AN INTERPRETATION OF FLUCTUATING MACRO POLICIES

TROY DAVIG AND ERIC M. LEEPER

ABSTRACT. This paper estimates simple regime-switching rules for monetary policy and tax policy over the post-war period in the United States and imposes the estimated policy process on a standard dynamic stochastic general equilibrium model with nominal rigidities. The estimated joint policy process produces a unique stationary rational expectations equilibrium in a simple New Keynesian model. We characterize policy impacts across regimes.

Policy regime changes are like the weather: everyone talks about them but few people do anything about them. This paper does something about them. It estimates simple Markov-switching rules for monetary policy and tax policy over the post-war period in the United States. When imposed on a standard dynamic stochastic general equilibrium model with nominal rigidities, the estimated policy process produces a unique stationary rational expectations equilibrium. In that equilibrium, shocks to (lump-sum) taxes always affect aggregate demand for reasons articulated by the fiscal theory of the price level. The paper’s view that monetary and fiscal policies are subject to on-going fluctuations in regime puts on the table a new interpretation of macro policies and their impacts over the past six decades.1

Date: January 18, 2005. This is a preliminary draft. Department of Economics, The College of William and Mary, tadav3@wm.edu; Department of Economics, Indiana University and NBER, eleeper@indiana.edu. We thank Jon Faust, Jinill Kim, Jim Nason, Jürgen von Hagen, Tack Yun, Tao Zha, and seminar participants at the Federal Reserve Board for comments, and especially Hess Chung for many helpful discussions and suggestions.

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Estimated rules separate policy behavior into distinct periods of active and passive monetary and fiscal regimes that accord well with narrative accounts of policy and with fixed-regime estimates of policy rules.\footnote{We apply the terminology in Leeper (1991). \textit{Active monetary policy} is periods when the response of the interest rate is more than one-for-one to inflation and \textit{passive monetary policy} is periods when that response is less than one-for-one. Analogously, \textit{passive fiscal policy} occurs when the response of taxes to debt exceeds the real interest rate and \textit{active fiscal policy} occurs when taxes do not respond sufficiently to debt to cover real interest payments. In many models, a unique stationary equilibrium requires one active and one passive policy.} Monetary policy follows a Taylor (1993) rule and fiscal policy adjusts taxes as a function of government debt and other variables. The estimates uncover periods of active monetary/passive fiscal behavior, the policy mix typically assumed to prevail in monetary studies, along with episodes of passive monetary/active fiscal behavior, the mix associated with the fiscal theory of the price level. Remaining periods combine passive monetary with passive fiscal policy or active monetary with active fiscal behavior.

Because simple policy rules that undergo recurring regime changes fit U.S. data well, it is natural to examine the implications of embedding on-going regime change in a standard dynamic stochastic general equilibrium model. Conventional fixed-regime theories, in which regime is permanent and regime change is always a surprise that \textit{a priori} agents believe to be impossible, provide awkward interpretations of the evidence. By that view, the empirical evidence suggests the economy is lurching among periods of indeterminacy (passive/passive), non-existence of equilibrium (active/active), or unique equilibria with completely different characteristics (active monetary/passive fiscal or passive monetary/active fiscal) \cite{ClaridaGaliGertler2000,Sala2004} for such interpretations.

Our interpretation of policy behavior views the post-war period as one of on-going regime change. Recognizing that policies change regime according to some probability law, private agents embed the stochastic process for regime in their expectation functions and decision rules. There can exist a unique stationary equilibrium even though policies can go through periods where both are passive or both are active. To provide a coherent interpretation of the history of policy behavior, we impose the estimated policy process directly on an off-the-shelf model with nominal rigidities, using a calibration that follows Rotemberg and Woodford (1997). The process governing regime change and the policy rules estimated from U.S. data produce a unique stationary equilibrium in that model. In that equilibrium, exogenous disturbances to monetary and tax policy induce agents to believe that, at initial prices, their wealth has changed. Responses to perceived changes in wealth determine the impacts of those shocks. The model implies that\footnote{We apply the terminology in Leeper (1991). \textit{Active monetary policy} is periods when the response of the interest rate is more than one-for-one to inflation and \textit{passive monetary policy} is periods when that response is less than one-for-one. Analogously, \textit{passive fiscal policy} occurs when the response of taxes to debt exceeds the real interest rate and \textit{active fiscal policy} occurs when taxes do not respond sufficiently to debt to cover real interest payments. In many models, a unique stationary equilibrium requires one active and one passive policy.}
• shocks to lump-sum taxes always affect aggregate demand, inflation, and output, regardless of the prevailing policy regime;
• \textit{i.i.d.} tax shocks are propagated for many periods by the Fed's interest rate response to inflation;
• monetary shocks have conventional short-run impacts, but because their wealth effects are not neutralized, their long-run effects can differ in important ways;
• unique stationary equilibrium exists even conditional on active monetary/active fiscal behavior or passive monetary/passive fiscal behavior.

A regime-switching setup using an estimated policy process helps to reconcile the fiscal implications of conventional models with the empirical literature that finds large aggregate demand effects from tax policy [Blanchard and Perotti (2002), Mountford and Uhlig (2002), and Perotti (2004)]. Conventional monetary analysis assumes monetary policy is active and fiscal policy is passive [for example, Woodford (2003)]. With lump-sum taxes, the conventional policy mix produces Ricardian equivalence. But taxes are no longer irrelevant when regimes recur and the joint policy process is consistent with U.S. data. It also turns out that the switching environment generates substantial short-run aggregate demand effects from tax disturbances, an outcome that Poterba and Summers (1987) argue life-cycle approaches to breaking down Ricardian equivalence cannot produce.

Why treat regime change as recurring? A single piece of legislation set the macro policy agenda for the post-war era. The Employment Act of 1946 states that it shall be the goal of the Federal government “to promote maximum employment, production, and purchasing power.” Stein (1996) characterizes this act as the culmination of a developing consensus on the objectives of macro policies. No legislation since then has instructed monetary or fiscal authorities to behave differently. The Humphrey-Hawkins Act of 1978, for example, “did nothing at all—save commit the Federal Reserve chairman to a twice-a-year round of congressional testimony” [DeLong (1997, p. 271)]. Orphanides (2003a) argues there has been great consistency in the Fed’s objectives, at least since World War II. Those objectives are summarized in a 1977 amendment to the Federal Reserve Act as being “maximum employment, stable prices, and moderate long-term interest rates.” Without the creation of new policy institutions or changes in the legal mandates of existing institutions, there is nothing to prevent past policy behavior from recurring, and treating changes in policy behavior as once-and-for-all reforms is at best a working hypothesis. Constancy of the overarching macro policy objectives means that regime change, when it occurs, is the

\footnote{These results generalize and make empirically relevant the calibrated findings of Davig, Leeper, and Chung (2004).}
outgrowth of political pressures and personalities of policy makers—factors that are
temporary and likely to recur. This suggests the alternative working hypothesis of
on-going regime change.  

1. Estimated Policy Rules

We seek empirical characterizations of policy behavior that use simple rules of
the kind appearing in the policy literature, but allow for the possibility of on-going
changes in regime. Monetary and tax policies follow regimes that can switch indepen-
dently of each other. This section reports maximum likelihood estimates of monetary
and fiscal rules in which policy regime evolves according to a hidden Markov chain,

1.1. Specifications. For monetary policy, we estimate a standard Taylor speci-
cification, which Rotemberg and Woodford (1999) have shown is nearly optimal in
the class of models we consider in section 2. The rule makes the nominal interest rate,
r_t, depend only on inflation, \pi_t, and the output gap, x_t:

\[ r_t = \alpha_0(S_t^M) + \alpha_x(S_t^M)x_t + \alpha_R(S_t^M)\varepsilon_t^R, \]  

where \( S_t^M \) is the monetary policy regime. Regime evolves according to a Markov
chain with transition matrix \( P^M \). \( r \) and \( \pi \) are net rates. We allow for four states,
with the parameters restricted to take only two sets of values, while variances may
take four different values. \( P^M \) is a 4 × 4 matrix.

Unlike monetary policy, there is no widely accepted simple specification for fiscal
policy. We model some of the complexity of tax policy with a rule that allows for
the revenue impacts of automatic stabilizers, some degree of pay-as-you-go spending,
and a response to the state of government indebtedness. The rule links revenues net
of transfer payments, \( \tau_t \), to current government purchases, \( g_t \), the output gap, and
debt held by the public, \( b_{t-1} \). The variables \( (\tau_t, g_t, b_t) \) are measured as shares of GDP.
The specification is:

\[ \tau_t = \gamma_0(S_t^F) + \gamma_b(S_t^F)b_{t-1} + \gamma_x(S_t^F)x_t + \gamma_g(S_t^F)g_t + \sigma_\tau(S_t^F)\varepsilon_t^\tau, \]  

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4We restrict attention to recurring regime change also for computational and econometric
convenience.

5Examples of estimated fiscal rules include Bohn (1998), Taylor (2000), Fatas and Mihov (2001),
Auerbach (2003), Cohen and Follette (2003), Ballabriga and Martinez-Mongay (2004), and Claeyss
(2004a,b).
where $S^F_t$ is the fiscal policy regime, which obeys a Markov chain with transition matrix $P^F$ for the two fiscal states. Both (1) and (2) allow for heteroskedastic errors, which Sims and Zha (2004) emphasize are essential for fitting U.S. time series.  

Let $S_t = (S^M_t, S^F_t)$ denote the joint monetary and fiscal policy state. The joint distribution of policy regimes evolves according to a Markov chain with transition matrix $P = P^M \otimes P^F$, whose typical element is $p_{ij} = \Pr[S_t = j | S_{t-1} = i]$, where $\sum_j p_{ij} = 1$. With independent switching, the joint policy process has eight states.

1.2. Estimation Results. We use U.S. quarterly data from 1948:2 to 2004:1. To obtain estimates of (1) that resemble those from the Taylor rule literature, we define $\pi_t$ to be the inflation rate over the past four quarters. Similarly, estimates of (2) use the average debt-output ratio over the previous four quarters as a measure of $b_{t-1}$.

The nominal interest rate is the three-month Treasury bill rate in the secondary market. Inflation is the log difference in the GDP deflator over four quarters. The output gap is the log deviation of real GDP from the Congressional Budget Office’s measure of potential real GDP. All fiscal variables are for the federal government only. $\tau$ is federal tax receipts net of total federal transfer payments as a share of GDP, $b$ is the market value of gross marketable federal debt held by the public as a share of GDP, and $g$ is federal government consumption plus investment expenditures as a share of GDP. All variables are converted to quarterly rates.

Parameter estimates are reported in table 1 (standard errors in parentheses) and estimated transition matrices are in table 2.

For monetary policy, associated with each set of feedback parameters is a high- and a low-variance state. Monetary policy behavior breaks into periods when it responds strongly to inflation (active policy), as dictated by the Taylor principle, and periods when it does not (passive policy). In the active, volatile periods, the standard deviation is 3.7 times higher than in the active, docile periods; in passive periods, the standard deviations differ by a factor of seven. Passive regimes respond twice as strongly to the output gap, which is consistent with the Fed paying relatively less slack.

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6Ireland (2001), Leeper and Roush (2003), and Sims and Zha (2004) argue that allowing money growth to enter the monetary policy rule is important for identifying policy behavior. To keep to a specification that is comparable to the Taylor rule literature, we exclude money growth.

7We include a dummy variable to absorb the extreme variability in interest rates induced by credit controls in the second and third quarters of 1980. See Schreft (1990) for a detailed account of the economics and the politics of those controls.

8In the New Keynesian model of section 2, as Woodford (2003) shows for monetary policy, the presence of the output gap in the policy rules alters the roots of the linearized system. Because the gap has only a small effect on the roots, we retain the simple classifications of active and passive behavior.
attention to inflation stabilization. There are also important differences in duration of regime. Active regimes last about 15 quarters each, on average, while the duration of the docile passive regime is over 22 quarters; the volatile passive regime is most transient, with a duration of 11.6 quarters.

Tax policies fluctuate between responding by more than the quarterly real interest rate to debt (passive) and responding negatively to debt (active). The active policy is what one would expect from automatic stabilizers, which reduce revenues and raise debt as national income falls. Active policy reacts strongly to government spending, though by less than one-to-one, while passive policy reacts more weakly. In both regimes taxes rise systematically and strongly with the output gap, as one would expect from built-in stabilizers in the tax system. A stronger response to output under passive policy is consistent with active policy pursuing countercyclical objectives more vigorously. Assume that on average the same degrees of automatic stabilization and tax progressivity are in effect in active and passive periods. Because simultaneity between revenues and output biases downward the coefficient on the gap, a smaller coefficient is consistent with the idea that when fiscal policy is ignoring debt it is aggressively pursuing countercyclical tax policies.

1.3. Plausibility of Estimates. We consider four checks on the plausibility of the estimated rules. First, are the estimates reasonable on a priori grounds? We think they are, as the rules fluctuate between theoretically interpretable regimes. Monetary policy fluctuates between periods when it is active, satisfying the Taylor principle ($\alpha_\pi > 1$), and periods when it is passive ($\alpha_\pi < 1$). Passive tax policy responds to debt by a coefficient that exceeds most estimates of the quarterly real interest rate, while active tax policy lowers taxes when debt is high.

Second, how well do the estimated equations track the actual paths of the interest rate and taxes? We use the estimates of equations (1) and (2), weighted by the estimated regime probabilities, to predict the time paths of the short-term nominal interest rate, $R$, and the ratio of tax revenues to output, $\tau$, treating all explanatory variables as evolving exogenously. The predicted—using smoothed and filtered probabilities—and actual paths of $R$ and $\tau$ appear in figures 1 and 2. These fits are easily comparable to those reported by, for example, Taylor (1999a) for monetary policy.\textsuperscript{9} The interest-rate equation goes off track in the 1950s, suggesting that that

\textsuperscript{9}Orphanides (2003b) argues that the poor inflation performance from 1965-1979 was due, not to a weak response of policy to inflation, but to a strong response to poor estimates of the output gap available at the time. Using real-time data on the gap and inflation, he claims the fit of a conventional Taylor rule specification—1.5 on inflation, .5 on the gap, and a 2 percent real interest rate—is much improved when real-time data are used rather recent vintage data. Orphanides (2003a) extends this
period might constitute a third distinct regime. The tax rule tracks the revenue-output ratio extremely well, except in the last year or so when revenues dropped precipitously.

Third, do the periods estimated to be active and passive jibe with narrative accounts of policy history? The estimated marginal probabilities—smoothed and filtered—of the monetary and fiscal states are plotted in figures 3 and 5. All probabilities reported are at time $t$, conditional on information available at $t-1, \Omega_{t-1}$.

Figure 3 reports that, except for a brief active period in 1959-60, monetary policy was passive from 1948 until the Fed changed operating procedures October 1979 and policy became active. Monetary policy was consistently active except immediately after the two recessions in 1991 and 2001. For extended periods during the so-called “jobless recoveries” monetary policy continued to be less responsive to inflation for two or more years after the official troughs of the downturns. The passive episode in 1991 became active when the Fed launched its preemptive strike against inflation in 1994.

These results are broadly consistent with previous findings. From the beginning of the sample until the Treasury Accord of March 1951, Federal Reserve policy supported high bond prices to the exclusion of targeting inflation, an extreme form of passive monetary policy. Through the Korean War, monetary policy largely accommodated the financing needs of fiscal policy [Ohanian (1997) and Woodford (2001)]. Romer and Romer (2002) offer narrative evidence that Fed objectives and views about the economy in the 1950s were very much like those in the 1990s, particularly in its overarching concern about inflation. But Romer and Romer (2002, p. 123) quote Chairman William McChesney Martin’s congressional testimony, in which he explained that “the 1957-58 recession was a direct result of letting inflation get substantially ahead of us.” The Romers also mention that FOMC “members felt they had not reacted soon enough in 1955 [to offset the burst of inflation]” (p. 122). To buttress their narrative case, the Romers estimate a forward-looking Taylor rule from 1952:1-1958:4. They conclude that policy was active: the response of the interest rate to inflation

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10 We tried a three-regime specification, but rejected it because the response of the interest rate to output was negative.
12 An appendix considers alternative specifications of the policy process that increase the duration of the active monetary policy regime by re-labeling these periods active.
was 1.178 with a standard error of .876. Our estimate of this response coefficient in passive regimes is .522, which is less than one standard error below the Romers’ point estimate. The Fed might well have intended to be vigilant against inflation, but it appears not to have acted to prevent the 1955 inflation. More importantly for our purposes, the data cannot sharply distinguish between active and passive monetary policy behavior through most of the 1950s. The brief burst of active monetary policy late in 1959 and early in 1960 is consistent with the Romers’ (2002) finding that the Fed raised the real interest rate in this period to combat inflation. From 1960-1979, monetary policy responded weakly to inflation, while since the mid-1980s the Fed has reacted strongly to inflation, a pattern found in many studies [Taylor (1999a), Clarida, Gali, and Gertler (2000), Romer and Romer (2002) and Lubik and Schorfheide (2004)].

Figure 4 plots the nominal interest rate, inflation, the monetary regime, and the NBER business cycle dates. Active monetary periods are denoted by AM and passive period by PM. No pattern emerges linking recessions to monetary regime, suggesting that regime changes do not merely reflect changes in policy behavior over the business cycle.

Estimates of the tax rule in (2) reveal substantially more regime instability than for monetary policy. Over the post-war period, there were 12 fiscal regime changes, with tax policy spending 55 percent of the time in the active regime. Figure 5 shows that the model associates tax policy with regimes that accord well with narrative histories. Fiscal policy was active in the beginning of the sample. Despite an extremely high level of debt from World War II expenditures, Congress overrode President Truman’s veto in early 1948 to cut taxes. Although, as Stein (1996) recounts the history, legislators argued that cutting taxes would reduce the debt, the debt-GDP ratio rose while revenues as a share of GDP fell. In 1950 and 1951 policy became passive, as taxes were increased and excess profits taxes were extended into 1953 to finance the Korean War, consistent with the budget-balancing goals of both the Truman and the Eisenhower Administrations. From the mid-50s, through the Kennedy tax cut of 1964, and into the second half of the 1960s, fiscal policy was active, paying little attention to debt. There followed a period of about 15 years when fiscal policy fluctuated in its degree of concern about debt relative to economic conditions.

President Carter signed a bill to cut taxes to stimulate the economy in early 1979, initiating a period of active fiscal policy that extended through the Reagan Administration’s Economic Recovery Plan of 1981. By the mid-1980s, the probability of passive tax policy increased as legislation was passed in 1982 and 1984 to raise revenues in response to the rapidly increasing debt-output ratio. Following President
Clinton’s tax hike in 1993, fiscal policy switched to being passive through the 2001 tax cut. President Bush’s tax reductions in 2002 and 2003 made fiscal policy active again.

Some readers might regard as perverse the negative response of taxes to debt in the active fiscal regime. A negative correlation arises naturally over the business cycle, as recessions automatically lower revenues and raise debt. Figure 6 plots the debt-output and net taxes-output ratios along with dashed vertical lines marking NBER business cycle peaks and troughs, and solid vertical lines marking fiscal regime switches between active (AF) and passive (PF) behavior. Two active fiscal regimes appear to be generated by recessions: the late 1940s and 1973:4-1975:1 almost exactly coincide with the cycle. But there are extended periods of active behavior, which include but do not coincide with recessions [1955:4-1965:2 and 1978:4-1984:3]. There are also instances in which recessions occur during periods of passive fiscal policy [1990:3-1991:1 and 2001:1-2001:4]. Taken together these results suggest that the tax rule does more than simply identify active regimes with economic downturns.

Favero and Monacelli (2003) estimate switching regressions similar to (1) and (2). They also find that monetary policy was passive from 1961 to 1979. In contrast to our results, Favero and Monacelli do not detect any tendency to return to passive policy following the 1991 and 2001 recessions, though they do estimate one regime, which emerges from 1985:2-2000:4 and 2002:2-2002:4, in which monetary policy is only borderline active. It is difficult to compare their estimates of fiscal policy to ours because Favero and Monacelli use the net-of-interest deficit as the policy variable. Although common in empirical work on fiscal policy, this has the drawback of confounding spending and tax policies. Moreover, for the purpose of obtaining a policy process to embed in a DSGE model, treating the deficit as the control variable is problematic because spending and tax shocks generally have very different impacts. Like us, though, Favero and Monacelli find that fiscal policy is more unstable than monetary policy.

1.4. Joint Policy Process. It is convenient, and does no violence to the qualitative predictions of the theory, to aggregate the four monetary states to two states. We aggregate the high- and low-variance states for both the active and the passive regimes, weighted by the regimes’ ergodic probabilities. An analogous transformation is applied to the estimated variances. The resulting transition matrix is

\[ P^M = \begin{bmatrix} .9505 & .0495 \\ .0175 & .9825 \end{bmatrix} \]
and variance are $\sigma_R^2(S_t = \text{Active}) = 4.0576e - 6$ and $\sigma_R^2(S_t = \text{Passive}) = 1.8002e - 5$. Combining this transition matrix with the one estimated for fiscal policy yields the joint transition matrix

$$P = P^M \otimes P^F = \begin{bmatrix}
.8908 & .0597 & .0464 & .0031 \\
.0494 & .9011 & .0026 & .0469 \\
.0164 & .0011 & .9208 & .0617 \\
.0009 & .0166 & .0511 & .9314
\end{bmatrix}. \quad (4)$$

Probabilities on the main diagonal are $P[AM/PF | AM/PF]$, $P[AM/AF | AM/AF]$, $P[PM/PF | PM/PF]$, and $P[PM/AF | PM/AF]$. The transition matrix implies that all states communicate and each state is recurring, so the economy visits each one infinitely often. The probabilities of the joint distribution of policies, appearing in figure 7, are computed using (4).

Figure 7 shows that the joint probabilities of policies also correspond to periods that have been noted in the literature. Both policies were passive in the early 1950s, when the Fed supported bond prices (and gradually phased out that support) and fiscal policy was financing the Korean War [Ohanian (1997) and Woodford (2001)]. From the late 1960s through most of the 1970s, both policies were again passive. Arguing this, Clarida, Gali, and Gertler (2000) claim the policy mix allowed for bursts of inflation and output from self-fulfilling expectations. Using data only from 1960-1979, it is easy to see how one might reach this conclusion. The early-to-mid-1980s, when monetary policy was aggressively fighting inflation and fiscal policy was financing interest payments with new debt issuances, gets labeled as doubly active policies. Finally, the mid-1980s on is largely a period of active monetary and passive fiscal policies, as most models of monetary policy assume [Clarida, Gali, and Gertler (2000), Rotemberg and Woodford (1997) and the papers in Bryant, Hooper, and Mann (1993) and Taylor (1999b)].

The economy has spent the most time in a combination of passive monetary with active fiscal policies, the mix associated with the fiscal theory of the price level [Leeper (1991), Sims (1994), Woodford (1995), and Cochrane (1999)]. A substantial fraction of the sample combined passive monetary and tax policies, a combination, which if it were expected to persist forever, would leave the equilibrium undetermined in many theoretical models. Almost equal fractions of time are spent in the active monetary/passive fiscal regime—the policy mix assumed to prevail in most monetary analyses—and the active/active mix. If the active/active combination were expected to last forever, theory suggests that no equilibrium would exist, because private agents’ transversality conditions or some feasibility conditions are violated.
Taken together, the marginal and joint probabilities paint a picture of post-war monetary and fiscal policies that is broadly in accord with both narrative accounts and fixed-regime policy rule estimates. A final check on the plausibility of the estimates asks if estimated policies make sense when they are embedded in a conventional DSGE model. Section 4 answers this question in detail.13

2. A Conventional Model with Nominal Rigidities

This is a standard model with monopolistic competition and sticky prices in goods markets.14 We extend the model to include lump-sum taxes and nominal government debt.

2.1. Households. The representative household chooses \( \{C_t, N_t, M_t, B_t\} \) to maximize

\[
E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{C_{t+i}^{1-\sigma}}{1-\sigma} - \frac{N_{t+i}^{1+\eta}}{1+\eta} + \frac{\delta (M_{t+i}/P_{t+i})^{1-\kappa}}{1-\kappa} \right]
\]

with \( 0 < \beta < 1, \sigma > 0, \eta > 0, \kappa > 0, \chi > 0 \) and \( \delta > 0 \). \( C_t \) is a composite consumption good that combines the demand for the differentiated goods, \( c_{jt} \), using a Dixit and Stiglitz (1977) aggregator:

\[
C_t = \left[ \int_0^1 \frac{c_{jt}}{\theta} \, dj \right]^{\theta-1}, \quad \theta > 1.
\]

The household chooses \( c_{jt} \) to minimize expenditure on the continuum of goods indexed by the unit interval, leading to the demand functions for each good \( j \)

\[
c_{jt} = \left( \frac{p_{jt}}{P_t} \right)^{-\theta} C_t,
\]

where \( P_t \equiv \left[ \int_0^1 p_{jt}^{1-\theta} \, dj \right]^{1-\theta} \) is the aggregate price level at \( t \).

13A key limitation of the estimates stems from the absence of identification. This approach follows closely existing empirical work on simple policy rules, which usually does not estimate the rules as parts of a fully specified model. We are reassured in doing this by the model-based maximum likelihood estimates of Ireland (2001) and Lubik and Schorfheide (2004), which are very close to single-equation estimates of Taylor rules. It is noteworthy, though, that in an identified monetary VAR, Sims and Zha (2004) conclude that monetary policy was consistently active since 1960; they do not consider fiscal behavior.

The household’s budget constraint is

\[ C_t + \frac{M_t}{P_t} + E_t \left( Q_{t,t+1} \frac{B_t}{P_t} \right) + \tau_t \leq \left( \frac{W_t}{P_t} \right) N_t + \frac{M_{t-1}}{P_t} + \frac{B_{t-1}}{P_t} + \Pi_t, \]  
where \( \tau_t \) is lump-sum taxes/transfers from the government to the household, \( B_t \) is one-period nominal bonds, \( Q_{t,t+1} \) is the stochastic discount factor for the price at \( t \) of one unit of composite consumption goods at \( t + 1 \), and \( \Pi_t \) is profits from the firm, which the household owns. The household maximizes (5) subject to (8) to yield the first-order conditions

\[ \chi \frac{N_t^\sigma}{C_t^\sigma} = \frac{W_t}{P_t} \]  
(9)

\[ Q_{t,t+1} = \beta \left( \frac{C_t}{C_{t+1}} \right)^\sigma. \]  
(10)

If \( R_t \) denotes the risk-free gross nominal interest rate between \( t \) and \( t + 1 \), then absence of arbitrage implies the equilibrium condition

\[ E_t \left[ Q_{t,t+1} \frac{P_t}{P_{t+1}} \right] = \frac{1}{R_t}, \]  
(11)

so the first-order conditions imply that real money balances may be written as

\[ \frac{M_t}{P_t} = \delta^\kappa \left( \frac{R_t}{R_t - 1} \right)^{-1/\kappa} C_t^{\sigma/\kappa}. \]  
(12)

We assume the government demands goods in the same proportion that households do, so the government’s demand is

\[ g_{jt} = \left( \frac{p_{jt}}{P_t} \right)^{-\theta} G_t, \]  
(13)

where \( G_t = \left[ \int_0^1 \frac{G_t^\theta}{g_{jt}^\theta} \, dj \right]^{\frac{1}{\theta}} \).

2.2. Firms. A continuum of monopolistically competitive firms produce goods using labor. Production of good \( j \) is given by

\[ y_{jt} = A_t N_{jt}, \]  
(14)

where \( A_t \) is an aggregate technology shock, common across firms.

From (7) and (13), the demand curve firm \( j \) faces is given by
\[ y_{jt} = \left( \frac{p_{jt}}{P_t} \right)^{-\theta} Y_t, \]  

where \( Y_t \) is defined by

\[ C_t + G_t = Y_t. \]  

Equating supply and demand for individual goods,

\[ A_t N_{jt} = \left( \frac{p_{jt}}{P_t} \right)^{-\theta} Y_t. \]  

The real profit flow of firm \( j \) at period \( t \) is

\[ \Pi_{jt} = \left( \frac{p_{jt}}{P_t} \right)^{1-\theta} Y_t - W_t N_{jt}. \]  

Following Calvo (1983), a fraction \( 1 - \varphi \) firms are permitted to adjust their prices each period, while the fraction \( \varphi \) are not permitted to adjust. If firms are permitted to adjust at \( t \), they choose a new optimal price, \( p_t^* \), to maximize the expected discounted sum of profits given by

\[
E_t \sum_{i=0}^{\infty} \varphi^i Q_{t,t+i} \left[ \left( \frac{p_t^*}{P_{t+i}} \right)^{1-\theta} - \Psi_{t+i} \left( \frac{p_t^*}{P_{t+i}} \right)^{-\theta} \right] Y_{t+i}. \]

where \( Q_{t,t+i} = \beta^i (C_t/C_{t+i})^\sigma \) and profits have been rewritten using (17). \( \Psi_t \) is real marginal cost, defined as

\[ \Psi_t = \frac{W_t}{A_t P_t}. \]

The first-order condition that determines \( p_t^* \) is

\[
E_t \sum_{i=0}^{\infty} \varphi^i Q_{t,t+i} \left[ (1 - \theta) \left( \frac{p_t^*}{P_{t+i}} \right) + \theta \Psi_{t+i} \right] \left( \frac{1}{P_t^*} \right) \left( \frac{p_t^*}{P_{t+i}} \right)^{-\theta} Y_{t+i} = 0
\]

Using the definition of \( Q_{t,t+i} \) and rearranging, (21) is

\[
\frac{p_t^*}{P_t} = \left( \frac{\theta}{\theta - 1} \right) \frac{E_t \sum_{i=0}^{\infty} (\varphi \beta)^i (Y_{t+i} - G_{t+i})^{-\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\theta} \Psi_{t+i} Y_{t+i}}{E_t \sum_{i=0}^{\infty} (\varphi \beta)^i (Y_{t+i} - G_{t+i})^{-\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\theta-1} Y_{t+i}}.
\]

Denote (22) as
\[
\frac{p^*_t}{P_t} = \left( \frac{\theta}{\theta - 1} \right) \frac{K_{1t}}{K_{2t}},
\]

where \(K_{1t}\) denotes the numerator and \(K_{2t}\) denotes the denominator. Note that these two expressions have the following recursive representations:

\[
K_{1t} = (Y_t - G_t)^{-\sigma} \Psi_t Y_t + \varphi \beta E_t K_{1t+1} \left( \frac{P_{t+1}}{P_t} \right)^{\theta},
\]

and

\[
K_{2t} = (Y_t - G_t)^{-\sigma} Y_t + \varphi \beta E_t K_{2t+1} \left( \frac{P_{t+1}}{P_t} \right)^{-\theta-1}.
\]

Solving (23) for \(p^*_t\) and using the result in the price index

\[
P_t^{1-\theta} = (1 - \varphi)(p^*_t)^{1-\theta} + \varphi P_t^{1-\theta},
\]

yields

\[
\pi_t^{\theta-1} = \frac{1}{\varphi} - \frac{1 - \varphi}{\varphi} \left( \frac{K_{1t}}{K_{2t}} \right)^{1-\theta},
\]

where \(\mu \equiv \theta/(\theta - 1)\).

### 2.3. Aggregation.

We assume that individual labor services may be aggregated linearly to produce aggregate labor:

\[
N_t = \int_0^1 N_{jt} dj.
\]

Linear aggregation of individual market clearing conditions implies

\[
A_t N_t = \Delta_t Y_t,
\]

where \(\Delta_t\) is a measure of relative price dispersion defined by

\[
\Delta_t = \int_0^1 \left( \frac{p_{jt}}{P_t} \right)^{-\theta} dj.
\]

Now the aggregate production function is given by

\[
Y_t = \frac{A_t}{\Delta_t} N_t.
\]

It is natural to define aggregate profits as the sum of individual firm profits, \(\Pi_t = \int_0^1 \Pi_{jt} dj\), so (18) and (25) imply that the aggregate profit flow can be expressed as
\[ \Pi_t = Y_t - \frac{W_t}{P_t} N_t. \]  

(32)

Substituting (32) into the household’s budget constraint, (8), and combining the result with the government’s budget constraint, yields the aggregate resource constraint

\[ \frac{A_t}{\Delta_t} N_t = C_t + G_t. \]  

(33)

We now derive the law of motion of relative price dispersion. From the definition of price dispersion, (30) and (26), relative price dispersion evolves according to

\[ \Delta_t = (1 - \varphi) \left( \frac{p^*_t}{P_t} \right)^{-\theta} + \varphi \pi_t^\theta \Delta_{t-1}, \]  

(34)

where \( \pi_t = P_t/P_{t-1} \).

2.4. **Policy Specification.** Monetary and fiscal policies follow (1) and (2), where the error terms are taken to be standard normal and i.i.d. The processes for \( \{G_t, \tau_t, R_t, M_t, B_t\} \) must satisfy the government budget identity

\[ G_t = \tau_t + \frac{M_t - M_{t-1}}{P_t} + \frac{B_t - R_{t-1}B_{t-1}}{P_t}. \]  

(35)

given \( M_{-1} > 0 \) and \( R_{-1}B_{-1} \).

3. **Calibration and Solution Method**

Parameters describing preferences, technology and price adjustment for the benchmark nonsynchronous switching model are specified to be consistent with other work, such as Rotemberg and Woodford (1997) and Woodford (2003). We calibrate the model at a quarterly frequency. The markup of price over marginal cost is set to 15 percent, implying \( \mu = \theta(1 - \theta)^{-1} = 1.15 \) and 66 percent of firms are unable to reset their price each period, implying \( \varphi = .66 \). The quarterly real interest rate is set to 1 percent, implying \( \beta = .99 \). Each intermediate goods producing firm has access to a production function with constant returns to labor. The technology parameter, \( A \), is chosen to normalize the deterministic steady state level of output to be 1.

The coefficient on real balances in the utility function, \( \delta \), is set to ensure that velocity in the deterministic steady state, defined as \( cP/M \), matches average U.S. monetary base velocity at 2.4. This value comes from the period 1959-2004 and uses the average real expenditure on non-durable consumption plus services. The parameter governing the interest elasticity of real money balances, \( \kappa \), is set to 2.6, a

Reaction coefficients in the policy rules are taken from the estimates in Table 1 and the four-state joint transition matrix (4). The intercepts in the policy rules govern the deterministic steady state values of inflation and debt-output in the computational model. Intercepts are set so the deterministic steady state is common across all regimes and match their sample means from 1948:2-2004:1. Those values, annualized, are \( \pi = 3.43\% \) and \( b = .3525 \). Government purchases, as a share of output, are fixed in the model at its mean value of .115.

We compute the solution using the monotone map method, based on Coleman (1991). The algorithm requires a set of initial candidate decision rules that reduce the system to a set of non-linear expectational first-order difference equations. The complete model consists of the first-order necessary conditions from the representative agent and firms’ optimization problem, constraints, specification of policy, the price adjustment process, and the transversality conditions on real balances and bonds. The solution is a set of functions that map the state, \( \Theta_t = \{b_{t-1}, w_{t-1}, \Delta_{t-1}, \theta_t, \psi_t, S_t\} \), into values for the endogenous variables.

Implementation of the algorithm begins by conjecturing an initial set of rules, which we take to be the solution from the models fixed-regime counterpart. Specifically, we take the solutions from fixed-regime model with AM/PF and PM/AF policies as the initial rules for the corresponding regimes in the nonsynchronous switching model. For the AM/AF and PM/PF regimes there are no stationary, unique fixed-regime counterparts, so we use the solution from the PM/AF fixed-regime model to initialize the algorithm. To ensure the solution is not sensitive to initial conditions, we also use the solution from the AM/PF regime and weighted averages of the two. Further perturbations of the initial rules have no effect on the final solution, suggesting the solution is locally unique. The appendix more fully draws out connections between determinacy and uniqueness in linear models with convergence of the monotone map algorithm.

Taking the initial rules for labor, \( \hat{h}^N(\Theta_t) = N_t \), and the functions determining the firm’s optimal pricing decision, \( \hat{h}^{K_1}(\Theta_t) = K_{1,t} \) and \( \hat{h}^{K_2}(\Theta_t) = K_{2,t} \), we find values using a nonlinear equation solver for \( N_t, K_{1,t}, K_{2,t} \) such that

\[
C_t^{-\sigma} = \beta R_t E_t \left[ \pi_{t+1} \left( \hat{h}^C(\Theta_{t+1}) \right)^{-\sigma} \right], \quad (36)
\]

\[
K_{1,t} = C_t^{1-\sigma} \Psi_t + \varphi \beta E_t \hat{h}^{K_1}(\Theta_{t+1}), \quad (37)
\]

\[
K_{2,t} = C_t^{1-\sigma} + \varphi \beta E_t \hat{h}^{K_2}(\Theta_{t+1}), \quad (38)
\]
where \( h^C(\Theta_{t+1}) = (A/\Delta_t)\tilde{h}^N(\Theta_t) - g \). Given \( N_t, K_{1,t}, K_{2,t} \), we compute the endogenous variables. Note that \( \Delta_t, b_t \) and \( w_t = R_t b_t + \left( \frac{M_t}{T_t} \right) \) are states at \( t+1 \). Gauss-Hermite integration is used over possible values for \( \theta_{t+1}, \psi_{t+1} \) and \( S_{t+1} \), yielding values for \( E_t \left[ \pi_{t+1} G_{t+1}^{-\gamma} \right], E_t K_{1,t+1}, E_t K_{2,t+1} \) and reduces the above system to three equations in three unknowns. The (net) nominal interest rate is restricted to always be positive.

When solving the above system, the state vector and the decision rules are taken as given. The system is solved for every set of state variables defined over a discrete partition of the state space. This procedure is repeated until the iteration improves the current decision rule at any given state vector by less than some \( \epsilon \). Local uniqueness is more formally discussed in appendix B, which considers a relation between numerical converge using the monotone map and uniqueness in a linear univariate framework. While the regime-switching model is greatly more complex than a linear univariate model, the monotone map algorithm does converge for the estimated parameter settings. Using parameters that suggest indeterminacy, such as setting all regimes to PM/PF, does result in failure of the algorithm to converge. To argue for local uniqueness, we start with a wide set of initial rules, all of which converge to the equilibrium we report in the following section.

4. Equilibrium with Policy Switching

4.1. Preliminaries. In discussing notions of stationarity in regime-switching frameworks, the following definitions, adapted from Francq and Zakoian (2001) are useful. Below, \( b_0 \) represents initial real debt, \( S \) is the set of all possible policy regimes and

\[
b_t = E_t \left[ \lim_{T \to \infty} Q_{t,T} b_T \right],
\]

where \( Q_{t,T} \) is a stochastic discount factor for pricing arbitrary (possibly state-contingent) financial claims.

**Definition 1.** For \( b_0 < \infty, S_0 \in S \) and a sequence \( \{S_t\}_{t=0}^\infty \) that evolves according to \( P^M \otimes P^F \), a globally stable macro policy implies the unconditional expectation of discounted debt is zero, \( E[b_t] = 0 \).

**Definition 2.** For \( b_0 < \infty, S_0 = j \) and a sequence \( \{S_t = j\}_{t=0}^\infty \) for all \( t \) a locally stable macro policy implies the conditional expectation of discounted debt is zero, \( E[b_t | S_t = j] = 0 \).

A zero expected present value of debt is equivalent to the intertemporal equilibrium condition
where $x$ and $z$ are the expected present values of the primary surplus and seigniorage, defined in the model as:

$$x_t = E_t \sum_{s=0}^{\infty} \left( \prod_{j=0}^{s} \pi_{t+j+1} R_{t+j}^{-1} \right) (\tau_{t+s+1} - g)$$

and

$$z_t = E_t \sum_{s=0}^{\infty} \left( \prod_{j=0}^{s} \pi_{t+j+1} R_{t+j}^{-1} \right) \left( m_{t+s+1} - m_{t+s} \pi_{t+s}^{-1} \right).$$

Cochrane (1999, 2001) refers to (39) as a “debt valuation” equation because $b_t = B_t/P_t$ and fluctuations in $x$ or $z$ can induce jumps in $P_t$, which alter the real value of debt to keep it consistent with expected policies. In practice, we check whether the expected present value of debt is zero following an exogenous shock by computing $x$ and $z$ and comparing their sum to $b$. All the results satisfy this restriction.

4.2. Fixed Policy Regimes. As a first step in the analysis of the nature of equilibrium under the estimated policy process, we consider a first-order approximation of the nonlinear model around its deterministic steady state when policy regime is permanent. We use Juillard’s (2003) Dynare program to check for existence and uniqueness in each of the estimated policy regimes. Locally unique equilibria exist under both active monetary/passive fiscal and passive monetary/active fiscal policy regimes. No equilibrium exists when both policies are active and multiple equilibria exist when both policies are passive. This suggests that at least locally, when regimes are fixed, our labelling of estimated policies as “active” and “passive” is consistent with theory.

Active monetary and passive fiscal policies produce the equilibrium studied in the vast recent literature on monetary policy rules [Taylor (1999b) and Woodford (2003)]. An $i.i.d$. monetary expansion generates a one-period decrease in the nominal interest rate and $i.i.d$. contemporaneous increases in inflation and output. Ricardian equivalence holds, so a debt-financed tax cut merely substitutes current for future taxation.

Under passive monetary and active fiscal policies, $i.i.d$. policy disturbances generate (at initial prices) wealth effects whose impacts are propagated by the weak, but positive, response of monetary policy to inflation. Changes in wealth arise because tax policy does not adjust future taxes in order to neutralize wealth effects. A monetary expansion reduces the nominal interest rate and debt (via an open-market purchase), which reduces wealth and aggregate demand. Inflation falls and, after an initial
increase, output falls. This outcome is perverse relative to conventional theories, which are predicated on active monetary and passive fiscal policies. Lower current taxes increase wealth and demand, which raises inflation and output. Tax effects are consistent with traditional Keynesian theories. In both cases, the transitory policy shock has persistent impacts because monetary policy passively adjusts the interest rate with inflation.\footnote{Woodford (1998) discusses a variant of this regime.}

4.3. Characterization of Equilibrium. One feature of the regime-switching model is that monetary and fiscal policy are free to switch independently of one another. An immediate implication is that the model temporarily permits policy combinations with passive monetary and passive fiscal policies, as well as active monetary and active fiscal policies. A passive-passive policy combination gives rise to indeterminate equilibrium in fixed-regime versions of the model, admitting the possibility for sunspot shocks that affect equilibrium allocations. An active-active policy combination results in a nonstationary path for real debt, implying no stationary equilibrium exists. Solving the model with computation methods, the model converges to a locally unique equilibrium with dynamics that imply a globally stationary path for real debt.

To study the equilibrium properties of the model, we choose to first look at monetary and fiscal shocks conditional on regime, then look at monetary and fiscal shocks arising from Monte Carlo experiments.

4.4. Nonlinear Impulse Response Analysis. Although the decision rules reflect agents’ knowledge of the true switching policy process, to separate the impacts of regime from the impacts of \textit{i.i.d.} policy shocks, we consider a stylized experiment. Assume the economy is in a regime-dependent stationary equilibrium. Perturb the error term in a policy rule and solve for equilibrium time paths, conditioning on remaining in the prevailing regime. We compute the paths of variables relative to their regime-dependent stationary values. On impact, the response of endogenous variable $k$ for this conditional impulse response is given by

$$
\phi_k(\theta, \psi|\Theta_J) = h^k(b_J, w_J, \Delta_J, \theta, \psi, J) - h^k(b_J, w_J, \Delta_J, 0, 0, J)
$$

where $b_J$, $w_J$ and $\Delta_J$ are regime-dependent stationary values for regime $J$ and $\Theta_J$ is the regime-dependent stationary state for regime $J$. As the notation suggests, the effects of policy shocks in the regime-switching model depend on initial conditions. This experiment is highly stylized and useful for understanding the dynamics of the model, but not for establishing the average response to a shock. Following initial
impact, the value of variable $k$ after $n$ periods for this conditional impulse response is given by

$$
\phi_{k,n}(\theta, \psi|\Theta_J) = h^k(b_{n-1}, w_{n-1}, \Delta_{n-1}, 0, 0, J) - h^k(b_J, w_J, \Delta_J, 0, 0, J)
$$

for $n > 1$, with $b_J \equiv b_0$. (43) is still a function of the initial shocks, $\theta$ and $\psi$, because the impulse responses are state (or history) dependent.

Also of interest is the average response of a variable to a shock, where the mean is computed over future regimes. For such an experiment, the impact period is computed the same as above, but then the generalized impulse response of variable $k$ after $n$ periods is given by

$$
\phi_{k,n}(\theta, \psi|\Theta_J) = E_J[h^k(b_{n-1}, w_{n-1}, \Delta_{n-1}, 0, 0, J)] - h^k(b_J, w_J, \Delta_J, 0, 0, J)
$$

where $E_J$ denotes expectations taken conditional on $S_t = J$ and $\theta_n = \psi_n = 0$ for $n > 1$. The difference between the two types of impulse response is that (42) and (43) holds the regime constant throughout, whereas and (44) averages over future realizations of regimes, yielding the average response to a policy shock. The averaging is done using Monte Carlo simulations where draws for the regime are taken from the estimated Markov chain. The responses are then averaged across each simulation.

4.4.1. A Fiscal Expansion. Figure 8 reports paths following a two-standard deviation cut in taxes in period 5, conditional, in turn, on the stationary means for the AM/PF, PM/AF, and PM/PF regimes. The figure reports the conditional impulse responses given by (42) and (43). Regardless of the prevailing regime, an i.i.d. cut in lump-sum taxes financed by new sales of nominal government debt generates wealth effects that increase aggregate demand, inflation, and output. In each regime—even AM/PF, where taxes rise by more than the real interest in response to debt—the higher real value of debt is associated with increases in the expected present values of both net-of-interest surpluses and seigniorage.

Wealth effects do not stem from a changes in the resources available to the economy, such as arise productivity or government purchases. Instead, tax cuts generate the wealth effects by increasing the path of consumption the representative household believes it can afford at initial prices and interest rates. Under an initially passive fiscal policy, agents increase demand because they incorporate the probability of switching to an active fiscal regime where taxes will not rise sufficiently to repay the increase in real debt. In the event of a switch to an active fiscal policy, the higher level of real debt arising from the fiscal expansion leads to a still lower level of lump-sum taxes, given the negative coefficient on lagged debt. When fiscal policy is initially active, agents incorporate the probability of staying in the active regime. In either case, debt is rationally viewed by agents as adding to net wealth because there is positive
probability that future lump-sum taxes will not sufficiently rise to completely repay outstanding debt.

There is positive probability that an increase in the real value of debt arising from a fiscal expansion may not be retired in the future with higher lump-sum taxes. However, regardless of what regime materializes in the future, the government policy process must finance purchases and outstanding debt plus interest without persistently running primary deficits [McCallum (1984)]. Consequently, the government must back a fiscal expansion with a combination of discounted primary surpluses and seigniorage. Figure 8 highlights the increase in seigniorage following the fiscal expansion. Seigniorage in fixed regime models does not respond to tax induced fiscal expansions because debt is completely backed by future lump-sum taxes. In a fixed-regime PM/AF model, Woodford (1998) points out that unexpected inflation and a lower real rate following a fiscal expansion are additional factors that allow the government to finance outstanding debt with lower primary surpluses. The rise in the present value of seigniorage implies that future money creation will, at some point in the future, be used to retire outstanding government debt. In equilibrium, agents then rationally view debt as an increase in net wealth, have incentives to economize on money holdings due to the positive probability of an increase in future money creation, and therefore, increase aggregate demand.

In all regimes, the i.i.d. tax cut is propagated for many periods when monetary policy raises the nominal interest rate in response to the expansion in aggregate demand. When monetary policy is passive, with a relatively small response to inflation ($\alpha_\pi = .522$ in the estimates), the impact on inflation lasts about 5 years. A stronger response of monetary policy to inflation, as occurs when monetary policy is active ($\alpha_\pi = 1.31$), spreads the impacts of a tax cut over many more years.

The reason active monetary policy induces more serial correlation in inflation and output arises from the speed of seigniorage extraction. As $\alpha_\pi$ declines, the shorter the time horizon over which seigniorage is extracted, as Davig, Leeper, and Chung (2004) show. Combining slow debt repayment with an active monetary authority, which extracts seigniorage over a long time horizon, generates a very persistent increase in debt. The wealth effects arising from higher debt then generate persistence in output

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16Franq and Zoakian (2001) show in a general framework that Markov-switching processes can be locally explosive, yet globally stable. Davig (2004) shows that a properly restricted Markov-switching process for discounted debt can be locally explosive, yet globally stable and therefore, satisfy the transversality condition for debt. Given that the ergodic mean of debt from a long simulated series ($N = 500,000$) from the regime-switching New Keynesian model has a finite mean that is close to the deterministic steady state, we conclude the debt is globally stable and the transversality condition is satisfied.
and inflation. Passive monetary policy extracts seigniorage more quickly, repays debt more quickly and reduces the persistence in output and inflation.

It is a common view that because the fiscal theory of the price level emphasizes the fiscal financing consequences of alternative policies, just as Sargent and Wallace (1981) do, the theory relies heavily on debt monetization and, therefore, is irrelevant for understanding the inflation process in most developed countries [King (1995) and Castro, Resende, and Ruge-Murcia (2003)]. The model’s results contradict this perception. Across all regimes, the deterministic steady state levels of inflation and seigniorage are identical: inflation matches the average for the United States and seigniorage is only 0.8 percent of output, which is in line with King’s (1995) estimates for the United States.

4.4.2. A Monetary Expansion. Responses to an i.i.d. monetary expansion appear in figure 9. For the stationary regimes, the short-run impacts are completely conventional in this class of models: output and inflation expand. Under the constant AM/PF policy, there is no propagation of inflation and output and the effects disappear after the initial impact. In a fixed-regime setup, active monetary and passive fiscal policies neutralize the wealth effects of an open-market purchase by adjusting lump-sum taxes enough to fully finance the increase in the real value of debt. This does not occur in the switching setup using the estimated policy rules in any of the stationary regimes. So, the injection of liquidity is coupled with a reduction in wealth because the expected present value of future taxes does not decline by the full amount of the decline in debt. However, the monetary expansion increases the nominal supply for the impact period and with the presence of nominal rigidities, increases aggregate demand. The speed at which inflation and output rise back to their initial levels is determined by how rapidly wealth returns to its initial level.

As the figure shows, lower wealth reduces demand, inflation, and output over longer horizons. The figure makes it appear that those wealth effects are more moderate when monetary policy is active than when it is passive but as table 4 shows, in the long run monetary policy shocks are more potent when monetary policy is active.

4.4.3. Generalized Impulse Responses. The previous section holds the regime constant before, during and after the policy shock. The resulting dynamics differ from fixed-regime counterparts because agents’ decision rules incorporate the probability of switching regimes. This section allows future regimes to vary stochastically after the policy shock and is more consistent with the notion that regime changes do occur and have important implications for the dynamics of the economy.
Generalized impulse response function condition on the regime dependent stationary values for debt, price dispersion, and real wealth, then shock either fiscal policy or monetary policy and draw regime from the estimated four-state Markov chain. The results report the average at each date across all draws. For example, we report dynamic responses for inflation, \( \{\pi_j\}_{j=0}^T \), by computing the following sample analog of (44) at each \( t \)

\[
\pi_t(\Theta_0) = \frac{1}{N} \sum_{i=1}^{N} \pi_{i,t}(\Theta_{i,t}),
\]

where \( \Theta_0 \) denotes a particular initial state. We set \( N = 10,000 \) and turn off shocks to \( \theta_t \) and \( \psi_t \) for all \( t \) following the shock.

For the three stationary regimes, Figure 10 shows the mean and one standard deviation bands of the generalized impulse responses following a fiscal expansion. The first 4 periods condition on the stationary mean and a particular regime, period 5 imposes the shock and continues to condition on regime, period 6 onward then makes random draws over regimes. While the properties of the responses depend on the initial conditions, of interest is that a one-period fiscal expansion generates a hump-shaped response of inflation and output, even under an initial AM/PF policy. The responses resemble those arising from identified VARs focusing on the effects of fiscal policy [Blanchard and Perotti (2002)]. For regimes with passive fiscal policy, the inflation and output responses are stronger for the periods immediately following the shock, but die out more quickly.

The dynamic properties of the generalized impulse responses rest with the evolution of regimes. Given that regimes are relatively persistent—the transition probability of staying in the same regime is greater than .9 for all regimes—the dynamics for the few periods following the shock resemble those from the impulse responses that condition on a constant regime. For example, a fiscal expansion in the AM/PF regime leads to a small, but very persistent output and inflation response. The PM/PF regime generates a larger output and inflation response the period of the shock, but the responses are much less persistent than in the AM/PF regime. Consider the generalized impulse response in the PM/PF regime, there is a strong output and inflation response that is more persistent than when conditioning on this regime. The additional persistence arises from realizations of active fiscal regimes in the periods following the shock, where the higher level of debt is very slowly repaid. Averaging over many realizations of the different regime paths results in a more persistent response than when conditioning on the regime for all periods.
The effects of innovations to the interest rate are short-lived, as they are in this class of models with \textit{i.i.d.} innovations. Adding regime switching does not induce a propagation mechanism as it does with tax innovations.

4.4.4. \textit{AM/AF Regime}. This section assesses the implications of a shift to the regime where both monetary and fiscal policy are active and consequently, debt is nonstationary. The active fiscal regime has taxes respond negatively to debt. An important aspect of the analysis for the AM/AF regime regards the initial conditions, since they matter for the dynamics. A shift to the AM/AF regime results in a large tax cut if the initial level of debt is roughly below the ergodic mean, whereas the same shift causes a tax increase if initial debt is above the ergodic mean.

Responses under various initial conditions are given in figure 12. One interesting feature of the dynamic response under the high level of initial debt, set to .9, is the hump-shape response of inflation and output. The switch to AM/AF under a high level of debt reduces taxes and causes an increase in aggregate demand, which is partially offset by an active monetary authority. These factors work to keep prices relatively stable in the period of the switch. Given the persistence of the AM/AF regime, inflation for several periods is quite low. Eventually, there is a switch to passive monetary policy, allowing the aggregate demand effects to have a stronger impact on inflation. The eventual realization of the passive monetary policy, again averaged over many simulations, causes the hump-shaped response.

To better understand the dynamics of how regimes evolve, figure 13 plots the proportion of runs where each regime was in place at each point in time. For example, the upper-right graph shows that at \( t = 1 \), the AM/AF regime was in place for all 10,000 runs of the simulation, since this is initial condition for every simulation. After 10 quarters, approximately 45\% of the simulations still had the AM/AF regime in place, 20\% in the AM/PF regime, 11\% in the PM/PF regime and 24\% in PM/AM. So, following a tax cut, it is the eventual movement, on average, to a regime with a passive monetary policy that generates the hump-shaped response. As an implication, we should see the hump get pushed further out as the persistence of the AM/AF regime increases.

5. Ergodic Properties

The ergodic properties of the model reveal an interesting tradeoff between inflation or output variability and debt-output variability. Active monetary policy generates lower inflation and output variability relative to a passive monetary policy. Although the effects of shocks under active monetary policy are very persistent, the impact responses of output and inflation to shocks are much lower relative to those under a
passive monetary policy. Under passive monetary policy, shocks are not as persistent, but have large impacts on inflation and output. The resulting variability of these variables are then higher than under active monetary policy.

A passive monetary policy yields lower debt-output variability, but at the expense of higher inflation and output variability. Given the debt-output ratio is only important for welfare insofar as it affects consumption, leisure and real balances, it seems these findings are consistent with work that finds active monetary policy as welfare maximizing.

6. Concluding Remarks

Estimation of a monetary policy rule with regime-dependent parameters, including the shock variance, endogenously splits the sample of postwar U.S. data between active and passive policy regimes. The timing of regimes accord with other studies, such as Clarida, Gali, and Gertler (2000), that exogenously break the sample around 1979, where pre-1979 monetary policy is passive and then active (although, we do find episodes post-1979 when monetary policy was passive). Estimation of a fiscal policy rule with regime-dependent parameters, also including the shock variance, delineates between active and passive policy regimes. The regimes alternate frequently throughout U.S. history and can be linked to policy reforms, instead of just reflecting changes in policy arising from economic conditions.

Having estimates of the switching policy rules in hand, we progress to embed them in an otherwise standard model. The resulting model admits the possibility of switching to policy combinations that imply indeterminacy (PM/PF) and explosiveness (AM/AF) under fixed-regimes. Solving the model with computation methods, the model converges to a locally unique equilibrium with dynamics that imply a globally stationary path for real debt. The resulting dynamics of the regime-switching model differ from their fixed-regime counterparts, primarily in the response of output and inflation to fiscal expansions. Tax cuts under AM/PF policy generate an increase in inflation and output, responses not observed in fixed-regime models due to Ricardian equivalence. Moreover, the response of output and inflation to i.i.d. tax shocks under AM/PF policy are extremely persistent. The PM/PF regime exhibits similar dynamics to the AM/PF following a tax cut, though output and inflation exhibit much less inflation. Debt is nonstationary in the AM/AF regime. Monetary policy shocks look similar to their fixed regime counterparts.

One interesting trade-off that emerges is between inflation or output variability and debt-output variability. Under an active monetary policy, inflation and output variability is less than under a passive monetary policy, but at the cost of higher
debt-output variability. Passive monetary policy yields low debt variability, but at the expense of high inflation and output variability. Given the debt-output ratio is only important for welfare insofar as it affects consumption, leisure and real balances, it seems these findings are consistent with work that finds active monetary policy as welfare maximizing. We leave formal welfare analysis in a regime-switching environment for future work.

A final point connects the regime-switching model with existing empirical work on fiscal policy. Simulating data from a fixed-regime NK model and then fitting a VAR to the resulting data, and using identification methods similar to Blanchard and Perotti (2002), tax shock do not affect output and inflation. However, the identified structural VAR in Blanchard and Perotti reports strong output effects following a tax induced fiscal expansion. Simulating data from the regime-switching model and then fitting a VAR, using the same identification as with the fixed regime, yields positive output response to tax innovations. Thus, giving taxes a role in DSGE models that empirical work suggests they should have.
Appendix A. An Analytical Example\textsuperscript{17}

This section lays out a simplified theoretical model with monetary and fiscal policy switching that yields an analytical solution. The model establishes that when monetary and fiscal policies experience on-going regime change, several principles that guide the development of policy models today are called into question. For tractability, it is necessary to deviate somewhat from the specifications of the estimated rules, (1) and (2). We show that:

- equilibrium is determinate (in the class of bounded equilibria) even if there are periods in which both monetary and fiscal policy are passive;\textsuperscript{18}
- a stationary equilibrium can exist even if there are periods when both monetary and fiscal policy are active;
- tax shocks always matter for aggregate demand and inflation, even if the Taylor principle is satisfied in some periods;
- the response of inflation to an \textit{i.i.d.} monetary policy disturbance is serially correlated, even if the Taylor principle is satisfied;
- the impact and persistence of policy shocks is greater the stronger is the long-run response of monetary policy to inflation.

Consider an endowment version of Sidrauski (1967). A representative household has logarithmic, time-separable preferences over consumption and real money balances. The household can hold money and one-period nominal government debt that pays a gross nominal interest rate of $R_t$ on debt that matures in period $t+1$. With constant government purchases, $g$, in equilibrium, private consumption, $c$, will also be constant. The agent pays real lump-sum taxes in the amount $T_t$ each period. The model implies a Fisher equation

\begin{equation}
1/R_t = \beta E_t[1/\pi_{t+1}],
\end{equation}

where $0 < \beta < 1$ is the discount factor, $\pi_{t+1}$ is the gross rate of inflation between $t$ and $t+1$, and the expectation is taken with respect to a set $\Omega_t$ that contains information dated $t$ and earlier, including the history of regimes up to $t$. The money demand function is

\begin{itemize}
  \item equilibrium is determinate (in the class of bounded equilibria) even if there are periods in which both monetary and fiscal policy are passive;\textsuperscript{18}
  \item a stationary equilibrium can exist even if there are periods when both monetary and fiscal policy are active;
  \item tax shocks always matter for aggregate demand and inflation, even if the Taylor principle is satisfied in some periods;
  \item the response of inflation to an \textit{i.i.d.} monetary policy disturbance is serially correlated, even if the Taylor principle is satisfied;
  \item the impact and persistence of policy shocks is greater the stronger is the long-run response of monetary policy to inflation.
\end{itemize}

\textsuperscript{17}This section was written by Hess Chung.

\textsuperscript{18}The active/passive regime categorization has a useful dynamic interpretation. If monetary policy behavior provides a boundary condition for the inflation process, as it does when it obeys the Taylor principle, then monetary policy is \textit{active}; otherwise it is \textit{passive}. Analogously, if fiscal policy behavior provides a boundary condition for the real debt process, as it does under the fiscal theory of price level determination, then fiscal policy is \textit{active}; otherwise it is \textit{passive}.
where \( m_t = M_t / P_t \) is real money balances.

Monetary policy adjusts the nominal interest rate in response to inflation

\[
R_t = \alpha_0 + \alpha (S^M_t) \pi_t + \theta_t, \quad \theta_t \sim i.i.d.
\]

and fiscal policy adjusts taxes in response to the real value of total government liabilities

\[
T_t = \gamma_0 + \gamma (S^F_t) (b_{t-1} + m_{t-1}) + \psi_t, \quad \psi_t \sim i.i.d.
\]

\( \theta_t \) and \( \psi_t \) are exogenous shocks. \( S^M_t \) and \( S^F_t \) are random monetary and fiscal regimes that follow independent Markov processes. Assume there are \( N^M \) monetary states and \( N^F \) fiscal states. Denote the processes’ transition probabilities by \( p^Z_{ij} = P[S^Z_{t} = j | S^Z_{t-1} = i] \) for \( Z = M, F \). Further assume that \( E_t \gamma (S^F_{t+1}) = \gamma \), with \( |\beta^{-1} - \gamma| > 1 \), so that on average (in the long run) fiscal policy is active. In equilibrium, policy choices must obey the government budget identity.

Let \( l_t = b_t + m_t \) and define the forecast error

\[
\eta_{t+1} = \frac{1}{\pi_{t+1}} \frac{\pi_{t+1}}{E_t[1/\pi_{t+1}]}.
\]

An equilibrium is a set of stochastic processes for \( \{l_t, \pi_t\} \) satisfying

\[
\pi_{t+1} = \beta \left[ \frac{\alpha_0 + \alpha (S^M_t) \pi_t + \theta_t}{\eta_{t+1}} \right],
\]

\[
l_t = \left[ \frac{\eta_t}{\beta} - \gamma (S^F_t) \right] l_{t-1} - \frac{\eta_t}{\beta} c + D - \psi_t,
\]

where \( D = g - \gamma_0 \), equilibrium money balances, (46), and transversality conditions for \( m \) and \( b \).

Canzoneri, Cumby, and Diba (2001) show that an infinite number of inflation processes are consistent with transversality in this type of economy. However, there is a unique equilibrium with bounded government liabilities. Iterate on (51) and use the fact that \( E_t \eta_{t+1} = 1 \) to obtain equilibrium liabilities

\[
l_t = c \left( \frac{\beta^{-1} - D/c}{\beta^{-1} - \gamma - 1} \right).
\]

Solving for the forecast error yields
$$\eta_t = 1 + \frac{(1 - \beta D/c)[\gamma(S^F_t) - \gamma]}{1 + \gamma - D/c} + \frac{\beta}{c} \left(\frac{\beta^{-1} - \gamma - 1}{1 + \gamma - D/c}\right) \psi_t,$$

implying that tax disturbances, both to the feedback parameter and to the \textit{i.i.d.} shock, always affect aggregate demand and inflation.

We can derive restrictions on policy behavior that deliver a stable equilibrium inflation process. After iterating on (50), inflation evolves according to

$$E_t \pi_{t+k} = E_t \left[\prod_{j=0}^{k-1} \frac{\beta \alpha(S^M_{t+j})}{\eta(S^F_{t+j+1})} \pi_t\right] + \text{other terms},$$

(54)

where $1/\bar{\eta}(S^F_{t+k}) = E_t[1/\eta(S^F_{t+k}, \psi_{t+k}) | S^F_{t+k}]$ because, by (53), $\eta$ is a function solely of the fiscal state and the \textit{i.i.d.} fiscal shock. Stability of the inflation process depends on the behavior of random products that can be characterized by simple recursive formulas. Given the independence of $S^M$ and $S^F$, write

$$E_t \left[\prod_{j=0}^{k-1} \frac{\beta \alpha(S^M_{t+j})}{\bar{\eta}(S^F_{t+j+1})}\right] = \beta^{k-1} \left(E_t \prod_{j=0}^{k-1} \alpha(S^M_{t+j})\right) \left(E_t \prod_{j=0}^{k-1} \frac{1}{\bar{\eta}(S^F_{t+j+1})}\right).$$

(55)

Each of the products on the right side of (55) can be written recursively as

$$E_t \left[\prod_{j=1}^{k} \alpha(S^M_{t+j}) \mid S^M_t = m\right] = \sum_{s=1}^{N^M} p^M_{ms} \alpha(s) E_{t+1} \left[\prod_{j=1}^{k-1} \alpha(S^M_{t+j+1}) \mid S^M_{t+1} = s\right],$$

(56)

where $p^M_{ms}$ is the probability the monetary policy regime will move from state $s$ to state $m$. Hence, growth rates are determined by the eigenvalues of the matrices $\Gamma^M_{ij} = p^M_{ij} \alpha(j)$ and $\Gamma^F_{ij} = p^F_{ij} \frac{1}{\eta(j)}$. Let the eigenvalues of $\Gamma^M$ be $\lambda^M_i$ and the eigenvalues of $\Gamma^F$ be $\lambda^F_i$. Then the inflation process is stable if $|\lambda^M_i \lambda^F_j| < 1$ for all $i, j$.

An example is helpful. Suppose that monetary policy switches between regime 1, with $\alpha(1) > 1$ and regime 2, with $\alpha(2) = 0$. Then the eigenvalues are 0 and $p^M_{11} \alpha(1)$. If, given the active fiscal policy process, $p^M_{11}$ is sufficiently small, then inflation may be stable even though $\alpha(1)$ exceeds 1.

Taken together, (50) and (54) make it clear that \textit{i.i.d.} monetary and fiscal shocks have persistent effects on inflation. The size and persistence of the effects rise with $\alpha(S^M_t)$, the responsiveness of monetary policy to inflation. We have also shown that a unique equilibrium can exist even if there are periods when both monetary and fiscal policy are passive or both are active. A finding like Clarida, Galí, and Gertler’s (2000) that monetary policy was passive before 1979 is not sufficient to infer the equilibrium
was indeterminate. Similarly, a finding that in some period both policies were active does not imply the government was insolvent [Kremers (1988, 1989)].

APPENDIX B. UNIQUENESS OF EQUILIBRIUM

B.1. Relation Between Convergence and Uniqueness. In this environment, where regimes can switch nonsynchronously, we are able to consider two policy mixes that are problematic to interpret in fixed-regime environments. When policy regimes are permanent, the combination of passive monetary/passive fiscal policy does not determine the equilibrium and allows for self-fulfilling sunspot equilibria. If both policies are active, and believed to be permanent, the transversality condition for debt is violated and no equilibrium exists. But when regimes are subject to on-going change, according to a known probability law, the issues of existence and uniqueness depend on the entire policy process (as well as the rest of model), as the analytical example in section ?? illustrates.

Analytical expressions are not available once the estimated policy process is embedded in the model. Instead, to provide some insight into why convergence of the monotone map algorithm may imply uniqueness in our nonlinear multivariate case, we show the result for a linear univariate example.

An analytical solution is available for the linear univariate case, making transparent the reason that Coleman’s (1991) algorithm succeeds or fails to find a solution. Consider the model

\[ y_t = a E_t y_{t+1} + \theta_t. \] (57)

This equation has two endogenous variables, \( y_t \) and \( E_t y_{t+1} \). Interest typically is in stationary solutions, requiring that the boundary condition \( \lim_{t \to \infty} E_t [y_{t+1}] = 0 \) be satisfied. When \( |a| < 1 \), a unique stationary rational expectations equilibrium exists, with solution \( y_t = \theta_t \). When \( |a| > 1 \), we can define the expectational error \( \eta_t = y_t - E_{t-1} [y_t] \), where \( \eta_t \) is a martingale difference sequence with \( E_t [\eta_{t+1}] = 0 \), and rewrite (57) as

\[ y_t = a^{-1} y_{t-1} - a^{-1} \theta_{t-1} + \eta_t. \] (58)

We can introduce sunspot shocks that may or may not be correlated with \( \theta_t \), resulting in errors in beliefs that are unrelated to fundamentals. Sunspot shocks are consistent with a stationary equilibria and rational expectations. In this model, there are no restrictions except that the sunspot shock have finite variance. Given \( y_0, \theta_0 \), and a sequence of sunspots shocks \( \{s_t\}_{t=0}^{\infty} \), the resulting equilibrium is consistent with (57) and (58). The sequence for the sunspot shocks is not unique, so any specified sequence defines an equilibrium.
Coleman’s algorithm requires specification of the state space. One clear implication of (57) is that the state space is different whether there is determinacy or indeterminacy. Under determinacy, a set of values for $\theta_t$ is an adequate representation of the state space. Specifying $y_{t-1}$ and $\theta_{t-1}$ as additional states, as may be suggested by (58) is redundant, since the equilibrium is $y_{t-1} = \theta_{t-1}$. However, with indeterminacy, $y_{t-1}$ and $\theta_{t-1}$ are required, since the equilibrium is no longer required to be $y_{t-1} = \theta_{t-1}$. If $y_{t-1} \neq \theta_{t-1}$, we know some extrinsic uncertainty affected the system at period $t$. An alternative method of defining the state is to include a lagged sunspot shock as a state variable. Under determinacy, extrinsic uncertainty has no effect on equilibrium allocations.

The monotone map requires an initial conjecture for the decision rules, which are then substituted into the nonlinear system. Using numerical integration methods, a nonlinear model can be reduced to a system of nonlinear equations for a given state that is solved using standard numerical nonlinear equation solving routines. Constructing a complete solution requires repeated iteration over the entire state space until the solution is updated by less than some $\epsilon$.

For (57), define the initial guess as

$$y_t = \hat{h}^0 (\theta_t)$$

and define the state space as $\Theta = \{\theta_0, \ldots, \theta_N\}$. For a given point in the state space, say $\theta_0$, $\hat{h}^0 (\theta_t)$ is updated by evaluating the expectation using numerical quadrature then solving the following equation for $y_t$

$$y_t = a \sum_{s \in \Omega} \phi(\theta_s) \hat{h}^0 (\theta_s) + \theta_0,$$

where $\Omega$ is the set of abscissa used in the integration, $\theta_t \sim N(0, \sigma^2)$ and $\phi(\cdot)$ denotes the normal density function. Since the domain of $\phi(\cdot)$ is the real line, the domain is truncated where, say, min($\Omega$) = $-4\sigma$ and max($\Omega$) = $4\sigma$. Alternatively, a transformation can be made that allows integration over the entire real line; Gauss-Hermite integration is one example. Note that the set of abscissa used in the integration may differ from the state space. The value for $y_t$ that solves (60) represents an updated mapping from the current state to the decision rule, $y_t = \hat{h}_j^1 (\theta_t)$. Iteration continues until $\max \left( \left| \hat{h}_j^j (\theta_i) - \hat{h}_j^{j-1} (\theta_i) \right| \right) < \epsilon$ for $i = 1, 2 \ldots N$.

The algorithm diverges unless the initial guess, $\hat{h}^0 (\theta_t) = 0$ for all $\theta_t \in \Theta$. With $a > 1$, solving (60) for $y_t$ implies $y_t > \hat{h}^0 (\theta_t)$. Consequently, $y_t > \hat{h}_j^1 (\theta_t)$ for all $j$, meaning that given any state $\theta_t \in \Theta$, $y_t$ increases after every iteration and results in a failure of the algorithm to converge. With $a < 1$, $y_t$ becomes smaller after each
iteration until it converges to $y_t = 0$, the deterministic steady state. Thus, in this simple context, convergence of Coleman’s algorithm implies a unique solution.
FLUCTUATING MACRO POLICIES

References


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Table 1. Monetary and Tax Policy Estimates. Log likelihood values: Monetary = −1014.737; Fiscal = −765.279.

\[
P^M = \begin{bmatrix}
.9349 & .0651 & .0000 & .0000 \\
.0000 & .9324 & .0444 & .0232 \\
.0093 & .0000 & .9552 & .0355 \\
.0000 & .0332 & .0529 & .9139 \\
\end{bmatrix},
P^F = \begin{bmatrix}
.9372 & .0628 \\
.0520 & .9480 \\
\end{bmatrix}
\]

Table 2. Monetary and Fiscal Policy Transition Matrices

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<th>( R )</th>
<th>( x )</th>
<th>( b )</th>
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<td>5.64</td>
<td>3.65</td>
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Table 3. Cumulative Effects of Tax Shock

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Table 4. Cumulative Effects of Monetary Shock
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Table 5. Summary Statistics for Simulated Data. $\sigma$ is the standard deviation and $\rho$ is the first-order autocorrelation. $\pi$ and $R$ are in annual percentage points; $b$ is the debt-output ratio at an annual rate; $x$ is in percent.
Figure 1. Actual and predicted paths of the nominal interest rate from estimates of the monetary policy rule, equation (1) using smoothed and filtered probabilities.
Figure 2. Actual and predicted paths of the tax-output ratio from estimates of the tax policy rule, equation (2), using smoothed and filtered probabilities.
Figure 3. Smoothed (solid line) and filtered (dashed line) probabilities.
**Figure 4.** Nominal interest rate (solid line) and inflation rate (dotted-dashed line). Solid vertical lines mark monetary regimes. Dotted vertical lines mark NBER business cycle peaks; dashed vertical lines mark troughs.
Figure 5. Smoothed (solid line) and filtered (dashed line) probabilities.
Figure 6. Net taxes (solid line) and lagged debt (dotted-dashed line). Solid vertical lines mark fiscal regimes. Dotted vertical lines mark NBER business cycle peaks; dashed vertical lines mark troughs. (Taxes have been rescaled to have the same mean as debt.)
Figure 7. Smoothed (solid line) and filtered (dashed line) probabilities.
Figure 8. Responses to an i.i.d. tax cut, conditional on remaining in the prevailing regime.
Figure 9. Responses to an i.i.d. monetary contraction, conditional on remaining in the prevailing regime.
Figure 10. Responses to an i.i.d. tax cut, given the regime at the date of the shock and drawing from regime over the forecast horizon.
Figure 11. Responses to an \textit{i.i.d.} monetary contraction, given the regime at the date of the shock and drawing from regime over the forecast horizon.
Figure 12. Paths following a shift to AM/AF regime, conditional on alternative initial levels of government debt.
Figure 13. Proportion of draws that a given regime is in place, based on 10,000 draws.
Figure 14. Distributions based on 500,000 draws from regime and policy shocks, sorted by regime.