THE ROLE OF CONTRACTING SCHEMES FOR THE WELFARE COSTS OF NOMINAL RIGIDITIES OVER THE BUSINESS CYCLE*

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Abstract

What is the role of contracting schemes for the welfare costs of nominal rigidities over the business cycle? We examine 5 different modeling schemes of nominal rigidities that all have the same average duration of contracts. We find that Calvo (1983) wage and price contracts *may* deliver welfare costs that are 3-4 times higher than Taylor (1980) contracts. However, that result is sensitive to the monetary policy rule. It does not hold generally, that Calvo (1983) contracts yields higher welfare cost than Taylor (1980) or Wolman (1999) contracts. The sticky information scheme of Mankiw and Reis (2002) is shown to deliver welfare costs of nominal rigidities that are smallest of all schemes and typically smaller than 0.07 of period consumption, the Lucas (2003) cost of consumption variability. We discuss the implications of modeling capital mobility. We show further that our key results also hold in the more complex model of Canzoneri, Cumby, and Diba (2004) which has better empirical support.

JEL codes: E52, E32
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1 Introduction

The welfare costs of business cycle fluctuations have been an important topic for macroeconomic research since the seminal paper by Lucas (1987). Lucas (2003) focused on the variability of consumption and has argued that the costs of cyclical fluctuations are very small, roughly 0.07 percent of steady state consumption. Recently, a number of studies have employed new Keynesian monetary models with sticky prices and wages to find that the welfare costs could be much larger. Canzoneri, Cumby, and Diba (2004) show that the costs of nominal inertia may be 20-60 larger than the costs of consumption variability computed by Lucas (2003).

Nominal rigidities can be modeled in many different ways in New Keynesian models. The contribution of this paper is to analyze the role of wage and price contracting schemes for the welfare costs of nominal rigidities.[1] Most monetary models adopt the scheme of Calvo (1983) for introducing nominal stickiness. Despite its popularity, the scheme has several unappealing properties. Collard and Dellas (2005) point out that *"a well-known but unfortunate feature of this scheme is that there exists a non-zero set of firms that almost never adjust their prices"*. Similarly, Bergin and Tchakarov (2003) note that *"under Calvo pricing, some firms are forced to retain prices arbitrarily far away from the optimal price, which can influence the welfare implications in a way viewed by some as unreasonable"*. The main contribution of this paper is to compute the welfare costs of nominal rigidities across several popular modeling devices for stickiness in nominal contracts. In doing so, we address an issue identified by Woodford (2003), who writes that " *to deal with other, possibly more realistic distributions of intervals between price changes, is an important topic for future research*."

This paper is motivated by recent work of Kiley (2002) and Ascari (2004). These authors find that Calvo (1983) contracts imply much larger dispersion of output across firms in the steady state induced by trend inflation and therefore lower steady welfare than Taylor (1980) contracts of equal average duration. An important and natural question is whether the use of the Calvo (1983) scheme has similar implications for quantifying the welfare costs of nominal rigidities over the business cycle rather than in the steady state. Monetary general equilibrium models that feature steady state inflation often assume that agents can index their contracts to the trend inflation rate, thereby completely bypassing the point made in Ascari (2004). It is an open question, to what extent the findings of Kiley (2002) and Ascari (2004) are also relevant for the costs of nominal rigidities over the business cycle in an environment without trend inflation or equivalently with full indexation to the trend inflation rate.

We build on the paper by Erceg, Henderson, and Levin (2000) who analyze welfare in a model with nominal stickiness in wage and price setting. The models of nominal rigidities we consider are the following: Random price adjustment of Calvo (1983), a truncated version of the Calvo (1983) scheme, N period overlapping contracts as in Taylor (1980), a more general scheme with time varying adjustment probabilities as suggested by Wolman (1999) and finally variants of the sticky information scheme of Mankiw and Reis

[1] Roberts (1995) has shown that Taylor (1980) and Calvo (1983) contracts as well as Rotemberg (1982) quadratic adjustment costs give rise to similar Phillips curves. This does not imply that the welfare costs of nominal rigidities must be similar, as the Phillips curves are similar only after linearization, whereas welfare measures take into account some of the nonlinearity of the model.

(2002). All of these schemes imply that the welfare cost of nominal rigidities stem from the dispersion of differentiated goods across producers or of differentiated labour across workers. Thus we consider a homogenous family of modeling devices for nominal rigidities.[2] When comparing these schemes we require an equal expected duration over which newly set contracts or information sets are fixed.[3] In such a way no model implies more exogenous stickiness on average. While this metric is typically assumed in the literature, it is not the only obvious choice. We also report results when choosing the average age in a cross-section of contracts as our metric, taking into account recent criticism by Dixon and Kara (2005) .

The schemes suggested by Taylor (1980), Wolman (1999) all relax at least one unpleasant assumption of the Calvo (1983) contracts. Taylor (1980) contracts truncate the horizon in the expectation formation of price and wage setters, which is infinite in the scheme of Calvo (1983). Wolman (1999) additionally relaxes the assumption that adjustment probabilities are independent of time elapsed since last adjustment. Finally, the Mankiw and Reis (2002) scheme provides a tractable framework for considering limited availability of information, often seen as one possible micro-foundation for sticky nominal contracts.

Our main findings can be summarized as follows. Overall, we find that the welfare costs or nominal rigidities depend very strongly on how these rigidities are modeled. These costs range from 0.01 of one per cent of period consumption to 0.32 of one per cent, depending on the monetary policy rule in place and on the particular contracting scheme chosen. We find that Calvo (1983) contracts *may* imply much higher welfare costs of nominal rigidities than schemes with a narrower support of the distribution of intervals between price or wage changes but equal mean. In the baseline parameterization of the model, Calvo (1983) contracts give rise to welfare costs of nominal rigidities that are more than 3 times as high as for Taylor (1980) contracts. This supports the view that the use of Calvo (1983) contracts may influence welfare calculations in a very special way. However, the finding that Calvo (1983) contracts imply higher welfare costs is not general. Contrary to the results by Kiley (2002) for the steady state effects of inflation, Calvo (1983) contracts *may* also imply similar or even slightly smaller welfare costs over the business cycle than Taylor (1980) contracts. The intuition for this result is as follows. When the age distribution has larger support, there are some agents that charge prices set farther in the past. Their contract price tends to be more out of line with current economic conditions. While that effect typically increases the welfare costs, those agents setting prices today will look farther into the future and also charge a price that reflects less on current economic conditions. Therefore widening the support of the age distribution as done in the Calvo (1983) scheme has an ambiguous impact on relative price dispersion and therefore on welfare.

When restricting the ability to obtain current information rather than directly restricting the ability to reset contracts, we find that the welfare costs of nominal rigidities are typically much smaller. For all considered monetary policy rules and for all considered

[2]An often used scheme missing from this comparison is Rotemberg (1982) quadratic adjustment costs. We do not consider it here, since it does not belong to the family of rigidities that imply dispersion of output across firms or hours across workers. Furthermore, the derivation of welfare based loss function is not as easily done as with the other schemes.

[3]The duration is equal to the inverse of the frequency of contract adjustment.

age distributions of information sets, the welfare costs of nominal rigidities are smaller than the cost of consumption variability computed by Lucas (2003), i.e. less than 0.07 of one per cent of consumption.

In the final section we perform a robustness check that explores the role of an economy wide rental market for capital. The absence of a rental market for capital represents a source of real rigidity identified by Ball and Romer (1990) and Kimball (1995) to be important for the business cycle analysis of monetary models.[4] We show that imposing firm specific capital has two opposite effects on the welfare costs of nominal rigidities that may be largely offsetting. A given amount of relative price dispersion is more costly in welfare terms since there is more dispersion of labour across firms. However, specific capital makes firms more reluctant to changes prices, since marginal cost now depends on the price they charge. Hence, there is less price dispersion in equilibrium. We find that the relative welfare costs of nominal rigidities across the considered contracting schemes do not depend strongly on how we model capital.

The paper proceeds as follows. Section 2 introduces the setup of the model. In section 3, we describe our alternative wage and price setting schemes. Section 4 collects the log-linearized necessary conditions for equilibrium and the model's baseline calibration. Section 5 computes the welfare costs of nominal rigidities for our considered schemes. In section 6 we take up the aforementioned robustness check. Finally, section 7 briefly summarizes the findings and concludes. An appendix contains the sensitivity analysis with respect to key parameters.

2 Model

The model is based on Erceg, Henderson, and Levin (2000). In particular, capital is in fixed supply in the aggregate. In the baseline model, we assume that there exists an economy-wide rental market that allows capital to move freely between firms. This assumption is relaxed in a robustness check. We assume further that subsidies exist that completely offset the effects of monopolistic competition in the steady state in order to avoid involved derivations of the loss function that would arise with an inefficient steady state.[5] Finally, following Erceg, Henderson, and Levin (2000) households can fully insure against the idiosyncracies in income streams arising from nominal wage rigidities. While this may bias the welfare costs of nominal rigidities, it greatly facilitates the analysis.

[4] For an early treatment see Henderson (1973)

[5] Recently Benignio and Woodford (2004) have shown that not assuming the subsidies, the quadratic welfare criterion used here and the *relative* weights attached to the elements in that criterion still apply as long as there are no government purchases. When government purchases are present and not unreasonably large, the relative weights in the welfare function change only slightly and the output gap must be defined differently. However, the *levels* of the weights will increase in the severity of the steady state distortion no matter whether government purchases are present or not. Since government purchases are absent in our model, we do not introduce any bias into the analysis of the relative welfare costs of different contracting schemes by conveniently assuming subsidies, whereas the levels are affected by our assumption.

2.1 Households

There is a continuum of households with unit mass indexed by h. Households are infinitely lived, supply labour $N_t(h)$ and receive nominal wages $W_t(h)$, consume final goods $C_t(h)$, and purchase state contingent securities $B_t(h)$. Furthermore, they receive lump sum transfers T_t as well as profits $\Gamma_t(h)$ from the monopolistic retailers and hold nominal money balances $M_t(h)$. The utility function is assumed to be separable in consumption, real money balances and leisure. The representative household's problems is:

$$\max_{\{B_t,M_{t+1},C_t,N_t\}_{t=0}^{\infty}} E_t \sum_{i=0}^{\infty} \beta^{t+i} \left[\frac{C_t^{1-\sigma}}{1-\sigma} + H\left(\frac{M_{t+i}(h)}{P_{t+i}}\right) - \frac{N_t(h)^{1+\chi}}{1+\chi}\right]$$

$$\text{s.t.} \quad C_t(h) = (1+\tau_w)\frac{W_t(h)}{P_t}N_t(h) + T_t(h) + \Gamma_t(h) - \frac{M_{t+1}(h) - M_t(h)}{P_t} - \frac{\delta_{t+1,t}B_t(h) - B_{t-1}(h)}{P_t}.$$

The function $H(\cdot)$ depicts the utility flow arising from real money balances, often used as a shortcut for a more explicit role for money, such as a cash-in-advance constraint. We assume that the utility from real money balances is arbitrarily small. Following Woodford (2003) we omit the first-order condition for money holdings. Given that the we model monetary policy as an interest rate rule, this equation merely serves to back out the quantity of money that supports a given nominal interest rate. B_t is a row vector of state contingent bonds, where each bond pays one unit in a particular state of nature in the subsequent period. The column vector $\delta_{t+1,t}$ represents the price of these bonds. The role of the bonds is to allow perfect consumption insurance against labour income risk stemming from nominal rigidities in wage setting. τ_w is a wage subsidy used to offset the steady state effects of monopolistic competition in the labour market. The first order conditions for consumption and state contingent bond holdings give rise to the standard Euler equation. Note that consumption is perfectly insured against idiosyncratic labour income risk and therefore consumption is no longer indexed by h.

$$C_t^{-\sigma} = E_t\beta\left\{\frac{P_t}{P_{t+1}}C_{t+1}^{-\sigma}\right\}R_t^n. \tag{1}$$

Here, R_t^n is the nominal interest rate on a non-contingent bond.

A continuum of households supply differentiated labour $N_t(h)$, which is aggregated according to the Dixit-Stiglitz form:

$$L_t = \left[\int_0^1 [N_t(h)]^{\frac{\kappa-1}{\kappa}}\, dh\right]^{\frac{\kappa}{\kappa-1}}, \quad k > 1. \tag{2}$$

The demand function for differentiated labour is:

$$N_t(h) = \left[\frac{W_t(h)}{W_t}\right]^{-\kappa} L_t. \tag{3}$$

Here W_t is the Dixit-Stiglitz wage index.

2.2 Production

Firms in the final good sector produce a homogeneous good, Y_t, using intermediate goods, $Y_t(z)$, as input in production. There is a continuum of intermediate goods of unit mass. The production function that transforms intermediate goods into final output is given by

$$Y_t = \left[\int_0^1 Y_t(z)^{\frac{\epsilon-1}{\epsilon}} dz\right]^{\frac{\epsilon}{\epsilon-1}}. \tag{4}$$

where $\epsilon > 1$. The solution to the problem of optimal factor demand yields the following demand function for variety z

$$Y_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\epsilon} Y_t. \tag{5}$$

A continuum of monopolistically competitive intermediate goods firms indexed by $z \in [0, 1]$ and owned by consumers uses labour $L_t(z)$ and capital $K_t(z)$ to produce output according to the following constant returns technology:

$$Y_t(z) = A_t L_t(z)^{1-\alpha} K_t(z)^{\alpha}. \tag{6}$$

A_t denotes total factor productivity, which follows an exogenous stochastic process. Capital is freely mobile across firms rather than being firm specific. Firms rent capital from households in a competitive market on a period by period basis after they observe the aggregate shocks. Firm z chooses $L_t(z)$ and $K_t(z)$ to minimize total cost subject to meeting demand

$$\min_{K_t(z), L_t(z)} w_t^r L_t(z) + Z_t K_t(z) \quad \text{s.t.} \quad A_t L_t(z)^{1-\alpha} K_t(z)^{\alpha} - Y_t = 0. \tag{7}$$

Here, w_t^r is the real wage and Z_t the real rental rate for capital. Let $X_t(z)$ denote the Lagrange multiplier with respect to the constraint, i.e. real marginal cost. The first order conditions with respect to $L_t(z)$ and $K_t(z)$ are given by:

$$w_t^r = (1-\alpha) X_t(z) A_t K(z)_t^{\alpha} L(z)_t^{-\alpha}, \tag{8}$$
$$Z_t = \alpha X_t(z) A_t K(z)_t^{\alpha-1} L(z)_t^{1-\alpha}. \tag{9}$$

The first order conditions imply that all firms choose the same capital to labour ratio, therefore marginal cost X_t is equalized across firms.

2.3 The efficiency gap and monetary policy

In order to compute welfare, it is useful to consider the solution under perfectly flexible wages and prices. Up to a first-order approximation in log deviations[6] flexible price output is given by:

$$\widehat{Y}_t^* = \left[\frac{1+\chi}{\chi + \alpha + (1-\alpha)\sigma}\right] \widehat{A}_t. \tag{10}$$

[6]Throughout this text, for any variable X_t, $\bar{X}$ denotes its steady value and $\hat{X}_t \equiv \log X_t - \log \bar{X}$

One can use this equation together with the firm's first-order condition for labour demand, to derive a key equation for this model. This equation links marginal cost to the output gap, $Y_t - Y_t^*$, and the gap between the average marginal rate of substitution between consumption and labour and the real wage:

$$\widehat{X}_t = \left[\frac{\chi + \alpha}{1 - \alpha} + \sigma\right] \left(\widehat{Y}_t - \widehat{Y}_t^*\right) - \left[\chi \widehat{L}_t + \sigma \widehat{C}_t - \widehat{w^r}_t\right]. \tag{11}$$

When there are no nominal rigidities in the labour market, the marginal rate of substitution between consumption and labour is equal to the real wage and the last term in brackets vanishes. We then recover the condition from sticky price models that marginal cost is log-linearly related to the output gap. When prices are perfectly flexible as well, the marginal product of labour is equal to the real wage, i.e. log marginal cost is zero. It follows that the output gap is zero.

Monetary policy is assumed to follow an interest rate as suggested by Henderson and McKibbin (1993) or Taylor (1993). We consider the following functional form that is often used in the empirical description of central bank behaviour:

$$\hat{R}_t^n = (1 - \alpha_2)\alpha_0 \left(\hat{Y}_t - \hat{Y}_t^*\right) + (1 - \alpha_2)\alpha_1 \hat{\pi}_t + \alpha_2 \hat{R}_{t-1}^n + v_t. \tag{12}$$

Here v_t is an i.i.d. monetary policy shock capturing non-systematic central bank behaviour.

3 Alternative wage and price setting schemes

There are many different ways in which nominal rigidities can be modeled in New Keynesian models. The apparatus of Calvo (1983) and Yun (1996) has emerged as a widely used standard in the analysis of welfare effects of monetary policy, see e.g. Pappa (2004), Kollmann (2004), Kollmann (2002), Erceg, Henderson, and Levin (2000) or Rotemberg and Woodford (1997).

Recently, Ascari (2004) and Kiley (2002) have pointed out that Calvo (1983) pricing implies much more price dispersion induced by steady state inflation than Taylor (1980) pricing. Ascari (2004) considers a standard New Keynesian model with trend inflation and Calvo pricing, where those firms that are not re-optimizing their price cannot adjust for trend inflation. He computes that a trend inflation rate of 5 % generates a steady state loss[7] of output relative to the zero inflation case of 11.5 % with Calvo (1983) pricing, but only of 0.5% with Taylor (1980) contracts that have the same average duration over which prices are expected to be fixed. The reason is that trend inflation translates into much more price dispersion with Calvo (1983) pricing. We are motivated by these findings to undertake a systematic quantitative comparison of the welfare effects of monetary policy over the business cycle across different assumptions about wage and price setting. In order not to pick up any effects already emphasized by Kiley (2002) and Ascari (2004) and in keeping with much of the new Keynesian literature, we analyze an environment with zero trend inflation.

[7] The loss reported here refers to the case of firm-specific capital and a markup of 10%. The loss is only 3% when there is an economy wide rental market for capital.

We next turn to the description of the alternative price and wage setting schemes. We can subsume 4 contracting schemes under the general framework provided by Wolman (1999). The sticky information schemes requires a separate exposition.

3.1 4 sticky contracting schemes

Following Wolman (1999), we nest 4 considered models of price and wage setting as special cases of the following set-up. Limited ability to reset prices is described by a vector α. The j-th element of that vector is the probability that a firm adjusts its price in period t, conditional on the previous adjustment having occurred in period $t-j$. One can deduce from α a vector ω, denoting the fraction of firms charging prices set in period $t-j$. The Wolman (1999) scheme is very flexible, it encompasses Taylor (1980) overlapping contracts as a special case and can approximate Calvo pricing for large J.

There are two key differences between the Calvo (1983) price setting scheme and the scheme proposed by Wolman (1999). The first is that Calvo (1983) has an infinite tail: for any integer J there is a nonzero fraction of firms that last adjusted their price J periods ago. In the scheme considered here, J is finite. Furthermore, under Calvo (1983) pricing the probability that a firm adjusts its price in any given period is independent of the time elapsed since the previous adjustment. A firm that has not adjusted its price for say 10 quarters is just as likely to keep the price fixed in period t as a firm that has last adjusted its price in period $t-1$. Both the infinite tail and the constant adjustment probabilities of Calvo (1983) are often considered to be unrealistic. Nevertheless, the Calvo (1983) scheme is often used under the implicit assumption that these particularities do not influence welfare computations by much.

We compare the Calvo scheme for wage and price setting to 3 alternative contracting specifications. To allow for a fair comparison, we require that the expected lifetime of a newly set contract is equal across schemes. In such a way, no contracting scheme imposes more exogenous stickiness on average. Using the general notation above the expected lifetime of a newly set contract, D, is

$$D \equiv \sum_{k=1}^{J} k\alpha_k \Pi_{j=0}^{k-1}(1-\alpha_j) \quad \text{with:} \quad \alpha_0 \equiv 0. \tag{13}$$

For Calvo (1983) pricing, $J=\infty$, $\alpha_j=(1-\theta)$ for $j=1,2,...$ and $\theta \in [0,1)$. The duration is given by:

$$D = (1-\theta)\sum_{k=1}^{\infty}\theta^{k-1}k = \frac{1}{1-\theta}. \tag{14}$$

We consider overlapping contracts in the spirit of Taylor (1980), a truncated version of the Calvo (1983) scheme and a more general upward sloping scheme with time dependent adjustment probabilities following Wolman (1999). We assume that no price or wage is fixed for longer than 10 quarters. Prices are assumed to be fixed on average for 2 quarters, while wages are fixed for 4 quarters on average. The choice of these durations is motivated in section 4. The vectors of adjustment probabilities is depicted in the table 1. In that table the suffix p refers to price setting and w to wage setting.

Table 1: Conditional adjustment probabilities

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9	α_{10}
Taylor p	0	1	0	0	0	0	0	0	0	0
Taylor w	0	0	0	1	0	0	0	0	0	0
Wolman p	0.45	0.5	0.57	0.65	0.75	0.80	0.85	0.9	0.95	1
Wolman w	0.1	0.18	0.25	0.3	0.39	0.5	0.6	0.7	0.9	1
truncated p	θ_p	θ_p	θ_p	θ_p	θ_p	θ_p	θ_p	θ_p	θ_p	1
truncated w	θ_w	θ_w	θ_w	θ_w	θ_w	θ_w	θ_w	θ_w	θ_w	1

Price and wage setting is staggered under each of these schemes. At any point in time the aggregate price or wage index is determined by the prices of J^p cohorts of firms or wages of J^w cohorts of households indexed by when they last updated their contract. Under Taylor pricing there are two cohorts of firms that charge prices lasting for exactly 2 periods. At any point in time half of the prices were determined yesterday and half today. Similarly for 4 period wage setting. Under the Wolman scheme, the time profile of the conditional adjustment probability is increasing. This is an ad hoc way of capturing the notion that the gain from adjusting prices when firms face fixed adjustment costs should increase the longer prices have not been adjusted. The truncated version of the infinite horizon Calvo (1983) scheme has a time invariant adjustment probability $\theta_p = 0.5$ for price setting and $\theta_w = 0.232$ for wage setting that applies for periods 1 through 9. 10 periods after last adjustment, all contracts can be renegotiated.

While all contracting schemes imply that the newly set contracts have the same expected lifetime, the schemes differ with respect to the average time elapsed since last adjustment (the age[8] of contracts) at any given point in time as has recently been pointed out by Dixon and Kara (2005). Calvo (1983) contracts have the largest average age, Taylor (1980) contracts have the smallest average age. The following two graphs plot the implied age distribution of wage and price contracts.[9]

[10]

3.2 The log-linearized price and wage setting conditions

In any given period, a firm faces a certain probability of receiving a signal that allows that firm to reset its price. Firms that do not receive the signal, carry on the prices posted in the last period and satisfy any demand at that price. Given the vector of adjustment

[8]The average age is defined as follows. Let ω_j denote the fraction of contracts at any point in time that were last adjusted j periods ago. The average age is $\sum_{j=1}^{J} \omega_j j$. The ω_j can be derived from the vector α via recursions described in Wolman (1999).

[9]Note that graphs show that the fraction of contracts that is one period old (i.e. newly set contracts) is equal across schemes. The same fraction of contracts are updated in each period, implying an equal average expected duration of a newly set contract. This follows from the fact that the average updating frequency is the inverse of the average duration.

[10]At any given point in time, the average age of a price (wage) contracts is 2 (4) for Calvo (1983) contracts, 1.99 (3.54) for the truncated Calvo scheme, 1.84 (2.99) for Wolman contracts and 1.5 (2.5) for Taylor contracts.

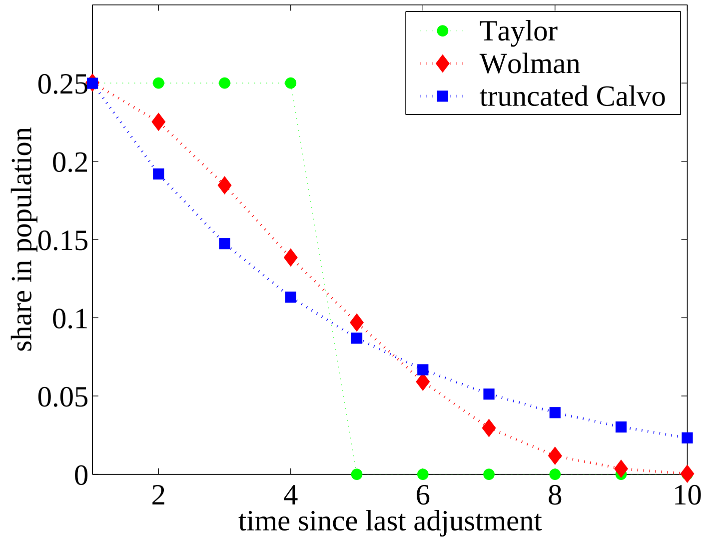

Figure 1: Age distribution of wage contracts

probabilities α, firms compute the probability that the price set today will still be in place j periods from now, labeled ϕ^p_j. The firms problem is:

$$\max_{P^*_t(z)} \mathrm{E}_t \sum_{i=0}^{J^p} \phi^p_i \beta^i C_{t+i}^{-\sigma} \left\{ (1+\tau_p) \left[\frac{P^*_t(z)}{P_{t+i}} \right]^{1-\epsilon} Y_{t+i} - X_{t+i} \left[\frac{P^*_t(z)}{P_{t+i}} \right]^{-\epsilon} Y_{t+i} \right\}. \tag{15}$$

Here, τ_p is a sales subsidy suitably chosen as to offset the steady state effects of monopolistic competition $(1+\tau_p \equiv \frac{\epsilon}{\epsilon-1})$. The first-order necessary condition for the optimal nominal price P^*_t expressed in terms of the stationary variable $p^*_t \equiv \frac{P^*_t}{P_t}$ is:

$$p^*_t = \frac{E_t \sum_{j=0}^{J^p-1} \phi^p_j \beta^j C_{t+j}^{-\sigma} X_{t+j} \left(\pi_{t,t+j}\right)^{\epsilon} Y_{t+j}}{E_t \sum_{j=0}^{J^p-1} \phi^p_j \beta^j C_{t+j}^{-\sigma} \left(\pi_{t,t+j}\right)^{\epsilon-1} Y_{t+j}}. \tag{16}$$

Here, $\pi_{t,t+j} \equiv \Pi_{k=1}^{j} \pi_{t+k}$, i.e inflation between periods t and $t+j$ with $\pi_{t,t+j} \equiv 1$ Log-linearizing this around a steady state with zero price inflation yields:

$$\widehat{p^*}_t = E_t \sum_{j=0}^{J^p-1} \frac{\beta^j \phi^p_j}{\nu^p} \hat{X}_{t+j} + \sum_{j=1}^{J^p-1} \frac{\gamma^p_j}{\nu_p} \widehat{\pi}_{t+j}, \tag{17}$$

$$\text{with: } \nu^p = \sum_{j=0}^{J^p-1} \beta^j \phi^p_j, \quad \gamma^p_j = \sum_{k=j}^{J^p-1} \beta^k \phi^p_k. \tag{18}$$

From the definition of the price index $P_t \equiv \left[\int_0^1 P_t(z)^{1-\epsilon} dz \right]^{\frac{1}{1-\epsilon}}$ we obtain:

$$0 = \sum_{j=0}^{J^p-1} \omega^p_j \widehat{p^*}_{t-j} - \sum_{j=0}^{J^p-2} \vartheta^p_j \widehat{\pi}_{t-j} \quad \text{with:} \quad \vartheta^p_j = \sum_{k=j+1}^{J^p-1} \omega^p_k. \tag{19}$$

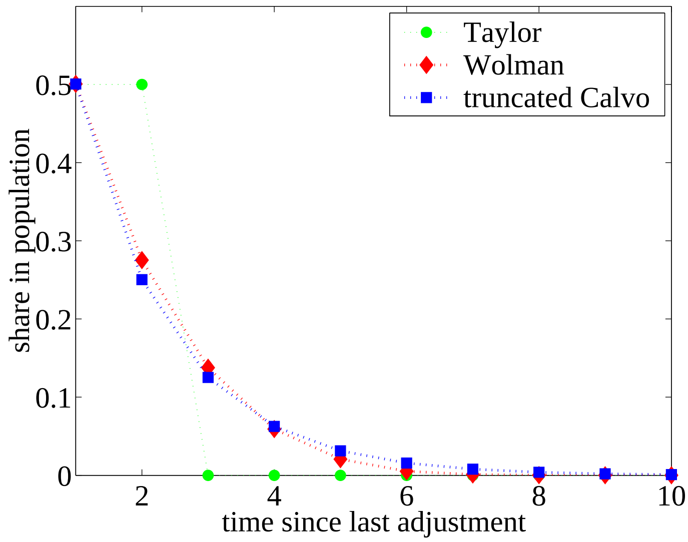

Figure 2: Age distribution of price contracts

One can show that it is not necessary to keep track of the relative quantities of intermediate goods produced by the different cohorts of firms in order to solve the model. A relation between aggregate inputs and aggregate output similar to what was pointed out by Yun (1996) for the case of Calvo (1983) pricing can be derived:

$$Y_t = \frac{A_t}{D_t}\bar{K}^\alpha L_t^{1-\alpha} \quad \text{with:} \quad D_t \equiv \sum_{j=0}^{J^p-1} \omega_j^p \left(\frac{P^*_{t-j}}{P_t}\right)^{-\epsilon}. \tag{20}$$

For this pricing scheme it is also possible to show that the price distortion term D_t can be ignored for a log-linear analysis around a steady state with zero inflation. One can log-linearize the price distortion term together with the aggregate price index to show that $\hat{D}_t = 0$, if there is no trend inflation.

The wage setting problem is analogous. Given the vector of adjustment probabilities, households maximize utility through choice of the wage subject to the demand curve implied by the labour aggregator. Let ϕ_j^w denote the probability that a wage set in period t is still in place in period $t+j$. The first order condition for the wage can be expressed in terms of the stationary variable $w_t^* \equiv \frac{W_t^*}{W_t}$ as:

$$w_t^* = \left(\frac{E_t \sum_{j=0}^{J_w-1} \phi_j^w \beta^j \left(\pi^w_{t,t+j}\right)^{\kappa(1+\chi)} L_{t+j}^{1+\chi}}{E_t \sum_{j=0}^{J_w-1} \phi_j^w \beta^j \left(\pi^w_{t,t+j}\right)^{\kappa} L_{t+j} C_{t+j}^{-\sigma} \frac{w^r_{t+j}}{\pi^w_{t,t+j}}} \right)^{\frac{1}{1+\kappa\chi}}. \tag{21}$$

Here, $\pi^w_{t,t+j} \equiv \Pi_{k=1}^j \pi^w_{t+k}$, i.e wage inflation between periods t and $t+j$ and $\pi^w_{t,t} \equiv 1$. At any point in time there are J_w such conditions determining the aggregate wage index, corresponding to the first-order condition of wage setters in the current and previous $J-1$ periods.

The first order conditions log-linearized around a steady state with zero wage inflation is:

$$\widehat{w^*}_t = E_t \frac{1}{(1+\kappa\chi)} \sum_{j=0}^{J^w-1} \frac{\beta^j \phi_j^w}{\nu^w} \hat{\mu}_{t+j} + \sum_{j=1}^{J^w-1} \frac{\gamma_j^w}{\nu_w} \widehat{\pi^w}_{t+j}, \tag{22}$$

$$\text{with: } \nu^w = \sum_{j=0}^{J^w-1} \beta^j \phi_j^w, \quad \gamma_j^w = \sum_{k=j}^{J^w-1} \beta^k \phi_k^w. \tag{23}$$

We use $\hat{\mu}_t$ as shorthand for the difference between the marginal rate of substitution and the real wage: $\hat{\mu}_t \equiv \chi \hat{L}_t + \sigma \hat{C}_t - \hat{w}_t^r$. From the definition of the wage index $W_t \equiv \left[\int_0^1 W_t(h)^{1-\kappa} dh\right]^{\frac{1}{1-\kappa}}$ we obtain

$$0 = \sum_{j=0}^{J^w-1} \omega_j^w \widehat{w^*}_{t-j} - \sum_{j=0}^{J^w-2} \vartheta_j^w \widehat{\pi^w}_{t-j} \quad \text{with:} \quad \vartheta_j^w = \sum_{k=j+1}^{J^w-1} \omega_k^w. \tag{24}$$

The familiar framework of Calvo contracts obtains as a special case of this general scheme. In particular, $J^p = \infty$ and the adjustment probability is time invariant $\alpha_j = 1 - \theta_p$, for $j = 1, 2, ...$. Similarly for wage setting, there is a time invariant adjustment probability θ_w. One could numerically simulate the economy with Calvo (1983) contracts by truncating the infinite sums for some large integer J^p and J^w. However, one can exploit the recursive representation of the infinite geometric sums to obtain the well-known wage and price Phillips curves.

$$\widehat{\pi}_t = \frac{(1-\theta)(1-\beta\theta)}{\theta} \widehat{X}_t + \beta \mathrm{E}_t \widehat{\pi}_{t+1}, \tag{25}$$

$$\widehat{\pi^w}_t = \frac{(1-\theta_w)(1-\beta\theta_w)}{(1+\kappa\chi)\theta_w} \widehat{\mu}_t + \beta \mathrm{E}_t \widehat{\pi^w}_{t+1}. \tag{26}$$

3.3 A variant of Mankiw and Reis (2002): Sticky information

We build a N-period version of the sticky information scheme by Mankiw and Reis (2002). As with Wolman (1999) contracts, the arrival of new information is described by a vector α, whose elements are the same as in Table 1 for our earlier analysis. The generic entry α_j denotes the probability that a firm that last updated its information set in period $t - j$ receives a signal to update the information set in period t. In section 5 the adjustment probabilities referred to the signal to be allowed to change the contract price. Hence, price setters must choose one price which does well on average over the weighted future periods. Sticky information firms can change their price in any period. We can think of the problem of a sticky information agent that receives the most recent information set as choosing a whole path for the contract wage or price to be applicable in the subsequent periods unless the agent receives another update of the information set.

Equilibrium sequences for allocations and prices in the model with sticky information must then satisfy (32), (35) - (40) as well as the following conditions stemming from wage and price setting. There are J^w cohorts of households[11], indexed by how many periods

[11] We assume that sticky information pertains only to the households choice of the nominal wage, not to the choice of consumption and state contingent bonds.

ago they last received an update of their information set. The first order condition of the optimal nominal wage chosen by cohort j conditional on the information set at time $t-j$ and applicable in t denoted with $W_{t,t-j}$ is:

$$0 = \mathrm{E}_{t-j}\left[\frac{N_{t,t-j}^{1+\chi}}{W_{t,t-j}} - C_t^{-\sigma} N_{t,t-j} P_t^{-1}\right]. \tag{27}$$

Here, $N_{t,t-j}$ is labour supplied by cohort j, i.e. $N_{t,t-j} \equiv \left(\frac{W_{t,t-j}}{W_t}\right)^{-\kappa} L_t$. Solving this for the optimal wage in terms of the stationary variable $w^*_{t,t-j} \equiv \frac{W_{t,t-j}}{W_{t-j}}$ yields:

$$\left(w^*_{t,t-j}\right)^{1+\kappa\chi} = \frac{\mathrm{E}_{t-j}\left\{L_t^{1+\chi}\left(\pi^w_{t-j,t}\right)^{\kappa(1+\chi)}\right\}}{\mathrm{E}_{t-j}\left\{C_t^{-\sigma} L_t \left(\pi^w_{t-j,t}\right)^{\kappa-1} w^r_t\right\}}. \tag{28}$$

For price setting the optimality condition is:

$$0 = E_{t-j}\left\{C_t^{-\sigma}\left(\frac{P^*_{t,t-j}}{P_t}\right)^{-\epsilon}\frac{Y_t}{P_t}\left[1 - X_t\left(\frac{P^*_{t,t-j}}{P_t}\right)^{-1}\right]\right\}. \tag{29}$$

We define the log-linearized optimality in conditions in terms of stationary variables $\hat{w}^*_{t,t-j} \equiv \hat{W}^*_{t,t-j} - \hat{W}_{t-j}$ and $\hat{p}^*_{t,t-j} \equiv \hat{P}^*_{t,t-j} - \hat{P}_{t-j}$:

$$(1+\kappa\chi)\hat{w}^*_{t,t-j} = E_{t-j}\sum_{k=0}^{j-1}(1+\kappa\chi)\hat{\pi}^w_{t-k} - \hat{w}^r_t + \chi\hat{L}_t + \sigma\hat{Y}_t, \tag{30}$$

$$\hat{p}^*_{t,t-j} = E_{t-j}\sum_{k=0}^{j-1}\hat{\pi}_{t-k} + \hat{X}_t. \tag{31}$$

4 The key equations and model calibration

In this section, we collect necessary conditions that must be satisfied by equilibrium sequences of allocations and prices for the case of Calvo (1983) wage and price setting. The model has 7 endogenous variables: price inflation π_t, wage inflation π^w_t, labour L_t, output Y_t, marginal cost X_t, nominal interest rate R^n_t and the real wage w^r_t. Market clearing requires $C_t = Y_t$, henceforth we drop C_t. The model's equilibrium conditions are summarized in the following box.

$$
\begin{aligned}
\widehat{Y}_t &= E_t\widehat{Y}_{t+1} - \sigma^{-1}\left(\widehat{R}^n_t - \widehat{\pi}_{t+1}\right), && (32)\\
\widehat{\pi^w}_t &= \frac{(1-\theta_w)(1-\beta\theta_w)}{(1+\kappa\chi)\theta_w}\widehat{\mu}_t + \beta \mathrm{E}_t\widehat{\pi^w}_{t+1}, && (33)\\
\widehat{\pi}_t &= \frac{(1-\theta)(1-\beta\theta)}{\theta}\widehat{X}_t + \theta \mathrm{E}_t\widehat{\pi}_{t+1}, && (34)\\
\widehat{Y}_t &= \widehat{A}_t + (1-\alpha)\widehat{L}_t, && (35)\\
\widehat{w^r}_t &= \widehat{X}_t + \widehat{A}_t - \alpha\widehat{L}_t, && (36)\\
\widehat{w^r}_t &= \widehat{w^r}_{t-1} + \widehat{\pi^w}_t - \widehat{\pi}_t, && (37)\\
\widehat{X}_t &= \left[\frac{\chi+\alpha}{1-\alpha} + \sigma\right]\left(\widehat{Y}_t - \widehat{Y^*}_t\right) - \left[\chi\widehat{L}_t + \sigma\widehat{Y}_t - \widehat{w^r}_t\right], && (38)\\
\hat{R}^n_t &= (1-\alpha_2)\alpha_0\left(\hat{Y}_t - \hat{Y}^*_t\right) + (1-\alpha_2)\alpha_1\hat{\pi}_t + \alpha_2\hat{R}^n_{t-1} + v_t, && (39)\\
\widehat{A}_t &= \rho\widehat{A}_{t-1} + u_t. && (40)
\end{aligned}
$$

Upper case letters denote the aggregate of the respective lower case variables. (32) is the consumption Euler equation. (33) and (34) are the wage and price Phillips curves. (35) is the log-linearized aggregate production function. (36) is the firm's labour demand function. (37) is an identity defining the change of the real wage. (38) links marginal cost to the output gap and the "wage gap". (39) is the monetary policy rule. Finally, the last equation is the exogenous stochastic process for total factor productivity.

We now turn to the calibration. A time period is taken to be a quarter. The average duration of newly set price contracts is set to 2 quarters. This number is somewhat smaller than the average duration in most recent calibrated monetary models[12]. It reflects survey evidence by Bills and Klenow (2004) pointing to much more price flexibility in the U.S. economy than typically assumed in the literature. Bills and Klenow (2004) find that half of all prices last 5 months or less when excluding temporary sales. For wage contracts we follow Canzoneri, Cumby, and Diba (2004) and much of the literature and set the average duration to 4 quarters. A recent micro study by Rich and Tracey (2004) suggests that U.S. wage contracts last on average 10 quarters, with a significant fraction of contracts negotiated annually. We do not calibrate the average contract duration to the data for the purpose of comparisons with the existing studies on wage stickiness, that almost universally assume annual contract duration.

The time preference rate is matched to yield an annual gross real interest rate of 1.03, i.e. $\beta = 1.03^{-0.25}$. The markup is calibrated to 15% in both goods and labour markets, resulting in $\kappa = \epsilon = 7.66$. This value is intermediate in the range employed by other studies. Keen (2004) and Collard and Dellas (2005) set the price markup to 10% while Kollmann (2004) assumes 20%. Rabanal and Rubio-Ramirez (2005) calibrate a wage markup of 20%. The labour share in production $1-\alpha$ is set to the standard value 0.65. We assume a Frisch (constant marginal utility of wealth) elasticity of labour supply of $\frac{1}{3}$ (implying $\chi = 3$), which is in line with most empirical estimates ranging between 0 and

[12]For instance, Kollmann (2004), Keen (2004) and Pappa (2004) assume an average duration of 4 quarters.

0.5 as surveyed in Blundell and MaCurdy (1999). We set the coefficient of relative risk aversion σ to 2. This is a compromise between experimental evidence by Barsky, Juster, Kimball, and Shapiro (1997) who find a mean risk aversion of roughly 4 and the value of 1 commonly chosen in real business cycle models. The exogenous process for technology follows an AR(1) with autoregressive parameter equal to 0.9. The innovation has standard deviation equal to 0.00852. Following, Canzoneri, Cumby, and Diba (2004), the standard deviation of the monetary policy shock is set to 0.0025. Since the welfare costs of nominal rigidities are highly sensitive to the monetary policy rule in place, we consider 3 different rules. The reaction coefficients describing these three rules are summarized in Table 2. Rule 1 is our estimated on quarterly U.S. data over the sample 1980:1-2004:2.[13] We

Table 2: The considered interest rate rules

	α_0	α_1	α_2
Rule 1: estimated	0.09	2.13	0.78
Rule 2: estimated, no output gap	0.00	1.90	0.77
Rule 3: Taylor (1993)	0.50	1.50	0.00

refer to the model with this monetary policy rule as our baseline model. Rule 2 is an estimated rule where the response to the output gap is restricted to zero. The reason is that the output gap is typically only marginally significant or even insignificant and that the adverse welfare effects of reacting to a wrong measure of the gap can be very large, see Schmitt-Grohé and Uribe (2004). We finally consider the classical Taylor (1993) rule as a prominent benchmark from the literature.

The model is too stylized to estimate its parameters or rigorously calibrate them to the data. However, we note that the overall amount of volatility present in the model is roughly comparable to the data. For instance, the baseline calibration implies that the standard deviation of consumption and hours worked in the model are 0.017 and 0.012, respectively. Canzoneri, Cumby, and Diba (2004) report that these numbers are 0.016 and 0.014 in detrended U.S. data. Extensive sensitivity analysis with respect to the key parameters is documented in the appendix.

The linearized model is solved using the MATLAB codes provided by Klein (2000) and Juillard (2001).

[13] Data is taken from the FRED2 database. The interest rate is the end of quarter federal funds rate. (http://research.stlouisfed.org/fred2/series/FEDFUNDS/downloaddata/FEDFUNDS.txt) It is converted to quarterly decimal rate by dividing by 400. The output gap is constructed as the log difference between quarterly real GDP and potential real GDP as constructed by the Congressional Budget Office.
This data is available at http://research.stlouisfed.org/fred2/series/GDPC96/downloaddata/GDPC96.txt and http://research.stlouisfed.org/fred2/series/GDPPOT2/downloaddata/GDPPOT2.txt, respectively. Inflation is constructed as the log of the first difference of the GDP price index available at http://research.stlouisfed.org/fred2/series/GDPCTPI/downloaddata/GDPCTPI.txt. All data except potential GDP are seasonally adjusted. All coefficients are significant at standard levels.

5 Welfare costs of nominal rigidities

This section computes welfare costs of business cycle fluctuations stemming from nominal rigidities in wage and price setting. The welfare measure is the expected lifetime utility of a randomly drawn household. We follow Rotemberg and Woodford (1999) and assume that utility flow from real money balances is negligible. Define period utility $\mathbb{W}_t$ as

$$\mathbb{W}_t \equiv \frac{C_t^{-\sigma}}{1-\sigma} + \int_0^1 \frac{N_t^{1+\chi}(h)}{1+\chi} dh. \tag{41}$$

Let $\mathbb{W}_t^*$ denote period utility under perfectly flexible wages and prices. The consumption equivalent welfare measure $\mathbb{L} \equiv -\mathrm{E}\sum_{t=0}^{\infty} \beta^t \left(\mathbb{W}_t - \mathbb{W}_t^*\right) / \left(U_C \bar{C}\right)$ can be approximated up to second order by the following weighted sum of second moments:

$$\mathbb{L} = \mathrm{E}\sum_{t=0}^{\infty} \beta^t \left[\tilde{\lambda}_0 \sum_{j=0}^{J^p-1} \omega_j^p \left(\widehat{p^\diamond}_{t-j}\right)^2 + \tilde{\lambda}_1 \left(\hat{Y}_t - \hat{Y}_t^*\right)^2 + \tilde{\lambda}_2 \sum_{j=0}^{J^w-1} \omega_j^w \left(\widehat{w^\diamond}_{t-j}\right)^2\right], \tag{42}$$

$$\text{with: } \tilde{\lambda}_0 = \frac{1}{2}\epsilon, \quad \tilde{\lambda}_1 = \frac{1}{2}\left(\frac{\chi+\alpha}{1-\alpha} + \sigma\right), \quad \tilde{\lambda}_2 = \frac{1}{2}(1-\alpha)(\kappa^{-1}+\chi)\kappa^2.$$

E denotes unconditional expectation. Here, $\widehat{p^\diamond}_{t-j} \equiv \widehat{P^*}_{t-j} - \widehat{P}_t$ and similarly for $\widehat{w^\diamond}_{t-j}$. $\mathbb{L}$ gives the onetime increase in consumption, expressed as a percentage of period consumption in the steady state, necessary to make agents as well off in a world with nominal rigidities as in a world with perfectly flexible wages and prices. Since this loss function is free of first moments, it can be accurately (up to second-order) evaluated by considering a linear approximation to the model's equilibrium conditions.[14]

For Calvo (1983) contracts, we have $J^P = J^W = \infty$ and we could compute welfare by truncating at some large integer. Instead we work with the well-known equivalent representation:

$$\mathbb{L} = \mathrm{E}\sum_{t=0}^{\infty} \beta^t \left[\lambda_0 \hat{\pi}_t^2 + \lambda_1 \left(\hat{Y}_t - \hat{Y}_t^*\right)^2 + \lambda_2 \widehat{\pi^w}_t^2\right]. \tag{43}$$

Here, the weights are given by

$$\lambda_0 = \frac{\epsilon\theta}{2(1-\theta)(1-\theta\beta)}, \quad \lambda_1 = \frac{1}{2}\left(\frac{\chi+\alpha}{1-\alpha} + \sigma\right), \quad \lambda_2 = \frac{\theta_w \kappa^2 (1-\alpha)(\kappa^{-1}+\chi)}{2(1-\theta_w)(1-\theta_w\beta)}.$$

[14] It should be noted that to check the welfare calculations done here using the linear quadratic method of Woodford (2003, chapter 6), the full nonlinear model was solved using a second-order approximation. The welfare numbers computed in such a way were extremely similar to the ones presented using the linear quadratic method. Differences in welfare computations appeared to be mainly due to numerical approximation errors. The main reason to use the linear quadratic method of Woodford (2003) and Benigno and Woodford (2005) is that it becomes feasible to analyze models with a large number of state variables. Some of the sticky information schemes cannot be solved numerically using second-order techniques due to the large number of state variables. However, one can still analyze welfare for those schemes using the linear-quadratic approach.

Note that the weight on wage dispersion ($\tilde{\lambda}_0$) is roughly 15 times larger than the weight on price dispersion ($\tilde{\lambda}_2$) for our benchmark calibration. The higher weight attached to wage dispersion depends on the labour supply elasticity. With sticky wages labour supply is demand determined. The inverse of the labour supply elasticity signals how much compensation the household requires for supplying an extra unit of labour. When wages are sticky, households are induced to vary their labour supply without any such compensation taking place.[15] Therefore, it is clear that the inverse of the labour supply elasticity is closely related to the welfare cost of wage inflation. For instance, setting $\chi = 1$ brings the weight on wage inflation relative to price inflation down to 3.1 for our benchmark calibration. Other important parameters determining the weights on wage and price dispersion are the elasticities κ and ϵ. The higher these parameter, the more substitutable are different varieties of labour in production. Since welfare is decreasing in the dispersion of goods or labour across varieties, a given amount of dispersion of relative prices results in lower welfare when goods are more substitutable.

We next address whether choosing the average contract duration rather than the average age as a metric to make contracting schemes comparable influences our welfare comparison systematically.

5.1 Average duration or average age?

Choosing the average contract duration as our metric for making different schemes comparable implies that the average time elapsed since last adjustment in a cross-section (the average age) is different across schemes.[16] In particular Calvo (1983) contracts have a higher average age in a cross-section than Taylor (1980) contracts, despite the fact that the expected duration of a contract is the same. For instance, our Calvo (1983) wage contracts with $\theta = \frac{3}{4}$ are on average 4 quarters old, whereas our 4 periods Taylor (1980) contracts are on average 2.5 quarters old. It is therefore of interest to find out how the welfare costs of nominal rigidities depend on the average contract age.

We show that the welfare costs of nominal rigidities under Calvo (1983) contracts are non-monotonic in the average age of contracts. The following graph plots the welfare costs of nominal rigidities as a function of the probability θ that the random signal to change the contract is not received. Both the average age in a cross section and the average duration of Calvo (1983) contracts are equal to $\frac{1}{1-\theta}$.

Two effects influence the welfare costs, when agents can adjust their contracts less often. More agents keep their contract fixed and therefore do no respond optimally to current period shocks. That effect tends to increase the welfare costs. However, those agents that currently receive a signal to change their contract are attaching more weight to economic conditions farther into the future as their contract is fixed for a longer period. They therefore tend to also be less responsive to current economic conditions and that effect works to decrease relative price dispersion. The result is a non-monotonic relation depicted in the above figures. Apparently, the two mentioned effects interact strongly

[15] An indirect compensation takes place, since firms make higher profits when workers are paid a wage that is off their labour supply schedule.

[16] For the Calvo (1983) contracts both the expected lifetime of a newly set contract and the average age of contracts in a cross-section are equal to the inverse of the reset probability. For N period Taylor (1980) contracts the expected lifetime is N and the average age is $\frac{N+1}{2}$.

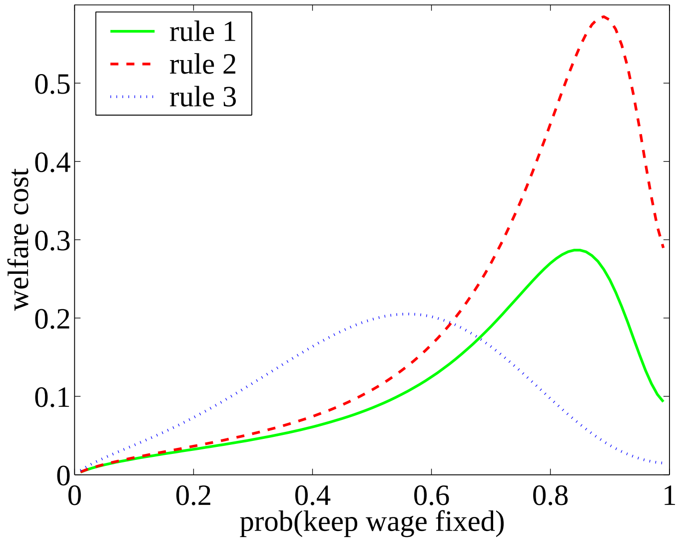

Figure 3: Welfare costs with sticky wages and flexible prices

with monetary policy as the location of the peak depends on the monetary policy rule in place.[17]

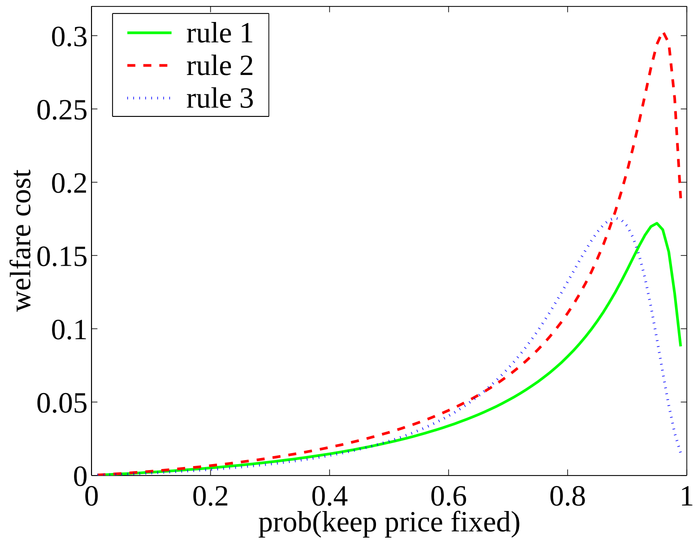

Figure 4: Welfare costs with sticky prices and flexible wages

The message of this simple plot is as follows. When comparing welfare costs across different contracting scheme holding fixed the *average contract duration* we are in effect comparing schemes with different average ages in a cross-section. However, on average

[17]If the shock has zero autocorrelation, than economic conditions in the period of the shock are most different from those in the following periods. Therefore, the peak of the plot of the welfare costs with respect to the average contract duration should occurs earlier than with highly correlated shocks. Numerical results support this and therefore tend to support our line of intuitive argument.

older Calvo (1983) contracts do not necessarily imply higher welfare costs than on average younger Calvo (1983) contracts, as the plot shows.

This hump shaped relation between contract duration and the welfare costs of nominal rigidities is not specific to Calvo (1983) contract. Similar plots hold for Taylor (1980) contracts, but the shape and the location of the peak differ.

5.2 Welfare costs of sticky contracting schemes

Table 3 displays the welfare cost of nominal inertia for the considered contracting schemes. In that table, we express the welfare cost in terms of a per period compensation[18] and as a fraction of 1 per cent. We consider two comparisons. In the first one, we hold the average duration of newly set contracts constant across schemes as is typical in the literature (see Kiley (2002)). In the second one, we modify the vector α of adjustment probabilities such that the average age in a cross section is equal across schemes. The particular average we choose is that of 2 period Taylor price contracts and 4 period Taylor wage contracts, i.e. 1.5 and 2.5. Note that requiring equal average age as the Taylor scheme involves shortening the average duration of the other schemes.

Table 3: Welfare costs of nominal rigidities - mobile capital

rule	$\mathbb{L}$ (Calvo)	$\mathbb{L}$ (tr. Calvo)	$\mathbb{L}$ (Taylor)	$\mathbb{L}$ (Wolman)
same average duration				
Rule 1	0.215	0.156	0.063	0.110
Rule 2	0.324	0.217	0.078	0.146
Rule 3	0.135	0.160	0.154	0.180
same average age in cross section				
Rule 1	0.125	0.117	0.063	0.097
Rule 2	0.166	0.154	0.078	0.127
Rule 3	0.199	0.197	0.154	0.190

The first two rows of Table 3 show that the welfare costs of nominal rigidities may differ strongly across the considered contracting schemes when the average duration is held fix. This supports the view that the exact nature of price and wage contracting schemes can be important for the quantification of the welfare costs of nominal rigidities. In particular, under Taylor (1980) contracts the welfare costs are roughly of the same magnitude as suggested by Lucas (2003), approximately 0.07 of one per cent of period consumption. However, for the Calvo (1983) scheme they are roughly 3-4 times higher. The truncated Calvo (1983) scheme and the Wolman (1999) scheme that have a 10 period support of the age distribution of contracts deliver costs that are somewhat in between the costs implied by the Taylor (1980) and the Calvo (1983) scheme.

However, considering rule 3 shows that the Calvo (1983) scheme need not always have the highest welfare costs. This is in stark contrast to the unambiguous results for the case

[18]Given our unconditional welfare measure, per period welfare compensation and lifetime compensation are proportional with proportionality factor $1-\beta$

of trend inflation in the comparison between Calvo (1983) and Taylor (1980) contracts in the work of Kiley (2002). In his steady state analysis, the prices farther out in tail of the age distribution of the Calvo (1983) scheme differ more and more from the aggregate price level due to the trend in the general price level. The Taylor (1980) scheme truncates that tail and therefore necessarily implies smaller welfare costs. The conjecture by Kiley (2002) that this basic insight would carry over to a welfare comparison over the business cycle[19] cannot be supported by our analysis. Must a contracting scheme that features a wider support of the age distribution *necessarily* imply more dispersion of relative prices or relative wages? Without trend inflation (or with full indexation to the trend inflation rate) our results suggest the answer is no.

The intuition is as follows and similar to the one in the previous subsection. When the age distribution has larger support, there are some agents that charge prices set farther in the past. Their contract price tends to be more out of line with current conditions. While that typically increases the welfare costs, those agents setting prices today will look farther into the future and also charge a price that reflects less on current economic conditions. Therefore widening the support of the age distribution as done in the Calvo (1983) scheme has an ambiguous effect on relative price dispersion and therefore on welfare. Whether contracts with a large support of the age distribution or a small support have higher welfare costs depends on the exact model dynamics, in particular on the monetary policy rule and on the persistence of the shocks.

Finally, we note that by requiring that all schemes have an equal average age in a cross-section, the difference in welfare costs across schemes is somewhat smaller. In particular, the Calvo (1983) scheme now yields the highest welfare costs throughout all 3 monetary policy rules considered.[20]

5.3 Welfare costs of sticky information schemes

The loss function for the sticky information model is the again of the form (42). In that formula, one must simply substitute $\widehat{p^*}_{t-j}$ with the expectation based on the information $t-j$ periods ago of the optimal price and $\widehat{w^*}_{t-j}$ with the expectation based on the information $t-j$ periods ago of the optimal wage. In the following table we display the welfare losses associated with different monetary policy rules and different assumptions about the arrival rates of new information. These arrival rates are the ones described in Table 1.

Table 4 shows that the welfare costs of Mankiw and Reis (2002) price and wage setting are typically very small. Apparently, the amount of price and wage dispersion induced by the exogenous restrictions on the ability to reset contracts is much higher than the dispersion induced by restricting the arrival of new information. Qualitatively, this result should not be surprising. After all sticky information agents will typically choose to post a new price or wage even when they do not receive any new information about the state of the world. They use optimally the information available in the period they last updated their

[19] Kiley (2002, p.291) conjectures that the gains from inflation stabilization policies in the model of Erceg, Henderson, and Levin (2000) may be overstated due the choice of Calvo (1983) contracts.

[20] Extensive sensitivity analysis suggests that this is the case for a lot of parameter configurations, but not for all. Even when requiring an equal average age, Calvo (1983) contracts may for some parameter configuration yield higher costs than Taylor (1980) contracts.

Table 4: Welfare costs of nominal rigidities - sticky information

rule	$\mathbb{L}$ (tr. Calvo)	$\mathbb{L}$ (Taylor)	$\mathbb{L}$(Wolman)
same average duration			
Rule 1	0.045	0.021	0.036
Rule 2	0.058	0.024	0.044
Rule 3	0.039	0.010	0.029
same average age			
Rule 1	0.038	0.021	0.032
Rule 2	0.047	0.024	0.038
Rule 3	0.037	0.010	0.026

information set to infer about the state of the world today. Thus, the sticky information framework typically involves less contract rigidity. The quantitative differences pointed out here are important since more ad-hoc models of nominal rigidities are sometimes justified by arguing that agents have imperfect capacities to constantly compute and post the optimal price. Our comparison suggests it may well make a difference for welfare comparisons whether one explicitly models imperfect ability to process information as in Mankiw and Reis (2002) or resorts to more ad-hoc models of price setting.

6 Robustness

So far we have assumed that capital is freely mobile across firms and can be re-allocated as to equalize the shadow value of capital. Danthine and Donaldson (2002), Woodford (2003, p.166) and others have pointed out that capital cannot be costlessly and instantaneously relocated across firms. Danthine and Donaldson (2002) view it as unreasonable that it is too costly to post a new price tag, but that it is costless to unbolt machinery and ship it between firms.[21]. Here, we check how the welfare cost of nominal rigidities across the considered schemes depends on the mobility of capital.

For the purpose of business cycle analysis, capital might better be modeled as being completely fixed at the firm level on a quarterly frequency. The problem of the firm is to choose its optimal nominal price $P_t^*(z)$ subject to the demand curve and the production function to maximize

$$\max_{P_t^*(z)} \mathrm{E}_t \sum_{i=0}^{J^p} \phi_{t+i}^p \beta^i C_{t+i}^{-\sigma} \left\{ (1+\tau_p) \left[\frac{P_t^*(z)}{P_{t+i}} \right]^{1-\epsilon} Y_{t+i} - Z_{t+i} K(z) - w_{t+i}^r L_{t+i}(z) \right\}. \quad (44)$$

[21] Furthermore, Eichenbaum and Fischer (2004) show that departing from the assumption of perfect capital mobility is necessary to reconcile the Calvo (1983) model with the data. Sveen and Weinke (2004) and Woodford (2005) further discuss the implications of modeling capital for the equilibrium dynamics in sticky prices models

The first order condition for this problem is:

$$\mathrm{E}_t \sum_{i=0}^{J^p} \phi^p_{t+i} \beta^i C^{-\sigma}_{t+i} \left\{ (1+\tau_p)(1-\epsilon) \left[\frac{P^*_t}{P_{t+i}} \right]^{1-\epsilon} Y_{t+i} + \frac{\epsilon}{1-\alpha} w^r_{t+i} L_{t+i}(z) \right\} = 0. \quad (45)$$

With firm specific capital and Calvo (1983) pricing, the Phillips Curve is now given by:

$$\widehat{\pi}_t = \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{(1-\alpha)}{(1-\alpha+\alpha\epsilon)} \widehat{X}^a_t + \beta \mathrm{E}_t \widehat{\pi}_{t+1}. \quad (46)$$

Here, $\hat{X}^a_t$ is average marginal cost. Since capital is fixed at the firm level, the capital-labour ratio differs across firms and so does marginal cost. While the slope of the price Phillips curve becomes much flatter, the weight on price inflation in the loss function becomes larger. In particular, the slope of the Phillips curve decreases by a factor $\frac{(1-\alpha)}{(1-\alpha+\alpha\epsilon)}$, while the weight in the welfare function increases by exactly the inverse of that factor. A given amount of inflation variability is roughly 5 times more costly than with mobile capital for our benchmark values of α and ϵ. The same correction factors to the relation between marginal cost and inflation and the welfare function must be applied for the more general price setting model of Wolman (1999).

The reason why the weight in the welfare increases is as follows. Firm specific capital implies that a given dispersion in relative quantities results in a much bigger dispersion of labour across firms. Capital is fixed at the firm level, the firm can only adjust labour to vary production. Since labour has decreasing marginal product in production at the level of the individual firm, the dispersion of labour across firms is welfare reducing.[22] Therefore, the weight attached to price inflation rises strongly with firm specific capital. To understand why the slope of the Phillips curve falls, consider the problem of a firm that receives a signal to change its price. By setting a lower price than the fixed price firms it can attract additional demand. But with firm specific capital marginal cost depends on the firms' own level of production while it only depends on the aggregate production level with a common rental market. Therefore, the firm will choose a relative price that deviates from unity by less when marginal cost depends on own output and therefore on its relative price. As a result the variance of price inflation is smaller with firm specific capital.[23]

Table 5 displays the welfare costs of nominal rigidities for the case of immobile capital. Whether we model capital as fixed at the firm level or assume a rental market does not have a large effect on the welfare cost for the benchmark calibration of this model. This result depends crucially on the concavity of the production function with respect to labour, i.e. on α. The welfare loss with firm specific capital increases monotonically as α approaches unity, but remains bounded when we assume a rental market. For α close to unity the welfare costs with firm specific capital can therefore be much larger than under the rental market assumption. The table shows that the loss with firm specific capital is generally somewhat lower for the policy rules considered, but the difference is not huge.

[22] If $\alpha = 0$ production is linear in labour and capital does not enter the production function. One can double check that our correction factor seems correct, by noting that it reduces to unity in that case as intuition suggests it should.

[23] See Kimball (1995) for a discussion of assumption on factor markets on price setting and Ball and Romer (1990) for the seminal paper on real rigidities.

Table 5: Welfare costs of nominal rigidities - immobile capital

rule	$\mathbb{L}$ (Calvo)	$\mathbb{L}$ (tr. Calvo)	$\mathbb{L}$ (Taylor)	$\mathbb{L}$ (Wolman)
same average duration				
Rule 1	0.190	0.145	0.066	0.111
Rule 2	0.287	0.204	0.082	0.147
Rule 3	0.129	0.152	0.152	0.184
same average age				
Rule 1	0.124	0.117	0.066	0.099
Rule 2	0.165	0.155	0.082	0.129
Rule 3	0.190	0.189	0.152	0.184

In broad terms, the picture emerging from Table 3 is similar to that from Table 5. In particular, how we model capital does not seem to matter much for the relative welfare costs of our 4 considered schemes.

7 Summary and conclusion

Our main findings have already been summarized in the introduction. Here, we only briefly sketch the papers main contributions.

Overall, we find that the welfare costs of nominal rigidities depend very strongly on the particular scheme used for introducing nominal rigidities. Contrary to the finding of Kiley (2002) for the case of trend inflation we do not find that the Calvo (1983) schemes *necessarily* overstates the welfare costs of nominal rigidities relative to contracting schemes with a smaller support of the age distribution. In the baseline parameterization of the model, Calvo (1983) contracts give rise to welfare costs of nominal rigidities that are more than 3 times as high as for Taylor (1980) contracts. However, there are monetary policy rules for which Calvo (1983) contracts *may* also imply similar or even slightly smaller welfare costs over the business cycle than Taylor (1980) contracts. The intuition for this result is as follows. When the age distribution has larger support, there are some agents that charge prices set farther in the past. Their contract price tends to be more out of line with current economic conditions. While that effect typically increases the welfare costs, those agents setting prices today will look farther into the future and also charge a price that reflects less on current economic conditions. Therefore widening the support of the age distribution as done in the Calvo (1983) scheme has an ambiguous impact on relative price dispersion and therefore on welfare.

We show how the welfare costs of nominal rigidities and the relative welfare costs across modeling schemes depend surprisingly little on capital mobility. As discussed, the reason is that immobile capital strongly increases the welfare cost of a given amount of dispersion of relative prices but also generates less equilibrium price dispersion. We finally analyze the welfare costs implied by the sticky information scheme of Mankiw and Reis (2002). We apply the same restrictions on the availability of information costs as we did in our earlier analysis on the ability to change contracts. The resulting welfare costs of the sticky information scheme however are much smaller.

Sensitivity Analysis

In this subsection we document the sensitivity analysis undertaken. First, we show how the welfare costs of nominal rigidities depend on the definition of the output gap. When the central banks reacts to a gap measure defined as the deviation of output from the steady state, we have the following table:

Table 6: Welfare costs of nominal rigidities - deviation from SS

rule	$\mathbb{L}$ (Calvo)	$\mathbb{L}$ (tr. Calvo)	$\mathbb{L}$ (Taylor)	$\mathbb{L}$ (Wolman)
same average duration				
Rule 1	0.381	0.2711	0.0953	0.1850
Rule 2	0.324	0.2167	0.0783	0.1457
Rule 3	1.043	1.2619	1.2648	1.4489
same average age in cross section				
Rule 1	0.210	0.1952	0.0953	0.1582
Rule 2	0.166	0.1541	0.0783	0.1255
Rule 3	1.635	1.6181	1.2648	1.5553

Next, we present surface plots of the welfare costs of nominal rigidities for the Calvo (1983) scheme and the of the Calvo (1983) relative to the Taylor (1980) when varying two parameters while keeping the others at their benchmark value. We show that the welfare costs of nominal rigidities are increasing in the inverse of the Frisch elasticity of labour supply χ, the coefficient of relative risk aversion σ as well as the substitutability among differentiated labour κ and among differentiated goods ϵ. The economic explanation for this has been given in the text, it is not repeated here.

However, for any value of these key parameters governing the level of the welfare cost and given our benchmark monetary policy rule the welfare costs implied by Calvo (1983) contracts are always roughly 2-4 times higher than those of Taylor (1980) contracts.

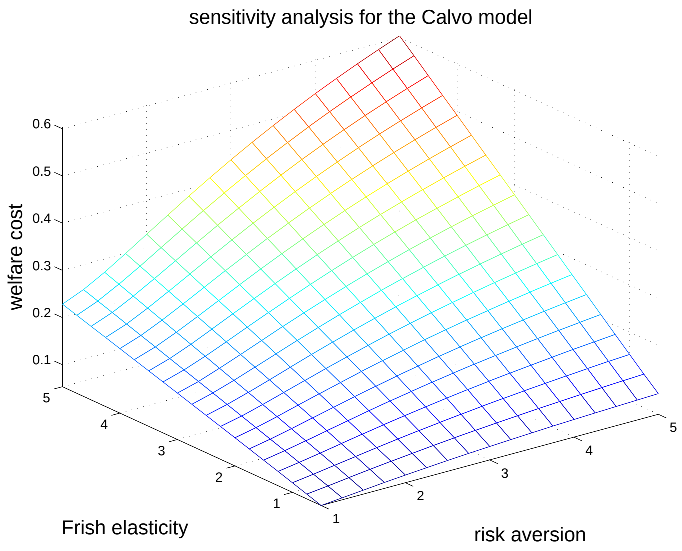

Figure 5: Sensitivity 1: Inverse of Frisch elasticity and relative risk aversion

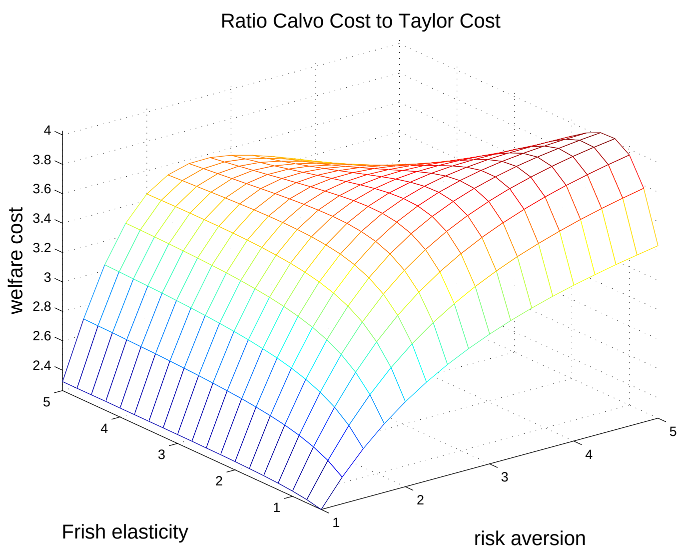

Figure 6: Sensitivity 1: Welfare costs of Calvo relative to Taylor

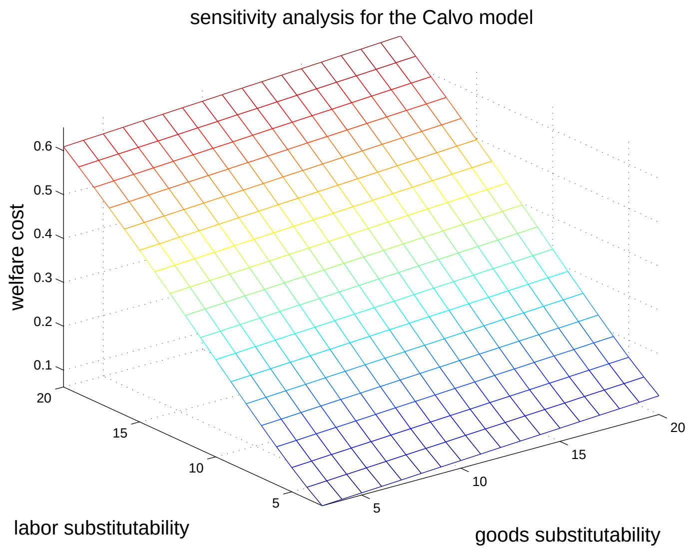

Figure 7: Sensitivity 2: Substitutability of differentiated labour and goods

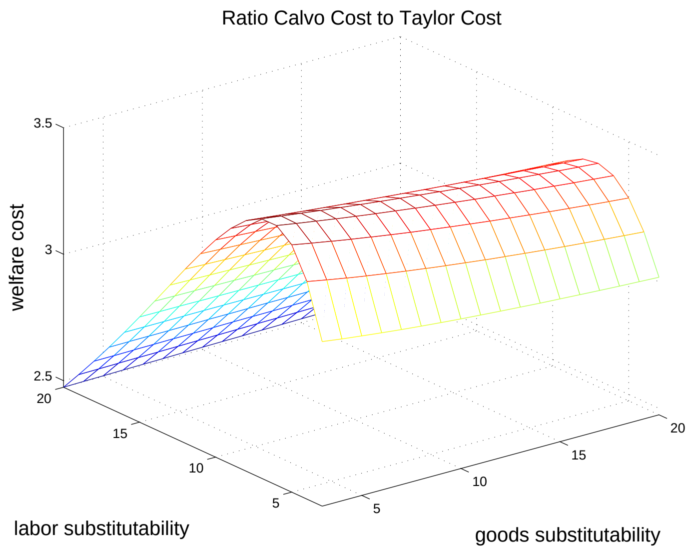

Figure 8: Sensitivity 2: Welfare costs of Calvo relative to Taylor

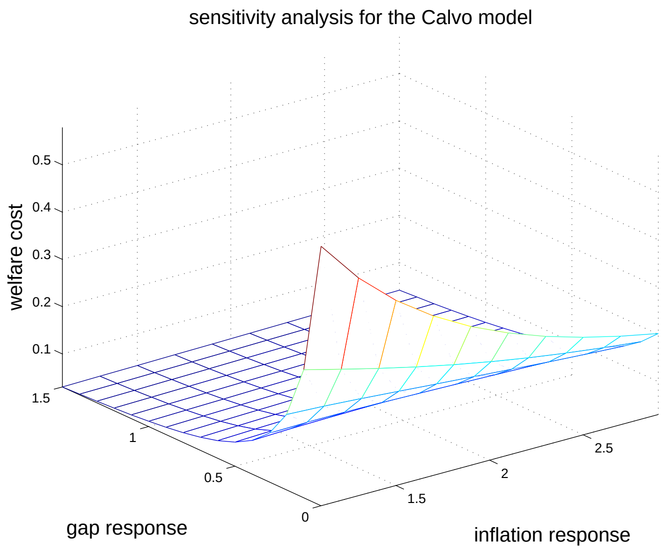

Figure 9: Sensitivity 3: Inflation response and Output gap response

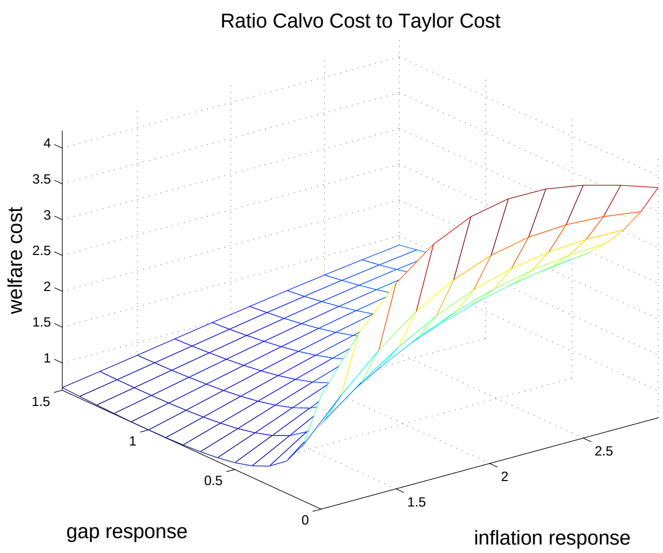

Figure 10: Sensitivity 3: Welfare costs of Calvo relative to Taylor

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