Unsolicited Commercial Email: An Attention Resource

Perspective

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Abstract
The growth of unsolicited commercial email (UCE) imposes increasing costs on organizations and causes considerable aggravation on the part of email recipients. A thriving anti-spam industry addresses some of the frustration. This paper contributes to our understanding of the UCE phenomenon by drawing on scholarly work in areas of marketing and resource ownership and use. Adapting the tragedy of the commons to the email context, we identify a causal structure that drives the direct e-marketing industry. Computer simulations indicate that although filtering may be an effective method to curb UCE arriving at individual inboxes, it is likely to increase the aggregate volume, thereby boosting overall costs. The analysis advances understanding of the digital commons, the economics of UCE, and has practical implications for the direct e-marketing industry.

Keywords: SPAM, unsolicited commercial email, UCE, system dynamics, filtering, computer simulation

Introduction
Computer-mediated communication has become one of the accepted channels in the mix of outlets that modern companies rely on to advertise their products (Figure 1). Electronic mail (email) advertising had nearly a billion dollars in revenue in 2001 and is predicted to reach several billion dollars within a few years (Martin, Durme et al. 2003). Reputable commercial establishments, such as J.C. Penney, Barnes and Noble, and Borders use email for communicating with customers (Martin, Durme et al. 2003). The marketing industry’s search for an optimal portfolio of online and traditional advertising (Kover 1999; Sheehan and Doherty 2001) will eventually evolve into integrated marketing communication programs (Brackett and Carr 2001).

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A cleverly designed direct marketing campaign contributes to overall sales (Chiang, Chhajed et al. 2003). Email is more attractive than regular mail due to its lower mailing cost, wider reach, convenience, and faster responses (Mehta and Sivadas 1995; Sheehan and McMillan 1999; Martin, Durme et al. 2003). The cost of sending email is $5 to $7 per one thousand messages, while it is $500 to $700 for the same volume of regular mail – two orders of magnitude greater (Martin, Durme et al. 2003). Moreover, digital marketing campaigns are easier to customize, which can produce better response rates than for mail campaigns (Ansari and Mela 2003). Timing is also an issue. It takes five to ten days to receive a response with email, versus ten to fifteen days with postal surveys (Sheehan and McMillan 1999). Finally, by including hyperlinks, email allows a degree of interactivity not afforded by conventional direct mail campaigns (Martin, Durme et al. 2003).

**Figure 1: Shares of total marketing expenditure in 2003 by channel. The UK case. Adapted from DMIS 2004.**

**Drawbacks of Direct E-Marketing**

Despite the many benefits to senders of direct e-marketing campaigns, there are pernicious impacts on consumers, email providers, and organizations. Many users are angry and frustrated, having to sift through mountains of what they perceive to be unsolicited commercial email in their inboxes. Email administrators struggle to maintain high service quality in the face of increasing server loads, storage requirements, and security threats. Of the roughly 31 billion daily emails sent globally, about 12.4 billion (41 percent) are considered UCE – MSN alone blocks 2.4 billion per day. The average email user receives 4.5 adult content emails per day, 16 percent of users change their email address due to UCE saturation, and 4.5 seconds of corporate time is wasted per spam message. With some users receiving hundreds of UCE messages per day, it’s no wonder that frustration is growing. Research conducted by the OECD examined the costs of UCE, finding that in June 2004, the annual spam cost per employee exceeded $1,900

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and the annual lost productivity per employee equaled 3.1 percent (Anonymous 2004). Corporations are burdened by the financial and intangible costs of spam, and managers struggle to find solutions to UCE (Corbitt 2004). They fear that the situation is likely to become more grave in the future (Fallows 2003).

*The Quest For Eyeballs: Attention as a Scarce Resource*

Over thirty years ago, Nobel laureate Herbert Simon (1971) observed that *attention* is a scarce resource in an information-rich society. In the parlance of modern theory, the attention of workers can be viewed as a strategic asset (Warren 2002) that determines the long-term success of an organization. According to Simon (1971, p. 44): “The design principle that attention is scarce and must be preserved is very different from a principle of “the more information the better.” When more information arrives than individuals can process, an *information overload* (Simon 1971) occurs, and the likelihood of organizational failure increases.

Examples of information overload are abundant. The first category of information overload, when there is more information than can be analyzed by given resources, is demonstrated by recent reports about FBI’s inability to process hours of sensitive communication that may have high intelligence value (Lichtblau 2004). Perlow (1999) gives an example of a situation in which information overload occurs due to the loss of attention resource. Perlow describes a software company in which employees were continually distracted throughout the day from their primary tasks. The reduced amount of attention devoted to productive activities led to chronic project time overruns.

The attention squeeze and information overload are exacerbated by the onslaught of UCE, whether viewed from an individual, organizational, or macro level. At an individual level, spam is increasing tremendously for some email users. At an organizational level, spam as a percentage of regular email is substantial (Melville, Plice et al. 2005). On an aggregate level, industry reports suggest a steady upward trend for UCE volume. Brightmail, for example, estimates that spam as a percentage of total email grew from 49 percent in June 2003 to 65 percent in June 2004, a 33 percent annual increase. More somber news is that spam is moving beyond email to other platforms, including instant messaging (spim), blogs, and mobile text messaging. Given these dire trends and the limited time an individual spends on email, the email recipient is bound to experience information overload.

We now extend the concept of attention as a limited resource to the realm of the digital commons. We then construct a causal model of the UCE industry and use the model to analyze one popular abatement mechanism: filtering. The last section summarizes results and outlines further extensions.

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**Tragedy of the digital commons**

Rapidly increasing spam volume is a result of decisions made by many self-interested agents involved in digital marketing. These participants vie for one common resource: the attention of email recipients. A typical outcome for a situation in which many profit-seeking agents compete for the same scarce resource is resource overuse. The suboptimal outcome is often described as the *tragedy of the commons* (Hardin 1968). A vivid example is overexploitation of fish stock in the ocean. In an online world in which many resources are still open to all: “Management of the digital commons is perhaps the most critical issue of market design that our society faces” (McFadden 2001, pp. 61).

*The Physical Commons*

When self-interested decisions concerning a common scarce resource degrade the quality or quantity of the resource (think fish in oceans), there is said to be a *tragedy of the commons* (Hardin 1968, Mankiw 2001, Mas-Colell, Whinston et al. 1995). A common resource is typically identified as one having the following two properties: (i) it is rival, that is, when it is used, less is available for others; and (ii) it is nonexclusive, that is, no one can be barred from using it. Individuals seek disproportionate private gains through the use of the resource but do not bear the full cost. The oceans, forests, grazing lands, the atmosphere, outer space, and highways are all susceptible to problems of the commons. History is replete with examples of resource degradation by rational, self-interested individuals: fish in oceans, oil reserves, etc.

*The Digital Commons*

The commons problem, however, is not limited to the physical world. Members of early Usenet discussion groups in the 1980s faced analogous circumstances: the groups were open to everyone and a small set of users could degrade the environment for all. In this context “pollution” sprang from various sources – excessive posting or posts that were off topic, offensive, or contained advertising – and lowered the value for all. The notion of *virtual commons* was thus applied to an online common resource whose misuse by the few degraded the value of the resource for the many (Kollock and Smith 1996).

To formalize the application of commons logic to the Internet, two conditions are necessary (Regan 2002). First, the Web must be a “place,” just as the earth is a place. The Internet is commonly and consistently recognized as a place for conducting a wide array of economic and social activity. Everyday metaphors provide evidence in this regard, with terms such as “going online,” “size of the internet,” “internet storm,” “virtual community,” and “virus” illustrating the mapping of the physical to the virtual. The place metaphor is also a fundamental concept used in Internet law: “the cyberspace as place metaphor operates as one of the most compelling theories of how we have regulated cyberspace to date, and how we are likely to regulate it in the future” (Hunter 2003, pp. 446).
The second necessary condition of online commons is that it must contain resources (analogous to fish stock in oceans) characterized by sharing, the lack of clearly defined private ownership, overuse, and negative externalities. A common resource is shared by many and private ownership is unclear or non-existent, just as no one owns the depths of the oceans and the fish stock in it. The Internet has many resources that involve sharing, including public discussion groups, peer-to-peer file sharing networks, and email. Also, these areas of cyberspace are characterized by the fact that no one can be barred from using them. There is the presence of overuse by rational individuals leading to pollution that affects all. In this context, email is a common good (Regan 2002). Spam is clearly the result of rational businesspersons whose private revenues exceed costs to society. Spam exhibits signs of a negative externality, which results in production that is higher than society desires. This is evidenced by the bombardment of email addresses with spam and the resulting financial and non-pecuniary costs borne by each of the millions of users, their respective email administrators, and employers. Table 1 draws parallels between UCE and fish population, which is a canonical common resource suffering from the tragedy of the commons. As the online commons is not a biological system, we take care in drawing the analogy homomorphically, i.e., by “paying attention to the peculiarities of the digital environment as well” (Greco and Floridi 2004).

<table>
<thead>
<tr>
<th>Common Resource</th>
<th>Fisheries</th>
<th>UCE</th>
</tr>
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<tbody>
<tr>
<td>Fish stock in oceans</td>
<td>Fish as much as possible</td>
<td>Send as much UCE as possible</td>
</tr>
<tr>
<td>Fishing expeditions</td>
<td>Marketing campaigns</td>
<td></td>
</tr>
<tr>
<td>Over fishing</td>
<td>Information overload</td>
<td></td>
</tr>
</tbody>
</table>

Dynamic analysis of filtering

Conventional commons problems such as over-fishing have been modeled as dynamic systems (e.g. FishBanks interactive computer simulation⁵). The system is comprised of at least two agents whose quests for private gain reinforce each other until curtailed by limits in the environment. In the case of fishing, each agent will maximize revenue or profit until the system is overrun and fish stocks become depleted. Our approach is to adapt this model to the case of the online commons, specifically, UCE. To the best of our

⁵ http://www.unh.edu/ipssr/Lab/FishBank.html
knowledge, this approach to studying spam is unique and it allows leveraging what we already know about physical commons to the problems of online commons.

We model the UCE value chain as having four participants: 1) inbox owners, 2) harvesters, 3) operators, and 4) sponsors. The inbox population is the set of feasible recipients of unwanted commercial e-mail. Harvesters are in the business of discovering inboxes and compiling them into lists of e-mail addresses, which they sell to UCE operators. UCE operators administer spam campaigns, which promote products from sponsors. Finally, sponsors support campaigns based on their success rate. We now explain the dynamic processes relating these four value chain participants illustrated in Figure 2.

**Address Harvesting**

To receive UCE, an inbox address must be discovered by a harvester. There are hundreds of ways to collect inbox addresses (Brain 2004). One of them is via directory harvest attacks (DHA), in which automated programs query email servers for the existence of millions of commonly designated usernames\(^6\). A study by the Center for Democracy and Technology (Center for Democracy & Technology 2003) reports that harvesters are also very effective at gathering email addresses posted on the web. There are also clearly illegal harvesting techniques, such as when an AOL employee was recently arrested for stealing the email addresses of 92 million AOL users (Swartz 2004a). The employee then sold the list to an operator of an online gambling business in Las Vegas for $100,000. That person in turn repackaged and resold the addresses to spammers for over a million dollars. Considering the many ways in which harvesters add email addresses to their lists, it is reasonable to assume that it is only a matter of time before an email account is discovered (see Figure 2). We model this by including an average inbox discovery delay. Delays, including the discovery delay, are shown in Figure 2 as two short lines crossing an arrow.

**Attention and Information Overload**

The attention resource can be measured in terms of time (Simon 1971). According to a recent survey conducted by the American Management Association, an employee typically spends about a quarter of her day on email (Swartz 2004b). Employees whose inboxes have been discovered possess a limited *attention resource*. The total demand for attention from regular and UCE email is proportional to their respective volumes delivered to inboxes. Assuming that regular email has a higher priority than spam, the time left for UCE is the difference between the *attention resource* and the *attention devoted to regular email* (see Figure 2). If the arriving volume of electronic messages is greater than what an individual is comfortable handling, then, using Herbert Simon’s terminology, *information overload* occurs.

\(^{6}\) See http://www.postini.com/services/what_are_dhas.html
Response Rate

Advertisers have known about the negative relationship between advertising volume that an individual is exposed to and the response rate to advertisements (Rudolph 1947; Starch 1966; Houston and Scott 1984). Houston and Scott (1984), for example, statistically showed a negative convex relationship between advertising readership and the number of pages in a journal. Recent research shows that the negative relationship holds equally well for electronic marketing. Martin et al. (2003) found that in the case of permission-based advertising for a company operating from Finland, the likelihood of visiting a link advertised within an email decreased as volume of email from the company increased. A recent survey by the Pew Internet & American Life Project (Rainie and Fallows 2004) found a decline in the readership of UCE while the UCE volume increased. Anecdotal evidence from UCE operators (Hansell 2003) also confirms the existence of a negative relationship between the amount of spam that a finite group receives and the response rate. This is indicated by the loop in Figure 2 comprising UCE volume - attention required by UCE - information overload - response rate.

Profitability

For a given overall response rate, the total number of responses a company receives increases with its share in the email volume (in Figure 2, this is captured by positive links between UCE Volume from Operator and Responses to UCE from Operator). More responses imply more revenue (see Figure 2). More revenue means more profit. Greater profit implies that with some delay (shown as two short lines crossing an arrow in Figure 2) more budget is allocated for UCE by a sponsor and thus expenditure on UCE volume increases. The UCE volume that a sponsor can buy for a given expenditure is inversely proportional to the UCE price that an operator charges for sending electronic messages.

It is clear from the graph in Figure 2 that there is a tendency to step up UCE volume while profits from UCE campaigns increase. This is captured by two positive Sponsor Profit Loops. Starting new campaigns is easy and quick thanks to specialized software packages (Lemke 2003). An example of such a tool is iBuilder from VerticalResponse7. Hence, campaigns have low marginal cost, and therefore cost recovery is unimportant (Kraut, Sunder et al. 2002). A campaign requires a very low response rate to break even: 0.001 percent is often sufficient (Fallows 2003: 26). The causality acting through the response rate forms the Attention Limit Loop (Figure 2), which checks the exponential growth of spam.

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7 http://www.verticalresponse.com/
Results: Base Case

We conducted computer simulations for a fictitious organization with 10,000 inboxes. The organization could be, for example, a company with employees or an email provider hosting email accounts. We allow only a small portion of the accounts to be initially known to spammers. In the model, we assume that UCE budget is proportional to the profitability of UCE campaigns; that is, the more profitable the UCE, the greater budget will be allocated for e-marketing. Table 2 summarizes the assumptions of the model. Parameter values were suggested by published surveys and anecdotal evidence from experts. Appendix offers a detailed mathematical description of the model.
Table 2: Model assumptions

<table>
<thead>
<tr>
<th>Item</th>
<th>Implementation</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Organization size</td>
<td>10000 inboxes</td>
<td>Assumption</td>
</tr>
<tr>
<td>Initial population of discovered inboxes</td>
<td>10 Inboxes</td>
<td>Assumption</td>
</tr>
<tr>
<td>Time spent on email by an employee</td>
<td>2 Hours/day</td>
<td>Swartz 2004b</td>
</tr>
<tr>
<td>Average regular email volume</td>
<td>20 Messages/day</td>
<td>Assumption</td>
</tr>
<tr>
<td>Base price of sending 1,000 UCE messages</td>
<td>$5 per 1000 messages</td>
<td>Martin, Durme et al. 2003</td>
</tr>
<tr>
<td>Response rate</td>
<td>A declining function of UCE volume</td>
<td>See Response Rate section above</td>
</tr>
<tr>
<td>Email marketing budget</td>
<td>Allocated proportionally to the past profit from the UCE campaign</td>
<td>Assumption</td>
</tr>
</tbody>
</table>

Figure 3 shows the base run for the simulated two years of life of our fictitious organization. As expected, given the fixed number of inboxes, the total number of discovered inboxes grows monotonically and asymptotically toward the total inbox population (Figure 3a). Within two years, more than 80% of the inboxes have been discovered. Positive profits accrued through spam campaigns (Figure 3b) encourage sponsors to allocate even more resources for electronic marketing (Figure 3c and Figure 2). Greater UCE budgets allow each sponsor to spend more on email (sponsor’s expenditure in Figure 2), which contributes to the growth of the UCE volume from operator A and B (Figure 2). Hence, global UCE volume grows as well (Figure 3d and Figure 2). The UCE volume that arrives to an individual inbox also grows exponentially (Figure 3e), which is consistent with real life examples (see Footnote 4).
**Results: The Impact of Spam Filtering**

When foreseeing an approaching information overload, Herbert Simon suggested in the early 1970s that filtering might be a possible solution (Simon 1971). Filtering of unsolicited email has proven to be capable of reducing demand for attention: users report lesser burden of spam at work than on their personal email accounts due to active email screening efforts at work (Fallows 2003). The popularity of this solution feeds the growth of a new and active anti-spam software industry. The method, however, has its flaws. Many inbox users fear that aggressive filtering may lead to some legitimate email being discarded. A survey by the Pew Internet Project (Fallows 2003: 29) found that about one third of the respondents feared their incoming email might be blocked, and 13 percent were convinced that it happened to them. About a quarter of respondents feared that their outgoing emails might be filtered out by the intended recipient.

In this section we study the effect of filtering on UCE volume using the computer model introduced earlier. We assume that the organization starts filtering email in the third year. To address fears that legitimate email may be discarded, the organization discards only UCE-suspects that it is most confident about. An interested reader can find a detailed account of the filtering procedures followed by a typical medium-sized organization in Melville et al. (2005).
Figure 4 shows a simulation that extends the run in Figure 3 for five more years (2555 days total). The figure compares the case when filtering is used to the case when no filtering is implemented. Under each scenario, during the seven-year period, harvesters discover all inboxes (Figure 4 a). Figures 7b – 7e depict the case when the organization performs no active filtering with dotted lines. As in the base case simulation discussed earlier, driven by the two Sponsor Profit Loops (see Figure 2), each operator continues to increase UCE production. This leads to the growth in global volume of unsolicited messages (Figure 4 b), which in turn contributes to the increase in spam arriving to individual accounts (Figure 4 c). Eventually, attention required by UCE outgrows attention available for UCE and information overload becomes more strongly felt (see Figure 2). Email recipients, who are overwhelmed by increasing volumes of spam (Figure 4 c), tend to delete most of it, thus driving the overall response rate down (see Figure 2 and ‘no filtering’ case in Figure 4 d). The declining response rate leads to lower revenue and lower profit (see Figure 2 to trace the logic and Figure 4 e for the resulting profit).
trajectory). With some delay, the declining financial performance of the electronic marketing campaigns affects the sponsors’ **UCE budgets** (Figure 2). As a result, the **UCE volume** tapers off (later portions of Figure 4 b).

Let us now examine the effect of filtering, which is clearly visible starting in year three. Filtering reduces the burden from UCE on an individual by lowering **UCE volume arriving to an individual inbox** (solid line in Figure 4 c). Feeling less overwhelmed by incoming spam, readers tend to read a greater fraction of incoming messages, thus increasing the overall **response rate** (solid line in Figure 4 d). Better response rates drive profitability upwards (solid line in Figure 4 e). Better profitability encourages greater **UCE budgets** (Figure 2), thus allowing sponsors to spend more on email marketing. Greater expenditure by each sponsor boosts overall **UCE volume** (filtered case in Figure 4 b). Hence, an interesting result is that, while lowering the burden of spam on individuals (Figure 4 c), filtering is likely to increase overall volume of UCE (Figure 4 b).

**Discussion**

In this paper we addressed the growing problem of unsolicited commercial email. Adopting the viewpoint that in an information-rich society **attention** is a limited resource – a notion proffered by Herbert Simon (1971) – allowed us to describe the problem of spam in terms of a common resource. The common resource framework is well understood in economic literature and has been helpful in explaining many phenomena that lead to the overexploitation of limited resources. The situation of overexploitation of a resource by self-interested agents is generally referred to as the **tragedy of the commons**. The framework has been applied to other Internet-related problems. When applied to the virtual world, the phenomenon has been dubbed the **tragedy of the digital commons**.

Filtering has been proposed as a possible solution to **information overload** long before email became popular (Simon 1971). Filtering, however, may impose costs that exceed the benefit (Cranor and Lamacchia 1998). The benefit is the reduction of spam volume arriving to an individual inbox. However, as our discussion in the previous sections showed, the use of filtering is likely to stimulate production of spam. Greater spam volume consumes more of organizations’ bandwidth and processing resources (Melville, Plice et al. 2005). Secondly, organizations and spam senders iteratively improve their filtering and electronic marketing tools, respectively, with no clear end to, or winner of, such an arms race. The continuous anti-spam effort is costly. Thirdly, inbox owners bear the cost too because false positives during spam filtering lead to the deletion of desired email. From a theoretical standpoint, this reduces consumer surplus (Loder, Van Alstyne et al. 2004). Table 3 summarizes benefits and drawbacks of the filtering solution.
<table>
<thead>
<tr>
<th>Benefit</th>
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<tbody>
<tr>
<td>Lower UCE volume to an inbox</td>
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<tr>
<td>Drawbacks</td>
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<tr>
<td>Greater global volume of UCE</td>
</tr>
<tr>
<td>Emergence of the costly “anti-spam arms race”</td>
</tr>
<tr>
<td>False-positives lower consumer surplus</td>
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</table>

Our analysis has several practical implications. Because of its effectiveness at the individual level, filtering will inevitably increase in popularity. Considering that many individuals still do not use filtering software and only 28 percent of small and medium size enterprises have active anti-spam measures in place (PR Newswire 2004), the growth in the filtering effort is likely to be very significant. As our simulations suggest, proliferation of filtering will further boost the global UCE volume. It will also encourage an arms race between IT managers and the filtering software companies on the one side and spam operators on the opposing side. Owners of filtering software companies are well positioned to benefit financially from the situation. Consumers and IT managers, on the other hand, will pay for the new upgrades of the filtering software and for the addition of processing and bandwidth capacity needed to handle the increased UCE volume. Moreover, greater reliance on filtering will make the task of legitimate electronic advertisers harder, as without the reliance on obfuscating tricks used by spammers, direct marketers are easy targets for filtering. Hence, to remain effective, the direct marketing industry will need to do the following: (i) foster relationships with customers; (ii) be sensitive to the optimal emailing frequency; (iii) limit email to customers who consented to receiving such communication; (iv) target and customize email, so as to make email communication useful to the recipient; (v) use email as only one dimension in a comprehensive marketing campaign; and (vi) actively distinguish itself from UCE.
Appendix

Below are the model equations.

**Address Harvesting**

- Average time to discovery: \( t_d \)
- Undiscovered inboxes: \( I_u \)
- Discovered inboxes: \( I_d \)
- Total inboxes: \( I = I_u + I_d \)
- Fraction of inboxes discovered: \( i = I_d / I \)

\[
\frac{d}{dt} I_d = I_u / t_d
\]

**Attention and Information Overload**

- Time an employee can spend on email: \( T \)
- Attention resource of an organization: \( A = T \cdot I \)
- Discovered attention resource: \( A_d = i \cdot A \)
- Time it takes to read an email: \( e \)
- Regular email volume to discovered inboxes: \( E_r \)
- Attention devoted to regular mail: \( A_r = e \cdot E_r \)
- Attention available for UCE: \( A_{UCE} = A_d - A_r \)
- UCE volume: \( E_{UCE} = E_a + E_b \)
- Attention required by UCE: \( D = e \cdot E_{UCE} \)
- Information overload: \( O = D / A_{UCE} \)

**Response Rate**

- Maximum response rate: \( r_{\max} \)
- Response rate: \( r = r_{\max} \cdot f(O) \)
- \( 0 \leq f(O) \leq 1, \quad f'(O) < 0 \)

**Profitability**

- Sponsor A’s profit: \( \pi = M_d - C_d \)
- Budget duration: \( \tau \)
- Average daily profit: \( (d / dt) \overline{\pi} = (\pi - \overline{\pi}) / \tau \)
- Sponsor A’s expenditure: \( C_d = B_d / \tau \)
- Price of UCE: \( p \)
- UCE volume from operator A: \( E_a = C_d / p \)
- Responses to UCE from operator A: \( R_d = r \cdot E_d \)
- Average revenue per sale: \( m \)
- Sponsor A’s revenue: \( M_d = m \cdot R_d \)
- Marketing fraction: \( a \)
- Adjustment of spam budget: \( B_s = a \cdot \overline{\pi} \)
- Sponsor A’s UCE budget: \( (d / dt) B_A = B_s - C_d \)
References


