Money, Inventories and Underemployment in Deflationary Recessions*

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Abstract

This paper investigates monetary shocks and the rôle of inventories with respect to the occurrence of deflationary recessions. We propose a non-tâtonnement approach involving temporary equilibria with rationing in each period and price adjustment between successive periods. By amplifying spillover effects inventories imply that, following a restrictive monetary shock, the economy may converge to a quasi-stationary Keynesian underemployment state, in which case money is persistently non-neutral. Contrary to conventional wisdom, this is favored by sufficient downward flexibility of the nominal wage. The model is applied to the current deflationary Japanese recession, and we propose an economic policy to overcome it.

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1. Introduction

Why money affects output and why it has long lasting effects have long been the two central questions for the business cycle literature, if not for all of macroeconomics. This is especially so because, as stressed for instance by Blanchard (2000), the empirical evidence is irremediably at odds with the conclusions of the flexible-price models which still represent the approach most commonly shared by economists.

If prices were fully flexible, an increase in nominal money would immediately induce a proportional increase in the price level offsetting any pressure on demand and output, and money would be neutral even in the short run. Prices and wages, however, do not change instantaneously: they exhibit a certain degree of stickiness and individual price changes tend to be staggered, which makes the adjustment process of the price level more or less slow. During the process, aggregate demand and output are higher than their original values, and the change in the money stock has real effects. Eventually, most economists maintain, the price level will adjust proportionally to the increase in the nominal money stock, so that demand and output will be back at their original levels, and money neutrality will be restored. Before this occurs, real and nominal rigidities, lying behind the slow adjustment of prices and wages, are the causes of the temporary non-neutrality of money. Since the beginning of the Nineties, the New Keynesian literature (see e.g. Ball and Romer, 1990, and Blanchard, 1987 and 1990) has emphasized that monetary shocks determine large aggregate effects when small frictions in nominal adjustment are supplemented by real rigidities.[1]

[1] Much of the recent research in macroeconomics has concentrated on imperfections of labor, goods, and financial markets responsible for the emergence of real rigidities and nominal stickiness and on their relevance for economic fluctuations.

Many causes of real rigidities have been investigated in the literature: among others, efficiency wages (see, for example, Solow, 1979, and Shapiro and Stiglitz, 1984), implicit contracts (Azariadis, 1975, and Baily, 1974), countercyclical mark-ups (Stiglitz, 1984, Rotemberg and Saloner, 1986, and Rotemberg and Woodford, 1991), inventories (Blinder, 1982), social customs (Akerlof, 1980, and Romer, 1984), strategic interactions and coordination failure (Ball and Romer, 1991), credit markets imperfections (Bernanke and Gertler, 1989 and 1995, Holmström and Tirole, 1997, 1998, and Kiyotaki and Moore, 1997) and increasing returns (Kiyotaki, 1988, and Diamond, 1982). Attention has been devoted as well to the sources of nominal stickiness focusing, for instance, on menu cost or near rationality (e.g. Mankiw, 1985, and Akerlof and Yellen, 1985), staggered contracts (Calvo, 1983) and uncertainty and risk aversion (Weinrich, 1997).

In the Nineties, Keynesian features - like the nominal and real stickiness just mentioned - have been incorporated into the dynamic general equilibrium framework typical of the business cycle

In this paper we aim to show that both claims presented in the theoretical literature - about the long-run money neutrality and the effectiveness of price flexibility to lead the economy quickly back to the pre-shock state - do not necessarily hold. On the contrary, money can affect the output level in the long run and price and wage flexibility can foster achieving this result, while wage rigidity may prove a good recipe to avoid or overcome permanent underemployment and to restore Walrasian equilibrium.

Our framework is that of a discrete-time dynamic non-tâtonnement macroeconomic model, building on Bignami, Colombo and Weinrich (2004) and on Colombo and Weinrich (2003a). The economy consists of an overlapping generations consumption sector, of a production sector characterized by an atemporal production function, and of a government that finances public expenditure by means of a tax levied on firms' profits. Within each period, prices are fixed and a consistent allocation is obtained by means of temporary equilibrium with stochastic rationing whereas prices are adjusted between successive periods according to the strength of rationing or disequilibrium on each market in the previous period.[2] This approach permits to account for the fact that in any economy with decentralized price setting, the "adjustment of the general level of prices in terms of the numeraire is likely to be slow relative to a (fictional) economy with an auctioneer", as emphasized by Blanchard (2000, p. 1393). It is important to stress that the way we model the price (wage) adjustment mechanism allows us to account quite naturally for different degrees of price and wage flexibility. Although our adjustment mechanism is given exogenously - and thus it may be considered *ad hoc* - it allows us to assess the impact of price and wage reactions to shocks generated by different underlying conceptual models. In other words, it is "agnostic" enough to provide a framework to study the impact of real and nominal rigidities in the New Keynesian tradition, as well as to investigate the consequences for price and wage adjustment of the presence of uncertainty (e.g. about the entity of monetary transfers as in Lucas and Woodford, 1993, or about information that becomes

literature, originating what has been named the New Neoclassical Synthesis (see, for example, Jeanne (1998) and the references in there).

[2] A natural idea is to relate the adjustment of prices to the size of the dissatisfaction of agents with their (foregone) trades. A reliable measure of such a dissatisfaction requires stochastic rationing, since - as opposed to deterministic rationing - it is compatible with manipulability of the rationing mechanism and therefore provides an incentive for rationed agents to express demands that exceed their expected trades, as argued by Green (1980), Svensson (1980), Douglas Gale (1979, 1981) and Weinrich (1982, 1984, 1988). For a definition of manipulability see for example Böhm (1989) or Weinrich (1988).

public only at the end of the process as in Eden, 1994), or confusion (as in Lucas, 1972).

The novelty of the economy developed here with respect to the one considered in our previous papers is that we abandon the simplifying assumption that there are no inventories. In the present paper inventories are possible and stored goods may be sold in periods subsequent to the period of their production. More precisely, at the beginning of each period the stock of inventories carried by each firm is simply given by the firm's output that remains unsold at the end of the previous period. In this sense, inventories are not used as "strategic" decision variables by firms, which makes our treatment of inventories different from, and simpler than, most of the accounts present in the recent literature (see, for instance, Blinder and Fischer, 1981, Blinder, 1982 and Bental and Eden, 1996). However, in our model as well, the explicit consideration of inventories adds a further propagation mechanism for shocks and amplifies the importance of the spillover effects among markets.

To highlight one of the main results of the paper, consider a restrictive monetary shock that, starting from a Walrasian equilibrium, reduces aggregate demand, inducing excess supply on the goods market and, consequently, a reduction in the goods price. The decrease in aggregate demand reduces labor demand and gives rise to an excess supply on the labor market as well, i.e. to Keynesian unemployment. Whenever the nominal wage is rigid downward, the real wage and the real money stock increase until the economy leaves the state of Keynesian unemployment to enter a state of Classical unemployment, that is excess demand on the goods market and excess supply on the labor market. At this point the goods price starts to increase again, determining a reduction of the real wage and of the real money stock until the economy converges back to the Walrasian equilibrium.

The process changes quite dramatically when there is downward wage flexibility. In this case, the monetary shock determines a reduction of the nominal wage that, if it is large enough, implies a decrease of the real wage, too. The presence of inventories reinforces this reduction, by increasing the fall of labor demand which in turn depresses labor income and aggregate demand. The real wage continues to fall although ever more slowly. Eventually the economy converges to a quasi-stationary Keynesian state with a constant low real wage, permanent unemployment and permanent deflation of the nominal variables. Therefore, contrary to the previous literature, downward nominal wage flexibility favors a lasting impact of monetary shocks whereas imposing downward nominal wage rigidity appears to be a viable policy to prevent the emergence of recessions or at least limit their extent and duration.

Moreover, we suggest, by means of numerical simulations, that such recessionary quasi-stationary equilibria are locally stable while the stationary Walrasian equilibrium is locally unstable. Specifically, reductions in the real money stock or in real profits, and increases in the stock of inventories, destroy the full employment equilibrium and cause the economy to converge to a quasi-stationary Keynesian equilibrium.

Besides the theoretical underpinnings on the role and consequences of inventories dynamics the paper is of interest from a policy perspective. Our setup is, in fact, able to account for the dynamic behavior of economies that are trapped in situations of underemployment or underutilization of the productive capacity. This proves very useful in evaluating the impact of alternative policy measures aimed at restoring full employment. In this respect, we use our economy as a test bank to investigate the deflationary behavior of the Japanese economy since the early Nineties of the past century and to evaluate the performance of different monetary policies designed to stimulate the economy. More precisely, the recessionary Keynesian equilibrium of our economy seems to reproduce quite well the recent experience of the Japanese economy, and therefore it provides a suitable framework to discuss the impact of different economic policies.[3] In particular, we focus on policy measures requiring simultaneous fiscal and monetary expansionary stimuli based on tax cuts directly financed by the central bank to check whether they are effective in restoring full employment in our model economy.[4] By operating a reduction of the tax rate and by maintaining unchanged both the government's budget deficit and the aggregate demand (by means of a monetary expansion), our analysis suggests that the stationary (and locally stable) long run employment level increases monotonically with the decrease in the tax rate. This confirms the efficacy of such policies that, provided they are of the right

[3] Although there are signs of recovery with respect to the drop in prices of the mid Nineties, according to OECD statistics (OECD Main Economic Indicators, October 2004), the Japanese consumer price index (base 2000=100) fell from 99.3 in 2001 to 98 in the second quarter 2004. Similarly, the producer price index (base 2000=100) fell from 97.7 to 95.6 in the same period. At the same time, the standardized unemployment rates increased steadily from 5% in 2001 to 5.3 % in 2003, to drop to 4.6% in the second quarter 2004.

[4] Similar policies have been called for by many economists and recently implemented by the Bank of Japan which adopted a policy of quantitative monetary easing. Policies equivalent to those outilined in the paper, for example, have been advocated by Ben Bernanke (see *The Economist*, June 21st, 2003). Along the same lines, Auerbach and Obstfeld (2003) made a case for the efficacy of large open-market purchases of domestic government debt as a way out of the recession for the Japanese economy.

magnitude, should be capable to restore full employment.

The remainder of the paper is organized as follows. In section two we present the model and describe the behavior of consumers, producers and the government. Section three focuses on temporary equilibria with rationing and proves the existence and uniqueness of equilibrium allocations. In section four we set up the dynamic system. Section five presents numerical simulations and discusses the impact of fiscal and monetary shocks. Section six investigates the Japanese deflationary recession and discusses policy measures to overcome it. Finally, section seven concludes, while proofs, some technical results and the complete dynamic system are given in the appendices.

2. The Model

We consider an economy in which there are n OLG-consumers, n' firms and a government. Consumers offer labor inelastically when young and consume a composite consumption good in both periods. That good is produced by firms using an atemporal production function whose only input is labor. The government levies a proportional tax on firms' profits to finance its expenditure for goods. Nevertheless, budget deficits and surpluses may arise and are made possible through money creation or destruction.

2.1. Timing of the Model

In period $t-1$ producers obtain an aggregate profit of Π_{t-1} which is distributed at the beginning of period t in part as tax to the government ($tax\Pi_{t-1}$) and in part to young consumers ($(1-tax)\,\Pi_{t-1}$), where $0 \leq tax \leq 1$. Also at the beginning of period t old consumers hold a total quantity of money M_t, consisting of savings generated in period $t-1$. Thus households use money as a means of transfer of purchasing power between periods.[5]

Let X_t denote the aggregate quantity of the good purchased by young consumers in period t, p_t its price, w_t the nominal wage and L_t the aggregate quantity

[5] We assume that, although the good is storable for firms, it is not so for consumers: they do not have access to firms' storage technology the cost of which is worthwhile to be borne for large quantities only. Moreover, even if the good were storable by consumers, this would not be convenient for them in case next period's price is lower than the current period's one. Thus our main results, which regard deflationary recessionary equilibria, would not be influenced anyway.

of labor. Then

$$M_{t+1} = (1 - tax)\,\Pi_{t-1} + w_t L_t - p_t X_t.$$

Denoting with G the quantity of goods purchased by the government and taking into account that old households want to consume all their money holdings in period t, the aggregate consumption of young and old households and the government is $Y_t = X_t + \frac{M_t}{p_t} + G$. Using that $\Pi_t = p_t Y_t - w_t L_t$, considering $\Pi_t - \Pi_{t-1} = \Delta M_t^P$ as the variation in the money stock held by producers before they distribute profits and denoting with $\Delta M_t^C = M_{t+1} - M_t$ the one referring to consumers, we obtain the usual accounting identity, i.e. $\Delta M_t^C + \Delta M_t^P = p_t G - tax\Pi_{t-1} =$ budget deficit.

Denoting with S_t the aggregate amount of inventories carried over by firms to period t and with Y_t^p the aggregate amount of goods produced in period t, there results $S_{t+1} = Y_t^p + S_t - Y_t$.

2.2. The Consumption Sector

In his first period of life each consumer born at t is endowed with labor ℓ^s and an amount of money $(1 - tax)\,\Pi_{t-1}/n$ while his preferences are described by the utility function $u\,(x_t, x_{t+1}) = x_t^h x_{t+1}^{1-h}, 0 < h < 1$, where x denotes consumption.[6] In solving his decision problem the young household has to meet the budget constraints

$$0 \leq x_t \leq \omega_t^i,\ 0 \leq x_{t+1} \leq \left(\omega_t^i - x_t\right)/\theta_t\ , i = 0, 1$$

where $\theta_t = p_{t+1}/p_t$ and

$$\omega_t^0 = \frac{1 - tax}{p_t}\frac{\Pi_{t-1}}{n} \quad \text{and} \quad \omega_t^1 = \omega_t^0 + \frac{w_t}{p_t}\ell^s$$

denote the consumer's real wealth when he is unemployed and employed, respectively. Implicit in this formulation is that rationing on the labor market is of the all-or-nothing type and that the labor market is visited before the goods market.

On the goods market the young household succeeds to buy its quantity demanded x_t^d with probability γ_t^d and is rationed to zero with probability $1 - \gamma_t^d$, where $\gamma_t^d \in [0, 1]$ is a rationing coefficient that the household perceives as given but that will be determined in equilibrium. Hence, the expected value of x_t is $\gamma_t^d x_t^d$, meaning that rationing is proportional and thus manipulable.

A household may also be rationed when old. Assuming again 0/1-rationing, the probability that it expects in period t *not* to be rationed in period $t + 1$

[6] See Colombo and Weinrich (2003b) for a more general approach to the consumer's problem.

is denoted by δ_{t+1}^{e}. Denoting moreover with θ_t^e the expected value of θ_t, the effective demand x_t^{di}, $i = 0, 1$, is obtained by maximizing the expected utility $\gamma_t^d \delta_{t+1}^e x_t^h \left(\left(\omega_t^i - x_t \right) / \theta_t^e \right)^{1-h}$. The solution is $x_t^{di} = h\omega_t^i$. Thus the young consumer's effective demand is independent of γ_t^d, δ_{t+1}^e and θ_t^e but it does depend on the real income ω_t^i and hence on whether the consumer has been employed.

The aggregate supply of labor is $L^s = n\ell^s$. Denoting with L_t^d the aggregate demand of labor and with $\lambda_t^s = \min \left\{ \frac{L_t^d}{L^s}, 1 \right\}$ the fraction of young consumers that will be employed, the aggregate demand of goods of young consumers is

$$\begin{aligned} X_t^d &= \lambda_t^s n x_t^{d1} + \left(1 - \lambda_t^s\right) n x_t^{d0} \\ &= h\left(1 - tax\right) \frac{\Pi_{t-1}}{p_t} + h \frac{w_t}{p_t} \lambda_t^s L^s \equiv X^d \left(\lambda_t^s; \frac{w_t}{p_t}, \frac{\left(1 - tax\right) \Pi_{t-1}}{p_t} \right). \quad (2.1) \end{aligned}$$

The total effective aggregate demand of the consumption sector is then obtained by adding old consumers' aggregate demand $m_t = M_t/p_t$ and government demand G:

$$Y_t^d = X^d \left(\lambda_t^s; \alpha_t, \left(1 - tax\right) \pi_t \right) + m_t + G$$

where $\alpha_t \equiv w_t/p_t$, $\pi_t \equiv \Pi_{t-1}/p_t$ and $m_t \equiv M_t/p_t$.

2.3. The Production Sector

Each of the n' identical firms uses an atemporal production function $y_t^p = f(\ell_t) = a\ell_t^b, a, b > 0$. Having transferred stocks from the previous period and being thus endowed with inventories s_t at the beginning of period t, the total amount supplied by a firm is $y_t^s = y_t^p + s_t$. As with consumers, firms too may be rationed, by means of a rationing mechanism analogous to that assumed for the consumption sector.

Denoting the single firm's effective demand of labor by ℓ_t^d, the quantity of labor effectively transacted is ℓ_t^d with probability λ_t^d and 0 with probability $1 - \lambda_t^d$, where $\lambda_t^d \in [0, 1]$. It is obvious that $E\ell_t = \lambda_t^d \ell_t^d$. On the goods market the rationing rule is assumed to be

$$y_t = \begin{cases} y_t^s, & \text{with prob. } \sigma\gamma_t^s \\ d_t y_t^s, & \text{with prob. } 1 - \sigma\gamma_t^s \end{cases},$$

where $\sigma \in (0, 1)$, $\gamma_t^s \in [0, 1]$ and $d_t = \left(\gamma_t^s - \sigma\gamma_t^s\right) / \left(1 - \sigma\gamma_t^s\right)$. σ is a fixed parameter of the mechanism whereas λ_t^d and γ_t^s are perceived rationing coefficients taken as given by the firm the effective value of which will be determined in equilibrium.

The definition of d_t implies that $Ey_t = \gamma_t^s y_t^s$ which, in particular, it is independent of σ. It is obvious that $E\ell_t = \lambda_t^d \ell_t^d$.

The firm's effective demand $\ell_t^d = \ell^d(\gamma_t^s; \alpha_t)$ is obtained from maximizing its expected profit $\gamma_t^s \left[f\left(\ell_t^d\right) + s_t\right] - \alpha_t \ell_t^d$ subject to

$$0 \le \ell_t^d \le \frac{d_t}{\alpha_t}\left[f\left(\ell_t^d\right) + s_t\right]$$

while its effective supply is $y_t^s = f\left(\ell_t^d\right) + s_t$. The upper bound on labor demand reflects the fact that the firm must be prepared to finance labor service purchases even if rationed on the goods market (since the labor market is visited first it will know whether it is rationed on the goods market only after it has hired labor). In general the solution depends on this constraint but it is not binding (see Appendix 1, Lemma 1) if we make the assumption $b \le 1 - \sigma$. In this case labor demand is

$$\ell_t^d = \ell^d(\gamma_t^s; \alpha_t) = \left(\frac{\gamma_t^s ab}{\alpha_t}\right)^{\frac{1}{1-b}}. \tag{2.2}$$

Notice that labor demand is independent of s_t. The aggregate labor demand then is $L_t^d = n'\ell^d(\gamma_t^s; \alpha_t) \equiv L^d(\gamma_t^s; \alpha_t)$ and, because only a fraction λ_t^d of firms can hire workers, the aggregate supply of goods is

$$Y_t^s = \lambda_t^d n' f\left(\ell^d(\gamma_t^s; \alpha_t)\right) + S_t \equiv Y^s\left(\lambda_t^d, \gamma_t^s; \alpha_t, S_t\right). \tag{2.3}$$

3. Temporary Equilibrium Allocations

For any given period t we can now describe a feasible allocation as a temporary equilibrium with rationing as follows.

Definition 3.1. *: Given a real wage α_t, a real profit level π_t, real money balances m_t, inventories S_t, a level of public expenditure G and a tax rate tax, a list of rationing coefficients $\left(\gamma_t^d, \gamma_t^s, \lambda_t^d, \lambda_t^s, \delta_t, \varepsilon_t\right) \in [0,1]^6$ and an aggregate allocation $\left(\overline{L}_t, \overline{Y}_t\right)$ constitute* a temporary equilibrium *if the following conditions are fulfilled:*

(1) $\overline{L}_t = \lambda_t^s L^s = \lambda_t^d L^d(\gamma_t^s; \alpha_t)$;

(2) $\overline{Y}_t = \gamma_t^s Y^s\left(\lambda_t^d, \gamma_t^s; \alpha_t, S_t\right) = \gamma_t^d X^d\left(\lambda_t^s; \alpha_t, (1 - tax)\pi_t\right) + \delta_t m_t + \varepsilon_t G$;

(3) $(1 - \lambda_t^s)\left(1 - \lambda_t^d\right) = 0$; $(1 - \gamma_t^s)\left(1 - \gamma_t^d\right) = 0$;

(4) $\gamma_t^d(1 - \delta_t) = 0$; $\delta_t(1 - \varepsilon_t) = 0$.

Conditions (1) and (2) require that expected aggregate transactions balance. This means that agents have correct perceptions of the rationing coefficients $\gamma_t^d, \gamma_t^s, \lambda_t^d$ and λ_t^s. Equations (3) formalize the short-side rule according to which at most one side on each market is rationed. The meaning of the coefficients δ_t and ε_t in equations (2) and (4) is that also old households and/or the government can be rationed. However, according to condition (4) this may occur only after young households have been rationed (to zero).

As shown in the table below it is possible to distinguish different types of equilibrium according to which market sides are rationed: excess supply on both markets is called *Keynesian Unemployment* $[K]$, excess demand on both markets *Repressed Inflation* $[I]$, excess supply on the labor market and excess demand on the goods market *Classical Unemployment* $[C]$ and excess demand on the labor market with excess supply on the goods market *Underconsumption* $[U]$.

	K	I	C	U
λ_t^s	<1	$=1$	<1	$=1$
λ_t^d	$=1$	<1	$=1$	<1
γ_t^s	<1	$=1$	$=1$	<1
γ_t^d	$=1$	<1	<1	$=1$
δ_t	$=1$	≤ 1	≤ 1	$=1$
ε_t	$=1$	≤ 1	≤ 1	$=1$

Of course there are further intermediate cases which, however, can be considered as limiting cases of the above ones. In particular, when all the rationing coefficients are equal to one, we are in a Walrasian Equilibrium.[7]

Existence and uniqueness of temporary equilibrium are established by the following proposition.

Proposition 3.2. *For any quadruple of variables $(\alpha_t, m_t, \pi_t, S_t)$, with α_t strictly positive and m_t, π_t and S_t non-negative, and any non-negative pair of policy parameters (G, tax), there exists a unique temporary equilibrium allocation $\left(\overline{L}_t, \overline{Y}_t\right)$. $\overline{L}_t$ is given by*

$$\overline{L}_t = \min\left\{\widetilde{L}\left(\alpha_t, \pi_t, m_t, S_t, G, tax\right), L^d\left(1, \alpha_t\right), L^s\right\} \equiv \mathcal{L}\left(\alpha_t, \pi_t, m_t, S_t, G, tax\right) \tag{3.1}$$

[7] For an illustration of equilibrium regimes and their representation in the $p-w$ plane, see Colombo and Weinrich (2003b).

where $\widetilde{L}(\alpha_t, \pi_t, m_t, S_t, G, tax)$ *is the unique solution in* L *of*

$$\alpha_t\left(\frac{1}{b}-h\right)L+\frac{\alpha_t}{ab}\left(\frac{L}{n'}\right)^{1-b}S_t=h\left(1-tax\right)\pi_t+m_t+G \tag{3.2}$$

and

$$L^d\left(1,\alpha_t\right)=n'\left(\frac{ab}{\alpha_t}\right)^{\frac{1}{1-b}}. \tag{3.3}$$

$\overline{Y}_t \equiv \mathcal{Y}(\alpha_t, \pi_t, m_t, S_t, G, tax)$ *is determined as follows. If* $\overline{L}_t = \widetilde{L}(\cdot)$*, then* $\overline{Y}_t = \frac{\alpha_t}{b}\overline{L}_t + \frac{\alpha_t}{ab}\left(\frac{\overline{L}_t}{n'}\right)^{1-b}S_t$*, and if* $\overline{L}_t = L^d(1, \alpha_t)$*, then* $\overline{Y}_t = \frac{\alpha_t}{b}L^d(1,\alpha_t) + S_t$*. Finally, if* $\overline{L}_t = L^s$*, then* $\overline{Y}_t = \min\left\{\frac{\alpha_t}{b}L^s + S_t, h(1-tax)\pi_t + h\alpha_t L^s + m_t + G\right\}$.

Proof. See Appendix 2.

For the sake of illustration let us consider a situation of Keynesian Unemployment. This type of equilibrium involves rationing of households on the labor market and of firms on the goods market. It is given by a pair $(\lambda_t^s, \gamma_t^s)$ such that

$$\begin{aligned}\overline{L}_t &= \lambda_t^s L^s = L^d\left(\gamma_t^s\right)\\ \overline{Y}_t &= \gamma_t^s Y^s\left(1,\gamma_t^s\right) = X^d\left(\lambda_t^s\right)+m_t+G\end{aligned}$$

(where we have suppressed all arguments that are not rationing coefficients).

The consumption sector supplies the amount of labor $L^s > \overline{L}_t$ and demands the quantity of goods $Y_t^d = \overline{Y}_t$ whereas firms demand labor $L_t^d = \overline{L}_t$ and supply $Y_t^s > \overline{Y}_t$ of goods. It follows that $\lambda_t^s = \overline{L}_t/L^s, \gamma_t^s = \overline{Y}_t/Y_t^s$ and $\lambda_t^d = \gamma_t^d = 1\ (= \delta_t = \varepsilon_t)$, which are just the values that led households and firms to express their respective transaction offers. Thus their expectations regarding these rationing coefficients are confirmed. Nevertheless, due to the randomness in rationing at an individual agent's level, effective aggregate demands and supplies of rationed agents exceed their actual transactions. Moreover, as indicated earlier, these excesses can be used to get an indicator of the strength of rationing. Since there is zero-one rationing on the labor market, $1-\lambda_t^s = (L^s - \overline{L}_t)/L^s$ is the ratio of the number of unemployed workers and the total number of young households. Regarding the goods market, in a K-equilibrium $\overline{Y}_t - \gamma_t^s Y^s(1,\gamma_t^s) = 0$, and therefore

$$\frac{d\left(1-\gamma_t^s\right)}{d\overline{Y}_t}=-\frac{1}{Y_t^s+\gamma_t^s\frac{\partial Y^s}{\partial\gamma_t^s}}<0$$

since $\frac{\partial Y^s}{\partial \gamma_t^s}(1,\gamma_t^s) = n' f'\left(\ell^d\left(\gamma_t^s\right)\right) \frac{d\ell^d}{d\gamma_t^s} > 0$. So a decrease in $\overline{Y}_t$ (for example due to a reduction of government spending), and thus an aggravation of the shortage of aggregate demand for firms' goods, is unambiguously related to an increase in $1-\gamma_t^s$ which can therefore be interpreted as a measure of the strength of rationing on the goods market. A similar reasoning justifies the use as rationing measures of the terms $1-\lambda_t^d$ and $1-\gamma_t^d$ in the other equilibrium regimes.

4. Dynamics

So far our analysis has been essentially static. For any given vector $(\alpha_t, \pi_t, m_t, S_t, G, tax)$ we have described a feasible allocation in terms of a temporary equilibrium with rationing. To extend now our analysis to a dynamic one we must link successive periods one to another. This link will of course be given by the adjustment of prices but also by the changes in the stock of money and in profits. Regarding the latter, this is automatic by definition of these variables and equations (3.1) to (3.3), i.e.

$$\Pi_t = p_t \mathcal{Y}(\alpha_t, \pi_t, m_t, S_t, G, tax) - w_t \mathcal{L}(\alpha_t, \pi_t, m_t, G, tax),$$

$$\begin{aligned} M_{t+1} &= (1-tax)\Pi_{t-1} + w_t \overline{L}_t - p_t \overline{Y}_t + \delta_t M_t + \varepsilon_t p_t G \\ &= (1-tax)\Pi_{t-1} - \Pi_t + \delta_t M_t + \varepsilon_t p_t G. \end{aligned}$$

$$S_{t+1} = Y^s\left(\lambda_t^d, \gamma_t^s; \alpha_t, S_t\right) - \mathcal{Y}(\alpha_t, \pi_t, m_t, S_t, G, tax).$$

As for the adjustment of prices and wages we assume that, whenever an excess of demand (supply) is observed, the price rises (falls). In terms of the rationing coefficients observed in period t, this amounts to

$$p_{t+1} < p_t \Leftrightarrow \gamma_t^s < 1; \; p_{t+1} > p_t \Leftrightarrow \gamma_t^d < 1,$$

$$w_{t+1} < w_t \Leftrightarrow \lambda_t^s < 1; \; w_{t+1} > w_t \Leftrightarrow \lambda_t^d < 1.$$

More precisely, in our simulation model we have specified these adjustments as follows:

$$p_{t+1} = \begin{cases} \left[1-\mu_1\left(1-\gamma_t^s\right)\right] p_t & \text{if } \gamma_t^s < 1 \\ \left[1+\mu_2\left(1-\frac{\gamma_t^d+\delta_t+\varepsilon_t}{3}\right)\right] p_t & \text{if } \gamma_t^d < 1 \end{cases} \tag{4.1}$$

$$w_{t+1} = \begin{cases} \left[1-\nu_1\left(1-\lambda_t^s\right)\right] w_t & \text{if } \lambda_t^s < 1 \\ \left[1+\nu_2\left(1-\lambda_t^d\right)\right] w_t & \text{if } \lambda_t^d < 1 \end{cases} \tag{4.2}$$

where μ_1, μ_2, ν_1, $\nu_2 \in [0,1]$. Then the adjustment equations for the real wage are

$$\alpha_{t+1} = \begin{cases} \frac{1-\nu_1(1-\lambda_t^s)}{1-\mu_1(1-\gamma_t^s)}\alpha_t & \text{if } \left(\overline{L}_t, \overline{Y}_t\right) \in K \\ \frac{1-\nu_1(1-\lambda_t^s)}{1+\mu_2\left(1-\frac{\gamma_t^d+\delta_t+\varepsilon_t}{3}\right)}\alpha_t & \text{if } \left(\overline{L}_t, \overline{Y}_t\right) \in C \\ \frac{1+\nu_2\left(1-\lambda_t^d\right)}{1+\mu_2\left(1-\frac{\gamma_t^d+\delta_t+\varepsilon_t}{3}\right)}\alpha_t & \text{if } \left(\overline{L}_t, \overline{Y}_t\right) \in I \\ \frac{1+\nu_2\left(1-\lambda_t^d\right)}{1-\mu_1(1-\gamma_t^s)}\alpha_t & \text{if } \left(\overline{L}_t, \overline{Y}_t\right) \in U \end{cases} \tag{4.3}$$

whereas θ_t is given by

$$\theta_t = \begin{cases} 1-\mu_1\left(1-\gamma_t^s\right) & \text{if } \left(\overline{L}_t, \overline{Y}_t\right) \in K \cup U \\ 1+\mu_2\left(1-\frac{\gamma_t^d+\delta_t+\varepsilon_t}{3}\right) & \text{if } \left(\overline{L}_t, \overline{Y}_t\right) \in C \cup I \end{cases} . \tag{4.4}$$

The dynamics of the model in real terms is given by the sequence $\{(\alpha_t, m_t, \pi_t, S_t)\}_{t=1}^{\infty}$, where α_{t+1} is as in (4.3) and, using equations (3.1) to (3.3),

$$\pi_{t+1} = \begin{cases} \frac{1-b}{\theta_t(1-hb)}\left[h\left(1-tax\right)\pi_t + m_t + G\right] & \text{if } \left(\overline{L}_t, \overline{Y}_t\right) \in K \\ \frac{1-b}{\theta_t}n'\left(\frac{\alpha_t}{ab}\right)^{\frac{b}{b-1}}\left(\frac{1}{a}\right)^{\frac{1}{b-1}} & \text{if } \left(\overline{L}_t, \overline{Y}_t\right) \in C \\ \frac{\alpha_t}{\theta_t}\frac{1-b}{b}L^s & \text{if } \left(\overline{L}_t, \overline{Y}_t\right) \in I \\ \frac{1}{\theta_t}\left[h\left(1-tax\right)\pi_t + m_t + G - \alpha_t\left(1-h\right)L^s\right] & \text{if } \left(\overline{L}_t, \overline{Y}_t\right) \in U \end{cases} .$$

The case U is derived as follows:

$$\begin{aligned} \pi_{t+1} &= \frac{\Pi_t}{p_{t+1}} = \frac{p_t\left[h\left(1-tax\right)\pi_t + h\alpha_t L^s + m_t + G\right] - w_t L^s}{p_{t+1}} \\ &= \frac{1}{\theta_t}\left[h\left(1-tax\right)\pi_t + h\alpha_t L^s + m_t + G - \alpha_t L^s\right]. \end{aligned}$$

Finally,

$$m_{t+1} = \frac{1}{\theta_t}\left[\delta_t m_t + \varepsilon_t G + \left(1-tax\right)\pi_t\right] - \pi_{t+1}$$

and

$$S_{t+1} = \lambda_t^d n' a \left(\frac{\gamma_t^s ab}{\alpha_t} \right)^{\frac{b}{1-b}} + S_t - \overline{Y}_t.$$

What has still to be determined here are the values of the rationing coefficients $\left(\gamma_t^d, \gamma_t^s, \lambda_t^d, \lambda_t^s, \delta_t, \varepsilon_t\right)$. This will be done in Appendix 3 where there will be also given the corresponding explicit equations of the complete dynamic system.

5. Simulations

The non-linear dynamic system describing our economy cannot be fully studied by means of analytical tools only. This is due to the fact that the system is four-dimensional, with state variables α_t, m_t, π_t and S_t. Moreover, since there are four nondegenerate equilibrium regimes, the overall dynamic system can be viewed as being composed of four subsystems each of which may become effective through endogenous regime switching.

In order to get some insights in these dynamics we resort to numerical simulations.[8] The basic parameter set specifies values for the technological coefficients (a and b), the exponent of the utility function (h), the labor supply (L^s) and the total number of producers in the economy (n'), for the price adjustment speeds downward and upward (respectively μ_1 and μ_2) and the corresponding wage adjustment speeds (ν_1 and ν_2). We also specify initial values for the real wage, real money stock, real profit level and inventories (α_0, m_0, π_0 and S_0), and values for the government policy parameters (G and tax). Choosing in addition an initial value p_0 for the goods price, we can moreover keep track of the development of the nominal variables by using (4.1) to determine p_t for any t from which follow $w_t = \alpha_t p_t$ and $M_t = m_t p_t$.

Assuming the parameter values $a = 1$, $b = 0.85$, $h = 0.5$, $L^s = 100$ and $n' = 100$, a stationary Walrasian equilibrium is obtained for

$$\alpha^* = 0.85, \quad m^* = 46.25, \quad \pi^* = 15, \quad S^* = 0, \quad G^* = 7.5, \quad tax^* = 0.5, \qquad (5.1)$$

with trading levels $L^* = Y^* = 100$. For the adjustment speeds of prices out of Walrasian equilibrium we set $\mu_1 = \mu_2 = \nu_2 = 0.1$ whereas ν_1, the downward

[8] Our numerical analysis is using programs written for this paper's purposes based on the packages GAUSS and MACRODYN . MACRODYN has been developed at the University of Bielefeld. See Böhm,V., Lohmann, M. and U. Middelberg (1999), MACRODYN – a dynamical system's tool kit, version x99.

speed of wage adjustment, will be varied between 0 and 0.1. This includes the case $\nu_1 = 0$ in which the wage rate is rigid downwards.

We consider a restrictive monetary shock determining a reduction in the initial money stock to $m_0 = 40$, keeping all other parameters and initial values at their Walrasian levels. Having set $p_0 = 1$, this is equivalent to a reduction in the nominal money stock from $M_0 = 46.25$ to $M_0 = 40$. Since m_0 is the demand of old agents at time $t = 0$, aggregate demand is reduced. Consequently there is excess supply on the goods market (and a reduction in the goods price) and, as firms adjust to the reduced transaction level on the goods market, they reduce their labor demand. Thus there is excess supply on the labor market, too, and the economy enters in a state of Keynesian unemployment. What happens next depends on whether the nominal wage is flexible downwards. If not, the real wage and the real money stock increase - as shown in Figure 5.1 - until the economy reaches a state of Classical unemployment, with excess demand on the goods market and excess supply on the labor market. Thereafter the price starts increasing, which determines a reduction of the real wage until the system is back at the Walrasian equilibrium. With the nominal wage rigid downwards the restrictive money shock has had a temporary but not lasting effect on economic activity.

The picture changes when downward wage flexibility is allowed. If the decrease in the wage rate is larger than the decrease in the goods price, the real wage decreases, and it may continue to decrease permanently approaching a limit level below the Walrasian real wage. The lower real wage diminishes labor income of workers which diminishes aggregate goods demand which in turn keeps employment below full employment. The dynamical system converges to a quasi-stationary Keynesian state with permanent deflation of all nominal variables but constant real magnitudes.[9] The nominal money stock shrinks because, due to the falling government spending in nominal terms, the government is permanently realizing a budget surplus: $\Delta M = p_t G - tax\Pi_{t-1} < 0$. These facts are illustrated in Figure 5.2 which shows time series for $\nu_1 = 0.025$. The restrictive monetary shock has caused a permanent decrease in employment and output.

Inventories are important here as their presence amplifies the fall of labor demand by firms, further depressing real labor income and aggregate demand. In fact, when aggregate demand is diminished due to a decrease in m_0, inventories become positive and rise further as excess supply on the goods market builds up. As $\gamma^s = \overline{Y}_t / Y^s\left(\lambda_t^d, \gamma_t^s; \alpha_t, S_t\right)$ by (2) of Definition 3.1 and S_t influences Y^s

[9] A state is *stationary* if all variables are constant; it is *quasi-stationary* if all real variables are constant but the nominal variables may change.

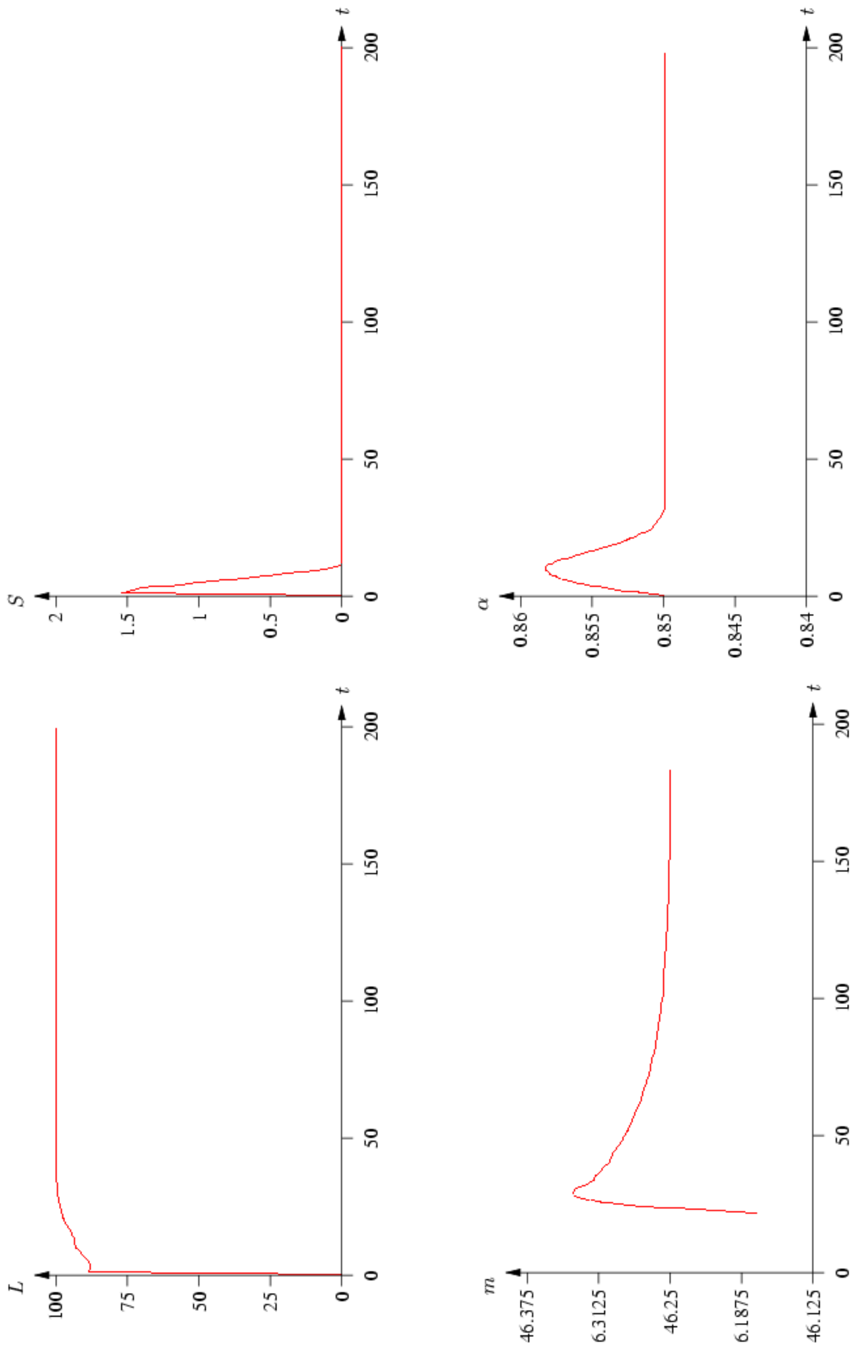

Figure 5.1: Time series when $\nu_1 = 0$ and $m_0 = 40$.

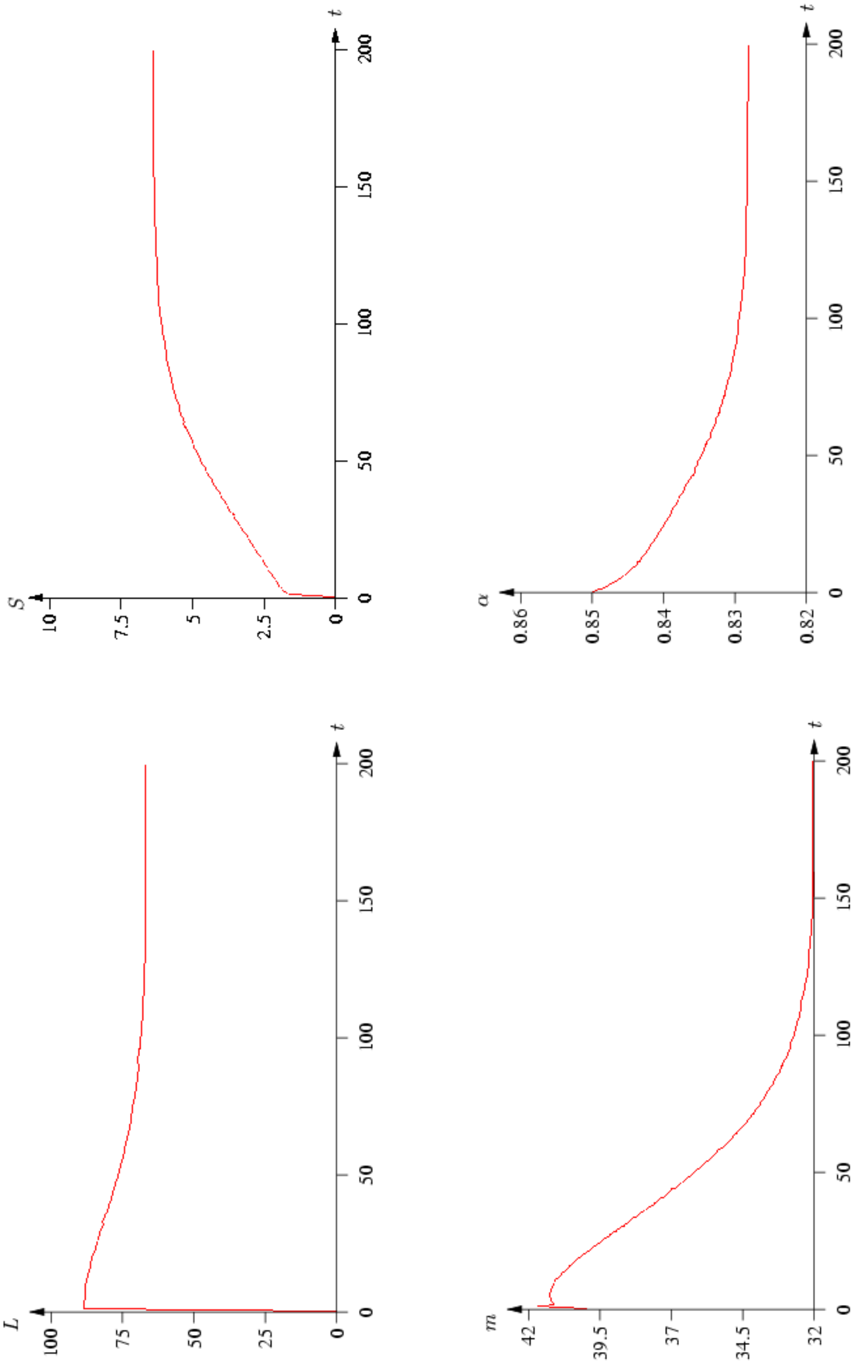

Figure 5.2: Time series when $\nu_1 = 0.025$ and $m_0 = 40$.

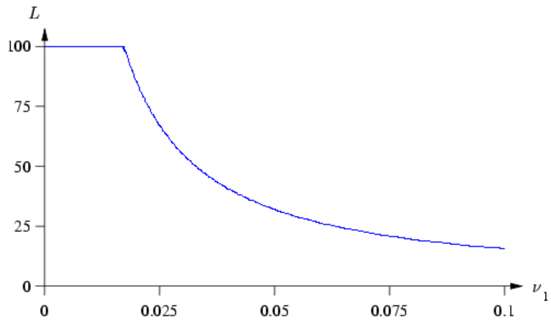

Figure 5.3: Stationary employment values when $m_0 = 40$.

positively by (2.3), an increase in S_t reduces the sales expectation ratio γ^s which by (2.2) diminishes the labor demand of firms and thus increases further the excess supply on the labor market. Therefore the downward flexible wage rate decreases more than would be the case without inventories. Indeed, setting $S_t \equiv 0$ changes the outcome in the scenario of a monetary shock, with the economy returning to the Walrasian equilibrium (see Colombo and Weinrich, 2003b). The real wage decreases initially but then the decrease in the goods price dominates the one in the nominal wage, and the real wage moves back to its Walrasian level, as do all the other variables.

At this point the natural question is which downward wage flexibility is needed to drive the economy into a permanent recession or even depression. The answer is given in the bifurcation diagram of Figure 5.3. From there it can be seen that approximately until $\nu_1 = 0.018$ the economy is capable of returning to the full employment after the monetary shock, whereas for speeds of wage adjustment larger than this the economy gets trapped in underemployment.

The fact that a restrictive monetary shock may lead to a Keynesian quasi-stationary state as limit of the dynamic system's trajectory raises the question of the stability of such a state. Analogously, the stability of the stationary Walrasian state may be investigated. Numerical simulations suggest that the quasi-stationary Keynesian unemployment state is locally stable, whereas the stationary Walrasian equilibrium is locally unstable. Specifically, reductions in the real money stock or in real profits, and increases in the stock of inventories, destroy the full employment equilibrium and cause the economy to converge to a quasi-stationary Keynesian equilibrium.[10]

6. Policy and the Japanese Deflationary Recession

As recalled in the Introduction, the performance of the Japanese economy in the last decade with prolonged recession, unemployment, overcapacity/excess inventories and falling prices and nominal wages fits into our scenario of a quasi-stationary state with Keynesian unemployment. Thus we are challenged to apply the insights from our theoretical model to the Japanese case.

The reasons why Japan has been in trouble for so long (and in some respects still is) are not unanimously shared by economists.[11] On the one hand it is argued that Japan's deflation has been largely structural and that the money-transmission system was not working because banks, saddled with bad loans, were not able to lend more than they actually did. So the priority was to fix the banking system. On the other hand, a standard Keynesian argument is that, when an economy is in a liquidity trap, a fiscal stimulus can boost demand. Japan's public debt appears, however, to be too big already and thus to finance a fiscal stimulus in a conventional way seems not possible. An alternative approach has been suggested by Ben Bernanke, namely, that the government enact tax cuts and the Bank of Japan finance them directly, paying for the forgone tax revenue to the government, so that the debt burden does not change.[12]

In the framework of our model we can emulate Bernanke's proposal by reducing the tax rate from $tax^* = 0.5$ to a new value tax so that the income of (young) consumers out of profit after taxation is

$$(1 - tax)\pi = (1 - tax^*)\pi + \Delta m,$$

[10] See Colombo and Weinrich (2003b) for a numerical analysis and discussion of the point.

[11] The strenght of the actual recovery is still to be fully assessed, being so much exposed to esternal shocks and economic conditions elsewhere, notably in China and the US. Furthermore, deflationary pressures have not been fully eliminated yet.

[12] In this way the "Bank of Japan would mitigate the usual concerns about rising debt: debt purchases by the central bank rather than the private sector implies no net increase in debt service and hence no future tax increases. Consumers should then be more willing to spend rather than save any tax cut. It also gets around the Bank of Japan's concern about the blocked money-transmission mechanism: a joint monetary and fiscal boost will increase spending regardless of the health of banks" (*The Economist*, June 21st 2003, p. 74).

with

$$\Delta m = (tax^* - tax)\pi.$$

Moreover, if the central bank pays for the reduction in taxes paid by consumers, the government's tax income is (as before)

$$tax \cdot \pi + \Delta m = tax^* \cdot \pi.$$

The government's budget deficit in real terms can then be written

$$G^* - tax^* \cdot \pi = G^* - tax \cdot \pi - \Delta m = (G^* - \Delta m) - tax \cdot \pi.$$

This is equivalent to a simultaneous balanced reduction in government spending to $G = G^* - \Delta m$ and in taxes.

As for the dynamic performance of the economy, starting from the quasi-stationary Keynesian unemployment state and setting government spending more precisely to $G = tax \cdot \pi^*$, with π^* the Walrasian value of real profits so that the government's budget is balanced at any Walrasian equilibrium, the result is displayed in Figure 6.1. The figure shows that a reduction in the tax rate monotonicly increases the long-run stationary locally stable value of employment. Moreover, at a value of tax approximately equal to 0.17, stationary Walrasian equilibrium with full employment is reached. Note that the horizontal lines in Figure 6.1 refer to the stationary employment values for values of $tax \leq 0.17$ and $tax = 0.2, 0.3, ..., 1$.[13]

The proposed policy, however, is a standard balanced-budget fiscal policy (in the form of a tax reduction) which, as expected from textbook economics, in the short-run determines an all but welcome reduction of employment below an already low stationary initial level. To avoid this, a simultaneous increase in the money stock m_0 can be used, ensuring that subsequent employment values increase monotonically to full employment.[14] This shows that the combined measure of tax reduction and expansive monetary policy works well in our model economy. A policy of quantitative monetary easing is indeed what is currently implemented

[13] The stationary values corresponding to $tax = 0.5 = tax^*$ are the limit values of the simulation shown in Figure 5.2 and are (approximately)

$$\overline{\alpha} = 0.8281, \quad \overline{m} = 31.9263, \quad \overline{\pi} = 15.7889, \quad \overline{S} = 6.4060,$$

with a stationary employment level $\overline{L} = 66.9342$.

[14] For a detailed analysis of the dynamics see Colombo and Weinrich (2003b).

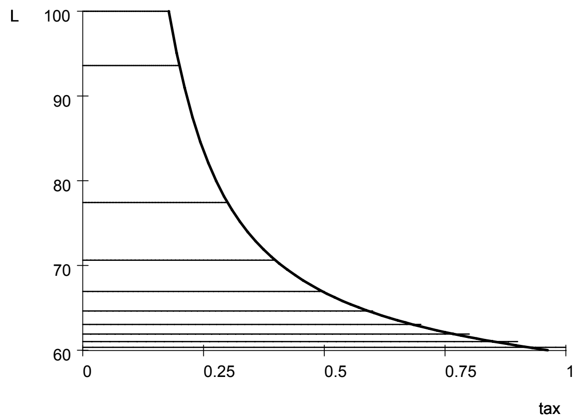

Figure 6.1: Stationary locally stable values of employment depending on different balanced-budget tax quotas.

by the Bank of Japan and likely to be maintained until consumer-price inflation is expected to turn positive again.

Our model economy can also provide a useful analytical setup to evaluate the current debate about the Bank of Japan's best "exit strategy" from its current expansive policy once the economy will be back on a solid track. Many observers believe that the Bank of Japan should have set an inflation target to reassure financial markets that it will not increase interest rates too early — hampering recovery, as it has done in the past — on the one hand, and will not allow inflation to get out of control, on the other hand.[15] Ito and Mishkin (2004), however, propose to set a price-level target instead, arguing it to be more adequate when an economy is suffering deflation for it implies "a compensating period of higher-than-normal inflation", hence having a "bigger effect in reducing real interest rates" and helping "repair balance sheets", as recently stressed by *The Economist* (October 2, 2004). Our setting offers a theoretical framework to think about this issue, as the idea of fixing a price level target is intrinsically paired with the necessity to define and link a sequence of consistent (dis)equilibrium allocations as provided for by the concept of temporary equilibrium with rationing.

[15] See Ito (2004) for a political economy analysis of why inflation targeting has not been adopted.

7. Conclusions

In this paper we have presented a non-tâtonnement dynamic macroeconomic model involving temporary equilibria with fixprices and stochastic rationing in each period, and price adjustment between periods. The model allows for trade also when prices are not at their market clearing levels, and consistent allocations are described in every period, obeying at the same time a well defined dynamics. This approach has enabled us to study, in a general-equilibrium setting, the dynamic functioning of an economy in which disequilibrium phenomena like underemployment, inflation and excess productive capacities are allowed to occur. These disequilibrium situations typically arise because the adjustment of prices to market imbalances is not instantaneous but proceeds with finite speed only; thus their functioning as an allocation device is imperfect, though not nil. As a consequence, quantity adjustments have to take place which complement prices in their task of making trades feasible.

On the other hand, the fact that prices do adjust in our model renders possible to also work out the possible negative effects of too large a price and wage flexibility. If aggregate demand is insufficient, price and wage flexibility together with the possibility of a declinining nominal money stock (due to government surpluses) may lead to a quasi-stationary situation in which there is permanent deflation of nominal variables but all real variables - among which most importantly employment - remain constant. This is so if the decrease in nominal money is proportional to the one in price and wage, because then the real stock of money held by consumers does not change. Thus it is possible that, in addition to the real wage, also the real wealth of households remains constant or, in other words, there is no real-balance effect. Vice versa, if the nominal wage is rigid downwards, then the real wage is eventually bound to increase, and aggregate-demand deficiency cannot persist in the long run. It is worth emphasizing that these results depend crucially on the possibility of modelling the quantity spillover effects between markets in disequilibrium, which in turn is rendered possible using as modelling strategy the non-tâtonnement approach and the adoption of the concept of equilibrium with quantity rationing.

Finally, the recessionary (Keynesian) equilibrium emerging in our economy resembles closely to what we have been witnessing for Japan since the beginning of the Nineties and until very recently, with increasing unemployment rates and decreasing prices and wages. Our framework provides therefore for a valid test bank to check the efficacy of alternative economic policies designed to escape the crisis, and for a conceptual setup to interpret the ongoing debate about the policies that are (or should be) implemented by the Bank of Japan. In the paper, we have focused explicitly on a mix of expansionary fiscal and monetary policies along the lines recently proposed by Ben Bernanke, concluding that they point in the right direction for restoring full employment, provided they are of the appropriate magnitude.

Appendix 1: Lemma 1.

Lemma 1. When $b \leq 1-\sigma$, the solution to the firm's maximization problem is independent of the constraint $\ell_t^d \leq \frac{d_t}{\alpha_t}\left[f\left(\ell_t^d\right)+s_t\right]$.

Proof. The first order condition for an interior solution of the firm's problem is

$$\gamma^s f'(\ell) = \alpha \Leftrightarrow \gamma^s \frac{bf(\ell)}{\ell} = \alpha \Leftrightarrow \ell = \gamma^s \frac{bf(\ell)}{\alpha}.$$

Moreover the inequalities $\frac{1}{b} \geq \frac{1}{1-\sigma} \geq \frac{1-\gamma^s\sigma}{1-\sigma}$ yield $1 \leq \frac{1-\sigma}{b(1-\gamma^s\sigma)}$. From this follows

$$\ell \leq \frac{\gamma^s(1-\sigma)}{1-\gamma^s\sigma}\frac{1}{\gamma^s}\frac{1}{b}\ell = d\frac{1}{\gamma^s}\frac{1}{b}\ell = d\frac{1}{\gamma^s}\frac{1}{b}\gamma^s\frac{bf(\ell)}{\alpha} = \frac{d}{\alpha}f(\ell),$$

which proves our claim. ■

Appendix 2: Proof of Proposition 3.2.

Since we hold $\{\alpha_t, m_t, \pi_t, S_t\}$ and (G, tax) fixed, we omit them whenever possible as arguments in the subsequent functions. Define the set

$$\overline{H} \equiv \left\{\left(\lambda^s L^s, \gamma^d X^d(\lambda^s)\right) \mid \left(\lambda^s, \gamma^d\right) \in [0,1]^2\right\}$$

and its subsets $\overline{H}^K = \overline{H}\mid_{\gamma^d=1,\lambda^s<1}$, $\overline{H}^I = \overline{H}\mid_{\gamma^d<1,\lambda^s=1}$, $\overline{H}^C = \overline{H}\mid_{\gamma^d<1,\lambda^s<1}$ and $\overline{H}^U = \overline{H}\mid_{\gamma^d=1,\lambda^s=1}$. Using the terminology introduced by Honkapohja and Ito (1985), we derive from these the consumption sector's trade curves

$$\overline{H}_0^K = \overline{H}^K + \{(0, m_t+G)\} = \left\{\left(\lambda^s L^s, X^d(\lambda^s)+m_t+G\right) \mid \lambda^s \in [0,1)\right\},$$

$$\overline{H}_0^I = \left\{\left(L^s, \gamma^d X^d(1)+m_t+G\right) \mid \gamma^d \in (0,1)\right\} \cup \{(L^s, \delta m_t+G) \mid \delta \in (0,1]\}$$
$$\cup\{(L^s, \varepsilon G) \mid \varepsilon \in [0,1]\},$$

$$\overline{H}_0^C = \left\{\left(\lambda^s L^s, \gamma^d X^d(\lambda^s)+m_t+G\right) \mid \left(\lambda^s, \gamma^d\right) \in [0,1)\times(0,1)\right\}$$
$$\cup\{(\lambda^s L^s, \delta m_t+G) \mid (\lambda^s, \delta) \in [0,1)\times(0,1]\} \cup \{(\lambda^s L^s, \varepsilon G) \mid (\lambda^s, \varepsilon) \in [0,1)\times[0,1]\}.$$

and

$$\overline{H}_0^U = \overline{H}^U + \{(0, m_t+G)\} = \left\{\left(L^s, X^d(1)+m_t+G\right)\right\}.$$

Similarly, starting from

$$\overline{F} \equiv \left\{\left(\lambda^d L^d(\gamma^s), \gamma^s Y^s\left(\lambda^d, \gamma^s\right)\right) \mid \left(\lambda^d, \gamma^s\right) \in [0,1]^2\right\}$$

we define the production sector's trade curves as $\overline{F}^K = \overline{F}\mid_{\lambda^d=1,\gamma^s<1}$, $\overline{F}^I = \overline{F}\mid_{\lambda^d<1,\gamma^s=1}$, $\overline{F}^C = \overline{F}\mid_{\lambda^d=1,\gamma^s=1}$and $\overline{F}^U = \overline{F}\mid_{\lambda^d<1,\gamma^s<1}$. To derive them, we begin with noticing that

$$\gamma^s Y^s\left(\lambda^d, \gamma^s; \alpha_t, S_t\right) = \frac{\alpha_t}{b}\lambda^d L^d\left(\gamma^s_t; \alpha_t\right) + \gamma^s S_t. \tag{7.1}$$

Indeed, by (2.3)

$$\gamma^s Y^s\left(\lambda^d, \gamma^s; \alpha_t, S_t\right) = \gamma^s\left[\lambda^d n' f\left(\ell^d\left(\gamma^s_t; \alpha_t\right)\right) + S_t\right]$$

whereas from $f(\ell) = a\ell^b$ follows $f'(\ell) = b\frac{f(\ell)}{\ell}$, which implies $f(\ell) = \frac{1}{b}f'(\ell)\,\ell$. Therefore

$$\gamma^s Y^s\left(\lambda^d, \gamma^s; \alpha_t, S_t\right) = \gamma^s\left[\lambda^d n'\frac{1}{b}f'\left(\ell^d\left(\gamma^s_t; \alpha_t\right)\right)\ell^d\left(\gamma^s; \alpha_t\right) + S_t\right].$$

But $\gamma^s f'\left(\ell^d\left(\gamma^s; \alpha_t\right)\right) = \alpha_t$ from any producer's optimizing behavior, and thus

$$\gamma^s Y^s\left(\lambda^d, \gamma^s; \alpha_t, S_t\right) = \frac{\alpha_t}{b}\lambda^d n'\ell^d\left(\gamma^s; \alpha_t\right) + \gamma^s S_t = \frac{\alpha_t}{b}\lambda^d L^d\left(\gamma^s_t; \alpha_t\right) + \gamma^s S_{t.}$$

This implies immediately that

$$\overline{F}^C = \left\{\left(L^d\left(1; \alpha_t\right), \frac{\alpha_t}{b}L^d\left(1; \alpha_t\right) + S_t\right)\right\}.$$

Consider now

$$\overline{F}^K = \left\{\left(L^d\left(\gamma^s; \alpha_t\right), \gamma^s Y^s\left(1, \gamma^s; \alpha_t, S_t\right)\right) \mid \gamma^s \in [0,1)\right\}.$$

Then (7.1) yields

$$\gamma^s Y^s\left(1, \gamma^s; \alpha_t, S_t\right) = \frac{\alpha_t}{b}L^d\left(\gamma^s_t; \alpha_t\right) + \gamma^s S_t.$$

On the other hand, (2.2) implies

$$\gamma^s = \frac{\alpha_t}{ab}\left(\ell^d\left(\gamma^s_t; \alpha_t\right)\right)^{1-b} = \frac{\alpha_t}{ab}\left(\frac{L^d\left(\gamma^s_t; \alpha_t\right)}{n'}\right)^{1-b}$$

and therefore

$$\gamma^s Y^s\left(1, \gamma^s; \alpha_t, S_t\right) = \frac{\alpha_t}{b}L^d\left(\gamma^s_t; \alpha_t\right) + \frac{\alpha_t}{ab}\left(\frac{L^d\left(\gamma^s_t; \alpha_t\right)}{n'}\right)^{1-b} S_{t.}$$

Since $L^d\left(\gamma^s_t; \alpha_t\right)$ is strictly increasing in γ^s_t, this yields

$$\overline{F}^K = \left\{\left(L, \frac{\alpha_t}{b}L + \frac{\alpha_t}{ab}\left(\frac{L}{n'}\right)^{1-b} S_{t.}\right) \mid 0 \le L < L^d\left(1; \alpha_t\right)\right\}. \tag{7.2}$$

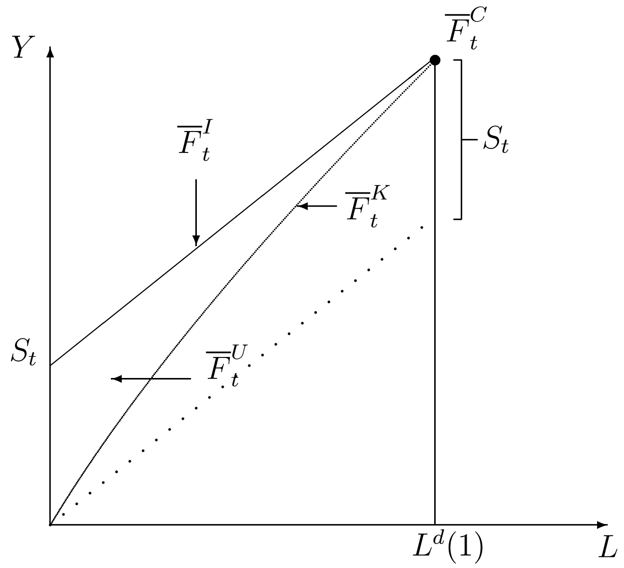

Figure 7.1: The producers' trade curves

Consider next

$$\overline{F}^I = \left\{ \left(\lambda^d L^d (1; \alpha_t), Y^s \left(\lambda^d, 1; \alpha_t, S_t \right) \right) \mid \lambda^d \in [0,1) \right\}.$$

By (7.1) $Y^s \left(\lambda^d, 1; \alpha_t \right) = \frac{\alpha_t}{b} \lambda^d L^d (1; \alpha_t) + S_{t-1}$ and therefore

$$\overline{F}^I = \left\{ \left(L, \frac{\alpha_t}{b} L + S_t \right) \mid 0 \leq L < L^d (1; \alpha_t) \right\}.$$

Since $\frac{\alpha_t}{ab} \left(\frac{L}{n'} \right)^{1-b} = \gamma^s \leq 1$, $\overline{F}^K$ is positioned below $\overline{F}^I$.

Finally consider $\overline{F}^U$. It is given by

$$\overline{F}^U = \left\{ \left(\lambda^d L^d (\gamma^s; \alpha_t), \frac{\alpha_t}{b} \lambda^d L^d (\gamma_t^s; \alpha_t) + \frac{\alpha_t}{ab} \left(\frac{L^d (\gamma_t^s; \alpha_t)}{n'} \right)^{1-b} S_t \right) \mid \left(\lambda^d, \gamma^s \right) \in [0,1)^2 \right\} \tag{7.3}$$

Comparing with $\overline{F}^K$ and $\overline{F}^I$, it is clear that $\overline{F}^U$ is the set of points contained between $\overline{F}^K$ and $\overline{F}^I$. Figure 7.1 illustrates the producers' trade curves.

Using the consumption sector's and the production sector's trade curves and indicating with S^c the closure of the set S, we now note that a pair $\left(\overline{L},\overline{Y}\right) \in R_+^2$ is a temporary equilibrium allocation if and only if it is an element of the set

$$Z = \left(\left(\overline{H}_0^K\right)^c \cap \left(\overline{F}^K\right)^c\right) \cup \left(\left(\overline{H}_0^I\right)^c \cap \left(\overline{F}^I\right)^c\right) \cup \left(\left(\overline{H}_0^C\right)^c \cap \left(\overline{F}^C\right)^c\right) \cup \left(\left(\overline{H}_0^U\right)^c \cap \left(\overline{F}^U\right)^c\right).$$

To show existence of an equilibrium is equivalent to showing that Z is not empty. To this end consider first the locus

$$\left(\overline{H}_0^K\right)^c = \left\{\left(\lambda_t^s L^s, X^d\left(\lambda_t^s\right) + m_t + G\right) \mid \lambda_t^s \in [0,1]\right\}$$

and recall that

$$X^d\left(\lambda_t^s\right) = nh\left(\lambda_t^s \omega_t^1 + \left(1-\lambda_t^s\right)\omega_t^0\right) = h\left(1-tax\right)\pi_t + h\alpha_t \lambda_t^s L^s.$$

Defining the function

$$\Gamma_t\left(L\right) = h\left(1-tax\right)\pi_t + h\alpha_t L + m_t + G,\ \ L \geq 0,$$

we see that $\left(\overline{H}_0^K\right)^c$ is the part of the graph of Γ_t for which $L \leq L^s$.

Next consider again the production sector's trade curves. From (7.2) we conclude that the locus $\left(\overline{F}^K\right)^c$ is the part of the graph of the function

$$\Delta_t\left(L\right) = \frac{\alpha_t}{b}L + \frac{\alpha_t}{ab}\left(\frac{L}{n'}\right)^{1-b} S_t,\ \ L \geq 0,$$

for which $L \leq L^d(1)$. Notice that the graphs of the functions Γ_t and Δ_t always intersect. Indeed, $\Gamma_t'\left(L\right) = h\alpha_t$ and $\Gamma_t\left(0\right) = h\left(1-tax\right)\pi_t + m_t + G > 0$,whereas $\Delta_t'\left(L\right) \geq \frac{\alpha_t}{b} > h\alpha_t$ (since $1/b > 1 > h$) and $\Delta_t\left(0\right) = 0$. Setting $\Delta_t\left(L\right) = \Gamma_t\left(L\right)$ yields (3.2) with the unique solution denoted $\widetilde{L}\left(\alpha_t, \pi_t, m_t, G, tax\right)$. Therefore the equilibrium level on the labor market is

$$\overline{L}_t = \min\left\{\widetilde{L}\left(\alpha_t, \pi_t, m_t, G, tax\right), L^d\left(1, \alpha_t\right), L^s\right\} = \mathcal{L}\left(\alpha_t, \pi_t, m_t, S_t, G, tax\right)$$

whereas the one the goods market is, by definition of the function $Y\left(\cdot\right)$,

$$\overline{Y}_t = \mathcal{Y}\left(\alpha_t, \pi_t, m_t, S_t, G, tax\right).$$

This shows that the equilibrium allocation

$$\left(\overline{L}_t, \overline{Y}_t\right) = \left(\mathcal{L}\left(\alpha_t, \pi_t, m_t, S_t, G, tax\right), \mathcal{Y}\left(\alpha_t, \pi_t, m_t, S_t, G, tax\right)\right)$$

exists and is uniquely defined. ■

Appendix 3: The explicit complete dynamic system

The dynamic system is given by four different subsystems, one for each of the equilibrium types K, I, C and U, and endogenous regime switching. For given (G, tax), any list $(\alpha_t, \pi_t, m_t, S_t)$ gives rise to a uniquely determined equilibrium allocation $(\overline{L}_t, \overline{Y}_t)$ being of one of the above types (or of an intermediate one). More precisely, equation (3.1) allows us to characterize the type of equilibrium defined in Table 1: if $\overline{L}_t = \widetilde{L}(\alpha_t, \pi_t, m_t, S_t, G, tax)$, the resulting equilibrium is of type K or a limiting case of it. If $\overline{L}_t = L^d(1, \alpha_t)$, type C or a limiting case of it occurs. Finally, if $\overline{L}_t = L^s$, an equilibrium of type I or a limiting case results if $\frac{\alpha_t}{b} L^s + S_t \leq h(1 - tax)\pi_t + h\alpha_t L^s + m_t + G$; otherwise the equilibrium is of type U. Regime switching may occur because $(\overline{L}_t, \overline{Y}_t)$ may be of type $T \in \{K, I, C, U\}$ and $(\overline{L}_{t+1}, \overline{Y}_{t+1})$ of type $T' \neq T$.

The above discussion and Proposition 3.2 allow us to determine the expressions of those rationing coefficients which are possibly smaller than one. This is summarized in the following corollary of Proposition 3.2.

Corollary 7.1. *In case K, $\lambda_t^s = \frac{\overline{L}_t}{L^s}$ and $\gamma_t^s = \frac{\alpha_t}{ab}\left(\frac{\overline{L}_t}{n'}\right)^{1-b}$. In case C, $\lambda_t^s = \frac{\overline{L}_t}{L^s}$ and, in case I, $\lambda_t^d = \frac{L^s}{L^d(1,\alpha_t)}$. Moreover, in both these latter cases,*

$$\left(\gamma_t^d, \delta_t, \varepsilon_t\right) = \begin{cases} \left(\frac{\overline{Y}_t - m_t - G}{h(1-tax)\pi_t + h\alpha_t \overline{L}_t}, 1, 1\right) & \text{if } \overline{Y}_t \geq G + m_t \\ \left(0, \frac{\overline{Y}_t - G}{m_t}, 1\right) & \text{if } G + m_t > \overline{Y}_t \geq G \\ \left(0, 0, \frac{\overline{Y}_t}{G}\right) & \text{if } \overline{Y}_t < G \end{cases}$$

Finally, in case U $\gamma_t^s = \frac{1}{S_t}\left(\overline{Y}_t - \frac{\alpha_t}{b}\overline{L}_t\right)$ and $\lambda_t^d = \overline{L}_t / L^d(\gamma_t^s; \alpha_t)$.

Proof. We start with case U. Then, by (7.3) it must be true that

$$\left(\overline{L}_t, \overline{Y}_t\right) = \left(\lambda^d L^d(\gamma^s; \alpha_t), \frac{\alpha_t}{b}\lambda^d L^d(\gamma_t^s; \alpha_t) + \frac{\alpha_t}{ab}\left(\frac{L^d(\gamma_t^s; \alpha_t)}{n'}\right)^{1-b} S_t\right).$$

Moreover by (2.2)

$$L^d(\gamma^s; \alpha_t) = n'\left(\frac{\gamma_t^s ab}{\alpha_t}\right)^{\frac{1}{1-b}}.$$

Therefore

$$\frac{\alpha_t}{b}\lambda^d L^d(\gamma_t^s; \alpha_t) + \frac{\alpha_t}{ab}\left(\frac{L^d(\gamma_t^s; \alpha_t)}{n'}\right)^{1-b} S_t = \overline{Y}_t$$

$\Leftrightarrow$

$$\frac{\alpha_t}{b}\lambda_t^d L^d\left(\gamma_t^s;\alpha_t\right)+\gamma_t^s S_t=\overline{Y}_t$$

Recalling that $\lambda^d L^d\left(\gamma^s;\alpha_t\right)=\overline{L}_t$ and solving for γ_t^s yields the claimed expression.

In all cases, the values of λ_t^s and λ_t^d are immediate by definition. The value of γ_t^s in case K can be obtained using equation (2.2). Finally, γ_t^d, δ_t, ε_t are determined by means of Definition 3.1 and (2.1). ■

We can now give the explicit equations of all subsystems of the dynamical system.

Keynesian unemployment system

Employment level: $\overline{L}_t=\widetilde{L}\left(\alpha_t,\pi_t,m_t,S_t,G,tax\right).$
Output level: $\overline{Y}_t=\frac{\alpha_t}{b}\overline{L}_t+\frac{\alpha_t}{ab}\left(\frac{\overline{L}_t}{n'}\right)^{1-b}S_t.$
Rationing coefficients: $\lambda_t^s=\frac{\overline{L}_t}{L^s}$, $\lambda_t^d=1$, $\gamma_t^s=\frac{\alpha_t}{ab}\left(\frac{\overline{L}_t}{n'}\right)^{1-b}$, $\gamma_t^d=1$, $\delta_t=\varepsilon_t=1.$
Price inflation: $\theta_t=1-\mu_1\left(1-\gamma_t^s\right).$
Real wage adjustment: $\alpha_{t+1}=\frac{1-\nu_1\left(1-\lambda_t^s\right)}{1-\mu_1\left(1-\gamma_t^s\right)}\alpha_t.$
Real profit: $\pi_{t+1}=\frac{1}{\theta_t}\left(\overline{Y}_t-\alpha_t\overline{L}_t\right)=\frac{1-b}{\theta_t(1-hb)}\left[h\left(1-tax\right)\pi_t+m_t+G\right].$
Real money stock: $m_{t+1}=\frac{1}{\theta_t}\left[m_t+G+\left(1-tax\right)\pi_t\right]-\pi_{t+1}.$
Inventories: $S_{t+1}=n'a\left(\frac{ab\gamma_t^s}{\alpha_t}\right)^{\frac{b}{1-b}}+S_t-\overline{Y}_t.$

Repressed inflation system

$\overline{L}_t=L^s.$
$\overline{Y}_t=\frac{\alpha_t}{b}\overline{L}_t+S_t.$
$\lambda_t^s=1$, $\lambda_t^d=\frac{L^s}{L^d(1,\alpha_t)}$; $\gamma_t^s=1.$
If $\overline{Y}_t\geq G+m_t$, then $\gamma_t^d=\frac{\overline{Y}_t-m_t-G}{h(1-tax)\pi_t+h\alpha_t\overline{L}_t}$, $\delta_t=\varepsilon_t=1;$
if $G+m_t>\overline{Y}_t\geq G$, then $\gamma_t^d=0$, $\delta_t=\frac{\overline{Y}_t-G}{m_t}$, $\varepsilon_t=1;$
if $\overline{Y}_t<G$, then $\gamma_t^d=\delta_t=0$, $\varepsilon_t=\frac{\overline{Y}_t}{G}.$
$\theta_t=1+\mu_2\left(1-\frac{\gamma_t^d+\delta_t+\varepsilon_t}{3}\right).$
$\alpha_{t+1}=\frac{1+\nu_2\left(1-\lambda_t^d\right)}{1+\mu_2\left(1-\frac{\gamma_t^d+\delta_t+\varepsilon_t}{3}\right)}\alpha_t.$
$\pi_{t+1}=\frac{1}{\theta_t}\left(\overline{Y}_t-\alpha_t\overline{L}_t\right)=\frac{\alpha_t}{\theta_t}\frac{1-b}{b}L^s.$
$m_{t+1}=\frac{1}{\theta_t}\left[\delta_t m_t+\varepsilon_t G+\left(1-tax\right)\pi_t\right]-\pi_{t+1}.$

$S_{t+1} = \lambda_t^d n' a \left(\frac{ab}{\alpha_t}\right)^{\frac{b}{1-b}} + S_t - \overline{Y}_t.$

Classical Unemployment System

$\overline{L}_t = L^d(1, \alpha_t)\,.$
$\overline{Y}_t = \frac{\alpha_t}{b}\overline{L}_t + S_t.$
$\lambda_t^s = \frac{\overline{L}_t}{L^s}$, $\lambda_t^d = 1$, $\gamma_t^s = 1$;
if $\overline{Y}_t \geq G + m_t$, then $\gamma_t^d = \frac{\overline{Y}_t - m_t - G}{h(1-tax)\pi_t + h\alpha_t \overline{L}_t}$, $\delta_t = \varepsilon_t = 1$;
if $G + m_t > \overline{Y}_t \geq G$, then $\gamma_t^d = 0$, $\delta_t = \frac{\overline{Y}_t - G}{m_t}$, $\varepsilon_t = 1$;
if $\overline{Y}_t < G$, then $\gamma_t^d = \delta_t = 0$, $\varepsilon_t = \frac{\overline{Y}_t}{G}$.
$\theta_t = 1 + \mu_2 \left(1 - \frac{\gamma_t^d + \delta_t + \varepsilon_t}{3}\right).$
$\alpha_{t+1} = \frac{1 - \nu_1(1-\lambda_t^s)}{1 + \mu_2\left(1 - \frac{\gamma_t^d + \delta_t + \varepsilon_t}{3}\right)}\alpha_t$
$\pi_{t+1} = \frac{1}{\theta_t}\left(\overline{Y}_t - \alpha_t \overline{L}_t\right) = \frac{1-b}{\theta_t} n' \left(\frac{\alpha_t}{ab}\right)^{\frac{b}{b-1}} \left(\frac{1}{a}\right)^{\frac{1}{b-1}}.$
$m_{t+1} = \frac{1}{\theta_t}\left[\delta_t m_t + \varepsilon_t G + (1 - tax)\,\pi_t\right] - \pi_{t+1}.$
$S_{t+1} = n' a \left(\frac{ab}{\alpha_t}\right)^{\frac{b}{1-b}} + S_t - \overline{Y}_t.$

Underconsumption

$\overline{L}_t = L^s.$
$\overline{Y}_t = h(1 - tax)\,\pi_t + h\alpha_t L^s + m_t + G.$
$\lambda_t^s = 1$, $\lambda_t^d = \frac{L^s}{L^d(\gamma_t^s, \alpha_t)} = \frac{(ab\gamma_t^s)^{1/(1-b)} L^s}{n' \alpha_t^{1/(1-b)}}$;
$\gamma_t^s = \frac{\alpha_t}{ab}\left(\frac{\overline{L}_t}{n'}\right)^{1-b}$, $\gamma_t^d = 1$, $\delta_t = \varepsilon_t = 1.$
$\theta_t = 1 - \mu_1 (1 - \gamma_t^s)\,.$
$\alpha_{t+1} = \frac{1 + \nu_2\left(1 - \lambda_t^d\right)}{1 - \mu_1(1 - \gamma_t^s)}\alpha_t.$
$\pi_{t+1} = \frac{1}{\theta_t}\left(\overline{Y}_t - \alpha_t \overline{L}_t\right) = \frac{1}{\theta_t}\left[h(1 - tax)\,\pi_t + m_t + G - \alpha_t (1-h) L^s\right].$
$m_{t+1} = \frac{1}{\theta_t}\left[m_t + G + (1 - tax)\,\pi_t\right] - \pi_{t+1}.$
$S_{t+1} = \lambda_t^d n' a \left(\frac{\gamma_t^s ab}{\alpha_t}\right)^{\frac{b}{1-b}} + S_t - \overline{Y}_t.$

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