Tied Versus Untied Foreign Aid: Consequences for a Growing Economy

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Abstract

This paper contrasts the effects of tied and untied foreign aid programs on the welfare and macroeconomic performance of a small open economy. We show that the acceptance of tied aid inevitably obligates the recipient economy to undertake certain internal structural adjustments, and the flexibility it possesses to undertake these adjustments eventually determines the effectiveness of the aid program. The economic consequences of tied and untied aid programs, their relative merits from a welfare standpoint, and the transitional dynamics depend crucially upon several characteristics of the recipient economy that summarize this flexibility. These include: (i) the costs of installing public capital relative to private capital (intertemporal adjustment costs), (ii) the substitutability between factors of production (intratemporal adjustment costs), (iii) the flexibility of labor supply (work effort), (iv) the recipient's degree of access to the world financial markets (capital market imperfections), and (v) the recipient's opportunities for co-financing infrastructure projects by domestic resources.

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1. Introduction

Official development assistance, in the form of foreign aid or unilateral capital transfers, represents an important channel through which wealth is transferred from rich, developed nations to poorer, underdeveloped economies. Both the magnitude and the scope of these international transfers have increased significantly over the last four decades. For example, total flows of official development assistance from members of the OECD and OPEC countries have increased from about $6 million in 1965 to over $59 million in 1999. By that time these funds represented between 3-5 percent of the Gross National Income of the recipient low and middle income countries, and financed between 10-20 percent of their gross capital formation.[1]

One issue of concern for both donors and recipients is how foreign aid should be spent in an economy with scarce resources. Guided either by self-interest, or to prevent the possibility of mismanagement of external funds, donor countries often impose restrictions on how such aid can be used by the recipient. This has given rise to a long-standing debate, both in academic and policy circles, as to whether international transfers should be "tied" ("productive") or "untied" ("pure"). As Bhagwati (1967) points out, tied external assistance can take several forms. It may be linked to a (i) specific investment project, (ii) specific commodity or service, or (iii) procurement in a specific country. Recent studies by the World Bank, however, point out that over time, a larger proportion of foreign aid has become "untied" with respect to requirements for procuring goods and services from the donor country, but it has become more "tied" in the sense of being linked to investments in public infrastructure projects (telecommunications, energy, transport, water services, etc). For example, between 1994 and 1999, the proportion of official development assistance that was "untied" in the sense of not being subject to restrictions by donors on procurement sources rose from 66 percent to about 84 percent. At the same time, between two-thirds and three-fourths of official development assistance was either fully or partially tied to public infrastructure projects (see footnote 1). A recent example is the European Union's assistance programs, both to its member nations as well as aspiring members. These programs tied the flow of aid to the accumulation of

[1] World Bank (1994, 2001).

public capital, and were aimed at building up infrastructure in the recipient nation, thereby enabling it to attain a strong positive short-run growth differential relative to the EU average, achieve higher and sustainable living standards in alignment with EU standards, and ultimately gain accession to EU membership.[2]

The move toward tying more aid to public investment has been dictated mainly by the growing infrastructure requirements of developing countries. Most economists agree that investment in public infrastructure raises the productivity and efficiency of the private sector and, as a consequence, provides a crucial channel for economic growth, development, and higher living standards.[3] But financing the required investment in infrastructure has proven to be a challenging task for developing countries. Most such countries have significantly restricted public sector borrowing after the debt-crisis of the early 1980s, while at the same time their infrastructure requirements have increased steadily. A 1994 World Bank study has estimated these requirements to be $200 billion a year. Facing binding fiscal constraints, governments in developing countries have turned to external financing, in the form of tied unilateral capital transfers, as a significant source of financing public investment. This paper analyzes the consequences of such tied external assistance programs for the growth and macroeconomic performance of a developing economy.

Is tied foreign aid always beneficial for the recipient economy? To answer this question, one must acknowledge the fact that the acceptance of a tied transfer by the recipient inevitably obligates it to undertake some internal structural adjustments, and the flexibility it has to do this will determine the effectiveness (or otherwise) of the aid program. Thus the consequences of tied versus untied aid on the growth path of the economy, and their relative merits from a welfare standpoint, depend crucially upon a number of key structural characteristics that summarize this flexibility. These characteristics include: (i) the costs associated with installing the publicly provided capital

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[2] Greece, Ireland, Spain, and Portugal received unilateral capital transfers tied to public investment projects under the Structural Funds Program between 1989-1999. A similar tied transfer program, called Agenda 2000, has been initiated for eleven aspiring member nations (Central Eastern European Countries), and is expected to continue until 2006.

[3] Theoretical and empirical interest in the impact of public capital on private capital accumulation and economic growth originated with the work of Arrow and Kurz (1970) and more recently with Aschauer (1989a, 1989b). Most of this literature has focused on closed economies, using both the Ramsey model and the AK endogenous growth framework; see e.g. Futagami, Morita, and Shibata (1993), Baxter and King (1993), Fisher and Turnovsky (1998). Turnovsky (1997) extends Futagami et. al. to a small open economy and introduces various forms of distortionary taxation, as well as the possibility of both external and internal debt financing. Devarajan, Xie, and Zou (1998) address the issue of whether public capital should be provided through taxation or through granting subsidies to private providers.

(intertemporal adjustment costs), (ii) the substitutability between public and private capital in production (intratemporal adjustment costs), (iii) the degree of access to the world financial market (financial adjustment costs), and (iv) the opportunities for co-financing infrastructure projects by domestic resources, like the domestic government or private sector.[4]

These issues have been analyzed in the context of an endogenous growth model by Chatterjee, Sakoulis, and Turnovsky (2003), and Chatterjee and Turnovsky (2003). But these papers share one crucial restriction, namely that labor supply in the recipient economy is fixed inelastically. This paper redresses this shortcoming by introducing endogenously supplied labor. We show that since labor is also a (variable) factor of production like the two types of capital, the potential for substitution along the labor margin is also important in determining the dynamic effects of untied aid, and adds a further dimension to the debate on whether or not aid should be tied. Just as the endogeneity of labor supply has proven to be crucial in determining the nature of the economy's dynamic response to demand shocks in other contexts, introducing flexible labor supply is important in determining the dynamic response to an untied foreign aid shock, thus providing new insights into the contrasting responses to tied and untied foreign aid shocks.[5]

To capture the issues we wish to address, the model has a number of key characteristics. First, the production conditions are sufficiently flexible to accommodate both intratemporal and intertemporal factor substitution. We specify the former by assuming that labor interacts with public capital to yield "labor efficiency units", which then interact with private capital in accordance with a constant elasticity of substitution (CES) production function. Intertemporal substitution is captured by assuming that new investment in both types of capital involves convex installation costs. Indeed, the impact of foreign aid on the evolution of the economy depends not only on the short-run degree of factor substitutability, but also on the relative costs of adjustment of the two types of capital.

In the case of tied aid, the assistance is linked to the accumulation of public capital, and thus

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[4] It is entirely plausible that the large financial needs of developing countries for infrastructure investment will not be met by the flow of external assistance, and hence domestic co-financing assumes a lot of significance. Recently, in a panel study of 56 developing countries and six four-year periods (1970-93), Burnside and Dollar (2000) find that foreign aid is most effective when combined with a good policy environment in the recipient economy. Previously, Gang and Khan (1990) report that most bilateral aid for public investment in LDCs is tied, and is given on the condition that the recipient government devotes certain resources to the same project.

[5] In the Ramsey model, for example, an increase in government consumption expenditure causes immediate crowding out of private consumption, when labor supply is fixed; otherwise it leads to transitional dynamics; see Turnovsky (2000).

provides an important stimulus for private capital accumulation and growth. We assume further that public investment in infrastructure is financed both by the domestic government as well as via the flow of international transfers, thereby incorporating the important element of domestic co-financing, characteristic of the European Union's and other bilateral aid programs that are tied to specific public investment projects. In both cases, the transfers are assumed to be linked to the scale of the recipient economy and therefore are consistent with maintaining an equilibrium of sustained (endogenous) growth. The model is sufficiently general to include the possibility of a third source of financing public infrastructure, namely the private sector of the economy. By taxing private firms, and spending a fixed proportion of those taxes in financing new infrastructure, the government can ensure the private sector's participation in building up the economy's stock of infrastructure.[6]

We also assume that the small open economy faces restricted access to the world capital market in the form of an upward-sloping supply curve of debt, according to which the country's cost of borrowing depends upon its debt position, relative to its capital stock, the latter serving as a measure of its debt-servicing capability. This assumption is motivated by the large debt burdens of most developing countries, which give rise to the potential risk of default on international borrowing. Indeed, evidence suggesting that more indebted economies pay a premium on their loans from international capital markets to insure against default risk has been provided by Edwards (1984).

Thus, the paper extends and contributes to the literature on foreign aid and macroeconomic performance in several important directions. First, it analyzes the role of tied development assistance as a mode of financing public investment and its effect on the transitional adjustment path and its sensitivity to the structural conditions of a growing open economy. Second, by relaxing the assumption of inelastic labor supply, it provides new insights into how the representative agents' incentive to choose between work and leisure responds to tied and untied aid shocks, and how, in turn, that response impacts on the macroeconomic evolution of the economy, both in the short run as well as over time. Third, since it is likely that external assistance and borrowing will not meet the total financial needs for public investment, domestic participation by both the government and the

[6] The efficient use of infrastructure is a further important issue. For example, Hulten (1996) shows that inefficient use of infrastructure accounts for more than 40 percent of the growth differential between high and low growth countries.

private sector is also important. This paper specifically characterizes the consequences of domestic co-financing of public investment and outlines the trade-offs faced by a recipient government when it responds to a flow of external assistance from abroad. Finally, the question we address is also closely related to the "transfer problem", one of the classic issues in international economics, dating back to Keynes (1929) and Ohlin (1929). This early literature was concerned with "pure" transfers, which could be in the form of an unrestricted gift or as debt-relief. By contrast, our analysis focuses on "productive" transfers, the use of which is tied to public investment. The formulation we develop parameterizes the transfer so that we can conveniently identify the pure transfer and the productive transfer as polar cases.[7]

Given the complexity of the model, most of the analysis is conducted numerically. The main results of our analysis are the following. The effects of foreign aid depend critically on whether it is tied or untied. Chatterjee et al. (2003) and Chatterjee and Turnovsky (2003) show that when labor supply is inelastic, even though a tied aid shock generates a dynamic adjustment, an untied aid shock has no dynamic consequences for the recipient economy, and leads only to instantaneous increases in consumption and welfare. In contrast, we show that allowing the agent to adjust his work effort leads to fundamental differences in the economy's response. Under the assumption of flexible labor supply, both types of aid generate dynamic responses, albeit dramatically different in nature. In the case of untied aid, they occur primarily through the labor-leisure choice and the effect this has on the marginal rate of substitution between consumption and leisure and on the real wage rate. For plausible parameters, the economy's dynamic adjustment occurs rapidly with little effect on the stocks of capital. However, although the economy's current account and welfare improve in the long run, the reduced work effort and higher consumption leads to a decline in the equilibrium growth rate. On the other hand, an aid program that is tied to investment in public capital generates a much more gradual dynamic adjustment, one that is exactly opposite in nature to that following an untied aid program. There are significant trade-offs in welfare between the short run and the long run, as the agent increases his work effort and initially substitutes away from consumption toward

[7] Much of the discussion of the transfer problem (untied aid) focuses on the welfare effects, doing so in a static framework. A recent paper by Djajic, Lahiri, and Raimondos-Møller (1999) analyzes the welfare effects of temporary untied foreign aid in a two country-two period model.

investment. The benefits of this substitution are realized only gradually over time, as the investment in public capital enhances productivity in the recipient economy and thereby increases consumption. The implied long-run changes in the relative capital stocks in the recipient economy are dramatic.

The magnitude and the direction of the transitional dynamics and long-run effects depend crucially upon the elasticity of substitution between the two types of capital in the recipient economy. Our analysis suggests that tied aid is more effective in terms of its impact on long-run growth and welfare for countries that have low substitutability between factors of production. This finding has important policy implications, especially in light of recent empirical evidence suggesting that the elasticities of substitution for less developed or poor countries are significantly below unity.[8] We find that the welfare gains from a particular type of aid program (tied or untied) are sensitive to the costs of installing public capital and capital market imperfections, even for small changes in the degree of substitutability between inputs. Economies in which the elasticity of substitution between the two types of capital and the installation costs are relatively high are likely to find tied transfers to be welfare-deteriorating. For such economies untied aid will be more appropriate.

The remainder of the paper is structured as follows. Section 2 sets out the analytical structure and summarizes the macrodynamic equilibrium. Section 3 conducts numerical simulations and considers their implications, while Section 4 performs substantial sensitivity analysis. Section 5 briefly addresses the issue of co-financing, while Section 6 concludes and provides some policy advice. An Appendix provides the technical details underlying the derivation of the macrodynamic equilibrium.

2. The Analytical Framework

We begin by spelling out the building blocks of the model.

2.1. Private Sector

We consider a small open economy populated by an infinitely-lived representative agent who produces and consumes a single traded commodity. The agent has a unit of time, a fraction l of

[8] See Duffy and Papageorgiou (2000).

which can be devoted to leisure, and the balance, 1 – l to labor supply. Output, Y, of the commodity is produced using the Constant Elasticity of Substitution (CES) production function

$$Y = \alpha\left[\eta\left\{ (1-l)K_G\right\}^{-\rho} + (1-\eta)K^{-\rho}\right]^{-1/\rho} \tag{1a}$$

where K denotes the representative agent's stock of private capital, and K_G denotes the stock of public capital. The latter provides an externality, interacting with the agent's labor supply to yield labor measured in efficiency units, $(1-l)K_G$. The quantity $s \equiv 1/(1+\rho)$ measures the intratemporal elasticity of substitution between private capital and "efficiency units of labor" in production. The production function has constant returns to scale in both the private factors of production, K and $(1-l)$, and the accumulating factors, K, K_G, enabling it to support an equilibrium of ongoing growth with both private factors being paid their respective marginal physical products.

The agent consumes this good at the rate C, yielding utility over an infinite horizon represented by the isoelastic utility function:[9]

$$U \equiv \int_0^{\infty} \frac{1}{\gamma}\left(Cl^{\theta}\right)^{\gamma} e^{-\beta t} dt \tag{1b}$$

where θ represents the relative importance of leisure in utility. The agent also accumulates physical capital, with expenditure on a given change in the capital stock, I, involving adjustment (installation) costs specified by the quadratic (convex) function

$$\Psi(I,K) = I + h_1\frac{I^2}{2K} = I\left(1 + h_1\frac{I}{2K}\right) \tag{1c}$$

This equation is an application of the familiar cost of adjustment framework, where we assume that the adjustment costs are proportional to the *rate* of investment per unit of installed capital (rather than its level). The linear homogeneity of this function is necessary for a steady-state equilibrium having ongoing growth to be sustained. The net rate of capital accumulation is thus

$$\dot{K} = I - \delta_K K \tag{1d}$$

[9] The exponent γ is related to the intertemporal elasticity of substitution e, by $e = 1/(1-\gamma)$, with $\gamma = 0$ being equivalent to a logarithmic utility function.

where δ_K denotes the rate of depreciation of private capital.

Agents may borrow internationally on a world capital market. The key factor we wish to take into account is that the creditworthiness of the economy influences its cost of borrowing from abroad. Essentially, we assume that world capital markets assess an economy's ability to service debt costs and the associated default risk, the key indicator of which is the country's debt-capital (equity) ratio. As a result, the interest rate countries are charged on world capital markets increases with this ratio. This leads to the upward sloping supply schedule for debt, expressed by assuming that the borrowing rate, $r(N/K)$, charged on (national) foreign debt, *N,* relative to the stock of private capital, *K*, is of the form:

$$r(N/K) = r^* + \omega(N/K); \quad \omega' > 0 \tag{1e}$$

where r^* is the exogenously given world interest rate, and $\omega(N/K)$ is the country-specific borrowing premium that increases with the nation's debt-capital ratio. The homogeneity of the relationship is required to sustain a balanced growth equilibrium.[10]

The agent's decision problem is to choose consumption, labor supply, and the rates of capital and debt accumulation, to maximize intertemporal utility (1b) subject to the flow budget constraint

$$\dot{N} = C + r\left(N/K\right)N + \Psi\left(I, K\right) - (1-\tau)Y + \overline{T} \tag{2}$$

where *N* is the stock of debt held by the private sector, τ is the income tax rate, and $\overline{T}$ denotes lump-sum taxes.[11] It is important to emphasize that in performing his optimization, the representative agent takes the borrowing rate, $r(.)$ as given. This is because the interest rate facing the debtor

[10] A rigorous derivation of (1e) presumes the existence of risk. Since we do not wish to model a full stochastic economy, we should view (1e) as representing a convenient reduced form, one supported by empirical evidence; see e.g. Edwards (1984) who finds a significant positive relationship between the spread over LIBOR (e.g. r^*) and the debt-GNP ratio. Aizenman and Turnovsky (2002) provide a formal justification for the relationship (1e) in a world in which lenders are subject to default risk. Various formulations of reduced form relationships such as (1e) can be found in the literature. The original formulation by Bardhan (1967) expressed the borrowing premium in terms of the absolute stock of debt; see also Obstfeld (1982), Bhandari, Haque, and Turnovsky (1990). Other authors such as Sachs (1984) also argue for a homogeneous function such as (1e). We have also adopted different specifications, including the Edwards (1984) formulation, $r = r(N/Y)$, and $r = \left(N/(K + K_G)\right)$, so that both private and public capital serve as collateral. In both cases similar results to those reported are obtained.

[11] It is natural for us to assume $N > 0$, so that the country is a debtor nation. However, it is possible for $N < 0$ in which case the agent accumulates credit by lending abroad. For simplicity, interest income is assumed to be untaxed.

nation, as reflected in its upward sloping supply curve of debt, is a function of the economy's *aggregate* debt-capital ratio, which the individual agent assumes he is unable to influence.

The optimality conditions with respect to the individual's choices of C, l, and I are

$$C^{\gamma-1}l^{\theta\gamma} = \lambda \tag{3a}$$

$$\theta C^{\gamma}l^{\theta\gamma-1} = \lambda\left(1-\tau\right)\frac{\partial Y}{\partial\left(1-l\right)} \tag{3b}$$

$$1+h_1\left(I/K\right)=q \tag{3c}$$

where λ is the shadow value of wealth in the form of internationally traded bonds, q' is the shadow value of the agent's private capital stock, and $q = q'/v$ is defined as the market price of private capital in terms of the (unitary) price of foreign bonds. Equation (3a) equates the marginal utility of consumption to the shadow value of wealth, while (3b) equates the marginal utility of leisure to the shadow value of after-tax income foregone, where $\partial Y/\partial\left(1-l\right) \equiv (\eta/\alpha^{\rho})(Y^{1+\rho}/\left(1-l\right)\left\{\left(1-l\right)K_G\right\}^{\rho})$ is the marginal product of labor and equals the equilibrium wage rate. It is well known that as θ increases, the equilibrium elasticity of labor supply declines. The third equation equates the marginal cost of an additional unit of investment, inclusive of the marginal installation cost, $h_1 I/K$ to the market value of capital. Equation (3c) may be immediately solved to yield the following expression for the growth rate of private capital

$$\frac{\dot{K}}{K}\equiv\psi_K=\psi_K(q)=\frac{q-1}{h_1}-\delta_K \tag{3c'}$$

Applying the standard optimality conditions with respect to N and K leads to the usual arbitrage relationships, equating the rates of return on consumption and investment in private capital to the costs of borrowing from abroad

$$\beta-\frac{\dot{\lambda}}{\lambda}=r\left(N/K\right) \tag{4a}$$

$$\frac{\alpha\left(1-\tau\right)\left(1-\eta\right)}{q}\left[\eta\left\{\left(1-l\right)\left(K_G/K\right)\right\}^{-\rho}+\left(1-\eta\right)\right]^{-(1+\rho)/\rho}+\frac{\dot{q}}{q}+\frac{\left(q-1\right)^2}{2h_1q}-\delta_K=r\left(N/K\right) \tag{4b}$$

Finally, in order to ensure that the agent's intertemporal budget constraint is met, the following transversality conditions must hold:

$$\lim_{t\to\infty} \lambda B e^{-\beta t} = 0;\ \lim_{t\to\infty} q' K e^{-\beta t} = 0. \tag{4c}$$

2.2 Public Capital, Foreign Aid, and National Debt

The resources for the accumulation of public capital come from two sources: domestically financed government expenditure on public capital, $\bar{G}$, and a program of capital transfers or foreign aid, *TR*, from the rest of the world. We therefore postulate

$$G \equiv \bar{G} + \phi TR \qquad\qquad 0 \le \phi \le 1 \tag{5}$$

where ϕ represents the degree to which the transfers from abroad are tied to investment in the stock of public infrastructure. The case $\phi = 1$ implies that transfers are completely tied to investment in public capital, representing a "productive" transfer. In the other polar case, $\phi = 0$, transfers are completely unrestricted and hence represent a "pure" transfer, of the Keynes-Ohlin type.

We assume that the gross accumulation of public capital, *G*, is also subject to convex costs of adjustment, similar to that of private capital[12]

$$\Gamma(G, K_G) = G\left(1 + (h_2/2)(G/K_G)\right).$$

In addition, the public capital stock depreciates at the rate, δ_G, so that its net rate of accumulation is

$$\dot{K}_G = G - \delta_G K_G . \tag{6}$$

To sustain an equilibrium of on-going growth, both domestic government expenditure on

[12]Note that there are different ways of specifying how aid is tied. The specification (5) does so by relating it to the accumulation of new public capital. An alternative formulation is to tie the aid to total investment costs, inclusive of installation costs, replacing (5) by $\Omega(G, K_G) - \Omega(\bar{G}, K_G) = \phi TR$. The difference between these two specifications is minor and is as follows. Equation (5) implies that to the extent that the transfer is tied in this way $(\phi > 0)$, a larger transfer increases installation costs that must be financed by some other domestic source, leading to the crowding out of private consumption, and thus reducing the benefits from the foreign transfer. According to the alternative specification, higher installation costs imply that more of the transfer is committed to installing the capital, leaving less available for the accumulation of new capital. These two specifications thus imply analogous tradeoffs between the rate of accumulation of new public capital and its associated installation costs, and thus have similar implications. Since there is no compelling evidence favoring one formulation over the other, we adopt (5), which turns out to be marginally simpler.

infrastructure ($\bar{G}$) and the flow of aid from abroad must be tied to the scale of the economy

$$\bar{G} = \bar{g}Y, \text{ and } TR = \sigma Y, \quad 0 < \bar{g} < 1, \ \sigma > 0, \quad 0 < \bar{g} + \sigma < 1$$

Substituting into (5) and then into (6), we can rewrite the latter in the following form

$$\dot{K}_G = \left(\bar{g} + \sigma\phi\right)Y - \delta_G K_G \, ; \tag{6'}$$

and thus the growth rate of public capital is given by

$$\frac{\dot{K}_G}{K_G} \equiv \psi_G = \left(\bar{g} + \sigma\phi\right)\frac{Y}{K_G} - \delta_G \, . \tag{6}$$

The government sets its tax and expenditure parameters to continuously maintain a balanced budget:

$$\tau Y + TR + \bar{T} = \Gamma(G, K_G) \tag{7}$$

The national budget constraint, (the current account) is obtained by combining (7) and (2),

$$\dot{N} = r\left(N/K\right)N + C + \Psi\left(I, K\right) + \Gamma\left(G, K_G\right) - Y - TR \, . \tag{8}$$

Equation (8) states that the economy accumulates debt to finance its total expenditures on public capital, private capital, consumption, and interest payments net of output produced and transfers received. It is immediately apparent that higher consumption or investment raises the rate at which the economy accumulates debt. The direct effect of a larger unit transfer on the growth rate of debt is given by $(\phi - 1) + \left(h_2/K_G\right)\phi G$. An interesting observation is that the more transfers are tied to public investment (the higher ϕ), the lower the decrease in the growth rate of debt. When transfers are completely tied to investment in infrastructure, i.e., ϕ=1, debt increases due to higher installation costs. However, the indirect effects induced by the change will still need to be taken into account.

2.3. Macroeconomic Equilibrium

The steady-state equilibrium of the economy has the characteristic that all real aggregate quantities grow at the same constant rate, and that the labor allocation, *l*, and the relative price of capital, *q*, are constant. We show in the Appendix how the equilibrium dynamics of the system can be conveniently expressed in terms of the following stationary variables, $z \equiv K_G/K$, $n \equiv N/K$, both normalized by the stock of private capital, and l and q. The equilibrium system can be described by:

$$\frac{\dot{z}}{z} = \left(\bar{g} + \sigma\phi\right)\frac{y}{z}\delta_G - \left(\frac{(q-1)}{h_1} - \delta_K\right) \tag{9a}$$

$$\frac{\dot{n}}{n} = r(n) + \frac{1}{n}\left[c + \frac{q^2-1}{2h_1} + \left\{\left(\bar{g}+\sigma\phi\right) - \left(1+\sigma\right)\right\}y + \frac{h_2}{2}\left(\bar{g}+\sigma\phi\right)^2\frac{y^2}{z}\right] - \left(\frac{(q-1)}{h_1} - \delta_K\right) \tag{9b}$$

$$\dot{q} = r(n)q - \alpha(1-\tau)(1-\eta)\left[(1-\eta) + \eta k^{-\rho}\right]^{-(1+\rho)/\rho} - \frac{(q-1)^2}{2h_1} + \delta_K q \tag{9c}$$

$$\dot{l} = \left(\frac{\{1+\Omega(z,l)\}\{\beta - r(n)\} + (1-\gamma)\{\Omega(z,l)(1+\rho)\psi_K + (1-\rho\,\Omega(z,l))\psi_G\}}{\left[\{\gamma(1+\theta)-1\}\{1+\Omega(z,l)\} - (1-\gamma)(1+\rho)\Omega(z,l)\left(\frac{l}{1-l}\right)\right]}\right)l \tag{9d}$$

where

$$\Omega(z,l) \equiv \left((1-\eta)/\eta\right)\left[(1-l)z\right]^{\rho} \tag{10a}$$

$$\frac{Y}{K} \equiv y = y(z,l) = \alpha\left[(1-\eta) + \eta\left\{(1-l)z\right\}^{-\rho}\right]^{-1/\rho} \tag{10b}$$

$$\frac{C}{K} \equiv c = c(z,l) = \frac{(1-\tau)}{\theta}\left(\frac{l}{1-l}\right)\left(\frac{1}{1+\Omega}\right)y \tag{10c}$$

and the growth rates of the two types of capital are

$$\frac{\dot{K}}{K} \equiv \psi_K = \frac{q-1}{h_1} - \delta_K \tag{11a}$$

$$\frac{\dot{K}_G}{K_G} \equiv \psi_G = \left(\bar{g} + \sigma\phi\right)\frac{y}{z} - \delta_G. \tag{11b}$$

Equations (9a) – (9d) provide an autonomous set of dynamic equations in z, n, l, and q of which, two (k,n) are state variables, while the remaining two (q,l) are "jump" variables, free to respond instantaneously to new information as it becomes available. Once z and l are known, the output-capital ratio and the consumption-output ratio are determined in accordance with (10b) and (10c).

The economy reaches steady state when $\dot{z} = \dot{n} = \dot{l} = \dot{q} = 0$. Applying these conditions in (9a) - (9d) we can determine $\tilde{z}$, $\tilde{q}$, $\tilde{n}$, and $\tilde{l}$, along with the steady-state interest rate $\tilde{r}(\tilde{n})$ and the long-run growth rate $\tilde{\psi}$. Given $\tilde{z}$ and $\tilde{l}$, (10c) yields $\tilde{c}$. The explicit solution for the steady-state equilibrium is set out in the Appendix. Because this system is highly non-linear, it need not be consistent with a well-defined steady-state equilibrium with $\tilde{z} > 0, \tilde{c} > 0$. Our numerical simulations, however, yield well-defined steady-state values for all plausible specifications of all the structural and policy parameters of the model.

Linearizing (9a) – (9d) around the steady-state yields the local dynamic system

$$\dot{\underline{\mathrm{X}}} = \Lambda \left(\underline{\mathrm{X}} - \underline{\tilde{X}} \right) \tag{12}$$

where $\underline{\mathrm{X}}' = (z, n, l, q)$, $\underline{\tilde{X}}' = (\tilde{z}, \tilde{n}, \tilde{l}, \tilde{q})$, and Λ represents the coefficient matrix of the linearized system, defined explicitly in the Appendix. The determinant of the coefficient matrix of (12) can be shown to be positive under the condition that $\tilde{r}(\tilde{n}) > \tilde{\psi}$ i.e., the steady-state interest rate facing the small open economy must be greater than the steady-state growth rate of the economy. Imposing the transversality condition (4c), we see that this condition is indeed satisfied. Since (12) is a fourth-order system, a positive determinant implies that there could be 2 or 4 positive (unstable) roots. In order to yield a saddlepoint-stable solution, we require that there be two unstable roots, to match the two jump variables. Our numerical simulations yield saddle-point stable behavior for all plausible ranges of parameters, with two positive (unstable) and two negative (stable) roots, the latter being denoted by μ_1 and μ_2, with $\mu_2 < \mu_1 < 0$.

Equations (9) and (10) represent "core" dynamic equations from which other key variables, including the various growth rates, may be derived. We have already noted how the growth rates of the two capital goods can be immediately inferred from (11a) and (11b). In addition, the growth rates of consumption and output are given by

$$\frac{\dot{C}}{C} \equiv \psi_C = \frac{r(n) - \beta + \gamma\theta(1/l)\left[F(z,n,q,l)/G(z,l)\right]}{1-\gamma} \tag{11c}$$

$$\frac{\dot{Y}}{Y} \equiv \psi_Y = \frac{1}{1+\Omega(z,l)}\left[\Omega(z,l)\psi_K + \psi_G - \frac{\dot{l}}{1-l}\right] \tag{11d}$$

where $F(.), G(.)$ are defined in the Appendix. Although the growth rates diverge during the transition, they ultimately converge to the common equilibrium rate $\tilde{\psi}_K = \tilde{\psi}_G = \tilde{\psi}_C = \tilde{\psi}_Y = \tilde{\psi}$.

3. The Dynamic Effects of Foreign Aid: A Numerical Analysis

Due to the complexity of the model, we will employ numerical methods to examine the dynamic effects of a foreign aid or a transfer shock. We begin by calibrating a benchmark economy, using the following parameters representative of a small open economy, which starts out from an equilibrium with zero transfers/aid.

The Benchmark Economy

Preference parameters:	γ= -1.5, β= 0.04, $\theta = 1$
Production parameters:	α= 0.6, η= 0.2, h_1 =15, h_2 = 15
Elasticity of substitution in production:	s = 1
Depreciation rates:	δ_K = 0.05, δ_G = 0.05
World interest rate:	r^* = 0.06,
Premium on borrowing:	a = 0.15[13]
Policy parameters:	τ = 0.15, $\bar{g}$= 0.05
Transfers:	σ = 0, ϕ= 0

Our choices of preference parameters β, γ, and depreciation rates, δ_K, δ_G, the world interest rate, r^* are standard, while α is a scale variable. The productive elasticity of public capital $\eta = 0.2$ is consistent with the empirical evidence (see Gramlich, 1994). But given the introduction of labor in efficiency units, this implies the productive elasticity of labor is also 0.2, while that of private capital is 0.8. An inevitable feature of calibrating a Romer (1986) - type AK model is that keeping the size of the externality plausible, while maintaining the assumption of constant returns to scale in the private factors, imposes constraints on the elasticities on labor and private capital. In order to reconcile these elasticities with the empirical evidence on the income shares of labor and private capital, it is necessary to interpret K as an amalgam of physical and human capital, with $(1-l)$ describing "raw" unskilled labor; see Rebelo (1991). The borrowing premium $a = 0.15$ is chosen to

[13] The functional specification of the upward sloping supply curve that we use is: $r(n) = r^* + e^{an} - 1$. Thus, in the case of a perfect world capital market, when a = 0, $r = r^*$, the world interest rate.

ensure a plausible equilibrium national debt to output ratio. The elasticity on leisure, θ, is the crucial determinant of the equilibrium labor-leisure allocation and has too been set to ensure that this is empirically plausible. The tax rate is set at $\tau = 0.15$, while the rate of government expenditure on public investment is assumed to be $\bar{g} = 0.05$. The choice of adjustment costs is less obvious and our choice $h_1 = 15$ lies in the consensus range of 10 to 16.[14] We have also assumed smaller values of h_i, with little change in results. Note also that the equality of adjustment costs between the two types of capital serves as a plausible benchmark.

Setting $s = 1$, the benchmark technology is thus Cobb-Douglas, and the equilibrium derived from the above parameter specification is reported in Table 1, Row 1.[15] It implies a steady-state ratio of public to private capital of 0.25; the consumption-output ratio is 0.60, the debt to GDP ratio of 0.42, leading to an equilibrium borrowing premium of 2.13% over the world rate of 6%. The capital-output ratio is around 2.5, while 78% of the agent's time is allocated to leisure, consistent with empirical evidence, yielding a long-run growth rate of around 1.65%. This equilibrium is a reasonable characterization of a small-medium indebted economy, experiencing a modest steady rate of growth and having a relatively small stock of public capital. The critical parameters upon which we focus are (i) the elasticity of substitution in production, *s*, and (ii) the elasticity of leisure in utility, θ. In Table 2, we allow these to vary between $s = 0.8 - 1.6$, $\theta = 0.5 - 5$, for both tied and untied transfers, respectively.[16] We also consider changes in h_2 and *a*.

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[14]For example, Origueira and Santos (1997) choose h_1 = 16 on the grounds that it generates a plausible speed of convergence leads to Auerbach and Kotlikoff (1987) assume h_1 = 10, recognizing that this is at the low values of estimates, while Barro and Sala-i-Martin (1995) propose a value above 10. Values of h_i in this range yield equilibrium values of the "Tobin q" in the empirically plausible range of 1.3 - 1.4.

[15] The original specification of the CES production function was in terms of capital and raw labor. Extensive empirical evidence on the elasticity of substitution between these two inputs was produced during the 1960's and 1970's. Berndt (1976) provides a reconciliation between alternative estimates for the aggregate production function, concluding that they generally range between around 0.8 and 1.2, thus suggesting that the Cobb-Douglas serves as a reasonable benchmark. In a recent panel study of 82 countries over a 28-year period, Duffy and Papageorgiou (2000) find that they can reject the Cobb-Douglas specification for the entire sample in favor of the CES function. They also report that the degree of substitution between inputs (in their case human and physical capital) may vary with the stages of development. Empirical evidence on the substituability between public and private capital is sparse. Lynde and Richmond (1993) introduce public and private capital into a more general translog production function for U.K. manufacturing and find that the Cobb-Douglas specification is rejected.

[16]Because the derivation of the CES production function involves two arbitrary constants of integration, there are many different specifications of the CES function, which differ by how these constants are determined; see Klump and Preisler (2000). In our specification, we assume that these constants are set so that α, η remain constant, independent of the elasticity of substitution. But it is also possible for these parameters to depend upon *s*, in which case as *s* is changed, these parameters would change correspondingly.

3.1. A Permanent Increase in the Flow of Foreign Aid: Long Run Effects

We now consider the introduction of a permanent foreign aid flow to the above benchmark Cobb-Douglas economy. Specifically, the foreign aid is tied to the scale of the recipient economy, and increases from 0% of GDP in the initial steady-state to 5% of GDP in the new steady-state (an increase in σ from 0 to 0.05). However, this aid may be tied to new investment in public capital (ϕ = 1), representing the case of a "productive" transfer, or it may be untied (ϕ = 0), representing the case of a "pure" transfer from abroad. The long-run and short-run responses of key variables in the recipient economy are reported in Rows 2 and 3 in Tables 1a and 1b. In addition, the final columns in the tables summarize the effects on long-run welfare, $[\Delta W]$, and short-run welfare, $[\Delta W(0)]$, both measured by the optimized utility of the representative agent where C and l are evaluated along the equilibrium path. These welfare changes are measures of equivalent variation, calculated as the percentage change in the initial stock of capital necessary to maintain the level of welfare unchanged following the particular shock. The differences between the two types of transfer are dramatic.

We first consider the long-run effects of an increase in foreign aid (Table1a) and then discuss the short-run transitional dynamics generated by this shock (Table 1b and Figure 1).

3.1.1 Tied Aid

The long run impact of a tied capital transfer is reported in Row 2 of Table 1a. Since the aid is tied directly to investment in public capital, in the new steady state the ratio of public to private capital increases dramatically from 0.25 to 0.54, as a consequence of the investment boom in infrastructure. The increase in the stock of public capital increases the marginal productivity of private capital and labor, thereby leading to a positive, though lesser, accumulation of private capital and increasing employment time from 0.220 to 0.232. Although the transfer stimulates consumption through the wealth effect, (like the pure transfer) the higher long-run productive capacity has a greater effect on output, leading to a decline in the long-run consumption-output ratio from 0.60 to 0.563. The higher productivity raises the long-run growth rate to 2.31%, while long-run welfare improves by 7.96%, as indicated in the last column of Row 2. The increased accumulation of both private and public capital lead to a higher demand for external borrowing as a means of financing new investment in private capital and the installation costs of public capital. This results in an increase in the steady state debt-output ratio from 0.42 to 0.62, raising the borrowing premium to nearly 3.8%. However, this higher debt relative to output is sustainable since it is caused by higher investment demand rather than by higher consumption demand. The long run increase in the economy's productive capacity (as measured by the higher stocks of public and private capital, and output) ensures that the higher debt is sustainable.[17]

3.1.2 Untied Aid

A permanent pure transfer shock, i.e., an aid flow not tied to any investment activity, has precisely the opposite qualitative effects. With the exception of the effects on consumption and leisure, the changes are much smaller in magnitude. Being untied, the transfer is devoted to debt reduction, which allows an increase in consumption, with the debt to income ratio declining to 0.396 and the consumption-output ratio rising to around 0.65. The increase in consumption raises the marginal utility of leisure, increasing the fraction of time devoted to leisure from around 0.78 to 0.793. With the aid being untied, there is no incentive to invest and the ratio of public to private capital remains virtually unchanged. With the shift toward more consumption and leisure, productivity of both types of capital decline and the equilibrium growth rate is marginally reduced from 1.65% to 1.60%, leading to an overall increase in welfare of around 7.71%, marginally less than for the tied transfer.

3.2 Transitional Dynamics

3.2.1 Tied Aid

The transitional adjustment paths following the increase in tied aid are illustrated in Figure 1 for the benchmark economy. Fig. 1.1 illustrates the stable adjustment-locus in z-n space, indicating how z and n both generally increase together during the transition.

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[17] This view has also been expressed by Roubini and Wachtel (1998).

The immediate effect of the tied transfer is to raise the growth of public capital, to above 8% on impact, thereby raising the productivity of both private capital and labor. Given the cost of borrowing, the higher return to capital causes an instantaneous upward jump in the shadow price of private capital, q, from its initial benchmark level of 2 to 2.04, thereby inducing a corresponding increase in private investment. At the same time, the higher productivity of labor induces an immediate, but slight, decline in leisure from 0.780 to 0.777. While the upward jump in q reduces the rate of return on private capital, the increase in labor raises the return. On balance, the former slightly dominates and immediately after its initial increase, q begins to drop slightly to around 2.03, after the first five periods. Leisure drops steadily toward its new equilibrium level of 0.768, so that after a few periods its positive productivity effect dominates, and q begins to rise monotonically toward its new equilibrium level of 2.10; see Figs. 1.2 and 1.3.

The introduction of the tied transfer leads to an initial short-run decline in the consumption-output ratio [Fig. 1.4]. This is because the short-run substitution from leisure to labor both increases output and reduces the marginal utility of consumption. Thereafter, as the larger capital stocks are reflected in more output, the consumption-output ratio continues to decline monotonically toward its new long-run equilibrium value; leisure and the consumption-output ratio move together. The contrasting time paths of the four growth rates, ψ_K, ψ_G, ψ_Y, and ψ_C during the transition toward their common long-run growth rate, of 2.31% is strikingly illustrated in Fig. 1.6. With public capital being directly stimulated by the transfer, its growth rate jumps initially to over 8.3% before gradually declining. By contrast, private capital increases only very gradually from 1.95% to 2.31% during the transition, as the accumulation of public capital enhances its productivity. The growth rate of output is a weighted average of the growth rates of the two capital stocks (plus the temporary growth of labor, which is small) and thus it immediately increases sharply to 3.5% with the transfer. On the other hand, the only influence on the initial growth rate of consumption is the (small) effect that operates through the labor supply and the labor-leisure choice, raising its growth rate from 1.65% to 1.87%. Thereafter it responds only gradually, in response to the accumulation of assets in the economy. It always lies below the growth rate of output, so that C/Y is falling, as noted in Fig. 1.4. However, the level of consumption is still growing, albeit at a modest rate.

The final aspect of the dynamics concerns the debt-output ratio. Starting at 0.416, the short-run increase in output leads to a very slight initial decline in the debt-output ratio, after which it increases monotonically through time. This is because the accumulation of public capital raises the average productivity of private capital, while the accumulation of both types of capital raises the need to borrow from abroad. However, the higher debt, being backed by a higher productive capacity through the tied transfer is sustainable.

3.2.2 Untied Aid

The transitional dynamics following an untied transfer are illustrated in Figure 2 and three points should be made at the outset. First, the existence of the transitional dynamics depends crucially upon the endogeneity of labor supply. If labor supply is inelastic, then an untied transfer has no dynamic effects and the economy moves instantaneously to its new steady-state; see Chatterjee et al. (2003). Second, the dynamics are in sharp contrast to those of the tied transfer, being more or less a reversal. This reflects the fact, noted in Table 2 that the long-run responses of the economy are generally in the opposite direction. Third, the dynamics are generally much more rapid than in response to a tied transfer. Thus, for example, labor supply and the consumption-output ratio almost complete their entire adjustment on impact.

Fig. 2.1 illustrates the transitional adjustment paths for the two state variables, debt/private capital and public capital/private capital. We see that on receipt of the transfer, these move in opposite directions, implying that on impact the debt-capital ratio begins to decline, while the public-private capital ratio begins to increase. Indeed, the untied transfer is initially applied primarily to debt reduction, which allows an immediate substantial increase in consumption, increasing the marginal utility of leisure, and thus inducing an immediate sharp reduction in labor supply.

The main impact of an untied transfer is on consumption, leisure, and debt reduction, as illustrated in Figs. 2.3, 2.4 and 2.5. Its initial impact is to raise the marginal utility of leisure causing a reduction in labor supply, and hence in the productivity of private capital, public capital, and in q. The receipt of the untied transfer has a slightly less adverse short-run effect on the growth rate of public capital, reducing it to 1.57%, slightly above that of private capital, 1.55%. As z increases, the productivity of public capital declines relative to private capital, causing their relative growth rates to reverse. After four periods the growth rate of private capital exceeds that of public capital and z begins to decline with n. The decline in q is partially reversed during the subsequent transition as the relative stock of public to private capital declines.

Although, the overall intertemporal welfare gains for the two types of transfers are comparable (7.96% vs. 7.71%), this contrast in the dynamics leads to a sharp contrast in the time-profile that the benefits yield. In the case of the tied transfer, the initial commitment toward public investment involves consumption losses and less leisure, leading to a short-run welfare loss of around 1.53%. Over time, as the fruits of the investment are borne, and the economy becomes more productive, consumption increases rapidly. Welfare increases rapidly over time, with subsequent gains offsetting the initial losses, resulting in an overall intertemporal welfare increase of nearly 8%. The response to an untied transfer is a much more uniform increase in consumption and leisure, resulting in an almost constant improvement in welfare, though of a slightly smaller magnitude.

4. Sensitivity Analysis

The contrast between the tied and untied transfers is striking. It is therefore important to determine how sensitive this is to the chosen parameter values for the benchmark economy. This is investigated in the results summarized in Tables 2 – 4, where the sensitivity in the following dimensions is considered:

(i) elasticity of substitution in production (s);

(ii) flexibility of labor supply (θ);

(iii) cost of installing public capital, relative to that of private capital (h_2);

(iv) cost of borrowing from international capital markets (a).

4.1 Elasticity of substitution in production (s) versus flexibility in labor supply (θ)

Table 2 presents a grid summarizing the changes in key variables in response to equal amounts of tied aid and untied aid, respectively, as the elasticity of substitution in production, s, varies between 0.8 and 1.6, while θ runs between 0.5 and 5. One interesting feature is that the effects of the tied transfer on the growth rate and welfare, in particular, are highly sensitive to relatively minor deviations from the benchmark value of $s = 1$ (Cobb-Douglas). Thus, for example, if a researcher estimates $s = 1$ with a standard error of 0.1 – a tight estimate - and if $\theta = 1$, then, with 95% probability the implied increase of 0.66 percentage points on the growth rate could be as high as 0.98 or as low as 0.45. It is important to stress that a sustained difference in the growth rate of nearly half a percentage point accumulates to a substantial difference in economic performance. This can be seen from the spread on the implied welfare gain of 7.97%, which is even larger, ranging between 21.1% or as low as 0.53%.

Looking though the two panels of Table 2, the following observations can be made.

(i) The tendency for tied and untied transfers to have opposite long-run effects is robust to variations in s and θ.

(ii) Tied transfers have substantially greater long-run effects on variables involving asset accumulation, than do untied transfers. The effects on consumption and leisure are comparable in magnitude (though opposite in direction).

(iii) Increasing s reduces the positive effect of a tied transfer on the growth rate, while reducing the negative effect on the consumption-output and capital-output ratios. It decreases the adverse effect of an untied transfer on the growth rate, while reducing the positive effect on the consumption-output and capital-output ratios.

(iv) Increasing θ reduces the positive effect of a tied transfer on the growth rate, and reduces the adverse effect on the consumption-output and capital-output ratios. It increases the adverse effect of an untied transfer on the growth rate, while reducing the positive effect on the consumption-output and capital-output ratios.

(v) Both the short-run and the intertemporal welfare gains of an untied transfer are relatively insensitive to both s and θ. For plausible ranges of the parameters a 5% untied transfer leads to a uniform welfare gain of about 7-8%, measured as an equivalent variation in initial capital.

(vi) In contrast, both the short-run and long-run welfare gains from a tied transfer are highly sensitive to both parameters, particularly for low elasticities of substitution. Long-run welfare gains decline with s and increase with θ for low elasticities of substitution. For high elasticities of substitution, the tied transfer yields both short-run and long-run losses, the former being relatively independent of θ, and the latter increasing with θ.

Results (v) and (vi) are two key findings, and the intuition underlying them is as follows. A pure transfer has little effect on the stocks of public or private capital; virtually all that happens is that consumption increases, raising the C/Y ratio. The higher elasticity of substitution raises the level of output attainable from given stocks of capital, thereby raising consumption and welfare approximately uniformly. If the transfer is tied, the transfer increases the rate of investment in public capital. With a low elasticity of substitution this requires an approximately corresponding increase in private capital, leading to a large increase in output, consumption, and benefits. As the elasticity of substitution increases, the higher public capital is associated with a larger decline of private capital, so that the increase in output, consumption, and welfare declines. This is exacerbated by the fact that for a high elasticity of substitution, the tied transfer generates a large increase in the real wage and its growth rate, leading to substantial substitution toward labor (welfare-reducing).

One of the most interesting results in Table 2 is the contrast in the response of leisure for $s = 0.8$ from those obtained for higher values of s, in the case of the tied transfer. As noted previously, for the Cobb-Douglas production function, by increasing the productivity of labor, a tied transfer will tend to encourage more labor and cause a shift away from leisure, an effect that is exacerbated as the elasticity of substitution increases. For low s, this effect is reversed, and the intuition can be seen most clearly by focusing on the polar case of the fixed coefficient production function, $s = 0$. In this case, private capital, K, and labor in efficiency units, $(1-l)K_g$, need to change proportionately. Since a tied transfer leads to an increase in the relative stock, $z \equiv K_g/K$, this must be accompanied by a decrease in labor supply (i.e. an increase in leisure) in order for $(1-l)z$ to remain constant and for production to remain efficient. As s increases through low values this effect continues, although it declines in size as the production function becomes more flexible.

Moreover, since $s = 0.8$ leads to a short-run increase in leisure, it is actually associated with small short-run welfare gains, rather than losses, as before. These compound to large gains over time as the investment comes to fruition.`

The above results suggest that insofar as its effect on long run growth and welfare is concerned, a tied aid program is more effective in countries with a low elasticity of substitution in production. This observation complements the recent findings of Duffy and Papageorgiou (2000) that less developed or poor countries have elasticities of substitution that are significantly below unity and developed or richer countries have elasticities that are significantly above unity. In such a scenario, our analysis shows that a tied aid program may be more effective for poor countries than for their richer counterparts.

4.2. Transitional Dynamics

We have recomputed the transitional paths allowing for variations in θ and in s. Changing θ has little effect on the qualitative nature of the transitional paths, just as it does on the steady state. The more interesting changes result from increasing the elasticity of substitution and are illustrated in Figs. 3 and 4.

Fig. 3 compares the transitional time paths for leisure and the consumption-output ratio, following a tied transfer, for the three values $s = 0.5,1,1.6$. As already observed, for the benchmark economy, l and C/Y move together. For a low elasticity of substitution ($s = 0.5$), leisure generally increases, for reasons discussed in Section 4.1. The initial increase in leisure increases the marginal utility of consumption, so that C/Y initially increases, after which it declines steadily. This implies that l and C/Y move in opposite directions throughout the transition. For a high elasticity of substitution, l initially declines and continues to decline during the transition, just as in the benchmark case. But in this case, the initial decline in l is sufficiently sharp to cause a sharp decline in initial consumption, $C(0)$. The C/Y ratio overshoots its long-run response, and thus rises during the transition, implying again that l and C/Y move in opposite directions throughout the transition.

Fig. 4 illustrates the sensitivity of the dynamic adjustments of the basic state variables, z and n, to an increase in s. Panel I illustrates the case of tied transfers. As the elasticity of substitution increases, the curvature of the adjustment path increases. The higher the degree of substitution between the two types of capital, the more the transfer increases the initial growth rate of public capital relative to that of private capital.[18] At the same time, the rate of debt accumulation increases, raising borrowing costs. Over time, as the growth rate of public capital declines and that of private capital increases, foreign borrowing and borrowing costs fall. For a very high elasticity of substitution, we get very rapidly increasing debt and borrowing costs during the early phases of the transition. However, over time, these inhibit borrowing and debt eventually declines. In the limiting case where the two types of capital are perfect substitutes, n ultimately returns to its initial level.

Panel II illustrates the case of untied transfers. The main point to observe is that the initial period of an increasing public-private capital ratio, which prevails only briefly for the benchmark case, is much more prolonged for a low elasticity of substitution, while for a high elasticity of substitution z declines uniformly along with n.

4.3 Welfare Sensitivity to Investment Costs and Capital Market Imperfections

Tables 3 and 4 conduct further sensitivity analysis. Specifically, they compare the welfare gains from tied and untied aid programs, focusing on the tradeoffs between (i) the cost of installing public capital (h_2),relative to that of private capital, and (ii) the cost of borrowing from international capital markets (a). Table 3 examines the sensitivity of this tradeoff to the elasticity of substitution, while Table 4 relates it to the elasticity of labor supply.

Specifically, Table 3 addresses the following question: For a given cost of installing public capital, what are the respective gains from a tied and an untied aid program when (i) the cost of borrowing increases (measured by an increase in a across a row), and (ii) the elasticity of substitution increases (measured by an increase in s down a column). Thus, a = 0.05 implies a low cost of borrowing from international capital markets, and a = 1 implies that the agent has virtually no access to international capital markets. The range of s we consider is from 0.8 to 1.2 We consider three values for investment costs for public capital, with h_2 = 1, 10, and 20 signifying low, medium, and high costs of installing public capital. For example, in Table 3A, when h_2 = 1, a =

[18] Care must be exercised in comparing the slopes of the $n-z$ loci in Figs. 1.1 – 2.1, as the units vary.

0.05, and s = 0.8, the welfare gain from an untied transfer is 6.39%, while from a tied transfer it is 36.38%. The following observations can be drawn from Tables 3 and 4.

(i) The previous findings that an increase in the elasticity of substitution: (i) always increases the welfare gains resulting from a *pure* transfer, and (ii) reduces the welfare gains resulting from a *tied* transfer, hold for all a and h_2. The fact that the latter may lead to a welfare loss, if the installation costs associated with public capital are sufficiently large is evident. With very high installation costs, the tied transfer is committing the recipient economy to devote a large portion of its resources to the costly task of installation, thereby making it worse off.

(ii) An increase in θ always reduces the welfare gains resulting from a *pure* transfer. It reduces the gains from a tied transfer as long as a is low and installation costs are not too high. In all other cases, it will raise the welfare gains resulting from a *tied* transfer.

(iii) Over the range of values included in Table 3, the benefits from pure aid decrease with the cost of borrowing. As s declines the sensitivity to borrowing costs declines, and in fact for $s = 0.5$ (not reported), the benefits from pure aid actually increase with a. As long as tied aid yields positive benefits, these decrease with the cost of borrowing. In the cases where s, h_2 are both high, so that tied aid leads to welfare losses, these losses decline with the cost of borrowing.

(iv) Irrespective of the cost of borrowing, the elasticity of substitution, or the elasticity of leisure in utility, the benefits from tied aid decrease, and those from untied aid increase, with installation costs.

(v) When installation costs are high (h_2 = 20), an untied transfer is better than a tied transfer even for s = 1 (the Cobb-Douglas case).

(vi) If we consider s = 1 as a benchmark values, then even small deviations of s from the benchmark (in the range 0.8-1.2), lead to moderate variations in welfare changes from untied aid programs, irrespective of the cost of installation and finance. The welfare changes are much more dramatic in the case of tied transfers.

5. Co-financing and Welfare Gains

Several aid programs call for co-financing by the domestic government. In Table 5 we compare the welfare effects of the tied and pure aid programs with two alternative forms of co-financing. In the first, the government receives a tied aid flow of 2.5% of its income, which it must match with an equal increase in its expenditure; in the second it must match an untied aid flow. In all four cases, the economy is experiencing a 5% increase in expenditure.

For low or medium elasticity of substitution the tied transfer (TT) is superior to the pure transfer (PT), where as for a high s this ordering is reversed, as we have seen. In all cases the matched tied transfer (MTT) is dominated by TT. This is because the MTT involves making the size of the government sector too large. While the matched pure transfer (MPT) is never dominant, it is superior to PT in the case where $s = 0.8$ and it is superior to TT for $s = 1.2$.[19] The rankings are much less sensitive to variations in θ, although the tied transfer begins to dominate the pure transfer as θ increases from 0.75 to 1, for the Cobb-Douglas production function.

We can also show that a *tied* aid of a given amount, coupled with an equivalent *decrease* in domestic government expenditure, is equivalent to an *untied* transfer of an equivalent amount. This is important since it implies that by combining the transfer with the appropriate expenditure and tax mix, the recipient economy can choose an equilibrium path and associated level of welfare that is independent of any constraints imposed by the donor country.

6. Conclusions

The receipt of foreign aid inevitably involves some structural adjustment by the recipient economy. In this paper we have investigated this issue in an endogenous growth model of a small open economy, where the margins along which the economy may adjust include (i) the intratemporal substitution of private capital for public capital, (ii) labor supply, and (iii) the intertemporal margins of private and public investment, and foreign borrowing. Our analysis has shown how the introduction of endogenous labor supply introduces a dynamic response to an untied transfer,

[19] Chatterjee, Sakoulis, and Turnovsky (2003) address the question of optimal co-financing in the case of the Cobb-Douglas production. The analogous exercise can be pursued here.

although virtually opposite in its time profile from the response to a tied transfer.

We find that the long-run impact of a tied aid program and the direction of transitional dynamics it generates depend crucially upon the elasticity of substitution in production and much less sensitive to the elasticity of labor supply. Our numerical simulations suggest that tied aid is more effective in economies with a low degree of substitution between factors of production. Moreover, the welfare gains from a tied or untied aid shock are sensitive to the substitutability of inputs, capital market imperfections, and costs of adjustment.

What role does the endogeneity of labor supply play? One of the striking features of the welfare results in Table 1 is that the differences between the intertemporal welfare gains for tied versus untied transfers are very small, at least for the benchmark parameters (7.96% vs. 7.71%). This contrasts with the substantially larger differences obtained in our earlier work with inelastic labor supply (9.83% vs. 8.32%).[20] The reason is that in the present framework any transfer impacts welfare in two ways, first through the consumption flows and second through labor supply. Both tied and untied transfers lead to increases in consumption, -- albeit having very different time profiles -- leading to improvements in intertemporal welfare. However, the two types of transfer have contrasting effects on leisure. The untied transfer is leads to an increase in leisure, adding to the consumption benefits, while the untied transfer leads to reduced leisure, partially offsetting the consumption benefits. Thus for the benchmark economy the endogeneity of labor supply tends to reduce the welfare advantage of a tied transfer.

Consider the column $s = 1$ in Table 2. For low values of θ (0.5, 0.75), labor supply is highly elastic, so that the response of leisure to either shock is sufficiently large in magnitude for it to dominate the welfare derived from consumption, in which case the untied transfer is superior from the welfare standpoint. For high values of θ, the response of leisure is small. The consumption effects dominate so that the tied transfer is clearly superior. For a low elasticity of substitution the leisure effect is positive for both types of transfers and the tied transfer is clearly superior even if θ is low and labor is highly elastic.

Finally, our results carry some important policy advice. They suggest that when donors

[20] See Chatterjee and Turnovsky (2003)

decide on whether a particular aid program should be tied to an investment activity, careful attention should be paid to the recipient's opportunities for substitution in production, its access to world capital markets, and the costs of installing the particular type of capital to which the aid will be tied. Overall, the effects of a tied transfer are highly sensitive to the specific structural characteristics of the recipient economy. It is perfectly possible for a tied transfer to have a presumably unintended adverse effect on the recipient economy, if that economy is structurally different from what the donor believed. On the other hand, the benefits of a comparable untied transfer are remarkably robust with respect to the same structural characteristics. Thus, if the donor economy does not possess the detailed information, particularly about the production characteristics of the recipient economy, giving untied aid is a less risky strategy.

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Appendix

The objective of this Appendix is to set out the dynamic macroeconomic equilibrium in terms of the four stationary variable, z, n, q, l. We begin by dividing (3b) by (3a), while noting the definition of the real wage, $\partial Y/\partial(1-l)$, enabling us to write the marginal rate of substitution between C and l:

$$\frac{C}{Y} = \frac{\eta(1-\tau)}{\alpha^{\rho}\theta}\left(\frac{l}{1-l}\right)\left[\frac{Y}{(1-l)K_G}\right]^{\rho} \tag{A.1}$$

Recalling $\Omega(z,l) \equiv \Omega = \left((1-\eta)/\eta\right)\left[(1-l)z\right]^{\rho}$ and the definitions of c and y, we can express the output-capital and consumption-capital ratios in the form

$$Y/K \equiv y = y(z,l) = \alpha\left[\left(1-\eta\right) + \eta\left\{\left(1-l\right)z\right\}^{-\rho}\right]^{-1/\rho} \tag{A.2a}$$

$$c = c(z,l) = \frac{\left(1-\tau\right)}{\theta}\left(\frac{l}{1-l}\right)\left[\frac{1}{1+\Omega}\right]y \tag{A.2b}$$

Then, differentiating the optimality condition, (3a), the marginal rate of substitution condition, (A.1), the production function (1), and the definition of k all with respect to time, yields

$$\left(\gamma-1\right)\frac{\dot{C}}{C} + \gamma\theta\frac{\dot{l}}{l} = \frac{\dot{\lambda}}{\lambda} = \beta - r\left(N/K\right) \tag{A.3a}$$

$$\frac{\dot{C}}{C} - \frac{\dot{Y}}{Y} = \frac{\dot{l}}{l} + (1+\rho)\frac{\dot{l}}{1-l} + \rho\left(\frac{\dot{Y}}{Y} - \frac{\dot{K}_G}{K_G}\right) \tag{A.3b}$$

$$\frac{\dot{Y}}{Y} = \frac{1}{1+\Omega}\left[\Omega\frac{\dot{K}}{K} + \frac{\dot{K}_G}{K_G} - \frac{\dot{l}}{1-l}\right] \tag{A.3c}$$

Combining these three equations and recalling (3c') and (6), we can express the dynamics of labor supply by the following differential equation:

$$\dot{l} = \frac{F\left(z,n,q,l\right)}{G\left(z,l\right)} \tag{A.4}$$

where

$$F(z,n,q,l) \equiv \left[\{1+\Omega(z,l)\}\{\beta - r(n)\} + (1-\gamma)\{\Omega(z,l)(1+\rho)\psi_K + (1-\rho\Omega(z,l))\psi_G\}\right] l,$$

$$G(k,l) \equiv \left[\{\gamma(1+\theta)-1\}\{1+\Omega(z,l)\} - (1-\gamma)(1+\rho)\Omega(z,l)\left(\frac{l}{1-l}\right)\right],$$

$$\psi_K = \frac{\dot{K}}{K} = \frac{(q-1)}{h_1} - \delta_K$$

$$\psi_G = \frac{\dot{K}_G}{K_G} = g\frac{Y}{K_G} - \delta_G = \alpha g(1-l)\left[\eta\{1+\Omega(k)\}\right]^{-1/\rho} - \delta_G$$

Using (A.3a) and (A.4) we can express the growth rate of consumption as

$$\frac{\dot{C}}{C} \equiv \psi_C = \frac{r(n) - \beta + \gamma\theta(1/l)\left[F(z,n,q,l)/G(z,l)\right]}{1-\gamma} \tag{A.5}$$

The equilibrium dynamics can now be represented by the following fourth order system in the stationary variables , n , z, q, l:

$$\frac{\dot{z}}{z} = \frac{\dot{K}_G}{K_G} - \frac{\dot{K}}{K} = (\bar{g}+\sigma\phi)\frac{y}{z}\delta_G - \left(\frac{(q-1)}{h_1} - \delta_K\right) \tag{A.6a}$$

$$\frac{\dot{n}}{n} = \frac{\dot{N}}{N} - \frac{\dot{K}}{K} = r(n) + \frac{1}{n}\left[c(z,l) + \frac{q^2-1}{2h_1} + \{(\bar{g}+\sigma\phi) - (1+\sigma)\}y + \frac{h_2}{2}(\bar{g}+\sigma\phi)^2\frac{y^2}{k}\right] - \left(\frac{(q-1)}{h_1} - \delta_K\right) \tag{A.6b}$$

$$\dot{q} = r(n)q - \alpha(1-\tau)(1-\eta)\frac{y^{1+\rho}}{\alpha^{\rho}} - \frac{(q-1)^2}{2h_1} + \delta_K q \tag{A.6c}$$

$$\dot{l} = \frac{F(l)}{G(l)} = \frac{\left[\{1+\Omega\}\{\beta - r(n)\} + (1-\gamma)\{\Omega(1+\rho)\psi_K + (1-\rho\,\Omega)\psi_G\}\right]}{\left[\{\gamma(1+\theta)-1\}\{1+\Omega\} - (1-\gamma)(1+\rho)\Omega\left(\frac{l}{1-l}\right)\right]} l \tag{A.6d}$$

Steady-state equilibrium is attained when $\dot{k} = \dot{n} = \dot{l} = \dot{q} = 0$, so that

$$\frac{\dot{C}}{C} = \frac{\dot{K}}{K} = \frac{\dot{K}_G}{K_G} = \frac{\dot{Y}}{Y} = \frac{\dot{N}}{N} = \tilde{\psi}.$$

Using (A.1) and (A.4a)-(A.4e), we can express the steady-state in the following form:

$$\left(\bar{g}+\sigma\phi\right)\frac{\tilde{y}}{\tilde{z}}-\delta_G=\frac{\tilde{q}-1}{h_1}-\delta_K \tag{A.7a}$$

$$r\left(\tilde{n}\right)+\frac{1}{\tilde{n}}\left[\tilde{c}(\tilde{z},\tilde{l})+\frac{\tilde{q}^2-1}{2h_1}+\left\{\left(\bar{g}+\sigma\phi\right)-\left(1+\sigma\right)\right\}\tilde{y}+\frac{h_2}{2}\left(\bar{g}+\sigma\phi\right)^2\frac{\left(1-\tilde{l}\right)\tilde{y}^2}{\tilde{k}}\right]=\frac{\left(\tilde{q}-1\right)}{h}-\delta_K \tag{A.7b}$$

$$r\left(\tilde{n}\right)\tilde{q}-\alpha\left(1-\tau\right)\left(1-\eta\right)\left[\left(1-\eta\right)+\eta\tilde{k}^{-\rho}\right]^{-(1+\rho)/\rho}-\frac{\left(\tilde{q}-1\right)^2}{2h_1}+\delta_K\tilde{q}=0 \tag{A.6c}$$

$$\frac{r\left(\tilde{n}\right)-\beta}{1-\gamma}=\frac{\left(q-1\right)}{h_1}-\delta_K \tag{A.6d}$$

$$\frac{\tilde{c}}{\tilde{y}}=\frac{\left(1-\tau\right)}{\theta}\left(\frac{\tilde{l}}{1-\tilde{l}}\right)\left(\frac{1}{1+\tilde{\Omega}}\right) \tag{A.6e}$$

(A.6a)-(A.6e) can be solved to yield the equilibrium values of $\tilde{z},\ \tilde{n},\ \tilde{q}$,and $\tilde{l}$. Given these values, we can solve for the steady-state output-capital and consumption-capital ratios

$$\tilde{y}=\alpha\left[\left(1-\eta\right)+\eta\left\{\left(1-\tilde{l}\right)\tilde{z}\right\}^{-\rho}\right]^{-1/\rho} \tag{A.7a}$$

$$\tilde{c}=\frac{\left(1-\tau\right)}{\theta}\left(\frac{l}{1-l}\right)\left[\frac{1}{1+\tilde{\Omega}}\right]\tilde{y} \tag{A.7b}$$

Linearizing (A.5) around the steady state (A.6) the local dynamics are represented by the fourth order system

$$\begin{pmatrix}\dot{z}\\ \dot{n}\\ \dot{q}\\ \dot{l}\end{pmatrix}=\begin{pmatrix}a_{11} & 0 & -\left(\tilde{z}/h_1\right) & a_{14}\\ a_{21} & \tilde{n}r'(\tilde{n})+r(\tilde{n})-\tilde{\psi} & (\tilde{q}-\tilde{n})/h_1 & a_{24}\\ a_{31} & \tilde{q}r'(\tilde{n}) & r(\tilde{n})-\tilde{\psi} & a_{34}\\ a_{41} & a_{42} & a_{43} & a_{44}\end{pmatrix}\begin{pmatrix}z-\tilde{z}\\ n-\tilde{n}\\ q-\tilde{q}\\ l-\tilde{l}\end{pmatrix} \tag{A.8}$$

where,

$$s=\frac{\tilde{y}}{(1-\tilde{l})\tilde{z}};\ \ a_{11}=\left(\bar{g}+\sigma\phi\right)\left(\frac{\tilde{y}}{\tilde{z}}\right)\left(\eta\left(\frac{s}{\alpha}\right)^{\rho}-1\right);\ \ a_{14}=-\left(\bar{g}+\sigma\phi\right)\frac{\eta}{\alpha^{\rho}}\tilde{z}\tilde{s}^{1+\rho},$$

$$a_{21}=\tilde{c}_z(\tilde{z},\tilde{l})+\frac{\eta}{\alpha^{\rho}}\left[\left(\bar{g}+\sigma\phi\right)-\left(1+\sigma\right)\right]\left(\frac{\tilde{y}}{\tilde{z}}\right)\tilde{s}^{\rho}+\frac{h_2}{2}\left(\bar{g}+\sigma\phi\right)^2\left(\frac{\tilde{y}}{\tilde{z}}\right)^2\left[2\eta\left(\frac{\tilde{s}}{\alpha}\right)^{\rho}-1\right]$$

and $\quad \tilde{c}_z\left(\tilde{z},\tilde{l}\right)=\left(\frac{1-\tau}{\theta}\right)\left(\frac{\tilde{l}}{1-\tilde{l}}\right)\left(\frac{\tilde{y}}{\tilde{z}}\right)\left(\frac{1}{1+\tilde{\Omega}}\right)\left(\eta\left(\frac{s}{\alpha}\right)^{\rho}-\rho\frac{\tilde{\Omega}}{1+\tilde{\Omega}}\right),$

$$a_{24}=\tilde{c}_l\left(\tilde{z},\tilde{l}\right)-\frac{\eta}{\alpha^{\rho}}s^{(1+\rho)}\left[\left(\bar{g}+\sigma\phi\right)-\left(1+\sigma\right)-h_2\left(\bar{g}+\sigma\phi\right)^2\left(\frac{\tilde{y}}{\tilde{z}}\right)\right]\tilde{z}\,,$$

and $\quad \tilde{c}_l\left(\tilde{z},\tilde{l}\right)=\left(\frac{1-\tau}{\theta}\right)\left(\frac{1}{1+\tilde{\Omega}}\right)\left(\frac{\tilde{l}}{1-\tilde{l}}\right)\left[\frac{\left(\tilde{y}/\tilde{l}\right)}{1-\tilde{l}}+\rho\left(\frac{\tilde{\Omega}}{1+\tilde{\Omega}}\right)\left(\frac{\tilde{y}}{1-\tilde{l}}\right)-\frac{\eta}{\alpha^{\rho}}s^{1+\rho}\tilde{z}\right],$

$$a_{31}=-\frac{\eta}{\alpha^{2\rho}}\left(1+\rho\right)\left(1-\eta\right)\left(1-\tau\right)s^{\rho}\,\frac{\tilde{y}^{(1+\rho)}}{\tilde{z}},\quad a_{34}=\frac{\eta}{\alpha^{2\rho}}\left(1+\rho\right)\left(1-\eta\right)\left(1-\tau\right)\frac{\tilde{y}^{(1+\rho)}}{\left(1-\tilde{l}\right)}s^{\rho}$$

$$a_{41}=\frac{\left(1-\gamma\right)\ \left(1-\rho\tilde{\Omega}\right)\ \left(\bar{g}+\sigma\phi\right)\ \ \left(\tilde{y}/\tilde{z}\right)\ \left[\left(\eta/\alpha^{\rho}\right)\ s^{\rho}-1\right]}{G\ \left(\tilde{l}\right)}\ \frac{\tilde{l}}{\tilde{z}};\,a_{42}=-\frac{\left(1+\tilde{\Omega}\right)r'\left(\tilde{n}\right)\tilde{l}}{G\left(\tilde{l}\right)},$$

$$a_{43}=\frac{\left(1-\gamma\right)\left(1+\rho\right)\tilde{\Omega}\,\tilde{l}}{h_1\,G\left(\tilde{l}\right)},\ a_{44}=-\frac{\eta}{\alpha^{\rho}}\left(1-\gamma\right)\left(\bar{g}+\sigma\phi\right)\left(1-\rho\tilde{\Omega}\right)\left(\frac{\tilde{l}}{G\left(\tilde{l}\right)}\right)s^{(1+\rho)}.$$

Table 1: Permanent Foreign Aid Shock

Benchmark Equilibrium: Cobb-Douglas production function (*s* = 1)

a. Long-run Effects

	$\tilde{K}_G/\tilde{K}$	$\tilde{r}$ %	$\tilde{l}$	$\widetilde{C/Y}$	$\widetilde{N/Y}$	$\tilde{\psi}$ %	$\Delta(W)$ %
Benchmark Equilibrium $\sigma = 0, \phi = 0,$ $\bar{g} = 0.05, \tau = 0.15$	**0.253**	**8.13**	**0.780**	**0.602**	**0.416**	**1.65**	**--**
Tied aid $\sigma = 0.05, \phi = 1,$ $\bar{g} = 0.05, \tau = 0.15$	0.542	9.77	0.768	0.563	0.622	1.87	7.96
Untied aid $\sigma = 0.05, \phi = 0,$ $\bar{g} = 0.05, \tau = 0.15$	0.252	7.99	0.7934	0.653	0.396	1.64	7.71

b. Short-run Effects

	$l(0)$	$\frac{C(0)}{Y(0)}$	$\psi_K(0)$ %	$\psi_G(0)$ %	$\psi_Y(0)$ %	$\psi_C(0)$ %	$\Delta(W(0))$ %
Benchmark Equilibrium $\sigma = 0, \phi = 0,$ $\bar{g} = 0.05, \tau = 0.15$	**0.780**	**0.602**	**1.65**	**1.65**	**1.65**	**1.65**	**--**
Tied aid $\sigma = 0.05, \phi = 1,$ $\bar{g} = 0.05, \tau = 0.15$	0.777	0.594	1.95	8.33	3.48	1.87	-1.53
Untied aid $\sigma = 0.05, \phi = 0,$ $\bar{g} = 0.05, \tau = 0.15$	0.7925	0.650	1.55	1.57	1.54	1.60	8.32

Table 2

A. Sensitivity of Permanent Responses to the Elasticities of Substitution (*s*) and Leisure (*q*)

(i) Tied Aid Shock: *f* = 0, *q* = 0 to 0.05

	s = 0.8					***s* = 1**					*s* = 1.2					*s* = 1.6				
	$d\tilde{l}$	$d\left(\widetilde{\frac{C}{Y}}\right)$	$d\left(\widetilde{\frac{N}{Y}}\right)$	$d\left(\widetilde{\frac{K}{Y}}\right)$	$d\tilde{y}$	$d\tilde{l}$	$d\left(\widetilde{\frac{C}{Y}}\right)$	$d\left(\widetilde{\frac{N}{Y}}\right)$	$d\left(\widetilde{\frac{K}{Y}}\right)$	$d\tilde{y}$	$d\tilde{l}$	$d\left(\widetilde{\frac{C}{Y}}\right)$	$d\left(\widetilde{\frac{N}{Y}}\right)$	$d\left(\widetilde{\frac{K}{Y}}\right)$	$d\tilde{y}$	$d\tilde{l}$	$d\left(\widetilde{\frac{C}{Y}}\right)$	$d\left(\widetilde{\frac{N}{Y}}\right)$	$d\left(\widetilde{\frac{K}{Y}}\right)$	$d\tilde{y}$
q = 0.5	0.009	-0.059	0.323	-0.513	1.01	-0.017	-0.042	0.169	-0.396	0.695	-0.04	-0.032	0.088	-0.319	0.499	-0.073	-0.023	0.018	-0.227	0.289
q = 0.75	0.009	-0.056	0.373	-0.575	0.997	-0.014	-0.040	0.189	-0.424	0.674	-0.03	-0.031	0.095	-0.325	0.470	-0.054	-0.023	0.019	-0.208	0.252
***q* = 1**	0.009	-0.054	0.419	-0.630	0.984	-0.012	-0.039	0.206	-0.446	0.657	-0.026	-0.029	0.101	-0.327	0.446	-0.041	-0.024	0.019	-0.192	0.223
q = 2	0.007	-0.046	0.572	-0.814	0.939	-0.007	-0.035	0.254	-0.507	0.608	-0.015	-0.028	0.111	-0.327	0.385	-0.017	-0.025	0.017	-0.147	0.158
q = 5	0.005	-0.029	0.912	-1.22	0.840	-0.003	-0.029	0.335	-0.608	0.537	-0.006	-0.026	0.119	-0.315	0.309	-0.005	-0.026	0.013	-0.094	0.094

(ii) Untied Aid Shock: *f* = 0, *q* = 0 to 0.05

	s = 0.8					***s* = 1**					*s* = 1.2					*s* = 1.6				
	$d\tilde{l}$	$d\left(\widetilde{\frac{C}{Y}}\right)$	$d\left(\widetilde{\frac{N}{Y}}\right)$	$d\left(\widetilde{\frac{K}{Y}}\right)$	$d\tilde{y}$	$d\tilde{l}$	$d\left(\widetilde{\frac{C}{Y}}\right)$	$d\left(\widetilde{\frac{N}{Y}}\right)$	$d\left(\widetilde{\frac{K}{Y}}\right)$	$d\tilde{y}$	$d\tilde{l}$	$d\left(\widetilde{\frac{C}{Y}}\right)$	$d\left(\widetilde{\frac{N}{Y}}\right)$	$d\left(\widetilde{\frac{K}{Y}}\right)$	$d\tilde{y}$	$d\tilde{l}$	$d\left(\widetilde{\frac{C}{Y}}\right)$	$d\left(\widetilde{\frac{N}{Y}}\right)$	$d\left(\widetilde{\frac{K}{Y}}\right)$	$d\tilde{y}$
q = 0.5	0.017	0.052	-0.024	0.036	-0.059	0.019	0.051	-0.014	0.031	-0.047	0.02	0.05	-0.008	0.027	-0.037	0.019	0.049	-0.002	0.020	-0.024
q = 0.75	0.015	0.052	-0.031	0.046	-0.066	0.016	0.051	-0.018	0.037	-0.051	0.016	0.05	-0.009	0.030	-0.039	0.013	0.049	-0.002	0.019	-0.022
***q* = 1**	0.014	0.052	-0.038	0. 055	-0.069	0.014	0.051	-0.020	0.042	-0.053	0.013	0.05	-0.011	0.032	-0.039	0.010	0.049	-0.002	0.018	-0.020
q = 2	0.009	0.052	-0.058	0.080	-0.073	0.008	0.051	-0.028	0.053	-0.055	0.007	0.05	-0.012	0.034	-0.037	0.004	0.050	-0.002	0.014	-0.015
q = 5	0.005	0.050	-0.102	0.133	-0.070	0.004	0.051	-0.038	0.067	-0.051	0.003	0.05	-0.014	0.034	-0.031	0.001	0.050	-0.001	0.009	-0.010

Table 2

B. Sensitivity of Short-run and Long-run Welfare Responses to the Elasticities of Substitution (*s*) and Leisure (q)

(i) Tied Aid Shock: f = 1, q = 0 to 0.05

	s = 0.8		**s = 1**		s = 1.2		s = 1.6	
	% $\Delta[W(0)]$	$\Delta(W)$	% $\Delta[W(0)]$	$\Delta(W)$	% $\Delta[W(0)]$	$\Delta(W)$	% $\Delta[W(0)]$	$\Delta(W)$
q = 0.5	3.64	17.08	-2.30	6.90	-6.95	0.79	-12.78	-5.84
q = 0.75	4.47	19.26	-1.86	7.50	-6.80	0.65	-12.64	-6.42
q = 1	5.14	21.08	**-1.53**	**7.96**	-6.71	0.53	-12.56	-6.82
q = 2	7.03	26.46	-0.69	9.15	-6.55	0.16	-12.52	-7.68
q = 5	10.34	35.66	0.45	10.79	-6.49	0.43	-12.50	-8.55

(ii) Untied Aid Shock: f = 0, q = 0 to 0.05

	s = 0.8		**s = 1**		s = 1.2		s = 1.6	
	% $\Delta[W(0)]$	$\Delta(W)$	% $\Delta[W(0)]$	$\Delta(W)$	% $\Delta[W(0)]$	$\Delta(W)$	% $\Delta[W(0)]$	$\Delta(W)$
q = 0.5	7.74	7.01	8.43	7.95	8.94	8.60	9.59	9.40
q = 0.75	7.60	6.70	8.37	7.81	8.94	8.57	9.66	9.47
q = 1	7.49	6.47	**8.32**	**7.71**	8.94	8.55	9.70	9.52
q = 2	7.22	5.89	8.21	7.50	8.94	8.54	9.78	9.64
q = 5	6.93	5.20	8.09	7.28	8.95	8.52	9.81	9.72

Table 3

Welfare Sensitivity to Installation Costs of Public Capital (relative to that of private capital), Capital Market Imperfections, and Degree of Substitutability ($\sigma = 0$ to $\sigma = 0.05$) (Numbers reported are percentage changes in intertemporal welfare)

A. Low Installation Costs ($h_1 = 10$, $h_2 = 1$)

	$a = 0.05$		$a = 0.15$		$a = 1$	
	$\phi = 0$	$\phi = 1$	$\phi = 0$	$\phi = 1$	$\phi = 0$	$\phi = 1$
$s = 0.8$	6.39	36.38	6.25	28.02	6.24	26.72
$s = 1$	9.46	23.06	**7.65**	**15.22**	7.15	14.43
$s = 1.2$	12.75	15.50	8.66	8.21	7.69	7.53

B. Medium Installation Costs ($h_1 = 10$, $h_2 = 10$)

	$a = 0.05$		$a = 0.15$		$a = 1$	
	$\phi = 0$	$\phi = 1$	$\phi = 0$	$\phi = 1$	$\phi = 0$	$\phi = 1$
$s = 0.8$	6.66	32.28	6.47	23.41	6.45	22.43
$s = 1$	10.01	16.65	**7.96**	**9.89**	7.40	9.79
$s = 1.2$	13.70	6.32	9.03	2.29	7.97	2.61

C. High Installation Costs ($h_1 = 10$, $h_2 = 20$)

	$a = 0.05$		$a = 0.15$		$a = 1$	
	$\phi = 0$	$\phi = 1$	$\phi = 0$	$\phi = 1$	$\phi = 0$	$\phi = 1$
$s = 0.8$	6.98	27.27	6.74	17.73	6.69	17.24
$s = 1$	10.70	8.51	**8.33**	**3.35**	7.97	4.22
$s = 1.2$	14.95	-5.75	9.49	-4.90	8.30	-3.24

Table 4

Welfare Sensitivity to Installation Costs of Public Capital (relative to that of private capital), Capital Market Imperfections, and Sensitivity of Utility to Leisure
($\sigma = 0$ to $\sigma = 0.05$)
(Numbers reported are percentage changes in intertemporal welfare)

A. Low Installation Costs ($h_1 = 10$, $h_2 = 1$)

	$a = 0.05$		$a = 0.15$		$a = 1$	
	$\phi = 0$	$\phi = 1$	$\phi = 0$	$\phi = 1$	$\phi = 0$	$\phi = 1$
$\theta = 0.75$	9.98	23.82	7.77	15.08	7.18	14.26
$\boldsymbol{\theta} = \mathbf{1}$	9.46	23.06	**7.65**	**15.22**	7.15	14.43
$\theta = 1.25$	9.07	22.44	7.57	15.34	7.13	14.57

B. Medium Installation Costs ($h_1 = 10$, $h_2 = 10$)

	$a = 0.05$		$a = 0.15$		$a = 1$	
	$\phi = 0$	$\phi = 1$	$\phi = 0$	$\phi = 1$	$\phi = 0$	$\phi = 1$
$\theta = 0.75$	10.58	16.82	8.09	9.50	7.43	9.46
$\boldsymbol{\theta} = \mathbf{1}$	10.01	16.65	**7.96**	**9.89**	7.40	9.79
$\theta = 1.25$	9.57	16.41	7.86	10.21	7.38	10.05

C. High Installation Costs ($h_1 = 10$, $h_2 = 20$)

	$a = 0.05$		$a = 0.15$		$a = 1$	
	$\phi = 0$	$\phi = 1$	$\phi = 0$	$\phi = 1$	$\phi = 0$	$\phi = 1$
$\theta = 0.75$	11.35	7.88	8.47	2.65	7.73	3.71
$\boldsymbol{\theta} = \mathbf{1}$	10.70	8.51	**8.33**	**3.35**	7.70	4.22
$\theta = 1.25$	10.20	8.80	8.22	3.91	7.67	4.63

Table 5

Co-financing Tradeoffs
(Numbers reported are percentage changes in intertemporal welfare)

	$s = 0.8$				$s = 1$				$s = 1.2$			
	TT	**PT**	**MTT**	**MPT**	**TT**	**PT**	**MTT**	**MPT**	**TT**	**PT**	**MTT**	**MPT**
$\theta = 0.75$	19.26	6.70	14.79	10.69	7.50	7.81	2.90	4.50	0.65	8.57	-4.05	0.88
	TT > MTT> MPT> PT				**PT > TT >MPT> MTT**				**PT > MPT>TT>MTT**			
$\boldsymbol{\theta = 1}$	21.08	6.47	16.70	11.61	**7.96**	**7.71**	**3.40**	**4.73**	0.53	8.55	-4.15	0.81
	TT > MTT> MPT> PT				**TT > PT >MPT>MTT**				**PT > MPT>TT>MTT**			
$\theta = 1.25$	22.66	6.29	18.34	12.40	8.34	7.64	3.81	4.91	0.42	8.54	-4.25	0.74
	TT > MTT> MPT> PT				**TT > PT >MPT>MTT**				**PT > MPT>TT>MTT**			

TT : Tied Transfer ($\sigma = 0.05, \phi = 1, \overline{g} = 0.05$)
PT : Pure Transfer ($\sigma = 0.05, \phi = 0, \overline{g} = 0.05$)
MTT : Matched Tied Transfer ($\sigma = 0.025, \phi = 1, \overline{g} = 0.075$)
MPT : Matched Pure Transfer ($\sigma = 0.025, \phi = 0, \overline{g} = 0.075$)

Figure 1. Tied Transfer, s =1

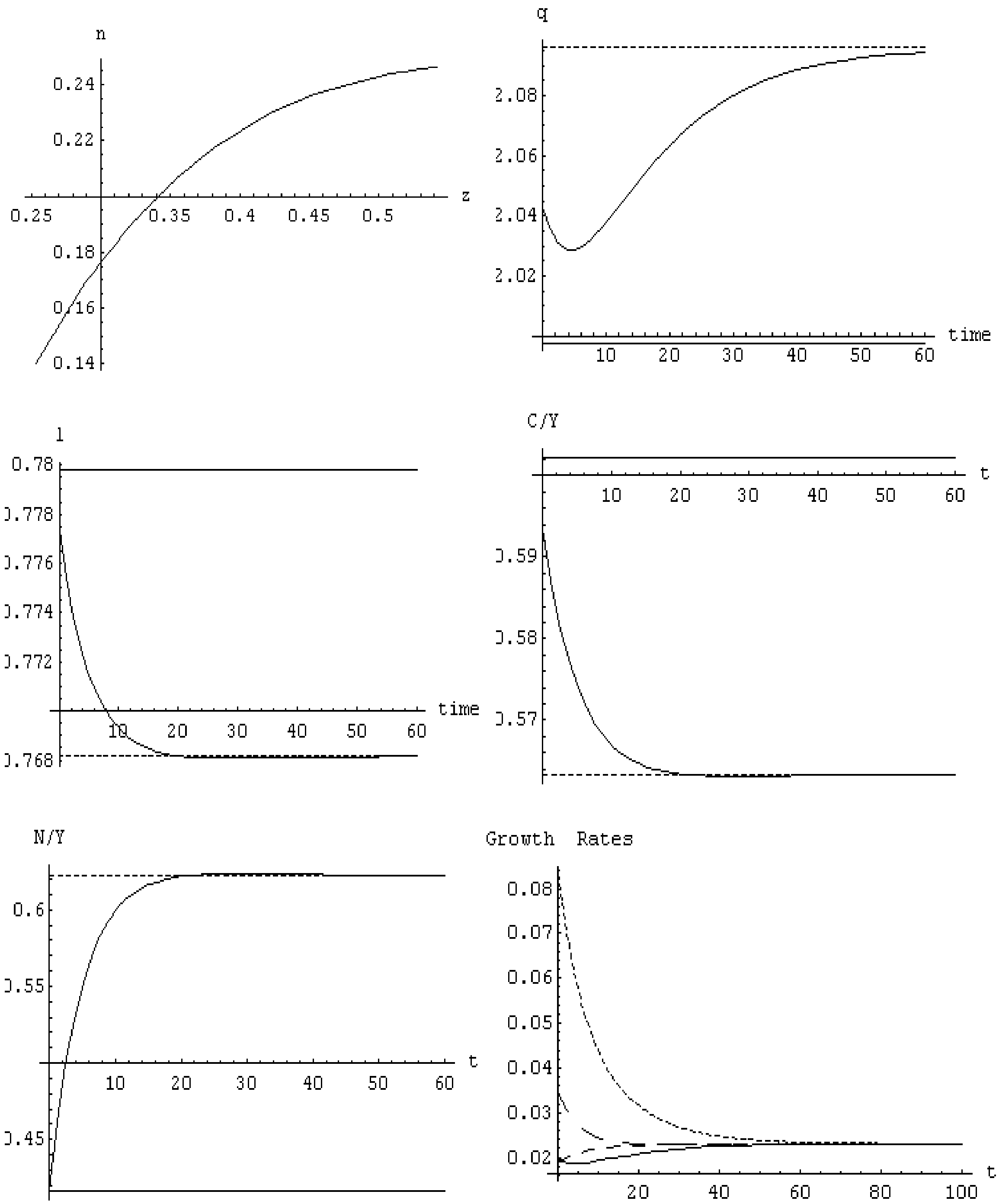
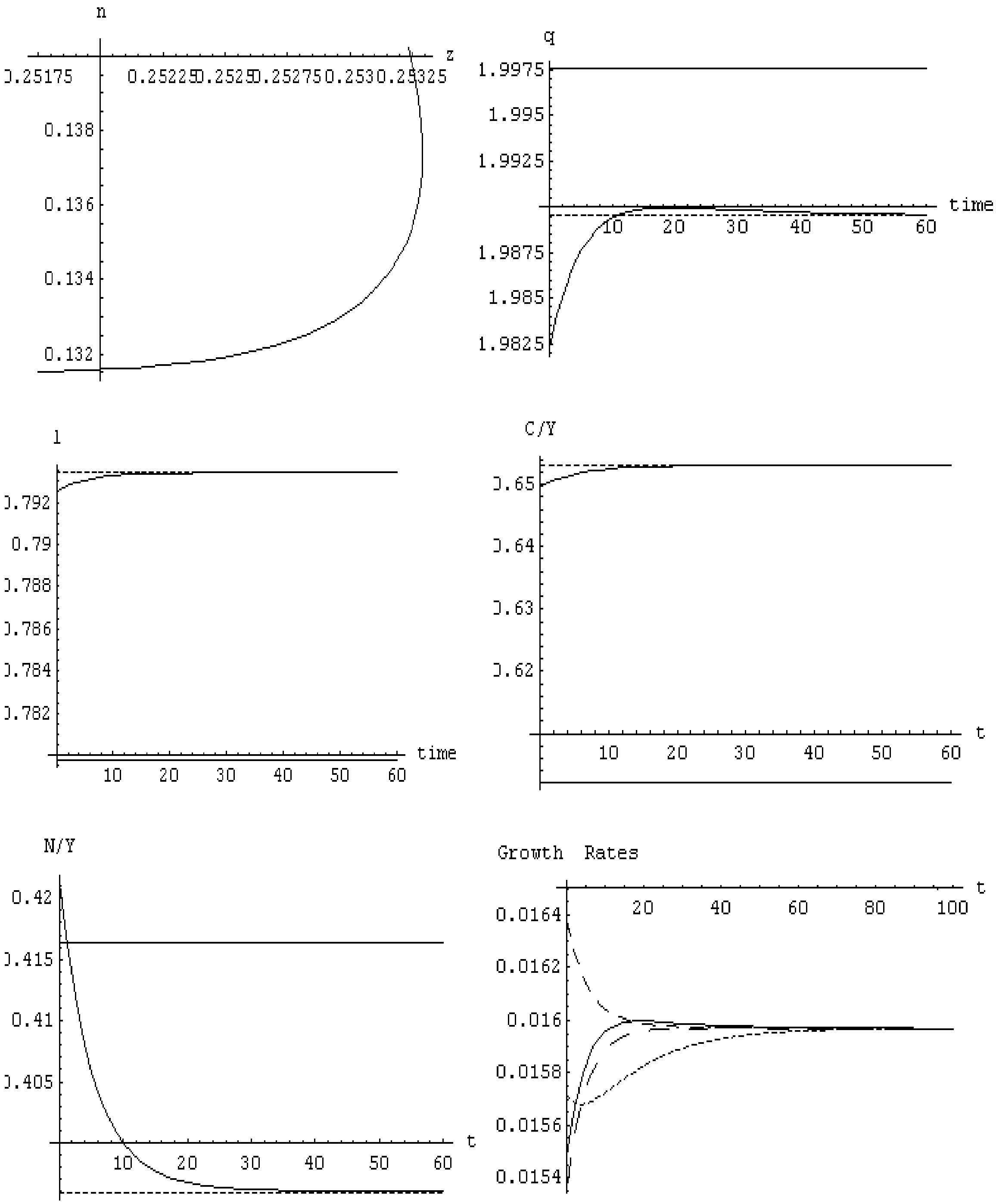
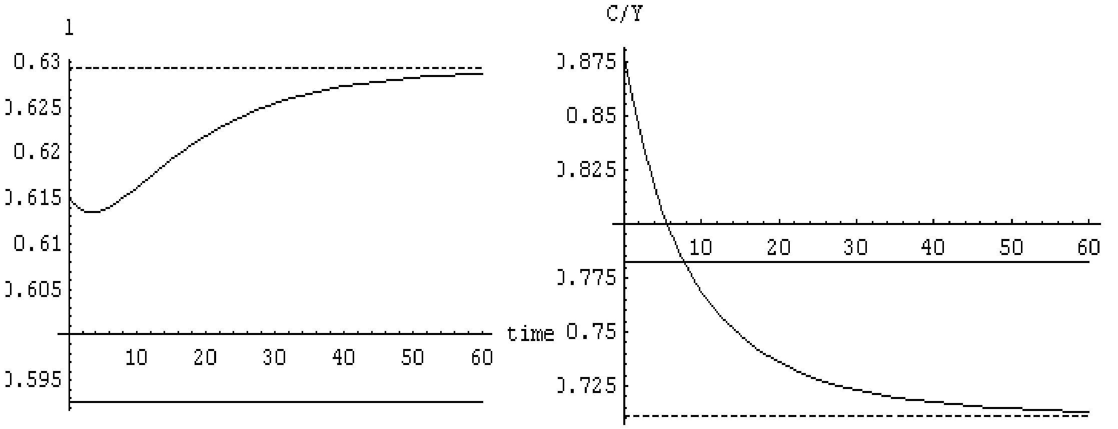
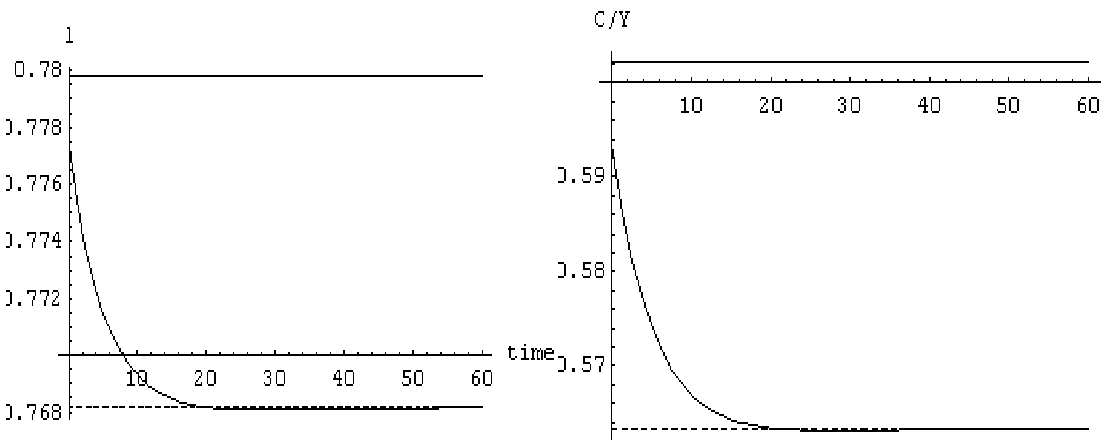


Figure 2. Untied Transfer, s =1

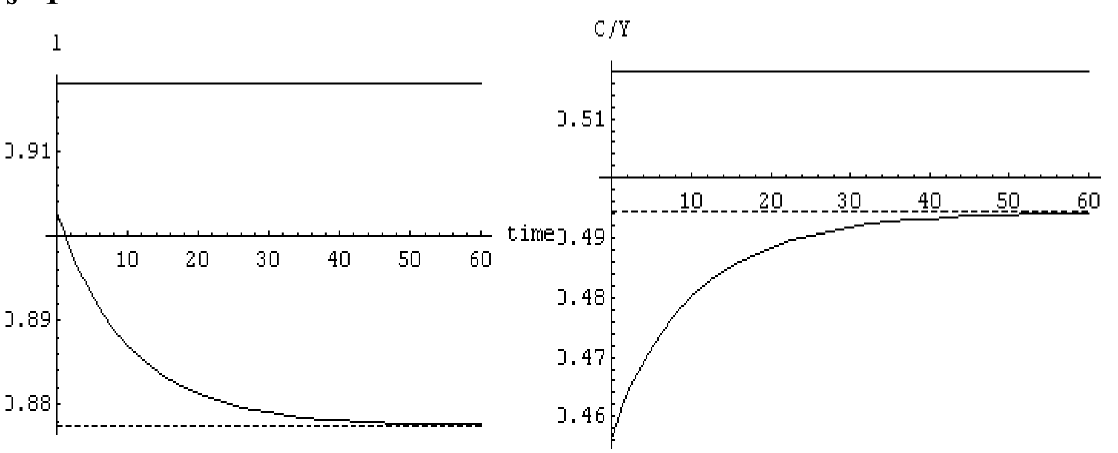




s=0.5

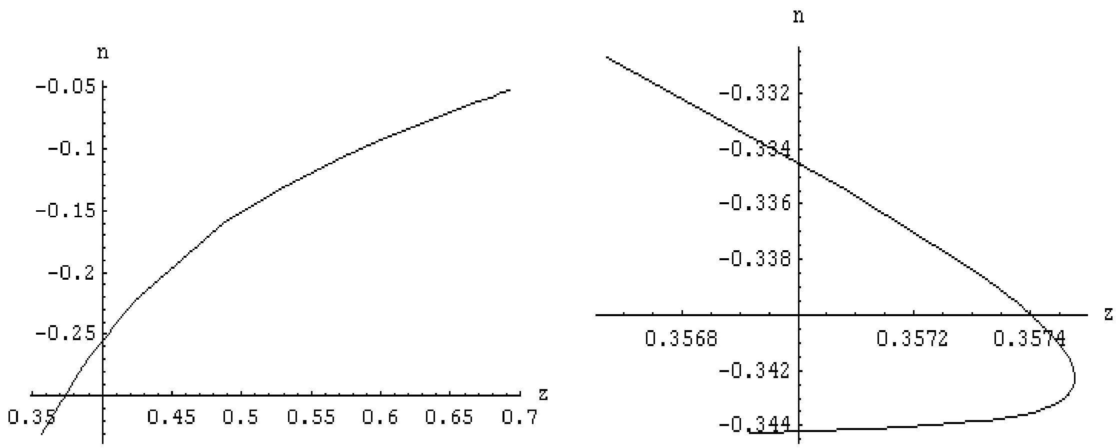


s =1

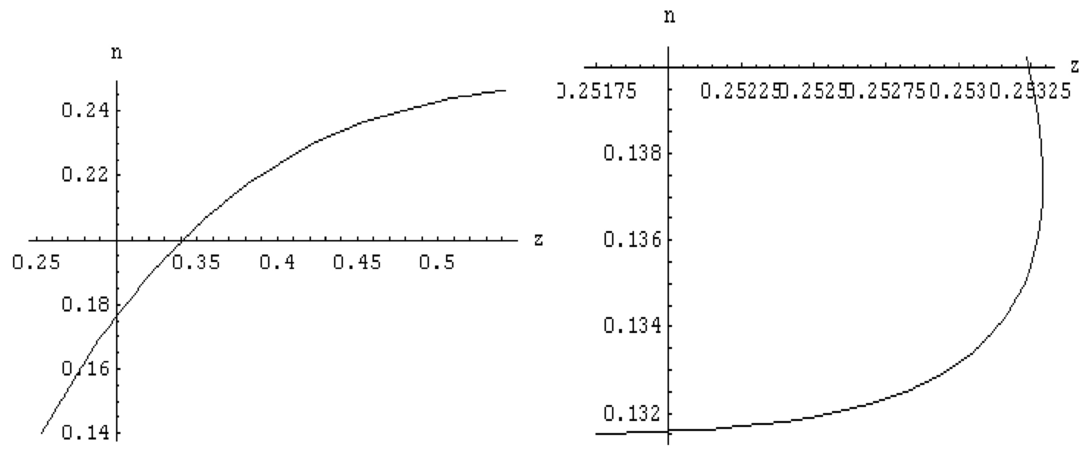


s=1.6

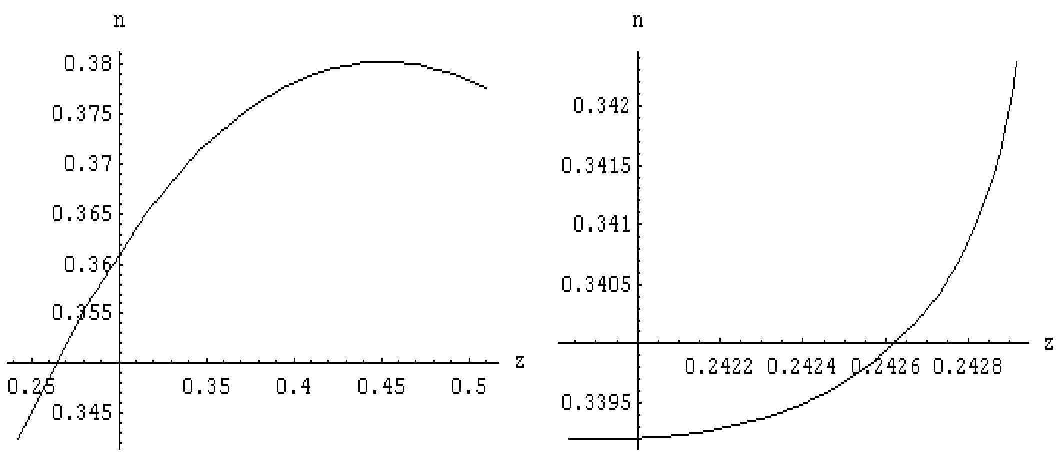
Fig 3: Dynamics of Labor-consumption sensitivity to Elasticity of Substitution



s=0.5



s= 1



s=1.6

I. Tied Tansfers **II Pure Transfers**

Fig 4: Sensitivity of Basic Dynamics to elasticity of Substitution