The Inflation Aversion of the Bundesbank: A State Space Approach*

Vladimir Kuzin
Goethe-University Frankfurt†

February 11, 2004

Keywords
Taylor rule; state space models; Markov switching models; Kalman filter

JEL classification: C22, E58

Abstract

A simple backward-looking Taylor rule is estimated in a time-varying coefficient framework with quarterly German data for the period 1975-1998. Markov switching models and the Kalman filter are used to extract the unobservable paths of the coefficients. The main finding is that the inflation aversion of the Bundesbank was not constant over time and exhibits some sudden and large shifts during the period of monetary targeting. There are phases with low and with high inflation aversion. This could for example explain why the estimated value of the inflation coefficient in backward-looking Taylor rules often does not exceed one and so violates the implications of theoretical monetary policy models. Moreover, the results provide evidence that the Bundesbank followed the so-called "opportunistic approach" to disinflation.

*This paper evolves from the author's diploma thesis at the Free University of Berlin. The author thanks his supervisor, Prof. Dr. Jürgen Wolters, for his support and helpful comments. He is also grateful to Ulrich Fritsche, Sven Schreiber and Uwe Hassler for helpful comments.

†Statistics and Econometric Methods, Goethe-University Frankfurt, Gräfstr. 78, D-60054 Frankfurt, Germany, Tel: +49.69.798.25332, Fax: +49.69.798.23662, email: kuzin@wiwi.uni-frankfurt.de.

1 Introduction

Over the last decade the new-keynesian models with microeconomic background gained a lot of popularity. This recent development leads also to growing interest in issues of optimal design of monetary policy and theoretically founded monetary policy rules. In small new-keynesian models monetary policy rules often have structure, which is very similar to the well known rule of Taylor (1993).

In the Taylor rule the interest rate is a function only of inflation gap and output gap. Taylor (1993) found that this very simple rule is able to describe the behavior of the Fed in the period 1987-1992 relatively well. Plenty of papers followed the publication of Taylor (1993), whose authors reported econometric estimations of Taylor-like monetary policy rules for different countries. Econometric research in this area raised the questions of structural stability of the estimated monetary policy rules, because structural breaks in the estimated policy rules may reflect changes in operating procedures and design of monetary policy. Clarida, Gali, Gertler (1999), for example, estimate a monetary policy rule for the samples before and after the Volcker disinflation period and find that the reaction function of the Fed has remarkably changed after the Volcker disinflation period. The estimation results show that since the Volcker period the Fed reacts much more aggressively to the changes in the inflation gap. Also Sims (2001) estimates a monetary policy reaction function of the Fed, permitting for several possible patterns of time variation in both its coefficients and its disturbance variances. The variation is estimated using Markov switching models as evolving in a stochastic, repeating pattern, not as evolution from one style of policy to another. In contrast to Clarida, Gali, Gertler (1999) the regime shifts estimated by Sims (2001) do not last very long and appear to reflect temporary shifts in the level of policy activism, not systematic improvement. Demers, Rodrguez (2001) investigate the stability of the Taylor rule under the period 1963(2) to 1999(4) using Canadian data. They show that the monetary rule cannot be evaluated over this period without taking into account parameter instability and structural changes, reflecting changes in monetary policy preferences. Using Kalman filter to estimate the monetary rule, evidence is found that the parameters are suffering from important changes during the period of the first oil shock, the late seventies and early eighties, thus capturing the adoption of the explicit inflation targeting policy by the Bank of Canada.

The aim of this paper is to investigate the possible changes of the monetary policy of the

Bundesbank in the period of monetary targeting. The focus of the Bundesbank policy on the inflation control is well known and therefore the emphasis of the current econometric analysis is put only on the changes in the reaction to inflationary developments. The research strategy is very simple. Firstly, a linear Taylor-like reaction function with partial adjustment is estimated. The linear specification with dynamic adjustment serves as a benchmark for the following state space specifications. Secondly, the coefficient, which is responsible for the Bundesbank reaction to inflationary development, is allowed to vary over time in the state space framework. Then the results of the linear and state space estimations are compared. The variation of only one coefficient in the equation makes the estimation more stable and easy to interpret than the variation of all coefficients. In particular the Markov switching models and Kalman filter are used to estimate the changes in the inflation aversion of the Bundesbank.

The reason for the choice of Markov switching models and Kalman filter in the current empirical investigation is the extraordinary flexibility of this class of models. In both models the dynamics of state variables is assumed to be exogen, which allows to avoid the choice of an explicit transition function and transition variables like in STR- or in SETAR-models. Thus, no explicit assumptions about how the Bundesbank changes the design of its own policy over time are necessary in the empirical investigation. In the case of this paper this is not a disadvantage, because no estimation of a model with any predictive power in relation to the changes in the monetary policy is purposed.

The layout of the article is as follows. Section 2 describes the model and econometric tools used for its estimation. Section 3 provides a brief description of the data used for the estimations and its properties. The empirical results are collected in section 4. The final section provides a possible interpretation of the results and some concluding remarks.

2 The model and econometric tools

2.1 The model

The structure and parameters of the Taylor rule are well known:

$$i_t = \bar{r} + \pi_t + 0.5(\pi_t - \bar{\pi}) + 0.5x_t \tag{1}$$

or in other form

$$i_t = \bar{r} + \bar{\pi} + 1.5(\pi_t - \bar{\pi}) + 0.5x_t \tag{2}$$

where $\bar{r}$ and $\bar{\pi}$ are the constant values of the real interest rate and the inflation target of the central bank under consideration. i_t is the short-term nominal interest rate, which is assumed to be monetary policy instrument of a central bank under consideration. Finally π_t and x_t are inflation and output gap. Taylor (1993) shows that this monetary policy rule yields a good performance describing the behavior of the Fed in the period 1987-1992.

The functional structure of the Taylor rule is supported through theoretical implications of the new-keynesian models. For example, Clarida, Gali, Gertler (1999) assume a quadratic loss function of a central bank and forward looking behavior of economic agents and show that the optimal policy rule has the following form

$$i_t = \gamma_\pi E_t \pi_{t+1} + \gamma_g g_t \tag{3}$$

where $\gamma_\pi > 1$, $\gamma_g > 0$ and π_t describes the deviation of inflation from the target of the central bank.[1] g_t could be interpreted as a demand shock. The result that $\gamma_\pi > 1$ has crucial empirical implications and means that a rise of the short nominal rate in response to a rise of the inflation gap is high enough to cause also a rise in the real interest rates. Thus the estimated value of γ_π can be used as a very simple measure of the monetary policy "quality". A good monetary policy should put emphasis on fighting inflation and so lead to an estimated value of $\gamma_\pi > 1$.

The version of the equation (2), which can be estimated, has three unknown parameters

$$i_t = \alpha + \beta(\pi_t - \bar{\pi}) + \gamma x_t + u_t \tag{4}$$

where $\alpha = \bar{r} + \bar{\pi}$. No forward-looking specification are considered and estimated in this paper. The reason for this restriction is the concentration on the estimation of time-varying coefficients, while most empirical studies incorporating forward-looking behavior are accomplished under the assumption of parameter constance.

The assumption of the constant inflation target $\bar{\pi}$ often turns out to be not very realistic. This assumption is slightly relaxed in the context of the current analysis. The inflation goal published officially by the Bundesbank is used as a measure for $\bar{\pi}$, denoted below by $\bar{\pi}_t$. The equation (4) is rewritten accordingly

$$i_t = \bar{r} + \bar{\pi}_t + \beta(\pi_t - \bar{\pi}_t) + \gamma x_t + u_t \tag{5}$$

where the real interest rate $\bar{r}$ is estimated directly. The problem is that the estimated residuals $\hat{u}_t$ often contain strong autocorrelation of the first order. This phenomenon is called interest

[1] In this case i_t denotes the deviation from the long-run equilibrium value of the nominal short rate.

rate smoothing. There is no satisfactory theoretical explanation for this effect and therefore in most econometric estimations the problem is solved simply by adding a lagged interest rate as a regressor. The equation (5) is transformed as follows to incorporate the interest rate smoothing:

$$i_t = [\bar{r} + \bar{\pi}_t + \beta(\pi_t - \bar{\pi}_t) + \gamma x_t] \cdot (1 - \rho) + \rho i_{t-1} + u_t \tag{6}$$

In the case of the autocorrelation of an order higher than one (6) is easily generalized in the following way:

$$i_t = [\bar{r} + \bar{\pi}_t + \beta(\pi_t - \bar{\pi}_t) + \gamma x_t] \cdot (1 - \rho) + \rho_1 i_{t-1} + \ldots + \rho_p i_{t-p} + u_t \tag{7}$$

where $\rho = \sum_{j=1}^{p} \rho_j$ is a persistence measure. The specification (6) or its generalized version (7) is estimated with different econometric methods. The results are reported in section 4.

2.2 Markov switching models

In the Markov switching framework a subset of the parameter set of an econometric model is modelled as a function of a real valued discrete unobservable Markov chain. Thus, one cannot say with certainty, in which state the system is at an assigned time t. Only a probability to be in state j at the time t can be inferred. A good introduction about the theory and application of Markov switching model is available for instance in Hamilton (1994). On the other side Krolzig (1997) provides a very deep and detailed discussion of this class of econometric models. In the remaining part of the subsection a brief formal description of the Markov switching framework is provided.

Let $\{s_t\}_{t=1}^{\infty}$ be a sequence of discrete random variables with sample space $\Omega = \{1, 2, \ldots, N\}$. The sequence $\{s_t\}_{t=1}^{\infty}$ is called to be a Markov chain of the first order, if the following statement is true:

$$Pr\{s_t = j | s_{t-1} = i, s_{t-2} = k, \ldots\} = Pr\{s_t = j | \mathfrak{F}_{t-1}\} = Pr\{s_t = j | s_{t-1} = i\} = p_{ji}$$

where $\mathfrak{F}_{t-1}$ denotes the information set available until $t-1$. The Markov chain is called a Markov chain of the first order, because, like in case of an autoregressive process of order one, only the first lag of the state influences the future of the process. The set of all transition probabilities p_{ji} can be summarized into the so called transition matrix $\{p_{ji}\}_{1 \leq j,i \leq N}$, where $\sum_{j=1}^{N} p_{ji} = 1$.

A Markov switching model posseses the following structure

$$y_t = x_t'\varphi(s_t) + u_t,\ \ u_t \sim i.i.d.(0, \Sigma(s_t)) \tag{8}$$

where s_t is an unobservable first order Markov chain discussed above and the vector x_t includes all exogenous and lagged endogenous regressors. An important assumption is exogenity of the Markov chain s_t. Under assumption of known or any given parameters Θ some extraction algorithm is needed to gain some information about the evolution of the state s_t over time. We denote $Y_s = \{y_1, \ldots, y_s, x_1, \ldots, x_s\}$ as all available information until s for this purpose. The probabilities $Pr\{s_t = j|Y_{t-1}, \Theta\}$, $Pr\{s_t = j|Y_t, \Theta\}$ and $Pr\{s_t = j|Y_T, \Theta\}$ are called *forecasted*, *filtered* and *smoothed* probabilities respectively. These probabilities are calculated for each data point t and collected into time series, which can be used for economic interpretation of the empirical results. These time series contain all information about the state of model (8) conditional on s and parameter values Θ. Obviously, the smoothed probabilities $Pr\{s_t = j|Y_T, \Theta\}$ are the best inference about the unobserved state s_t, because they contain *all* information available to an econometrician. A nonlinear recursive algorithm for the extraction of *filtered* probabilities can be derived using the calculation rule for conditional probabilities:

$$Pr\{A|B\} = \frac{Pr\{A \cap B\}}{Pr\{B\}}$$

Another recursive algorithm uses the *filtered* probabilities as input to calculate *smoothed* probabilities. For the derivation and deeper discussion of both algorithms it is referred to HAMILTON (1994) or KROLZIG (1997) to keep the extent of the current paper within a limit.

The *forecasted* probabilities $Pr\{s_t = j|Y_{t-1}, \Theta\}$ can be used to derive the conditional density of the dependent variable

$$f(y_t|Y_{t-1}, \Theta) = \sum_{j=1}^{N} Pr\{s_t = j|Y_{t-1}, \Theta\} \cdot f(y_t|s_t = j, Y_{t-1}, \Theta) \tag{9}$$

which leads to the conditional likelihood function

$$L(\Theta) = \prod_{t=1}^{T} f(y_t|Y_{t-1}, \Theta). \tag{10}$$

The ML-estimates are now obtained through the maximization of the conditional loglikelihood

$$logL(\Theta) = \sum_{t=1}^{T} log \sum_{j=1}^{N} Pr\{s_t = j|Y_{t-1}, \Theta\} \cdot f(y_t|s_t = j, Y_{t-1}\Theta). \tag{11}$$

To perform the estimations, the equation (6) under assumption of normality of u_t is directly used for the evaluation of the loglikelihood (11). As mentioned above, only the coefficient responsible for the reaction of a central bank to inflation gap is allowed to vary over time. So the model (6) in the Markov switching case is written as follows

$$i_t = [\bar{r} + \bar{\pi}_t + \beta(s_t)(\pi_t - \bar{\pi}_t) + \gamma x_t] \cdot (1-\rho) + \rho i_{t-1} + u_t,\ u_t \sim N(0, \sigma^2) \tag{12}$$

where s_t is a first order Markov chain as stated above. The number of regimes is assumed to be two, $s_t \in \{1, 2\}$. The BFGS maximization procedure implemented in OX[2] is used to obtain the ML-estimates of specification (12).

2.3 Kalman filter

In the remaining part of the section a brief formal description of Kalman filter and corresponding state space framework is provided. The concept of Kalman filter was developed by KALMAN (1960, 1963) and is based on the state space representation of dynamic systems. In the state space framework it is possible to extract the moments of unobservable stochastic components from observable stochastic processes.

Let y_t be a vector of observable variables and α_t an unobservable vector of state variables. Then the state space representation of a linear dynamic system can be written down as follows

$$y_t = c_t + Z_t\alpha_t + \varepsilon_t \tag{13}$$

$$\alpha_{t+1} = d_t + T_t\alpha_t + v_t \tag{14}$$

where c_t, d_t are vectors and Z_t, T_t matrices, which are also allowed to be time dependent. ε_t and v_t are i.i.d. errors processes allowed to be correlated. Equation (13) is called the signal equation and equation (14) is the corresponding state equation. Because the vector process α_t is unobservable, statistical inference about α_t is needed. The terminology used below is similar to the case of Markov switching. The set of all available information until the time s is denoted by $Y_s = \{y_1, \ldots, y_s, x_1, \ldots, x_s\}$, where x_t is the vector of exogenous variables at the time t. Given some value of the population parameter vector Θ and Y_s, the Kalman filter and smoother allows to calculate conditional moments of the state vector at the time t. According

[2] The programming language OX 3.30 CONSOLE was used to write the estimation procedures, downloaded from `http://www.nuff.ox.ac.uk/Users/Doornik/`

to the formal description of the Markov switching framework we distinguish between *forecasted*, *filtered* and *smoothed* inference about the state α_t, denoted as follows

$$a_{t|t-1} = E[\alpha_t|Y_{t-1}, \Theta] \tag{15}$$

$$a_{t|t} = E[\alpha_t|Y_t, \Theta] \tag{16}$$

$$a_{t|T} = E[\alpha_t|Y_T, \Theta]. \tag{17}$$

Conditional covariance matrices can be assigned to each expectation term above

$$P_{t|t-1} = Var[\alpha_t|Y_{t-1}, \Theta] \tag{18}$$

$$P_{t|t} = Var[\alpha_t|Y_t, \Theta] \tag{19}$$

$$P_{t|T} = Var[\alpha_t|Y_T, \Theta]. \tag{20}$$

For the derivation and deeper discussion of Kalman filter, it can referred for example to HAMILTON (1994) or DURBIN, COOPMAN (2001). As in the case of Markov switching the filtering algorithm can be used for the evaluation of the likelihood function under some distributional assumption, for example normal distribution

$$y_t|Y_{t-1} \sim N(a_{t|t-1}, P_{t|t-1})$$

which leads to the loglikehood function

$$logL(\Theta) = \sum_{t=1}^{T} logf(y_t|Y_{t-1}, \Theta). \tag{21}$$

For the empirical purposes of this paper it is assumed that the coefficient β follows a random walk. Now the equation (6) can be written as follows

$$i_t = [\bar{r} + \bar{\pi}_t + \beta_t(\pi_t - \bar{\pi}_t) + \gamma x_t] \cdot (1 - \rho) + \rho i_{t-1} + u_t \tag{22}$$

$$\beta_{t+1} = \beta_t + v_t, \;\; E[u_t v_t] = 0 \tag{23}$$

No correlation between u_t and v_t is allowed, otherwise the unsystematic monetary policy would influence the parameters of systematic monetary policy in the long run, which is not a very feasible assumption. The ML-estimation is performed using EVIEWS 4.1, which allows the estimation of a relatively large class of state space models.

3 Data

All estimations are performed for the period 1975(1)-1998(4) with quarterly German data. The day-to-day German market rate (so called *overnight rate*) is considered as the monetary policy instrument of the Bundesbank and is the dependent variable in all equations.

The output gap is an unobservable time series and has to be extracted from the available observable time series of the German GDP. The time series of the GDP used for the calculation of the output gap is seasonally adjusted by using X12-ARIMA (adjusted for outliers and calender effects) and is based on the ESVG 95 standard for GDP calculation. This time series, which was calculated backwards for West Germany until 1970, is available from the German Statistical Office since August 2003. From 1970 to 1990 the data are West German data, from 1991 onwards the data for Germany until the second quarter 2003 is used. The time series were chained using the relationship of the 1991 values of German real GDP to West German real GDP as a conversion factor. To calculate the output gap, the time series of the real GDP in logs is filtered using Hodrick-Prescott filter with the smoothing parameter $\lambda = 1600$. The difference between the real GDP in logs and the potential output is the resulting output gap. As a measure of inflation one-year growth rates[3] of the the German CPI time series are employed.

To perform the estimation of the policy rule with time dependent inflation target $\bar{\pi}_t$, the inflation target of the Bundesbank is used. It can be derived from the Bundesbank's formulation of its target for monetary growth as published in its monthly reports. These figures until 1993 are for example collected in Clarida (1996) and were also employed in this paper. The period from 1994 until 1998 was filled with figures from the monthly reports of the Bundesbank. Both time series, π_t and $\bar{\pi}_t$, are displayed in figure (1).

4 Empirical Results

As outlined above three specifications are estimated. The first specification is a special case of term (7) and is estimated with nonlinear least squares. Then, the Markov switching specification (12) and the state space model (22) follow. The results are reported below. *Durbin-Watson test statistic*, *Breusch-Godfrey Serial Correlation LM test statistic*, *Q-statistic* and *Jarque-Bera*

[3]Calculated as one-year percentage changes, $\pi_t = \frac{P_t - P_{t-4}}{P_{t-4}} \cdot 100$

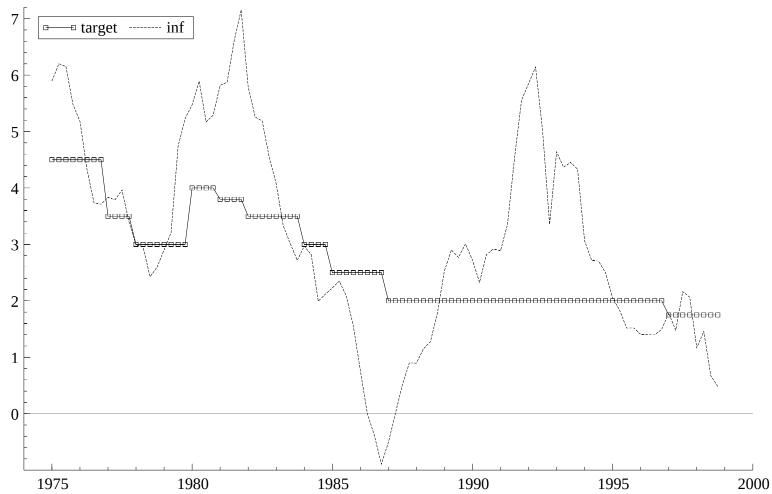

Figure 1: Inflation rate and inflation target

test statistic are denoted by DW, LM, Q and JB. The corresponding *p-values* are in braces. Standard errors, which corresponds to the estimated coefficients, can be found in brackets. Firstly, the estimated version of (7) is presented. The first and fourth lagged interest rate are found to be significant.

$$Estimation\ sample: 1976(1) - 1998(4)$$

$$\hat{i}_t = [\underset{(0.41)}{3.09} + \bar{\pi}_t + \underset{(\mathbf{0.31})}{\mathbf{0.65}}(\pi_t - \bar{\pi}_t) + \underset{(0.40)}{1.00} x_t] \cdot 0.15 + \underset{(0.06)}{1.00} i_{t-1} - \underset{(0.05)}{0.15} i_{t-4}$$
$$R^2 = 0.94,\ AIC = 1.80,\ DW = 2.04,\ LM(4) = 1.14[0.89],\ JB = 113.71[0.00] \quad (24)$$

The estimated value of the inflation coefficient is smaller than one, $\hat{\beta} < 1$. This result is unsatisfactory from the theoretical point of view. It indicates, that the Bundesbank does not sufficiently rise the short nominal rate i_t in response to a rise in the inflation gap and therefore no rise in the real interest rates results. Furthermore, the *Jarque-Bera* test statistic indicates nonnormality of residuals, which can be explained through the presence of outliers in the residuals.

Now, the Markov switching estimation is presented. In this case only the first lag of the short rate was significant and sufficient to remove the autocorrelation structure from the residuals.

$$Estimation\ sample: 1975(2) - 1998(4)$$

$$
\hat{i}_t = \begin{cases} [\underset{(0.42)}{2.44} + \bar{\pi}_t + \underset{(\mathbf{0.33})}{\mathbf{0.24}}(\pi_t - \bar{\pi}_t) + \underset{(0.40)}{1.64} x_t] \cdot 0.13 + \underset{(0.03)}{0.87} i_{t-1}, \text{ if } s_t = 1 \\ [\underset{(0.42)}{2.44} + \bar{\pi}_t + \underset{(\mathbf{2.37})}{\mathbf{9.34}}(\pi_t - \bar{\pi}_t) + \underset{(0.40)}{1.64} x_t] \cdot 0.13 + \underset{(0.03)}{0.87} i_{t-1}, \text{ if } s_t = 2 \end{cases}
$$

$$
logL = -71.15, \ AIC = 1.67, \ \hat{P} = \begin{bmatrix} 0.94 & 0.50 \\ 0.06 & 0.50 \end{bmatrix} \tag{25}
$$

$$
Q(1) = 2.67[0.10], \ Q(4) = 4.90[0.30], \ JB = 0.75[0.69]
$$

The estimated transition matrix is denoted by $\hat{P}$. The examination of the estimated coefficients shows, that the estimated regimes are very different. First of all, there is huge difference in the inflation aversion of the Bundesbank between these two regimes. In regime 1 ($s_t = 1$) there is no significant reaction to inflationary development at all. In contrast, regime 2 ($s_t = 2$) exhibits a very strong reaction to the inflation gap. Also a remarkable difference in the persistence of two regime is apparent. The conditional probability to stay in regime 1, $Pr\{s_t = 1|s_{t-1} = 1\}$, is 0.94 and in regime 2, $Pr\{s_t = 2|s_{t-1} = 2\}$, only 0.50. That means a much shorter expected duration of the high inflation aversion regime. It is also worth noting that in contrast to the nonlinear least squares estimation the *Jarque-Bera* test statistic indicates normality of the residuals. Obviously, the estimation of regime 2 with short duration removes some outliers from the residuals and on the other hand it simplifies the lag structure of the estimated policy rule. The fourth lag of the short-term interest rate is no more significant. In the figure (2) the time series of *smoothed* probabilities for regime 2 are displayed. However, the smoothed probabilities should be compared with the macroeconomic variables used in the estimation to obtain a possible economic interpretation. *Overnight* rate, inflation and rescaled smoothed probabilities from the previous figure are collected in figure (3). Obviously, the switches to the regime of high inflation aversion corresponds very well with the periods of strong interest rate rises: two times in the end of seventies and begin of eighties and one time in the end of eighties. Moreover, the figure (3) shows that these switches could have been caused by the sharp rises of inflation. Surprisingly, the deflation period in the second half of eighties does not lead to any sharp rise of the smoothed probabilities of regime 2. This leads to the conclusion, that the Bundesbank reacted asymmetrically to the fluctuations in the inflation gap. The Bundesbank pays a lot of attention to the positive deviations from the inflation target. On the other hand there are no signs, that the Bundesbank is very concerned about the negative deviations from its own target.

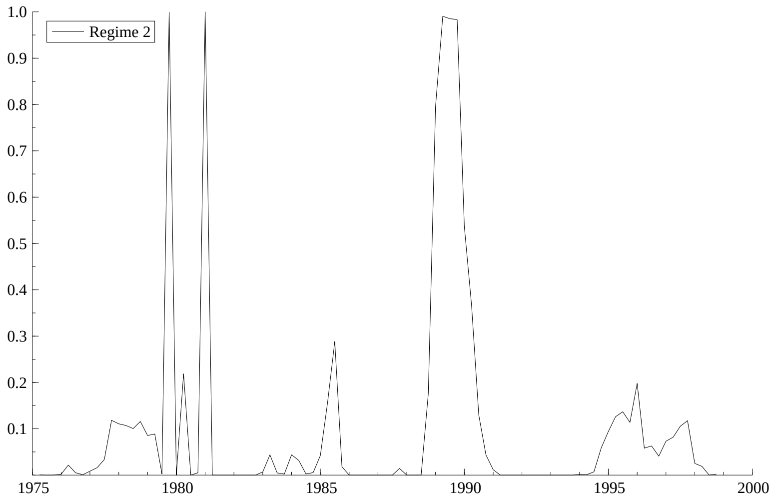

Figure 2: Smoothed probabilities for regime 2 from the equation (25)

The state space modelling with Kalman filter supports the results of the Markov switching estimation presented above. As outlined before it is assumed that the inflation coefficient in the monetary policy rule, according to the specification (22), follows a random walk (without drift). The model was estimated also with some other specifications for the state variable, like a stationary AR(1)-process, however, the estimation became unstable as the parameter number in the state equation growths while the the results remain equivalent from the qualitative point of view. Thus, only the estimated specification with a random walk as the state variable is presented in (26) below.

$$\begin{aligned} &Estimation\ sample : 1975(2) - 1998(4) \\ \hat{i}_{t|T} &= [\underset{(0.48)}{2.18} + \bar{\pi}_t + \hat{\beta}_{t|T}(\pi_t - \bar{\pi}_t) + \underset{(0.39)}{0.61}\, x_t] \cdot 0.15 + \underset{(0.05)}{0.85}\, i_{t-1} \\ &\hat{\sigma}^2_{signal} = e^{\hat{\gamma}_1},\ \hat{\gamma}_1 = -\underset{(0.16)}{1.53} \\ &\hat{\beta}_{t+1|T} = \hat{\beta}_{t|T} + \hat{v}_{t|T},\ \hat{\sigma}^2_v = e^{\hat{\gamma}_2},\ \hat{\gamma}_2 = \underset{(0.68)}{0.37} \\ &logL = -90.58,\ AIC = 2.01 \\ &Q(1) = 0.18[0.67],\ Q(4) = 5.29[0.26],\ JB = 17.83[0.00] \end{aligned} \tag{26}$$

First of all, the notation has to be explained. The smoothed state variable β_t given the estimated parameters is denoted by $\hat{\beta}_{t|T}$. In other words, $\hat{\beta}_{t|T}$ can be described as the estimated conditional expectation $E[\beta_t|Y_T, \hat{\Theta}]$ in termini of definition (18). Accordingly, the adjusted

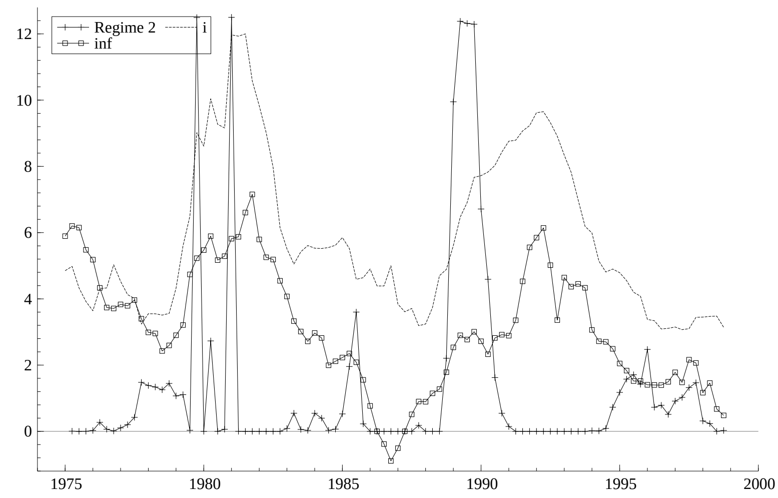

Figure 3: Overnight rate, inflation (boxes) and smoothed probabilities (pluses).

value of the nominal rate $\hat{i}_t$ is denoted by $\hat{i}_{i|T}$ if the smoothed state $\hat{\beta}_{t|T}$ is used to calculate it. In the same manner, $\hat{\beta}_{t|t-1}$ and $\hat{\beta}_{t|t}$ lead to $\hat{i}_{i|t-1}$ and $\hat{i}_{i|t}$.

Obviously, only a graphical illustration of the smoothed state $\hat{\beta}_{t|T}$ permits an economic interpretation of the estimated model (26). In the figure (4) the smoothed state variable $\hat{\beta}_{t|T}$, its confidence bounds and the smoothed probabilities (in other scaling) from (25) for comparison purposes are displayed. The confidence bounds of the smoothed state in the figure (4) are calculated as follows. Firstly, squared root of the diagonal elements of the estimated state covariance matrices $\hat{P}_{t|T}$ is calculated. $\hat{P}_{t|T}$ is defined as an estimated version of (18), $\hat{P}_{t|T} = Var[\beta_t|Y_T, \hat{\Theta}]$. The square root of the diagonal elements can be interpreted as root mean squared errors of $\hat{\beta}_{t|T}$. Secondly, the confidence bounds themselves are calculated as $\hat{\beta}_{t|T} \pm 2RMSE$. The confidence bounds calculated in such way tend to underestimate the uncertainty of the extracted state variable, because the uncertainty of the parameter estimation is not included.

The resulting accordance of the Markov switching and Kalman filter results is apparent. The fluctuations of the smoothed probabilities and smoothed state variable $\hat{\beta}_{t|T}$ are very similar. As in the case of the Markov switching specification there are two periods, when the reaction of the Bundesbank to inflationary developments is very strong: the time around 1980 and around 1990. Also the Kalman filter results can be interpreted in termini of an asymmetric reaction

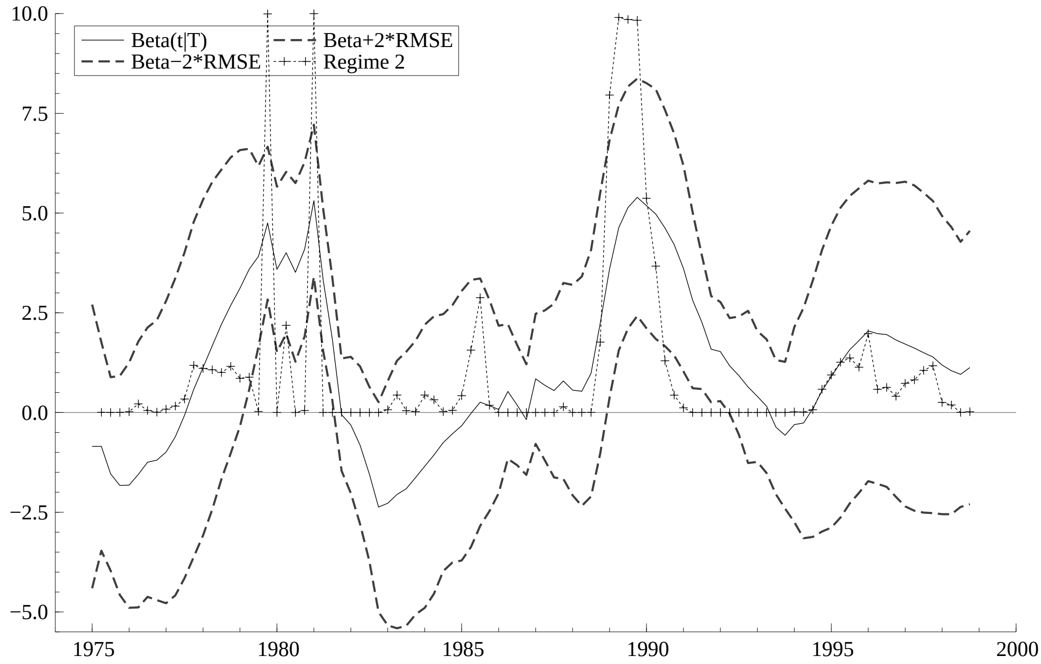

Figure 4: Smoothed state $\hat{\beta}_{t|T}$ from (26), smoothed probabilities from (25).

function of the Bundesbank. No significant rise of the smoothed state $\hat{\beta}_{t|T}$ is observed during the deflation phase in the second half of the eighties.

5 Concluding Remarks

The main empirical result of this paper is the instability of the Bundesbank reaction to inflationary developments in Germany. But there is evidence from both empirical methods used in the paper, that this is an instability only from a linear point of view because structural breaks found in the data do not have any irreversible nature and seem to be caused through the nonlinearity of the Bundesbank reaction function.

The instability and possible nonlinearity in the estimations can be interpreted as an asymmetry in the reaction function of the Bundesbank. In this case the Bundesbank strongly reacts to the positive deviation from its own inflation target. On the other hand during the deflation phase in the second half of the eighties no significant reaction to this development could be found.

The results in some other papers supports the results of the current work. For example, Clarida, Gertler (1996) found the estimated policy rule of the Bundesbank to be asymmetric. Clarida, Gertler (1996) estimate a Taylor-like reaction function with monthly German data and use the German industrial production to perform the output gap. They construct a dummy, which is one if the inflation gap is positive and zero otherwise. The estimated value of the inflation coefficient is $\hat{\beta} = 1.60$ in the periods of high inflation and $\hat{\beta} = 0.28$ otherwise. There are some possible economic explanations for the asymmetric reaction to the inflation fluctuations. For example, for some reasons the Bundesbank could have asymmetric preferences for fighting inflation. From the theoretical point of view the loss function has in this case no quadratic symmetric form. The implications of such theoretical setup can be found, for example, in Nobay, Peel (1998), Cuckierman (1999) or Gerlach (1999).

An asymmetry of the Bundesbank reaction function in respect to the inflationary developments may be caused by the existence of short-run nonlinear convex Phillips curve. The convexity of the Phillips curve implies that at any given point on the curve, the inflation increase associated with an incremental decline in the unemployment rate exceeds the inflation decline associated with an equal rise in the unemployment rate. The main difference between the linear and convex Phillips curves that in case of convexity, the short-run tradeoff facing policymakers is a function of the state of the economy: a one percentage point decline in the unemployment rate leads to a smaller increase in inflation given high unemployment rates than in case of low unemployment rates. There is some theoretical evidence that the convexity of Phillips curve may induce an asymmetric form of the loss function that the central bank chooses to minimize, which could result in the so called "opportunistic approach" to disinflation. The "opportunistic approach" to disinflation is a monetary policy strategy in which the central bank is fighting against any incipient rise in inflation, but waits for the next favorable inflation shock to lower inflation toward the target, rather than seeking to actively lower inflation in a manner that pushes the unemployment rate higher. The results of this paper may suggest the theoretical result. As outlined above the switches to the high inflation aversion regime could be caused by the sharp or long rises of inflation.[4] On the other hand there are no significant reaction to inflation during the phases of of its decline.

Some remarkable examples for the "opportunistic approach" can be found in Blinder (1997). A theoretical model of monetary policy incorporating opportunistic disinflation strategies was

[4]See figure (3).

introduced by ORPHANIDES AT AL. (1996A, B). The implications of the short-run convex Phillips curve are also investigated by TAMBAKIS (1998) and SCHALING (2000). DOLADO (2001) estimates a monetary policy rule under assumption of the convex Phillips curve.
An estimation with monthly data or a forward-looking specification could provide a starting point for further research. Another possibility to extend the current work is a comparison with the estimated monetary policy rules in other countries.

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