Distributional Effects of Monetary Policies
in a New Neoclassical Model with Progressive Income Taxation*

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Abstract:
In our dynamic optimizing sticky price model, agents are heterogenous with regard to their assets and their income. Unanticipated inflation redistributes income and wealth. In order to model the wealth distribution, we study a 60-period OLG model with aggregate uncertainty. A positive technology shock increases the concentration of wealth as measured by the Gini coefficient considerably. In particular, a one percent increase of the technology level results in a one percent increase of the Gini coefficient. An unexpected expansionary monetary policy is found to reduce the inequality of the wealth distribution. In addition, we find that the business cycle dynamics in the OLG model in response to both a technology shock and a monetary shock are different from those in the corresponding representative-agent model.

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1 Introduction

The effect of inflation on the distribution of income and wealth is a matter of ongoing concern for economists. Quite recently, the relationship between inflation and income distribution has received renewed interest. Easterly and Fischer (2001) as well as Romer and Romer (1998) point out in their empirical analysis that inflation hurts the very poor. Galli and van der Hoeven (2001) provide a survey of the empirical literature. Most of the studies find a negative effect of inflation on the equality of the income distribution; however, in many cases, the effect is not significant once some basic control variables are included in the regression analysis. Moreover, there is no evidence on the empirical effects of inflation on the distribution of wealth, basically for the reason that there is hardly any high-frequency micro data for wealth. Given that the correlation of wealth and income is rather low empirically due to the life-cycle effect of savings,\footnote{The reason is simple. In general, the wealth-rich people, e.g. the retired people, are not necessarily the high-income people. Labor earnings peak in the early 50s of one’s life-time (see, e.g. Hansen, 1963). Díaz-Giménez et al. (1997) report a correlation of US earnings (income) and wealth equal to 0.230 (0.321) using data from the 1992 Survey of Consumer Finances.} we are also uncertain with regard to the empirical effects of inflation on the distribution of wealth.

In recent years, due to the progress in the computer technology, there are some, even though very few quantitative studies of the distributional effects of inflation that are based on general equilibrium models. These include studies by Erosa and Ventura (2002) and Heer and Süssmuth (2003).\footnote{Relatedly, Blattarchya (2001) studies the effect of higher inflation on the distribution on income. In his work, inflation increases the external finance premium.} These two studies focus on the effects of a change in the long-run inflation rate and, therefore, their studies amount to a comparative steady-state analysis of the endogenous wealth distributions. In both papers, a rise in the anticipated long-run inflation rate results in a rise of the wealth inequality. Erosa and Ventura (2002) emphasize the inflation’s effect on the composition of the consumption good bundle. Higher inflation results in an increase of the consumption of the credit good at the expense of the consumption of the cash good, and richer agents have lower credit costs. Heer and Süssmuth (2003) model the observation that not all agents have access to the stock market and, therefore, that poorer agents are less likely to hold assets whose real return is not reduced through higher inflation. All these studies, however, refrain from modelling the
short-run effects of inflation.

In order to model the distribution of wealth, we study an Overlapping Generations (OLG) model. The life-cycle motive of savings has been described as a prominent factor in the explanation of the wealth heterogeneity, among others by Huggett (1996). In order to model the short-run effects of monetary policy, we assume that prices are sticky and adjust as in Calvo (1983). Following an unexpected rise of the money growth rate, we observe unexpected inflation. Prices and markups adjust endogenously in our economy. As a consequence, both factor prices and the distribution of income change. In addition, income is taxed progressively in our model. If there is unexpected inflation, the real tax burden of the income-rich agents increases. We also consider the effect of inflation on pensions. In particular, unexpected inflation results in a reduction of real pensions as the government adjusts the pensions for higher-than-average inflation only with a lag. Consequently, following an expansionary monetary policy with an unexpected rise of inflation, the non-interest income distribution becomes more equal, while the effect on the total income distribution depends on the distribution of interest income (and, hence, the distribution of wealth).

The paper is organized as follows. In section 2, we describe the OLG model. The model is calibrated in section 3, where we also present the algorithm for our computation. The results are presented in section 4. We will present the effect of a monetary shock and a productivity shock on the distribution of wealth. Furthermore, we compare the heterogeneous-agent with the corresponding representative-agent economy and make the interesting observation that the business cycle dynamics in these two models are not identical. Section 5 concludes. The appendix describes the corresponding representative-agent Ramsey model and the solution method for the dynamics of the OLG model.

2 The model

The model is based on the stochastic Overlapping Generations (OLG) model with elastic labor supply and aggregate productivity risk, augmented by a government sector and the monetary authority. The model is an extension of Rios-Rull (1996).

Four different sectors are depicted: households, firms, the government, and the monetary authority. Households maximize discounted life-time utility with regard to their intertemp-
poral consumption, capital and money demand, and labor supply. Firms in the production sector are competitive, while firms in the retail sector are monopolistically competitive and set prices in a staggered way a la Calvo (1983). The intermediate good firms produce using labor input and capital. The government taxes income progressively and spends the revenues on government pensions and transfers. Both aggregate productivity and monetary policy are stochastic.

2.1 Households

Households live 60 periods. Each generation is of measure $1/60$. The first 40 periods, they are working, the last 20 periods, they are retired and receive pensions. The $s$-year old household holds real money holdings $M_s^t / P_t$ and capital $k_s^t$ in period $t$. He maximizes expected life-time utility at age 1 in period $t$ with regard to consumption $c_i^t$, labor supply $n_i^t$, and next-period money balances $M_{t+1}^t$, and real capital $k_{t+1}^{i+1}$:

$$E_t \sum_{s=1}^{60} \beta^{s-1} u \left( c_{i+\tau-s}^t, \frac{M_{t+\tau-s}^t}{P_{t+\tau-s}^t}, 1 - n_{i+\tau-s}^t \right),$$

where $\beta$ is a discount factor and expectations are conditioned on the information set of the household at time $t$. Instantaneous utility $u \left( c, \frac{M}{P}, 1 - n \right)$ is assumed to be:

$$u \left( c, \frac{M}{P}, 1 - n \right) = \begin{cases} 
\gamma \ln c + (1 - \gamma) \ln \frac{M}{P} + \eta_0 \frac{(1-n)^{1-\eta}}{1-\eta} & \text{if } \sigma = 1 \\
\left( \frac{c(\frac{M}{P})^{1-\gamma}}{1-\sigma} \right)^{1-\sigma} + \eta_0 \frac{(1-n)^{1-\eta}}{1-\eta} & \text{if } \sigma \neq 1 
\end{cases}$$

where $\sigma > 0$ denotes the coefficient of relative risk aversion.\footnote{We follow Castañeda et al. (2004) in our choice of the functional form for the utility from leisure. In particular, this additive functional form implies a relatively low variability of working hours across individuals that is in good accordance with empirical evidence.} The agent is born without capital $k_1^t = 0$.\footnote{In order to avoid that utility is not defined at age 1, we do not choose $m_1^1 \equiv 0$, but rather endow the household at age 1 with a small constant amount of money, $m_1^1 = \psi$. In our computation, we pick $\psi = 0.001$. However, the choice of $\psi$ does not have any quantitative effect on our results.}

The $s$-year old working agent faces the following nominal budget constraint in period $t$:...

\[ \text{3}\]
\begin{equation}
P_t \left( k_{t+1}^{s+1} - k_i^s \right) + \left( M_{t+1}^{s+1} - M_i^s \right) + P_t c_i^s = P_t r_t k_i^s + P_t w_t \epsilon^s n_i^s + P_t \Omega_t - P_t \tau_t \left( \frac{P_t y_i^s}{P_{t-1} \pi} \right), \\
\quad s = 1, \ldots, 40. \tag{3}
\end{equation}

Individual productivity \( \epsilon^s \) depends on age \( s \). The working agent receives income from effective labor \( \epsilon^s n_i^s \) and capital \( k_i^s \) as well as government transfers \( tr_t \) and profits \( \Omega_t \) which are spent on consumption \( c_i^s \) and next-period capital \( k_{t+1}^{s+1} \) and money \( M_{t+1}^{s+1} \). He pays taxes on his nominal income \( P_t y_i^s \):

\[ P_t y_i^s = P_t r_t k_i^s + P_t w_t \epsilon^s n_i^s. \]

The government adjust the tax income schedule at the beginning of each period for the average rate of inflation in the economy which is equal to the non-stochastic steady state rate \( \pi \). Therefore, nominal income is divided by the price level, \( P_{t-1} \pi \), and the tax schedule \( \tau(\cdot) \) is a time-invariant function of (deflated) income with \( \tau' > 0 \). Notice that when we have unanticipated inflation, \( \pi_t = \frac{P_t}{P_{t-1} \pi} > \pi \), the real tax burden increases and we have cold progression.

The nominal budget constraint of the retired worker is given by

\begin{equation}
P_t \left( k_{t+1}^{s+1} - k_i^s \right) + \left( M_{t+1}^{s+1} - M_i^s \right) + P_t c_i^s = P_t r_t k_i^s + Pen_t + P_t tr_t + P_t \Omega_t - P_t \tau_t \left( \frac{P_t y_i^s}{P_{t-1} \pi} \right), \quad s = 41, \ldots, 59.
\end{equation}

with the capital stock and money balances at the end of the life at age \( s = 60 \) being equal to zero, \( k_i^{61} = m_i^{61} = 0 \), because the household does not leave bequests, and \( m_i^{41} = m_i^{42} = \ldots = m_i^{60} = 0 \). \( Pen_t \) are nominal pensions and are distributed lump-sum. Again, the government adjusts pensions each period for expected inflation according to \( Pen_t = pen \ P_{t-1} \pi \), where \( pen \) is constant through time. If inflation is higher then expected, \( \pi_t > \pi \), the real value of pensions declines.

The real budget constraint of the \( s \)-year old household is given by

\begin{equation}
k_{t+1}^{s+1} + m_{t+1}^{s+1} \pi_{t+1} = \begin{cases} 
(1 + r_t)k_i^s + m_i^s + w_t \epsilon^s n_i^s + tr_t + \Omega_t - \tau_t \left( \frac{y_i^s \pi_t}{\pi} \right) - c_i^s, & s = 1, \ldots, 40, \\
(1 + r_t)k_i^s + m_i^s + \frac{pen \pi}{\pi_t} + tr_t + \Omega_t - \tau_t \left( \frac{y_i^s \pi_t}{\pi} \right) - c_i^s, & s = 41, \ldots, 59.
\end{cases} \tag{4}
\end{equation}
where we define \( m_t \equiv \frac{M_t}{P_t} \).

The necessary conditions for the households with respect to consumption \( c_t^s, s = 1, \ldots, 60 \), next-period capital \( k_{t+1}^{s+1} \), and next-period money \( m_{t+1}^{s+1} \) at age \( s = 1, \ldots, 59 \) in period \( t \) are as follows:

\[
\lambda_t^s = u_c \left( c_t^s, \frac{M_t^s}{P_t}, 1 - n_t^s \right) = \gamma \left( c_t^s \right)^{(1-\sigma)-1} \left( m_t^s \right)^{(1-\gamma)(1-\sigma)-1}, \tag{5}
\]
\[
\lambda_t^s = \beta E_t \left[ \lambda_{t+1}^{s+1} \left( 1 + \tau_{t+1} \left( \frac{y_t^{s+1}}{\pi_t} \right) \frac{\pi_{t+1}}{\pi_t} \right) \right], \tag{6}
\]
\[
\lambda_t^s = \beta E \left[ \frac{\lambda_{t+1}^{s+1}}{\pi_{t+1}} \frac{1 - \tau'}{\tau'} \left( \frac{c_t^{s+1}}{\pi_t} \right)^{(1-\sigma)} \frac{\pi_{t+1}}{\pi_t} \right] \tag{7}
\]

The optimal labor supply of the workers at age \( s = 1, \ldots, 40 \) is given by:

\[
\lambda_t^s w_t e^s \left[ 1 - \tau' \left( \frac{y_t^s}{\pi_t} \right) \frac{\pi_{t+1}}{\pi_t} \right] = u_n \left( c_t, \frac{M_t^s}{P_t}, 1 - n_t^s \right) = \eta_0 (n_t^s)^{-\eta}. \tag{8}
\]

### 2.2 Production

The description of the production sector is similar to Bernanke et al. (1999). A continuum of perfectly competitive firms produce a differentiated intermediate goods. These goods are used by monopolistically competitive firms in order to produce the final good. The final goods producer sets prices according to Calvo (1983).

#### 2.2.1 Final Goods Firms

The firms in the final goods sector produce the final good with a constant returns to scale technology using the intermediate goods \( Y_i(j), j \in [0, 1] \) as an input:

\[
Y_i = \left( \int_0^1 Y_i(j) \frac{d_j}{\tau} \, dj \right)^{\frac{1}{\tau-1}}. \tag{9}
\]
Profit maximization implies the demand functions:

\[ Y_i(j) = \left( \frac{P_i(j)}{P_l} \right), \quad (10) \]

with the zero-profit condition

\[ P_l = \left( \int_0^1 P_l(j)^{1-\epsilon} \, dj \right)^{1/\epsilon}. \quad (11) \]

### 2.2.2 Intermediate Goods Firms

The intermediate good is produced with aggregate capital \( K_l \) and aggregate effective labor \( N_l \) according to:

\[ Y_i(j) = z_l K_l(j)^{\alpha} N_l(j)^{1-\alpha}. \quad (12) \]

The aggregate technology level \( z_l \) follows the AR(1) process:

\[ \ln z_l = \rho_z \ln z_{l-1} + \varepsilon_{zt}, \quad (13) \]

where \( \varepsilon_{zt} \) is i.i.d., \( \varepsilon_{zt} \sim N(0, \sigma_z^2) \).

The firms choose \( K_l \) and \( N_l \) in order to maximize profits. Profit maximization of the intermediate goods’ producers implies:

\[ r_l = \frac{1}{x_l} \alpha z_l K_l^{\alpha-1} N_l^{1-\alpha}, \quad (14) \]

\[ w_l = \frac{1}{x_l} (1 - \alpha) z_l K_l^{\alpha} N_l^{-\alpha}, \quad (15) \]

where \( x_l \) denotes the mark-up of the goods prices over the wholesale price.

### Calvo Price Setting

Let \( \phi \) denote the fraction of producers that keep their prices unchanged. Profit maximization of symmetric firms leads to a condition that can be expressed as a dynamic equation for the aggregate inflation rate:
\[ \hat{\pi}_t = -\kappa \hat{x}_t + \beta E_t \{ \hat{\pi}_{t+1} \} . \]  

(16)

with \( \kappa \equiv (1 - \phi)(1 - \beta \phi)/\phi > 0 \) and \( \hat{\pi}_t \) is the percent deviation of the gross inflation rate from its non-stochastic steady state level \( \bar{\pi} \).

2.3 Monetary Authority

Nominal money grows at the exogenous rate \( \theta \):

\[ \frac{M_{t+1} - M_t}{M_t} = \theta_t. \]  

(17)

The seignorage is transferred lump-sum to the government:

\[ Seign_t = M_{t+1} - M_t. \]  

(18)

The growth rate \( \theta_t \) follows the process:

\[ \hat{\theta}_t = \rho \hat{\theta}_{t-1} + \varepsilon_{\theta_t}, \]  

(19)

where \( \varepsilon_{\theta_t} \) is assumed to be i.i.d., \( \eta_t \sim N(0, \sigma^2_\theta) \).

2.4 Government

Nominal government expenditures consists of pensions \( Pen_t \), and government lump-sum transfers \( P_T R_t \) to households. Government expenditures are financed by an income tax \( Tax_t \) and seignorage:

\[ P_T R_t + Pen_t = Tax_t + Seign_t. \]  

(20)

The income tax structure is chosen to match the current income tax structure in the US most closely. Gouveia and Strauss (1994) have characterized the US effective income tax function in the year 1989 with the following function:

\[ \tau(y) = a_0 \left( y - \left(y^{-a_1} + a_2 \right)^{-\frac{1}{a_1}} \right), \]  

(21)
and estimate the parameters with $a_0 = 0.258$, $a_1 = 0.768$ and $a_2 = 0.031$. We use the same functional form for our benchmark tax schedule. The average nominal income in 1989 amounts to approximately $50,000.\footnote{We follow Castañeda et al. (2004).}

### 2.5 Equilibrium conditions

1. Aggregate and individual behavior are consistent, i.e. the sum of the individual consumption, effective labor supply, wealth, and money is equal to the aggregate level of consumption, effective labor supply, wealth, and money, respectively:

   \[ C_t = \sum_{s=1}^{60} c_t^s, \]

   \[ N_t = \sum_{s=1}^{40} n_t^s e^s, \]

   \[ K_t = \sum_{s=1}^{60} k_t^s, \]

   \[ m_t = \sum_{s=1}^{60} m_t^s. \]

2. Households maximize life-time utility (1).

3. Firms maximize profits.

4. The goods market clears:

   \[ z_t K_t^\alpha N_t^{1-\alpha} = C_t + K_{t+1} + (1 - \delta) K_t. \]

5. The government budget (20) balances.

6. Monetary growth (17) is stochastic and seignORAGE is transferred to the government.

7. Technology is subject to a shock (13).

The non-stochastic steady state and the log-linearization of the model at the non-stochastic steady state are described in more detail in the appendix. In addition, we will compare our OLG model with the corresponding representative-agent model which is also presented in the appendix.
3 Calibration and Computation

The model is calibrated with regard to the characteristics of the US postwar economy. We use standard values for the parameters of the model.

3.1 Preferences

$\beta$ is set equal to 0.99 implying a non-stochastic steady state annual real rate of return equal to $r = 2.71\%$. The relative risk aversion coefficient $\sigma$ is set equal to 2.0. $\eta_0$ is set so that the average labor supply is approximately equal to $1/3$, $\bar{\bar{n}} \approx 1/3$. Furthermore, we choose $\eta = \gamma = 7.0$ which implies a conservative value of 0.3 for the Frisch labor supply elasticity.\footnote{The estimates of the Frisch intertemporal labor supply elasticity $\eta_{n,w}$ implied by microeconometric studies and the implied values of $\gamma$ vary considerably. Macurdy (1981) and Altonji (1986) both use PSID data in order to estimate values of 0.23 and 0.28, respectively, while Killingsworth (1983) finds an US labor supply elasticity equal to $\eta_{n,w} = 0.4$. Domeij and Floden (2001), however, argue that these estimates are biased downward due to the omission of borrowing constraints.} $\gamma$ is chosen so that the (annualized) average velocity of money $PY/M$ is equal to the velocity of M1 during 1960-2001, which is equal to 5.18. For the values $\eta_0 = 0.09$ and $\gamma = 0.9815$, average labor is equal to 0.331 and the velocity of money amounts to 5.19.

3.2 Government

Pensions are constant. We choose a non-stochastic replacement ratio of pensions relative to net wage earnings equal to 30%, $\zeta = \frac{p_{NM}}{1 - \bar{\bar{n}}\bar{n}}$, where $\bar{n}$ and $\bar{n}$ are the average labor supply and the income tax rate of the average income in the non-stochastic steady state of the economy. The calibration of the tax schedule follows Gouveia and Strauss (1994).

3.3 Monetary authority

The inflation rate is set equal to $\bar{\pi} = 1.06$. Money growth follows an AR(1)-process. For the postwar US economy, we estimate $\rho_0 = 0.5$ and $\sigma_\pi^2 = 0.02$ with the help of annual data.
3.4 Production

The production elasticity of capital $\alpha = 0.36$ and the annual depreciation rate $\delta = 0.04$ are taken from Hansen (1985). The annual fraction of producers that do not adjust their prices in any year is set equal to $1 - \phi = 0.75^4 = 0.3164$. Bernanke et al. (1999) use a quarterly value of 0.75. Following empirical evidence presented by Basu and Fernand (1997), we set the average mark-up at the amount of $\bar{\epsilon} = 1.2$ implying a constant elasticity of substitution between any two intermediate goods equal to 6.0. The parameters of the AR(1) for the technology are set equal to $\rho_z = 0.814$ and $\sigma_z = 0.024$. These parameters correspond to annual frequencies by a quarterly AR(1) process for the Solow residual with parameter 0.95 and 0.00763, which are the parameters in Prescott (1986).

3.5 Computation

In order to compute business cycle dynamics of the model, we first need to compute the non-stochastic steady state of the model. Secondly, we log-linearize the model around the non-stochastic steady state. The following algorithm describes the computation of the non-stochastic steady state:

1. Make initial guesses of the aggregate capital stock $K$, aggregate effective labor $N$, transfers $tr$, and aggregate real money $M/P$.

2. Compute the values of $w$ and $r$ that solve the firm’s Euler equations. Compute the pensions.

3. Compute the household’s decision functions.

4. Compute the steady-state distribution of the state variable $\{k^*,m^*\}$.

5. Update $K$, $N$, $tr$, and $M/P$, and return to step 1 until convergence.

Thereafter, we log-linearize the model around the non-stochastic steady. This linear rational expectations model can be solved by, e.g., applying the method of Blanchard and Kahn (1980), (see King, Plosser, and Rebelo, 1988) or of King and Watson (2002). A detailed description of the algorithm for the computation of a decentralized OLG-model is presented in Heer and Mauffner (2004), Chapter 7.2.2.
4 Results

In this section, we present the results on the distribution effects of inflation. We will describe the benchmark model and look at the effects of a productivity shock and a monetary shock on the distribution. In addition, we compare the heterogeneous-agent to the representative-agent case and will find out that the two economies display different behavior, a result that is different from the one in the non-monetary models of Krussell and Smith (1998) and Ríos-Rull (1996).

Our OLG model displays the behavior that is typical for this kind of model. The wealth-age profile is hump-shaped (not illustrated). Furthermore, labor supply attains a maximum at around age 30 because the age-specific productivity is rather low at young ages. Labor supply also attains its maximum prior to the maximum in the hourly wages because older agents have higher wealth and work fewer hours. In addition, consumption is increasing over the life-time as the discount rate is smaller than the interest rate.\footnote{In order to imply a more realistic consumption-age profile, we may have introduced stochastic survival probabilities; in this case, consumption declines at old age. However, our quantitative results are not sensitive to this modelling choice and, therefore, we kept the model as simple as possible.}

In our economy, income and wealth are distributed unequally. The heterogeneity of income, however, is a little lower than observed empirically. In particular, the Gini coefficient of total gross income and total net income amount to 0.34 and 0.33, respectively. For the US economy, Díaz-Giménez et al. (1997) find a value of 0.51 for households aged 36-50, while Henle and Ryscavage (1980) estimate an average US earnings Gini coefficient for men of 0.42 in the period 1958-77.\footnote{Income transfers are excluded in the respective definition of earnings.} The main reason for our low Gini values is the neglect of productivity heterogeneity within generations.\footnote{In future work, we are planning to introduce this kind of heterogeneity. At this moment, however, we do not know of any numerical method that is able to handle the large state-space that results from this more realistic modelling.} As a consequence, the distribution of wealth is also much more equal than observed empirically. In our model, the Gini coefficient of wealth amounts to 0.43, whereas Greenwood (1983), Wolff (1987), Kessler and Wolff (1992), and Díaz-Giménez et al. (1997) estimate Gini coefficients of the wealth distribution for the US economy in the range of 0.72 (single, without dependents, female household head) to 0.81 (nonworking household head).\footnote{Huggett (1996) shows that we are able to replicate the empirically observable heterogeneity of wealth.} In summary, our model is able to replicate a large

\begin{itemize}
  \item In order to imply a more realistic consumption-age profile, we may have introduced stochastic survival probabilities; in this case, consumption declines at old age. However, our quantitative results are not sensitive to this modelling choice and, therefore, we kept the model as simple as possible.
  \item Income transfers are excluded in the respective definition of earnings.
  \item In future work, we are planning to introduce this kind of heterogeneity. At this moment, however, we do not know of any numerical method that is able to handle the large state-space that results from this more realistic modelling.
  \item Huggett (1996) shows that we are able to replicate the empirically observable heterogeneity of wealth.
\end{itemize}
portion, but not the total heterogeneity of the income and wealth distribution that is observed empirically. Given the current state of numerical methods, however, this is the best we can achieve.\footnote{Please see Heer and Mauffner (2004) for a discussion of the current state of computational methods and the curse of dimensionality.}

### 4.1 Productivity shock and distribution

In figure 1, the impulse response functions of aggregate variables to a technology shock $\varepsilon_{z,1} = 1$ are presented. In the first row, the percentage deviations of the variables technology level $z_t$, output $Y_t$, capital $K_t$, and effective employment $N_t$ are graphed, in the second row, we illustrate the percentage deviations of real money $m_t$, gross inflation $\pi_t$, the mark-up $x_t$, and profits $\Omega_t$, while in the third row, you find the responses of the real interest $r_t$, the wage rate $w_t$, and the Gini coefficient of wealth. Following an unexpected increase of the technology level by 1%, output and employment instantaneously increase by 1.04% and 0.07%, respectively. Inflation declines and most of the price adjustment takes place in the first two periods as we consider a rather long period of one year. Both the interest rate and the wage rate increase so that the distribution of total income (including transfer income) is becoming more unequal. Similarly, the rich agents benefit from lower tax rates as the deflation reduces the real tax burden. In addition, the real values of pensions increases.

As the maximum of the wealth holdings is at age 60 and 61 (corresponding to the model periods 40 and 41), this effect also increase the inequality of wealth. As a consequence, inequality increases in the economy and the Gini coefficient of wealth increases by more than one percentage (please see the picture in the third row and the third column of figure 1).

In figure 2, we compare the behavior of our heterogeneous-agent economy with the corresponding representative-agent economy, that is described in more detail in the appendix. Surprisingly, the two economies do not behave alike. In particular, the response of employment to a positive technology shock is even negative in the representative-agent model. In a computable general equilibrium model if we introduce both life-cycle savings and individual earnings heterogeneity. Only the wealth concentration among the very rich agents is not well described in such a model. In order to replicate the wealth distribution of the top quintile, Quadrini (2000) models entrepreneurship endogenously.
This result is in contrast to the findings of Rios-Rull (1996) who studies the business-cycle behavior of an OLG model. He concludes that the OLG model behaves both qualitatively and quantitatively like the corresponding representative-agent Ramsey model. Different from him, we introduce Calvo price staggering. As a consequence, we emphasize that in more complicated general equilibrium models, the consideration of heterogeneity may be important for the study of business-cycle behavior.

4.2 Monetary shock and distribution

In figure 3, the impulse response functions of aggregate variables to a monetary growth shock $\epsilon_{\theta,1} = 1$ are presented. Output increases only after a lag of one period. Employment even declines, which is a bit puzzling. Again, employment behaves differently in the representative-agent economy. The impulse response functions of the aggregate variables in the Ramsey model with Calvo price staggering are graphed in figure 4.

Following a monetary expansion, the distribution of wealth is becoming more equal. An increase of the money growth rate by 1% results in an instantaneous decline of the Gini coefficient by 0.01%. Hence the distribution effect of unexpected inflation is small in our
Figure 2: Effects of a technology shock: representative-agent economy

... economy. The main reason for this effect can again be understood by studying the response of inflation to a monetary expansion. The unexpected jump in the inflation rate reduces the real values of pensions and increases the real tax burden of the rich. As a consequence, the distribution of income and hence wealth becomes more equal.

At this point, it is reasonable to give a fair evaluation of our model and its implications. We have emphasized above that due to computational constraints our model is only able to replicate part of the income and wealth inequality that is observed empirically. In addition, we find that our model displays deficiencies in replicating the response of employment in response to a monetary shock, in contrast to the underlying representative agent model. Furthermore, we do not model all channels through which unexpected inflation may affect the distribution of income and wealth. For example, we neglect housing which is an important asset in the study of the distribution of wealth, of course. Again, at the current state of computational methods and computer technology, we are unable to consider all these effects in one computable general equilibrium model simultaneously.
Figure 3: Effects of a monetary shock

Figure 4: Effects of a monetary shock: representative-agent economy
5 Conclusion

So far, only the long-run distribution effects of monetary policy have only been analyzed in computable general equilibrium models, as e.g. in Erosa and Ventura (2002) or Heer and Süssmuth (2003). The short-run effect of unexpected inflation on the distribution of wealth, which many economists believe to be more important than the change of the long-run inflation rate (as long as higher inflation rates are not subject to higher volatility), have not received any attention yet, to the best of our knowledge. In this paper, we presented a model framework for the analysis of the distribution effects of unanticipated inflation. Our framework can only be regarded as a first step to a fully-fledged analysis of the short-run distribution effects of monetary policy. Nevertheless, our model can serve as a benchmark case for future work that may include a more sophisticated modelling of the idiosyncratic earnings process and may even allow for a third asset besides money and capital, namely housing. In particular, we suggested a framework that replicates the following important channels of monetary policy on the distribution of income and wealth: 1) cold progression, 2) inflation-dependent pensions, and 3) the response of prices, and hence the change in the mark-ups, interest rates, wages and, ultimately, the incomes of the individuals.

Our results can be summarized as follows. A monetary expansion is associated with a rise in inflation and lower income heterogeneity because income is taxed progressively and the tax rates only adjust to the higher inflation rate with a lag. As a consequence, wealth is also distributed more equally, even though the quantitative effect is negligible. For this reason, our results do not give support to the idea that monetary policy should be concerned with its distribution effects. Again, we emphasize that we have neglected housing from our analysis.\textsuperscript{12} In addition, we also find another very important result with relevance for the literature on computable general equilibrium. The business cycle dynamics of the heterogeneous-agent economy with sticky prices differ from those of the corresponding representative-agent economy. As a consequence, the focus of the business cycle literature on the Ramsey model may not be justified for the study of all questions alike. We made this observation only in the case of a monetary economy with sticky prices. In a Walrasian economy, we find that the OLG model and the representative-agent economy behave in a similar way, a result that is in good accordance with those of Rios-Rull (1996).

\textsuperscript{12}The effect will most likely be dominant in countries such as Italy where mortgage payments are flexible and tied to the short-run interest rate.
6 Appendix

6.1 Non-stochastic steady state

In the stationary state state (constant money growth $\bar{\theta}$ and $z_t \equiv 1$), the following equilibrium conditions hold:

1. $\pi = 1 + \bar{\theta}$
2. $\bar{x} = \frac{\bar{\theta}}{\epsilon}$.
3. $r = \frac{1}{\bar{x}} [K^{\alpha-1} N^{1-\alpha} - \delta]$
4. $w = \frac{1}{\bar{x}} (1 - \alpha) K^\alpha N^{-\alpha}$.
5. $\Omega = \left(1 - \frac{1}{\bar{x}}\right) K^\alpha N^{1-\alpha}$.
6. $\text{seign} = \frac{\text{seign}_t}{P_t} = \bar{\theta} m_t$.

6.2 Log-linearization around the non-stochastic steady state

Log-linearization of first-order conditions around the non-stochastic steady state results in:

\[
\tilde{L}^s_t = (\gamma(1 - \sigma) - 1) \tilde{c}_t + (1 - \gamma)(1 - \sigma)\tilde{m}_t, \quad s = 1, \ldots, 60. \tag{22}
\]

\[
\tilde{L}^s_t = E_t \left\{ \tilde{L}^{s+1}_{t+1} + \frac{r}{1 + r(1 - \tau'(y^{s+1}))} \left( (1 - \tau'(y^{s+1})) \hat{r}_{t+1} - \tau''(y^{s+1})y^{s+1} \left( \hat{m}_{t+1} + \hat{n}_{t+1} \right) - \tau'(y^{s+1})\hat{m}_t \right) \right\} \quad s = 1, \ldots, 59,
\]

where $y^s_t = r_t k^s_t + u_t n^s_t e^s$ for $s = 1, \ldots, 40$:

\[
\tilde{y}^s_t = \frac{r_k^s}{y^s} \left( \tilde{r}^s_t + \tilde{k}^s_t \right) + \frac{u_n^s e^s}{y^s} \left( \tilde{w}_t^s + \tilde{n}_t^s \right) \tag{24}
\]
and \( y_t^s = r_t k_t^i \) for \( s = 41, \ldots, 60: 
\[ \bar{y}_t^s = \bar{r}_t + \bar{k}_t^i \] 
(25)

\[
\pi \lambda^s \tilde{x}_t^s + \pi \lambda^s E_t \{ \hat{\pi}_{t+1} \} = \beta \lambda^{s+1} \tilde{x}_{t+1}^{s+1} + \beta (1 - \gamma) \left( e^{s+1} \right)^{\gamma (1-\sigma)} \left( m^{s+1} \right)^{(1-\gamma)(1-\sigma)-1} \cdot 
\left( \gamma (1 - \sigma) c_{t+1}^{s+1} + (1 - \gamma) (1 - \sigma) - 1 \right) m_{t+1}^{s+1}, \quad s = 1, \ldots, 59. 
\] 
(26)

\[
\tilde{\lambda}_t^s + \tilde{w}_t - \frac{1}{1 - \tau'(y^s)} \left( \tau''(y^s) y^s \left( \bar{y}_t^s + \bar{\pi}_t \right) + \tau'(y^s) \bar{\pi}_t \right) = \eta \frac{n^s}{1 - n^s} \hat{\pi}_t^s, \quad s = 1, \ldots, 40. 
\] 
(27)

Furthermore, we need to log-linearize the working household’s budget constraint around the steady state for the one-year old with \( k^1 = 0 \):

\[
k^{2+1} k_{t+1}^{s+1} + m^{2+1} \pi \left( m_{t+1}^{s+1} + \hat{\pi}_{t+1} \right) = w n^t e^t \left( \bar{w}_t + \bar{m}_t^t \right) + tr \bar{r}_t + \Omega \bar{\Omega}_t - \tau'(y^s) y^s \left( \bar{y}_t^s + \bar{\pi}_t \right) - c^s c_{t}^s 
\] 
(28)

and for \( s = 2, \ldots, 40: 
\[
k^{s+1} k_{t+1}^{s+1} + m^{s+1} \pi \left( m_{t+1}^{s+1} + \hat{\pi}_{t+1} \right) = (1 + r) k^s \tilde{k}_t^s + r k^s \tilde{r}_t + m^s \tilde{m}_t^s + w n^s e^s \left( \bar{w}_t + \bar{m}_t^s \right) + tr \bar{r}_t + \Omega \bar{\Omega}_t - \tau'(y^s) y^s \left( \bar{y}_t^s + \bar{\pi}_t \right) - c^s c_{t}^s 
\] 
(29)

Log-linearization of the retired agent’s budget constraint around the non-stochastic steady state results in:

\[
k^{s+1} k_{t+1}^{s+1} + m^{s+1} \pi \left( m_{t+1}^{s+1} + \hat{\pi}_{t+1} \right) = (1 + r) k^s \tilde{k}_t^s + r k^s \tilde{r}_t + m^s \tilde{m}_t^s - pen \hat{\pi}_t + tr \bar{r}_t + \Omega \bar{\Omega}_t - \tau'(y^s) y^s \left( \bar{y}_t^s + \bar{\pi}_t \right) - c^s c_{t}^s 
\] 
(30)

Finally, consumption at age \( s = 60 \) is given by:

\[
\epsilon^{60,j} c_{t}^{60,j} = (1 + r) k^{60,j} \tilde{k}_t^{60,j} + r k^{60,j} \tilde{r}_t + m^{60,j} \tilde{m}_t^{60,j} - pen \hat{\pi}_t + tr \bar{r}_t + \Omega \bar{\Omega}_t - \tau'(y^{60,j}) y^{60} \left( \bar{y}_t^{60,j} + \bar{\pi}_t \right) 
\] 
(31)

\[
+ tr \bar{r}_t + \Omega \bar{\Omega}_t - \tau'(y^{60,j}) y^{60} \left( \bar{y}_t^{60,j} + \bar{\pi}_t \right) 
\] 
(32)
Therefore, we have the following $60 + 60 + 40 + 3 + 3 = 166$ controls: $c^*_{it}$, $s = 1, \ldots, 60$, $n^*_{it}$, $s = 1, \ldots, 40$, $y^*_{it}$, $s = 1, \ldots, 60$, $u_2, r_t, x_t, K_t, N_t, m_t$, $59 + 1 = 60$ costates $\lambda^*_{it}$, $s = 1, \ldots, 59$, $\pi_t$, and $59 + 59 = 118$ predetermined variables $k^*_{it}$, $s = 2, \ldots, 60$, $m^*_{it}$, $s = 2, \ldots, 60$.

We also need to compute the aggregate capital stock, employment, and money balances:

From $K_t = \left( \sum_{s=2}^{\infty} \frac{k^*_{it}}{m^*_{it}} \right)$, we get:

$$
\hat{K}_t = \sum_{s=2}^{\infty} \frac{k^*_{it}}{K} \frac{1}{60} \hat{k}^*_{it}.
$$

Similarly, log-linearization of $N_t = \left( \sum_{s=1}^{40} \frac{n^*_{it} e^*_{it}}{60} \right)$ implies:

$$
\hat{N}_t = \sum_{s=1}^{40} n^*_{it} \frac{1}{N} \frac{1}{60} \hat{n}^*_{it}.
$$

Aggregate money $m_t = \left( \sum_{s=2}^{\infty} \frac{m^*_{it}}{m_{it}} \right)$ can be decomposed as follows:

$$
\hat{m}_t = \sum_{s=2}^{\infty} \frac{m^*_{it}}{m} \frac{1}{60} \hat{m}^*_{it}.
$$

The wage rate is given by the marginal product of labor:

$$
\hat{w}_t = \hat{z}_t - \hat{x}_t + \alpha \hat{K}_t - \alpha \hat{N}_t
$$

Similarly, the real interest rate can be log-linearized as follows:

$$
\hat{r}_t = \hat{z}_t - \hat{x}_t + (1 - \alpha) \hat{N}_t - (1 - \alpha) \hat{K}_t
$$

Calvo Price Stagerring implies:

$$
\hat{\pi}_t = -\kappa \hat{x}_t + \beta E_t \{ \hat{\pi}_{t+1} \}.
$$

Profits $\Omega_t = \left( 1 - \frac{1}{x_t} \right) z_t K_t^\alpha N_t^{1-\alpha}$ are given by:

$$
\hat{\Omega}_t = \frac{1}{x - 1} \hat{x}_t + \hat{z}_t + \alpha \hat{K}_t + (1 - \alpha) \hat{N}_t.
$$

The government budget ?? is given by:

$$
tr \ tr_1 = \sum_{s=1}^{\infty} \hat{\tau} (y^*) y^* (\hat{y}^*_{it} + \hat{\pi}_t) + (\hat{\theta} m) (\hat{\theta} + \hat{m}_t) + pen \hat{\pi}_t
$$
Money growth is defined as follows:

\[
\hat{m}_{t+1} + \hat{m}_{t+1} - \hat{m}_t = \frac{\theta}{1 + \theta} \hat{t}_t
\]

Finally, we have the law of motion for the exogenous state variables \( z_t \):

\[
\hat{z}_t = \rho \hat{z}_{t-1} + \varepsilon_{zt}
\]

and \( \theta_t \):

\[
\hat{\theta}_t = \rho_0 \hat{\theta}_{t-1} + \varepsilon_{\theta t},
\]

\[\text{(37)}\]

\[\text{(38)}\]

6.3 Local stability of the non-stochastic steady state

In order to conduct a local stability analysis, it is convenient to express our system of stochastic difference equations as follows:

\[
C_u u_t = C_s \lambda \begin{pmatrix} s_t \\ \lambda_t \end{pmatrix} + C_z z_t,
\]

\[\text{(39)}\]

\[
D_s \lambda E_t \begin{pmatrix} s_{t+1} \\ \lambda_{t+1} \end{pmatrix} + F_s \lambda \begin{pmatrix} s_t \\ \lambda_t \end{pmatrix} = D_u E_t u_{t+1} + F_u u_t + D_z E_t z_{t+1} + F_z z_t.
\]

\[\text{(40)}\]

Therefore, we define:
\[
\begin{pmatrix}
\hat{c}_1 \\
\hat{c}_2 \\
\vdots \\
\hat{c}_t \\
\hat{c}_{t+1} \\
n_1 \\
\vdots \\
n_t \\
n_{t+1}
\end{pmatrix}
= 
\begin{pmatrix}
\hat{y}_1 \\
\hat{y}_2 \\
\vdots \\
\hat{y}_t \\
\hat{y}_{t+1} \\
\hat{r}_1 \\
\vdots \\
\hat{r}_t \\
\hat{\bar{w}}_1 \\
\hat{\bar{K}}_1 \\
\hat{\bar{N}}_1 \\
\hat{\bar{m}}_1 \\
\hat{\bar{\Omega}}_1
\end{pmatrix}
\begin{pmatrix}
\hat{k}_1 \\
\hat{k}_2 \\
\vdots \\
\hat{k}_t \\
\hat{k}_{t+1} \\
\hat{m}_1 \\
\vdots \\
\hat{m}_t \\
\hat{m}_{t+1} \\
\hat{\pi}_1 \\
\hat{\pi}_2 \\
\vdots \\
\hat{\pi}_{t+1}
\end{pmatrix},
\begin{pmatrix}
\hat{x}_t \\
\lambda_1^t \\
\lambda_2^t \\
\vdots \\
\lambda_{t+1}^t \\
\hat{\pi}_t
\end{pmatrix},
\begin{pmatrix}
\hat{z}_1 \\
\hat{\theta}_1
\end{pmatrix}
\]

The dynamic system has exactly \( ns = 118 \) eigenvalues within the unit circle for both flexible \((\hat{x}_t = 0)\) and sticky prices \((35)\). The equilibrium is saddlepoint stable.

### 6.4 The representative agent model

We compare the behavior of the heterogeneous-agent OLG model with the behavior of the corresponding representative-agent Ramsey model. The representative household maximizes his infinite life-time utility

\[
\sum \beta^t u(c_t, M_t/P_t, 1 - n_t)
\]

subject to

\[
k_{t+1} + m_{t+1} \pi_{t+1} = (1 + r_t(1 - \tau))k_t + m_t + (1 - \tau)w_t n_t + tr_t + \Omega_t - c_t.
\]
Of course, in the representative-agent model we are unable to model progressive taxes and cold progression. In addition, the government does not provide pensions to the households. The rest of the model is unchanged.

For the household, we can derive the first-order conditions:

\[ \lambda_t = u_e \left( c_t, \frac{M_t}{P_t}, 1 - n_t \right) = \gamma \left( c_t \right)^{\gamma(1-\sigma)-1} \left( m_t \right)^{(1-\gamma)(1-\sigma)}, \tag{41} \]

\[ \lambda_t = \beta E_t \left[ \lambda_{t+1} \left( 1 + \tau_{t+1} \left( 1 - \tau \right) \right) \right], \tag{42} \]

\[ \lambda_t = \beta E \left[ \frac{\lambda_{t+1}}{\pi_{t+1}} + \frac{u_{M/P} \left( c_{t+1}, \frac{M_{t+1}}{P_{t+1}}, 1 - n_{t+1} \right)}{\pi_{t+1}} \right] = \beta E \left[ \frac{\lambda_{t+1}}{\pi_{t+1}} + \frac{\left( 1 - \gamma \right) \left( c_{t+1} \right)^{\gamma(1-\sigma)} \left( m_{t+1} \right)^{(1-\gamma)(1-\sigma)-1}}{\pi_{t+1}} \right], \tag{43} \]

\[ \lambda_t w_t (1 - \tau) = u_n \left( c_t, \frac{M_t}{P_t}, 1 - n_t \right) = B \left( n_t \right)^{\eta-1}. \tag{44} \]

In the representative agent model, we assume a constant income tax rate \( \tau(y) = \tau y \) at the amount of \( \tau = 10\% \) which is approximately equal to the average tax in the OLG model. Furthermore, we assume exogenous government spending which amount to 7\% of GDP, \( g = 0.07y \). The remaining parameters are set as follows: \( \beta = 0.96, B = 73.0, \) and \( \gamma = 0.975. \) Notice that in order to compare the representative-agent model with the OLG model, for the period, we choose years rather than quarters (as commonly applied in the business cycle literature).

### 6.4.1 Linearization

The log-linearized representative-agent model is characterized by the following equations:

\[
\bar{\lambda}_t = (\gamma (1 - \sigma) - 1) \bar{c}_t + (1 - \gamma) (1 - \sigma) \bar{m}_t,
\]

\[
\bar{\lambda}_t = E_t \left\{ \bar{\lambda}_{t+1} + \frac{r (1 - \tau)}{1 + r (1 - \tau)} \bar{y}_{t+1} \right\},
\]

\[
\pi \lambda \bar{\lambda}_t + \pi \lambda E_t \{ \bar{m}_{t+1} \} = \beta \lambda \bar{\lambda}_{t+1} + \beta (1 - \gamma) c^{\gamma(1-\sigma)} m^{(1-\gamma)(1-\sigma)-1} \left( (1 - \sigma) \bar{c}_{t+1} + ((1 - \gamma) (1 - \sigma) - 1) \bar{m}_{t+1} \right),
\]

\[\text{See also Heer and Maurer (2004).}\]
\[
\dot{\lambda}_t + \dot{w}_t = (\eta - 1)\dot{n}_t, \\
\dot{k}_{t+1} = \delta \dot{k}_t + (1 - \delta)\dot{k}_t, \\
\dot{y}_t = \dot{z}_t + \alpha \dot{k}_t + (1 - \alpha)\dot{n}_t, \\
\ddot{y}_t = \frac{c}{y} \dot{c}_t + \frac{i}{y} \dot{i}_t, \\
\ddot{w}_t = \dot{z}_t - \dot{x}_t + \alpha \dot{k}_t - \alpha \dot{n}_t, \\
\ddot{r}_t = \dot{z}_t - \dot{x}_t + (1 - \alpha)\dot{n}_t - (1 - \alpha)\dot{k}_t.
\]

Calvo Price Stagerring:
\[
\ddot{\pi}_t = \Theta \dot{x}_t + \beta E_t \{ \ddot{\pi}_{t+1} \}.
\]

Profits:
\[
\ddot{\Omega}_t = \frac{1}{x - 1} \dot{x}_t + \dot{y}_t
\]

The government budget:
\[
tr \ddot{tr}_t = (\theta m) \left( \ddot{\pi}_t + \dot{m}_t \right) + \tau \frac{y}{x} (\dot{y}_t - \dot{x}_t)
\]

Money growth rule:
\[
\dot{m}_{t+1} + \ddot{\pi}_{t+1} - \dot{m}_t = \frac{\theta}{1 + \theta} \dot{\pi}_t
\]
References


