Abstract

This paper studies optimal tax policy design problem by employing a two-country dynamic general equilibrium model with incomplete asset markets. We investigate the possibility of welfare-improving active, contingent tax policies (tax rates respond to changes in productivity) on consumption, and capital and labor income taxes. Unlike the conventional wisdom regarding stabilization policies, procyclical factor income tax policy is optimal in open economy. Procyclical tax policy generates efficiency gains by correcting market incompleteness. Optimal tax policy under cooperative equilibrium is similar to that under the Nash equilibrium and welfare gains from tax policy coordination is quite small.

- JEL Classification: F4, E6;
- Key Words: optimal tax, procyclical, countercyclical, stabilization, cooperation.

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1 Introduction

Under certain circumstances, fiscal policy can be effectively used for stabilization purposes. An example is monetary union such as the European Union where stabilizing monetary policy is not available for regional shocks. Another case when monetary policy is ineffective is a deflationary economy with zero or negative real interest rate such as Japan in the late 1990s. In order to properly use active fiscal policy rules under such circumstances, it is important to obtain accurate welfare implications of fiscal policies.

This paper studies optimal tax policy design problem using an open economy model with incomplete asset markets. In our model, a stabilization problem exists because of distortionary taxes within each country and incomplete asset markets across countries where sovereign bonds are the only internationally-traded asset. We develop a two-country single-good dynamic stochastic general-equilibrium model to analyze the effects of tax policy (on consumption, and capital and labor income taxes) on the welfare level of each individual country as well as of the world. Each country faces productivity shocks and tax policy is active and contingent in the sense that governments change tax rates in response to the realized productivity in the economy. Governments maintain balanced budget in each period by using lump-sum transfers.

We first use the closed economy setup and analyze welfare effects of contingent tax policy versus fixed (exogenous) tax policy. We derive the optimal level of tax rate adjustment to productivity shocks and calculate the amount of welfare gains from the optimal tax policy. Next, using an open economy model with incomplete asset markets, we calculate optimal tax policies and examine how much they are different from the closed economy case. We analyze welfare effects of domestic tax policies on both domestic and foreign countries and derive the non-cooperative Nash equilibrium. Finally, we investigate the possible gains from tax policy coordination by deriving cooperative equilibrium. If non-cooperative and cooperative equilibria are different, then there is room for welfare improvement via tax policy coordination. These results can provide realistic implications on potential welfare gains from tax policy coordination.

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1 See Feldstein (2002) for the discussion on the positive role of discretionary fiscal policy in this case.

2 Our search for ‘optimal’ tax policy is by assuming a certain parametric family of tax policy rules and optimizing over the parameters of the rule. This exercise is similar to Mendoza and Tesar (1998) in that we consider welfare consequences of ad-hoc changes in taxes. Note that this is different from defining optimal tax policy as the best possible tax rate responses to disturbances, as in Chari et al. (1994).
welfare effects of international policy coordination.

Three main contributions of this paper are as follows. First, we adopt an open-economy framework. The literature on welfare analysis of tax policy has focused on closed-economy. However, these results can dramatically change under open economy because tax policies can have significant effects on other countries through various channels such as the world interest rate and capital flows. Second, we analyze tax policies in a stochastic setup, which has been used extensively for the analysis of monetary policy (e.g., Obstfeld and Rogoff 2002, and Canzoneri, Cumby and Diba 2002). Most papers in the literature have analyzed tax policies in a deterministic setup and focused on the effects of permanent changes in tax policies or tax policy reform. However, certain economic phenomena should be analyzed under the stochastic framework. For example, recent discussion in the European Union about the role of fiscal policies as absorbers of asymmetric shocks is an example due to the stochastic nature of such shocks. Third, in order to capture the nonlinear dynamics of the model which matters for welfare analysis, we solve the model using a second-order accurate solution method. We adopt the second-order perturbation method following Kim, et al. (2004). It is crucial to adopt a second-order method in calculating the level of welfare because the conventional method of linearization, such as the one used in King, Plosser and Rebelo (1988), can produce inaccurate welfare calculation as documented in Kim and Kim (2003).

Our main findings are as follows. In closed economy, optimal tax policy is countercyclical for consumption and capital income taxes, while optimal labor income tax policy is slightly procyclical. However, these results change in the open economy setup; optimal capital and labor income tax policy becomes procyclical. For example, the optimal tax response to a 1% increase in productivity, assuming no tax policies in the foreign country, is to decrease capital income tax rate by 0.8% or to decrease labor income tax rate by 0.7%.

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3Papers with the closed economy setup include Greenwood and Huffman (1991), McGrattan (1994), and Chari et al. (1994). In many cases, tax policies aiming for the stabilization of the economy produce allocation distortions that outweigh the stabilization gains and therefore reduce welfare. Tax policies can be welfare-improving if the economy is already subject to other distortions such as imperfect competition or externalities, e.g., Easley et al. (1993) and Hairault et al. (2001).

4For example, Baxter (1997) and Kollmann (1998) examined the effects of taxes as well as government spending to explain the twin deficits and the U.S. trade balance, respectively.

Optimality of procyclical tax policy is analogous to the procyclical nature of optimal monetary policy when shocks are from the supply side, as shown in Ireland (1996), Obstfeld and Rogoff (2002), and Kim and Henderson (2002). We also compare optimal tax policy under the Nash equilibrium and the cooperative equilibrium. Results show that the non-cooperative and cooperative tax policies are similar in all three taxes and welfare gains from tax policy coordination is quite small.

The remaining of the paper are as follows. Section 2 describes a two-country model of a production economy with capital and labor. We also explain the second-order accurate solution method. Section 3 reports simulation results for welfare implications of optimal tax policy in both closed and open economies. We analyze two versions of the open economy model: small open economy and two-country models. In order to help interpret the welfare results, we examine impulse responses of main macro variables to a positive productivity shock with and without contingent tax policies. Section 4 provides the results of tax policy transmission and coordination. We compare non-cooperative Nash equilibrium and cooperative equilibrium and calculate potential welfare gains from tax policy coordination. Finally, section 5 offers the conclusion of the paper.

2 The Model

The economy consists of two countries which have the identical preference and production technology. There is a single nondurable tradable good serving as the numeraire. Each country consists of a representative household, a representative firm, and a government. Households decide the level of consumption, leisure, investment, and bond holdings subject to budget constraints. Bond holdings and investment are subject to adjustment costs. We assume that the international financial market is incomplete in the sense that agents can trade only state-non-contingent bonds.

The government is described as a sequence of government spending and tax rates on consumption, capital income and labor income. The entire amount of tax revenue, net of fixed government spending, is distributed to households as lump-sum transfers in each period. The transfers can be negative and in this case they operate as lump-sum taxes. The use of lump-sum transfers allows us to avoid potential additional distortions from adjusting other tax rates to balance the budget. The only source of disturbances in the economy is productivity shocks which can be correlated across countries. Foreign variables are denoted by asterisks and their behavior is symmetric.
to the home country when not specified.

2.1 Households and Firms

Households enter the market owning one unit of labor at time $t$ with pre-determined capital and bond holding. The household receives its wage and rental income from firms, and its interest income out of risk-free bonds.

Household in each country maximizes the expected lifetime utility given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t, \text{ where } U_t = \frac{C_t^\theta (1 - L_t)^{1-\theta}}{1 - \sigma},$$

(1)

where $C_t$ is the level of consumption, and $1 - L_t$ the amount of leisure. Households in both countries have the same discount factor $\beta$.

The budget constraint of household is given by:

$$(1 + \tau_c)C_t + I_t + B_t + \frac{\zeta}{2} (B_t)^2 = (1 - \tau_l)w_t L_t + [(1 - \tau_{kt})r_t + \tau_{kt}\delta] K_t + R_{t-1}B_{t-1} + T_t,$$

(2)

where $B_t$ denotes the quantity of international bonds purchased in period $t$ maturing in $t+1$, $R_t$ is the gross interest rate on bonds, $r_t$ is the rental rate, $w_t$ is the wage rate, and $\tau$ represents tax rates ($\tau_c =$ consumption tax rate, $\tau_k =$ capital income tax rate and $\tau_l =$ labor income tax rate). Note that there is a depreciation allowance, $\tau_{kt}\delta K_t$, and bond holdings are subject to quadratic holding costs, $\frac{\zeta}{2} (B_t)^2$. $T_t$ is the lump-sum transfer (tax) to the household which amounts to the budget surplus (deficit).

As in Kim (2003), households accumulate capital according to the following equation:

$$K_{t+1} = \left[ \delta (I_t/\delta)^{1-\phi} + (1 - \delta) K_t^{1-\phi} \right]^{\frac{1}{1-\phi}}.$$

(3)

A zero $\phi$ implies no adjustment costs. A positive $\phi$ implies the presence of adjustment costs and $\phi = 1$ corresponds to a loglinear capital accumulation equation.\(^7\)

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\(^6\)Using the bond holding adjustment costs allows us to avoid the nonstationarity problem in the small open economy model with incomplete markets. See Kim and Kose (2003) for a detailed discussion on this issue.

\(^7\)See Kim (2003) for comparison of this with other specifications of investment adjustment costs.
For firms, the production function follows a Cobb-Douglas form with labor and capital,
\[ Y_t = A_t L_t^\alpha K_t^{1-\alpha}. \] (4)
While labor cannot move across countries, investment in the domestic country can be financed by foreign capital. A No-Ponzi-Game condition is imposed on the household’s borrowing.

Productivity variable \( A_t \) and \( A^*_t \), representing stochastic components of the production functions of the two countries, follow a symmetric vector Markov process:
\[
\begin{bmatrix}
\log(A_t) \\
\log(A^*_t)
\end{bmatrix} = \begin{bmatrix}
\rho & \nu \\
\nu & \rho
\end{bmatrix} \begin{bmatrix}
\log(A_{t-1}) \\
\log(A^*_{t-1})
\end{bmatrix} + \begin{bmatrix}
\varepsilon_t \\
\varepsilon^*_t
\end{bmatrix}. \] (5)

where \( E(\varepsilon_t) = E(\varepsilon^*_t) = 0, E(\varepsilon^2_t) = \sigma^2_\varepsilon, E((\varepsilon^*_t)^2) = \sigma^2_\varepsilon^*, \) and \( \rho(\varepsilon_t, \varepsilon^*_t) = \psi \) for all \( t \). \( \rho \) is the persistence of productivity shocks and \( \nu \) represents the spillover effects. A non-zero \( \psi \) means that the innovations are contemporaneously correlated across countries.

### 2.2 Government

Government income includes tax revenues as well as bond holding adjustment costs and government spending \( G_t \) is assumed to be fixed and unproductive. The government does not issue any debt and balances its budget in each period by rebating all the tax revenue to households. That is, the level of the government transfer satisfies
\[
\tau_{ct} C_t + \tau_{lt} w_t L_t + \tau_{kt}(r_t - \delta) K_t + \frac{\zeta}{2} (B_t)^2 = G_t + T_t \] (6)

Domestic equilibrium is restricted by the optimizing behavior of the household and the firm, and the government policy regarding tax and transfer. The country’s resource constraint is
\[ Y_t + R_{t-1} B_{t-1} = C_t + I_t + G_t + B_t. \] (7)

For the world equilibrium, the model requires bond market-clearing condition that bonds should be in zero net supply:
\[ B_t + B^*_t = 0. \] (8)

The equations describing the equilibrium are listed in the Appendix.

In the benchmark case of exogenous tax policy, the tax rates are fixed at the steady state level (denoted with \( \bar{\tau} \)). Active (contingent) tax policy means
that governments change tax rates according to the observed current-period productivity.\footnote{Another possible form of tax policy is to change tax rate in response to the changes in directly observable data such as output. However, both types of policies give similar results.} That is, tax policies are represented by the parameter \( \eta \) in

\[
\tau_t = \bar{\tau} + \eta A_t
\]  

(9)

where the sign of \( \eta \) indicates whether the tax policies are countercyclical (if positive) or procyclical (if negative).\footnote{This definition of procyclical and countercyclical policy is slightly different from that used in monetary policy literature where cyclicality of policy is determined by the reaction to the output gap or output itself, not productivity as in this paper.} Absolute value of \( \eta \) represents the sensitivity of tax policy (i.e. how much tax rate should be changed to a unit change in productivity).

We measure welfare gains by calculating the change in welfare when the government implements active tax policies to the benchmark economy where both countries face stochastic productivity shocks but tax rates are fixed at the steady state level (\( \eta = 0 \) for all three taxes). Welfare is measured in terms of consumption units, a common measure in business cycle literature as in Lucas (1987). The certainty equivalent consumption is based on the conditional expectation of expected lifetime utility.\footnote{It is important to use conditional mean, instead of unconditional mean, in order to correctly capture the dynamic transitional effects of policy changes. See Kim et al. (2003) for more on this.}

\section*{2.3 Calibration}

As for calibration, we use the conventional parameter values for annual data. We use the annual data because tax rates do not vary much on a quarterly basis. Capital depreciation rate, \( \delta \), is 0.1 per year. Labor share, \( \alpha \), is 0.6 and the consumption share parameter, \( \theta \), is set to match the steady state share of time devoted to market activities, 0.4. The representative agent’s discount factor, \( \beta \), is 0.95 so that the steady state annual real interest rate is equal to 5\%. We set the utility curvature parameter, \( \sigma \), which determines the household’s coefficient of relative risk aversion at 2. The elasticity of bond holding costs, \( \zeta \), is set at \( 10^{-3} \) to allow only minimal effects from holding costs.\footnote{This number is a little bit higher than the one normally used in the literature. This is to improve the accuracy of approximation. As \( \zeta \) decreases, the model becomes more nonstationary and the accuracy of approximation decreases dramatically.} Government spending is fixed at the level that allows balanced budget under the steady state. Finally, we need to decide the parameter
value for $\phi$ in capital adjustment costs. We set it at 0.2 to match the volatility of investment in the data. Most previous studies reported that productivity measures are highly persistent. For volatility of productivity shocks, we follow Backus et al. (1992) and Baxter and Crucini (1995) and assume that $\sigma_\varepsilon = 0.852\%$. We experiment with different values for other productivity parameters ($\rho$, $\nu$, $\Psi$) for simulations.

Measuring aggregate tax rates is a complex and difficult task and there is little consensus on effective tax rate measures. In this paper, we use the aggregate effective tax rates calculated by Mendoza et al. (1994).\textsuperscript{12} They calculate effective tax rates for G-7 countries by dividing actual tax payments by corresponding national accounts. These effective tax rates reflect government policies on tax credits, deductions, and exemptions as well as information on statutory tax rates. These tax rates also reflect the private sector’s behavior on tax payment over time. Moreover, they are consistent with the concept of aggregate tax rates at the national level and with the assumption of representative agents. These estimates, however, can be sensitive to cyclical factors and shocks to tax revenues and bases.

Table 1 reports the properties of tax rates of G-7 countries. Average tax rates are 12%, 31% and 36% for consumption, labor income and capital income tax, respectively. We use these values as steady state tax rates. We also estimate persistence of tax rates assuming an AR(1) structure. Table 1 shows that all tax rates are highly persistent. The average persistence for G-7 countries are 0.84, 0.91 and 0.81 for consumption, labor income and capital incomes taxes, respectively. The standard deviation of the tax rates are 1.4%, 4.4% and 5.7% for consumption, labor income and capital incomes taxes, respectively. Capital income taxes are more volatile than the other two taxes, especially in Japan and UK (9.9% and 9.5%, respectively). Compared to the productivity shocks, tax shocks are as much as or more volatile on average (estimated standard deviation of productivity shocks are around 1% in general for OECD countries). Even though our focus is on the normative side, these numbers indicate that the tax policies that are more than unit elastic to the productivity shocks are within the range of empirical observation.

\textsuperscript{12}Their method is in the same line with Lucas (1990) and Razin and Sadka (1994). A number of papers have used this method to construct data on tax rates. See, for example, Mendoza and Tesar (1998). Another widely-used alternative for tax rate data is aggregate marginal tax rates. See Mendoza et al. (1994) for a detailed explanation and comparison of different computation methods.
2.4 Solution Method

We adopt a second-order accurate solution method to correctly calculate the level of welfare. The accuracy of the conventional linearization method, as in King, Plosser and Rebelo (1988), is widely known to be satisfactory in computing second moments such as variances and correlation coefficients. However, the linearization method can generate inaccurate results in terms of welfare calculations, especially in open-economy models. We follow Kim et al. (2004) and adopt the second-order perturbation method to compute the level of welfare.

3 Welfare Implications of Tax Policy

This section analyzes welfare implications of active, contingent tax policy under both closed and open economies. We derive optimal response of tax rates against productivity shock and measure maximum welfare gains compared to fixed tax policy. We use two types of open economy models. First, we analyze a small open economy model with incomplete markets where the world interest rate is exogenously given and fixed. Next, we analyze the two country setup where the interest rate is endogenously determined by bond market clearing condition. The small open economy model can be considered as an extension of the two country model with infinite number of countries. We use the two country model to analyze the effects of tax policy transmission and coordination in the next section.

3.1 Closed Economy

In the closed economy, active tax policy can be welfare improving because governments should finance fiscal spending (which is positive and exogenously given) by collecting distortionary taxes. That is, the steady state tax rates are positive, which introduce distortions in the static and intertemporal optimal conditions. Therefore, contingent tax policies can improve welfare by reducing distortions in optimal conditions. We first calculate the level of welfare when tax rates are fixed at the steady state level and then measure potential welfare gains when government adopt active tax policy from the benchmark fixed tax case.

\footnote{Kim and Kim (2003) showed that the conventional linearization can be inaccurate as to generate a paradoxical result that the level of welfare under autarky is higher than that of the complete markets economy using a two-country model.}
Table 2 reports optimal $\eta_s$ for each tax with different values of $\rho$ (persistence of productivity shock). First, optimal tax policy is countercyclical for consumption ($5.5 \sim 6.3$) and capital income taxes ($1.8 \sim 3.9$), while it is slightly procyclical (almost acyclical) for labor income tax ($-0.2 \sim -0.4$). Countercyclical tax policy means that governments lower tax rates when the economy is hit by a negative productivity shock. Fluctuations of tax rates according to these optimal policies are within the range of empirical observations in table 1.

Welfare gains from consumption tax policy is the largest of the three, while labor income tax policy brings almost negligible gains. When productivity shock is very persistent ($\rho = 0.95$), maximum welfare gains from active tax policy are 0.14%, 0.03%, and 0.004% of permanent consumption for consumption, capital income and labor income taxes, respectively. These gains decrease as shocks become less persistent. As productivity shocks become less persistent, all three tax policies become more countercyclical ($\eta_s$ increase).

Countercyclical consumption tax policy directly reduces the volatility of consumption (reduces the level of increase in consumption with positive productivity shock) and therefore improves welfare (stabilization gains). Countercyclical capital income tax policy mitigates positive responses of investment, output and therefore consumption when facing positive productivity shock, and eventually reduces volatility of consumption and improves welfare. On the other hand, labor income tax works through substitution effect with consumption. Positive productivity shock increases consumption but at the same time reduces labor input through a substitution effect. Procyclical labor income tax policy (reducing tax rates with positive shock), however, increases labor input and mitigates the substitution effects of labor. This brings stabilization gains to the economy. However, this effect is quite negligible as we can see in quite low level of welfare gains from labor income tax policy in Table 2.

3.2 Open Economy

In the open economy model with bond trading, there is another source of distortions in the economy; incomplete asset markets. Incomplete markets distort intertemporal prices compared to complete markets economy. Therefore, active tax policy can increase welfare in both ways; correcting market

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14 Other parameter values also affect optimal $\eta_s$ but the effects are not significant in most cases.
incompleteness (efficiency gains) and reducing distortions from taxes (stabilization gains). We first analyze the case of a small open economy where the world interest rate exogenously given and fixed.

### 3.2.1 Small Open Economy Model

Table 2 compares optimal tax policies and welfare gains in closed and open economies. First, optimal tax response $\eta$ for capital and labor income taxes become procyclical in the small open economy model. Optimal $\eta_k$ decreases to $-0.4 \sim -1.6$. It becomes more procyclical when shocks become less persistent and the difference of optimal $\eta_k$s between open and closed economies widens. Welfare gains from optimal capital income tax policy is significantly lower than those in the closed economy model ($0.001 \sim 0.004$ vs. $0.007 \sim 0.025$). Optimal $\eta_l$, originally procyclical (negative $\eta$) in the closed economy, further decreases and becomes more procyclical when shocks are persistent ($\eta_l = -7.4$ when $\rho = 0.95$). When shocks are less persistent ($\rho = 0.85$), optimal $\eta_l$ becomes same in both closed and open economies at $-0.2$. Optimal $\eta$ for consumption tax becomes less countercyclical, decreasing to $2.1 \sim 3.7$ (it was $5.5 \sim 6.3$ in the closed economy).

In the open economy, the current account works as buffer stock against productivity shocks and plays a role for consumption smoothing (other than investment channel which exists in the closed economy as well). The level of consumption smoothing achieved in the open economy is larger than that in the closed economy and therefore the role of business cycle stabilizing policies is reduced. In the case of consumption tax where the optimal tax policy is countercyclical in the closed economy, governments—when facing positive shocks—do not have to increase tax rates as much as in the closed economy case to stabilize business cycles. With positive shocks, agents can smooth consumption by accumulating international bonds (lending to foreign countries). Therefore, optimal consumption tax becomes less countercyclical.

Another channel of welfare gains is through mitigating market incompleteness. The results in Table 2 show that optimal tax policy for both factor income taxes becomes procyclical in the small open economy model, implying that procyclical factor income taxes mitigate market incompleteness. Lowering tax rates with positive productivity shocks generates efficiency gains by stimulating agents to produce more in a more productive state and these efficiency gains exceed potential allocation distortions.\(^{15}\)

\(^{15}\)It is interesting that, in a different framework, Yakadina (2002) also finds similar behavior of optimal capital income tax rates in response to technology shocks.
channel is not available in closed economy where extra output should be consumed domestically. To understand the mechanism through which procyclical tax policy improves welfare by correcting market incompleteness, we draw impulse responses to a positive productivity shock of the economy with and without contingent tax policy. We also add impulse responses of the complete markets economy (with tax rates fixed at the same steady state) where shocks are completely shared across countries. If procyclical tax policy moves the impulse responses of the incomplete markets economy towards those of the complete markets economy, we can argue that procyclical tax policy improves welfare through correcting market incompleteness.

Figure 1 is the impulse responses in the case of capital income tax policy ($\rho = 0.9$, and $\eta_k$ is set at 0 and $-0.9$ for fixed and active tax policy cases, respectively). Procyclical capital income tax policy increases investment and labor input, and therefore output and consumption compared to the case of fixed tax policy. These responses become close to those of the complete markets model. The most significant effects are on investment. With procyclical capital income tax policy, investment rises almost 50% more than in the case with fixed tax policy for the first several years. In the economy with incomplete asset markets (without active tax policy), agents facing a high productivity shock do not invest as much as they should were they in the world with complete asset markets. Therefore, the procyclical tax policy shifts the behavior of the incomplete markets economy towards the complete markets equilibrium, resulting in a higher level of welfare. The fact that capital income tax becomes procyclical in open economy indicates that the efficiency gains from improving on market incompleteness (by adopting procyclical tax policy) exceeds stabilization gains from reducing volatility (by adopting countercyclical tax policy).

In Figure 2, we show the case of labor income tax policy ($\rho = 0.9$, and $\eta_l$ is set at 0 and $-0.7$ for fixed and active tax policy cases, respectively). Similar to Figure 1, under procyclical labor income tax policy, domestic residents increase their working hours more than in the fixed tax policy case in response to a positive productivity shock, resulting in higher output and consumption. As is the case with capital income tax, the incomplete asset markets imply that agents facing a high productivity shock do not work as much as they should were they in the world with complete asset markets. Therefore, the procyclical tax policy moves the outcome of the incomplete

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16 It is well known that output, investment and labor input respond more to productivity shocks in the complete markets economy, compared to the incomplete markets economy. See Baxter and Crucini (1995) and Kim et al. (2003) for details.
markets economy towards the complete markets equilibrium.

These results provide interesting implications for optimal monetary policy literature. A number of studies have shown that optimal monetary policy is procyclical with supply shocks (productivity shocks), while the optimal policy is countercyclical with demand shocks. Procyclical interest rate policy improves welfare by reducing distortions from rigidities in the economy, when hit by supply shocks. In this paper, the sources of distortions are different as our model has no nominal or real rigidities and the only distortions are from distortionary taxes and market incompleteness. Even with different sources of distortions, this model produces the same result that optimal capital and labor income tax policy is procyclical with supply shocks.

3.2.2 Two-country model

In the two country world, interest rate is endogenously determined by bond market clearing condition. It is well known that interest rate is a negative function of current world output; when world output increases temporarily, interest rate decreases (see Kim et al., 2003, for example). With positive shocks, agents would accumulate bonds for consumption smoothing purpose. However, increasing demand for bonds increases bond price (lowers interest rate), which lowers the amount of bond trading. Under the benchmark parameter values, endogenous interest rate (in the two country model) reduces the amount of bond trading to the one-third of the level achieved in the case of fixed interest rate (in the small open economy model).

Table 2 shows optimal tax policies derived in the two country model. Since endogenous interest rate reduces the amount of consumption smoothing, optimal tax policy becomes more countercyclical (or less procyclical) than the small open economy. For all cases, optimal tax policy $\eta$ of the two country model increases from that in the small open economy model, but it is still lower than the closed economy model. Since the behavior of the two country model lies between those of the closed and small open economy models, optimal tax policies should be between the two values. For example, optimal $\eta$ for consumption tax is $3.4 \sim 3.9$ which is more countercyclical than the small open economy case ($2.1 \sim 3.7$) but less countercyclical than the closed economy case ($5.5 \sim 6.3$). For consumption tax, lower level of consumption smoothing means that government should apply more countercyclical policy to reduce volatility of consumption compared to the case with fixed interest rate. Optimal $\eta_l$ is identical to the case of small open economy except when shock is very persistent ($\rho = 0.95$) where optimal $\eta_l$ becomes less procyclical than in the case of a small open economy. Optimal
\(\eta_k\) even becomes positive when shocks are persistent. Welfare gains from stabilization (by imposing countercyclical tax policy) dominates efficiency gains from mitigating market incompleteness (by imposing procyclical tax policy).

### 3.3 Sensitivity Analysis

Table 3 shows how optimal \(\eta_s^*\) and maximum welfare gains change when parameter values change. In this table, we use the two country model with the following parameter values: \(\rho\) (shock persistence) = 0.9, \(\zeta\) (bond holding adjustment cost parameter) = 0.001, \(\nu\) (shock spillovers) = 0, \(\Psi\) (contemporaneous cross-country correlation of shocks) = 0.

We first examine the case when bond holding adjustment costs increase (\(\zeta = 0.1\)). With higher adjustment costs, agents do not trade bonds as much as in the benchmark case and the behavior of the economy approaches that of the closed economy. Therefore, optimal \(\eta\) increases (become more countercyclical or less procyclical). Eventually as \(\zeta\) increases, incomplete markets model becomes close to the closed economy model and optimal \(\eta\) would be same as those in the closed economy model.

Next, we experiment by increasing spillovers of productivity shocks across countries (positive \(\nu\)). An increase in \(\nu\) has the same effects as increasing persistence of shocks \(\rho\). Therefore optimal \(\eta_s^*\) when \(\rho = 0.9\) and \(\nu = 0.08\) become almost identical to the optimal \(\eta_s^*\) with \(\rho = 0.95\) and \(\nu = 0.17\). Finally, we experiment by increasing contemporaneous correlation of shocks \(\Psi = 0.5\). An increase in \(\Psi\) has similar effects as increasing shock persistence. Therefore, optimal \(\eta_s^*\) become similar to those with high shock persistence and welfare gains of optimal tax policy also increase.

### 4 Non-cooperative and cooperative equilibrium

In this section, we relax the assumption of fixed tax rates in foreign country and analyze optimal tax policy when foreign tax rates change. First, we vary the reaction of the foreign country’s tax policy and draw the best response curve of the domestic country and find the non-cooperative Nash equilibrium. Next, we calculate the cooperative equilibrium and analyze welfare gains from tax policy coordination. We set the shock persistence parameter \(\rho\) at 0.85.

\(^{17}\)See Kim et al. (2003) for detailed explanation of the relationship between shock persistence and spillovers in incomplete market models.
Figure 3 shows the welfare gains of active capital income tax policy when foreign tax rate is fixed ($\eta_k^* = 0$). In this case, domestic welfare is maximized when $\eta_k = -0.9$, a decrease in capital income tax rate by 0.9% with a 1% increase in productivity. The maximum welfare gains are quite small at 0.001% of permanent consumption, as shown in Table 4. We observe slightly positive spillover effects of procyclical capital income tax policy in that foreign welfare gains are positive at 0.0006%. We can derive the non-cooperative Nash equilibrium by drawing best response curves of the two countries. For capital income tax, the best response curves are vertical and horizontal lines, meaning that optimal $\eta_k$ does not depend on foreign tax policy. Therefore, the Nash equilibrium is achieved when $\eta_k = \eta_k^* = -0.9$ and the welfare gains are 0.00165% which is higher than the domestic welfare gains when foreign country does not implement any tax policy. This is due to positive spillover effects.

This non-cooperative Nash equilibrium, however, does not maximize the world welfare. We define the cooperative equilibrium as the outcome when both countries use their tax policy to maximize the average of domestic and foreign welfare. For capital income tax, the cooperative equilibrium is achieved when $\eta_k = \eta_k^* = -1.1$, suggesting that the capital income tax policy should be more procyclical (only slightly) than the Nash equilibrium for the maximization of world welfare. The welfare gains at the cooperative equilibrium are 0.0017%. We measure the welfare gains from cooperation by taking the difference of welfare level between the Nash solution and the cooperative solution. In the case of capital income tax, the gains from cooperation is 0.00005% of permanent consumption.

Figure 4 plots the welfare gains of the two countries when the domestic government changes $\eta_l$ holding $\eta_l^*$ constant at zero. The maximum welfare gains are quite small at 0.0004% of permanent consumption, and it is achieved when $\eta_l = -0.2$, interpreted as a decrease in labor income tax rate by 0.2% with a 1% positive productivity shock. In this case, the procyclical labor income tax policy (negative $\eta_l$) produces negative spillover effects and decreases the level of foreign welfare. We can derive the Nash equilibrium by using the best response functions. The optimal tax policy stays unchanged when foreign tax policy moves from the ‘no response’ case, and the two reaction functions meet at $\eta_l = \eta_l^* = -0.2$. Because of the negative spillover, the welfare increases only by 0.00004% in the Nash equilibrium.

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\[18\] In our cooperative solution, each country’s tax rates respond to its own productivity shocks. It would of course create more welfare gains if tax rates respond to both countries’ productivity shocks.
The cooperative equilibrium is achieved when the two countries implement less procyclical policy at $\eta_l = \eta_l^* = -0.1$, and the size of welfare gain is 0.0002%. Welfare gains from cooperation is 0.00016%, which is larger than the case of capital income tax but still small in the absolute term. Figure 5 shows the welfare gains of consumption tax policy. With no foreign tax policy ($\eta_c^* = 0$), optimal $\eta_c$ is at 3.4 with welfare gains of 0.015%. The Nash equilibrium is at $\eta_c = \eta_c^* = 3.7$ with welfare gains of 0.0192%. Cooperative equilibrium is at $\eta_c = \eta_c^* = 4.0$ and the welfare gains from cooperation is 0.0001%.

In sum, optimal tax policy under the Nash equilibrium and the cooperative equilibrium are quite similar in all three taxes. The cooperative equilibrium brings positive welfare gains compared to the Nash equilibrium but the absolute amount of welfare gains are quite small.

5 Conclusion

The conventional idea in the literature is that optimal tax policy is countercyclical rather than procyclical. We have shown that this proposition is true only in the closed economy setup where countercyclical tax policy brings stabilization gains. In the open economy model where agents can trade international bonds for consumption smoothing purpose, optimal factor income tax policies become procyclical. Procyclical tax policy stimulates agents to produce more in a more productive state and agents can take advantage of this extra output through international lending and borrowing. In other words, procyclical tax policy moves responses of the incomplete markets economy (to productivity shock) close to those of the complete markets economy, thereby improving welfare. For capital and labor income taxes, these efficiency gains from procyclical tax policy exceed stabilization gains from countercyclical tax policy, which makes optimal tax policy procyclical. We also show that optimal tax policies under the Nash equilibrium and the cooperative equilibrium are similar and welfare gains from tax policy coordination are quite small.

In general, welfare gains from active tax policies are quite small compared to welfare gains of tax policy reform that changes tax rates permanently, as considered in Mendoza and Tesar (1998, 2001). This is because the tax policies considered in this paper are fine-tuning in that tax rates can only respond to business cycles (changes in productivity) in the economy. However, it is less difficult to implement such policies compared to the permanent changes in tax rates. Moreover, active tax policies can play an
important role in stabilizing an economy where monetary policy cannot be used for the stabilization purpose, such as in the member countries of the European Union.
A Appendix

A.1 The first-order conditions

The domestic economy is described by the following 12 equations together with equations for productivity shocks and tax processes:

\[
0 = (1 - \sigma)U_t - \left[C_t^\phi (1 - L_t)^{1-\theta}\right]^{1-\sigma},
\]

\[
0 = Y_t - A_t L_t^\alpha K_t^{1-\alpha},
\]

\[
0 = \lambda_t C_t (1 + \tau_{ct}) - \theta(1 - \sigma)U_t,
\]

\[
0 = (1 - \tau_{lt})\lambda_tw_t(1 - L_t) - (1 - \theta)(1 - \sigma)U_t,
\]

\[
0 = K_{t+1} - \left[\delta (I_t/\delta)^{1-\phi} + (1 - \delta)K_t^{1-\phi}\right]^{1/\phi},
\]

\[
0 = \beta R_t E_t (\lambda_{t+1}) - \lambda_t (1 + \zeta B_t),
\]

\[
0 = G_t + T_t - \tau_{ct} C_t - \tau_{lt} w_t L_t - \tau_{kt} (r_t - \delta)K_t - \frac{\zeta}{2} (B_t)^2,
\]

\[
0 = Y_t + R_{t-1} B_{t-1} - C_t - I_t - G_t - B_t,
\]

\[
0 = w_t L_t - \alpha Y_t,
\]

\[
0 = r_t K_t - (1 - \alpha) Y_t,
\]

\[
0 = \lambda_t - \mu_t \left[\delta (I_t/\delta)^{1-\phi} + (1 - \delta)K_t^{1-\phi}\right]^{1/\phi} \left(\frac{I_t}{\delta}\right)^{-\phi},
\]

\[
0 = \mu_t - \beta E_t \left[+\lambda_{t+1} (I_{t+1}/\delta)^{\phi} (K_{t+1})^{-\phi} + \lambda_{t+1} (1 - \tau_{k,t+1}) + \delta \tau_{k,t+1}\right],
\]

where \(\lambda_t\) and \(\mu_t\) are Lagrangian multipliers for the budget constraint and capital accumulation equation, respectively. There are foreign country analogues to the above equations. The world equilibrium is achieved by imposing the world resource constraint.
References


Table 1. Properties of estimated tax rates

<table>
<thead>
<tr>
<th></th>
<th>C-tax</th>
<th>L-tax</th>
<th>K-tax</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>0.12</td>
<td>0.25</td>
<td>0.43</td>
</tr>
<tr>
<td>France</td>
<td>0.20</td>
<td>0.42</td>
<td>0.24</td>
</tr>
<tr>
<td>Germany</td>
<td>0.16</td>
<td>0.38</td>
<td>0.27</td>
</tr>
<tr>
<td>Italy</td>
<td>0.13</td>
<td>0.41</td>
<td>0.29</td>
</tr>
<tr>
<td>Japan</td>
<td>0.05</td>
<td>0.22</td>
<td>0.35</td>
</tr>
<tr>
<td>UK</td>
<td>0.15</td>
<td>0.25</td>
<td>0.55</td>
</tr>
<tr>
<td>US</td>
<td>0.06</td>
<td>0.26</td>
<td>0.42</td>
</tr>
<tr>
<td>average</td>
<td>0.12</td>
<td>0.31</td>
<td>0.36</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>C-tax</th>
<th>L-tax</th>
<th>K-tax</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>0.76</td>
<td>0.92</td>
<td>0.87</td>
</tr>
<tr>
<td>France</td>
<td>0.96</td>
<td>0.98</td>
<td>0.86</td>
</tr>
<tr>
<td>Germany</td>
<td>0.62</td>
<td>0.89</td>
<td>0.85</td>
</tr>
<tr>
<td>Italy</td>
<td>0.90</td>
<td>0.95</td>
<td>0.79</td>
</tr>
<tr>
<td>Japan</td>
<td>0.92</td>
<td>0.97</td>
<td>0.94</td>
</tr>
<tr>
<td>UK</td>
<td>0.88</td>
<td>0.77</td>
<td>0.73</td>
</tr>
<tr>
<td>US</td>
<td>0.81</td>
<td>0.89</td>
<td>0.63</td>
</tr>
<tr>
<td>average</td>
<td>0.84</td>
<td>0.91</td>
<td>0.81</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>C-tax</th>
<th>L-tax</th>
<th>K-tax</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>0.012</td>
<td>0.052</td>
<td>0.050</td>
</tr>
<tr>
<td>France</td>
<td>0.026</td>
<td>0.062</td>
<td>0.038</td>
</tr>
<tr>
<td>Germany</td>
<td>0.011</td>
<td>0.045</td>
<td>0.037</td>
</tr>
<tr>
<td>Italy</td>
<td>0.017</td>
<td>0.046</td>
<td>0.050</td>
</tr>
<tr>
<td>Japan</td>
<td>0.006</td>
<td>0.047</td>
<td>0.099</td>
</tr>
<tr>
<td>UK</td>
<td>0.021</td>
<td>0.020</td>
<td>0.095</td>
</tr>
<tr>
<td>US</td>
<td>0.004</td>
<td>0.034</td>
<td>0.033</td>
</tr>
<tr>
<td>average</td>
<td>0.014</td>
<td>0.044</td>
<td>0.057</td>
</tr>
</tbody>
</table>

Note: C-tax: consumption tax rate, L-tax: labor income tax rate, K-tax: capital income tax rate. Persistence is calculated from AR(1) coefficient.
Table 2. Optimal tax policies in closed and open economies

<table>
<thead>
<tr>
<th></th>
<th>$\rho = 0.85$</th>
<th>$\rho = 0.9$</th>
<th>$\rho = 0.95$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>K-tax ($\eta_k$)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Closed</td>
<td>3.9 (0.007)</td>
<td>2.6 (0.012)</td>
<td>1.8 (0.025)</td>
</tr>
<tr>
<td>Two-country</td>
<td>-0.9 (0.001)</td>
<td>0.1 (0.00003)</td>
<td>0.7 (0.004)</td>
</tr>
<tr>
<td>Small open</td>
<td>-1.6 (0.005)</td>
<td>-0.8 (0.002)</td>
<td>-0.4 (0.001)</td>
</tr>
<tr>
<td><strong>L-tax ($\eta_l$)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Closed</td>
<td>-0.2 (0.0005)</td>
<td>-0.3 (0.001)</td>
<td>-0.4 (0.004)</td>
</tr>
<tr>
<td>Two-country</td>
<td>-0.2 (0.0004)</td>
<td>-0.7 (0.005)</td>
<td>-2.0 (0.05)</td>
</tr>
<tr>
<td>Small open</td>
<td>-0.2 (0.0004)</td>
<td>-0.7 (0.004)</td>
<td>-7.4 (0.33)</td>
</tr>
<tr>
<td><strong>C-tax ($\eta_c$)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Closed</td>
<td>6.3 (0.05)</td>
<td>6.1 (0.08)</td>
<td>5.5 (0.14)</td>
</tr>
<tr>
<td>Two-country</td>
<td>3.4 (0.02)</td>
<td>3.6 (0.03)</td>
<td>3.9 (0.18)</td>
</tr>
<tr>
<td>Small open</td>
<td>2.1 (0.004)</td>
<td>3.0 (0.02)</td>
<td>3.7 (0.09)</td>
</tr>
</tbody>
</table>

Note: Small open: Small open economy with fixed world interest rate.
Two-country: Two country model with endogenously determined world interest rate.

Numbers in this table are optimal $\eta_s$. Numbers in the parentheses are percentage welfare gains of active tax policy, over the benchmark case with fixed tax policy. Welfare gains are estimated as percentage changes in certainty equivalent consumption level, while the certainty equivalent consumption is calculated from conditional welfare changes over the benchmark economy.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>K-Tax($\eta_k$)</th>
<th>L-Tax($\eta_l$)</th>
<th>C-Tax($\eta_c$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two-country (benchmark)</td>
<td>0.1 (0.00003)</td>
<td>-0.7 (0.005)</td>
<td>3.6 (0.03)</td>
</tr>
<tr>
<td>Low capital mobility $\zeta = 0.1$</td>
<td>1.5 (0.005)</td>
<td>-0.4 (0.002)</td>
<td>5.5 (0.07)</td>
</tr>
<tr>
<td>Positive spillovers $\nu = 0.08$</td>
<td>0.6 (0.006)</td>
<td>-2.0 (0.02)</td>
<td>4.1 (0.12)</td>
</tr>
<tr>
<td>Correlated shocks $\Psi = 0.5$</td>
<td>0.4 (0.0004)</td>
<td>-0.9 (0.007)</td>
<td>3.9 (0.04)</td>
</tr>
</tbody>
</table>

Note: Numbers for the benchmark open economy are taken from table 2.
Table 4. Welfare effects of tax policy coordination

<table>
<thead>
<tr>
<th>Policy</th>
<th>$(\eta, \eta^*)$</th>
<th>Welfare gains (home, foreign, world)</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-tax</td>
<td>$(-0.9, 0)^1$</td>
<td>$(0.001, 0.0006, 0.0008)$</td>
</tr>
<tr>
<td></td>
<td>$(-0.9, -0.9)^2$</td>
<td>0.00165</td>
</tr>
<tr>
<td></td>
<td>$(-1.1, -1.1)^3$</td>
<td>0.0017</td>
</tr>
<tr>
<td>L-tax</td>
<td>$(-0.2, 0)^1$</td>
<td>$(0.0004, -0.0003, 0.00004)$</td>
</tr>
<tr>
<td></td>
<td>$(-0.2, -0.2)^2$</td>
<td>0.00004</td>
</tr>
<tr>
<td></td>
<td>$(-0.1, -0.1)^3$</td>
<td>0.0002</td>
</tr>
<tr>
<td>C-tax</td>
<td>$(3.4, 0)^1$</td>
<td>$(0.015, -0.0008, 0.0071)$</td>
</tr>
<tr>
<td></td>
<td>$(3.7, 3.7)^2$</td>
<td>0.0192</td>
</tr>
<tr>
<td></td>
<td>$(4.0, 4, 0)^3$</td>
<td>0.0193</td>
</tr>
</tbody>
</table>

1. Domestic tax policy only
2. Non-cooperative Nash equilibrium
3. Cooperative equilibrium

For 2 and 3, home, foreign and world welfare gains are identical due to the symmetry of countries.
Figure 1. Impulse responses to 1% increase in productivity: capital income tax ($\rho=0.9$, $\eta_k=-0.8$)
Figure 2. Impulse responses to 1% increase in productivity: labor income tax ($\rho=0.9$, $\eta_l=-0.7$)
Figure 3. Welfare effects of capital income tax policy at Home ($\eta^*=0$)

- Max(Home) at $\eta=-0.9$

Percentage Welfare Gains vs. Tax rate response $\eta$

Legend:
- Home
- Foreign
- World average
Figure 4. Welfare effects of labor income tax policy at Home ($\eta^*=0$)

- **Max(Home) at $\eta=-0.2$**
Figure 5. Welfare effects of consumption tax policy at Home ($\eta^*=0$)

- **Max(Home) at $\eta=3.4$**

Percentage Welfare Gains

- **Home**
- **Foreign**
- **World average**