Do European business cycles look like one?*

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Abstract

This paper analyzes if each European country presents business cycles that are similar enough to validate what some authors call the European cycle. Contrary to the majority of papers on business cycles, we concentrate on the appearance of the cycle, not on the synchronization. We provide a robust methodology for dating and characterizing business cycles and their phases and adopt the model-based cluster analysis to test the existence of an unique cluster (a common cycle) against more than one. We find evidence against a common cycle. Finally, we find no clear relation between similarities in business cycle appearance and synchronization across countries.

Keywords: Business cycle characteristics, economic integration, European Union enlargement, stationary bootstrap, model based cluster analysis.

JEL Classification: E32, F02, C22

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1 Introduction

In the literature of optimal monetary unions, it is well known that joining the European Monetary Union (EMU) does not necessarily imply an improvement for each of its members. For those countries joining the union, economic benefits and costs have been measured in different dimensions: studies on international income disparities, on the redistributive effects of structural and cohesion funds, on the impact of the relative ease of interregional movements of capital, labor and products, and on the effectiveness of stabilization policies in a new environment of monetary policy, that is left to a supranational authority, and of fiscal policy, that is restricted to the achievement of close-to-balance budget constraints imposed by the stability pacts, are good representative research lines of this increasing area.

With respect to the effects of the establishment of the union to the optimal control of the countries business cycle fluctuations, a growing attention is being devoted to examine the differences and similarities among their cycles. The theoretical argument behind this reasoning is that, when business cycles are heterogeneous, decisions made at a supranational level could be optimal only for a subset of countries, while other countries, with more particular business cycles, may suffer the stabilization decisions adopted for that subgroup of countries. Remarkably, the overwhelming majority of these empirical studies has exclusively concentrated on the analysis of just one property of the business cycle dynamics: its synchronicity. According to these studies, countries with strong linkages, in terms of business cycle correlations and concordances, are expected to face smaller costs of joining the union than those countries with relatively less synchronized cycles. Among others, examples are the studies of Agresti and Mojon (2001), Artis, Krolzig, and Toro (2002), Artis, Kontolemis, and Osborn (1997), Bergman (2004), Camacho, Perez-Quiros, and Saiz (2004), Croux, Forni, and Reichlin (2001), Dueker and Wesche (2003), Guha and Banerji (1998), and Harding and Pagan (2002).

However, we think that the basic concept behind those arguments need to be complemented. Even though synchronization of national business cycles is a relevant characteristic to analyze the timing of stabilization policies, it is a necessary but not sufficient condition to determine whether business cycles are close enough to consider that the costs of the union may or may not be negligible. For instance, within the previous literature, countries with synchronized cycles do not face apparent costs of joining the union in terms of their business cycle concordance. Nonetheless, if the shape of their cycles is different, policy reactions against recessions may be too accommodative for countries with deeper recessions and too tight for countries with smoother cycles. We then consider that the evaluation of business cycle similarities across countries must be extended to the analysis of
the appearance of the cycles. To our knowledge, the attention to analyze the extent to which the business cycle characteristics are the same across European countries has been minor and not statistically solid, mainly based on the description of some features of the cycle.

We consider that our paper contributes to this business cycle literature in different aspects. On the one side, we complement the business cycle synchronization literature by offering a careful and solid statistical framework to the analysis of business cycle characteristics. For this attempt, we adapt the stationary bootstrap method proposed by Politis and Romano (1994) to the analysis of the business cycle characteristics that are described in Harding and Pagan (2002a). In our opinion, this method is key to solve most of the criticisms that the studies on business cycle characteristics have received in the past. In particular, we show that our approach is specially useful to deal with short time series and that it reduces the end-of-sample problem of other standard proposals. We also think that we contribute to the literature because we are the first to comprehensively study the business cycle of the countries recently acceded to the European Union. This seems to be of increasing importance, from the prospect that these countries will be encouraged to qualify for participation in the Monetary Union. In addition, we innovate in the statistical approach developed to analyze and compare one to one the business cycle characteristics of each country. In this respect, we employ model-based clustering methods, as outlined in Fraley and Raftery (2002), to group with statistical techniques the countries in several clusters sharing similar business cycles characteristics. This allows us to address the question of whether the European business cycles are similar enough to consider just one European cycle. Finally, we are pioneer in trying to relate the distances across business cycle characteristics for each pair of countries with the distances across business cycle synchronization.

The paper is structured as follows. Section 2 proposes an appropriate framework to deal with a comprehensive study of the business cycle characteristics. For this attempt, this section revises the most prominent characterizations of the business cycle, develops a robust methodology to examine the business cycle characteristics, and describes a procedure to analyze the similitudes among the business cycle features of different economies. Section 3 describes the data, characterizes the business cycle of our sample of countries, analyzes the existence of an European cycle, and examines the relation between similitudes in business cycle features and business cycle synchronization across economies. Section 4 concludes.
2 A framework to analyze business cycle characteristics

This section attempts to construct an appropriate framework to the analysis of the similitudes and differences among the European countries’ business cycle characteristics. First, we need to select an appropriate set of features that allows to obtain a concrete and detailed description about the form of their cycles. Second, due to the potential dependence of the business cycle characteristics to the date of the cycles, we need to propose a robust business cycle turning points dating procedure that locates when these cycles begin and end. Finally, we require an statistical framework to examine the degree of similarity among these business cycles and whether groups of countries with comparable business cycle characteristics emerge.

2.1 The key features to describe the business cycle

The empirical literature on business cycles has identified a wide variety of business cycle characteristics. In this section, we review some of the main and more popular business cycle features. In our view, these features are capable to provide a complete description of the key features of the cycle, allowing the cycles of different countries to be comparable. In this respect, one of the first attempts to establish a definition of the business cycle was the work of Burns and Mitchell (1946), whose summary statements reveal the seminal descriptions of what have been nowadays elevated to the status of business cycle empirical regularities:

Business cycles are a type of fluctuation found in the aggregate economic activity of nations that organize their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions and revivals that merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic. In durations business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles of similar character with amplitudes approximating their own.

The Burn and Mitchel’s classical description of business cycles shows two key characteristics of the cycle, that have been highlighted in bold. The first one is referred to the business cycle duration of the fluctuations.\footnote{Note that, cycles shorter than one year are not considered as business cycle cycles but as seasonal cycles.} The duration of an expansion corresponds to the time spent between the trough, that is the lowest level of activity and marks the end of a recession, and the following peak, that is the highest point of activity and marks the end of an expansion. Similarly, the
duration of a recession is the time spent between a peak and the following trough. The second classical feature of the cycle is its amplitude. Although it entails some more ambiguity, Moore (1967) gives one of the clearest interpretations of the Burns and Mitchell’s description

... if a long expansion is interrupted by a decline, the decline should be recognized as a contraction if, and only if, it is as large as the smallest contraction in the historical record...

Moore is arguing that a decline may not be considered as a trough until it reaches certain depth. Then, the concept of amplitude refers to the profundity of the descent or ascent in the economic activity in terms of gains or losses in production, and consequently it has nothing to do with the timing of the cycle. Harding and Pagan (2002a) proposed two measures of the depth of expansions and recessions in cycles, the amplitude and the cumulation of expansions and recessions. The amplitude compares the logarithm of the log level of production at the turning points of the phase. In the case of expansions, the amplitude represents the percentage gained in terms of production during the period of expansion, and in case of recessions, the measure may be interpreted as the percentage lost. The other measure proposed by these authors, the cumulation, corresponds to the cumulated gain or loss and consists on the sum of the amplitudes for each period of the phase. This measure can be interpreted as the loss in wealth in that economy. The intuition behind these two measures can be easily understood with an example. Let the log level of production in a peak and a trough be equal to 100 and 75, respectively. In this recession there has been a loss of production of 25%. In addition, if this recession lasted only one period, the loss in wealth (measure as the accumulation of production) during the recession would also be of 25%. However, if it was a two periods recession, and assuming, for example a production log level of 80 for the first period of the recession, the cumulated loss in this case would be of 45%. As we can understand from this example, cumulation is a measure that combines duration, amplitude and shape of the expansions or recessions and tries to approximate the effect of the business cycle phase on the wealth. Figure.1 provides a stylized presentation of the actual path followed by a measure of production in recessions (Chart 1) and expansions (Chart 2). The bases and heights of the triangles represent the duration and amplitude of typical business cycle phases. The shaded areas refer to the cumulated gain or loss, whose discrete approximation is the sum of the amplitudes in each period of recessions and expansions, respectively.

Another important dimension in the study of the business cycle appearance is the shape of the expansions and recessions. In particular, it has special relevance to compare if they are more or less

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2In the last scenario, the lost (measured as amplitude) of the first period is 20% and the lost of the second one is 25%. Therefore, in terms of wealth, the recession produces a cumulated lost of 45%.
abrupt at the turn of the phase than at the end. To consider this aspect of the cycle, Harding and Pagan (2002a) develop a measure, called excess, that measures the departures of the actual path from the hypothetical path of the series if the transition between two consecutive turning points would have been linear. Defined in this way, the excess becomes an intuitive approximation to the second derivative of the log level of production, and thus, allows us to study the concavity or convexity of the cycle. To illustrate the relation between the sign of the excess, and the shape of the cycle, Figure 2 depicts the stylized pictures of typical expansions (top charts) and recessions (bottom charts). Convex (concave) actual paths are characterized by positive (negative) slopes and excesses, that are represented by the shaded areas. In addition, this may be related to the degree of abruptness with which production enters to and exits from turning points. In convex expansions and concave recessions, actual paths exhibit gradual changes in the slope at the beginning of the phase, but they become abrupt as the end of the phase comes. By the contrary, in concave expansions and convex recessions, actual paths starts the phase with steep changes and ends the phase smoothly.

In definitive, in the characterization of business cycles and their phases, we consider three relevant dimensions: length, depth and shape, that may be approximated by the measures of duration, amplitude, cumulation and excess. However it is straightforward to understand that all of them rely on an appropriate chronology of expansions and recessions. In the next subsection we revise several methods that allow us to mark the date of the expansions and recessions. Based on stationary bootstrap techniques, we propose a method of computing business cycle characteristics from business cycle dating rules that deals with some of the criticisms that these studies have received in the literature.

2.2 Dating the business cycle turning points

The primary step in analyzing the business cycle characteristics is to identify an appropriate chronology of turning points. In the US, the National Bureau of Economic Research (NBER) Business Cycle Dating Committee has been dating the expansions and recessions for the last fifty years and its decisions have been generally recognized as the official business cycle dates. On the contrary, in other countries, there is no widely accepted reference chronology of the classical business cycle.

Dating the turning points in countries other than US has been the source of many initiatives,
that can be broadly classified as nonparametric and parametric. Probably, the most popular ones are the nonparametric dating algorithms, since they have the attractive feature of not being dependent on applying an arbitrary parametric approach. Inside this category, the simplest rule has been proposed by Shiskin (1974), who suggest to consider that two consecutive negative GDP quarterly growth rates would determine the onset of a recession. In an attempt to stay close to the NBER when choosing the turning points, Bry and Boschan (1979) develop an algorithm that isolates the local minima and maxima in a series, subject to reasonable constraints on both the length and amplitude of expansions and contractions.\(^4\) Among other authors, Artis, Kontomelis, and Osborn (1997), Harding and Pagan (2002b), and Artis, Marcellino, and Proietti (2004a) suggest alternative refinements of the Bry-Boschan seminal dating algorithm. On the other hand, dating the turning points through parametric models has gained considerable attention during the last fifteen years. Among the possible parametric specifications, the most widely used has been the non-linear Markov switching specification of Hamilton (1989), while other alternatives as the threshold autoregressive processes of Tsay (1989), and the smooth transition autoregressive models of Teräsvirta (1994) have also been employed to distinguish the different phases of the business cycles.\(^5\)

Choosing a method among all of these proposals does not seem to be an easy task as long as none of them is exempt from problems.\(^6\) In any case, it turns out that the final decision about the turning point chronology, and consequently, about the conclusions on the business cycle characteristics, substantially relies on the dating mechanism adopted in the analysis. In this respect, Krozlig and Toro (2002) show that dating the turning points from nonparametric and parametric models may lead to major contradictions for different European economies.\(^7\) In addition, the same conclusion can be obtained when comparing between two different nonparametric dating rules, as documented, for instance, in the different turning point dates established for Hungary in independent studies by Camacho et al. (2004) and Artis et al. (2004b).\(^8\) Finally, as Artis et al. (2004a) conclude, nonparametric dating rules may face a high degree of uncertainty surrounding the signal estimates of some turning points.

\(^4\)For example, they enforce minimum lengths of expansions and recessions, and ensure that peaks and troughs alternate.

\(^5\)For a comprehensive coverage on major parametric techniques in business cycle regime switches identification, we refer interested readers to Camacho and Perez-Quirós (2002).

\(^6\)Nonparametric models have been criticized for using somehow ad hoc dating rules while parametric models have the inconvenience that make all the business cycle analysis to be determined by the underlying model.

\(^7\)Their Figure 1 provides a clarifying example. For the Italian economy, the nonparametric dating algorithm shows up a recession around 1983, but it does not appear with the Markov switching dating method.

\(^8\)Camacho et al. (2004) fail to detect the mid-nineties recession of Artis et al. (2004b). However, the latter conclude in their Figure 7 that there is just slight evidence to consider this period as recession.
Besides the selection of the methodology for dating the turning points, an additional drawback of analyzing business cycle fluctuations comes from the unavailability of sufficiently large samples in European countries times series. This problem is particularly dramatic for the recently acceded countries, for which data are restricted to the beginning of the nineties. This implies that the samples comprehend very few complete cycles (two or three in most cases) making impossible the statistical inference and therefore, not allowing a clear comparison across economies.

Why do these two problems may invalidate the analysis of the business cycle characteristics? Given the short size of the sample periods, if, for example, a long expansion is interrupted by a mild recession, captured by some methodologies but not for others, the business cycle characteristics associated with the different statistical methods are going to be dramatically different (even though the actual dating might be strongly correlated). In addition, if the sample is short, we might loose a valuable amount of information in the tails of the time series since we are not able to locate the first and the last turning points.

To overcome these drawbacks, a reasonable solution may be found in bootstrapping the original series. Originally proposed by Efron (1979), the bootstrap method consists on randomly resampling with replacement from the initial sample of observations and is ususally employed to calculate sampling errors and confidence intervals for the statistics. In our case, we are going to show how the bootstrap technique may be used to improve robustness in the analysis of business cycle characteristics. In the case of time series analysis, the bootstrap procedure should involve resampling methods to form pseudo-time series that retain the autocorrelation structure of the original data, basically by bootstraping the original series in blocks. Among the several methods developed in statistics for time series, we use the stationary bootstrap resampling scheme of Politis and Romano (1994) because this method is relatively less sensitive to the choice of the block length than other standard moving blocks bootstrap. The stationary bootstrap method consists on blocks bootstrap in which the first observation in each block is sampled from a discrete uniform distribution on \( \{1, \ldots, T\} \), where \( T \) is the sample size. The block length, \( l \), is randomly sampled from a geometric distribution, whose density function is

\[
P(l = k) = (1 - p)^{k-1} p, \quad (1)
\]

for \( k = 1, 2, \ldots \), and some \( p \in [0, 1] \), that refers to the probability of of incorporating one observation to the block.\(^9\) In this case, the expected size of each block is then given by

\[
E(l) = (1 - p)^{-1}. \quad (2)
\]

\(^9\)These authors show that the stationary bootstrap method leads to consistency and weak convergence of the resampling.

\(^{10}\)Note that, the expected size of each block is \( (1 - p)^{-1} \).
In short, the proposed way of using stationary bootstrap to compute the business cycle characteristics consists of generating, let’s say 10,000 bootstrapped time series from the original data. Each of these series comes from a concatenation of blocks of random size $l$. Now, we apply the Bry-Boschan algorithm to produce their respective 10,000 business cycle turning points chronologies. Each of them serves the basis for calculating one point estimate of the empirical distribution of the statistics that we have previously selected to identify the business cycle characteristics. Averaging from their empirical distributions mitigates the effects of spuriously selecting mild recessions from the dating algorithms to the characteristics of countries with time series with limited size. The problems related to the impossibility of making inference because of the short number of complete cycles and to the loss of the information for not using the first and the last observations are also solved.

This method may help us to overcome the main problems that are traditionally associated to the dating methods that form the basis to compute the business cycle characteristics. First, the stationary bootstrap may mitigate the dependence of the business cycle characteristics to the selected dating method. The difficulty of deciding across methods does not come from the fact that the dating rules are dramatically different from each other, but from the fact that, given the limited amount of whole phases available in standard time series, changing the characteristics obtained from one of these methods may seriously change the empirical results. Using stationary bootstrap, different algorithms may produce different results in some atypicals (of the 10,000 bootstrapped samples). However, we should expect the majority of the samples to retain the characteristics of the data generating process. Therefore these atypical characteristics are expected to be averaged out leading all the dating algorithms to give rise to similar business cycle characteristics. Second, due to the short number of observations that are available to the researchers, inference is prohibitive for business cycles characteristics that are obtained from standard dating rules. In addition, standard methods face the problem of losing the valuable information that is associated to the beginning and to the end of the sample. It is straightforward to see that these problems do not appear within the stationary bootstrap framework.

Even though the idea of bootstrapping might look interesting for business cycle research, we have to propose an experiment to illustrate that the stationary bootstrap is an appropriate tool for the analysis of business cycles characteristics. The main purpose is to show that a set of countries with randomly selected observations from the same data generating process may present dramatically different characteristics when looking at the observed realizations but not such as

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11 Among the nonparametric dating methods, we select the Bry-Boschan algorithm since it is the easiest to implement in our 10,000 replications.
important differences when looking at the bootstrapped samples. In addition, we want to show that the bootstrapped characteristics remain closer to the ones associated with the data generating process that have been used to sample the original data. For this attempt, we first apply the Bry-Boschan algorithm to our sample countries and compute the within expansions and within recessions averaged values of duration (41 and 14 months), amplitude (0.15 and −0.12), means (0.005 and −0.007), and the standard deviation (0.001). The averaged estimates of the probabilities of staying in expansions and recessions have been 0.976 and 0.940, respectively. Then, according to these averaged values obtained for our sample countries, we generate 10 samples of 200 observations from a Markov switching process.\textsuperscript{12}

In a first step, we proceed to date the ten generated series by means of the Bry-Boschan algorithm and then, to obtain the duration, amplitude and excess of recessions and expansions. The resulting characteristics, that are presented in the first ten rows of Table 1, show that, in spite of the fact that all of them have been generated from the same generating data process, there are considerable differences among them. The range of variation of the business cycle characteristics is usually larger than twice their expected values, leading in some cases to business cycle characteristics that clearly misrepresents the actual characteristics of the data generated process. For example, in the fifth generated series expansions are much longer, deeper and sharper, and recessions are much shorter, and smoother than in the rest of the generated samples and than in the data generating process. This example illustrates the high degree of uncertainty associated to some turning points dated with nonparametric algorithms by Artis et al. (2004a).

In a second step, we bootstrap the ten generated samples, obtaining 10,000 stationary bootstrap replications for each sample, by using a $p$ parameter of the geometric distribution of 0.976, that according to our averaged sample values, corresponds to an expected block size of 41 months.\textsuperscript{13} The resulting averaged business cycle characteristics are displayed at the bottom of Table 1. The dispersion of values for characteristics has descended dramatically, and the averaged values for all the replications are much closer to their expected values than in the case of computing these characteristics with standard methods. It is worth to note that, as in the case of the amplitude of recessions, the bootstrapped characteristic sometimes coincides with its expected value. These results show the usefulness of the stationary bootstrap to calculate robust business cycles characteristics.

\textsuperscript{12}Note that, by construction, we consider that the generated processes are linear in both phases of the cycle which lead to measures of excess equal to zero.

\textsuperscript{13}In the empirical analysis, we show that our results are robust to reasonable values of the block sizes.
2.3 Grouping countries with similar characteristics

In order to provide a complete framework to analyze the business cycle characteristics, we need to describe a principled statistical approach that allows us to summarize the results, to group countries with similar characteristics, and to test whether these countries exhibit business cycle characteristics similar enough to consider one reference cycle for all of them. For this attempt, we adopt the mixture models clustering approach described by Fraley and Raftery (2002).

To outline the strategy of clustering based on mixture models, let us consider that the population of interest may consist of $G$ different subpopulations. Given a sample of $N$ countries, let us collect the $d$ business cycle characteristics of any country $n$ in the $d$-dimensional vector $x_n$. Assume that each observation is a sample drawn from a probability distribution with joint density:

$$f(x|\tau_g, \mu_g, \Sigma_g) = \sum_{g=1}^{G} \tau_g \Phi(x|\mu_g, \Sigma_g),$$

where the $\tau_g$’s are the mixing proportions, with $\tau_g \geq 0$, and $\sum_{g=1}^{G} \tau_g = 1$, and $\Phi(x|\mu_g, \Sigma_g)$ is the $p$-dimensional Gaussian density, with $\mu_g$ and $\Sigma_g$ being its mean vector and covariance matrix, respectively. The goal of the mixture maximum likelihood method is to find the parameters $\tau_g$, $\mu_g$, and $\Sigma_g$, collected in $\tau$, $\mu$, and $\Sigma$, that maximize the likelihood:

$$L(\tau, \mu, \Sigma) = \prod_{n=1}^{N} f(x_n|\tau_g, \mu_g, \Sigma_g).$$

As the authors describe, the parameter estimates may be found through the expectation-maximization (EM) algorithm, that is a general approach to maximum likelihood in the presence of incomplete data. This algorithm initializes with an initial guess of $z_{ng}$, the posterior probabilities that country $n$ belongs to group $g$, given the maximum likelihood estimates $\tau$, $\mu$, and $\Sigma$. On the one hand, the M-step, consists on estimating the mixing proportions and means from the simple closed forms,

$$\tau_g = \frac{n_g}{N}, \text{ and } \mu_g = \frac{1}{n_g} \sum_{n=1}^{N} z_{ng} x_n,$$

with $n_g = \sum_{n=1}^{N} z_{ng}$. These authors show that the geometric properties (volume, shape and orientation) are governed by the covariances $\Sigma_g$. In particular, they propose a parametrization of the variances in terms of its eigenvalue decomposition:

$$\Sigma_g = \lambda_g D_g A_g D'_g,$$

$^{14}$In our case, we consider six business cycle characteristics that correspond to duration, amplitude and excess for expansions and recessions, respectively.
The parameter $\lambda_g$ governs the volume of the cluster. The matrix $A_g$ is a diagonal matrix such that $|A_g| = 1$, with the normalized eigenvalues of $\Sigma_g$ on the diagonal in decreasing order, and determines its shape. Finally, the matrix $D_g$ is formed by the eigenvectors of $\Sigma_g$ and determines its orientation. Due to the reduced number of sample observations, in this paper we assume that the clusters are spherical but have different volumes, that is $\Sigma_g = \lambda_g I$, where

$$\lambda_g = \frac{1}{\text{tr}(W_g)} \sum_{n=1}^{N} z_{ng} (x_n - \mu_g) (x_n - \mu_g)' .$$

In this respect, it is worth pointing out that Celeux and Govaert (1995) apply Monte Carlo simulations to show that this parsimonious version is capable of detecting many clustering structures even for small data sets. On the other hand, the E-step consists on computing the estimated posterior probabilities as follows:

$$z_{ng} = \frac{\tau_g \Phi(x_n | \mu_{ng}, \Sigma_g)}{\sum_{g=1}^{G} \tau_g \Phi(x_n | \mu_{ng}, \Sigma_g)} .$$

The EM algorithm is iterated until the relative difference between successive values of the likelihood falls below a small threshold. Finally, we assign country $n$ to group $g$ whenever the posterior probability that this country belong to group $g$ is maximum over the $G$ existing groups.

The mixture models clustering approach allows us to examine whether the countries of this paper present similar business cycle features. If these countries exhibit business cycle features that were similar enough to consider a common business cycle pattern then only one cluster should be enough to characterize their business cycle phases. On the contrary, two or more clusters would indicate the existence of separate groups with differentiated business cycle characteristics. Hence, the question of examining the similarities among the countries business cycle features may be reduced to compare two models $M_i$ and $M_j$ with $i$ and $j$ clusters, respectively. It is worth to note that standard likelihood ratio tests cannot be applied in this context due to the presence of nuisance parameters. In this respect, Fraley and Raftery (2002) base the decision of $M_i$ versus $M_j$ on the model that is more likely a posteriori. They define the Bayes factor as the ratio of the two integrated likelihoods, that is $B_{ji} = p(D|M_j)/p(D|M_i)$ and use the results of Kass and Raftery (1995) to propose that values $2 \ln(B_{ji})$ less than 2 correspond to weak evidence in favor of $M_j$, values between 2 and 6 to positive evidence, between 6 and 10 to strong evidence, and greater than 10 to very strong evidence. Finally, Roeder and Wasserman (1997) develop simulation experiments to show that, when the EM algorithm is used to find the maximum likelihood, a reliable rough equivalent to $2 \ln(p(D|M))$ is the Bayesian information criterion (BIC). And thus, this permits approximate $2 \ln(B_{ji})$ through the difference between their respective BICs:

$$2 \ln(B_{ji}) = 2 \ln(p(D/M_j)) - 2 \ln(p(D/M_i)) \approx BIC_j - BIC_i .$$
3 Empirical results

3.1 Data description

In this paper, we consider a sample of countries that covers the European countries that belong to the union prior to its recent enlargement: Belgium (BG), Denmark (DK), France (FR), Germany (BD), Greece (GR), Ireland (IR), Italy (IT), Luxembourg (IT), Netherlands (NL), Portugal (PT), Spain (ES), United Kingdom (UK), Austria (OE), Finland (FN) and Sweden (SD). In addition, with the exception of Malta for which the data were unavailable, we include the new members, that is, Cyprus (CY), Estonia (ET), Latvia (LA), Lithuania (LI), Poland (PO), Slovakia (SK), Slovenia (SL), the Czech Republic (CZ) and Hungary (HN), two negotiating countries, Romania (RO), and Turkey (TK), and four industrialized economies that are included as reference, Canada (CN), Japan (JP), Norway (NW) and the United States (US).

According to the traditional proposal of Burns and Mitchell (1946), the first best on the business cycle analysis consists on identifying the business cycles on the basis of a measure of the aggregate economic activity. This motivated us, in early versions of this paper, to construct a diffusion index of the economic activity following the lines of Stock and Watson (2002). However, owing to the lack of data availability that characterizes some of the new members of the union, we had to desist from our efforts after obtaining some misleading preliminary results. Additionally, we considered the development of experimental indexes of coincident indicators by averaging series of industrial production, personal income, sales, and employment, as proposed by Stock and Watson (1989). However, in an big proportion of countries the Kalman filter used in the index estimation assigned a negligible weight to the series other than industrial production.

Thus, we concentrate on the analysis of the (seasonally adjusted) Industrial Production (IP) index extracted from the OECD Main Economic Indicators and the IMF international Financial Statistics Databases. Note that, in contrast to the Gross Domestic Product (GDP) series, IP is available monthly, more statistically reliable, more homogeneous across countries, and covers longer samples for many countries. In addition, for many economies, GDP is not based on quarterly national accounts but annual and converted to quarterly by using indicators. Finally, our time series span from 1965:01 to 2004:03, but, due to data constraints, we need to start the sample in 1990:01 in those exercises that include the countries recently acceded.15

15 For those countries that experience atypical behaviors at their transition periods, we follow Blanchard (2003) to exclude two years of observations. For a detailed description, we refer the reader to Camacho et al. (2004).
3.2 Basic characteristics of the European business cycles

According to the previous discussion, Table 2a and Table 2b summarize the average and median values for six business cycles features, duration, amplitude and excess in expansions and recessions, computed for our set of thirty countries by means of the stationary bootstrap method on the first differences of their IP indexes.\footnote{Due to the high degree of similarity among the median (Table 2a) and the mean (Table 2b) values, it seems that the potential atypical replications does not affect the computation of the means substantially. However, to be more confident on our results, we base the analysis on the median values.} The stationary bootstrap is based on 10,000 replications with the \( p \) parameter on the geometric distribution of the block lengths of 0.97. This implies that the expected size of a typical block in the bootstrap is of 32 months, which coincides with the mode of the average duration of an expansion for a country in our sample.\footnote{Testing for the robustness of the analysis to the parameter \( p \), Appendix A shows the business cycle characteristics obtained from using different reasonable values of this parameter. According to these results, it seems that the role of this parameter is clearly marginal in defining the magnitude of the characteristics.} In addition, Figure 3 plots some charts that facilitates, through visual inspection, the description of the individual characteristics for each of these countries.

**Business cycle duration.** The median length of expansions and recessions and the percentage of time spent in each phase across countries are depicted in Chart 1 and Chart 2, respectively. It is noticeable that expansions come to last around 30 months meanwhile recessions endure about 15 months. Thus, according to a broadly accepted stylized fact in the business cycle literature, expansions appear to be longer than recessions. In concrete, a cycle spends more than 60\% of time in expansion. However, it is worthy to note that expansions have been considerably short lived in some of the countries that recently joined the union, as for example, Lithuania, Latvia and Cyprus, and that recessions have been also been short in the set of non European countries included in the analysis as reference. Of noticeable interest is the particularly strong asymmetric duration between the two phases of the cycle exhibited by Ireland, Hungary and Poland for which the percentage of time spent in expansions is roughly four times of that in recessions.

**Business cycle amplitude and cumulation.** These characteristics are studied in Chart 3 and 4 that display the median amplitude and cumulative gain or loss across countries. Again we observe symptoms of asymmetries across the phases of the cycle. Expansions are generally wider than recessions, and the cumulative gain in terms of production in the expansive phase is bigger than the experimented loss during the contraction. The case of Ireland is remarkable for the great magnitude of the amplitude and the cumulative gain during expansions. Once more, Hungary, and to less extent, Poland stand out for their pronounced business cycle asymmetries. In general, countries Eastern countries show wider and more severe recessions than other European countries.
Business cycle excess. To analyze the shape of the business cycle phases, Chart 5 and Chart 6 represent the excess for expansions and recessions, respectively. Positive excess and thus, convexity seems to dominate during expansive period. This means expansions start with smooth rates of growth and end with steep ones. However, during recessive phases none of the possibilities clearly dominates the other one: it seems that half of countries presents positive excess and the other half has negative excess. In terms of the shape of the cycle, it seems that the countries of the last enlargement do not exhibit country-specific business cycles characteristics, with the exception that they usually show concave recessions.

Prior to concluding this section, we want to examine how business cycle characteristics have evolved over time. For this attempt, we repeat the same analysis developed this section for two non-overlapping subperiods, from 1965.01 to 1989.12 and from 1990.01 to 2004.03. However, owing to data availability, we exclude the newcomers from this last analysis. As Table 3 stressed, a general finding of this exercise is that, with the exception of the excess, the degree of asymmetries decreases. Comparing the two sub-periods, expansions have shortened and turned into convex and on average, cycles have become smoother as the amplitude has reduced in both phases. These findings are in line with the literature on business cycles volatility reduction, as early documented by McConnell and Perez Quiros (2000). On the other hand, on average, the median of the excess of expansions switches from negative to positive, which suggests that the shape of expansions changes from concave to convex. This result goes in line with Kim and Murray (2002), who find that the existence the of the recovery phase of rapid growth detected by Sichel (1994) is no longer present in the last expansions.

3.3 Do European business cycles look like one?

Up to this point, we have examined some stylized facts about the European business cycles and their evolution over time. As stated in Table 2, there are some general business cycle facts that are shared by the major European economies, as, for example, the asymmetric duration of expansions and recessions. However, this table also points out that some business cycle characteristics differ widely across some countries.\(^\text{18}\) In this section, we investigate the degree of heterogeneity across the European countries’ business cycle characteristics.

The first immediate question that we should address is to examine whether these countries exhibit business cycle features that were similar enough to consider a common business cycle

\(^{18}\text{A significant example comes from the comparison of the business cycle characteristics between Ireland and UK. While both countries exhibit the same excess in recessions (0.07), the amplitude of a expansion is much higher in Ireland (0.45%) than in UK (0.06%).}\)
pattern as reference. On the basis of the mixture clustering approach described in Section 2, the analysis may be reduced to check the transformation of the Bayesian factor that allows to compare the likelihoods of forming just one group of European countries sharing the same (or with no statistically distinguishable differences) business cycle characteristics with the alternative scenario of two (or more) groups of countries with separate business cycle characteristics. In order to deal with this problem, Table 4 shows the BICs and the estimated clusters for several models from $M_1$, which considers only one cluster, to $M_5$, which considers five groups.\footnote{It is worthless the estimation of models with more than five clusters as there would not be enough observations to calculate all the parameters.} Comparing the model with one cluster with the model with two cluster, the transformation of the Bayes, $2\ln(B_{21})$, is 6.7 that is higher than 6 and thus, supports the conclusion that there is strong empirical evidence against the idea of one European business cycle.\footnote{In spite of considering non-European countries in our sample the conclusion is legitimate because we have repeated the analysis taking only into account European countries (see Appendix B), and we come to the same conclusions.}

The next step is to determine the optimal number of clusters within our sample of countries. According to Table 4, the maximum BIC value occurs for the four-group model, and the difference in the BICs between three-group and the four-group models is high enough to validate that there may be four groups of European countries with cohesive and separate business cycle characteristics. In this respect, Figure 4 shows some graphics that aid us in the detection of the patterns behind the formation of these four groups, whose averaged characteristics are summarized in Table 5. In particular, to examine duration and amplitude we use the spiderweb graphs, that represent the standardized characteristics for each cluster, using the mean and standard deviation of the overall data. In this way having zero as reference, we can determined if features of countries belonging to one cluster behave above or below the sample mean. In case the typified variables were higher than zero, it would indicate that those countries behave above the global mean in these variables. The same reasoning is true on the contrary case. However, to analyze the excess, we use bar graphs without standardizing since we are only interested in the sign of excess as approximation to the second derivative.

The first cluster is formed by some EU-enlargement countries, Cyprus, Estonia, Latvia, and Lithuania, and the two candidates, Romania and Turkey. According to the graphs plotted in Chart 1, it seems that the main characteristics of this group are the short duration of their expansions, that with the exception of Turkey are below the mean (represented by a thick hexagon), and the high amplitude of their recessions. Their expansions last 26 months and their recessions 17 on average. The amplitude of the expansions in absolute value is similar to the amplitude of the
recessions. This implies severe recessions to the extent of destroying the gain of expansions. The
second group includes United States, Canada, some Nordic countries and two EU-enlargement
countries, Slovakia and the Czech Republic. From the visual inspection of Chart 2, their cycles
are characterized by short and smooth recessions, and by convex expansions. In particular, they
have expansions of 33 months and recessions of 13 months. The amplitude of their expansions
is, in absolute value, twice the amplitude of their recessions. The excess in expansions is very
positive, that implies convex expansions (smooth at the beginning and abrupt at the end). So, this
group has long and deep expansions in relation to recessions. The third cluster, which contains
the majority of EU-15 countries, is formed by economies with low amplitude of both expansions
and recessions, as stated in Chart 3. These countries present a mean duration of expansions of
28 months and recessions of 18 months. In absolute value, the amplitude of expansions is slightly
superior than in recessions, but in general, both are very mild. The last cluster incorporates
Ireland, Hungary and Poland, that exhibit the more atypical business cycle characteristics. As
documented in Chart 4, their expansions are very long, wide, and convex, and their recessions very
short. In particular, their expansive phase lasts on average 44 months and contractions only 9
months. In these countries, expansions exhibit an amplitude whose magnitude is more than three
times that in recessions, and are very convex. Hence, they have obtained extreme positive benefits
from their expansions in the last years.

Finally, from our set of countries, we can obtain a simple measure of the similarity in appearance
of business cycles across these countries simply by calculating the Euclidean distance among their
characteristics.\textsuperscript{21} With these distances, we can apply Multidimensional scaling techniques (see
Cox and Cox, 1994) to represent the countries on a plane (such as a map). In this way, the
representation depicted in Figure 4 gives us a glimpse of how close are the cycles across countries
attending their similitudes in duration, amplitude and shape. The color circles represent the four
groups resulted from the cluster analysis. It is important to mention that, according to our previous
results, the fourth group is a little remote from the others reflecting that their business cycles are
somehow atypical.

In summary, we find no evidence in favor of the existence of just one European business cycle.
Cycles of European countries do not seem to be as similar as if they were only one cycle. Cluster
analysis identified four groups of countries attending to their similitudes in business cycles appear-
\textsuperscript{21} Considering \( x_{ij} \) the \( i \)-th characteristic of the \( d \)-dimensional vector of business cycle characteristics of country \( j \),
the Euclidean distance between countries \( A \) and \( B \) is:

\[
d_{A,B} = \sqrt{\sum_{i=1}^{d} (x_{i,A} - x_{i,B})^2}. \tag{10}\]

17
ance. Broadly speaking, the core EU countries belong to the same group and Ireland, along with Hungary and Poland, is a very exceptional case and is isolated from the rest of EU-15 countries. Countries that are neither EU countries nor accessing countries, as United States and Canada, form another group that is close to some Nordic countries, Spain and Luxemburg. Finally, the EU-enlargement countries are distributed among these four groups which reflects the high degree of heterogeneity among the business cycles of these countries.

3.4 Relation between similitudes in synchronicity and characteristics

Coming back to the issues that motivate this paper, we think that the literature on business cycles is devoting a disproportionate attention to business cycle synchronization in contrast to other characteristics that describe the form of the business cycle such as its duration, amplitude, and shape. In this respect, we consider that it may be interesting to study the potential relation between the two class of business cycle similitudes. For this attempt, we try to relate the results in present paper with our findings in our companion paper, Camacho et al. (2004), that analyzes the synchronicity across the business cycles of the same set of countries.

The country-by-country correlation coefficient between the synchronicity distances and the Euclidean distances of business cycle appearance (calculated as shown in the previous footnote) variables is 0.32. As expected, positive and statistically significant but substantially smaller than the value associated with a perfect correlation. It seems that, at least partially, these two distances move together but that there is a strong component of independent movement between them. In order to analyze which variables are common in explaining the behavior of these two distances, we use the variables selected in Camacho et al (2004) as explanatory of the synchronicity across economies and estimate the model by using the instrumental variable method. These variables are trade linkages, difference in the proportion of the industry sector in GDP, difference in public balance as proportion of GDP, difference in saving ratios, difference in the proportion of the agricultural sector in GDP and difference in labor productivity. We find that only trade and the proportion of agriculture are also significant in the equation of the characteristics. Some other variables as the public deficit even have the opposite sign as they obtained in the synchronicity equation. Finally, to conclude, we include in the synchronicity equation the distances across economies in their characteristics. The result is that, after controlling for all the explanatory variables that explain the distances across economies in synchronicity, using the same instrumental variables estimation, the distance in the characteristics are significant (t statistic of 3.15) although they only explain around 2% of the variance of the synchronicity distances. Therefore, business

22 For a description of the instruments see Camacho et al (2004).
cycle synchronization is no longer enough to consider that international business cycle are or are not similar, we need to include another dimension, the characteristics of the business cycle.

4 Conclusions

In this paper, we provide a comprehensive framework to analyze business cycle characteristics across a large set of countries with potential problems of data availability. First, we examine the minimum set of characteristics that are able to offer a complete description of the cycle. Second, we show how stationary bootstrap methods may be used to obtain robust business cycle characteristics from a time series. Our proposal minimizes typical problems of other studies on business cycles, such as the dependence of the results to the choice of a dating rule, and the short number of complete cycles observed in most of the countries. Finally, we adopt from other scientific disciplines an statistical method, the model based clustering approach, that allows us to put some new lights on the question about the existence of a cycle that might be representative of whole Euro economic area.

Applied to a set of countries of EU, EU-enlargement, and other industrialized countries that are used as reference, we find statistical evidence in contrast to the existence of one European cycle whose characteristics would be shared by the European countries. In addition, in the last decade we observe smoothing of cycles, and shorter and less abrupt expansions on average. Finally, we do not find a clear relation between similitude in business cycle synchronization and appearance.
References


Table 1. An experiment with stationary bootstrapping time series

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Notes. The observed data correspond to 10 generated samples from the same Markov switching process of 200 observations, with probability of expansion, probability of recession, within expansion mean, within recession mean, and standard deviation of 0.976, 0.940, 0.005, -0.007, and 0.001, respectively. The bootstrapped results correspond to 10,000 bootstrap replications of each these generated samples.
<table>
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Table 2a. Business cycle characteristics from stationary bootstrap

Median from 10,000 replications.
Table 2b. Business cycle characteristics from stationary bootstrap

Mean from 10,000 replications.

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Notes. Business cycle characteristics and the stationary bootstrap method are specified within the text. The probability parameter of the probability of geometric distribution is 0.97.
Table 3. Evolution of business cycle characteristics

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Notes. Business cycle characteristics and the stationary bootstrap method are specified within the text. The probability parameter of the probability of geometric distribution is 0.97.
Table 4. Determination of the number of clusters

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Notes. BIC refers to the Bayesian Information Criterion. B_{ji} is the Bayes factor.

Table 5. Average of business cycle characteristics for each cluster

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<td>Excess</td>
<td>Duration</td>
<td>Amplitude</td>
<td>Excess</td>
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Notes. Acronyms for these countries are specified in Section 2.
Appendix A.  Sensitivity Analysis to different values of $p$ for the geometric distribution

Table A.1. Median duration of expansions and recessions from 10,000 replications

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Table A.2. Median amplitude of expansions and recessions from 10,000 replications

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Table A.3. Median excess of expansions and recessions from 10,000 replications

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### Appendix B. Cluster analysis for European Union countries

#### Table B.1. Determination of the number of clusters

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<th>Model</th>
<th>BIC</th>
<th>$2 \times \ln(B_{ji})$</th>
<th>Decision</th>
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<tr>
<td>$M_1$</td>
<td>1 group</td>
<td>-459.40</td>
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<tr>
<td>$M_2$</td>
<td>2 groups</td>
<td>-460.29</td>
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<tr>
<td>$M_3$</td>
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Notes. BIC refers to the Bayesian Information Criterion. $B_{ji}$ is the Bayes factor.

#### Table B.2. Average of business cycle characteristics for each cluster

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<th>Clusters</th>
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<th></th>
<th>Recessions</th>
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<td></td>
<td>Duration</td>
<td>Amplitude</td>
<td>Excess</td>
<td>Duration</td>
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<td>IR, HN, PO</td>
<td>44.11</td>
<td>0.35</td>
<td>0.61</td>
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</tbody>
</table>

Notes. Acronyms for these countries are specified in Section 2.
Figure 1: Duration, amplitude, and cumulation

Chart 1. Recession

Chart 2. Expansion

Notes. Stylized representation of typical recessions (Chart 1) and expansions (Chart 2).

Figure 2: Excess cumulative movements and the shape of the cycle

Notes. Stylized representation of typical expansions (top charts) and recessions (bottom charts).
Figure 2: Analysis of the individual business cycle characteristics

Chart 1. Bar graph of duration of expansions (green) and recessions (red)

Chart 2. Percentage of time expended in expansions (green) and recessions (red)

Chart 3. Bar graph of amplitude of expansions (green) and recessions (red)
Figure 2: Analysis of the individual business cycle characteristics

Chart 4. Cumulation of expansions (green) and recessions (red)

Chart 5. Excess of expansions

Chart 6. Excess of recessions
Figure 3: Cluster analysis

Chart 1. Group 1: CY, LA, LI, ET, TK, RO

Chart 2. Group 2: OE, LX, FN, SD, DK, US, ES, CN, CZ, SK
Notes. Acronyms used for the countries are specified in Section 3.
Figure 4: Map of business cycle characteristics

Notes. Acronyms used for the countries are specified in Section 3. This map is the multidimensional scaling map based on the euclidean distance of the business cycle characteristics.