Can Miracles Lead to Crises? An Informational Frictions Explanation of Emerging Markets Crises*

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Abstract
Emerging market financial crises are abrupt and dramatic, usually occurring after a period of high output growth, massive capital flows, and a boom in asset markets. This paper develops an equilibrium asset pricing model with informational frictions in which vulnerability and the crisis itself are consequences of the investor optimism in the period preceding the crisis. The model features two sets of investors, domestic and foreign. Both sets of investors are imperfectly informed about the true state of the emerging economy. Investors learn from noisy signals which contain information relevant for asset returns and formulate expectations, or “beliefs”, about the state of productivity. Numerical analysis shows that, if preceded by a sequence of positive signals, a small, negative noise shock can trigger a sharp downward adjustment in investors’ beliefs, asset prices, and consumption. The magnitude of this downward adjustment and sensitivity to negative signals increase with the level of optimism attained prior to the negative signal. Moreover, with the introduction of informational frictions, asset prices display persistent effects in response to transitory shocks, and the volatility of consumption increases.

JEL Classification: F41, D82, G15
Keywords: financial crises, emerging markets, informational frictions, learning

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1 Introduction

...That this region [East Asia] might become embroiled in one of the worst financial crises in the postwar period was hardly ever considered-within or outside the region-a realistic possibility. What went wrong? Part of the answer seems to be that these countries became victims of their own success. This success had led domestic and foreign investors to underestimate the countries economic weaknesses. It had also, partly because of the large scale financial inflows that it encouraged, increased the demands on policies and institutions, especially but not only in the financial sector; and policies and institutions had not kept pace. The fundamental policy shortcomings and their ramifications were fully revealed only as the crisis deepened... IMF (1998)

The experience of the last decade suggests that emerging capital markets are vulnerable to significant shifts in investors’ confidence in both upward and downward directions. Downward shifts in confidence and financial market collapses are abrupt and often take place unexpectedly after a large boom. Table 1 documents the magnitude of these booms for several pre-crisis episodes: Argentina and Mexico in 1994, Korea in 1997, and Turkey in 2000. Taking Turkey as an example, the year before its financial crisis in 2001, the country boasted an average quarterly current account-to-GDP ratio of -5.1%, consumption growth of 4.5%, an increase in equity prices of 57% and GDP growth of 3%.1

It is widely agreed that overconfidence and informational problems are at least partially responsible for recent crisis episodes, as the above opening quote by International Monetary Fund on the Asian crisis suggests. Whether these frictions in international capital markets can be large enough to explain pre-crisis periods of bonanza and the depth of the crises remains an open question.

In this paper, we aim to answer this question by studying the quantitative predictions of a model in which optimism, due to investors’ underestimation of the weaknesses of emerging economies, acts as the driving force behind both the pre-crisis booms and the vulnerability that paves the way to financial turmoil and deep recessions. In the model, the pre-crisis bonanza is driven by a sequence of positive signals that investors interpret as an improvement in the true fundamentals of the economy. The crisis occurs as a sudden downward adjustment in investors’ expectations of the true fundamentals is triggered and their optimism suddenly fades.

1Calvo and Reinhart (2000) conclude that “Sudden Stops,” sharp negative reversals of capital flows, are usually preceded by a surge in capital inflows.
Furthermore, the magnitude of the adjustment increases with the level of optimism attained prior to the crisis.

Table 1: Magnitudes of pre-crisis booms

<table>
<thead>
<tr>
<th>Episode</th>
<th>GDP (%)</th>
<th>Private Consumption (%)</th>
<th>Equity Price (%)</th>
<th>CA/GDP (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina, 1994Q1-Q4</td>
<td>1.72</td>
<td>2.67</td>
<td>12.97</td>
<td>-1.08</td>
</tr>
<tr>
<td>Mexico, 1994Q1-Q4</td>
<td>3.43</td>
<td>6.69</td>
<td>18.53</td>
<td>-2.00</td>
</tr>
<tr>
<td>Korea, 1996Q4-1997Q3</td>
<td>3.67</td>
<td>5.14</td>
<td>1.04</td>
<td>-3.69</td>
</tr>
<tr>
<td>Turkey, 2000Q1-Q4</td>
<td>3.08</td>
<td>4.51</td>
<td>57.30</td>
<td>-5.12</td>
</tr>
</tbody>
</table>

The informational frictions that are the key ingredient of the model, are likely to be prevalent in emerging markets for several reasons. One is the lack of transparency in policy-making, and data reporting which manifests itself in the form of inaccurate or misleading data. In a report, the International Monetary Fund argued that this was a common thread running through several recent crisis episodes:

... A lack of transparency was a feature of the build-up to the Mexican crisis of 1994-95 and of the emerging market crises of 1997-98. In these crises, markets were kept in the dark about important developments and became first uncertain and then unnerved as a host of interrelated problems came to light. Inadequate economic data, hidden weaknesses in financial systems, and a lack of clarity about government policies and policy formulation contributed to a loss of confidence that ultimately threatened to undermine global stability ...(2001)

A second reason informational frictions pose particular challenges for emerging economies is the existence of high fixed costs associated with obtaining country-specific information and keeping up with the developments in emerging economies, as suggested by Calvo (1999). Such costs could arise due to idiosyncrasies affecting financial markets in these countries, including for example, each country’s unique institutions, policies, political environment, legal structure, etc. In addition, it might be optimal for international investors not to “buy” this information. Calvo and Mendoza (2000) provide two arguments for why this can be the case. First, if short

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2 Average quarterly changes in GDP, private consumption, equity prices and average quarterly current account-to-GDP ratios. GDP, and consumption are in constant prices, equity prices are in local currencies and are deflated using the CPI. Source: International Financial Statistics and corresponding countries’ central banks.
selling positions are limited, the benefit of paying for costly information declines as the number
of emerging economies in which to invest becomes sufficiently large. Second, if punishment
for poor performance is high, managers of investment funds may choose to mimic each other’s
behavior instead of paying for costly information.

The model in this paper features two types of investors, domestic and foreign, both of whom
trade a single emerging market asset. Domestic investors are consumer-investors who maximize
the expected present discounted value of their lifetime utility. Foreign investors specialize in
trading the emerging market asset, face trading costs, and maximize the expected present dis-
counted value of profits from investing. We model the informational frictions as follows. Both
sets of investors are imperfectly informed about the true state of current productivity, which
contains information relevant for predicting future returns on the emerging market asset. They
can only partially infer the true state of productivity by “learning” from publicly observed divi-
dends (or signals) and, they share the same information set. The dividends consist of two parts:
a persistent component, which we interpret as “true productivity”, and a transitory component,
which is a noise term that controls the accuracy of the signals. Modeled in this way, dividends
serve an informational role since a dividend payment is a noisy signal that contains information
about current and future realizations of productivity. Every period, foreign and domestic in-
vestors observe dividends, solve a signal extraction problem, and “learn” about productivity by
updating their expectations or “beliefs” regarding true productivity.

When investors turn pessimistic (optimistic), asset prices are driven below (above) the “fun-
damentals price,” which is defined as the expected present discounted value of dividends con-
ditional on full information. In these periods, asset prices and domestic investors’ consumption
display swings that are not associated with changes in true productivity. We find that a se-
quence of positive signals can cause a boom in both the asset market and in consumption, and
can be a source of economic vulnerability if true productivity is in fact low. If a negative signal
is realized at the peak of a boom of this nature and, as a result, “challenges” current prevailing
beliefs, an abrupt and large downward adjustment in asset prices and consumption takes place.
If, however, the same signal “confirms” prevailing beliefs, its impact is smaller.\footnote{Moore and Schaller (2002) establish the state dependence of responses to noisy signals. We borrow our terminology from them.}

Foreign and domestic investors trade due to differences in their objective functions particu-
larly their risk aversions, but not for speculation (given that they have the same beliefs). From
the domestic investors’ perspective, dividend shocks are important for two reasons. First, in order to intertemporally smooth consumption domestic investors would like to increase (decrease) their asset position in response to positive (negative) dividend shocks. Second, they play a critical informational role. In response to a negative dividend shock, changes in expectations due to the new information compounds the first effect, and as result, domestic investors reduce their demand for the emerging market asset. Foreign investors also reduce their demand for the asset in response to this shock, since they receive a negative signal regarding future productivity. In equilibrium, we find that domestic investors’ demand decreases by more than that of their foreign counterparts, therefore, domestic investors become net sellers in response to a negative dividend shock. This result leads to a procyclical current account on average. However, we also find that for a given dividend shock, the higher the expectations about future productivity, the lower are the domestic investors’ asset holdings since higher expectations induce foreign investors to bid more aggressively, compared to their risk-averse domestic counterparts, for the same asset. Hence, the higher the investment optimism, the more the emerging economy can attract foreign investment, and therefore the more likely the country is to develop a potentially sizable current account deficit. For a given dividend shock, the model can thus produce a current account deficit and booms in consumption and asset prices if investors are “sufficiently optimistic”.

The numerical analysis shows that with the introduction of informational frictions, the volatility of the emerging economy’s consumption increases by 2 percentage points compared to the “full information” setup. Uncertainty about true current productivity leads to increased uncertainty regarding future asset returns and a more volatile consumption profile for the risk averse domestic investors. Moreover, informational frictions produce persistence in response to transitory noise shocks. If investors turn pessimistic (optimistic) in response to a misleading signal, it takes several periods for them to correct their beliefs. The mechanism behind this result is the Bayesian learning process: the posteriors of one period are used in the calculation of the following period’s priors.

This paper is at the crossroads of two main strands of literature. The first is the literature on Sudden Stops and financial crises in open economies, and the second is that on informational frictions in finance. Most existing models of financial crises and Sudden Stops, focus on crash episodes, but not on the booms preceding the crashes that might indeed contain the seeds of the financial crises. In contrast, the model proposed in this paper accounts for the boom-bust cycles observed in emerging markets. Studies explaining Sudden Stops focus on financial frictions
and often utilize collateral constraints, (see, for example, Caballero and Krishnamurthy (2001), Paasche (2001), or Mendoza and Smith (2004)). Credit constraints are successful for producing amplification in the response of the economy to typical negative shocks. In this paper, however, business cycles can also be driven by changes in investor sentiment and amplification is at work in expansions as well as in recessions.

In the international finance literature, shifts in investor sentiment have usually been analyzed within the context of currency crises, often using sunspot models that produce multiple equilibria. In this paper, we take a different approach by considering a model with a unique equilibrium that can endogenously produce shifts in investors’ confidence and switches between good states and bad ones which allows us to predict when these shifts occur and how long it takes for the market to recover after a bust.

This paper is also related to the literature on learning in macro and finance. Particularly, Wang (1994), models dividends as noisy signals to analyze trading volume in stock markets, Albuquerque, Bauer and Schneider (2004) use noisy dividend signals to investigate the effects of investor sophistication on international equity flows, and Nieuwerburgh and Veldkamp (2004) use them to explain U.S. business cycle asymmetries in an RBC framework with asymmetric learning.

The rest of the paper proceeds as follows. We describe the model in Section 2, and in Section 3 we discuss the model’s solution procedure, calibration, and numerical results. Finally, Section 4 concludes.

2 Model

The economy has two classes of agents, foreign investors and domestic household-investors, who are identical within each class. The domestic households maximize expected lifetime utility by making consumption and asset holding decisions conditional on their information set, that includes the noisy signals about the true state of productivity. Foreign investors choose their asset positions in order to maximize the expected present discounted value of profits based on their beliefs about the state of productivity. Foreign investors also face trading costs associated with operating in the asset market. Neither domestic nor foreign investors observe the true realization of the stochastic productivity shock, which contains information relevant for forecasting the returns from the asset. They only observe dividends, which are noisy signals about the true
value of productivity. Foreign and domestic investors form their beliefs by solving a signal extraction problem.

2.1 Domestic Households’ Problem

Domestic households choose stochastic intertemporal plans for consumption, $c_t$, and asset holdings, $\alpha_{t+1}$, in order to maximize expected life-time utility conditional on the information available to them:

$$U = E_0 \left[ \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} | I_U^t \right]$$

subject to

$$c_t + \alpha_{t+1} q_t = \alpha_t (q_t + d_t)$$

taking asset prices, $q_t$, and the evolution of beliefs and their information set $I_U^t$ as given.\(^4\) $d_t$ denotes dividend payments of the emerging market asset, the parameter $\sigma$ is the coefficient of relative risk aversion of domestic investors and $\beta$ is the standard subjective discount factor.

At the beginning of each period, productivity shocks are realized and dividends are determined. Domestic investors make their decisions after observing dividends. The optimality conditions characterizing their decisions are:

$$\beta^t u'(c_t) - \lambda_t = 0$$

$$-\lambda_t q_t + E_t [\lambda_{t+1} (d_{t+1} + q_{t+1}) | I_U^t] = 0$$

where $\lambda_t$ denotes the Lagrange multiplier associated with the budget constraint. Combining these two first order conditions gives the Euler equation:

$$q_t u'(c_t) = \beta E_t \left[ (q_{t+1} + d_{t+1}) u'(c_{t+1}) | I_U^t \right].$$

This equation is familiar except that the expectations are taken conditional on the information set $I_U^t$.

\(^4\)We discuss the role of the expectation operator and the information structure in Section 2.3.
2.2 Foreign Investors’ Problem

As in Mendoza and Smith (2004), foreign investors choose \({\alpha^*}_{t+1}\) in order to maximize the expected present discounted value of their profits conditional on their information sets:

\[
E_0 \sum_{t=0}^{\infty} R^{-t} \left( \alpha_t^* (d_t + q_t) - \alpha^*_{t+1} q_t - q_t \frac{a}{2} (\alpha^*_{t+1} - \alpha^*_t + \theta)^2 | I_0^U \right)
\]  

(6)

where \(R\) is the gross world interest rate, \(1/a\) is the price elasticity of foreign investors’ demand, \(q_t \frac{a}{2} (\alpha^*_{t+1} - \alpha^*_t + \theta)^2\) is the total trading cost associated with buying and selling equities in the emerging economies, \(\theta\) is the recurrent cost. As in Aiyagari and Gertler (1999) and Mendoza and Smith (2004), we model the trading cost associated with buying and selling the asset as quadratic in the size of the asset trade.\(^5\) The first order condition of the foreign investors’ problem is:

\[
q_t \left(1 + a(\alpha^*_{t+1} - \alpha^*_t + \theta)\right) = R^{-1} E \left[d_{t+1} + q_{t+1}(1 + a(\alpha^*_{t+2} - \alpha^*_{t+1} + \theta)) | I_0^U \right].
\]  

(7)

We can solve the above first order condition forward to obtain:

\[
\alpha^*_{t+1} - \alpha^*_t = \frac{1}{a} \left( \frac{q_t^b}{q_t} - 1 \right) - \theta.
\]  

(8)

\(q_t^b\), called the belief price, is defined as the expected present discounted value of future dividends conditional on the current belief about productivity:

\[
q_t^b \equiv E[R^{-1}d_{t+1} + R^{-2}d_{t+2} + R^{-3}d_{t+3} + \ldots | I_0^U].
\]  

(9)

Intuitively, foreign investors adjust their asset holdings “partially” depending on the gap between the market price \(q_t\) and their belief price \(q_t^b\). How much of this gap is reflected in the asset demand is determined by \(1/a\).

\(^5\)This specification does not rule out buy & hold type of trading strategies. The foreign investors are allowed to buy and “watch” the market and sell when they find it profitable to so. The assumption that \(\theta \neq 0\) implies that “watching” the market also comes at a cost although it is less costly compared to trading. It is intuitive to assume that “watching” the market is costly as the investors still need to follow the developments in the emerging economy so as to determine the right time to sell. In Section 3.4, we do analyze the robustness of our results to this assumption by solving the case in which \(\theta = 0\).
2.3 Information Structure

Dividends are determined exogenously as follows:

\[ d_t = e^{z_t + \eta_t}. \]  

(10)

There are two types of uncertainty associated with dividends: persistent aggregate productivity shocks, \( z \), and noise, in the form of transitory, additive, Normal i.i.d. shocks, \( \eta \), with \( E[\eta] = -\frac{\sigma_\eta^2}{2} \) and \( E[\eta^2] = \sigma_\eta^2 \); \( \eta \sim N(-\frac{\sigma_\eta^2}{2}, \sigma_\eta^2) \).\(^6\) Aggregate productivity shocks follow a Markov process with two states and transition probability matrix \( P \). We denote the values \( z \) can take as \( z \in \{z^L, z^H\} \) and assume \( z^L < z^H \) without loss of generality.

**Assumption** \( P >> 0 \) (irreducible Markov chain) and \( P_{ii} \neq P_{ji} \) where \( P_{ij} \) is the probability of transiting from state \( i \) to state \( j \), \( i, j \in \{L, H\} \) and \( i \neq j \) (positive autocorrelation).

\( P >> 0 \) rules out absorbent states. \( P_{ii} = P_{ji} \) would imply that the probability of transiting to state \( i \) is the same regardless of the current state. Therefore, in this case, information regarding the current state would not be useful for forecasting the following period’s state (no autocorrelation).

We assume both sets of investors know the true distributions governing the productivity shocks \( z \) and the noise \( \eta \). They observe the dividends \( d \) at the beginning of each period, but do not observe the current or past values of the productivity shock \( z \) or the noise \( \eta \).\(^7\) Both investors use the information revealed by dividends in order to infer the realization of the productivity shock in the current period.\(^8\) Beliefs are defined as:

\[ \tilde{z}_t \equiv E[z_t | I^U_t] \]  

(11)

where \( I^U_t \) includes the entire history of dividends observed by the investors:

\[ I^U_t \equiv \{d_t, d_{t-1}, \ldots\}. \]  

(12)

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\(^6\)This specification for \( E[\eta] \) guarantees that changes in \( \sigma_\eta \) produce mean preserving spreads.

\(^7\)One can imagine that investors observe productivity with such a long lag that, once received, the information is no longer useful for predicting current productivity any more.

\(^8\)It is also possible to model different types of publicly observed signals, such as news reports, in addition to dividends. In any case, the model variables will be sensitive to the information content of the signals and this sensitivity will be qualitatively similar but quantitatively different depending on the informativeness of the publicly observed signals.
Throughout the paper we refer to this information structure as the “incomplete information” scenario. The belief $\tilde{z}_t$ is formed by updating the previous period’s belief $\tilde{z}_{t-1}$ using Bayes’ rule, as in Hamilton (1989), Moore and Schaller (2002), and Nieuwerburgh and Veldkamp (2003):

$$Pr(z_t = z^i|I^U_t) = \frac{f(d_t|z_t = z^i)Pr(z_t = z^i|I^U_{t-1})}{f(d_t|z_t = z^j)Pr(z_t = z^j|I^U_{t-1}) + f(d_t|z_t = z^i)Pr(z_t = z^i|I^U_{t-1})}$$

where $f$ is the conditional normal probability density that can be written as:

$$f(d_t|z_t = z^i) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(d_t - z^i)^2}$$

for $i, j \in \{H, L\}$ and $i \neq j$. Equation (13) is used to update the probability assigned to being in the high productivity state, incorporating the additional information revealed by $d_t$ at the beginning of period $t$. The priors that will be used in period $t + 1$ for updating beliefs are obtained by simply adjusting for the probability of a change in state from period $t$ to $t + 1$ using the Markov transition matrix known by investors. That is:

$$Pr(z_{t+1} = z^i|I^U_t) = Pr(z_t = z^i|I^U_t)P_{ii} + Pr(z_t = z^j|I^U_t)P_{ji}.$$  

(15)

Once the posteriors of the current period are calculated, beliefs are:

$$\tilde{z}_t = Pr(z_t = z^L|I^U_t)z^L + Pr(z_t = z^H|I^U_t)z^H.$$  

(16)

**Proposition 1** $0 < Pr(z_t = z^i|I^U_{t-1}) < 1$ and $0 < Pr(z_t = z^i|I^U_t) < 1$.

**Proof** See Appendix.

The interval to be considered for the prior and posterior probabilities is $(0, 1)$. The prior $Pr(z_0 = z^i|I^U_0)$ or the posterior $Pr(z_0 = z^i|I^U_t)$ can be set exogenously to “start” from 0 or 1. Afterwards, however, it can take these values with zero probability. From Equation (16), we know that beliefs are convex combinations of low and high values of productivity, with weights defined by the Bayesian posterior probabilities assigned to each state. Hence, beliefs are always higher than the low value of productivity and lower than the high value, $z^L < \tilde{z} < z^H$. This implies that agents can never be exactly sure about being in a particular state. In addition, they never underestimate (overestimate) productivity to be lower (higher) than the low (high) realization of the true productivity. This is an unappealing feature of learning with discrete
probabilistic processes. Also, as a result of this limitation, the standard deviation of beliefs is always less than or equal to that of productivity.

Equation (16) implies that beliefs are sufficient to backtrack the probabilities assigned to each state. Using Equation (16) and \( Pr(z_t = z^i|I_t^U) = 1 - Pr(z_t = z^j|I_t^U) \) for \( i, j \in \{H, L\} \) and \( i \neq j \), a given \( \tilde{z}_t \) can be mapped to a unique \( Pr(z_t = z^i|I_t^U) \). The assumption that provides this simplification is having two states for productivity. This simplification is crucial for the numerical analysis since probabilities assigned to each state are continuous endogenous state variables for the problem. Given the computational difficulty of handling continuous state variables, we assume two states for productivity and carry \( \tilde{z} \) as a state variable that is sufficient for backtracking the posterior probabilities assigned to each state of productivity.

![Figure 1: Density of \( d_t \) conditional on \( z_t \).](image)

We denote the evolution of investors’ beliefs as \( \tilde{z}_{t+1} = \phi(\tilde{z}_t, d_{t+1}) \). When investors make their decisions at date \( t \), \( d_{t+1} \) is not known, but its distribution conditional on \( z_{t+1} \) is known to both domestic and foreign investors. Figure 1 plots these conditional distributions for signal-to-noise ratios of 1.66 and 2.26, respectively. As the signal-to-noise ratio increases, the distribution of dividends conditional on the high and low productivity overlap less, as a result, dividends become more informative. In Figure 1, most of the conditional density is concentrated around the means when the signal-to-noise ratio is high (right panel). As \( \sigma_\eta \) decreases (or as the signal-to-noise ratio increases), these two conditional densities separate, and in the limit as \( \sigma_\eta \) approaches zero, the informational imperfection vanishes.

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9The signal-to-noise ratio is defined as \( \frac{z^E - z^L}{\sigma_\eta} \). We pick these particular values for the signal-to-noise ratios because they are also the ones used for the numerical analysis.
In Figure 2, we plot $\tilde{z}_{t+1} = \phi(\tilde{z}_t, d_{t+1})$ for three different values of $\tilde{z}_t$ where $d_{t+1}$ is on the horizontal axis and $\tilde{z}_{t+1}$ is on the vertical axis. The solid curve corresponds to $\tilde{z}_{t+1} = \phi(\min(\tilde{z}), d_{t+1})$; that is, it is the case where the investors are “almost sure” that the economy is in the bad state today. Similarly, the dashed curve shows $\tilde{z}_{t+1} = \phi(\max(\tilde{z}), d_{t+1})$, or the case in which they are optimistic. All other beliefs would be represented by curves that lie between the solid and dashed curves, such as the dotted curve, which shows the case in which the investors assign equal probability to each state, $\tilde{z}_{t+1} = \phi(\frac{z^H + z^L}{2}, d_{t+1})$.

![Figure 2: Next period’s beliefs $\tilde{z}_{t+1} = \phi(\tilde{z}_t, d_{t+1})$ for three different values of current beliefs $\tilde{z}_t$.](image)

**Proposition 2** If $P_{ii} < P_{ji}$ then $\phi(\tilde{z}_t, d_{t+1})$ is strictly increasing in both of its arguments.

**Proof** See Appendix.

$P_{ii} > P_{ji}$ corresponds to a scenario where knowing the current state would still be useful for forecasting future productivity: the information that the economy is in a particular state would reveal that the economy is more likely to transition to the other state than to stay in the same state in the subsequent period (negative autocorrelation). Although information is valuable and learning would still take place, we rule out the case $P_{ii} > P_{ji}$ in order to establish Proposition 2.

The elasticity of $\tilde{z}_{t+1}$ with respect to $d_{t+1}$ varies depending on $\tilde{z}_t$. When the investors assign a high probability to being in the low state ($\tilde{z}_t$ is low), a low realization of $d_{t+1}$ “confirms” the beliefs and as a result $\tilde{z}_{t+1}$ changes only marginally. On the other hand, if a high $d_{t+1}$ is
observed, the beliefs of investors are “challenged” and there is a large adjustment in the next period’s beliefs.

In order to see this, consider the following scenario. Assume that true productivity is low and that investors’ current beliefs are “almost correct”. In this case, \( \tilde{z}_t = \min \tilde{z} \), as depicted by “lowest beliefs” curve in Figure 2. The vertical line in Figure 2 marks the mean of the signals conditional on the economy being in the low state. Hence, a small negative noise shock is a realization of dividends to the left of this vertical line. If investors observe a negative noisy signal at \( t + 1 \), the response of beliefs to this signal is minimal (the solid curve is flat on the left side of the vertical line). On the other hand, if investors receive a sequence of misleading positive signals before the negative one, their optimism builds up and their beliefs can move to reach that reflected in dashed curve in Figure 2. When the economy ends up in this situation, the response to a small negative signal is large (the dashed curve is steep on the left side of the vertical line). Therefore, a stream of positive signals can move the economy to a vulnerable state in which a negative signal triggers a large downward adjustment. As investors turn optimistic, it is as if the economy is moving along the convex part of this curve. The point of maximum vulnerability lies at the intersection of the vertical line and the inflection point of the curve.

Figure 3 shows the numerical derivative of \( \phi(\tilde{z}_t, d_{t+1}) \) with respect to \( d_{t+1} \) around \( d_{t+1} = z^L \) as a function of \( \tilde{z}_t \). This derivative captures the response of the beliefs to a small, negative signal conditional on true productivity being low, and it approximates the “vulnerability” of the economy. Figure 3 illustrates that this derivative is a convex function. Hence, the response of beliefs to a negative signal increases at an increasing rate with the level of optimism attained prior to the negative signal. The convexity of the derivative of \( \phi(\cdot) \) is due to the assumption that true productivity is a discrete random variable. In the case of continuous random variables, learning takes place in a linear fashion, that is, the posteriors are a convex combination of the priors and the signal with weights that depend on the signal-to-noise ratio. In that case, this derivative would be linearly increasing in the level of optimism prior to the negative signal.

The quantitative analysis focuses on the model’s equilibrium which is defined as follows.

**Definition** A competitive equilibrium is given by allocations \( \alpha'(\alpha, \tilde{z}, d) \), \( c(\alpha, \tilde{z}, d) \), \( \alpha^{**}(\alpha, \tilde{z}, d) \) and asset prices \( q(\alpha, \tilde{z}, d) \) such that:

(i) Domestic households maximize \( U \) subject to their budget constraint and their information

10We approximate this derivative numerically with \( \frac{\phi(\tilde{z}, z^L) - \phi(\tilde{z}, z^L - \varepsilon)}{\varepsilon} \) for \( \varepsilon \) small and positive. In the figure, we plot this expression for different values of \( \tilde{z} \).
set, $I^U$, taking asset prices as given.

(ii) Foreign investors maximize the expected present discounted value of future profits conditional on their beliefs about the state of productivity, taking asset prices as given.

(iii) The goods and asset markets clear.

3 Quantitative Analysis

3.1 Computation

The dynamic programming representation of the domestic investors’ problem for $i,j \in \{L,H\}$ and $i \neq j$ is:

$$V(\alpha, \tilde{z}, d) = \max_{\alpha'} \{ u(\alpha(q + d) - \alpha'q)$$

$$+ \beta \left[ Pr(z = z^i|I^U)P_{ii} + Pr(z = z^j|I^U)P_{ji} \right] \int V(\alpha', \phi(\tilde{z}, d'), d')f(d'|z' = z^i)dd' \right. \}$$

$$+ \beta \left[ Pr(z = z^i|I^U)P_{ij} + Pr(z = z^j|I^U)P_{jj} \right] \int V(\alpha', \phi(\tilde{z}, d'), d')f(d'|z' = z^j)dd' \}.$$  \hspace{1cm} (17)

The solution algorithm includes the following steps:

1. Discretize the state space. We use 102 equally spaced nodes for $\alpha$ and 40 equally spaced nodes for $\tilde{z}$ in the intervals [.83, 1.00] and $[z^L, z^H]$ respectively. To discretize the noise component of dividends we use Gaussian quadratures with 20 quadrature nodes.

2. Evaluate the evolution of beliefs $\tilde{z}_{t+1} = \phi(\tilde{z}_t, d_{t+1})$ using Equations (13)-(16).
3. For a conjectured pricing function \( q^{old}(\alpha, \tilde{z}, d) \), solve the dynamic programming problem described in Equation 17 using value function iterations in order to get \( \alpha'(\alpha, \tilde{z}, d) \) and \( c(\alpha, \tilde{z}, d) \).

4. Calculate the foreign investors’ demand function using domestic investors’ asset demand function obtained in Step 3 and the market clearing condition in the asset market, \( \alpha^* + \alpha = 1 \).

5. Using foreign investors’ demand calculated in Equation (8), calculate new prices \( q^{new}(\alpha, \tilde{z}, d) \).

6. Update the conjectured prices with \( \xi q^{old}(\alpha, \tilde{z}, d) + (1 - \xi)q^{new}(\alpha, \tilde{z}, d) \) where \( \xi \) is a fixed relaxation parameter that satisfies \( \xi \in (0, 1) \) and is set close to 1 in order to dampen hog cycles.

7. Iterate prices until convergence according to the stopping criterion \( \max\{|q^{new} - q^{old}|\} < 0.00001 \) and get equilibrium asset prices \( q(\alpha, \tilde{z}, d) \).

To check the accuracy of the solution of the dynamic programming problem, we evaluate Euler equation residuals as described in Judd (1992). In order to do so, we solve for \( \hat{c} \) in the following Euler equation:

\[
q_t u'(\hat{c}_t) = \beta E_t[(q_{t+1} + d_{t+1})u'(c_{t+1})].
\]

Intuitively, we evaluate the consumption function that exactly satisfies the Euler equation implied by the solution of the dynamic programming problem. Then, we calculate \( 1 - (\hat{c}_t/c_t) \), which is a unitless measure of error. We find that the average Euler equation error is 0.0016.\(^{11}\)

Euler equation errors do not include the errors from the price iteration since the Euler equation must hold for any pricing function, not only the equilibrium pricing function. As a measure for the accuracy of the equilibrium price, we report the tolerance of the price iteration. Tolerance is defined as the maximum of the absolute value of the difference between prices evaluated in the last two consecutive iterations, \( \max\{|q^{new} - q^{old}|\} \). We iterate prices until tolerance is less than 0.00001.

3.2 Calibration

The model is calibrated quarterly for Turkey using data for the 1987:1-2005:2 period. We set the risk free interest rate to average US Treasury Bill rate, \( R = (1.0471)^{25} = 1.0115 \). We set \( \beta = 0.9886 \) and \( \sigma = 2 \) following the business cycles literature. We set the trading costs of the foreign investors to \( \{a = 0.001, \theta = 0.1\} \). With this calibration, total trading costs on average constitute 0.2589\% of foreign investors’ per period profits as specified in Equation (6) and 1.8845\% of the trade value. These costs are in line with the findings of Domowitz, Glen

\(^{11}\)Judd (1992) calls this measure the “bounded rationality measure,” and interprets an error of 0.0016 as a $16 error made on a $10,000 expenditure.
and Madhavan (2001) showing equity trading costs during the period 1996-1998 for a total of 42 countries among which 20 are emerging countries. They found that for emerging markets, trading costs are higher than the developed ones and they range between 0.58% (Brazil) and 1.97% (Korea) as percentage of trade value.

We estimate the parameters \( \{\sigma_\eta, z^H, z^L\} \) and Markov transition probabilities \( \{P_{HH}, P_{LL}\} \) using a Maximum Likelihood Estimation procedure similar to the one described in Hamilton (1989). For this exercise, we use quarterly GDP data for Turkey from 1987:1 to 2005:2 with a total of 74 observations. The data are from Central Bank of the Republic of Turkey’s web site and are in constant 1987 prices. They are logged, seasonally adjusted (using the Bureau of Economic Analysis’s X12 Method) and filtered with HP filter using a smoothing parameter of 1600.

### Table 2: Model parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>0.9881</td>
<td>Discount factor</td>
</tr>
<tr>
<td>( R )</td>
<td>1.0121</td>
<td>Risk free rate</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>2</td>
<td>Risk aversion coefficient</td>
</tr>
<tr>
<td>( P_{HH} )</td>
<td>0.8933</td>
<td>Transition probability from H to H</td>
</tr>
<tr>
<td>( P_{LL} )</td>
<td>0.6815</td>
<td>Transition probability from L to L</td>
</tr>
<tr>
<td>( z^L )</td>
<td>-0.0427</td>
<td>Productivity in state L</td>
</tr>
<tr>
<td>( z^H )</td>
<td>+0.0175</td>
<td>Productivity in state H</td>
</tr>
<tr>
<td>( \sigma_\eta )</td>
<td>0.0362</td>
<td>Standard deviation of noise</td>
</tr>
<tr>
<td>( \frac{z^H - z^L}{\sigma_\eta} )</td>
<td>1.6638</td>
<td>Signal-to-noise ratio</td>
</tr>
<tr>
<td>{( a, \theta )}</td>
<td>{0.001, 0.1}</td>
<td>Trading costs</td>
</tr>
</tbody>
</table>

We denote the observed GDP series as \( y_t \) for \( t \in \{1, 2, ..., T\} \) and the parameters to be estimated are \( \psi \equiv \{z^i, z^j, \sigma_\eta, P_{ii}, P_{jj}\} \). The algorithm used for the estimation is as follows:

1. Calculate the ergodic distribution of the Markov process, \( \pi = [\pi_i, \pi_j] \), using \( \pi_i = (1 - P_{jj})/(2 - P_{jj} - P_{ii}) \). \( \pi_j \) can be calculated using \( \pi_i + \pi_j = 1 \).

2. Calculate the conditional density:

\[
 f(y_t, \psi|y_t^{t-1}) = \frac{1}{\sqrt{2\pi\sigma_\eta}} \left( Pr(z_t = z^i|y_t^{t-1})e^{-\frac{(y_t - y_{t-1}^i)^2}{2\sigma_\eta^2}} + Pr(z_t = z^j|y_t^{t-1})e^{-\frac{(y_t - y_{t-1}^j)^2}{2\sigma_\eta^2}} \right) \tag{19}
\]
where $Pr(z_t = z^i|y^{t-1})$ denotes the posterior probability assigned to being in state $i$ conditional on the observed history of $y$ until period $t - 1$.

3. For $t = 1$, when no history is available, use the ergodic probabilities calculated in Step 1 instead of the conditional probabilities.

4. Update the prior probability $Pr(z_t = z^i|y^{t-1})$ using Bayesian updating Equations 13 and 15.

5. Repeat Steps 2-4 for $\forall t \in \{1, 2, \ldots, T\}$.

6. The log likelihood function is evaluated by simply adding the logged conditional density functions for all observations:

$$L(\psi) = \sum_{t=1}^{T} \ln f(y_t; \psi|y^{t-1}).$$

(20)

7. Maximize the log likelihood function:

$$\max_{\psi} L(\psi; y^T)$$

subject to $P_{ii} > 0$, $P_{jj} > 0$ and $P_{ii} > P_{ji}$ (see Assumption).

The estimates of the productivity shock are $\{z^H, z^L\} = \{0.0175, -0.0427\}$ which translate into $\{\exp(z^H), \exp(z^L)\} = \{1 + 0.0177, 1 - 0.0418\}$. The estimated transition probabilities are $P_{HH} = 0.8933$ and $P_{LL} = 0.6815$. The estimated persistent component variance is $\sigma_z = 0.0260$, and the estimated noise component variance is $\sigma_\eta = 0.0362$, the ratio of the two is $\frac{\sigma_z}{\sigma_\eta} = 0.7182$. With these parameters, the estimated signal-to-noise ratio is $\frac{z^H - z^L}{\sigma_\eta} = 1.6638$. The productivity shocks and the transition probability matrix approximate a Normal AR(1) process: $z_{t+1} = (0.0004) + (0.5763)z_t + \epsilon_{t+1}$, where $\sigma_\epsilon = 0.0213$. This calibration implies $\frac{\sigma_z}{\sigma_\eta} = 0.5888$ which constitutes another measure of information content of the signals.\textsuperscript{12}

For comparison, we estimate these parameters using constant price GDP data for the US for the same time period using the same estimation procedure. Not surprisingly, we find a lower variance for the persistent component ($\sigma_z = 0.0086$) as well as lower variance for the noise ($\sigma_\eta = 0.0060$) compared to those of Turkey. What determines the informativeness of the signals however, is the ratio of these two variances. So, we calculate signal-to-noise ratios for the US and compare them with those of Turkey. The signal-to-noise ratios estimated for the US are $\frac{z^H - z^L}{\sigma_\eta} = 2.7037$ (vs. 1.6638 for Turkey) and $\frac{\sigma_z}{\sigma_\eta} = 0.7258$ (vs. 0.5888 for Turkey). This finding

\textsuperscript{12}This, in fact, is the conventional measure of the information content of the signals when learning takes place about continuous as opposed to discrete variables.
suggests that if we were to solve the model calibrated to the US economy, the signals would be much more informative and therefore, the signal extraction problem faced by the investors would be easier.

3.3 Quantitative Findings

Figure 4 shows the ergodic distribution of the domestic investors’ asset position, $\alpha$, for a situation in which investors have full information (panel (a)) and in which investors have incomplete information (panel (b)) scenarios. The “full information” scenario corresponds to the case in which the information set of both investors is $I^f_t \equiv \{d_t, d_{t-1}, \ldots, z_t, z_{t-1}, \ldots\}$\(^{13}\). In both cases, the ergodic distributions are skewed to the left. The informational imperfection reduces the mean asset holdings of domestic investors. This is because the informational imperfection increases the uncertainty associated with future asset returns, and, hence, risk averse domestic investors are less inclined to demand risky assets.

![Figure 4: Ergodic distribution of domestic investors’ asset holdings, $\alpha$, in the case of (a) full information, and (b) incomplete information.](image)

The ergodic distribution of beliefs, $\tilde{z}$, is plotted in Figure 5. In this distribution, most of the mass is concentrated at the tails, or around $z^L$ and $z^H$. This result arises because beliefs usually being close to correct. The extent to which the mass is concentrated at the tails depends crucially on the signal-to-noise ratio. The more informative the signals, the less beliefs deviate from the truth and the more the ergodic distribution is concentrated at the tails. Another feature of this distribution is its skewness. Skewness is a result of the asymmetry of the Markov transition matrix. The high state is more persistent than the low, an asymmetry that both sets

\(^{13}\)One can model a full information scenario by setting $\sigma_\eta = 0$. However, doing so would alter the distribution of the dividend process. As a result, it would not be possible to distinguish changes in results that are due to full information per se from those due to the change in the distribution of the dividend process.
of investors acknowledge as they formulate their beliefs. Knowing that there are more periods in which the economy is in the high state than in the low state, investors’ beliefs are more likely to be close to $z^H$ than $z^L$.

Table 3 documents the long run moments of simulated and actual data.\textsuperscript{14} Consistent with Figure 4, average asset holdings of the domestic investors is higher in the full information scenario than in the incomplete information scenario (86.1 percent v. 84 percent). As a result of their greater asset holdings, domestic investors’ consumption is also higher on average in the full information scenario than in the incomplete information. In the full information case, higher average consumption and lower consumption volatility lead to a higher level of welfare compared to the case in which investors have only incomplete information.

Going from the full information setup to one with incomplete information, the standard deviations of consumption and the current account increase by 2.1 percentage points, and 0.4 percent, respectively. On the other hand, the standard deviation of asset prices falls by 0.87 basis points. The decline in the standard deviation of asset prices is due to beliefs being a convex combination of the low and high value of true productivity. (See Equation (16) and Proposition 1.)

The correlation between true productivity, $z$, and asset prices, $q$, falls from 0.9990 in the full information setup to 0.6678 in the incomplete information setup. This is due to booms-busts induced by the imperfection of information, which gives rise to misperceptions regarding the true state of productivity. In the full information case, all of the cycles are driven by changes in true productivity and noise shocks have negligible effects on asset prices. Although most of

\textsuperscript{14}We simulate the model for 10,000 periods and calculate the moments after dropping the first 1,000 observations.
<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Full Info.</th>
<th>Incomplete Info.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(d)$</td>
<td>1.0036</td>
<td>1.0032</td>
<td></td>
</tr>
<tr>
<td>$E(c)$</td>
<td>0.8642</td>
<td>0.8419</td>
<td></td>
</tr>
<tr>
<td>$E(\alpha)$</td>
<td>0.8609</td>
<td>0.8397</td>
<td></td>
</tr>
<tr>
<td>$E(q)$</td>
<td>83.1358</td>
<td>83.0617</td>
<td></td>
</tr>
<tr>
<td>$E(CA/d)$</td>
<td>-0.0001</td>
<td>0.0001</td>
<td></td>
</tr>
<tr>
<td>$\sigma(z)$</td>
<td>2.5884</td>
<td>2.5884</td>
<td>2.5884</td>
</tr>
<tr>
<td>$\sigma(\eta)$</td>
<td>3.6341</td>
<td>3.6341</td>
<td>3.6341</td>
</tr>
<tr>
<td>$\sigma(d)$</td>
<td>4.5694</td>
<td>4.5514</td>
<td>4.5514</td>
</tr>
<tr>
<td>$\sigma(c)/E(c)$</td>
<td>5.4597</td>
<td>2.1265</td>
<td>4.2168</td>
</tr>
<tr>
<td>$\sigma(q)/E(q)$</td>
<td>38.0997</td>
<td>0.0370</td>
<td>0.0283</td>
</tr>
<tr>
<td>$\sigma(CA/d)$</td>
<td>3.1168</td>
<td>3.6134</td>
<td>3.8935</td>
</tr>
<tr>
<td>$corr(d,c)$</td>
<td>0.6984</td>
<td>0.3153</td>
<td>0.4425</td>
</tr>
<tr>
<td>$corr(d,q)$</td>
<td>0.0718</td>
<td>0.5611</td>
<td>0.8327</td>
</tr>
<tr>
<td>$corr(d,CA)$</td>
<td>-0.4217</td>
<td>0.9019</td>
<td>0.5801</td>
</tr>
<tr>
<td>$corr(d,\alpha')$</td>
<td>0.0347</td>
<td>0.1655</td>
<td></td>
</tr>
<tr>
<td>$corr(\tilde{z},\tilde{z}_{-1})$</td>
<td>x</td>
<td>0.5532</td>
<td></td>
</tr>
<tr>
<td>$corr(z,q)$</td>
<td>0.9990</td>
<td>0.6678</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Long-run business cycle moments, simulated data is logged and HP filtered.

The booms and busts in the incomplete information scenario are also due to changes in true productivity, there is a significant number of optimism-pessimism driven cycles.

The autocorrelation coefficient of $\tilde{z}$ is 0.5532 which suggests that transitory shocks have persistent effects on beliefs. This occurs because investors cannot distinguish the component of shocks that is persistent from the component that is transitory. The belief updating structure is the key element in the model that induces persistence: the previous period’s posteriors are current period’s priors.

Another important observation from Table 3 is the decrease in the correlation between dividends and the current account going from full information to imperfect information (0.90 vs. 0.58). In response to a positive dividend shock, domestic investors would like to increase their
asset position so as to smooth consumption over time and in addition, their expectations for asset returns increase since they observe a positive signal. Foreign investors are modeled not to have a consumption smoothing motive therefore, for them only the second effect (positive signal) is present and this effect is in fact stronger than their domestic counterparts because they bid more aggressively for the asset when there is a positive signal due to their risk neutrality. Overall, we find that usually the first effect is greater than the second, and therefore, the model produces a procyclical current account. However, as we mentioned the procyclicality is lower compared to the full information scenario where only the first effect is present.

Figures 6 and 7 show simulated asset prices, productivity, and consumption under full information and incomplete information, respectively. In the full information case, swings in asset prices match the swings in productivity and shocks to the transitory component of dividends have minimal effects on prices. Without the information role of dividends, in response to a negative noise shock domestic investors would like to be net sellers so as to intertemporally smooth their consumption and thus at equilibrium asset prices fall. However, this effect on prices is small and not visible in the figure. In the incomplete information scenario, asset prices fluctuate both with true productivity and with transitory shocks. Noisy signals thus can induce cycles driven by misperceptions among investors regarding true productivity. In addition,
as mentioned before, the volatility of consumption increases substantially when we introduce informational imperfections.

In Figure 8, we plot the conditional forecasting functions starting from a state where investors are optimistic (first column) and where they are pessimistic (second column). In the optimistic scenario we set the state variables to \((\alpha, \tilde{z}, d) = (0.840, 0.017, 0.958)\): that is, beliefs are \(\tilde{z} = \max(\tilde{z})\); dividends are set to signal that the productivity is low; \(d = e^{\alpha t}\) and the domestic investors’ asset position is set to its long-run mean. The pessimistic scenario is set to start at \((\alpha, \tilde{z}, d) = (0.840, -0.042, 0.958)\). Hence, these scenarios are identical except for the initial beliefs.

In the figures for consumption and asset prices, the vertical axes show percentage deviations from long-run means. In the figure for the current account, the vertical axis shows the ratio of the current account to dividends in percentage terms. On impact in period one, the economy with optimistic investors is characterized by a current account deficit as well as a boom in consumption and asset prices. In period two, however, consumption falls sharply below its mean by 1.5% and the current account turns to a surplus of roughly 2.5%. The prices also adjust downwards but the adjustment is more gradual than those of consumption and the current account. After the
Figure 8: Forecasting functions conditional on $\alpha = 0.840$, $d = z^L$, $\tilde{z} = z^H$ (first column), and $\tilde{z} = z^L$ (second column).
second period, all variables slowly and monotonically converge to their long-run means.

The dynamics of the model economy starting with optimistic investors are similar to that of an emerging market in the period before a crisis. As documented in Section 1, pre-crisis periods are generally characterized by current account deficits as well as consumption and asset price booms. Our model is able to forecast a collapse in consumption and asset prices as well as reversal of the current account after this period of optimism.

The results in Table 3 suggested that the model produces a procyclical current account on average and in the imperfect information scenario this procyclicality is lower than in the full information case. Previously, we explained the model dynamics that lead to this result. The forecasting functions plotted in Figure 8 support the previous explanation and the results of Table 3. Particulary, the economy with optimistic investors has a current account deficit because, ceteris paribus, the higher the beliefs, the lower the current account.

<table>
<thead>
<tr>
<th>Probability (%)</th>
<th>Booms</th>
<th>Busts</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Prob} \left[ \text{Prob}(z_t = z^i</td>
<td>I^U_t, z_t = z^j) &gt; 0.5 \right] )</td>
<td>9.6800</td>
</tr>
<tr>
<td>( \text{Prob} \left[ \text{Prob}(z_t = z^i</td>
<td>I^U_t, z_t = z^j) &gt; 0.5</td>
<td>z_t = z^j \right] )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Duration (quarters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average duration</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Magnitude (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q )</td>
</tr>
<tr>
<td>( c )</td>
</tr>
<tr>
<td>( CA )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Magnitude (in std. deviations)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q )</td>
</tr>
<tr>
<td>( c )</td>
</tr>
<tr>
<td>( CA )</td>
</tr>
</tbody>
</table>

Table 4: Analysis of optimism (pessimism) driven booms (busts).

Table 4 analyzes optimism and pessimism driven cycles in terms of their frequency, average duration, and magnitude. In order to conduct the analysis, we use simulated data to identify periods in which investors assign a probability greater than 0.5 to productivity being high (low).
even though the true productivity is low (high) and call them optimism (pessimism) periods.\textsuperscript{15} In the second and third rows of Table 4, we report the ratio of the number of optimism (pessimism) periods to the total number of observations, and to the number of periods in which the state was low (high), respectively. We calculate the average duration by calculating the average length of the distinct optimism-pessimism periods. Given the inherent noisiness of signals obtained by calibrating the model to a typical emerging economy, this table reveals how often investors turn optimistic-pessimistic due to misleading signals, how long these periods last, and more importantly, whether and how much optimism (pessimism) periods are associated with booms (busts) in asset prices and consumption and current account deficits (surpluses).

Unconditionally, the model produces optimism driven booms with a 9.64\% probability, whereas it produces pessimism driven busts with a 4.23\% probability. Also, given that the true state is low, there is a 37.10\% probability that the investors are optimistic and similarly, conditional on the true productivity being high, the investors are pessimistic with 5.72\% probability. The former is more likely to happen because investors interpret positive signals to be more “credible” than negative signals due to the asymmetry of the Markov transition probability matrix. The optimism in response to a misleading positive signal is greater than the pessimism caused by a misleading negative signal with the same magnitude.

On average, the model predicts an average duration of 1.35 (1.21) quarters for the optimism (pessimism) driven booms (busts). These cycles are relatively short lived because these cycles hinge on the realization of a sequence of positive or negative signals.

In the same table, we also report the size of these booms-busts as percentage deviations from the value that corresponding variables would have taken if investors had correctly estimated the true productivity instead of being optimistic or pessimistic. The magnitude for the asset price boom is small when we look at it as percentage deviation because the equilibrium asset prices have low volatility. This magnitude is closer to what we observe in the data in terms of standard deviations. The boom periods are characterized by asset prices, consumption, and current account that are on average 2.36, 1.31, and 0.83 standard deviations above what they would have been if the investors were not optimistic. The over-pricing as well as over-consumption are evident in this table. Especially, the over-pricing of the emerging market asset is significant: during the booms on average we observe prices that are more than two standard deviations higher

\textsuperscript{15}Note that by doing so, we are picking up only those periods in which optimism and pessimism are due to misperceptions of investors.
that what they would have been if investors were not optimistic. Similarly, we see under-pricing and under-consumption during the busts, although their magnitudes are smaller in absolute value than those observed during booms due to the asymmetry of the Markov process.

3.4 Sensitivity Analysis

We document the long run business cycle moments of the model with different calibrations for the noisiness of the signals, $\sigma_\eta$, and trading costs, $a$ and $\theta$. The third column of Table 5 shows the results with $\sigma_\eta = 0.0265$ and we compare these results with those of the baseline model with $\sigma_\eta = 0.0362$ reproduced in the second column.\footnote{With $\sigma_\eta = 0.0265$ the signal-to-noise ratio increases to 2.26 from 1.66 in the baseline scenario.} With lower $\sigma_\eta$, the standard deviation of dividends, consumption and the current account fall by 85, 20, and 27 basis points, respectively. Average consumption among domestic investors increases due to the lower volatility of dividends and the associated decrease in uncertainty regarding future asset returns.

Lower $\sigma_\eta$ implies that the signals are more informative and credible. Therefore, learning is faster compared to the baseline scenario. This leads to less persistence in beliefs. The autocorrelation of beliefs drops down to 0.54 from 0.55 in the baseline model. In addition, the probability of optimism-pessimism driven cycles falls leading to a stronger correlation between asset prices and true productivity.

The fourth column of the same table presents the results for the scenario with higher per trade costs, $a = 0.002$. The standard deviation of prices, consumption, and the current account increase by 0.05, 26, and 67 basis points, respectively. Due to higher per trade costs on the foreign investors’ side, domestic investors hold more of the asset in equilibrium, leading to higher mean consumption but more volatile consumption.

Analysis of the scenario with no recurrent costs, $\theta = 0$, is reported in the fifth column. The results remain largely unchanged except for the slight drops in the current account volatility and the correlation of the current account with dividends.

4 Conclusion

The boom-bust cycles of emerging economies suggest that periods of apparent prosperity in these countries might contain the seeds of crises. This paper explores this possibility using an open economy equilibrium asset pricing model with imperfect information in which agents
do not know the true state of productivity in the economy. The main contribution of the paper is its ability to endogenously generate (a) periods of optimism characterized by booms in asset prices and consumption followed by sudden reversals, (b) sensitivity to negative signals that increases with, and arises from, investor optimism attained prior to the negative signal. These results are due to the fact that informational frictions generate a disconnect between country fundamentals and asset prices. That is, busts (booms) in asset markets can occur even though the fundamentals of the economy are strong (weak). Asset prices display persistence in response to transitory shocks since investors cannot perfectly identify the underlying state of productivity. Due to the additional uncertainty created by informational frictions, the volatility of the emerging economy’s consumption increases by 2 percentage points compared to the full
information scenario. In addition, periods with high levels of optimism are more likely to be associated with current account deficits than periods of pessimism.

Although the informational frictions introduced in this paper can produce booms and busts in asset prices and consumption due to shifts in investor confidence, these booms and busts are short lived and are of about the same size as regular business cycles. In addition, even though the introduction of imperfect information provides an improvement in terms of matching the volatility of consumption and the current account dynamics observed in the data, the model cannot account for the volatility of asset prices.

The role of informational frictions in understanding emerging market regularities is an area ready for further research. For instance, the model presented in this paper endogenously produces sensitivity to negative signals given an exogenous sequence of positive signals. We could think of producing an endogenous sequence of positive signals by introducing strategic information manipulation into the model, especially prevalent during the run-ups to crises. If there is initially some sensitivity due to short-term and/or dollarized debt, a policymaker might find it optimal to manipulate or screen the signals to send positive signals. However, this would come at a cost because, by taking out the negative signals and sending only positive ones, the sensitivity of the economy to a sudden downward adjustment would increase. This would create a feedback mechanism in which the policymaker, concerned about the country’s ability to continue borrowing in international markets, has a self-perpetuating incentive to hide negative information about the economy from the public.
References


Appendix

Throughout this section, we assume that $i, j \in \{L, H\}$ and $i \neq j$.

Proof of Proposition 1

Denote the prior $Pr(z_t = z^i|I_{t-1}) = p_t(i)$ and the Normal density function $f(d_t|z_t = z^i) = f(i)$ for $i \in \{L, H\}$.

Priors:

Evolution of $p_t(i)$ is characterized by:

$$p_t(i) = \frac{p_{t-1}(i)f(i)P_{ii} + [1 - p_{t-1}(i)]f(j)P_{ji}}{p_{t-1}(i)f(i) + [1 - p_{t-1}(i)]f(j)},$$

$•$ $p_t(i) = 1 \iff p_{t-1}(i)f(i)P_{ii} + [1 - p_{t-1}(i)]f(j)P_{ji} = p_{t-1}(i)f(i) + [1 - p_{t-1}(i)]f(j)$ and $p_{t-1}(i)f(i) + [1 - p_{t-1}(i)]f(j) \neq 0$. Given $P >> 0$ (see Assumption), the first condition is satisfied iff

$$p_{t-1}(i) = 0 \text{ and } f(j) = 0 \text{ or }$$

$$p_{t-1}(i) = 1 \text{ and } f(i) = 0,$$

both of which violate the second condition.

$•$ $p_t(i) = 0 \iff f(j)P_{ji} + p_{t-1}(i)[f(i)P_{ii} - f(j)P_{ji}] = 0$ and $p_{t-1}(i)f(i) + [1 - p_{t-1}(i)]f(j) \neq 0$. The first condition is satisfied iff
\[ f(j) = 0 \text{ and } f(i)P_{ii} = f(j)P_{ji}. \] These two hold iff \( f(j) = 0 \) and \( f(i) = 0 \), in which case the second condition above does not hold.

\[ f(j) = 0 \text{ and } p_{t-1} = 0. \] In this case, second condition is again violated.

See Liptser and Shiryaev (1977) Ch. 9 and David (1997) for the proof of entrance boundaries in continuous time.

**Posteriors:**

Rewrite Equation (13):

\[
Pr(z_t = z^i \mid I^U_t) = \frac{p_{t-1}(i)f(i)}{p_{t-1}(i)f(i) + [1 - p_{t-1}(i)]f(j)}.
\]

Clearly, all terms on the right hand side of the equation are positive: \( p > 0 \) (the proof above) and \( f > 0 \) (Normal distribution).

**Proof of Proposition 2**

First Argument: We need to show that \( \frac{\partial \phi(z_t, d_{t+1})}{\partial z_t} > 0 \) for all \( z_t \). Denote the posterior probabilities \( Pr(z_t = z^i \mid I^U_t) = \gamma_t \) and \( f(d_{t+1} \mid z_t = z^i) = f(i) \). We start with expressing \( \gamma_{t+1} \) as a function of \( \gamma_t \):

\[
\gamma_{t+1} = \frac{[\gamma_t P_{ii} + (1 - \gamma_t)P_{ji}]f(i)}{[\gamma_t P_{ii} + (1 - \gamma_t)P_{ji}]f(i) + [1 - \gamma_t P_{ii} - (1 - \gamma_t)P_{ji}]f(j)}.
\] (A-1)

Also:

\[
\frac{\partial \phi(z_t, d_{t+1})}{\partial z_t} = \frac{\partial z_{t+1}}{\partial z_t} \frac{\partial \gamma_{t+1}}{\partial \gamma_t} + \frac{\partial z_{t+1}}{\partial \gamma_t} \frac{\partial \gamma_{t+1}}{\partial \gamma_t}.
\] (A-2)

Remember that \( \tilde{z}_t = \gamma_t z^i + (1 - \gamma_t)z^j \), so we can calculate the first and the third expressions in the above equation:

\[
\frac{\partial \tilde{z}_{t+1}}{\partial \gamma_{t+1}} = z^i - z^j, \quad \frac{\partial \gamma_t}{\partial \tilde{z}_t} = \frac{1}{z^i - z^j}.
\] (A-3)

The second expression can be calculated using Equation (A-1). After some manipulation:

\[
\frac{\partial \gamma_{t+1}}{\gamma_t} = \frac{f(z^i)f(z^j)(P_{ii} - P_{ji})}{[\gamma_t P_{ii} + (1 - \gamma_t)P_{ji}]f(i) + [1 - \gamma_t P_{ii} - (1 - \gamma_t)P_{ji}]f(j)]^2.\] (A-4)

Plug in Equations (A-3) and (A-4) into Equation (A-2). To complete the proof, we need to establish \( f(z^i), f(z^j) > 0 \) and \( P_{ii} > P_{ji} \). \( f(z^i), f(z^j) > 0 \) for all \( z^i, z^j \) since the Normal distribution is unbounded. \( P_{ii} > P_{ji} \) follows from Assumption 2.3.
Second Argument: We need to show that $\frac{\partial \phi(.,d_{t+1})}{\partial d_{t+1}} > 0$. Write:

$$\frac{\partial \phi(.,d_{t+1})}{\partial d_{t+1}} = \frac{\partial \tilde{z}_{t+1}}{\partial d_{t+1}} = \frac{\partial \gamma_{t+1}}{\partial \gamma_{t+1}} \frac{\partial d_{t+1}}{\partial d_{t+1}}.$$  \hspace{1cm} (A-5)

Denote $A = P_{ij} + \gamma(P_{ii} - P_{ji})$. Then we can rewrite Equation (A-1):

$$\gamma_{t+1} = \frac{Af(i)}{Af(i) + (1 - A)f(j)} = \frac{1}{1 + \frac{1-A}{\gamma} \frac{f(j)}{f(i)}}.$$  \hspace{1cm} (A-6)

Write $f(i)$ and $f(j)$ explicitly:

$$f(j) = e^{\frac{1}{2\sigma^2}[(d_{t+1} - z_i)^2 - (d_{t+1} - z_j)^2]} = e^{\frac{(2d_{t+1} - z_i - z_j)(z_j - z_i)}{2\sigma^2}}.$$  \hspace{1cm} (A-7)

Then we can calculate its derivative with respect to $d_{t+1}$:

$$\frac{\partial [f(j)/f(i)]}{\partial d_{t+1}} = \frac{z^j - z^i}{\sigma^2} e^{\frac{(2d_{t+1} - z_i - z_j)(z_j - z_i)}{2\sigma^2}}.$$  \hspace{1cm} (A-7)

Rewrite Equation (A-5):

$$\frac{\partial \tilde{z}_{t+1}}{\partial d_{t+1}} = \frac{\partial \gamma_{t+1}}{\partial \gamma_{t+1}} \frac{\partial [f(j)/f(i)]}{\partial d_{t+1}}.$$  \hspace{1cm} (A-8)

We know the first expression from Equation (A-2). The second expression can be calculated using Equation (A-6):

$$\frac{\partial \gamma_{t+1}}{\partial [f(j)/f(i)]} = \frac{-(1 - A)/A}{(1 + [(1 - A)/A][f(j)/f(i)])^2}.$$  \hspace{1cm} (A-9)

Plugging in Equations (A-2), (A-7) and (A-9) into (A-8) and rearranging we get:

$$\frac{\partial \tilde{z}_{t+1}}{\partial d_{t+1}} = \left[\frac{z^i - z^j}{\sigma[1 + (1 - A)/A(f(j)/f(i))]}\right]^2 e^{\frac{(2d_{t+1} - z_i - z_j)(z_j - z_i)}{2\sigma^2}} > 0.$$  \hspace{1cm} (A-8)