Estimating the Intensity of Choice in a Dynamic Mutual Fund Allocation Decision

David Goldbaum*
Rutgers University - Newark
Bruce Mizrach
Rutgers University
January 2005

Abstract:

We estimate the intensity of choice parameter in heterogenous agent models in both a static and dynamic setting. Mean-variance optimizing agents choose among mutual funds of similar styles but varying performance. Actively managed funds have a lower Sharpe ratio than passive index funds, yet they attract a majority share of asset allocation. By estimating the relative growth of passive funds, we obtain a dynamic estimate of the intensity of choice calibrated to 10 years of mutual fund flows.

JEL: G11.

Keywords: heterogenous agents; intensity of choice, mutual funds.

* Goldbaum is with the Department of Economics, Rutgers University, Newark, NJ e-mail: goldbaum@andromeda.rutgers.edu and Mizrach is with the Department of Economics, Rutgers University, New Brunswick, NJ 08901, email: mizrach@econ.rutgers.edu. Any future revisions to this manuscript may be found at http://snde.rutgers.edu

I. Issues

Question being asked in the static model: Why do people still put new funds into the inferior product? We do not model the switching of funds from one type to the other and by using net inflow rather than gross inflow for measuring λ, we implicitly set transfers to zero. Thus, we should not discuss switching by investors until we introduce the dynamic model. The dynamic model explicitly models the flow from one fund to the other, so here it is appropriate to bring up switching.

The IOC parameter is interesting because within the ARED it determines whether or not the population process, and thus other dynamic processes in the model such as price, are stable or unstable. To what extent have we enabled a researcher to determine whether his or her model is stable or unstable based on our findings?

II. Introduction

There is growing recognition among economists that the individual interactions between agents matter when trying to explain a variety of aggregate economic phenomenon. The changing interactions of dynamic heterogeneous populations provided explanations for observed economic behavior that is often difficult to explain using more tradition models. This has been particularly true in financial markets where such phenomena as market bubbles, excess volatility of returns, volatility clustering of returns, and the success of technical trading rules have all been investigated using heterogeneous agents (include list of papers).

The Adaptive Rational Equilibrium Dynamic (ARED) introduced by Brock and Hommes (1997) is a frequently employed tool for modeling the evolutionary process of a dynamic population. The model applies to a population in which each agent chooses between a finite number of discrete options. The heterogeneity of the population manifests as the population becomes distributed among the available options. A key parameter of the ARED defines the population's "intensity of choice" (IOC). The parameter captures the strength with which the population responds to the measured benefit of being associated with one group over the others. Its value determines the nature of the population dynamics and thus the evolution of the model as a whole. The IOC often determines whether a model converges to a stable fixed point or produces unstable cycles or chaotic behavior. It is thus reasonable to strive for an estimate of the IOC parameter in order to know whether a given system is stable or unstable.

The literature has yet to attempt such an estimate. This paper address this deficiency by examining the distribution of the inflow of funds into and between actively managed and passively managed mutual funds. By narrowing the category of funds examined, we are able to limit the analysis to a population with a reasonably uniform investment objective.

(or your superior version of the same motivational paragraph: What the current literature lacks is a reliable estimate of ρ and even how to interpret the units of the parameter once a model has been properly calibrated. Our approach addresses both of these objections. We place the intensity of choice into a utility maximizing framework so that ρ can be measured in a standard economic context. An added advantage of the portfolio choice setting is that we can also easily transform utility differences into dollar returns to further assess their reasonableness.)

While the IOC parameter is intended to capture a descriptive feature of the population that is very likely stable and uniform across different settings, the scale of any IOC measure is unique to the setting under examination. In order to produce a useful measure that is relevance across a wide range of economic queries, we translate the estimate of the IOC into a monetary value representing the willingness of the population to bear a cost in order to avoid tightening the population distribution. We examine the same economic choice under a variety of utility specifications. Though this produces a wide range of values for the model specific IOC parameters, the cost measure is fairly uniform.

Section 1 defines the intensity of choice. Section 2 describes our baseline one-period model. Section 3 discusses data and parameters used to calibrate the models. Section 4 reports estimates of choice intensity in the one-period problem, and analyses the sensitivity of these estimates. We describe a more general dynamic model in Section 5. Section 6 re-estimates the model in this dynamic setting. Section 7 concludes and secures fame and fortune for the authors.

III. Intensity of Choice

The foundation for the ARED system is the randomized discrete choice framework of Manski and McFadden (1981). Consider an environment in which agents select from a set of K of discrete options from which each agent must make a single selection. The agents estimate the relative benefits of each choice. In empirical studies, not all of agents necessarily choose the option the model indicates to be the best choice for the individual. This is generally attributed to either unmodelled idiosyncratic components of the agents' utility function or to a random component in individual preferences. Either explanation introduces a degree of randomness allowing that the individual agent's ranking of the options may differ from the modeler's. Each agent can be thought of as having a private estimate of the value of each of the options.

Consider $K = 2$. Manski and McFadden establish that the probability individual i selects option 1 is

$$\Pr(x_i = 1) = \frac{e^{\rho U_{i,1}}}{e^{\rho U_{i,1}} + e^{\rho U_{i,0}}}. \tag{1}$$

The $U_{i,k}$, $k = 0$, 1 is the model indicated utility value of option k to individual i,

$$U_{i,k} = U_k + \sigma\varepsilon_{ik}. \tag{2}$$

The idiosyncratic component, $\sigma\varepsilon_i$, is IID with an Extreme Value distribution for ε_i, $E(\varepsilon_{ik}) = 0, V(\varepsilon_{ik}) = 1$. The parameter ρ is an inverse function of σ. The greater ρ the less unmodeled aspects play a role in determining individual choice and thus the more responsive are the investors to the model's measured difference in anticipated utility. The greater σ, the weaker is the model at predicting an agent's choice. At the extreme, as $\sigma \longrightarrow \infty$, the model has no predictive ability and the probability associated with each choice is 1/2 which is the result produced in (1) by $\rho = 0$. At the other extreme, if there is zero variance in the random component of choice, then the agent always selects the choice indicated by the model to be the favorite. Thus, $\Pr(x_i = k\ ) = 1$ for the indicated $U_{i1} > U_{i0}$, 0 or the indicated $U_{i1} < U_{i0}$, and 1/2 if $U_{i1} = U_{i0}$. This result is produced in (1) by $\rho \longrightarrow \infty$.

In a population of N, the law of large numbers ensures that for the proportion of the population selecting k, $n_k \longrightarrow \Pr(x_i = k)$ as $N \longrightarrow \infty$. For $0 \leq \rho < \infty$ there is a tail of the population that selects what the modeler measures to be the inferior option. A larger ρ represents a smaller σ producing a tighter population distribution around the modeler's $U_1 - U_0$ and thus a smaller tail of the population who believe $U_1 < U_0$.

In a dynamic setting in which the choice is repeated each period, let n_t indicate the proportion of the population choosing option 1 in period t. Often, the values of U_1 and U_0 are functions of n_t, thus there is a reduced form expression of n_{t+1} is a function of n_t (though not in this case). The evolution of n_t over time is affected by the value of ρ. Setting $\rho = 0$ ensures a fixed point with $n_t = 1/2$. As ρ increases, the system can become unstable, producing multiple equilibria or chaos. A large number of papers employ the ARED model to explore the range of dynamics produced ρ in different market environment.

We first approach this estimation in a one-period setting trying to describe the changes from the beginning to the end of our samp leperiod.Welatertry to model the annual flows.

IV. Static Model

Investors have two types of mutual funds in which they can place their wealth. The first are the more numerous actively managed funds in which investors hire managers to increase their returns. Recent literature has amply documented the underperformance of mutual fund managers[1]. Beginning with the Vanguard funds in 1976, a growing pool of money has been managed passively, with portfolio weights tied to an index. By far the most popular of these is the Standard and Poor's 500 index, an index of large capitalization stocks, that represents approximately 85% of the total U.S. market capitalization. Mostly due to their lower expenses, these passive funds have on average produced higher returns than their actively managed counterparts.

Each period new investors place wealth into one of the two fund types. Cumulative past performance serves as an indicator of the future value of placing wealth into the funds. Thus, the inflow into passive funds, λ_P, is a reflection of how sensitive investors are to the utility differences in the two options,

$$\lambda_P = \frac{e^{\rho U_P}}{e^{\rho U_P} + e^{\rho U_A}}, \tag{3}$$

$$\lambda_A = \frac{e^{\rho U_A}}{e^{\rho U_P} + e^{\rho U_A}} = 1 - \lambda_P.$$

Using

$$\lambda_P - \lambda_A = 2\lambda_P - 1 = \tanh(\rho(U_P - U_A)/2) \tag{4}$$

the value of ρ can be backed out from observations of λ_P, and estimates of U_P and U_A,

$$\rho = 2\tanh^{-1}(2\lambda_P - 1)/(U_P - U_A). \tag{5}$$

[1] The most comprehensive reference remains the study by Carhart (1997).

We will assume that agents are mean-variance optimizers. To estimate the intensity of choice, we will need data on expected returns and variances for active and passive funds and inflows into the two funds types. Fortunately, these data are readily available, and we discuss them in the next section.

V. Data and parameters

V.A Fund universe

To estimate the intensity of choice, we must find a group of agents with homogenous risk preferences. To do this, we look at a group of mutual funds that track closely the S&P 500 index. Morningstar groups these funds as an equity style called "Large Cap Blend." 209 funds in the Morningstar Principia database have 10-year track records from 1994 to 2003. We have data on assets under management for all but 10 of the funds, and two funds merged, leaving a final sample of 198. The group is diverse, including load and no-load funds. There are 161 active funds, ranging in asset size from Fidelity Magellan (FMAGX) with \$68.0 billion to Black Rock Select Equity (PCESX) with only \$1.9 million. Among the 37 passive funds are the Vanguard S&P 500 index (VFINX) with \$71.9 billion under management and the Black Rock Index Equity (PNESX) with \$70.8 million.

The underperformance of the actively managed funds is evident in our sample. No group of fund managers seems to display a consistent track record of stock picking, and the additional costs from trading, higher expenses and loads lead them to trail most index funds. In our 10-year sample, this is true with an average 10-year load adjusted performance of 9.70% versus 10.49% for the index funds. Over the 10 years, choosing the active funds will lead to a nearly 30% cumulative underperformance, or \$2,997 on a \$10,000 investment.

The index funds display a surprising degree of heterogeneity[2], with mean 10-year returns ranging from 11.13% for the Vanguard Institutional Index (VINIX) to MainStay Equity Index A (MSCEX) at 10.12%. The Vanguard S&P 500 Index has an expected 10-year return of 10.99%.

Even though the S&P 500 index, the benchmark for all the funds in this group, is a passive index, it has been quite volatile, especially in the last three bear market years. The index funds

[2] See Elton, Gruber and Busse (2004).

have a standard deviation of 21.39% versus only 19.44% for the active funds[3]. Using the average 3-month Treasury bill rate over the sample of 4.10%, the passive funds have a higher Sharpe ratio, 0.2880 versus 0.3197. For VFINX, the Sharpe ratio is 0.3212.

Based on the higher Sharpe ratio, many risk averse investors would prefer to hold the passive funds. In this class of funds though, the ratio of actively managed money to index money is more than 5 to 1 at the beginning of our sample i n1994.Astimegoesby,thehigherreturnsandgreaterinflows raise to ratio.

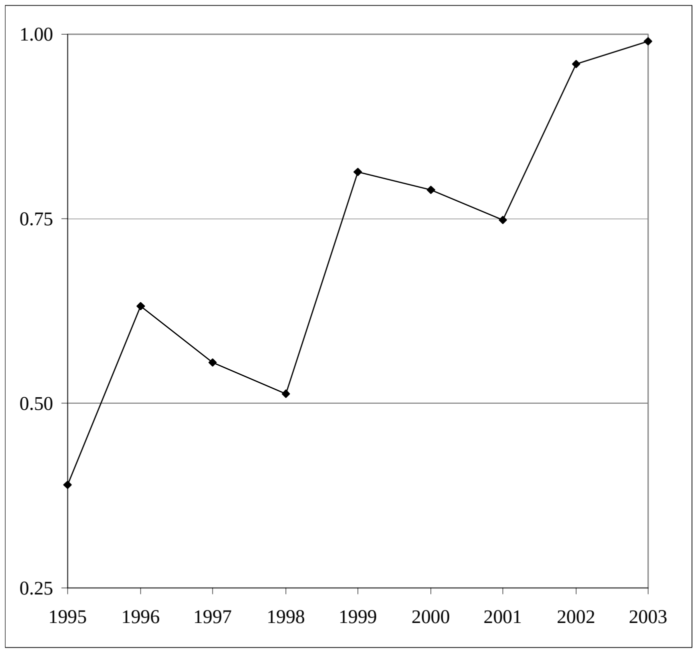

Figure 1: Ratio of Inflows to Passive Funds

We graph the relative inflows into the index funds in Figure 1, $\lambda_P = (\text{passive}/(\text{active}+\text{passive}))$. In years in which both types of funds experience outflows (2000, 2001 and 2002), we measure $1-(\text{passive}/(\text{active}+\text{passive}))$, to reflect the ratio of outflows. When the active flows are negative and the passive flows positive as in 2003, we set $\lambda_P = 0.99$.

Relative inflows rise from 0.39 in 1994 to 0.99 in 2003. By the end of the sample, assets in the active funds have risen to \$297.4 billion from \$110.4 billion, but the index funds have risen from \$20.5 billion to \$178.1 billion. The value for λ_P using the 10 years of cumulative flows is 0.9037.

[3] This is due to the higher cash levels in the active funds.Reference to this here.

Our objective in the next section that is to explain the ratio of invested assets in 2003 measuring the intensity of agent choice over the entire period. Later, we will use this estimate to calibrate annual flows into both groups of funds.

V.B Preferences

We consider four utility functions: (1) constant relative risk aversion $U = W^{1-\gamma}/(1-\gamma)$; (2) log utility $U = \ln(W)$; (3) exponential utility $U = -\exp(-\alpha W)/\alpha$; (4) quadratic utility $U = aW + bW^2$. We set $\gamma = 0.5$, $\alpha = 0.5$, $a = 10$ and $b = -1$. We then take a second order Taylor expansions around expected wealth, so that means and variances are sufficient statistics,

$$U \approx U(E[W]) - U''(E[W])V[W],$$

where $E[W]$ is expected wealth, $V[W]$ is the variance of wealth.

[Insert Table 1 Here]

Table 1 contains static estimates of the intensity of choice for the four utility functions. The range of estimates from 1.02 for the quadratic utility function to 28.46 for the exponential is indicative of the difficulty in drawing inferences from estimates in the literature. Fortunately, all four estimates can be analyzed in terms of the trade-off of between risk and return.

For all four utility functions, we will do two comparative statics experiments. The first will be to look at the change in expected return at the estimated intensity of choice required to increase fund inflows to the passive funds by 5%. The second experiment is to ask what change in required return is necessary to keep fund flows constant after a 5% decline in the intensity of choice. Since both these experiments are stated in terms of expected return, we can quantify them in dollars with some assumptions about investor wealth.

Despite a wide range of estimates for ρ, the changes in expected return $\Delta E_\lambda[W]$ in Table 1 differ slightly across utility functions. At the low end, for quadratic utility, a 0.4175% increase in expected return would raise average fund inflows by 5% to 95.37%. The high end estimate, for exponential utility, is just 3 basis points higher at 0.4478%. These rate of return differentials are within the more than 1.0056% cross-section range of the performance of the passive funds.

To put these estimates in dollars, we assume an initial wealth of $10,000. By 2003, 90% of investors are allocating their new wealth to passive funds. The intensity of choice estimates imply, in the CRRA case, that an additional $5.45 per year in return is needed to induce an additional

5% of investors to put their money in the passive funds. The 54.5 basis points differential seems reasonable compared to the 160 basis points transactions costs[4] on the average equity mutual fund.

The second experiment is to examine the increase in required return $\Delta E_\rho[W]$ needed to offset a 5% decline in the intensity of choice. This is essentially a sensitivity analysis for our results. The range of results across all four utility functions is again minimal, ranging from 0.0631% for exponential utility to 0.0652% for log utility. Assuming an initial wealth of $10,000, these imply very tight dollar estimates, with the inflows held constant by only $1.85 per year in the CRRA case.

VI. Dynamic model

(Revelation: The increase in ρ as dU increases is actually in the wrong direction from my intuition. The traders are not under reacting but over reacting.

work in this transition. The increasing value of the estimate of ρ over the sample period suggests another process is in play. The model accomodates with an increased measure of responsiveness on the part of the investors. This suggests the need a dynamic model that accounts for the flow that takes place as funds are transfered from the active to the passive mutuals. (My new objective for the dynamic model is to find the cost that is consistent with a fixed value of ρ (probably at the first year estimate if it is necessary to specify a value.))

As a new entrant into the investor's choice set, not only will passive funds attract a portion of the new funds, but some investors will look to transfer funds out of active funds into passive funds. Thus, the inflow of funds into both the active and passive funds represents both the distribution of new wealth and the redistribution of existing wealth. Consider the gross inflows in each year to be the result of informed investors who have researched their options and made selections according to the discrete choice model. With an estimate of the spread in utility, $U_P - U_A$, and of the intensity of choice, the rate at which investors transfer funds indicate the cost of transferring wealth.

There are two costs involved in transferring funds. First, an investor must spend resources evaluating the relative benefits of each fund type. Second, there may be fees involved with transferring the funds. A dynamic model of the proportion of wealth invested in active and passive funds will be developed and used to estimate these costs. The presumption of the model is that low intensity

---

[4] See John C. Bogle, "Mutual Fund Directors: The Dog that Didn't Bark," Speech on January 28, 2001.

of choice investors have little inclination check (or monitor, or respond to) the relative benefits of one investment strategy over the other. Thus, the investor may have funds in a low performing mutual fund but fail to recognize this or fail to respond, even if higher utility can be realized by switching funds, even after accounting for the costs.

In the dynamic model, at the beginning of each period there is a pool of investors who evaluate the relative performance of active versus passive funds. This pool is comprised of investors with new wealth to invest, investors who currently have wealth invested in active funds but have decided to check whether this is optimal for them, and investors who currently have wealth invested in passive funds but have decided to check whether this is optimal for them.

Let θ_A and θ_P represent the proportion of current investors in the active and passive funds respectively who decide to evaluate their investment options (@The input to deciding λ is different than the input to deciding θ. What will the difference be? >From meeting, θ includes the cost while λ does not. Are there other differences, such as a difference in the computation of U? We discussed that costs will come out before taking utility. @@Followup by Bruce: I think this is the \$5.45 estimate in the static case)

$$\theta_P = \frac{e^{\rho U_A(c)}}{e^{\rho U_P} + e^{\rho U_A(c)}} \tag{6}$$

$$\theta_A = \frac{e^{\rho U_P(c)}}{e^{\rho U_P(c)} + e^{\rho U_A}}$$

Of those who evaluate their options, λ_P of the static analysis remains the proportion who place wealth into the passive funds. (Note: can only treat everyone as equivalent if there are no fees involved in transferring wealth. If there is a fee, then have to treat the 3 groups separately).

Let x_A and x_P represent the dollar value of the funds in the active and passive funds respectively. With x_n representing the inflow of new wealth to the mutual funds market, the evolution of x over time is

$$\begin{aligned} x_{P,t+1} &= x_{P,t}(1+r_{P,t})(1-\theta_P) + (x_{P,t}(1+r_{P,t})\theta_P + x_{A,t}(1+r_{A,t})\theta_A + x_{n,t})\lambda_P \\ x_{A,t+1} &= x_{A,t}(1+r_{A,t})(1-\theta_A) + (x_{P,t}(1+r_{P,t})\theta_P + x_{A,t}(1+r_{A,t})\theta_A + x_{n,t})(1-\lambda_P) \end{aligned} \tag{7}$$

Let n_t represent the proportion of total wealth invested in mutual funds that are in the passively managed funds. By (7), the evolution of n_t over time is

$$n_{t+1} = \frac{(n_t(1-\theta_{Pt}+\theta_{Pt}\lambda_{Pt})(1+r_{pt}) + (1-n_t)(1+r_{At})\theta_{At}\lambda_{Pt} + \delta_t\lambda_P)}{n_t(1+r_{Pt}) + (1-n_t)(1+r_{At}) + \delta_t}$$

where $\delta_t = x_{n,t}/(x_{P,t}+x_{A,t})$. (Note: if have a net outflow, replace λ_{Pt} with $(1-\lambda_{Pt}) = \lambda_{At}$.)

(@Alternative version that I think is more consistent with the definition in (6). The transition equation below is version "A" in the spreadsheet while the original above is "B". The above set of equations have investors dump funds into a general pool along with new investment funds for further evaluation if θ is triggered. The set below has the investors decide to switch if θ is triggered. For further implications, see the included note)

$$x_{A,t+1} = x_{At}(1+r_{A,t})(1-\theta_A) + x_{Pt}(1+r_{P,t})\theta_P + x_{nt}(1-\lambda_P)$$

$$x_{P,t+1} = x_{Pt}(1+r_{P,t})(1-\theta_A) + x_{At}(1+r_{A,t})\theta_P + x_{nt}\lambda_P$$

$$n_{t+1} = \frac{(n_t(1+r_{pt})(1-\theta_{Pt}) + (1-n_t)(1+r_{At})\theta_{At} + \delta_t\lambda_P)}{n_t(1+r_{Pt}) + (1-n_t)(1+r_{At}) + \delta_t}$$

(Note: if have a net outflow, replace λ_{Pt} with $(1-\lambda_{Pt}) = \lambda_{At}$.))

(From here, as I understand the project, take ρ as given based on the static model results. Plug (3), (6) into **??**. Use data to estimate c. Alternately, could estimate both ρ and c. Allowing $\delta_t < 0$ is probably a good way to handle the net outflow years.)

VII. Dynamic Estimates

We will use time series of estimates in the spreadsheet. The standard errors make the estimates for 2002 and 2003 statistically significant which should offset any and all doubts of even the keenest referees.

VII.A discussion after playing with the Dynamic model in the spead-sheet.

The dynamic model allows investors to transfer wealth between mutual funds. Since some of the net inflow is the result of this transfer, a correct estimate of ρ would be lower. The fact that x_A is larger than x_P also means that the static model based estimate of ρ is biased upwards. While a reasonable model of the distribution of new funds, the ARED model may not be the best model of population dynamics to capture the transfer of wealth. The discussion will highlight some of the issues.

In the absence of any costs to transfering funds, $U_P - U_A > 0$. As a result, greater than half of the wealth in the actively managed funds should be considered for transfer to the passive. This is likely too large a number. Take the cost of research and transfer fees into account and can get $U_P(c) - U_A < 0$. Now, less than half of the investors with funds in the actively managed funds will consider transfering wealth. The value of ρ matters. If ρ is large, the periods in which $U_P(c) - U_A < 0$ a very low proportion of the population transfer wealth. Once $U_P(c) - U_A > 0$ is realized as the disparity grows, there is a sudden shift as almost all of the wealth is available for transfer. If ρ is small then regardless of $U_P(c) - U_A$ about half of the funds are available for transfer. Neither situation is particularly satisfactory. Basically, desirable parameters that might come close to matching the data would involve a medium value of ρ and a cost sufficiently high so that $U_P(c) - U_A < 0$ and θ_A remains below 1/2.

I feel that I have little guildance in selecting the correct ρ to use. The range of estimates from the different periods is sufficiently broad to be able to produce both extremes described above. I think that what I would prefer is to have some kind of Maximum Likelihood estimate of a single ρ and the cost done simultaniously in order to minimize the squared deviation of the fitted n_t from the actual. This capability is outside of my area of experties. Would such an estimate be possible?

VIII. Conclusion

Static estimates are within the range of typical mutual fund fees.

Dynamic estimates vary but are statistically significant by the end of the sample.

References

Brock, W. A. and C. Hommes, 1998, “Heterogenous beliefs and routes to chaos in a simple asset pricing model,” *Journal of Economic Dynamics and Control* 22, 1235-74.

Brock, W. A. and C. Hommes, 1997, “A rational route to randomness,” *Econometrica* 65, 1059-95.

Carhart, M., 1997, “On persistence in mutual fund performance,” *Journal of Finance* 52, 57-82.

Elton, E., M. Gruber, and J. Busse, 2004, “Are investors rational? choices among index funds,” *Journal of Finance* 59, 261-88.

Goldbaum papers that are relevant.

Manski, C.F., and D. McFadden, 1981. *Structural Analysis of Discrete Data with Econometric Applications* (MIT Press, Cambridge, MA).

Table 1

Intensity of Choice in the One Period Model

Utility function	$U_P - U_A$	ρ	$\Delta E_\lambda[W]$	$\Delta E_\rho[W]$
CRRA	0.1833	12.2181	0.4210%	0.0635%
Log	0.1123	19.9421	0.4361%	0.0652%
Exponential	0.0787	28.4636	0.4478%	0.0665%
Quadratic	2.1916	1.0219	0.4175%	0.0631%

U_P is the utility with the passive fund investment returns, and U_P is utility under the active. ρ is the estimate of the intensity of choice. Inflows λ_P are set at the 10-year average of 90.37%. $\Delta E_\lambda[W]$ is the change in expected return required to achieve a 5% increase in inflows to the passive funds. $\Delta E_\rho[W]$ is the increase in expected return needed to keep fund inflows constant after a 5% decline in the intensity of choice.