$ and sequences of capital and labour prices $\{r_t\}$, $\{w_t\}$, variables $\{k_t\}$, $\{n_t\}$, $\{v_t\}$, solve optimization problem of a representative firm subject to the job flow constraints;
- numbers of job matches created by efforts of workers and firms are equal and are described by matching function:

$$m_t u_t s_t = \mu_t v_t = M\left(v_t; u_t, s_t\right)$$

- equilibrium wage is determined by Nash bargaining and maximizes weighted (geometric) average of surpluses of workers and firms from an additional unit of employment, with weights corresponding to their bargaining powers (λ for the workers and $1-\lambda$ for the firms):

$$w_t^* = arg\max\{\left(w_t - U_t^{(2)}/U_t^{(1)}\right)^{\lambda}\left(F_t^{(2)} - w_t\right)^{1-\lambda}\} \tag{9}$$

The system of equations describing the dynamic equilibrium in this economy can be reduced to the following. Four intertemporal equations describe the laws of motion of the variables and prices[1]:

$$\begin{aligned}
k_{t+1} &= (1-\delta)k_t - c_t - \mathcal{C}(s_t) - \mathcal{V}(v_t) + F(f_t, n_t) && (10)\\
n_{t+1} &= (1-\theta)n_t + M(v_t; u_t, s_t) && (11)\\
\frac{\Omega_t^H}{\Psi_t} &= \frac{1}{r_{t+1}+1-\delta}\left[\left(1-\theta-\lambda\frac{M(v_t;u_t,s_t)}{u_t}\right)\frac{\Omega_{t+1}^H}{\Psi_{t+1}}\right. \\
&\quad \left. +\lambda F^{(2)}(k_{t+1}, n_{t+1})\right] && (12)\\
\Psi_t &= \beta(r_{t+1}+1-\delta)\Psi_{t+1} && (13)
\end{aligned}$$

[1] Condition (9) can be equivalently expressed as $\left(\frac{\Omega_t^H}{\Psi_t}\right)_{w_t^*} : \left(\Omega_t^F\right)_{w_t^*} = \lambda : (1-\lambda)$, i.e. in equilibrium shadow prices, in real terms, of an additional unit of employment to the workers and to the firms are proportional to their bargaining powers. This allows to eliminate the law of motion for Ω_t^F in the dynamic equilibrium.

The rest of the equations describe contemporaneous relations among the variables and prices:

$$\begin{aligned}
\frac{\Omega_t^H}{\Psi_t} &= \frac{s_t \mathcal{C}'(s_t)}{M(v_t; u_t, s_t)} && (14) \\
\Omega_t^H &= \frac{U^{(2)}(c_t, l_t) u_t}{M(v_t; u_t, s_t)} && (15) \\
\Psi_t &= U^{(1)}(c_t, l_t) && (16) \\
\Omega_t^F &= \frac{v_t \mathcal{V}'(v_t)}{M(v_t; u_t, s_t)} && (17) \\
\Omega_t^F &= \frac{1-\lambda}{\lambda} \frac{\Omega_t^H}{\Psi_t} && (18) \\
r_t &= F^{(1)}(k_t, n_t) && (19) \\
w_t &= (1-\lambda) \frac{U^{(2)}(c_t, l_t)}{U^{(1)}(c_t, l_t)} + \lambda F^{(2)}(k_t, n_t) && (20)
\end{aligned}$$

For the further analysis we will use the following functional forms for preferences and technologies: CES form for the instantaneous utility function $U(\cdot, \cdot)$:

$$U\left(c_t, l_t\right) = \frac{\left(c_t^\gamma l_t^{1-\gamma}\right)^{1-\sigma} - 1}{1-\sigma}, \tag{21}$$

Cobb-Douglas form for the production function:

$$F\left(k_t, n_t\right) = A_0 k_t^\alpha n_t^{1-\alpha} \tag{22}$$

and the “normalized” form for the matching function:

$$M\left(v_t; u_t, s_t\right) = \frac{v_t \cdot \left(u_t \cdot s_t\right)}{\left[v_t^l + \left(u_t s_t\right)^l\right]^{1/l}} \tag{23}$$

(the latter, unlike often used Cobb-Douglas form, ensures that matching rates, also interpreted as probabilities of finding a job on the worker's side and of filling a vacancy on the firm's side, are always between zero and one.) For the cost functions we assume linear form:

$$\begin{aligned}
\mathcal{C}(s_t) &= p_0 s_t. && (24) \\
\mathcal{V}\left(v_t\right) &= \phi_0 v_t && (25)
\end{aligned}$$

3 Steady state equilibrium and local dynamics

To compute the steady state, or the long-run, equilibrium we solve the system of equations for the first-order optimization conditions (10) - (25), constraints, and equilibrium conditions, assuming that all time-dependent variables take constant values. After eliminating shadow prices we obtain the following equations for the observable macroeconomic variables (for the steady state equilibrium values we omit time index):

$$\begin{aligned}
\delta k &= y - c - p_0 s - \phi_0 v \\
\theta n &= \frac{vus}{\left[v^l + (us)^l\right]^{1/l}} \\
\frac{p_0 s}{\theta n}\left[1 - \beta\left(1 - \theta - \lambda\frac{\theta n}{u}\right)\right] &= \beta\theta\lambda\frac{1-\alpha}{\alpha} rk \\
r &= \frac{1}{\beta} - 1 + \delta \\
\frac{p_0 s}{u} &= \frac{1-\gamma}{\gamma}\frac{c}{1-n-u} \\
\phi_0 v &= \frac{1-\lambda}{\lambda} p_0 s \\
r &= \alpha A_0\left(\frac{k}{n}\right)^{\alpha-1} \\
w &= (1-\lambda)\frac{p_0 s}{u} + \lambda\frac{1-\alpha}{\alpha}\frac{rk}{n}
\end{aligned}$$

It can be shown that this system of equations has a unique solution, and, hence, the steady state equilibrium in our model economy is unique.

To explore the dynamics of the economy in the vicinity of the steady state equilibrium we log-linearize the system of dynamic equations around the steady state values. Following the literature, we introduce the "hat" notation, $\hat{x}_t = (x_t - x)/x$, for the relative deviation of variable x_t from its steady state value x. After eliminating contemporaneous relations among "hat"-variables, we reduce the system of log-linearized dynamic and contemporaneous equations to the following system of four first-order linear difference equations for the variables $\hat{k}_t, \hat{n}_t, \hat{u}_t, \hat{c}_t$:

$$\begin{aligned}
\hat{k}_{t+1} &= \frac{1}{\beta}\hat{k}_t + \left(r\frac{1-\alpha}{\alpha} - \frac{p_0 s}{\lambda k}\frac{n}{1-n-u}\right)\hat{n}_t - \frac{p_0 s}{\lambda k}\frac{1-n}{n}\hat{u}_t \\
&\quad - \left(\frac{c}{k} + \frac{p_0 s}{\lambda k}\right)\hat{c}_t \\
\hat{n}_{t+1} &= \left(1 - \theta\frac{1-u}{1-n-u}\right)\hat{n}_t + \theta\left(\left(\frac{\theta n}{us}\right)^l + \frac{1-n}{1-n-u}\right)\hat{u}_t \\
&\quad + \theta\hat{c}_t \\
-\frac{1}{\beta}\left(\frac{\theta n}{us}\right)^l \hat{u}_t &= r(1-\alpha)\left(1 + \frac{\lambda\theta k}{p_0 s}\right)\hat{k}_{t+1} \\
&\quad - \left[r(1-\alpha)\left(1 + \frac{\lambda\theta k}{p_0 s}\right) + \lambda\theta\frac{n^2}{u(1-n-u)}\right]\hat{n}_{t+1}
\end{aligned}$$

$$-\left[\left(\frac{\theta n}{us}\right)^{l}(1-\theta)+\lambda\theta\frac{n}{1-n-u}\right]\hat{u}_{t+1}-\lambda\theta\frac{n}{u}\hat{c}_{t+1}$$

$$\begin{aligned}-\beta r(1-\alpha)\hat{k}_{t+1}+\left(\beta r(1-\alpha)-(1-\gamma)(1-\sigma)\frac{n}{1-n-u}\right)\hat{n}_{t+1}\\ -(1-\gamma)(1-\sigma)\frac{u}{1-n-u}\hat{u}_{t+1}-\left[1-\gamma(1-\sigma)\right]\hat{c}_{t+1}\\ =-(1-\gamma)(1-\sigma)\frac{n}{1-n-u}\hat{n}_{t}-(1-\gamma)(1-\sigma)\frac{u}{1-n-u}\hat{u}_{t}\\ -\left[1-\gamma(1-\sigma)\right]\hat{c}_{t}\end{aligned}$$

or, in the matrix form,

$$\hat{\mathbf{z}}_{t+1}=\mathcal{T}\hat{\mathbf{z}}_{t}$$

where $\hat{\mathbf{z}}_t = \left(\hat{k}_t, \hat{n}_t, \hat{u}_t, \hat{c}_t\right)'$, and $\mathcal{T}$ is the transition matrix of constant coefficients. Equivalently, using a contemporaneous relation

$$\hat{n}_t=\frac{1-n-u}{u}\hat{s}_t-\frac{1-n}{n}\hat{u}_t-\frac{1-n-u}{u}\hat{c}_t$$

this system of equations can be rewritten for the vector $\left(\hat{k}_t, \hat{s}_t, \hat{u}_t, \hat{c}_t\right)'$. Although the coefficients in the new system are more cumbersome, this representation is useful if we want to focus on how workers' beliefs regarding the tightness of the labour market affects evolution of the dynamic equilibrium in this economy through their choice of search efforts s_t and u_t.

4 Calibration and simulation results

To test the performance of the model we calibrate it to the post-WWII quarterly data from the U.S. economy and look whether the model economy is able to reproduce the patterns of the real economy. Here we use the standard approach of business cycles literature. The values of the following parameters were predetermined:

- Technology
  - Share of capital in production, $\alpha = 0.36$
  - Depreciation rate of capital, $\delta = 0.025$ (quarterly)
- Preferences
  - Discount factor, $\beta = 0.99$
  - Elasticity of substitution between consumption and leisure, $\gamma = 0.33$
  - Intertemporal elasticity of substitution, $\sigma = 1$

The following long-term averages of macroeconomics variables were used as the steady state equilibrium values:

- Average working hours, manufacturing sector, $n = 0.28$
- Average unemployment rate, $UR = 6\%$ (using the definitions of the model, $UR = u/(n+u)$)
- Average consumption to output ratio, $c/y = 0.6$

The following estimated values for the parameters were obtained from the previous literature:

- Average probability of filling a vacancy, $q = 0.9$ (per quarter) [1] ($q = M(\cdot)/v$)
- Job destruction rate, $\theta = 0.1$ (per quarter) [12]
- Curvature parameter in the matching function, $l = 1.27$ [12]

The magnitudes of the workers' bargaining power, λ, as well as marginal costs of search efforts, p_0 and ϕ_0, and the rest of the steady state equilibrium values were derived using the above parameterization. Specifically, rearranging the equations for the steady state equilibrium and using some definitions, we get

$$k = \left(\frac{A_0\alpha}{r}\right)^{\frac{1}{1-\alpha}} n \tag{26}$$

$$u = \frac{n \cdot UR}{1-UR} \tag{27}$$

$$s = \frac{\theta n}{u\left(1-q^l\right)^{1/l}} \tag{28}$$

$$v = \frac{\theta n}{q} \tag{29}$$

$$c = \frac{c}{y}\frac{rk}{\alpha} \tag{30}$$

$$p_0 = \frac{1-\gamma}{\gamma}\frac{c}{1-n-u}\frac{u}{s} \tag{31}$$

$$\phi_0 = \frac{1-\lambda}{\lambda}\frac{p_0 s}{v} \tag{32}$$

$$\lambda = \frac{1}{\beta\theta}\frac{1-\gamma}{\gamma}\frac{u}{1-n-u}\frac{1-\beta(1-\theta)}{1-\alpha-\frac{1-\gamma}{\gamma}\frac{n}{1-n-u}\frac{c}{y}} \tag{33}$$

With the above parameterization, the transition matrix $\mathcal{T}$ has one eigenvalue outside and three eigenvalues inside the unit circle. Hence, the steady state equilibrium is sink-type stable, and there exists a continuum of equilibrium paths converging to the steady state (indeterminacy). Therefore, volatility, or business cycles, in this economy can be generated endogenously, by extrinsic shocks to the beliefs of the agents, for example, regarding the tightness of the labour market. This result is robust to shifts in the parameters within a range of non-zero measure.

Unlike in the existing literature on endogenous business cycles, indeterminacy and possibility of endogenous fluctuations in this model arise without increasing returns to scale or monopolistic competition. Furthermore, due to the rich propagation mechanism in our model economy serially correlated fluctuations can be generated by serially independent shocks. The result is driven by inefficiency of the labour market. Previous models of business cycle with search in the labour market had to rely on real shocks with strong autocorrelation to generate basic features of the business cycles. In our model such features as significant positive autocorrelation in output growth, hump-shaped impulse response function of output, correlations between macroeconomic time series, and relative volatility of different macroeconomic variables, can be reproduced, with reasonable accuracy, using only i.i.d. extrinsic shocks (sunspots). The model is not free from some common drawbacks of endogenous business cycle models, - specifically, it generates volatility of working hours exceeding that of the output. However, contrary to the statement in [20] that extrinsic uncertainty alone cannot generate significant positive ACF of output growth and hump-shape impulse response function of output, our model reproduces both features. In the table below we show the results of computer simulations for the model economy, along with the data for the US economy and results obtained by other authors in different framework. To generate fluctuations we add a random (normally distributed around zero) i.i.d. component to u_t or s_t; the results for these two cases are very similar.

Table 1: Relative Volatility in Levels

	US	RBC	AND	BPS	PER	FGU	I	II	III
c	0.48	0.29	0.32	0.70	0.37	0.23	0.30	0.26	0.14
i	3.10	3.26	2.98	3.23	2.12	5.15	9.56	9.00	8.35
n	0.78	0.76	0.22	1.22	0.90	0.83	1.68	1.70	1.69

Table 2: Comovement in Levels

	US	RBC	AND	BPS	PER	FGU	I	II	III
c	0.78	0.86	0.91	0.34	0.80	0.78	0.62	0.78	0.68
i	0.89	0.99	0.99	0.88	0.99	0.99	0.45	0.49	0.57
n	0.83	0.98	0.96	0.74	0.99	0.98	0.99	0.99	0.99

In these Tables asterisk denotes insignificance; Roman numerals denote different specifications of our model. Namely, we used the following values for the

Table 3: Persistence in Levels

	US	RBC	BPS	PER	FGU	I	II	III
y	0.82	0.78	0.93	0.88	0.80	0.73	0.79	0.78
c	0.78	0.86	0.93	0.77	0.90	0.89	0.89	0.91
i	0.88	0.77	0.88	0.88	0.79	0.14	0.34	0.30
n	0.80	0.77	0.82	0.87	0.79	0.72	0.78	0.77

Table 4: Persistence in Growth Rates

	US	BW1	BW2	SG1	SG2	I	II	III
Δy	0.37	0.78	0.10	$*$	$*$	0.60	0.66	0.55
Δc	0.21	-0.05	0.59	0.19	0.17	0.74	0.78	0.77
Δi	0.49	0.56	0.10	0.16	0.69	-0.35	-0.27	-0.29
Δn	0.65	0.11	0.78	0.10	0.16	0.53	0.62	0.53

parameters n, σ, θ, and q (we also show the value of the workers' bargaining power, λ, derived in each of these specifications):
I: $n = 0.3, \sigma = 1.5, \theta = 0.1, q = 0.875, \lambda = 0.36$
II: $n = 0.28, \sigma = 1.5, \theta = 0.1, q = 0.9, \lambda = 0.22$
III: $n = 0.3, \sigma = 1.2, \theta = 0.08, q = 0.9, \lambda = 0.37$

We used the following abbreviations for the models in the literature with which we are comparing our results (for convenience, we indicate for each paper the source of volatility in the model economy, "extrinsic" for self-fulfilling beliefs and "intrinsic" for shocks to the production technology):
BW1: Benhabib, J. and Wen, Y. [6], intrinsic shocks
BW2: Benhabib, J. and Wen, Y. [6], extrinsic shocks
SG1: Schmitt-Grohé, S. [20], extrinsic shocks
SG2: Schmitt-Grohé, S. [20], extrinsic and intrinsic shocks
RBC: Hansen, G. [13], intrinsic shocks
AND: Andolfatto, D. [1], intrinsic shocks
BPS: Benhabib, J., Perli, R. and Sakellaris, P.[5], intrinsic shocks
PER: Perli, R. [17], extrinsic shocks
FGU: Farmer, R. and Guo, J.T. [10], extrinsic shocks

Once can see from Tables 1 to 4 that our model captures most of the basic features of the business cycles. Most importantly, it generates persistence in the effect of shocks when shocks are i.i.d., or have zero autocorrelation. Another important result is the significant positive autocorrelation in growth rates of the output, consumption, investment and working hours under i.i.d. shocks, - a business cycle phenomenon that the models in the previous literature failed to reproduce. Figures 1 to 3 show the response of macroeconomic variables to one-time shock to search efforts (u) in model specifications I, II, and III. One can see that one-time shock generates persistent volatility in the variables, which illustrated the rich propagation mechanism of the model.

5 Conclusions

In this paper we developed a model of an endogenous business cycle with search in the labour market. When search in labour market is costly, the employment is pre-determined and responds slowly to the disturbances in the economy. This mechanism generates persistence in the fluctuations of time series of macroeconomic variables. When equilibrium in the labour market is inefficient, the existence of thin market externalities provides room for indeterminacy and, hence, a possibility of a continuum of equilibrium paths around the steady state. In contrast with the existing literature on endogenous business cycles, indeterminacy of the equilibrium arises under constant returns to scale, perfect competition, and in the absense of any distortionary policies. Due to the internal propagation mechanism, i.i.d. shocks to the beliefs regarding the state of labour market (search efforts is the sunspot variable), without serially correlated shocks to fundamentals, generate serially correlated fluctuations in economic variables. We calibrate the model to the U.S. data on aggregate macroeconomic variables. In terms of predicting the relative volatility, persistense and comovement in macroeconomic time series the results obtained in our framework compare reasonably well to the results in the existing literature on the theory of business cycles. Also, our model explains autocorrelation function of output growth and hump-shaped impulse response function of output to one-time shock, which could not be explained by real business cycle models and various endogenous business cycle models based on increasing returns to scale and monopolistic competition.

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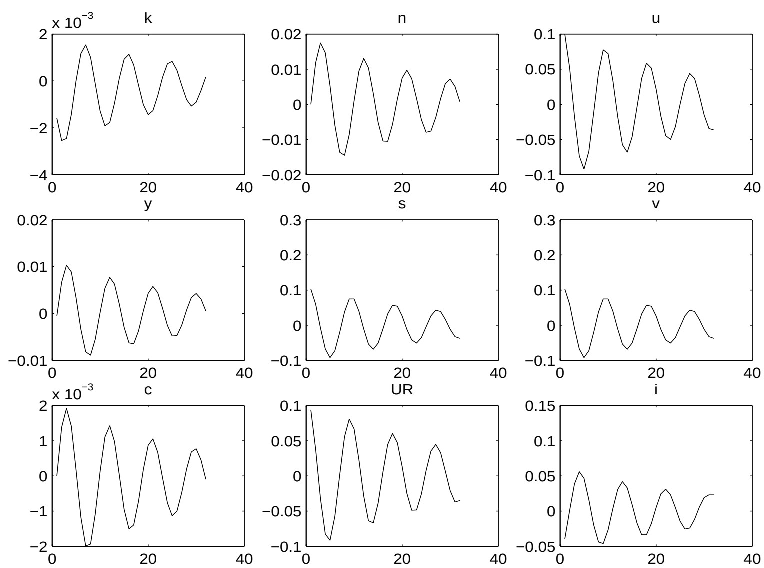

Figure 1: Response to one-time shock to u, model I

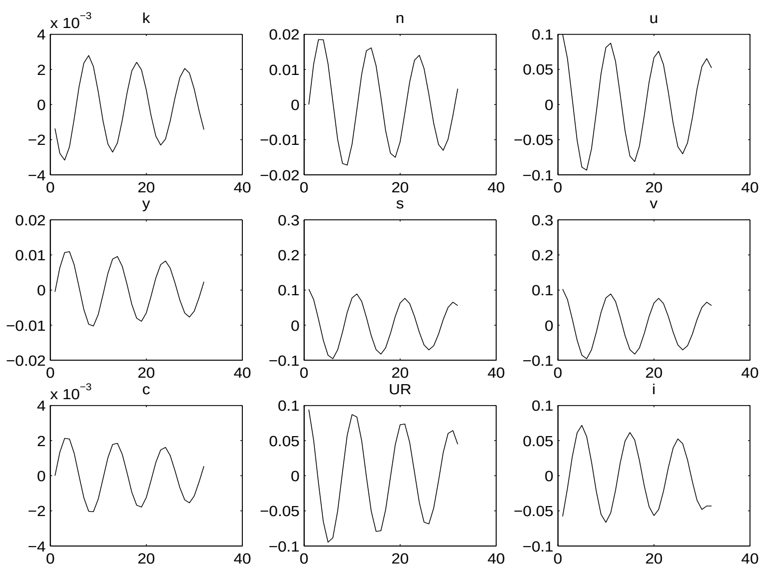

Figure 2: Response to one-time shock to u, model II

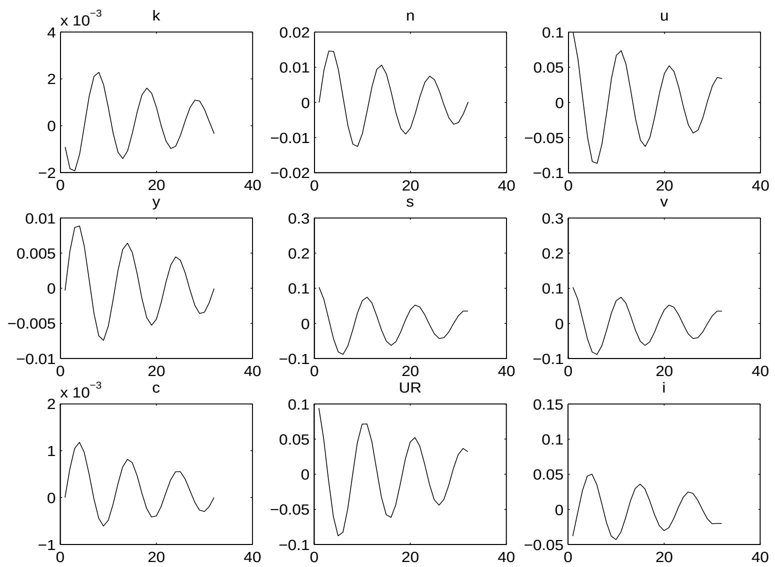

Figure 3: Response to one-time shock to u, model III