A Dynamic Tobit Model for the Open Market Desk's Daily Reaction Function*

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Abstract

A dynamic Tobit model with Time-varying parameters is proposed for the daily reaction function of the Open Market Desk of the US Federal Reserve. Such a model offers a more realistic depiction of the Desk's behavior than those of past contributions in the literature as it allows for both possible dynamics in the Desk's daily operations and for day-to-day time varying responses of the Desk to changing conditions in the reserves markets and in the short-term interest rates. Ensuing computational complications are overcome by employing Markov Chain Monte Carlo techniques for the estimation of the model. The results reveal a rich pattern of dynamic behavior by the Open Market Desk both inside the maintenance period and across maintenance periods and point towards a Desk which is highly adaptable to evolving conditions both in the economy in general and in the market for reserves in particular.

JEL Classification: C15, C22, C24, E4, E5

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1 Introduction

The historical evolution of US monetary policy has received substantial attention recently in the applied macroeconomic literature. A large portion of this literature estimates single equation reaction functions of the Federal Reserve (typically linking the Fed's policy instrument, the fed funds rate, to various measures of inflation and output) in an attempt to uncover structural breaks, regime shifts, and, in general, important episodes in US monetary policy and to also characterize their impact on the economy[1].

While estimating such reaction functions[2] allows us to quantify shifts in the broad direction of monetary policy and to characterize the overall policy stance of the monetary authority at different points in time, as this is decided upon by the Federal Open Markets Committee, this approach tells us little, if anything, about the way the Fed goes about implementing its chosen policies on a daily basis.

It is the task of the *Open Market Desk of the Federal Reserve Bank of New York* (henceforth "the desk") to implement FOMC directives on the desired target level for the federal funds rate, through *open market operations*. These open market operations (such as overnight *repurchase agreements*, or *RP*s) ensure that the banks' (and other depository institutions') demand for balances at the Federal Reserve are close to the supply of such balances at an interest rate consistent with the desired target level for the federal funds rate. Thus, the desk, by engaging in such OMOs on a daily basis, carries out the FOMC directives on the target for the policy instrument, thereby essentially ensuring that monetary policy is implemented in a way consistent with FOMC decisions[3].

Examining the daily behavior of the desk and its reaction to relevant market variables (such as reserve need levels, or the deviation of the fed funds rate from its target level) is of interest on its own, and can potentially yield insightful answers to interesting questions. For instance, to what

[1] Some of the papers from this large literature that estimate such reaction functions include Boivin (2004), Clarida, Galí, and Gertler (2000), Cogley and Sargent (2002), Dueker (1999), Monokroussos (2005), Orphanides (2004), Sims (1999), Taylor (1993), etc.

[2] Typically such reaction functions (also known as Taylor rules) are estimated using quarterly or monthly data.

[3] Some excellent sources for further details on the New York Fed, its trading desk, and its open market operations are, inter alia, Hamilton (1996), Meulendyke (1998) and Stigum (1989).

extent did the Fed contribute, through its daily conduct of monetary policy, to the destabilizing effects of the October 1987 stock market crash on the US economy being relatively muted? Or, why did the volatility of the federal funds rate decrease in the 1990's, despite a substantial reduction in the level of required reserve balances?

Nevertheless, and in contrast to the very extensive work that has been done estimating FOMC reaction functions, the literature on *desk reaction functions* is very small. Two papers that propose and estimate a reaction function for the desk are Feinman (1993) and Demiralp and Farley (2005). They both use the maintenance-period-average[4] reserve impact of daily open market operations as the dependent variable, and they model that as a function of various variables capturing conditions in the market for reserves (such as projected maintenance-period-average reserve needs, and the measures of the intraperiod distribution of the reserve need).

Both Feinman and Demiralp and Farley (henceforth DF) are careful in noticing, however, that a simple, linear-OLS framework using the above variables would not be appropriate as a model of the desk reaction function: The explanatory variables are "continuous", in the sense that they change daily, but despite these daily fluctuations of the explanatory variables, the desk very often abstains from any action during the maintenance period. Indeed, and as DF point out, recently the desk does not engage in even the most frequent type of open market operations, the overnight RPs more than 50% of the time.

Feinman and DF thus propose and estimate a (censored) Tobit model[5], where, in a standard manner, there is a linear latent equation, and the observed dependent variable is different from zero (and equal to the latent dependent variable) only when the latent variable is above (or below, depending on whether we have open market operations that add or drain reserves from the system)

[4] The maintenance period is a fourteen day period starting on a Thursday and ending on a Wednesday. Banks and other depository institutions are required to hold a certain average level of balances at the Federal Reserve, and this average level is calculated over the 14-day maintenance period. For instance, banks can compensate for large reserve deficiencies on a given day by holding excess reserve surpluses on other days within the same maintenace period.

[5] This is a censored tobit as we do have observations for the explanatory variables when the dependent variable is zero. The idea is that the dependent variable is subject to a constraint (there won't be any open market operations when only (relatively) small changes in reserve levels are called for), whereas the independent variables aren't subject to any constraints.

a certain threshold value[6].

However, and despite Feinman's and DF's judicious choice of a Tobit model for the desk's reaction function, there is arguably some substantial space for improvement. First, the model that the authors propose is a static Tobit. This does not allow for the possibility of dynamics and inertia in the desk's behavior. It is likely that the case for inertia in the desk reaction function is not as strong as that with the FOMC reaction function. After all, the desk, in its quest to implement FOMC directives, responds to daily market conditions with less concern for reputational effects, that would, in the face of uncertainty about data used and about the economy, contribute to inertial behavior and caution, as is the case with the FOMC; but this is still a possibility that ought to be investigated.

However, and while dynamics in the form of lags of the dependent variable may not be of central importance in the present context, there is little doubt that a different sort of dynamics ought to be a central feature in any realistic attempt to model the desk's behavior, namely dynamics in the form of evolving responses of the Fed towards changing conditions in the markets for reserves, both *within* each maintenance period, and *across* maintenance periods. Feinman and DF model such changing responses of the desk through time by attempting to identify structural breaks and by estimating reaction functions separately for each of the subsamples (determined by the structural breaks that have been identified). They model the desk's changing responses within a given maintenance period by introducing a host of dummy variables representing whether we're early or late in the period (Feinman) or different days and events inside the maintenance period (DF).

In this paper I propose and estimate a *dynamic Tobit with time varying parameters* that allows for more complete modeling of the dynamics in both of the dimensions described above. However, the addition of both dynamics and time varying parameters to a standard Tobit comes at the cost

[6] Feinman also proposes a multinomial logit model, where, in a similar fashion to the tobit model, the explanatory variables include information related to reserves and short-term interest rates that are available to the desk each morning, but the dependent variable is the *type* of open market operation conducted (rather than changes brought about to average levels of reserves, as in the tobit model). As DF correctly point out, however, logits rely on very restrictive distributional assumptions that may not be appropriate in the present context. Furthermore, I believe that little, if any, additional insight on the economics of desk behavior can be gained from such a model, that is not obtained from the estimation of the tobit model in the first place. Indeed, Feinman uses such estimated logit models primarily in in-sample prediction experiments.

of substantially increased computational complexity, if one attempts to estimate such a model in a standard, maximum likelihood context. I overcome these computational challenges by adopting a Markov Chain Monte Carlo framework that does not require computation of the likelihood and that is also outside the extremum context.

My findings confirm some of the Feinman and the DF conclusions and refute others. In particular, the results of this paper indicate that the desk exhibits inertial behavior only to a minimal degree: past changes in maintenance-period-average levels of reserves are not a significant determinant of the desk's actions during the present maintenance period. Furthermore, the paper's results broadly confirm the DF finding that the desk adjusted its behavior in the 1990's in order to better face the new environment created by lower reserve requirements in a way that contributed to lower funds rate volatility. Similarly, the results here are compatible with the Feinman findings on the desk's response towards daily reserve surpluses/deficiencies becoming weaker as the maintenance period draws closer to its end. The desk is shown to be focusing more on maintaining a desired maintenance-period average level of reserves and less on smoothing intra-period reserve fluctuations, and this contrast is even more pronounced as we get closer to maintenance-period ends. Finally, the results show that, in contrast to Feinman's claim of a unique, stark structural break in the desk response towards deviations of the fed funds rate from its target level during the stock market crash of October of 1987, the break during that period was neither unique, nor the largest one.

The rest of the paper is organized as follows: Section 2 provides a more detailed depiction of the desk behavior as a way of motivating the proposed model for the desk reaction function, and a detailed description of the model (both the benchmark dynamic Tobit model, as well as its extension that includes both dynamics and time varying parameters) and of the associated computational challenges, as well as the econometric techniques adopted so as to overcome these challenges. Section 3 describes the data used in the paper. Section 4 provides the results and their discussion, and Section 5 gives some concluding remarks. The details of all the MCMC algorithms employed in the paper are provided in the Appendix.

2 A Dynamic Tobit Model for the Open Market Desk's Daily Reaction Function

The desk's principal task is to intervene in the reserves market by engaging in Open Market Operations (OMOs) when this is called for so as to bring the demand for banks' and other depository institutions' balances at the Fed close to their supply at a level for the federal funds rate that is close to the target level dictated by the Federal Open Markets Committee, thus essentially implementing FOMC directives on the desired target level for the fed funds rate, the Fed's monetary policy instrument.

These OMOs are usually temporary, and occasionally permanent (or "outright") purchases or sales of US Government securities in the "open" market (also known as "secondary" market). Temporary OMOs are purchases or sales of such securities by the desk that are going to be reversed at a later point in time. They consist of operations that add reserves to the banking system or drain reserves from the banking system. Temporary add operations are *Repurchase Agreements* (or *RP*s), with which the Desk buys securities from dealers in the secondary market, who agree to repurchase them on a specified date at a specified price. The most frequent type of such add operations (and the most frequent type of OMOs in general) are overnight RPs, but the desk also uses RPs that are longer in duration (short-term RPs, which last for thirteen days or less, or long-term RPs, which last for fourteen days or more) in order to achieve its objectives for the availability of funds in the reserves market. The desk also resorts occasionally to temporary draining operations (*Matched Sale-Purchase* transactions, or *MSP*s, and *Reverse Repurchase Agreements* (or *Reverse RPs*), which, however are not as common as RPs.

As the desk's goal with all these OMOs is to bring the availability of reserves in the banking system to a desired level (thus indirectly also controlling the fed funds rate (and thus other short-term interest rates as well) according to FOMC directives), the natural candidate for a dependent variable for a desk reaction function is the change in the (maintenance-period average) level of reserves brought about by such OMOs[7]. The desk reaches decisions each day on what (if any) OMOs to conduct by reviewing all the relevant information that is available each morning and that summarize the conditions in the reserves and federal funds markets. Thus, obvious choices for important explanatory variables that ought to appear in a desk reaction function are the deviation of the daily fed funds rate from its target level, and the projected reserve availability for the maintenance period, probably in terms of both average levels over the maintenance period and of the distribution of reserve availability (again over the maintenance period).

Further details on all the variables and the data used are provided in the following section. There are three issues that are of central importance however and that motivate the specification proposed in this paper for the desk reaction function:

2.1 *Model Specification*

First, it is crucial to notice, as both Feinman and DF did, that while all the explanatory variables are "continuous" (in the sense that there are no constraints on the support space of these variables and they do fluctuate daily), the dependent variable is constrained. As noted above, the dependent variable is the changes in the (maintenance-period average) reserve levels brought about by OMOs. However, these OMOs do not take place every day of the maintenance period, rather, they take place relatively infrequently, as for instance, overnight RPs, which are the most frequent type of operations, happen less than 50% of the time. Thus, the dependent variable is constrained in a substantial way, as it is very often zero during the maintenance period. Given this structure, it is clear that a linear-OLS estimation approach is inappropriate for the estimation of the desk's reaction function. So, the model adopted here is a censored Tobit, where there is a linear latent equation (with a latent "change in maintenance period average level of reserves" being the dependent variable, and the explanatory variables being as described above) and the observed dependent variable is nonzero (and equal to the latent variable) only when the latent variable is above (for RPs) or below (for reverse RPs and MSPs) a certain threshold value.

[7] Indeed this is the dependent variable in both of the Feinman and DF specifications.

The second issue is that a static censored Tobit (the model used by both Feinman and DF), may be too restrictive a specification, given that we are in a time series context. While the case for inertia and dynamics for the desk reaction function may not be as strong as with the FOMC reaction function[8], we cannot rule out ex ante the possibility that the desk indeed exhibits such inertial behavior in the way it conducts its daily operations. The dynamics are modeled by introducing lags of the latent dependent variable as additional explanatory variables.

Thus, the **benchmark specification** is as follows:

$$y_t^* = \alpha + \rho(L)y_{t-1}^* + X_t\beta + \varepsilon_t^*, \text{ where:} \tag{1}$$

$$\begin{cases} y_t = 0 \text{ if } y_t^* \leq \delta \\ y_t = y_t^* \text{ if } y_t^* > \delta \end{cases}, \tag{2}$$

where, $t = 1, ..., T$ indexes time, y_t^* is the latent dependent variable (latent "change in maintenance period average level of reserves"), for period t, and similarly y_t is the (constrained) dependent variable for period t, X_t are the period-t explanatory variables (other than the latent lags), α, β (β is a $k \times 1$ vector, where, k is the number of explanatory variables) are the intercept and the coefficients of the explanatory variables, $\rho(L) = \rho_1 + \rho_2 L + ... + \rho_n L^{n-1}$, (where all of the roots of the associated polynomial $1 - \rho_1 L - \rho_2 L^2 - ... - \rho_n L^n$ lie outside the unit circle), $\varepsilon_t^* \sim N(0, \sigma^2)$, and δ is the threshold value that determines the cutoff point beyond which there will be an operation[9].

Note that, as is usually the case with censored Tobits, there are three alternative concepts for conditional mean functions to consider, and thus three corresponding concepts of marginal effects of the independent variables on the dependent variable. First, we have the *truncated mean*, which is the mean of the dependent variable conditioning on the truncation, with which we confine our

[8] After all, the desk reaches decisions every morning on what (if any) OMOs to conduct for the day mostly on the basis of daily information related to the markets for reserves and for short-term interest rates. Thus, arguments such as caution and reputational or signaling considerations on the part of the desk, that in the face of uncertainties about data estimates available in real time or the conditions in the markets would induce inertial behavior (as they do with the FOMC) do not seem to be that important in this context.

[9] The focus of the paper is exclusively on add operations (draining operations are very infrequent and are not considered in this paper, due to data limitations).

attention to *uncensored* observations only. Second, we have the *mean of the latent variable, y^**, and, third, we have the *censored mean*, where the mean is taken over the entire distribution (including both the censored and the uncensored parts). As is usually the case with such models, opinions differ on which of the three concepts of means and corresponding marginal effects are the useful ones[10], and of course the choice also depends on the context and on what question exactly the researcher is interested in answering[11].

In the present context, we wish to study the desk's reaction function (to changing market conditions), and thus we are primarily interested in the effects that the changing conditions in the markets (for reserves and for short-term interest rates) have on the desk's desire to intervene by affecting the availability of reserves in the banking system. The desk's urge to intervene fluctuates in magnitude on a daily basis, and does not result in actual interventions unless this urge is big enough[12], but it's always there in the first place; so the truncated means and corresponding marginal effects may be the least interesting of the three concepts. The censored mean and the mean of the latent variable are both smaller than the truncated mean[13], and the choice made in this paper is the mean of the latent variable and its corresponding marginal effects. The reason for this choice is both its computational simplicity (indeed, the marginal effects in the latent context are just the coefficients of the explanatory variables, and are thus easier to obtain than either the censored or the truncated marginal effects), and also the fact that the focus of the paper is policy analysis and a historical description of the evolution of the desk's reaction function, rather than, say, predicting the desk's next OMO (in which case the censored mean would be more appropriate)[14].

The third important issue that ought to be considered in a realistic specification for the desk's

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[10] Indeed, in the present context, Feinman considers the mean of the latent variable, and DF choose to focus on the truncated mean.

[11] See Greene (2002) for a very interesting discussion of the alternative concepts and their interpretations in various applied contexts.

[12] Or, (using terminology from the tobit model), it does not result in actual interventions unless the latent dependent variable is larger than the threshold value.

[13] That's quite intuitive, as the truncation is from below in the present context, but the exact expressions and proofs can be found in Greene (2002).

[14] Note that the latent and censored means (and corresponding marginal effects) are unlikely to differ much in practice, and also that the *ratios* of different explanatory variables marginal effects are identical for all three alternatives (truncated, censored, latent), and equal to the ratios of the corresponding coefficients of the explanatory variables (for a proof, see Greene (2002)).

reaction function (and that is not being adequately addressed in the benchmark specification of equations (1) and (2)) is that of the changing responses of the desk towards fluctuating conditions in the markets for reserves and for the fed funds rate, *both within* each maintenance period, and *across* maintenance periods, and through time. For example, and as Feinman and DF point out, the desk is more likely to let daily surpluses or deficiencies persist as the maintenance period draws closer to the end. An example of the desk's possibly changing behavior through time is the a structural break during the stock market crash of October 1987, for which Feinman provides evidence in his paper.

Feinman and DF account for such changes by testing for structural breaks and by estimating different reaction functions for each of the subperiods determined by such breaks and also by employing dummy variables for different days and events of the maintenance period. However, this split sample/dummy variable approach may be too crude and insufficient as a way of fully capturing a richer pattern of dynamics that could be present in the desk's reaction function.

Thus, the approach that this paper adopts in order to allow for a (possibly) rich pattern of dynamic behavior by the desk is a **dynamic Tobit with time varying parameters**. This time-varying parameter extension to the benchmark model is as follows:

$$y_t^* = \alpha_t + \rho_t(L)y_{t-1}^* + X_t\beta_t + \varepsilon_t^*, \text{ where:} \tag{1'}$$

$$\begin{cases} y_t = 0 \text{ if } y_t^* \leq \delta \\ y_t = y_t^* \text{ if } y_t^* > \delta \end{cases} . \tag{2}$$

The only difference from the benchmark model is that now the coefficients of the explanatory variables are time varying as well: As we can see from (1′) these coefficients are α_t, $\rho_t(L)$, and β_t , $t = 1, ..., T$.

The dynamics of the time varying parameters are modeled using driftless random walks. This is quite popular in the relevant time-varying parameter literature (see, inter alia, Boivin (2004) and Cogley and Sargent (2002)) as it allows for (possibly) very rich dynamic patterns without making restrictive assumptions on trends and stationarity. Specifically, the time varying parameter specification is as follows:

Let Φ_t, $t = 1, ..., T$ be an $(n + k + 1) \times 1$ vector[15] that includes the lags of the latent dependent variable, the intercept, and the coefficients of the explanatory variables. Then:

$$\Phi_t = \Phi_{t-1} + \omega_t, \tag{3}$$

with

$$\omega_t \tilde{} N(0, H^{-1}),$$

where $E(\varepsilon_t^* \omega_{js}) = 0$, for all $t, s = 1, ..., T$, and $j = 1, ..., n + k + 1$.

While both the benchmark model and its time-varying-parameter extension arguably provide a more realistic depiction of the desk's reaction function (over the static Tobits that have been used in the past), this comes at a cost as well, that of substantially increased computational complexity associated with estimating such models. The remainder of this section provides a discussion of the computational and estimation challenges and of the estimation strategy adopted to overcome these challenges:

2.2 *Estimation Strategy*

Estimation of static Tobits is standard (see, for instance, Greene (2002) for the specification of the likelihood and the MLE approach). However, estimation of dynamic Tobits is substantially more involved, computationally: The likelihood of Tobits is a mixture of discrete and continuous distributions. The continuous part corresponds to the nonlimit observations (in the context of equation (2), these correspond to time periods for which $y_t^* > \delta$), whereas the discrete part corresponds to the censored observations (in the context of equation (2), these correspond to time periods for which $y_t^* \leq \delta$).

[15] As can be seen from equations (1) and (1′), n is the number of lags of the latent dependent variable, and k is the number of explanatory variables.

The continuous part is standard[16]. All the computational complications arise because of the discrete part: The part of the likelihood that corresponds to each censored observation is a joint event probability. More specifically, let's say that the observation corresponding to period t_0 is censored. Then, the likelihood of that observation is a multiple integral, whose dimensionality is as high as the length of the censoring (up to period t_0) and whose integrand is a Gaussian pdf. The functional specification of the integrand, as well as the limits of integration for each of the multiple integrals are determined by equation (2).

The computational challenge arises because there is no closed-form solution for each of these multiple integrals, and such joint event probabilities are multiple integrals even when expressed as products of conditional probabilities (with the conditioning being on past periods' information sets). For more information on the multiple integral problem see, inter alia, Lee (1999), and Monokroussos (2004, 2005).

The dimensionality of the multiple integrals is too high (in the present context, where we have extensive consecutive periods with no OMOs, and thus, where we have extensive censoring) for traditional numerical methods to be feasible[17]. One possible way to estimate dynamic Tobits, as Lee (1999) demonstrates, is simulation-assisted estimation (in an extremum context, whereby the mode of the simulated likelihood is sought with numerical methods). This paper adopts instead a *Markov Chain Monte Carlo*[18] approach of a particular type, namely *Gibbs sampling with data augmentation* (see Tanner and Wong (1987), Albert and Chib (1993), Dueker (1999b), Dueker and Wesche (2003)). Indeed, there are several advantages to Gibbs sampling that are important in the present context and that make it a convenient and efficient estimation approach for estimating dynamic Tobits.

First, the set of parameters to be estimated is broken into subsets that are chosen in a such

[16]In the MLE context it corresponds to the classical, regression-style Gaussian likelihood. In a manner similar to that of linear autoregressive models, and as is illustrated in Chapter 5 of Hamilton (1994), the pdf corresponding to the continuous part can be expressed as a product of conditional pdfs.

[17]Traditional numerical integration methods, such as Gaussian quadrature can typically handle very low dimensionalities (such as, up to four or five) of multiple integrals.

[18]Excellent introductions to simulations and MCMC-based estimation and inference are Chib (2001), Geweke and Keane (2001), and Robert and Casella (1999).

a way that we know how to sample from the conditional distribution of each of the subsets (the conditional distribution is taken conditioning on the entire history of the data and on the other subsets). Thus, we do not have to worry any longer about the joint pdf of the entire parameter set and about the multiple integrals of the full likelihood.

Second, we are outside the extremum context, and thus we avoid the additional computational challenge of having to numerically optimize a simulated objective function. The point estimates of the parameters are taken to be the means of the simulated marginal posteriors of the parameters, and the confidence intervals are given by the quantiles of these posteriors[19].

Finally, this approach enables us to implement the time-varying-parameter extension to the benchmark model relatively easily, simply by adding additional blocks to those of the Gibbs sampler for the benchmark model.

An important advantage of Gibbs sampling (when compared to, for instance, the *Metropolis-Hastings* algorithm[20]) is that it is relatively easy to achieve convergence to the ergodic distribution of the Markov Chain of the simulated draws[21]. The issue of detecting convergence is to a substantial extent an open research question, as there is no consensus in the literature on what the best way to detect convergence is. The approach adopted here is that of McCulloch and Rossi (1994) whereby histograms or nonparametric estimates of the posteriors of the parameters are visually examined and compared when the Gibbs sampler is initiated from different starting values, with evidence of unimodality and of little effect of the starting values on the posteriors serving to establish convergence[22].

The Gibbs sampler for the benchmark model has $T + 2$ blocks: First, there is a block for the variance σ^2, and second, a block for the coefficients of the explanatory variables, α and β. Fi-

[19] This is a valid way to construct the confidence intervals given that the information equality holds in the present context (see Chernozhukov and Hong (2003) and Monokroussos (2004)).

[20] The *Metropolis-Hastings* algorithm is another MCMC technique that encompasses Gibbs sampling as a special case.

[21] In contrast to Gibbs sampling, the success of the Metropolis-Hastings algorithm depends on the (necessarily arbitrary) choice of a candidate distribution. Achieving, and also detecting convergence may be substantially harder (in practice) with Metropolis-Hastings than with Gibbs sampling and may thus offset any computational advantages the non-extremum MCMC approach may have over, say, simulated Maximum Likelihood (which requires numerical optimization of potentially very difficult objective functions).

[22] For further details on this approach see also Monokroussos (2004, 2005).

nally, there is the *data augmentation* step, which enables estimation of Limited Dependent Variable models in an MCMC context by exploiting their latent-variable representation, and specifically by generating (simulated samples of) these latent variables from their model-implied conditional distributions (see Tanner and Wong (1987), and Albert and Chib (1993)). The data augmentation step is implemented here with a *single-move Gibbs smoothing algorithm*, where each latent variable is generated in a separate block from its model-implied conditional distribution, with the conditioning being on all the other parameters, including all the other latent variables, and also on the entire history of the data. This conditioning on the entire history of the data necessitates the use of a smoothing algorithm for the generation of the latent variables, and these single-move Gibbs steps used here are a way of implementing the smoother without having to resort to nonlinear filters (or linear approximations of nonlinear filters). Dueker (1999b), and Dueker and Wesche (2003) adopt this approach for Probits of time series, and this paper adapts their approach to Tobits of time series. The specifics of how the single-move Gibbs sampler for the latent variables is implemented are provided in the Appendix (together with the details on the remaining blocks, namely for σ^2, α, and β).

The time-varying-parameter extension to the benchmark model is implemented by adding a single-move Gibbs sampling step, whereby all the time-varying parameters are generated in one block. In contrast to the latent variables, the smoothing algorithm here can be implemented within a linear state-space framework and the Kalman filter (with the measurement equation being equation $(1')$ and the transition equation being equation (3)). Finally the precision matrix H is also generated in a separate block. All the additional details on these additional blocks are also provided in the Appendix.

3 Data

The data on reserves and the fed funds rate used in the paper are at the daily frequency, and the period spanned is from January 1st, 1986 until June 4th, 1998[23]. The dependent variable and the explanatory variables constructed using this data (and used in the estimation of both the benchmark model and its time-varying-parameter extension) are as follows:

The Dependent Variable

The obvious choice of time period for the reserves market and for the specification of the desk's reaction function is the fourteen day maintenance period (starting on a Thursday and ending on a Wednesday), as banks have to meet legislated reserve requirements not on a daily basis, but on average over the maintenance period. Thus, quite naturally, the main focus of the desk's attention (in its quest to meet its objectives in the markets for reserves and for short term interest rates) is the maintenance period average level of reserves. This is the quantity that the desk seeks to affect through open market operations, and so the dependent variable for the desk's reaction function is the change in the maintenance period average level of reserves brought about by the desk's open market operations.

This quantity is not the par value of an OMO, but rather the *impact* that a given OMO has on the period-average level of reserves. Thus, and following Feinman and DF, to calculate the impact of temporary OMOs, I multiply the par value of the OMO by the number of days (counting holidays and weekends as well) spanned by the OMO and divide by fourteen. For outright transactions, I multiply by the number of days *remaining* (including the delivery day) in the maintenance period (and again divide by fourteen).

For instance, let's say that the desk injects reserves on a Monday into the banking system via an overnight RP of $14 billion. Then the dependent variable will be $1 billion. The same transaction executed on a Friday would give us a dependent variable of $3 billion. A reverse RP of the same

[23] This time period corresponds to the subset of the reserves dataset that is publicly available (as there is a confidentiality restriction on the reserve time series for the most recent years).

magnitude spanning three days would result in a dependent variable of -$3 billion[24]. An outright add transaction of $1 billion for securities delivered on day 8 of the maintenance period would result in a dependent variable of $500 million.

The empirical results presented in the next section are restricted exclusively to overnight RPs[25]. While this weakens to some extent the strength of the conclusions reached on the estimated reaction function of the desk, the results are still quite informative as overnight RPs are the most frequent operations conducted by the desk.

Explanatory Variables

Deviation of the federal funds rate from its target: As mentioned earlier, the desk conducts OMOs in order to bring the fed funds rate close to its target level designated by the FOMC; so the first explanatory variable is the deviation of the (morning[26]) fed funds rate from the target. Obviously, large deviations of the morning rate from its target increase the likelihood of an OMO by the desk.

Estimated Maintenance-Period-Average Reserve Need: The banks' (and other depository institutions') demand for reserves is given by the reserves that the banks are required to have, augmented by any desires the banks may have for excess reserves (on top of the minimum reserve requirements) minus deposits. The desk estimates every day an average demand for reserves over the maintenance period. This estimate minus the reserves borrowed from the discount window gives us the *nonborrowed reserve path*, which is the desk's primary objective. That quantity, (minus any nonborrowed reserves that are forecasted to exist for reasons other than the ones outlined above[27]) is the desk's *Estimated Maintenance-Period-Average Reserve Need* (henceforth, "*the need*"). This is the (projected) amount of reserves that must be added or subtracted (on average every day of the maintenance period) in order to reach the desk's objective. Both the Board and the New York Fed come up with estimates for the need, and the explanatory variable used in what follows is an

[24] Of course, in all these examples it is assumed that the transaction described is the only transaction of the day.

[25] The reason for this is data limitations; the present version of the data set available to the author does not include any MSPs or reverse RPs, whereas short term RPs are often affected by early withdrawals from the short-term RP contracts, and accounting for such early withdrawals is not feasible (with the present version of the data set).

[26] That is, at the beginning of the day and before any transactions take place.

[27] For further details see Feinman and DF.

average of these two estimates.

Distribution of Reserve Need over the Maintenance Period: The desk looks on a daily basis not just at the average (for any given day of the maintenance period) need, but also at the distribution of the need over the maintenance period, as it tries to smooth in general the daily reserves path over the duration of the period.

One variable that captures such distributional aspects of the reserve need is simply the (desk's estimate of the) *daily reserve deficiency (or surplus).* The daily reserve deficiency is the projected (nonborrowed) reserve availability for the day subtracted from the estimated maintenance period average reserve objective; the variable used here is, again, an average of the Board and New York Fed estimates. The estimated coefficient of such a variable can shed some light to the extent to which the desk tries to smooth the reserves path over the maintenance period. If the desk does exhibit such behavior, then we would expect that it would be more likely (ceteris paribus) to engage in an add OMO on a day in which reserves are projected to be deficient than on a day in which the desk expects a surplus.

Another aspect of the distribution of the reserve need over the maintenance period that the desk is likely to take into consideration is the *cumulative reserve deficiency (or surplus) to date*: This is based on the desk's estimates of available nonborrowed reserves that have accumulated since the beginning of the maintenance period. So, if, for instance, there is a deficiency in the amount of nonborrowed reserves that have accumulated, relative to what was expected on the basis of the desk's estimate of the nonborrowed reserve path, then the desk is more likely to proceed to an OMO that injects reserves into the system (again, ceteris paribus, and given the projected maintenance period average reserve need in particular).

4 Estimation Results and Discussion

The period examined is from January 1st, 1986 until June 4, 1998. However, there is a 1-year gap in the reserves data (1993-1994)[28]. I thus consider two separate subperiods in all the estimation exercises: The 1986-1992 period, and the 1994-1998 period[29]. The end of the first subperiod (which comes at the end of 1992) coincides with a structural break in the volatility of the fed funds rate detected by DF, and is thus further motivated on these grounds as well. However, I conducted experiments estimating the Time Varying Parameter model using the entire sample (including, that is, the 1993-1994 period) where the "possibility of a break (corresponding to the period 1993-1994) is allowed for"[30], and the results from these experiments are similar to those obtained from the Time Varying Parameter Model estimated separately in each of the two subperiods. Furthermore, these results are also similar to those obtained when the Time Varying Parameter model is estimated using the entire sample with no break allowed for the 1993-1994 period. This robustness is quite comforting and it suggests that the whole issue of a possible break in 1993-1994 is not an important factor in the present context. In the paper I report the results from the Time Varying Parameters for each of the two subperiods separately, mostly to facilitate the comparison with the results from the benchmark model and from the existing literature.

An additional break that has been detected in the literature is the October of 1987 stock market crash (further details on this break are available in Feinman). For the benchmark model, I account for this break by dividing the 1986-1992 period in two parts and by estimating the benchmark model separately for each of the pre- and post- crash subperiods. I allow for such a break in the Time Varying Parameter model as well by allowing again for the possibility of a jump in the evolution of the time-varying parameters. The details of this, as well as a discussion of the results from the estimation of both models are provided in what follows:

[28] This is due to a gap in the datasets used by Feinman and by DF.

[29] This period starts on May 5, 1994 and ends on June 4, 1998.

[30] The meaning of this phrase will become more specific in the subsection on Time Varying Parameter Model that follows.

4.1 Benchmark Model

The first conclusion we reach upon examining the estimated benchmark dynamic Tobit model (equations (1) and (2)) is that the dynamics in the desk reaction function are very weak or almost non-existent. Specifically, in experiments where both the (projected) daily reserve deficiency and the cumulative (to date) reserve deficiency are included as explanatory variables, the first lag of the latent dependent variable is not significant in any of the three subperiods (that is, the period until the October '87 crash, the period between the crash and through 1992, and the 1994-98 period). In runs with the daily reserve deficiency being the only explanatory variable (other than the average need) accounting for the distribution of the reserve need over the maintenance period, the first lag of the latent dependent variable is marginally significant for the 1986-1987 period, significant for the 1987-1992 period[31], and insignificant for the 1994-1998 period.

These results confirm the prior expectation that the dynamics in the desk reaction function are not as central a factor as they are in the case of the FOMC reaction function. For one thing, any dynamics induced by possible inertia in the desk behavior are hardly there, because the desk has much weaker incentives to resort to inertial behavior than the FOMC does. The FOMC's moves are heavily scrutinized by the financial markets and any signs of backtracking and policy reversals will have reputational costs and will undermine the effectiveness of the monetary authority. Faced with such risks, and in the face of uncertainty in real time about the shape of the economy and about the data summarizing current macroeconomic conditions, the FOMC has very strong incentives to be extra cautious and to implement its policy stance in a series of slow, stodgy steps that are highly (positively) autocorrelated, rather than instantaneously.

The desk's task is much easier in that respect; its task is to make sure that the FOMC directives are met on a daily basis by monitoring the reserves market and employing OMOs when interventions are called for. This is just implementing policy decisions, rather than policy setting; thus any signaling or reputational considerations (that would induce caution and inertia) are only of limited

[31] This possibly reflects the desk's tendency to resort to overnight RPs on consecutive days during the 1987-1992 period more often than in the other two periods.

relevance in the context of the desk's reaction function.

The results from estimating the benchmark model in each of the three periods (before and after 1992, and before and after October 1987) are presented in Table 1[32], and largely confirm our prior expectations.

Specifically, and as expected, the estimated coefficient of the maintenance-period-average reserve need is positive and significant in all of the three periods. This implies that the desk has been consistently monitoring the average reserve need within maintenance periods and across periods and has been addressing the projected average need by injecting funds into the banking system via overnight RPs.[33].

The coefficient of the daily reserve deficiency however, is much weaker than the average-need coefficient, and significant only during the 1994-1998 period. It does have the expected positive sign however for all three periods. The positive sign implies that, ceteris paribus, and in particular, for a certain deviation of the morning fed funds rate from its target, and given the maintenance period average reserve need, the desk is more likely to resort to an overnight RP, thus adding reserves, on days when it expects a reserve deficiency than on days when it expects a reserves surplus. Thus, there is some evidence that the desk tries to smooth the reserves path over the maintenance period; however, the estimates suggest that such smoothing considerations may not be as important a determinant of the desk's behavior as one might have thought.

Note, however, that the estimated coefficients of both the maintenance-period average reserve need and the daily reserve deficiency increase substantially as we move from the late eighties and early nineties to the mid- and late nineties. This transition coincides with the regime change in the market for reserves that DF document and discuss and suggest that, as DF claim, the desk changed its reaction function in order to adjust to these changing conditions.

More specifically, there was a decline in the reserve requirements in the early nineties, which,

[32]These results are from a specification without any lag of the latent dependent variable, because of the reasons outlined above. The results change very little when the first lag is added to the benchmark specification.

[33]Note that the magnitudes of the estimated coefficients are lower than those of Feinman. Note however that Feinman's dependent variable is based on all transactions conducted by the desk (both temporary and outright), whereas the results here are based only on overnight RPs.

contrary to what one might expect, did not have a lasting impact on short-term interest rates. The volatility in the federal funds rate increased at first, but then it actually declined. DF suggest that it was changes in the desk's reaction function that played a crucial role in controlling the volatility of short term interest rates. The evidence provided here, which show a desk that adopts a more pro-active behavior in response to changing conditions in the reserves market as we move from the early nineties to the late nineties point towards conclusions which are similar to those of DF.

Finally, another interesting observation that can be made from Table 1 is the big jump in the coefficient of the deviation of the morning fed funds rate from its target (henceforth, the "fed funds gap"), as we move from the pre-October '87 period to the post-stock-market-crash period. These results are similar to Feinman's findings, and possibly reflect the monetary authority's determination to vigorously implement its policies, control short-term interest rates and stem any destabilizing effects on the economy in the period following the crash.

However, and while these results are interesting, this split-sample approach may be too crude to capture possibly rich and complicated dynamics in the desk reaction function, both inside the maintenance period (which are obviously completely ignored by the benchmark model) and through the years. Thus, the remainder of this section discusses the results from estimating the time-varying parameter extension to the benchmark dynamic Tobit model.

4.2 Time Varying Parameter Model

The Time Varying Parameter (henceforth, "TVP") model is estimated for two periods, namely the 1986-1992 period, and the 1994-1998 period. The first period contains the October '87 crash where a structural break has been identified by Feinman. Using a driftless random walk such as that of equation (3) when in reality there is a structural break would be a misspecification which could lead to misleading conclusions. However, an additional advantage of the TVP approach is that such a structural break can be allowed for inside the TVP context and thus splitting the sample can be avoided. In the present context, this is done as follows:

The time varying parameters are generated as draws from a Normal distribution whose parameters are computed using the Kalman filter[34]. A natural way to allow for the possibility of a jump in the parameters during the stock market crash of October 19, 1987 is to draw the parameters for that date from a distribution with a much higher variance than the variance that would be implied by the updating equations of the Kalman filter (for that date). Doing that however results in an implausibly big spike for that particular date in the graphs of the time varying parameters.

This suggests that this setup is too restrictive; the break is insufficiently modeled by allowing for a large variance only on the exact day of the stock market crash, and the possibility of richer dynamics with a higher amplitude ought to be allowed for a larger period around October 19, 1987. The obvious way to implement this is by drawing not just the parameters corresponding to October 19, 1987, but also the parameters corresponding to many dates before and after October 19, 1987 from distributions with high variance. However, and as we can see from equation (3), the parameters follow a random walk, and the time series dimension is relatively large in the present context. If we use variances that are too high, or if the period over which the parameters are drawn from high-variance distributions is too wide, then the variance covariance matrix of the time varying parameters can become arbitrarily high. After some experimentation, I modeled the structural break corresponding to the stock market crash by drawing the parameters for the month of October, 1987 from distributions with variances that were ten times higher than the variances derived from the Kalman filter updating equations[35].

The graphs of the time varying parameters for the 1986-1992 period are presented in Figures 1A-4A, and those for the 1994-1998 period are in Figures 1B-4B[36]. These figures suggest a rich pattern of dynamics for the desk reaction function that cannot be revealed by standard split sample / dummy variable specifications. While the TVP model (and the rich dynamics it usually implies when confronted with the data in applied situations) is motivated ex ante for the context of the

[34] The details of the smoothing algorithm that generates the time varying parameters are described in the Appendix.

[35] I also conducted the robustness checks for the possibility of a break corresponding to the 1993-1994 gap in the data (discussed in the beginning of Section 4) following a similar approach. More specifically, I drew the parameters for the month of May of 1994 from distributions with variances that were ten times higer than the variances derived from the Kalman filter updating equations.

[36] All the figures present point estimates (solid lines), which are the means of the posterior distributions, and 90% confidence bands (dotted lines), which are constructed from the 5th and 95th percentiles of the posterior distribution.

desk and of its operations because of the institutional reasons discussed in DF and Feinman, and in Section 2 of this paper, it would still be interesting to see how the results obtained with the TVP model compare with the respective ones of the benchmark specification and of the existing literature. The rest of this section takes up this task:

First of all, the figures confirm the finding from the benchmark model that the maintenance-period-average reserve need is a more important determinant of the desk's behavior than the daily reserve deficiency. Indeed, as we can see from Figures 2A and 2B (that present the maintenance-period-average reserve needs for the two subsamples) and from Figures 3A and 3B (the respective figures for the daily reserve deficiency) the "need" estimates are around 10 times bigger than the "deficiency" estimates, and are also significant much more often than the deficiency estimates. This again suggests that the desk may be less concerned with smoothing the reserves path over the maintenance period than one might have thought, and more focused on meeting the average (over the maintenance period) need.

Similarly, we see from Figures 3A and 3B that the coefficient of the daily reserve deficiency slopes downwards quite more often, and much more gradually than it slopes upwards. Thus, there is a downward sloping pattern most of the time, gradual downward movements that are interrupted by (usually) abrupt upward movements, only to be followed again by gradual downward movements, etc. The implication of this pattern is that the desk's response towards daily reserve deficiencies becomes weaker as the maintenance period comes closer to its end and settlement day approaches. This confirms Feinman's findings and is also compatible with our intuition, since, as the maintenance period draws to a close it is less likely that high daily reserve deficiencies (or surpluses) will lead to a sustained pattern of high volatility in reserves and in the fed funds rate (simply because the reserve path for most of the maintenance period has already been determined).

Thus, in a nutshell, there is evidence that the desk has been focusing primarily on maintaining a desired average (over the maintenance period) level of reserves and not as much on smoothing intra-period reserve fluctuations, especially as we get closer to the end of the maintenance period.

Furthermore, we can see by observing the volatility of the coefficient of the daily reserve deficiency for the mid- to late nineties (Figure 3B) that is higher in the late nineties than earlier, that in a manner similar to what is suggested by the results of the benchmark model and by DF, the desk seems to have adopted a more agile and proactive stance towards the distribution of the reserve need over the maintenance period. As discussed in DF and earlier in this paper, this more interventionist approach of the late nineties in response to daily movements of the daily reserve deficiencies (or surpluses) possibly reflects the desk's adjusting behavior in the new environment of lower reserve requirements of the mid- to late nineties

One interesting case of the Time Varying Parameter estimates leading to substantially different conclusions than those reached with the benchmark model (and of the existing literature) is with the coefficient of the deviation of the morning fed funds rate from its target for the 1986-1992 period. As we can see in Figure 4A, while there is a jump in the estimated coefficients in October of 1987, this jump is far from being unique. It is confined only to the immediate period subsequent to the stock market crash, and is preceded and followed by jumps (and drops) of comparable magnitudes[37]. Furthermore, there is evidence that the desk's response towards the deviation of the fed funds rate from its target moves to consistently higher average levels only in the early nineties, rather than immediately after the crash (which is what is suggested by Feinman and by the benchmark results discussed earlier).

5 Concluding remarks

This paper proposes and estimates a dynamic Tobit model with time varying parameters for the reaction function of the Open Market Desk of the Federal Reserve that allows both for inertial behavior and for changing responses of the desk to evolving conditions in the markets for reserves and for short-term interest rates. The results reveal rich and complicated dynamics that are present in the desk reaction function (both inside the maintenance period and across maintenance periods

[37] However, when the TVP model is estimated over the entire period, that encompasses the 1993-1994 gap, again allowing for the possibility of a jump in the parameters for the month of October of 1987 (as discussed earlier), the magnitude of the jump (during October '87) in the coefficient of the deviation of the fed funds rate from its target increases substantially (regardless of whether we model the 1993-1994 gap with a jump as well or not). However, and even in that case, the October '87 jump is again neither the biggest one, nor is it unique.

and over the years) and that cannot be adequately modeled by split sample / dummy variable specifications.

The results of this paper indicate that the desk exhibits inertial behavior in the way it conducts its open market operations only to a minimal degree, in the sense that lagged changes in maintenance-period-average levels of reserves (brought about by past OMOs) are a weak and insignificant determinant of any changes in the respective average levels of reserves that the desk may choose to bring about during the current maintenance period.

Furthermore, the findings with the time varying parameters point towards a desk that focuses mostly on the maintenance-period average reserve need and is only moderately inclined to smooth intraperiod distribution patterns of reserve availability, especially as the maintenance period draws to a close.

Additionally, upon examining the evolution of the time varying parameters in the second half of the eighties and in the nineties, we conclude that the desk did not respond to the October 1987 stock market crash in a unique and unprecedented way, as suggested by constant parameter / split sample estimates. There was a jump in the desk's response towards deviations of the fed funds rate from its target, but only in the immediate aftermath of the crash, and this jump was neither the biggest one nor the only one in the eighties and in the nineties.

Finally, there is evidence that the desk adjusted its behavior in response to a new environment in the market for reserves (caused by lower reserve requirements) in the nineties by adopting a more agile approach and by being more responsive towards daily reserve deficiencies (than in the early nineties).

Appendix: MCMC Algorithms for the Benchmark Model and for the Time Varying Parameter Model

A few words on notation and terminology for what follows first: θ will be taken to mean in what follows all the variables *other than the ones being generated in the particular block under consideration.* Furthermore, y, y^*, X will denote the entire vector for the dependent variable, the latent dependent variable, and the entire matrix of explanatory variables, respectively, (periods $1, ..., T$), and y_t, y_t^*, X_t will denote the dependent variable, the latent dependent variable, and the explanatory variables for period t, respectively. The word "conditional" will be taken to mean *conditional on everything, except of course for the variable(s) being generated in the particular block under consideration.*

Algorithm for the Benchmark Model:

Generating the Variance: Inverted gamma distributions are convenient priors for the variance, since when multiplied by the conditional likelihood, they result in conditional posteriors which are also inverted gammas[38], that we know how to sample from:

So, if the prior for σ^2 is $IG(\frac{\nu_0}{2}, \frac{\delta_0}{2})$, where IG stands for inverted gamma, then the conditional posterior is also $IG(\frac{\nu_1}{2}, \frac{\delta_1}{2})$, where $\nu_1 = \nu_0 + T$, and $\delta_1 = \delta_0 + \varepsilon^{*\prime}\varepsilon^*$, where ε^* is the $T \times 1$ vector of latent error terms of equation (1).

Generating the coefficients of the explanatory variables: A flat prior for this block results in a Gaussian conditional posterior from which I can sample easily: In particular, this conditional posterior is, in a standard way, $N((X'X)^{-1}X'y^*, \sigma^2(X'X)^{-1})$. For a derivation of this, see, for instance, Albert and Chib (1993).

The required stationarity constraints on the coefficients of the lags of the latent dependent variable are implemented with rejection sampling, whereby draws from the posterior for the coefficients are taken until the constraints are satisfied, (and the draws are discarded when they do not satisfy the constraints).

[38] See, for instance, Kim & Nelson (1999) for the derivation of this.

Generating the latent dependent variables: I use a single-move smoothing algorithm here, which entails simulating each y_t^*, $t = 1, ..., T$, one by one in separate blocks, while also conditioning on all the data, and all the other parameters, including all the other latent variables, for each block. The algorithm is derived as follows:

Let $g(y_t^*|\theta, y, X)$ denote the conditional distribution of y_t^*, and let $\widetilde{y}_t^*$ denote all the latent variables for periods $1, ..., t$, and let $\widetilde{y}_{\neq t}^*$ denote all the latent variables *for all periods except for* t, and similarly let $\widetilde{y}_t$ denote all the dependent variables for periods $1, ..., t$. The dependence on the parameters other than the latent variables and on the explanatory variables is suppressed in what follows for convenience. Furthermore, for expositional purposes, I present the case of one lag for the latent variable. The proof for more than one lags is the same. So, we have that:

$$
\begin{aligned}
g(y_t^*|\widetilde{y}_{\neq t}^*, \widetilde{y}_T^*) &= g(y_t^*|\widetilde{y}_{\neq t}^*, \widetilde{y}_t, y_{t+1}, ..., y_T) \\
&= \frac{g(y_t^*, y_{t+1}, ..., y_T|\widetilde{y}_{\neq t}^*, \widetilde{y}_t)}{g(y_{t+1}, ..., y_T|\widetilde{y}_{\neq t}^*, \widetilde{y}_t)} \\
&= \frac{g(y_t^*|\widetilde{y}_{\neq t}^*, \widetilde{y}_t)g(y_{t+1}, ..., y_T|\widetilde{y}_{\neq t}^*, \widetilde{y}_t, y_t^*)}{g(y_{t+1}, ..., y_T|\widetilde{y}_{\neq t}^*, \widetilde{y}_t)} \\
&= g(y_t^*|\widetilde{y}_{\neq t}^*, \widetilde{y}_t) \\
&= g(y_t^*|\widetilde{y}_{t-1}^*, y_{t+1}^*, ..., y_T^*, \widetilde{y}_{t-1}, y_t) \\
&= \frac{g(y_t^*, y_{t+2}^*, ..., y_T^*|\widetilde{y}_{t-1}^*, y_{t+1}^*, \widetilde{y}_{t-1}, y_t)}{g(y_{t+2}^*, ..., y_T^*|\widetilde{y}_{t-1}^*, y_{t+1}^*, \widetilde{y}_{t-1}, y_t)} \\
&\propto g(y_t^*, y_{t+2}^*, ..., y_T^*|\widetilde{y}_{t-1}^*, y_{t+1}^*, \widetilde{y}_{t-1}, y_t) \\
&= g(y_t^*|\widetilde{y}_{t-1}^*, y_{t+1}^*, \widetilde{y}_{t-1}, y_t)g(y_{t+2}^*, ..., y_T^*|\widetilde{y}_{t-1}^*, y_t^*, y_{t+1}^*, \widetilde{y}_{t-1}, y_t) \\
&\propto g(y_t^*|\widetilde{y}_{t-1}^*, y_{t+1}^*, \widetilde{y}_{t-1}, y_t) \\
&= g(y_t^*|y_{t-1}^*, y_{t+1}^*, \widetilde{y}_t).
\end{aligned}
$$

Note that the transition from the 3rd line to the 4th line is valid as $y_{t+1}, ..., y_T$ do not depend on y_t^*, given $\widetilde{y}_{\neq t}^*$. Note also that the transition from the 5th line to the 6th line is valid (for the case of models with one lag for the latent variable) because the denominator of the fraction of the 5th line does not depend on y_t^*.

The pdf of the resulting distribution, namely $g(y_t^*|y_{t-1}^*, y_{t+1}^*, \widetilde{y}_t)$ can be obtained from the joint distribution of all the error terms where y_t^* appears. For the case with one latent lag, y_t^* appears in the equations giving ε_t^*, and ε_{t+1}^*. The joint pdf of the error terms is Gaussian, and ignoring for a moment the effect of conditioning on $\widetilde{y}_t$, it is easy to show[39] that y_t^* is distributed (given y_{t-1}^*, y_{t+1}^*) as $N(\frac{\beta' X_t + \beta_1 y_{t-1}^* + \beta_1 (y_{t+1}^* - \beta' X_{t+1})}{1+\beta_1^2}, \sigma^2)$, where β, X_t, X_{t+1} are defined here to exclude the latent lag and its coefficient, and β_1 is the coefficient of the latent lag.

The effect of conditioning on $\widetilde{y}_t$ is a truncation, and the form of the truncation is determined by equation (2):

If $\Delta y_t \in$ category j, then $y_t^* \in (y_{t-1} + c_{j-1}, y_{t-1} + c_j), \forall j, \forall t$.

Thus, the required sampling task is that of sampling from a (univariate) truncated normal. The best way of doing that is a combination of sampling from a uniform and inverting the truncated normal cdf. Specifically, I wish to simulate the latent dependent variable, which has a Normal cdf F with mean μ and variance σ^2, but that is truncated between a (being $-\infty$ in the context of the Tobit) and b (in the context of the Tobit and of equation (2) b is the threshold coefficient δ). Let $Z \sim uniform(0,1)$. Then $F^{-1}(Z) \sim F$. Therefore, and since $F(y_t^*) = \frac{\Phi(\frac{y_t^* - \mu}{\sigma}) - \Phi(\frac{a-\mu}{\sigma})}{\Phi(\frac{b-\mu}{\sigma}) - \Phi(\frac{a-\mu}{\sigma})}$, where Φ is the standard normal cdf, I simulate the latent dependent variable by sampling from: $\sigma\Phi^{-1}\{Z[\Phi(\frac{b-\mu}{\sigma}) - \Phi(\frac{a-\mu}{\sigma})] + \Phi(\frac{a-\mu}{\sigma})\} + \mu$.

Additional blocks needed for the Time Varying Parameters:

Generating the Time Varying Parameters: In contrast to the case of the latent variables, obtaining the smoothing algorithm for the Time Varying Parameters is standard because here we can usefully employ a state-space representation, with the Measurement Equation being the latent equation ((1′)), and with the Transition Equation being the driftless random walk for the TVP's (equation (3)), together with the Kalman filter.

Specifically, let $\widetilde{\Phi}_T = [\Phi_1 ... \Phi_T]'$. Let $Y_1, ..., Y_T$ denote the "data" (that is, the dependent variable,

[39] Just rewrite, in that joint pdf, each of the error terms as an expression of the latent variables that appear in the latent equation that corresponds to that error term.

the latent dependent variable, and the explanatory variables) for periods $1, ..., T$, respectively, and let $\widetilde{Y}_t$ denote all the data up to period $t, t = 1, ..., T$. Let $g(\widetilde{\Phi}_T|\theta, \widetilde{Y_T})$ denote the conditional distribution of $\widetilde{\Phi}_T$. Then, following Kim and Nelson (1999) I employ a multimove Gibbs-sampling approach, thus generating the entire $\widetilde{\Phi}_T$ as a block from its conditional distribution, $g(\widetilde{\Phi}_T|\theta, \widetilde{Y_T})$. The Markov property of the Φ_t's ensure that convenient simplifications occur in $g(\widetilde{\Phi}_T|\theta, \widetilde{Y_T})$, and in particular:

$$g(\widetilde{\Phi}_T|\theta, \widetilde{Y_T}) = g(\Phi_T|\theta, \widetilde{Y_T})g(\widetilde{\Phi}_{T-1}|\theta, \Phi_T, \widetilde{Y_T})$$
$$= g(\Phi_T|\theta, \widetilde{Y_T})g(\Phi_{T-1}|\theta, \Phi_T, \widetilde{Y_T})g(\widetilde{\Phi}_{T-2}|\theta, \Phi_{T-1}, \Phi_T, \widetilde{Y_T})$$
$$= ...$$
$$= g(\Phi_T|\theta, \widetilde{Y_T})g(\Phi_{T-1}|\theta, \Phi_T, \widetilde{Y_T})g(\Phi_{T-2}|\theta, \Phi_{T-1}, \widetilde{Y_T})...g(\Phi_1|\theta, \Phi_2, \widetilde{Y_T})$$
$$= g(\Phi_T|\theta, \widetilde{Y_T})g(\Phi_{T-1}|\theta, \Phi_T, \widetilde{Y_{T-1}})g(\widetilde{\Phi}_{T-2}|\theta, \Phi_{T-1}, \widetilde{Y_{T-2}})...g(\Phi_1|\theta, \Phi_2, \widetilde{Y_1})$$
$$= g(\Phi_T|\theta, \widetilde{Y_T}) \prod_{t=1}^{T-1} g(\Phi_t|\theta, \Phi_{t+1}, \widetilde{Y}_t)$$

As suggested by this last expression, I first need to generate Φ_T from $g(\Phi_T|\theta, \widetilde{Y_T})$, and then, given Φ_{t+1}, generate Φ_t from $g(\Phi_t|\theta, \Phi_{t+1}, \widetilde{Y}_t), t =, ..., T-1$. Thus, I first generate Φ_T from $g(\Phi_T|\theta, \widetilde{Y_T})\tilde{\ }N(\Phi_{T|T}, P_{T|T})$, and then Φ_t, for $t = T-1, .., 1$ from $g(\Phi_t|\theta, \Phi_{t+1}, \widetilde{Y}_t)\tilde{\ }N(\Phi_{t|t,\Phi_{t+1}}, P_{t|t,\Phi_{t+1}})$, where $\Phi_{T|T} = E(\Phi_T|\theta, \widetilde{Y_T})$, $P_{T|T} = Cov(\Phi_T|\theta, \widetilde{Y_T})$, $\Phi_{t|t,\Phi_{t+1}} = E(\Phi_t|\theta, \widetilde{Y}_t, \Phi_{t+1}) = E(\Phi_t|\theta, \Phi_{t|t}, \Phi_{t+1})$, $P_{t|t,\Phi_{t+1}} = Cov(\Phi_T|\theta, \widetilde{Y_T}, \Phi_{t+1}) = Cov(\Phi_T|\theta, \Phi_{t|t}, \Phi_{t+1})$. The updating terms $\Phi_{T|T}, P_{T|T}$, (and also all $\Phi_{t|t}, P_{t|t}, t = 1, ..., T$) can be derived in a standard way using the Kalman filter[40]. The same holds true for the terms $\Phi_{t|t,\Phi_{t+1}}$, and $P_{t|t,\Phi_{t+1}}$ since they can also be viewed as updating terms in which the updating is done not with Y_t, but with Φ_{t+1}, which has been generated, and thus can be considered as observed data.

The initial values, $\beta_{0|0}$ are arbitrary, with $P_{0|0}$ having large diagonal elements (so that large uncertainty is attached to $\beta_{0|0}$).

The reflecting barriers imposing the stability condition on the coefficients of the lags of the dependent variable are implemented with rejection sampling, done separately for each time period $t = 1, ..., T$.

[40] See Hamilton (1994), and Kim and Nelson (1999).

Generating the precision matrix H: The prior for H is Wishart, $W(\nu_0, H_0)$, where I set $\nu_0 = 0, H_0^{-1} = 0$, and then the conditional posterior for H is also Wishart, $W(\nu_1, H_1)$, where $\nu_1 = T + \nu_0$, $H_1 = [H_0^{-1} + \sum_{t=1}^{T}(\Phi_t - \Phi_{t-1})(\Phi_t - \Phi_{t-1})']^{-1}$.

A note on the computer code: All of the computer code for this paper was written in Gauss, Version 3.2.34. The seed was always fixed at 180303.

References

Albert, J., and S. Chib (1993): "Bayesian Analysis of Binary and Polychotomous Response Data," *Journal of the American Statistical Association*, June, Vol., 88, No. 422, pp. 669-679.

Boivin, J. (2004): "Has U.S. Monetary Policy Changed? Evidence from Drifting Coefficients and Real-Time Data," *mimeo, Columbia.*

Chernozhukov, V., and H. Hong (2003): "An MCMC approach to classical estimation," *Journal of Econometrics* (115), 293-346.

Chib, S. (2001): "Markov Chain Monte Carlo Methods: Computation and Inference." In: Heckman, J, and Leamer, E. (Eds.), *Handbook of Econometrics, Vol. 5*, North-Holland, Amsterdam, p. 3569-3649.

Clarida, R., Galí, J., and M. Gertler (2000): "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *Quarterly Journal of Economics* 115(1), 147-180.

Cogley, T., and T. Sargent (2002): "Drifts and Volatilities: Monetary Policies and Outcomes in the Post WWII U.S.," *mimeo, New York University.*

Demiralp, S., and D. Farley (2005): "Declining Required Reserves, Funds Rate Volatility, and Open Market Operations," *Journal of Banking and Finance* (29), 1131-1152.

Dueker, Michael (1999): "Measuring Monetary Policy Inertia in Target Fed Funds Rate Changes," *Federal Reserve Bank of St. Louis Review* 81(5): 3-9.

Dueker, Michael (1999b): "Conditional Heteroskedasticity in Qualitative Response Models of Time Series: A Gibbs Sampling Approach to the Bank Prime Rate," *Journal of Business and Economic Statistics* 17(4): 466-472.

Dueker, M., and K. Wesche (2003): "European Business Cycles: New Indices and their Synchronicity", *Economic Inquiry* 41(1): 116-131.

Feinman, J. (1993): "Estimating the Open Market Desk's Daily Reaction Function," *Journal of Money, Credit and Banking*, 25(2), 213-247.

Geweke, J., and M. Keane (2001): "Computationally Intensive Methods for Integration in Econometrics." In: Heckman, J, and Leamer, E. (Eds.), *Handbook of Econometrics, Vol. 5*, North-Holland, Amsterdam, p. 3463-3568.

Greene, William H. (2002): *Econometric Analysis*, Prentice Hall, 5th Edition.

Hamilton, James D. (1994): *Time Series Analysis*, Princeton University Press, Princeton.

Hamilton, James D. (1996): "The Daily Market for Federal Funds," *Journal of Political Economy*, 104(1), 26-56.

Kim, C., and C. Nelson (1999a): *State-Space Models With Regime Switching*, MIT Press, Cambridge.

Lee, Lung-fei (1999): "Estimation of Dynamic and ARCH Tobit Models", *Journal of Econometrics* 92: 355-390.

McCulloch, R., and P. Rossi (1994): "An Exact Likelihood Analysis of the Multinomial Probit Model," *Journal of Econometrics* 64: 207-240.

Meulendyke, Ann Marie (1998): *U.S. Monetary Policy and Financial Markets.* New York: Federal Reserve Bank of New York.

Monokroussos, G. (2004): "A Classical MCMC Approach to the Estimation of Limited Dependent Variable Models of Time Series," *mimeo, UCSD.*

Monokroussos, G. (2005): "Dynamic Limited Dependent Variable Modeling and US Monetary Policy," *mimeo, UCSD.*

Orphanides, A. (2004): "Monetary Policy Rules, Macroeconomic Stability and Inflation: A View from the Trenches," *Journal of Money, Credit and Banking*, 35(6).

Robert, C.P., and G. Casella (1999): *Monte Carlo Statistical Methods*, Springer.

Sims, C. (1999): "Drift and Breaks in Monetary Policy," *mimeo, Princeton University.*

Stigum, Marcia (1989): *The Money Market*, McGraw-Hill, 3rd Edition.

Taylor, John B. (1993): "Discretion Versus Policy Rules in Practice," *Carnegie-Rochester Conference Series on Public Policy* 39 (December 1993), 195-214.

Tanner, M.A., and W.H. Wong (1987): "The calculation of posterior distributions by data augmentation," *Journal of the Americal Statistical Association*, 82: 528-549.

Table 1: Results from the Benchmark Model

1986-1987							
variable	*mean*	*std. dev.*	*median*	*2.5% qntl.*	*5% qntl.*	*95% qntl.*	*97.5% qntl.*
intercept	33.4187	9.2532	33.1875	15.3603	18.1268	49.0429	51.2051
need	0.2404	0.0291	0.2406	0.1815	0.1923	0.2859	0.2945
deficiency	0.0047	0.0034	0.0048	-0.0017	-0.0009	0.0101	0.0113
fed funds dev	85.1683	33.9127	84.713	23.6575	31.1884	141.0264	152.4317

1987-1992							
variable	*mean*	*std. dev.*	*median*	*2.5% qntl.*	*5% qntl.*	*95% qntl.*	*97.5% qntl.*
intercept	26.761	4.6468	26.7344	17.4182	19.0165	34.501	35.7401
need	0.0823	0.0144	0.0824	0.0534	0.0578	0.1062	0.11
deficiency	0.0016	0.0017	0.0016	-0.0019	-0.0013	0.0044	0.005
fed funds dev	589.321	19.7663	589.9061	552.3822	558.1365	621.9905	626.8735

1994-1998							
variable	*mean*	*std. dev.*	*median*	*2.5% qntl.*	*5% qntl.*	*95% qntl.*	*97.5% qntl.*
intercept	21.3196	7.8402	21.3046	6.6183	8.2462	33.6368	36.4431
need	0.2474	0.0166	0.2478	0.2136	0.2202	0.2737	0.2812
deficiency	0.0104	0.002	0.0103	0.0062	0.0069	0.0135	0.0143
fed funds dev	586.884	31.1361	585.9598	526.6569	535.4788	639.6612	646.931

Note: This is the benchmark specification (excluding the 1st lag of the dependent variable and the cumulative reserve deficiency).
The three panels correspond to the three subperiods of the benchmark specification.
The first column is the mean of the posterior distribution.
The last 5 columns are quantiles of the posterior distribution.

Figure 1A: Intercept, 1986-1992

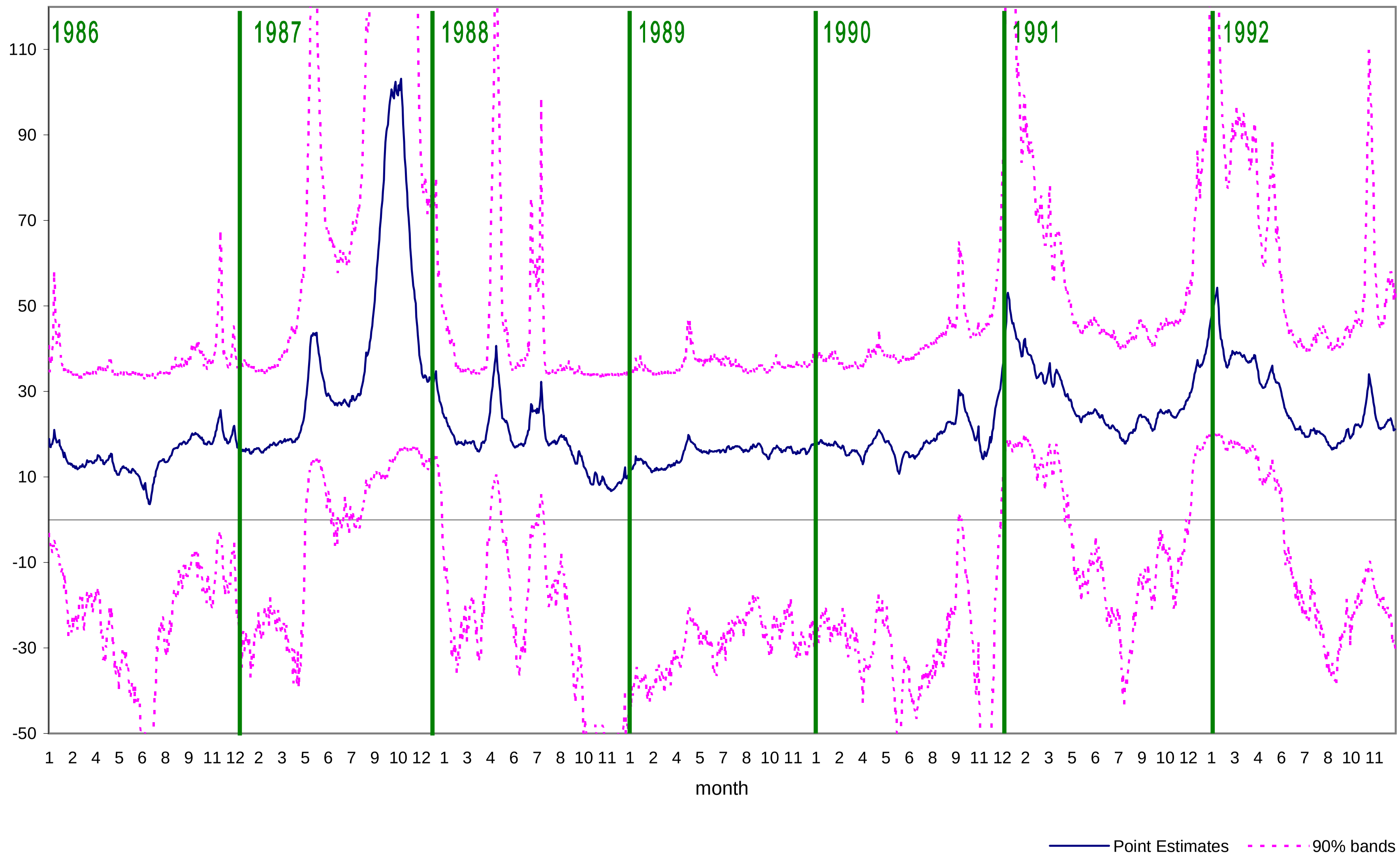

Figure 1B: Intercept, 1994-1998

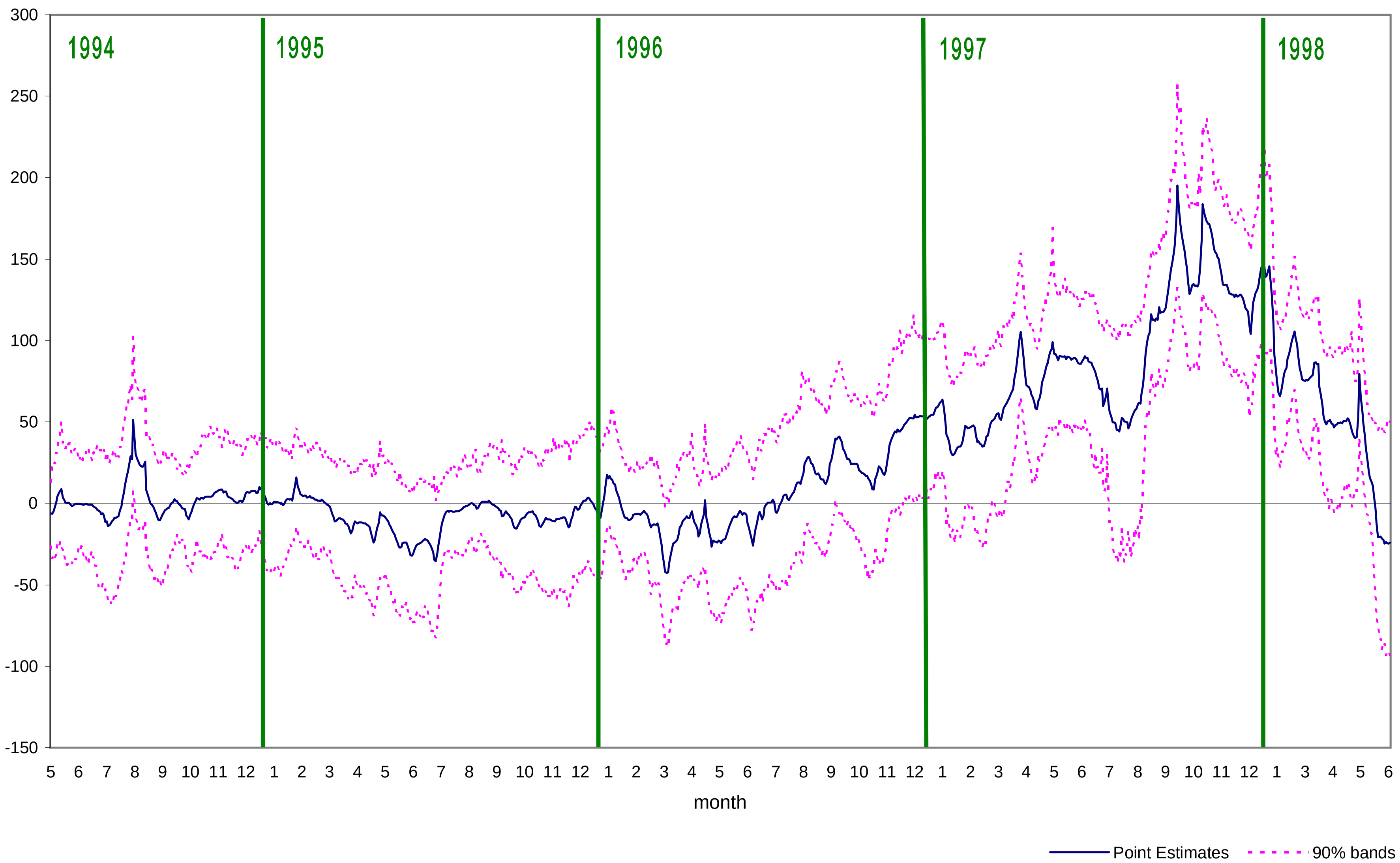

Figure 2A: Maintenance-Period Average Reserve Need, 1986-1992

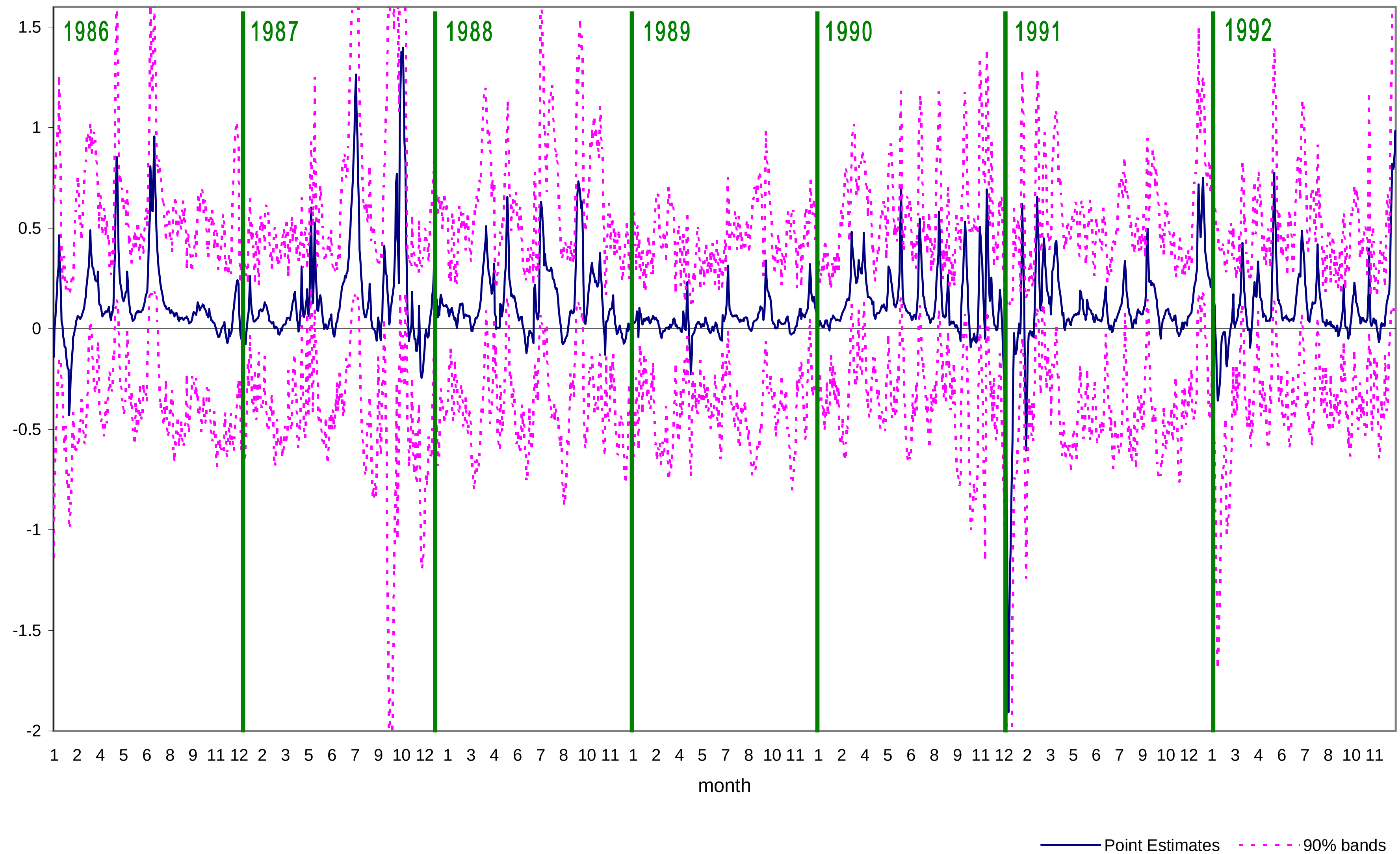

Figure 2B: Maintenance-Period Average Reserve Need, 1994-1998

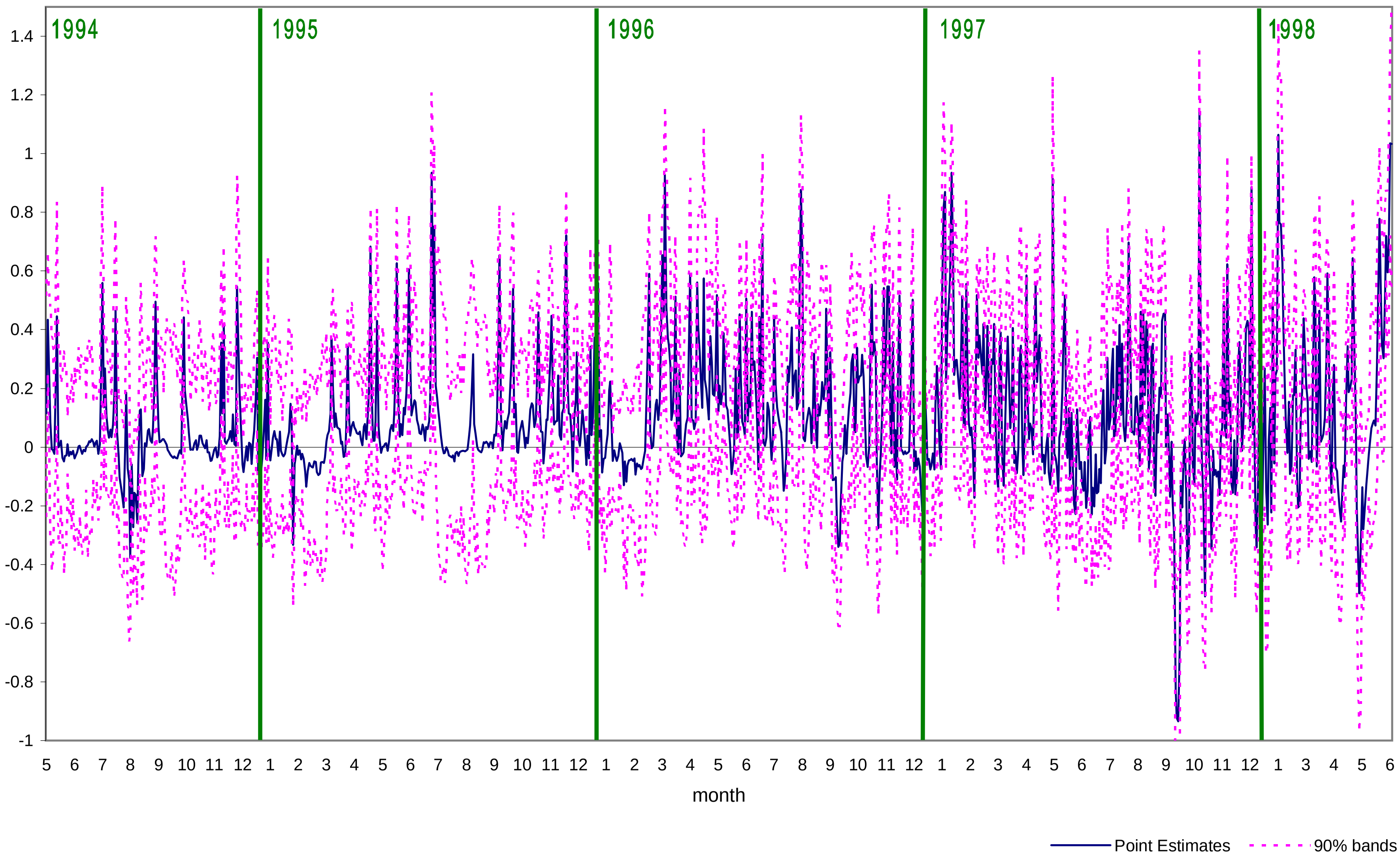

Figure 3A: Daily Reserve Deficiency, 1986-1992

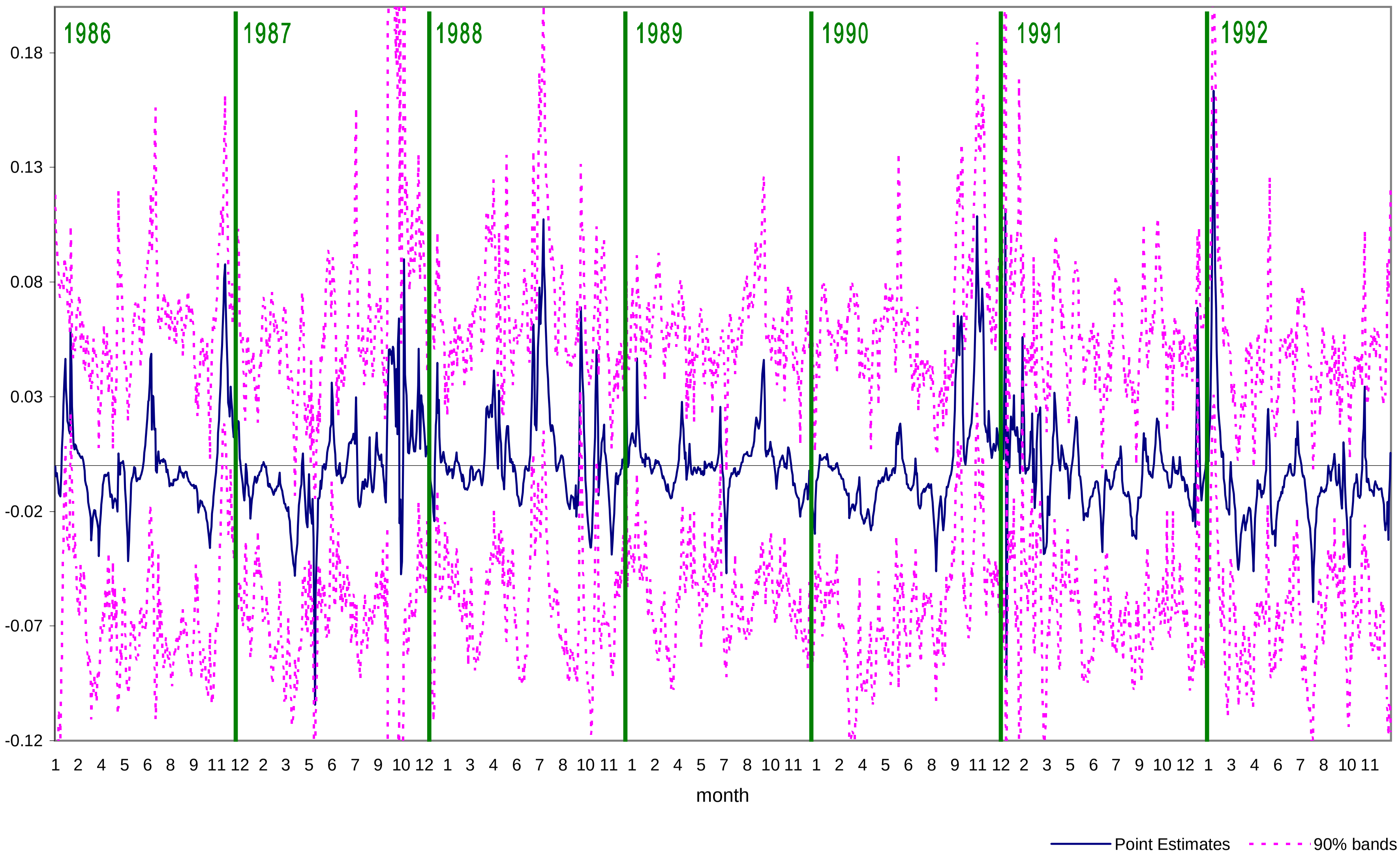

Figure 3B: Daily Reserve Deficiency, 1994-1998

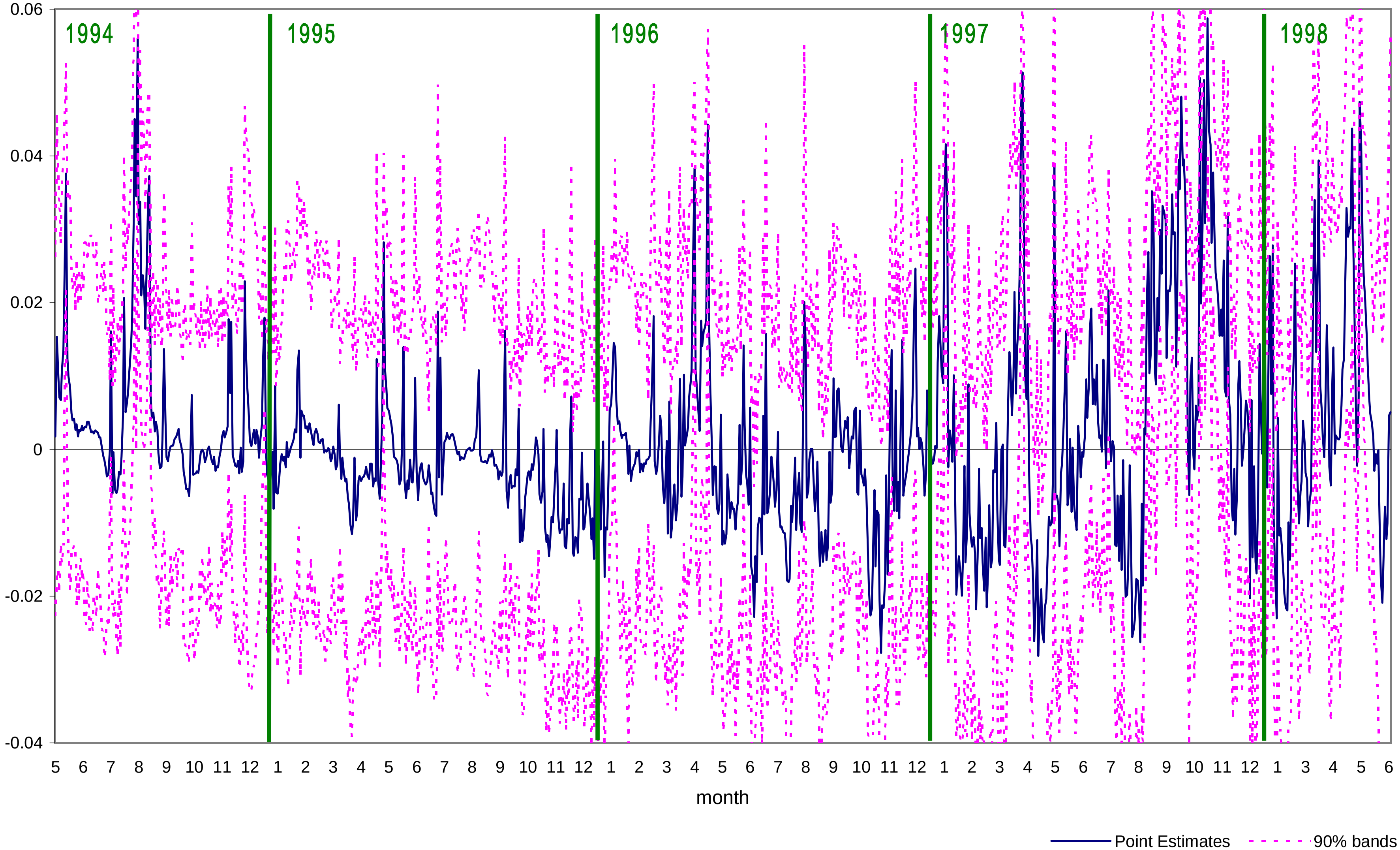

Figure 4A: Deviation of Fed Funds Rate from its Target, 1986-1992

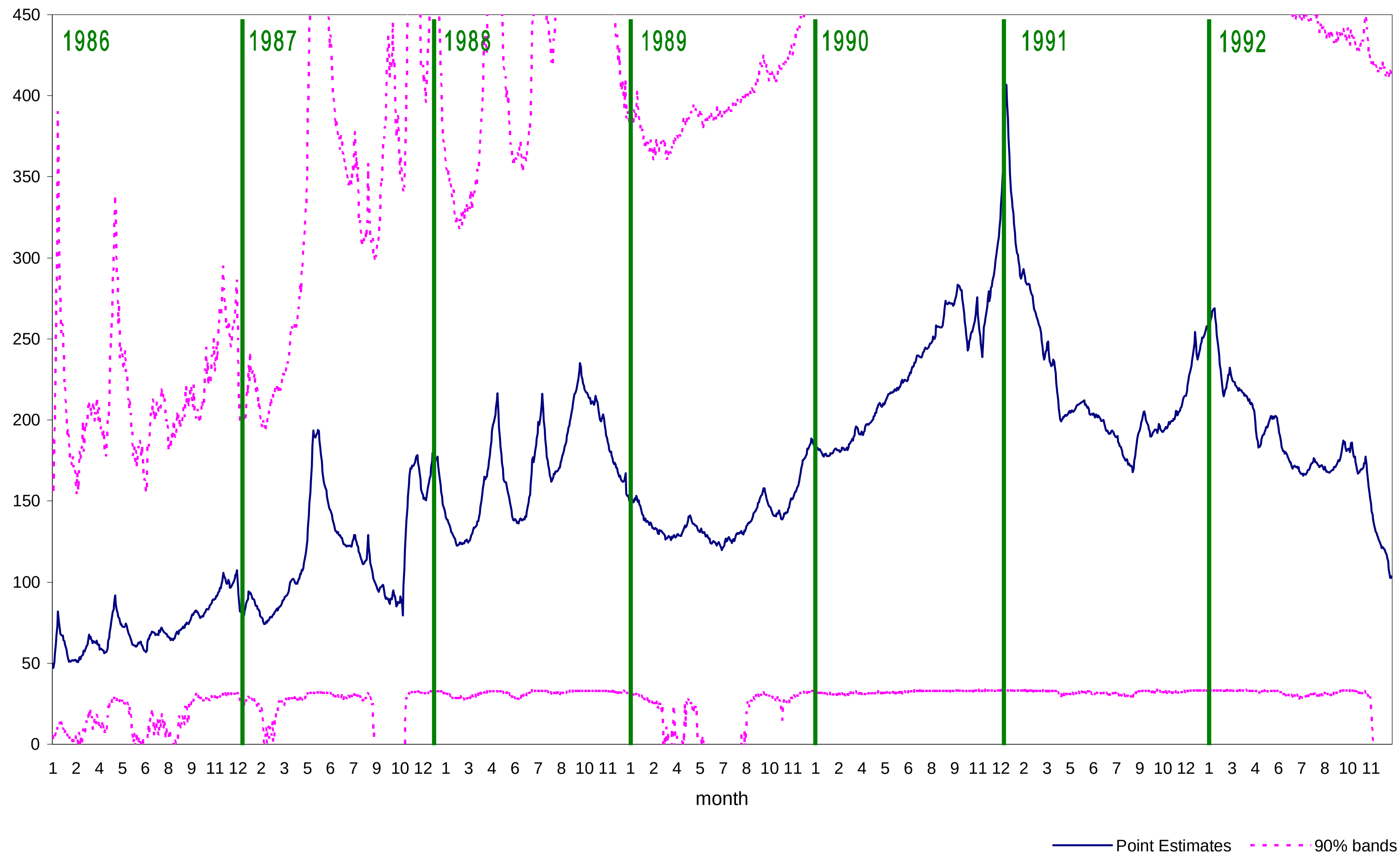

Figure 4B: Deviation of Fed Funds Rate from its Target, 1994-1998