

# Assessing the structural VAR approach to exchange rate pass-through

Ida Wolden Bache\*

Norges Bank (Central Bank of Norway)

P.O. Box 1179 Sentrum, N-0107 Oslo

ida-wolden.bache@norges-bank.no

First version: 15 August 2005

This version: 28 April 2006

## Abstract

A common approach to evaluate dynamic stochastic general equilibrium (DSGE) models is to compare the impulse responses functions from the DSGE model to impulse responses obtained from identified vector autoregressions (VARs). This paper uses Monte Carlo techniques to address the question: Are impulse responses of prices to a UIP shock a useful tool to evaluate DSGE models with incomplete exchange rate pass-through? The data generating process is a small open economy DSGE. The results suggest that (i) the estimates obtained from a VAR estimated in first differences exhibit a systematic downward bias, even when the VAR is specified with a large number of lags; (ii) by contrast, estimates derived from a low order vector equilibrium correction model are fairly accurate; but (iii) standard cointegration tests have low power to detect the cointegration relations implied by the DSGE model.

---

\*The views expressed in this paper are my own and do not necessarily reflect the views of Norges Bank. I would like to thank Hilde Bjørnland, Neil R. Ericsson, Hamid Faruquee, Egil Matsen, Bjørn Naug, Ragnar Nymoen, Alasdair Scott, Fredrik Wulfsberg and seminar participants at the University of Oslo, Sveriges Riksbank, and Norges Bank for helpful comments and suggestions. I have also benefitted from discussions with Gunnar Bårdsen, Øyvind Eitrheim, Eilev S. Jansen, and Tommy Sveen. Part of this research was undertaken while I was visiting the Bank of England.

# 1 Introduction

A common approach to evaluate dynamic stochastic general equilibrium (DSGE) models is to compare the impulse responses functions from the DSGE model to impulse responses obtained from identified vector autoregressions (VARs). The VAR responses, which rely only on minimal theoretical restrictions, are interpreted as ‘stylised facts’ that empirically relevant DSGE models should reproduce. Prominent examples are Rotemberg & Woodford (1997) and Christiano et al. (2005a) who estimate the parameters of their DSGE models by minimising the distance between the impulse responses to a monetary policy shock generated by an identified VAR and the responses to the monetary policy shock in the DSGE model. Choudhri et al. (2005) and Faruquee (2005), employ the same strategy to estimate NOEM of exchange rate pass-through, defining exchange rate pass-through as the impulse responses of a set of prices (import prices, export prices, producer prices, consumer prices) to a shock to the uncovered interest rate parity (UIP) condition.

Recently, several papers have examined the reliability of the structural VAR approach using Monte Carlo simulations. The basic idea in this literature is to generate artificial data from a DSGE model, construct impulse responses from a VAR estimated on the artificial data and ask whether the VAR recovers the DSGE model’s responses. Chari et al. (2005), Erceg et al. (2004) and Christiano et al. (2005b) assess the ability of a structural VAR to recover the impulse responses to a technology shock in an RBC model. Chari et al. (2005) conclude that a very large number of lags is needed for the VAR to well approximate their log-linearised RBC model. Erceg et al. (2004) find that, while the VAR responses have the same sign and shape as the true responses, quantitatively, the bias in the estimated responses could be considerable. Christiano et al. (2005b) reach a more optimistic conclusion. They find that the VAR does a good job in recovering the responses from the RBC model, particularly if the technology shock is identified using short-run restrictions. Kapetanios et al. (2005) estimate a five variable VAR on data generated from a small open economy model and derive impulse responses to productivity, monetary policy, foreign demand, fiscal and risk premium shocks. The results suggest that the ability of the VAR to reproduce the theoretical shock responses varies across shocks. In particular, a high lag order is required for the VAR to recover the responses to a risk premium shock and a domestic fiscal shock.

My paper extends this literature to assess the reliability of the structural VAR approach to estimating exchange rate pass-through. The motivating question is: Are impulse responses of prices to a UIP shock a useful tool to evaluate and estimate DSGE models with incomplete exchange rate pass-through? To address this question I generate a large number of artificial datasets from a small open economy DSGE model, estimate a VAR on the artificial data and compare the responses of prices to a UIP shock in the VAR and the DSGE model. The DSGE model that serves as the data generating process resembles the model considered by Choudhri et al. (2005), and incorporates many of the mechanisms for generating imperfect pass-through that have been proposed in the NOEM literature.

The specification of the DSGE model implies that the nominal exchange rate and nominal prices are non-stationary unit root process, but that relative prices and the real exchange rate are stationary. Given that exchange rate pass-through is usually defined in terms of levels of prices and the nominal exchange rate, a conjecture is that the magnitude of the bias in the estimated VAR responses will depend on whether the correct cointegration rank has been imposed during estimation. To test this conjecture I compare the performance of two different VAR specifications: A pure first differenced VAR and a VAR that includes the cointegration relations implied by the DSGE model. The former is by far the most

common specification in the structural VAR literature on exchange rate pass-through.<sup>1</sup> As a second exercise, I investigate whether an econometrician would be able to infer the true cointegration rank and identify the cointegration relations using Johansen (1988) maximum likelihood procedures. My findings can be summarised as follows: The estimates of exchange rate pass-through obtained from a VAR estimated in first differences are biased downwards. This is true even when the VAR is specified with a large number of lags. The bias is attributable to the fact that the finite order VAR is not a good approximation to the infinite order VAR implied by the DSGE model. By contrast, a low order vector equilibrium correction model (VEqCM) that includes the cointegration relations implied by the DSGE model is a good approximation to the data generating process. However, the results from the cointegration analysis raise doubts about whether in practice, an econometrician would be able to infer the cointegration properties implied by the DSGE model.

The paper is organised as follows. Section 2 lays out the DSGE model that serves as the data generating process in the Monte Carlo exercise. Section 3 discusses the mapping from the DSGE model to a VAR, and the results of the simulation experiments are presented in section 4. Section 5 concludes the paper.

## 2 The model economy

The model economy is a small open economy that produces two goods: a final consumption good and an intermediate good. The final good is non-tradable, while the intermediate good is sold both in the domestic and foreign markets. International goods markets are segmented and so intermediate goods firms have the option to price discriminate across markets. Both goods are produced using domestic labour and a basket of domestic and foreign intermediate goods as inputs. The model thus defines four different price indices: consumer prices, export prices, producer prices and import prices. The model derivation focuses on assumptions and mechanisms affecting the degree of exchange rate pass-through to these price indices.

### 2.1 Firms

The production structure is the same as considered by Choudhri et al. (2005). The home economy produces two goods: a non-tradable final consumption good and a tradable intermediate good. Firms in both sectors use domestic labour and a basket of domestic and imported intermediate goods as inputs. The assumption that imports do not enter directly into the consumption basket of households is consistent with the notion that all goods in the consumer price index contain a significant non-traded component. It follows that the direct effect of import prices on consumer prices will be muted, and this acts to limit the degree of exchange rate pass-through to consumer prices. The assumption that imported goods are used as inputs in the production of domestic goods implies a direct link between import prices and the production costs of domestic firms. The latter is potentially an important transmission channel for exchange rate changes in a small open economy.

#### 2.1.1 Final goods firms

**Technology and factor demand** There is a continuum of firms indexed by  $c \in [0, 1]$  that produces differentiated non-tradable final consumption goods. The market for final goods is characterised by monopolistic competition. The consumption good is produced

---

<sup>1</sup>See e.g., McCarthy (2000), Hahn (2003), Choudhri et al. (2005), and Faruquee (2005).

using the following Cobb-Douglas technology

$$C_t(c) = Q_t(c)^{\phi_c} H_t^c(c)^{1-\phi_c}$$

where  $C_t$  is final good output at time  $t$ ,  $\phi_c \in [0, 1]$  is the weight on intermediate goods, and  $H_t$  is a constant elasticity of substitution (CES) aggregate of differentiated labour inputs

$$H_t = \left[ \int_0^1 H_t(j)^{\frac{\theta_h-1}{\theta_h}} dj \right]^{\frac{\theta_h}{\theta_h-1}}, \quad (1)$$

where  $\theta_h > 1$  is the elasticity of substitution between labour types.  $Q_t$  is a composite intermediate good

$$Q_t = \left[ \alpha^{\frac{1}{\nu}} \left( Q_t^d \right)^{\frac{\nu-1}{\nu}} + (1-\alpha)^{\frac{1}{\nu}} \left( Q_t^m \right)^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}} \quad (2)$$

where  $\alpha \in [0, 1]$  is a parameter related to the degree of home bias in preferences, and  $\nu > 0$  denotes the elasticity of substitution between domestic and imported goods.  $Q_t^d$  and  $Q_t^m$  are quantity indices of differentiated domestic and foreign intermediate goods indexed by  $i \in [0, 1]$  and  $m \in [0, 1]$  respectively:<sup>2</sup>

$$Q_t^d = \left[ \int_0^1 Y_t^{dq}(i)^{\frac{\theta_t^y-1}{\theta_t^y}} di \right]^{\frac{\theta_t^y}{\theta_t^y-1}} \quad (3)$$

$$Q_t^m = \left[ \int_0^1 Y_t^{mq}(m)^{\frac{\theta_t^m-1}{\theta_t^m}} dm \right]^{\frac{\theta_t^m}{\theta_t^m-1}} \quad (4)$$

where  $\theta_t^y > 1$  and  $\theta_t^m > 1$  are the elasticities of substitution between varieties of domestic and intermediate goods in the domestic market. The elasticities of substitution are equal to the elasticities of demand for individual goods. Following e.g., Smets & Wouters (2003) and Adolfson et al. (2005), the demand elasticities are assumed to be time-varying.

**Price setting** The aggregate consumption index is defined as

$$C_t = \left[ \int_0^1 C_t(c)^{\frac{\theta_t^c-1}{\theta_t^c}} dc \right]^{\frac{\theta_t^c}{\theta_t^c-1}} \quad (5)$$

where  $\theta_t^c$  is the time-varying elasticity of substitution between individual goods. The corresponding price index is

$$P_t^c = \left[ \int_0^1 P_t^c(c)^{1-\theta_t^c} dc \right]^{\frac{1}{1-\theta_t^c}} \quad (6)$$

The demand for a single variety of the consumption good is

$$C_t(c) = \left( \frac{P_t^c(c)}{P_t^c} \right)^{-\theta_t^c} C_t \quad (7)$$

Nominal price stickiness is modelled using the quadratic adjustment cost framework of Rotemberg (1982).<sup>3</sup> Following Hunt & Rebucci (2005), I assume that there are costs associated both with changing the level of prices and with changing the inflation rate relative to past

<sup>2</sup>The corresponding price indices and demand functions are given in table 1.

<sup>3</sup>The list of NOEM papers which model price stickiness by assuming quadratic costs of price adjustment includes Bergin (2004), Corsetti et al. (2005), Laxton & Pesenti (2003), and Hunt & Rebucci (2005).

observed inflation. Specifically, adjustment costs are given by

$$\Upsilon_{t+l}^c(c) \equiv \frac{\phi_{c1}}{2} \left( \frac{P_{t+l}^c(c)}{\pi^c P_{t+l-1}^c(c)} - 1 \right)^2 + \frac{\phi_{c2}}{2} \left( \frac{P_{t+l}^c(c)/P_{t+l-1}^c(c)}{P_{t+l-1}^c/P_{t+l-2}^c} - 1 \right)^2 \quad (8)$$

where  $\pi^c$  is the steady-state gross inflation rate.

Since all firms in the economy are owned by households, future profits are valued according to the households' stochastic discount factor  $D_{t,t+l}$  (to be defined below). Firms set prices to maximise the expected discounted value of future profits subject to adjustment costs, that is they maximise

$$E_t \left[ \sum_{l=0}^{\infty} D_{t,t+l} (P_{t+l}^c(c) - \xi_{t+l}^c) \left( \frac{P_{t+l}^c(c)}{P_{t+l}^c} \right)^{-\theta_{t+l}^c} C_{t+l} (1 - \Upsilon_{t+l}^c(c)) \right] \quad (9)$$

subject to (8), and where  $\xi_t^c$  denotes nominal marginal costs. Note that the aggregate inflation dynamics implied by this model are similar to the inflation dynamics implied by the ?? model when firms index non-optimised prices to lagged inflation. If there were no costs associated with changing the rate of inflation (i.e.,  $\phi_{c2} = 0$ ), the linearised inflation equation would be purely forward-looking. If  $\phi_{c2} > 0$  lagged inflation enters the inflation equation. In the absence of costs of changing the price level (i.e.,  $\phi_{c1} = 0$ ) the inflation equation becomes a forward-looking equation in the first difference of inflation, similar to the Calvo model with full dynamic indexation considered by Christiano et al. (2005a) If prices were flexible (i.e.,  $\phi_{c1} = \phi_{c2} = 0$ ) firms would set the prices according to the familiar mark-up rule:

$$P_t^c = \frac{\theta_t^c}{\theta_t^c - 1} \xi_t^c \quad (10)$$

### 2.1.2 Intermediate goods firms

**Technology and factor demand** There is a continuum of intermediate goods firms indexed by  $i \in [0, 1]$  operating in a monopolistically competitive market. Intermediate goods are produced with the following technology

$$Y_t(i) = Z_t(i)^{\phi_y} H_t^y(i)^{1-\phi_y} \quad (11)$$

where  $Z_t(i)$  are units of the composite intermediate good used in the production of variety  $i$  of the domestic intermediate good

$$Z_t = \left[ \alpha^{\frac{1}{\nu}} \left( Z_t^d \right)^{\frac{\nu-1}{\nu}} + (1-\alpha)^{\frac{1}{\nu}} \left( Z_t^m \right)^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}} \quad (12)$$

where  $Z_t^d$  and  $Z_t^m$  are quantity indices of differentiated domestic and foreign intermediate goods, that is,

$$Z_t^d = \left[ \int_0^1 Y_t^{dz}(i)^{\frac{\theta_t^y-1}{\theta_t^y}} di \right]^{\frac{\theta_t^y}{\theta_t^y-1}} \quad (13)$$

$$Z_t^m = \left[ \int_0^1 Y_t^{mz}(m)^{\frac{\theta_t^m-1}{\theta_t^m}} dm \right]^{\frac{\theta_t^m}{\theta_t^m-1}} \quad (14)$$

**Price setting** International goods markets are assumed to be segmented. Intermediate goods firms thus have the option to set different prices in the domestic and foreign markets.

**Domestic market** The demand facing firm  $i$  in the domestic market is

$$Y_t^d(i) = \left( \frac{P_t^y(i)}{P_t^y} \right)^{-\theta_t^y} Y_t^d \quad (15)$$

where  $Y_t^d = Q_t^d + Z_t^d$  is the total demand for domestic intermediate goods from domestic final goods firms and intermediate goods firms. Firm  $i$ 's price setting problem in the domestic market is

$$\max_{P_t^y(i)} E_t \left[ \sum_{l=0}^{\infty} D_{t,t+l} (P_{t+l}^y(i) - \xi_{t+l}^y) \left( \frac{P_{t+l}^y(i)}{P_{t+l}^y} \right)^{-\theta_{t+l}^y} Y_{t+l}^d (1 - \Upsilon_{t+l}^y(i)) \right] \quad (16)$$

where  $\xi_{t+l}^y$  denote marginal costs, and the form of adjustment costs  $\Upsilon_{t+l}^y(i)$  is

$$\Upsilon_{t+l}^y(i) \equiv \frac{\phi_{y1}}{2} \left( \frac{P_{t+l}^y(i)}{\pi^y P_{t+l-1}^y(i)} - 1 \right)^2 + \frac{\phi_{y2}}{2} \left( \frac{P_{t+l}^y(i)/P_{t+l-1}^y(i)}{P_{t+l-1}^y/P_{t+l-2}^y} - 1 \right)^2 \quad (17)$$

where  $\pi^y$  is gross inflation rate in the steady-state.

**Foreign market** In the Obstfeld & Rogoff (1995) Redux model, international goods markets are integrated, and the law of one price holds continuously. Moreover, because prices are set in the currency of the producer (so-called producer currency pricing or PCP), exchange rate pass-through to import prices is immediate and complete. Betts & Devereux (1996) extended the Redux to allow for market segmentation and to allow a share of prices to be sticky in the currency of the buyer (so-called local currency pricing or LCP). Local currency pricing implies that import prices will respond only gradually in response to exchange rate changes, a feature consistent with the findings of a large empirical literature on exchange rate pass-through.<sup>4</sup> In this paper I follow Choudhri et al. (2005) and assume a proportion  $\varpi$  of domestic intermediate goods firms engages in PCP, and a proportion  $1 - \varpi$  engages in LCP. Both PCP and LCP firms have the option to price discriminate between foreign and domestic markets.<sup>5</sup>

Following Choudhri et al. (2005) the distribution of one unit of the domestic traded good to foreign firms requires the input of  $\delta_f$  units of foreign labour.<sup>6</sup> Let  $\bar{P}_t^{xp}(i)$  and  $\bar{P}_t^{xl}(i)$  be the (wholesale) prices set by a representative PCP firm and LCP firm respectively. The Leontief production technology and the zero profit condition in the perfectly competitive distribution sector implies that the (retail) prices paid by foreign firms for a type  $i$  domestic good satisfy

$$\frac{P_t^{xp}(i)}{e_t} = \frac{\bar{P}_t^{xp}(i)}{e_t} + \delta_f W_t^f \quad (18)$$

$$P_t^{xl}(i) = \bar{P}_t^{xl}(i) + \delta_f W_t^f \quad (19)$$

<sup>4</sup>See Campa & Goldberg (2005) for a recent study.

<sup>5</sup>In this paper  $\varpi$  is treated as an exogenous parameter. Several recent papers have examined the optimal choice of invoicing currency in the context of NOEM models (e.g., Devereux et al. (2004), Bacchetta & Wincoop (2005), and Goldberg & Tille (2005)). The choice is found to depend on several factors, including the exporting firm's market share in the foreign market, the degree of substitutability between foreign and domestic goods and relative monetary stability.

<sup>6</sup>Corsetti & Dedola (2005) were the first to introduce distribution costs in the NOEM framework.

where  $e_t$  is the nominal exchange rate and  $W_t^f$  is the foreign wage level. The existence of distribution sector thus implies that there will be a wedge between the wholesale and the retail price of imports in the foreign economy.

Letting  $\theta_t^x > 1$  denote the elasticity of substitution between domestic intermediate goods in the foreign economy, the aggregate export price index (in domestic currency) is

$$P_t^x = \left[ \varpi (P_t^{xp})^{1-\theta_t^x} + (1-\varpi)(e_t P_t^{xl})^{1-\theta_t^x} \right]^{\frac{1}{1-\theta_t^x}} \quad (20)$$

where  $P_t^{xp}$  and  $P_t^{xl}$  are the export price indices obtained by aggregating over PCP firms and LCP firms respectively. A representative PCP firm sets  $\bar{P}_t^{xp}(i)$  to maximise

$$E_t \left[ \sum_{l=0}^{\infty} D_{t,t+l} (\bar{P}_{t+l}^{xp}(i) - \xi_{t+l}^y) Y_{t+l}^{xp}(i) (1 - \Upsilon_{t+l}^{xp}(i)) \right] \quad (21)$$

subject to demand<sup>7</sup>

$$Y_{t+l}^{xp}(i) = \left( \frac{\bar{P}_{t+l}^{xp}(i)/e_{t+l} + \delta_f W_{t+l}^f}{P_{t+l}^x/e_{t+l}} \right)^{-\theta_{t+l}^x} Y_{t+l}^x, \quad (22)$$

and adjustment costs

$$\Upsilon_{t+l}^{xp}(i) \equiv \frac{\phi_{x1}}{2} \left( \frac{\bar{P}_{t+l}^{xp}(i)}{\bar{\pi}^{xp} \bar{P}_{t+l-1}^{xp}(i)} - 1 \right)^2 + \frac{\phi_{x2}}{2} \left( \frac{\bar{P}_{t+l}^{xp}(i)/\bar{P}_{t+l-1}^{xp}(i)}{\bar{P}_{t+l-1}^{xp}/\bar{P}_{t+l-2}^{xp}} - 1 \right)^2, \quad (23)$$

where  $\bar{\pi}^{xp}$  is the steady state inflation rate. The wedge between prices at the wholesale and retail levels implies that the price elasticity of demand as perceived by the exporter, will be a function of the exchange rate. To see this note that in the absence of price stickiness (i.e., if  $\phi_{x1} = \phi_{x2} = 0$ ) the optimal export price is

$$\bar{P}_t^{xp} = \frac{\theta_t^x}{\theta_t^x - 1} \xi_t^y + \frac{\delta_f}{\theta_t^x - 1} e_t W_t^f \quad (24)$$

In the case of no distribution costs ( $\delta_f = 0$ ), the export price in domestic currency is independent of the exchange rate, and the price setting rule collapses to the standard mark-up rule. Moreover, if the elasticities of demand are the same across countries (i.e.,  $\theta_t^x = \theta_t^y$ ) the firm sets identical prices to the home and foreign markets. The existence of distribution costs creates a motive for price discrimination across markets. With distribution costs the optimal mark-up varies positively with the level of the exchange rate. This can be seen more clearly by rewriting (24) as

$$\bar{P}_t^{xp} = \frac{\theta_t^x}{\theta_t^x - 1} \xi_t^y \left( 1 + \frac{\delta_f e_t W_t^f}{\theta_t^x \xi_t^y} \right) \quad (25)$$

In the face of an exchange rate depreciation, the exporter will find it optimal to absorb part of the exchange rate movement in her mark-up. From the point of view of the importing country, exchange rate pass-through to import prices at the docks is incomplete even in the absence of nominal rigidities.

<sup>7</sup>See table 1 for definitions of the sectoral price and quantity indices.

A representative LCP firm sets  $\bar{P}_t^{xl}(i)$  to maximise

$$\max_{\bar{P}_t^{xl}(i)} E_t \left[ \sum_{l=0}^{\infty} D_{t,t+l} \left( e_{t+l} \bar{P}_{t+l}^{xl}(i) - \xi_{t+l}^y \right) Y_{t+l}^{xl}(i) \left( 1 - \Upsilon_{t+l}^{xl}(i) \right) \right] \quad (26)$$

subject to demand

$$Y_{t+l}^{xl}(i) = \left( \frac{\bar{P}_{t+l}^{xl}(i) + \delta_f W_{t+l}^f}{P_{t+l}^x / e_{t+l}} \right)^{-\theta_{t+l}^x} Y_{t+l}^x \quad (27)$$

and adjustment costs

$$\Upsilon_{t+l}^{xl}(i) \equiv \frac{\phi_{x1}}{2} \left( \frac{\bar{P}_{t+l}^{xl}(i)}{\bar{\pi}^{xl} \bar{P}_{t+l-1}^{xl}(i)} - 1 \right)^2 + \frac{\phi_{x2}}{2} \left( \frac{\bar{P}_{t+l}^{xl}(i) / \bar{P}_{t+l-1}^{xl}(i)}{\bar{P}_{t+l-1}^{xl} / \bar{P}_{t+l-2}^{xl}} - 1 \right)^2 \quad (28)$$

where  $\bar{\pi}^{xl}$  is the steady state inflation rate. In the absence of adjustment costs (i.e., if  $\phi_{x1} = \phi_{x2} = 0$ ) the optimal price is

$$\bar{P}_t^{xl} = \frac{\theta_t^x}{\theta_t^x - 1} \frac{\xi_t^y}{e_t} + \frac{\delta_f}{\theta_t^x - 1} W_t^f \quad (29)$$

Thus when prices are flexible, LCP and PCP firms set the same price. The choice of price-setting currency only matters in a situation where nominal prices are rigid.

Finally, aggregate export demand is assumed to be given by

$$Y_t^x = \alpha_f \left( \frac{P_t^x / e_t}{P_t^f} \right)^{-\nu_f} Y_t^f \quad (30)$$

where  $\alpha_f$  is (approximately) the share of home goods and  $\nu_f$  the elasticity of substitution between home and foreign goods in the composite index of intermediate goods in the foreign economy,  $P_t^f$  is the foreign price level, and  $Y_t^f$  denotes aggregate demand for domestic intermediate goods in the foreign economy.

### 2.1.3 Foreign firms

Foreign intermediate goods firms are treated symmetrically. The distribution of one unit of the imported goods to domestic firms requires the input of  $\delta$  units of foreign labour. A subset proportion  $\varpi_f$  of firms engages in PCP, and a subset  $1 - \varpi_f$  engages in LCP. The zero profit condition in the distribution sector implies that the prices paid by domestic firms for a type  $m$  imported good will be

$$e_t P_t^{mp}(m) = e_t \bar{P}_t^{mp}(m) + \delta W_t \quad (31)$$

$$P_t^{ml}(m) = \bar{P}_t^{ml}(m) + \delta W_t \quad (32)$$

where  $W_t$  is the domestic wage rate. The aggregate import price index (in domestic currency) is

$$P_t^m = \left[ \varpi_f (e_t P_t^{mp})^{1-\theta_t^m} + (1 - \varpi_f) (P_t^{ml})^{1-\theta_t^m} \right]^{\frac{1}{1-\theta_t^m}} \quad (33)$$

where  $\bar{P}_t^{mp}$  and  $\bar{P}_t^{ml}$  are the price indices obtained by aggregating over PCP firms and LCP firms respectively<sup>8</sup>

<sup>8</sup>See table 1 for definitions of the sectoral price and quantity indices.



Let  $D_{t,t+l}^f$  denote the stochastic discount factor of foreign households. A representative foreign LCP firm sets  $\bar{P}_t^{ml}(m)$  to maximise

$$E_t \left[ \sum_{l=0}^{\infty} D_{t,t+l}^f \left( \frac{\bar{P}_{t+l}^{ml}(m)}{e_{t+l}} - \xi_{t+l}^f \right) Y_{t+l}^{ml}(m) \left( 1 - \Upsilon_{t+l}^{ml}(m) \right) \right] \quad (34)$$

subject to demand

$$Y_{t+l}^{ml}(m) = \left( \frac{\bar{P}_{t+l}^{ml}(m) + \delta W_{t+l}}{P_{t+l}^m} \right)^{-\theta_{t+l}^m} Y_{t+l}^m \quad (35)$$

where  $Y_t^m = Q_t^m + Z_t^m$  and adjustment costs are

$$\Upsilon_{t+l}^{ml}(m) \equiv \frac{\phi_{m1}}{2} \left( \frac{\bar{P}_{t+l}^{ml}(m)}{\bar{\pi}^{ml} \bar{P}_{t+l-1}^{ml}(m)} - 1 \right)^2 + \frac{\phi_{m2}}{2} \left( \frac{\bar{P}_{t+l}^{ml}(m)/\bar{P}_{t+l-1}^{ml}(m)}{\bar{P}_{t+l-1}^{ml}/\bar{P}_{t+l-2}^{ml}} - 1 \right)^2 \quad (36)$$

If  $\phi_{m1} = \phi_{m2} = 0$ , the first-order condition reduces to

$$\bar{P}_t^{ml} = \frac{\theta_t^m}{\theta_t^m - 1} e_t \xi_t^f + \frac{\delta}{\theta_t^m - 1} W_t \quad (37)$$

The foreign firm's optimal mark-up is a function of the exchange rate. Conditional on wages and foreign prices, exchange rate pass-through to domestic currency import prices at the wholesale level is incomplete, even if prices are perfectly flexible.

A representative foreign PCP firm sets  $\bar{P}_t^{mp}(m)$  to maximise

$$E_t \left[ \sum_{l=0}^{\infty} D_{t,t+l}^f \left( \bar{P}_{t+l}^{mp}(m) - \xi_{t+l}^f \right) Y_{t+l}^{mp}(m) \left( 1 - \Upsilon_{t+l}^{mp}(m) \right) \right] \quad (38)$$

subject to demand

$$Y_{t+l}^{mp}(m) = \left( \frac{e_{t+l} \bar{P}_{t+l}^{mp}(m) + \delta W_{t+l}}{P_{t+l}^m} \right)^{-\theta_{t+l}^m} Y_{t+l}^m \quad (39)$$

and adjustment costs

$$\Upsilon_{t+l}^{mp}(m) \equiv \frac{\phi_{m1}}{2} \left( \frac{\bar{P}_{t+l}^{mp}(m)}{\bar{\pi}^{mp} \bar{P}_{t+l-1}^{mp}(m)} - 1 \right)^2 + \frac{\phi_{m2}}{2} \left( \frac{\bar{P}_{t+l}^{mp}(m)/\bar{P}_{t+l-1}^{mp}(m)}{\bar{P}_{t+l-1}^{mp}/\bar{P}_{t+l-2}^{mp}} - 1 \right)^2 \quad (40)$$

where  $\bar{\pi}^{mp}$  is the steady-state inflation rate.

## 2.2 Households

The economy is inhabited by a continuum of symmetric, infinitely lived households indexed from  $j \in [0, 1]$  that derive utility from leisure and consumption of the final good. Households get income from selling labour services, from holding one-period domestic and foreign bonds, and they receive the real profits from domestic firms. The adjustment costs incurred by domestic firms are also rebated to households. Each household is a monopoly supplier of a differentiated labour service and sets the wage rate subject to labour demand

$$H_t(j) = \left( \frac{W_t(j)}{W_t} \right)^{-\theta_h} H_t, \quad (41)$$

and quadratic costs of wage adjustment. Following Hunt & Rebucci (2005) the adjustment costs are measured in terms of the total wage bill. The costs of adjusting wages are

$$\Upsilon_t^w(j) \equiv \frac{\phi_{w1}}{2} \left( \frac{W_t(j)}{\pi^w W_{t-1}(j)} - 1 \right)^2 + \frac{\phi_{w2}}{2} \left( \frac{W_t(j)/W_{t-1}(j)}{W_{t-1}/W_{t-2}} - 1 \right)^2 \quad (42)$$

where  $\pi^w$  is the steady-state (gross) growth rate of nominal wages.

The return on the foreign bond is given by  $\kappa_t R_t^f$ , where  $R_t^f$  is the gross nominal interest rate on foreign bonds, and  $\kappa_t$  is a premium on foreign bond holdings. The premium is assumed to be a function of the economy's real net foreign assets position

$$\kappa_t = \exp \left( -\psi \frac{e_t B_t^f}{P_t^c} + u_t \right) \quad (43)$$

where  $B_t^f$  is the aggregate holding of nominal foreign bonds in the economy, and  $u_t$  is a time-varying 'risk premium' shock.<sup>9</sup> The risk premium shock is assumed to follow an AR(1) process

$$\ln u_t = \rho_u \ln u_{t-1} + \varepsilon_{u,t} \quad (44)$$

where  $0 \leq \rho_u < 1$ , and  $\varepsilon_{u,t}$  is a white noise process. The specification of the risk premium implies that if the domestic economy is a net borrower ( $B_t^f < 0$ ), it has to pay a premium on the foreign interest rate. This assumption ensures that net foreign assets are stationary.<sup>10</sup>

Household  $j$ 's period  $t+l$  budget constraint is

$$\begin{aligned} & P_{t+l}^c C_{t+l}(j) + \frac{B_{t+l}(j)}{R_{t+l}} + \frac{e_{t+l} B_{t+l}^f(j)}{\kappa_{t+l} R_{t+l}^f} \\ &= (1 - \Upsilon_{t+l}^w(j)) W_{t+l}(j) H_{t+l}(j) + B_{t+l-1}(j) + e_{t+l} B_{t+l-1}^f(j) + \Pi_{t+l} \end{aligned} \quad (45)$$

where  $R_{t+l}$  is the (gross) nominal interest rate on domestic bonds,  $B_{t+l}(j)$  and  $B_{t+l}^f(j)$  are household  $j$ 's holdings of nominal domestic and foreign bonds, and the variable  $\Pi_{t+l}$  includes all profits accruing to domestic households and the revenue from nominal adjustment that is rebated to households.

The household chooses a sequence  $\{C_{t+l}(j), B_{t+l}(j), B_{t+l}^f(j), W_{t+l}(j)\}_{l=0}^{\infty}$  to maximise

$$E_t \left\{ \sum_{l=0}^{\infty} \beta^l \left( \ln \left( \frac{C_{t+l}(j) - \zeta C_{t+l-1}(j)}{1 - \zeta} \right) - \eta \frac{h_{t+l}^{1+\chi}}{1 + \chi} \right) \right\} \quad (46)$$

subject to the budget constraint (45). The parameter  $\zeta \in [0, 1)$  reflects the assumption of habit formation in consumption, and  $\chi \in (0, \infty)$  is the inverse of the Frisch elasticity of labour supply (i.e., the elasticity of labour supply with respect to real wages for a constant marginal utility of wealth). The parameter  $\eta > 0$  is a scale parameter and  $\beta \in (0, 1]$  is the

<sup>9</sup>As discussed by Bergin (2004) the mean-zero disturbance term  $u_t$  can be interpreted as a proxy for a time-varying risk premium omitted by linearisation, or as capturing the stochastic bias in exchange rate expectations in a noise trader model.

<sup>10</sup>See Schmitt-Grohe & Uribe (2003) for a discussion of alternative ways to ensure stationary net foreign assets in a small open economy. In the standard small open economy model with incomplete international asset markets, equilibrium dynamics have a random walk component. That is, transitory shocks have permanent effects on wealth and consumption.

subjective discount factor. The stochastic discount factor  $D_{t,t+l}$  is defined as

$$D_{t,t+l} = \beta^l \frac{C_t - \zeta C_{t-1}}{C_{t+l} - \zeta C_{t+l-1}} \frac{P_t^c}{P_{t+l}^c} \quad (47)$$

### 2.3 Monetary authorities

The central bank sets short-term interest rates according to the following simple feedback rule

$$R_t = \rho_R R_{t-1} + (1 - \rho_R) (R + \rho_\pi (\pi_t^c - \pi^c)) \quad (48)$$

where  $R$  is the steady-state level of the nominal interest rate, and the parameter  $0 < \rho_R < 1$  measures the degree of interest rate smoothing.

### 2.4 Market clearing

The market clearing conditions for the domestic labour market and the intermediate goods market are

$$H_t = H_t^y + H_t^c + H_t^m \quad (49)$$

$$Y_t = Y_t^d + Y_t^x \quad (50)$$

where  $H_t^m = \delta Y_t^m$ . Only foreign bonds are assumed to be traded internationally, and hence the domestic bond is in zero net supply at the domestic level (i.e.  $B_t = 0$ ). Net foreign assets evolve according to

$$\frac{e_t B_t^f}{\kappa_t R_t^f} = e_t B_{t-1}^f + \bar{P}_t^x Y_t^x - \bar{P}_t^m Y_t^m \quad (51)$$

### 2.5 Mark-up shocks

The model derived above only has one shock; the risk premium or UIP shock  $u_t$ . If the purpose is to estimate the DSGE model by matching impulse responses to a UIP shock, there is no need to introduce additional shocks. In fact, one of the advantages of the impulse response matching approach is that allows the researcher to leave most of the exogenous shocks unspecified (see Lubik & Schorfede, 2005). However, if the dimension of the VAR is greater than the number of shocks, a VAR fitted to data generated from the DSGE model will have a singular variance-covariance matrix. This is the stochastic singularity problem discussed by e.g. Ingram et al. (1994). One strategy for dealing with this problem is to add shocks until the number of shocks is at least as great as the number of variables in the VAR. This is the approach taken in this paper. Specifically, I introduce four mark-up shocks: the elasticities of substitution between varieties of goods are characterised by the following processes

$$\ln \theta_t^c = (1 - \rho_c) \ln \theta^c + \rho_c \ln \theta_{t-1}^c + \varepsilon_{c,t} \quad (52)$$

$$\ln \theta_t^y = (1 - \rho_y) \ln \theta^y + \rho_y \ln \theta_{t-1}^y + \varepsilon_{y,t} \quad (53)$$

$$\ln \theta_t^x = (1 - \rho_x) \ln \theta^x + \rho_x \ln \theta_{t-1}^x + \varepsilon_{x,t} \quad (54)$$

$$\ln \theta_t^m = (1 - \rho_m) \ln \theta^m + \rho_m \ln \theta_{t-1}^m + \varepsilon_{m,t} \quad (55)$$

where  $0 \leq \rho_i < 1$  and the  $\varepsilon_{i,t}$  are independent white noise processes,  $i = \{c, y, x, m\}$ . The motivation for adding this particular set of shocks is that the mark-up shocks have a direct effect on the price setting equations in the structural model and hence, on the variables included in the VAR. This turned out to be important to avoid a (near) singular

variance-covariance matrix. However, I do not attach a strong structural interpretation to the mark-up shocks. An alternative would be to add serially correlated errors to the observation equations in the state space representation. Such ‘measurement errors’ could be interpreted as capturing the effects of structural shocks that are omitted from the model or other forms of misspecification of the DSGE model.

## 2.6 Calibration

The calibration is guided by the following principles: First, the parameters should be within the range suggested by the literature.<sup>11</sup> Second, the model should loosely match the standard deviations and first-order autocorrelations of UK prices and exchange rates over the period 1980–2003.

Table 2 lists the values of the parameters in the baseline calibration of the model. The subjective discount factor is set to  $1.03^{-0.25}$  to yield a steady-state annualised real interest rate of 3%. The habit persistence parameter ( $\zeta$ ) is set to 0.85, which is close to the value chosen by Kapetanios et al. (2005) for the UK. There appears to be little consensus in the literature about the appropriate value for the inverse of the Frisch elasticity of labour demand ( $\chi$ ). Choudhri et al. (2005) choose an initial value of 0.5 for this parameter, but later allow it to vary between zero and infinity. In the baseline calibration in this paper, the inverse Frisch elasticity is set to 3, which is the same value used in Hunt & Rebucci (2005) in a version of the IMF’s Global Economy Model. The weight on leisure in the utility function ( $\eta$ ) is chosen to yield a steady-state level of labour supply equal to unity ( $H = 1$ ).

Based on the data for revenue shares of intermediate goods reported in Choudhri et al. (2005), the Cobb-Douglas shares of intermediate goods in the production functions for final goods and intermediate goods ( $\phi_c, \phi_y$ ) are set to 0.42 and 0.77 respectively. The share of domestic intermediate goods in the aggregate intermediate good ( $\alpha$ ) is set to 0.85, and the elasticity of substitution between domestic and foreign intermediate goods ( $\nu$ ) is 1.5. The range considered by the literature for the latter is quite large. Groen & Matsumoto (2004) use the value 1.5 in their calibrated model of the UK economy. The distribution cost parameters ( $\delta, \delta_f$ ) are set to 0.4, slightly higher than the 0.3 used by Hunt & Rebucci (2005).

The steady-state values of the elasticities of substitution between varieties of goods sold in domestic markets are set to 6. This implies a steady-state mark-up of 20% for final goods and domestic intermediate goods. Again these numbers are comparable to what has been used in other studies. Benigno & Thoenissen (2003) assume that the substitution elasticity between traded goods is 6.5, and Kapetanios et al. (2005) set the elasticity of substitution between varieties of domestic goods sold in domestic markets to 5. The elasticity of substitution between types of labour services is also set to 6, in line with the values in Hunt & Rebucci (2005) and Benigno & Thoenissen (2003). Finally, the elasticity of substitution between varieties of foreign goods sold in foreign markets is set to 15. This is based on the argument in Kapetanios et al. (2005) that domestic firms face more competitive demand conditions in foreign markets.

The annual domestic inflation target is 2%. The parameters in the monetary policy rule are taken from Kapetanios et al. (2005). The weight on interest rate smoothing in the monetary policy rule ( $\rho_r$ ) is 0.65 and the weight on inflation ( $\rho_\pi$ ) is 1.8.

In the baseline calibration the parameters determining the costs of changing the level of prices or wages ( $\phi_{c1}, \phi_{y1}, \phi_{m1}, \phi_{x1}, \phi_{w1}$ ) are set to zero. This implies that the weight on lagged inflation in the log-linearised equations for price and wage inflation is  $1/(1+\beta) \approx 0.5$  which is the maximum degree of structural inflation persistence achievable in this set-up.

<sup>11</sup>Reference to literature. DSGE models calibrated on UK data. GEM.

The adjustment costs parameters associated with changing the rates of change in prices and wages ( $\phi_{c2}, \phi_{y2}, \phi_{m2}, \phi_{x2}, \phi_{w2}$ ) are set to 400.

The share of PCP firms in the foreign economy ( $\omega_f$ ) is set to 0.4 while the share of PCP firms in exports ( $\omega$ ) is 0.6. Data on invoicing currency in UK trade from the years 1999 to 2002 show that the share of UK imports and exports that are invoiced in sterling is around 40% and 50% respectively.<sup>12</sup> To get short-run pass-through to import prices more in line with the empirical estimates I had to use a somewhat higher value for the share of PCP firms in the foreign economy than what is suggested by the data on invoicing currency. Admittedly, this is not entirely satisfying.

The steady-state levels of foreign output  $y_f$  and real wages  $w_f$  are normalised to unity. The implicit inflation target in the foreign economy ( $\pi_f$ ) is identical to the domestic inflation target. This implies that the rate of exchange rate depreciation is zero in the steady-state. Moreover, assuming that domestic and foreign households have the same subjective discount rates, the steady-state interest rates will be the same. This is consistent with a zero risk premium ( $\kappa = 1$ ) and zero net foreign assets ( $B_f = 0$ ) in the steady-state. The elasticity of substitution between foreign and domestic goods in the foreign economy ( $\nu_f$ ) is set to 1.5, the same as in the domestic economy.

The sensitivity of the risk premium to net foreign assets is set to 0.02. During the calibration process I found that setting this parameter too low caused the model to become non-invertible (see section 3). The parameters in the processes for the risk premium and the demand elasticities were chosen to make the standard deviation and autocorrelation of the inflation rates and exchange rate depreciation roughly match those in the data. Table 3 reports the standard deviations and the first-order autocorrelations from the model and UK data 1980q1–2003q4.

## 2.7 Model solution and properties

The model is solved using Dynare, which is a collection of Matlab routines for solving non-linear rational expectations models (see Juillard, 2005). As a first step Dynare computes a first order approximation (in logs) to the equilibrium conditions around a deterministic steady-state. The log-linearised equations are listed in appendix A

## 2.8 Is the model empirically relevant?

As a check on the calibration I examined whether the DSGE model is empirically relevant in the sense that the estimation of a VAR on artificial data generated from the DSGE model yields comparable results to those obtained when estimating a VAR on actual UK data using the same sample size, the same set of variables and the same identification scheme.

The estimated VAR includes the following variables: UK import prices of manufactures ( $P_t^m$ ), export prices of manufactures ( $P_t^x$ ), producer prices of manufactures ( $P_t^y$ ), consumer prices ( $P_t^c$ ), and a nominal effective exchange rate ( $e_t$ ).<sup>13</sup> An increase in the exchange rate  $e_t$  corresponds to a depreciation of sterling. The data are quarterly, covering the period 1980Q1–2003Q4, and all the price series are seasonally adjusted and measured in domestic currency. Variable definitions and sources are provided in appendix B.

In line with common practice in the literature the variables are differenced prior to estimation. The exchange rate shock is identified by placing the exchange rate first in a recursive ordering. Under this identification scheme exchange rate shocks have a contemporaneous effect on the price indices, but shocks to the price equations affect the exchange

<sup>12</sup>These numbers can be found on <http://customs.hmrc.gov.uk/>

<sup>13</sup>This is the same set of variables as considered by Faruquee (2005), with the exception that he also includes wages in the VAR. I have confirmed that the pass-through estimates reported in this section are robust to the inclusion of wages in the model.

rate with at least a one-period lag. This assumption could be justified by the existence of time lags in the publication of official statistics such as producer price and consumer price indices (see Choudhri et al., 2005).<sup>14</sup> Note that, if interest is only in the exchange rate shock, the ordering of the variables placed before or after the exchange rate is irrelevant.

Figure 1 plots the accumulated impulse responses of import prices, export prices, producer prices and consumer prices to a one standard deviation shock to the exchange rate. The responses are normalised on accumulated responses of the exchange rate. Exchange rate pass-through to import prices is 36% within the first quarter, increasing to 54% within one year and stabilising at around 67% after two years. The immediate response of export prices is somewhat lower; pass-through is 19% after one quarter, 47% after one year and increasing to 59% in the long run. The response of producer prices is smaller and more gradual; pass-through is 15% within one year and increases to 27% after five years. Long-run pass-through to consumer prices about 7% after three years. These estimates appear to be broadly in line with the estimates reported for the UK in other structural VAR studies such as McCarthy (2000), .

As a next step, I conducted the following simulation experiment: Using the log-linearised solution to the DSGE model as the data generating process, I simulated 5000 synthetic datasets of length  $T = 100$  for  $y'_t = \{\Delta \ln e_t, \Delta \ln \bar{P}_t^m, \Delta \ln \bar{P}_t^x, \Delta \ln P_t^y, \Delta \ln P_t^c\}$ . For each synthetic dataset I estimated a VAR(4) and computed the impulse responses to an exchange rate shock using the same recursive identification as above.<sup>15</sup> Figure 2 plots the pointwise mean of the normalised responses to an exchange rate shock. Exchange rate pass-through to import prices is 45% in the first quarter and stabilises at 75% after about 12 quarters. Pass-through to export prices is lower; 32% in the first quarter and 40% in the long-run. Short-run pass-through to producer and consumer prices is close to zero. After twenty periods pass-through is 22% and 9% respectively. All these estimates are all broadly similar to the estimates obtained using actual UK data.

### 3 Mapping from DSGE model to VAR

Adopting the notation in Fernández-Villaverde et al. (2005) the log-linear transition equations computed by Dynare can be expressed in state space form as

$$\begin{aligned} x_{t+1} &= Ax_t + Bw_t \\ y_t &= Cx_t + Dw_t \end{aligned} \tag{56}$$

where  $w_t$  is an  $m \times 1$  vector of structural shocks satisfying  $E[w_t] = 0, E[w_t w_t'] = I$  and  $E[w_t w_{t-j}] = 0$  for  $j \neq 0$ ,  $x_t$  is an  $n \times 1$  vector of state variables, and  $y_t$  is a  $k \times 1$  vector of variables observed by the econometrician. The eigenvalues of  $A$  are all strictly less than one in modulus, hence the model is stationary. In what follows I will focus on the case where  $D$  is square (i.e.,  $m = k$ ) and  $D^{-1}$  exists. The impulse responses from the structural

<sup>14</sup>The assumption that the exchange rate does not react within period to shocks to the price equations is not uncontroversial, however. As emphasised by Sarno & Thornton (2004), if foreign exchange markets are efficient, the exchange rate will by definition jump in response to news about fundamentals. The only way to achieve this using a recursive identification scheme is to order the exchange rate last.

<sup>15</sup>This identification scheme is not consistent with the DSGE model. However, the point of this exercise is to show that if I use a similar sample size and the same identification scheme I get results that are not too dissimilar from what was found using actual UK data. In the Monte Carlo experiments in section 4 I use an identification scheme which is compatible with the DSGE model.

shocks  $w_t$  to  $y_t$  are given by

$$y_t = d(L)w_t = \sum_{j=0}^{\infty} d_j L^j w_t \quad (57)$$

where  $d_0 = D$  and  $d_j = CA^{j-1}B$  for  $j \geq 1$ .

### 3.1 Invertibility

An infinite order VAR is defined by

$$y_t = \sum_{j=1}^{\infty} A_j y_{t-j} + G\nu_t \quad (58)$$

here  $E[\nu_t] = 0$ ,  $E[\nu_t \nu_t'] = I$ ,  $E[\nu_t \nu_{t-j}'] = 0$  for  $j \neq 0$ . The covariance matrix of the VAR innovations  $u_t = G\nu_t$  is  $E[G\nu_t \nu_t' G'] = GG' = \Sigma_u$ . Finally, the moving average representation of (58) is

$$y_t = c(L)v_t \quad (59)$$

where  $c(L) = \sum_{j=0}^{\infty} c_j L^j = (I - \sum_{j=1}^{\infty} A_j L^j)^{-1}G$ .

A potential source of discrepancies between the VAR impulse responses and the responses from the log-linearised solution to the DSGE model is that the moving average representation (57) is non-invertible. By construction, the moving average representation associated with the infinite order VAR (58) is fundamental in the sense that the innovations  $\nu_t$  can be expressed as a linear combination of current and past observations of  $y_t$ . However, there exists an infinite number of other non-fundamental moving average representations that are observationally equivalent to (59), but that cannot be recovered from the infinite order VAR. These moving average representations are non-invertible, meaning that they cannot be inverted to yield an infinite order VAR. In general, we cannot rule out the possibility that a DSGE model has a non-invertible moving average representation for a given set of observables.<sup>16</sup> That is, we cannot rule out the possibility that some of the roots of the characteristic equation associated with (57) are inside the unit circle. If this is the case, the impulse responses derived from an infinite order VAR will be misleading as the structural shocks cannot be recovered from the innovations of the VAR. Whether the moving average components of a model are invertible or non-invertible will in general depend on which variables are included in the VAR.<sup>17</sup>

Fernández-Villaverde et al. (2005) show that when  $D$  is square and  $D^{-1}$  exists, a necessary and sufficient condition for invertibility is that the eigenvalues of  $A - BD^{-1}C$  are strictly less than one in modulus. If this condition is satisfied then  $y_t$  has an infinite order VAR representation given by

$$y_t = \sum_{j=1}^{\infty} C(A - BD^{-1}C)^{j-1} BD^{-1} y_{t-j} + Dw_t, \quad (60)$$

The rate at which the autoregressive coefficients converge to zero is determined by the largest eigenvalue of  $A - BD^{-1}C$ . If this eigenvalue is close to unity a low order VAR is

<sup>16</sup>Lippi & Reichlin (1994) and Fernández-Villaverde et al. (2005) provide examples of economic models with non-invertible moving average components.

<sup>17</sup>Two special cases are worth noting. First, as can be seen from (56), if all the variables in  $x_t$  are observed by the econometrician (implying that  $A = C$  and  $B = D$ ), the process for  $y_t$  will be a VAR(1). Second, if all the endogenous state variables are included in  $y_t$  and the exogenous state variables follow a VAR(1) then  $y_t$  has a VAR(2) representation (see e.g. Kapetanios et al., 2005).

likely to be a poor approximation to the infinite order VAR. If one or more of the eigenvalues of  $A - BD^{-1}C$  are exactly equal to one in modulus, the model is still invertible, but  $y_t$  does not have a VAR representation. Fernández-Villaverde et al. (2005) refer to this as a ‘benign borderline case’. Often, roots on the unit circle indicate that the variables in the VAR have been overdifferenced (see Watson, 1994).

### 3.2 Identification

If the model is invertible, the impulse responses from the infinite order VAR (58) with  $Gv_t = Dw_t$  correspond to the impulse responses to the structural shocks in (57). In practice, however,  $D$  is unknown, and the econometrician is faced with an identification problem. As discussed in section ??, in the pass-through literature identification has typically been achieved by setting  $G = \Gamma_{tr}$ , where  $\Gamma_{tr}$  is the lower triangular Choleski factor of the estimated variance-covariance matrix of the VAR residuals,  $\hat{\Sigma}_u$ . However, this identification scheme is not consistent with the DSGE model set out in section 2, hence  $G = \Gamma_{tr}$  will yield biased estimates of the model’s impulse responses.<sup>18</sup> A prerequisite for estimating DSGE models by matching impulse responses, is that the identification restrictions imposed on the VAR are compatible with the theoretical model. In the simulation experiments in this paper I apply an identification scheme suggested by Del Negro & Schorfheide (2004). Using a QR decomposition of  $D$ , the impact responses of  $y_t$  to the structural shocks  $w_t$  can be decomposed into

$$\left( \frac{\partial y_t}{\partial w_t} \right)_{DSGE} = D = \Gamma_{tr}^* \Omega^* \quad (61)$$

where  $\Gamma_{tr}^*$  is lower triangular and  $\Omega^*$  satisfies  $(\Omega^*)' \Omega^* = I$ . The VAR is identified by setting  $G = \Gamma_{tr}^* \Omega^*$ . With this identification scheme the impact responses computed from the VAR will differ from  $D$  only to the extent that  $\Gamma_{tr}$  differs from  $\Gamma_{tr}^*$  (that is, only to the extent that the estimated variance-covariance matrix  $\hat{\Sigma}_u$  differs from  $DD'$ ). Thus, in the absence of misspecification of the VAR, the identification scheme succeeds in recovering the true impact responses.

## 4 Simulation experiments

This section presents the results of the simulation experiments. I consider two different VARs: a VAR in first differences of nominal prices and the exchange rate, and a VAR in relative prices and the first difference of consumer prices. The latter is equivalent to a VEqCM that includes the cointegration relations implied by the DSGE model as regressors. As a second exercise, I examine whether an econometrician who uses standard techniques for determining cointegration rank and for testing restrictions on the cointegration relations will be able to infer the cointegration properties of the DSGE model.

### 4.1 Monte Carlo design

I generate  $M = 5000$  datasets of lengths  $T = 1100$  and  $T = 1200$  using the state space representation of the log-linearised DSGE model as the data generating process.<sup>19</sup> Each

<sup>18</sup>Canova & Pina (1998) show that when the DSGE model does not imply a recursive ordering of the variables, the VAR responses to a monetary policy shock identified with a recursive identification scheme can be very misleading.

<sup>19</sup>To examine the sensitivity of the results to the number of Monte Carlo replications I conducted preliminary experiments using  $M = \{1000, 2000, \dots, 10000\}$  and found that the pointwise mean and standard deviations of the impulse responses obtained with  $M = 5000$  and  $M = 10000$  are essentially indistinguishable.



sample is initialised using the steady-state values of the variables. To limit the influence of the initial conditions, I discard the first 1000 observations in each replication and leave  $T = 100$  and  $T = 200$  observations for estimation of the VAR. The simulations are performed in Matlab, and the built-in function `randn.m` is used to generate the pseudo random normal errors. I use the same random numbers in all experiments. This is achieved by fixing the seed for the random number generator.

For each dataset I estimate a VAR and compute the accumulated responses of prices to a UIP shock. The UIP shock is identified using the Del Negro & Schorfheide (2004) identification scheme discussed in the previous section. The selection of lag order is an important preliminary step in VAR analyses. I report results for four different methods of lag order selection: The Akaike information criterion (AIC), the Hannan-Quinn criterion (HQ), the Schwarz criterion (SC) and the sequential likelihood-ratio test (LR) (see Lütkepohl, 1991 for a discussion). The LR test is implemented using the small sample correction suggested in Sims (1980) and a 5% significance level for the individual tests. In addition to results obtained for the different lag selection criteria, I report results for a fixed lag length ( $L = 2$  and  $L = 4$  for the VAR in first differences,  $L = 3$  and  $L = 5$  for the VEqCM and the VAR in levels).

Lütkepohl (1990) shows that, provided that the lag order goes to infinity with the sample size, orthogonalised impulse response functions computed from a finite order VAR are consistent and asymptotically normal, even if the true order of the process is infinite. In this sense, any discrepancies between the impulse responses from the VAR and the log linearised DSGE model can be attributed to a small sample size. It is nevertheless instructive to decompose the overall difference between the DSGE model's impulse responses and the VAR impulse responses into i) bias arising from approximating an infinite order VAR with a finite order VAR and ii) small sample estimation bias in the VAR coefficients for a given lag order. The first source of bias, which Chari et al. (2005) label the 'specification error', is given by the difference between the DSGE model's responses and those obtained from the population version of the finite order VAR for a given lag order. The coefficients in the population version of a finite order VAR can be interpreted as the probability limits of the OLS estimators or, what the OLS estimates would converge to if the number of observations went to infinity while keeping the lag order fixed (Christiano et al., 2005b). Fernández-Villaverde et al. (2005) provide formulas for these coefficients as functions of the matrices  $A, B, C$  and  $D$  in the state space representation (56). Hence, the magnitude of the specification error can be assessed without resorting to simulation exercises.<sup>20</sup> For a given lag order the bias arising from the specification error persists even in large samples. Regarding the small sample estimation bias, VAR impulse responses are non-linear functions of the autoregressive coefficients and the covariance matrix of the VAR residuals. It is well known that OLS estimates of the autoregressive coefficients in VARs are biased downward in small samples.

## 4.2 VAR in first differences

The first model I consider is a VAR in first differences of nominal prices and the exchange rate:

$$\Delta y_t = A_1 \Delta y_{t-1} + A_2 \Delta y_{t-2} + \dots + A_p \Delta y_{t-p} + \varepsilon_t \quad (62)$$

where

$$\Delta y_t' = \{\Delta \ln \bar{P}_t^m, \Delta \ln \bar{P}_t^x, \Delta \ln P_t^y, \Delta \ln P_t^c, \Delta \ln e_t\}.$$

---

<sup>20</sup>I am grateful to Jesús Fernández-Villaverde for providing the Matlab program `ssvar.m` which calculates the coefficients of the population version VAR.

With this vector of observables, the matrix  $A - BD^{-1}C$  has four roots equal to one, while the remaining roots are all smaller than one in modulus. This implies that the model is invertible, but technically, it does not have a VAR representation.

Table 4 reports the distribution of the lag orders chosen by the different lag order selection criteria for sample sizes  $T = 100$  and  $T = 200$ . The maximum lag length is set to five. As expected the SC is the most conservative and selects the lowest average lag order. For sample size  $T = 100$  the SC chooses lag order one in 71.5% of the replications. By contrast, the AIC and the HQ select a lag order of two in 90% of the replications. With a sample size of  $T = 200$  the average lag order selected increases for all criteria. The SC picks a lag order of two in 98% of the datasets, the AIC and the HQ select a lag order of two in 89% and 100% of the replications respectively. For both sample sizes, the LR test selects a somewhat higher lag order than the information criteria.

Figure 3 plots the outcome of the simulation experiment with  $T = 100$  and a fixed lag length  $L = 2$ . The solid lines represent the pointwise mean of the accumulated impulse responses, and the shaded areas correspond to the pointwise mean plus/minus 1.96 times the pointwise standard deviations. The starred lines correspond to a 95% interval for the pointwise responses, calculated by reading off the 2.5 and 97.5 percentiles of the ordered responses at each horizon. Finally, the circled lines depict the impulse responses from the DSGE model. Figure 4 plots the accumulated responses normalised on the exchange rate response.

Looking at the normalised responses we see that the VAR estimates of exchange rate pass-through are biased downwards. Whereas in the DSGE model exchange rate pass-through is nearly complete after twenty quarters, the mean of the VAR estimates of long-run pass-through is 72% for import prices, 35% for export prices, 15% for consumer prices and 20% for producer prices. From the bottom panel of figure 3 it is evident that the downward bias to some extent reflects that the exchange rate behaves almost like a random walk in the VAR, whereas there is significant reversion in the exchange rate towards the original level following a UIP shock in the DSGE model. The bias in the nominal exchange rate response is transmitted to import prices. By contrast, the estimated VAR responses of consumer and producer prices are smaller than the true responses. This suggests that the downward bias in the VAR estimates of pass-through to these prices would remain even if the VAR had accurately captured the exchange rate response. Figures 5 and 6 plot the outcome of an experiment with  $L = 2$  and  $T = 200$ . The biases in the impulse responses remain in the larger sample, the main effect of adding observations is to lower the standard deviations of the simulated responses.

Figures 7 and 8 decompose the overall bias into small sample bias and bias arising from approximating an infinite order VAR with a VAR(2). The latter is measured as the difference between the true impulse responses (circled lines) and the responses from the population version of a VAR(2) (solid line). It is evident that the dominant source of bias is the specification error. For a given lag order this bias persists in large samples. The small sample bias is measured as the difference between the responses from the population VAR(2) and the mean responses from the Monte Carlo experiments for  $T = 100$  (dotted line) and  $T = 200$  (crossed line). The impulse responses of exchange rate and import prices are biased downward in small samples. For these variables, the small sample bias and the specification error bias are of opposite signs. Hence, the effect of adding more observations is to increase the overall bias in the impulse responses. For consumer and producer prices, the opposite is true. For these variables the small sample bias reinforces the downward bias induced by the specification error.

Next, I examine how many lags are needed for the VAR to be able to recover the true impulse responses. Figures 9 and 10 show the impulse responses from the DSGE model (circled lines), together with the responses from the population version of the VAR for lag

orders  $L = \{2, 4, 10, 20\}$ . As expected, increasing the number of lags reduces the biases. However, even with as many as twenty lags, the VAR does not accurately capture the responses of prices to a UIP shock. Moreover, as we have seen, standard lag order selection criteria do not detect the need for longer lags.

Erceg et al. (2004) suggest measuring the bias in the impulse responses by the average absolute per cent difference between the mean response and the theoretical response for each variable, that is

$$bias_i^H = \frac{1}{H} \sum_{j=1}^H \left| \frac{r_{i,j}^{VAR} - r_{i,j}^{DSGE}}{r_{i,j}^{DSGE}} \right| \quad (63)$$

where  $r_{i,j}^{DSGE}$  and  $r_{i,j}^{VAR}$  are the DSGE model's responses and the mean across datasets of the VAR responses of variable  $i$  to a UIP shock at horizon  $j$  respectively. Tables 6 and 7 report the biases for  $H = 10$  and  $H = 20$  for different lag order criteria and sample sizes  $T = 100$  and  $T = 200$ . The results confirm that adding observations increases the bias in the responses of exchange rates and import prices, but reduces the biases the responses of consumer and producer prices. At both horizons and for both sample sizes the average bias is minimised for  $L = 4$ . The average bias is largest when the lag order is chosen to minimise the SC.

As a final point, note that a reduction in bias from estimating a higher order VAR may come at the cost of higher variance. Using VARs estimated by leading practitioners as data generating processes, Ivanov & Kilian (2005) find that underestimation of the true lag order is beneficial in very small samples because the bias induced by choosing a low lag order is more than offset by a reduction in variance. If the primary purpose of the VAR analysis is to construct accurate impulse responses, the authors recommend using the SC for sample sizes up to 120 quarters and the HQ for larger sample sizes. However, Ivanov & Kilian (2005) do not explore the case where the data generating process is an infinite order VAR in which case the trade-offs between bias and variance are likely to be different.

### 4.3 VEqCM

The fact that the monetary policy rule is specified in terms of inflation and not the price level induces a common stochastic trend in the nominal variables in the log-linearised DSGE model.<sup>21</sup> Hence, while nominal prices and the exchange rate contain a unit root, the real exchange rate and relative prices are stationary. Estimating a VAR in first differences implies a loss of efficiency, and in this sense it is not surprising that a VAR that omits the cointegration relations does a poor job in recovering the responses of the levels of prices and the exchange rate. Here I examine whether I obtain a better approximation of the DSGE model by estimating a VEqCM that includes the cointegration relations implied by the theoretical model. That is, I consider the system

$$\Delta y_t = \alpha \beta' y_{t-1} + A_1^* \Delta y_{t-1} + A_2^* \Delta y_{t-2} + \dots + A_p^* \Delta y_{t-p} + \varepsilon_t \quad (64)$$

with

$$\beta' y_{t-1} = \left\{ \begin{array}{c} \ln P_{t-1}^m - \ln P_{t-1}^c \\ \ln P_{t-1}^x - \ln P_{t-1}^c \\ \ln P_{t-1}^y - \ln P_{t-1}^c \\ \ln e_{t-1} + \ln P_{t-1}^f - \ln P_{t-1}^c \end{array} \right\}$$

<sup>21</sup>The foreign price level is stationary around a deterministic trend.

Estimating (64) is (nearly) the same as estimating a VAR in the real exchange rate, relative prices and consumer price inflation<sup>22</sup>, that is,

$$y_t^\dagger = A_1^\dagger y_{t-1}^\dagger + A_2^\dagger y_{t-2}^\dagger + \dots + A_p^\dagger y_{t-p+1}^\dagger + \varepsilon_t^\dagger \quad (65)$$

where

$$(y_t^\dagger)' = \left\{ \Delta \ln P_t^c, \ln(P_t^m/P_t^c), \ln(P_t^x/P_t^c), \ln(P_t^y/P_t^c), \ln(e_t P_t^f/P_t^c) \right\}$$

When the observation vector is  $y_t^\dagger$  all the roots of the matrix  $A - BD^{-1}C$  are smaller than one in modulus. Hence, the model is invertible, and  $y_t^\dagger$  has a VAR representation. Including the cointegration relations thus removes the unit roots in the moving average components that appear in the VARMA representation for the first differences. This is a common finding in the literature (see e.g., Del Negro et al., 2005).

Table 5 reports the distribution of the lag orders chosen by different selection criteria for  $T = 100$  and  $T = 200$ . The maximum lag length is six. The SC selects a lag length of two for both sample sizes. On average, the AIC chooses a higher lag order: for sample size  $T = 100$  the AIC chooses  $L = 2$  in 47.4% of the datasets and  $L = 3$  in 46.2% of the datasets. I have conducted simulation experiments for each of the selection criteria separately, but in the presentation of the results I will focus on the case  $L = 3$ . Figures 11 and 12 plot the outcome of the simulation experiment with  $T = 100$ . The VAR approximation to the DSGE model is good even with a moderate number of lags. This is confirmed in figures 13 and 14 which plot the responses computed from the population version of the VAR for lag orders  $L = \{2, 3, 20\}$ . There is some bias in the impulse responses for  $L = 2$ , but for  $L = 3$  the estimated responses are close to the true responses.

Figures 15 and 16 plot the impulse responses from the population version of the VEqCM(3) together with the true responses and the mean responses from a VEqCM(3) estimated on sample sizes  $T = 100$  and  $T = 200$ . In this case the small sample estimation bias is the dominant source of bias in the responses. For all prices except import prices the estimate of exchange rate pass-through is biased upwards, implying that for a given lag order, adding observations does not reduce the bias. This is confirmed in tables 6 and 7 which report the average biases over the first ten and twenty quarters respectively, for different lag order criteria and sample sizes  $T = 100$  and  $T = 200$ .

To summarise, provided the cointegration relations implied by the model are included as additional regressors, the state space representation of the log-linearised DSGE models can be approximated with a low order VAR. This raises the question of whether in practice the econometrician would be able to infer the cointegration rank and identify the cointegration relations using standard techniques.

#### 4.4 Cointegration analysis

This section asks the question: Will an econometrician armed with standard techniques be able to infer the correct cointegration rank and identify the cointegration relations implied by the theory?

The experiment is constructed as follows. I generate synthetic 5000 datasets of lengths  $T = 100$  and  $T = 200$  from the DSGE model. Series for the levels of the variables are obtained by cumulating the series for the first differences.<sup>23</sup> For a given synthetic dataset I

<sup>22</sup>The only difference is that an extra lag of  $\ln P_t^c$  is included in the latter from the inclusion of  $\Delta \ln P_{t-L}^c \equiv \ln P_{t-L}^c - \ln P_{t-L-1}^c$

<sup>23</sup>The initial values of the (log) levels of the variables are set to zero. Since the levels series are unit root processes and thus have infinite memory, dropping observations at the beginning of the sample does not reduce the dependence on the initial values.

estimate an unrestricted VAR in levels of the variables and determine the cointegration rank using the trace test for cointegration (see Johansen, 1988). Next, I test the restrictions on the cointegration space implied by the DSGE model using the standard LR test for known cointegration vectors (see Johansen, 1995).

The VAR is fitted with an unrestricted constant term and a restricted drift term. The specification of the deterministic terms is consistent with the data generating process. To see this, note that the monetary policy rule and the positive inflation target imply that nominal prices will have both a deterministic trend and a stochastic trend. Both trends are cancelled in the cointegrating relations which implies that relative prices are stationary around a constant mean. That is,  $\ln P_t^m - \ln P_t^c \sim I(0)$ ,  $\ln P_t^x - \ln P_t^c \sim I(0)$ , and  $\ln P_t^y - \ln P_t^c \sim I(0)$ . Since the inflation target in the foreign economy is assumed to be the same as the domestic inflation target, the process for the foreign price level contains the same deterministic trend as the domestic price level, and there is no linear trend in the nominal exchange rate. However, since the foreign price level is not included in the VAR, the fourth cointegration relation will be stationary around a deterministic trend. That is,  $\ln e_t - \ln P_t^c + 0.005t \sim I(0)$ .

Table 10 reports the distribution of lag orders chosen by the different selection criteria when the maximum lag length is set to six. The average lag order selected by the criteria is two or three for both sample sizes, with SC being the most conservative criterion.

The trace test is derived under the assumption that the errors are serially uncorrelated and normally distributed with mean zero. Good practice dictates that these assumptions be checked before testing for cointegration. Table 13 reports the rejection frequencies across 5000 datasets for the single-equation and vector tests for non-normality in the residuals described in Doornik & Hansen (1994). The rejection frequencies are close to the nominal 5% level for both sample sizes and across different lag order criteria. Table 14 reports the rejection frequencies for tests of no autocorrelation up to order five in the residuals. The test is the  $F$ -approximation to the Lagrange Multiplier (LM) test for autocorrelation described in Doornik (1996). For sample size  $T = 100$  and lag length  $L = 3$  the rejection frequencies for the single-equation tests are around 10%. The vector test rejects the null hypothesis in 23% of the datasets. Similar rejection frequencies are obtained for the AIC and the sequential LR tests. However, when a conservative criterion like the SC or HQ is used, the rejection frequencies are much higher. When the lag order is chosen to minimise the SC, the vector test rejects the null of no autocorrelation in 59.1% of the datasets. For all criteria except the SC, the rejection frequencies are lower in the larger sample  $T = 200$ . Below I report the outcome of the cointegration tests for all the lag order selection criteria. In practice, however, researchers often supplement the information criteria with tests for residual autocorrelation, and when there is a contradiction, overrule the lag order selected by the former. This suggests that less weight should be placed on the results obtained for the SC or HQ.

Table 11 shows the frequencies of preferred cointegration rank for different sample sizes and different methods of lag selection. The non-standard 5% critical values for the trace test are taken from MacKinnon et al. (1999). The numbers in parentheses correspond to the frequencies of preferred rank when the test statistic is adjusted using the small sample correction suggested by Reinsel & Ahn (1988). When  $T = 100$  and  $L = 3$  the correct cointegration rank is selected in only 2.7% of the datasets. In 17.5% of the datasets the trace test suggests that the rank is zero, in which case a model in first differences is appropriate. Using the small sample adjusted test statistics, the trace test chooses the correct rank in only 0.3% of the datasets. In 61.2% of the datasets the trace test would lead us to conclude that the variables are not cointegrated. The results are more encouraging when a sample size of  $T = 200$  is used. However, for  $L = 3$  the trace test still picks the true cointegration rank in only 31% of the replications.

When the lag order is endogenous, the correct rank is chosen most frequently when the

lag order is determined using the SC. For  $T = 100$  the correct rank is chosen in 20% of the datasets. With a sample size of  $T = 200$  the corresponding number is 53%. For the purpose of choosing the correct cointegration rank, it appears that a low lag order is beneficial.

As a second exercise, I examine how often the restrictions on the cointegration vector implied by the DSGE model are rejected when using the standard LR test for known cointegration vectors. Table 12 reports the rejection frequencies for the individual and joint tests of the following hypotheses

$$\ln P_t^m - \ln P_t^c \sim I(0), \ln P_t^x - \ln P_t^c \sim I(0), \ln P_t^y - \ln P_t^c \sim I(0) \text{ and } \ln e_t - \ln P_t^c + 0.005t \sim I(0)$$

The tests are conditional on the maintained hypothesis that the cointegration rank is 4 ( $r = 4$ ). For  $T = 100$  and  $L = 3$ , the rejection frequencies for the individual hypotheses are 20% when using a nominal test size of 5%. The rejection frequency for the joint hypothesis is 88%. These results raise doubts about whether, in practice, the econometrician will be able to identify the cointegration relations implied by the DSGE model.

Again it is instructive to see whether the results are driven by the specification error or by small sample estimation bias. In particular, it is of interest to see whether the frequent rejections of the autocorrelation tests are due to the omission of moving average terms or due to the fact that the autocorrelation tests are oversized in small samples. To assess this I redo the above Monte Carlo experiments, this time using the population version of a VEqCM(5) and a VEqCM(3) as the data generating processes. Table 16 reports the outcome of the trace test in this case. The results are similar to the results obtained when the log-linearised solution to the DSGE model is used as the data generating process. This finding suggests that the poor performance of the test is not due to approximating an infinite order VAR with a low order VAR, but is due to small sample problems. Interestingly, the same seems to hold for the autocorrelation test. When the data generating process is a VEqCM(3) and the estimated is a VAR(3) in levels of the data, the rejection frequencies of the autocorrelation tests are 10% for the single-equation tests and 23% for the vector test (see table 17) which suggests that the autocorrelation test is oversized in small samples. This is consistent with the Monte Carlo evidence presented in Brüggemann et al. (2004). Table 19 illustrates a well known result in the literature (see e.g. Gredenhoff & Jacobson, 2001), namely that the LR tests for restrictions on the cointegration space are oversized in small samples.

## 5 Concluding remarks

This paper has examined the ability of a structural VAR to recover the dynamic responses of a set of prices to a risk premium shock. The main results can be summarised as follows: The estimates of exchange rate pass-through obtained from a first differenced VAR are systematically biased downwards. The bias in the estimated responses can largely be attributed to the fact that a low order VAR is not a good approximation to the infinite order VAR implied by the DSGE model. Moreover, small sample estimation bias sometimes acts to offset the bias arising from the approximation error. When the cointegration relations implied by the DSGE model are included in the VAR, even a VAR with a modest number of lags is able to recover the true impulse responses. However, an econometrician using standard tests for cointegration rank and for testing restrictions on the cointegration space would in general not be able to infer the correct rank or identify the true cointegration relations.

## References

- Adolfson, M., Laséen, S., Lindé, J., & Villani, M. (2005). *Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-Through*. Working Paper 179, Sveriges Riksbank.
- Bacchetta, P. & Wincoop, E. V. (2005). A theory of the currency denomination of international trade. *Journal of International Economics*, 67, 295–319.
- Benigno, G. & Thoenissen, C. (2003). Equilibrium exchange rates and supply-side performance. *Economic Journal*, 113, 103–124.
- Bergin, P. R. (2004). *How Well Can the New Open Economy Macroeconomics Explain the Exchange Rate and the Current Account?* Working Paper 10356, NBER. Forthcoming, Journal of International Money and Finance.
- Betts, C. & Devereux, M. B. (1996). The exchange rate in a model of pricing-to-market. *European Economic Review*, 40, 1007–1021.
- Brüggemann, R., Lütkepohl, H., & Saikkonen, P. (2004). Residual autocorrelation testing for vector error correction models. Mimeo, available at <http://www.iue.it/Personal/Luetkepohl/ACTesting.pdf>.
- Campa, J. M. & Goldberg, L. S. (2005). Exchange rate pass-through into import prices. *The Review of Economics and Statistics*, 87, 679–690.
- Canova, F. & Pina, J. P. (1998). *Monetary Policy Misspecification in VAR Models*. Economics Working Paper 420, Universitat Pompeu Fabra. This revision: December 2002.
- Chari, V. V., Kehoe, P. J., & McGrattan, E. R. (2005). *A Critique of Structural VARs Using Real Business Cycle Theory*. Working Paper 631, Federal Reserve Bank of Minneapolis.
- Choudhri, E. U., Faruquee, H., & Hakura, D. S. (2005). Explaining the exchange rate pass-through in different prices. *Journal of International Economics*, 65, 349–374.
- Christiano, L. J., Eichenbaum, M., & Evans, C. L. (2005a). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy*, 113, 1–45.
- Christiano, L. J., Eichenbaum, M., & Vigfusson, R. (2005b). Assessing structural VARs. Mimeo, May 24.
- Corsetti, G. & Dedola, L. (2005). A macroeconomic model of international price discrimination. *Journal of International Economics*, 67, 129–155.
- Corsetti, G., Dedola, L., & Leduc, S. (2005). *DSGE Models of High Exchange-Rate Volatility and Low Pass-Through*. International Finance Discussion Paper 845, Federal Reserve Board.
- Del Negro, M. & Schorfheide, F. (2004). Priors from general equilibrium models for VARs. *International Economic Review*, 45, 643–673.
- Del Negro, M., Schorfheide, F., Smets, F., & Wouters, R. (2005). *On the Fit and Forecasting Properties of New-Keynesian Models*. Working Paper 491, ECB.
- Devereux, M. B., Engel, C., & Storgaard, P. E. (2004). Endogenous exchange rate pass-through when nominal prices are set in advance. *Journal of International Economics*, 63, 263–291.
- Doornik, J. A. (1996). Testing vector error autocorrelation and heteroscedasticity. Mimeo, University of Oxford <http://www.doornik.com/research/vectest.pdf>.
- Doornik, J. A. & Hansen, H. (1994). An omnibus test for univariate and multivariate normality. Mimeo, University of Oxford <http://www.doornik.com/research/normal2.pdf>.

- Erceg, C. J., Guerrieri, L., & Gust, C. (2004). *Can Long-Run Restrictions Identify Technology Shocks?* International Finance Discussion Papers 792, Board of Governors of the Federal Reserve System.
- Faruqee, H. (2005). Exchange-rate pass-through in the euro area. *IMF Staff Papers*. Forthcoming. Previous version available as IMF Working Paper 04/14.
- Fernández-Villaverde, J., Rubio-Ramirez, J. F., & Sargent, T. J. (2005). *A, B, C's (and Ds) for Understanding VARs*. Working Paper 2005-9, Federal Reserve Bank of Atlanta.
- Goldberg, L. S. & Tille, C. (2005). *Vehicle Currency Use in International Trade*. Working Paper 11127, NBER.
- Gredenhoff, M. & Jacobson, T. (2001). Bootstrap testing linear restrictions on cointegrating vectors. *Journal of Business and Economic Statistics*, 19, 63–72.
- Groen, J. J. J. & Matsumoto, A. (2004). *Real Exchange Rate Persistence and Systematic Monetary Policy Behaviour*. Working Paper 231, Bank of England.
- Hahn, E. (2003). *Pass-Through of External Shocks to Euro Area Inflation*. Working Paper 243, ECB.
- Hunt, B. & Rebucci, A. (2005). The US dollar and the trade deficit: What accounts for the late 1990s? *International Finance*, 8, 399–434.
- Ingram, B. F., Kocherlakota, N. R., & Savin, N. E. (1994). Explaining business cycles: A multiple-shock approach. *Journal of Monetary Economics*, 34, 415–428.
- Ivanov, V. & Kilian, L. (2005). A practitioner's guide to lag order selection for VAR impulse response analysis. *Studies in Nonlinear Dynamics & Econometrics*, 9, 1–34. Issue 2, Article 9.
- Johansen, S. (1988). Statistical analysis of cointegration vectors. *Journal of Economic Dynamics and Control*, 12, 231–254.
- Johansen, S. (1995). *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*. Oxford, UK: Oxford University Press.
- Juillard, M. (2005). *DYNARE Manual. Version 3.0*. Available at <http://www.cepremap.cnrs.fr/michel/dynare/>.
- Kapetanios, G., Pagan, A., & Scott, A. (2005). *Making a Match: Combining Theory and Evidence in Policy-Oriented Macroeconomic Modeling*. Working Paper 1/2005, Centre for Applied Macroeconomic Analysis (CAMA), Australian National University.
- Laxton, D. & Pesenti, P. (2003). Monetary rules for small, open, emerging economies. *Journal of Monetary Economics*, 50, 1109–1146.
- Lippi, M. & Reichlin, L. (1994). VAR analysis, nonfundamental representations, blaschke matrices. *Journal of Econometrics*, 63, 307–325.
- Lubik, T. & Schorfheide, F. (2005). A bayesian look at new open economy macroeconomics. Forthcoming, NBER Macroeconomics Annual 2005, Vol 20. This version: May 2005. <http://www.nber.org/books/macro20/lubik-schorfheide5-16-05.pdf>.
- Lütkepohl, H. (1990). Asymptotic distributions of impulse response functions and forecast error variance decompositions of vector autoregressive models. *The Review of Economics and Statistics*, 72, 116–125.
- Lütkepohl, H. (1991). *Introduction to Multiple Time Series Analysis*. Berlin: Springer-Verlag.
- MacKinnon, J. G., Haug, A. A., & Michelis, L. (1999). Numerical distribution functions of likelihood ratio tests for cointegration. *Journal of Applied Econometrics*, 14, 563–577.



- McCarthy, J. (2000). *Pass-Through of Exchange Rates and Import Prices to Domestic Inflation in some Industrialized Countries*. Staff Report 111, Federal Reserve Bank of New York.
- Obstfeld, M. & Rogoff, K. (1995). Exchange rate dynamics redux. *Journal of Political Economy*, 103, 624–660.
- Reinsel, G. C. & Ahn, S. K. (1988). *Asymptotic Properties of the Likelihood Ratio Test for Cointegration in the Non-Stationary Vector AR Model*. Technical report, Department of Statistics, University of Wisconsin.
- Rotemberg, J. J. (1982). Monopolistic price adjustment and aggregate output. *Review of Economic Studies*, 49, 517–531.
- Rotemberg, J. J. & Woodford, M. (1997). An optimization-based econometric model for the evaluation of monetary policy. *NBER Macroeconomics Annual*, 12(12), 297–346.
- Sarno, L. & Thornton, D. L. (2004). The efficient market hypothesis and identification in structural VARs. *Federal Reserve Bank of St. Louis Review*, 86, 49–60.
- Schmitt-Grohe, S. & Uribe, M. (2003). Closing small open economy models. *Journal of International Economics*, 61, 163–185.
- Sims, C. A. (1980). Macroeconomics and reality. *Econometrica*, 48, 1–48.
- Smets, F. & Wouters, R. (2003). An estimated dynamic stochastic general equilibrium model of the Euro area. *Journal of the European Economic Association*, 1, 1123–1175.
- Watson, M. W. (1994). *Vector Autoregressions and Cointegration*, volume 4 of *Handbook of Econometrics*, chapter 47, (pp. 2843–2915). Elsevier Science, Amsterdam.

## A Equilibrium conditions of log-linearised DSGE model

Letting variables with a hat denote percentage deviations from the deterministic steady state (i.e.,  $\widehat{X}_t = \ln X_t - \ln X$ ) the log-linearised equilibrium conditions can be written

$$\widehat{C}_t = \phi_c \widehat{Q}_t + (1 - \phi_c) \widehat{H}_t^c \quad (66)$$

$$\widehat{Q}_t^d = -\nu_q (\widehat{p}_t^y - \widehat{p}_t^q) + \widehat{Q}_t \quad (67)$$

$$\widehat{Q}_t^m = -\nu_q (\widehat{p}_t^m - \widehat{p}_t^q) + \widehat{Q}_t \quad (68)$$

$$\widehat{w}_t = \widehat{\vartheta}_t^c + \widehat{C}_t - \widehat{H}_t^c \quad (69)$$

$$\widehat{p}_t^q = \widehat{\vartheta}_t^c + \widehat{C}_t - \widehat{Q}_t \quad (70)$$

$$\widehat{p}_t^q = \alpha_q \left( \frac{p^y}{p^q} \right)^{1-\nu_q} \widehat{p}_t^y + (1 - \alpha_q) \left( \frac{p^m}{p^q} \right)^{1-\nu_q} \widehat{p}_t^m \quad (71)$$

$$\begin{aligned} \widehat{\pi}_t^c &= \frac{\beta(\phi_{c1} + \phi_{c2})}{\phi_{c1} + (1 + \beta)\phi_{c2}} E_t \widehat{\pi}_{t+1}^c + \frac{\phi_{c2}}{\phi_{c1} + (1 + \beta)\phi_{c2}} \widehat{\pi}_{t-1}^c \\ &+ \frac{\theta^c(\theta^c - 1)}{\phi_{c1} + (1 + \beta)\phi_{c2}} \widehat{\vartheta}_t^c - \frac{\theta^c}{\phi_{c1} + (1 + \beta)\phi_{c2}} \widehat{\theta}_t^c \end{aligned} \quad (72)$$

$$\widehat{Y}_t = \phi_y \widehat{Z}_t + (1 - \phi_y) \widehat{H}_t^y \quad (73)$$

$$\widehat{Z}_t^d = -\nu_z (\widehat{p}_t^y - \widehat{p}_t^z) + \widehat{Z}_t \quad (74)$$

$$\widehat{Z}_t^m = -\nu_z (\widehat{p}_t^m - \widehat{p}_t^z) + \widehat{Z}_t \quad (75)$$

$$\widehat{w}_t = \widehat{\vartheta}_t^y + \widehat{Y}_t - \widehat{H}_t^y \quad (76)$$

$$\widehat{p}_t^z = \widehat{\vartheta}_t^y + \widehat{Y}_t - \widehat{Z}_t \quad (77)$$

$$\widehat{p}_t^z = \alpha_z \left( \frac{p^y}{p^z} \right)^{1-\nu_z} \widehat{p}_t^y + (1 - \alpha_z) \left( \frac{p^m}{p^z} \right)^{1-\nu_z} \widehat{p}_t^m \quad (78)$$

$$\widehat{Y}_t^d = \left( \frac{Q^d}{Y^d} \right) \widehat{Q}_t^d + \left( \frac{Z^d}{Y^d} \right) \widehat{Z}_t^d \quad (79)$$

$$\widehat{Y}_t^m = \left( \frac{Q^m}{Y^m} \right) \widehat{Q}_t^m + \left( \frac{Z^m}{Y^m} \right) \widehat{Z}_t^m \quad (80)$$

$$\widehat{Y}_t = \left( \frac{Y^d}{Y} \right) \widehat{Y}_t^d + \left( \frac{Y^m}{Y} \right) \widehat{Y}_t^m \quad (81)$$

$$\begin{aligned} \widehat{\pi}_t^y &= \frac{\beta(\phi_{y1} + \phi_{y2})}{\phi_{y1} + (1 + \beta)\phi_{y2}} E_t \widehat{\pi}_{t+1}^y + \frac{\phi_{y2}}{\phi_{y1} + (1 + \beta)\phi_{y2}} \widehat{\pi}_{t-1}^y \\ &+ \frac{\theta^y(\theta^y - 1)}{\phi_{y1} + (1 + \beta)\phi_{y2}} (\widehat{\vartheta}_t^y - \widehat{p}_t^y) - \frac{\theta^y}{\phi_{y1} + (1 + \beta)\phi_{y2}} \widehat{\theta}_t^y \end{aligned} \quad (82)$$

$$\widehat{p}_t^x = \varpi \widehat{p}_t^{xp} + (1 - \varpi) \widehat{p}_t^{xl} \quad (83)$$

$$\widehat{p}_t^x = \left( \frac{\bar{p}^x}{p^x} \right) \widehat{p}_t^x + \delta_f \left( \frac{sw^f}{p^x} \right) (\widehat{w}_t^f + \widehat{s}_t) \quad (84)$$

$$\begin{aligned} \widehat{\pi}_t^{xl} &= \frac{\beta(\phi_{x1} + \phi_{x2})}{\phi_{x1} + (1 + \beta)\phi_{x2}} E_t \widehat{\pi}_{t+1}^{xl} + \frac{\phi_{x2}}{\phi_{x1} + (1 + \beta)\phi_{x2}} \widehat{\pi}_{t-1}^{xl} \\ &- \frac{\theta^x - 1}{\phi_{x1} + (1 + \beta)\phi_{x2}} \frac{\bar{p}^x}{\bar{p}^x - \vartheta^y} \frac{\bar{p}^x}{p^x} \left( \widehat{p}_t^{xl} - \left( \widehat{p}_t^x \right)^{opt} \right) \end{aligned} \quad (85)$$

$$\begin{aligned} \widehat{\pi}_t^{xp} &= \frac{\beta(\phi_{x1} + \phi_{x2})}{\phi_{x1} + (1 + \beta)\phi_{x2}} E_t \widehat{\pi}_{t+1}^{xp} + \frac{\phi_{x2}}{\phi_{x1} + (1 + \beta)\phi_{x2}} \widehat{\pi}_{t-1}^{xp} \\ &\quad - \frac{\theta^x - 1}{\phi_{x1} + (1 + \beta)\phi_{x2}} \frac{\bar{p}^x}{\bar{p}^x - \vartheta^y} \frac{\bar{p}^x}{p^x} \left( \widehat{p}_t^{xp} - \left( \widehat{p}_t^x \right)^{opt} \right) \end{aligned} \quad (86)$$

$$\bar{p}^x \left( \widehat{p}_t^x \right)^{opt} = \frac{\theta^x}{\theta^x - 1} \vartheta^y \left( \widehat{\vartheta}_t^y - \frac{1}{\theta^x - 1} \widehat{\theta}_t^x \right) + \frac{\delta_f}{\theta^x - 1} s w^f \left( \widehat{s}_t + \widehat{w}_t^f - \frac{\theta^x}{\theta^x - 1} \widehat{\theta}_t^x \right) \quad (87)$$

$$\widehat{Y}_t^x = -\nu_f \left( \widehat{p}_t^x - \widehat{s}_t \right) + \widehat{Y}_t^f \quad (88)$$

$$\widehat{H}_t^m = \widehat{Y}_t^m \quad (89)$$

$$\widehat{p}_t^m = \left( \frac{\bar{p}^m}{p^m} \right) \widehat{p}_t^m + \delta \left( \frac{w}{p^m} \right) \widehat{w}_t \quad (90)$$

$$\widehat{\bar{p}}_t^m = \varpi_f \widehat{p}_t^{mp} + (1 - \varpi_f) \widehat{\bar{p}}_t^{ml} \quad (91)$$

$$\begin{aligned} \widehat{\pi}_t^{ml} &= \frac{\beta(\phi_{m1} + \phi_{m2})}{\phi_{m1} + (1 + \beta)\phi_{m2}} E_t \widehat{\pi}_{t+1}^{ml} + \frac{\phi_{m2}}{\phi_{m1} + (1 + \beta)\phi_{m2}} \widehat{\pi}_{t-1}^{ml} \\ &\quad - \frac{\theta^m - 1}{\phi_{m1} + (1 + \beta)\phi_{m2}} \frac{\bar{p}^m}{\bar{p}^m - s\vartheta^f} \frac{\bar{p}^m}{p^m} \left( \widehat{p}_t^{ml} - \left( \widehat{p}_t^m \right)^{opt} \right) \end{aligned} \quad (92)$$

$$\begin{aligned} \widehat{\pi}_t^{mp} &= \frac{\beta(\phi_{m1} + \phi_{m2})}{\phi_{m1} + (1 + \beta)\phi_{m2}} E_t \widehat{\pi}_{t+1}^{mp} + \frac{\phi_{m2}}{\phi_{m1} + (1 + \beta)\phi_{m2}} \widehat{\pi}_{t-1}^{mp} \\ &\quad - \frac{\theta^m - 1}{\phi_{m1} + (1 + \beta)\phi_{m2}} \frac{\bar{p}^m}{\bar{p}^m - s\vartheta^f} \frac{\bar{p}^m}{p^m} \left( \widehat{p}_t^{mp} - \left( \widehat{p}_t^m \right)^{opt} \right) \end{aligned} \quad (93)$$

$$\bar{p}^m \left( \widehat{p}_t^m \right)^{opt} = \frac{\theta^m}{\theta^m - 1} s \vartheta^f \left( \widehat{s}_t + \widehat{\vartheta}_t^f - \frac{1}{\theta^m - 1} \widehat{\theta}_t^m \right) + \frac{\delta}{\theta^m - 1} w \left( \widehat{w}_t - \frac{\theta^m}{\theta^m - 1} \widehat{\theta}_t^m \right) \quad (94)$$

$$\widehat{C}_t = \frac{1}{1 + \zeta} E_t \widehat{C}_{t+1} + \frac{\zeta}{1 + \zeta} \widehat{C}_{t-1} - \frac{1 - \zeta}{1 + \zeta} \left( \widehat{R}_t - E_t \widehat{\pi}_{t+1}^c \right) \quad (95)$$

$$\widehat{R}_t = \widehat{R}_t^f + E_t \widehat{\Delta} e_{t+1} + \widehat{\kappa}_t \quad (96)$$

$$\begin{aligned} \widehat{\pi}_t^w &= \frac{\beta(\phi_{w1} + \phi_{w2})}{\phi_{w1} + (1 + \beta)\phi_{w2}} E_t \widehat{\pi}_{t+1}^w + \frac{\phi_{w2}}{\phi_{w1} + (1 + \beta)\phi_{w2}} \widehat{\pi}_{t-1}^w \\ &\quad - \frac{\theta_h - 1}{\phi_{w1} + (1 + \beta)\phi_{w2}} \left( \widehat{w}_t - \chi \widehat{H}_t - \frac{\widehat{C}_t - \zeta \widehat{C}_{t-1}}{1 - \zeta} \right) \end{aligned} \quad (97)$$

$$\widehat{\kappa}_t = -\psi s \widehat{b}_t^f + \widehat{u}_t \quad (98)$$

$$\widehat{R}_t = \rho_R \widehat{R}_{t-1} + \frac{(1 - \rho_R) \rho_\pi \pi}{R} \widehat{\pi}_t^c \quad (99)$$

$$\widehat{H}_t = \left( \frac{H^c}{H} \right) \widehat{H}_t^c + \left( \frac{H^y}{H} \right) \widehat{H}_t^y + \left( \frac{H^m}{H} \right) \widehat{H}_t^m \quad (100)$$

$$\frac{s\beta \widehat{b}_t^f}{\pi^f} = \frac{\widehat{s} b_{t-1}^f}{\pi^f} + \bar{p}^x Y^x \left( \widehat{p}_t^x + \widehat{Y}_t^x \right) - \bar{p}^m Y^m \left( \widehat{p}_t^m + \widehat{Y}_t^m \right) \quad (101)$$

$$\widehat{\theta}_t^c = \rho_c \widehat{\theta}_{t-1}^c + \varepsilon_{c,t} \quad (102)$$

$$\widehat{\theta}_t^y = \rho_y \widehat{\theta}_{t-1}^y + \varepsilon_{y,t} \quad (103)$$

$$\widehat{\theta}_t^x = \rho_x \widehat{\theta}_{t-1}^x + \varepsilon_{x,t} \quad (104)$$

$$\widehat{\theta}_t^m = \rho_m \widehat{\theta}_{t-1}^m + \varepsilon_{m,t} \quad (105)$$

$$\widehat{u}_t = \rho_u \widehat{u}_{t-1} + \widehat{\varepsilon}_{u,t} \quad (106)$$

$$\widehat{\pi}_t^y = \widehat{p}_t^y - \widehat{p}_{t-1}^y + \widehat{\pi}_t^c \quad (107)$$

$$\widehat{\pi}_t^m = \widehat{p}_t^m - \widehat{p}_{t-1}^m + \widehat{\pi}_t^c \quad (108)$$

$$\widehat{\pi}_t^m = \widehat{p}_t^m - \widehat{p}_{t-1}^m + \widehat{\pi}_t^c \quad (109)$$

$$\widehat{\pi}_t^{mp} = \widehat{p}_t^{mp} - \widehat{p}_{t-1}^{mp} + \widehat{\pi}_t^c - \widehat{\Delta}e_t \quad (110)$$

$$\widehat{\pi}_t^{ml} = \widehat{p}_t^{ml} - \widehat{p}_{t-1}^{ml} + \widehat{\pi}_t^c \quad (111)$$

$$\widehat{\Delta}e_t = \widehat{s}_t - \widehat{s}_{t-1} + \widehat{\pi}_t^c - \widehat{\pi}_t^f \quad (112)$$

$$\widehat{\pi}_t^w = \widehat{w}_t - \widehat{w}_{t-1} + \widehat{\pi}_t^c \quad (113)$$

$$\widehat{\pi}_t^x = \widehat{p}_t^x - \widehat{p}_{t-1}^x + \widehat{\pi}_t^c \quad (114)$$

$$\widehat{\pi}_t^{xl} = \widehat{p}_t^{xl} - \widehat{p}_{t-1}^{xl} + \widehat{\pi}_t^c - \widehat{\Delta}e_t \quad (115)$$

$$\widehat{\pi}_t^{xp} = \widehat{p}_t^{xp} - \widehat{p}_{t-1}^{xp} + \widehat{\pi}_t^c \quad (116)$$

Table 1: The price and quantity indices in the DSGE model

Quantity index	Price index	Demand functions
$Q_t = \left[ \alpha \frac{1}{\nu} (Q_t^d)^{\frac{\nu-1}{\nu}} + (1-\alpha) \frac{1}{\nu} (Q_t^m)^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}}$	$P_t^q = \left[ \alpha (P_t^y)^{1-\nu} + (1-\alpha) (P_t^m)^{1-\nu} \right]^{\frac{1}{1-\nu}}$	$Q_t^d = \alpha \left( \frac{P_t^y}{P_t^q} \right)^{-\nu} Q_t, \quad Q_t^m = (1-\alpha) \left( \frac{P_t^m}{P_t^q} \right)^{-\nu} Q_t$
$Q_t^d = \left[ \int_0^1 Y_t^{dq}(i) \frac{\theta_t^y - 1}{\theta_t^y} di \right]^{\frac{\theta_t^y}{\theta_t^y - 1}}$	$P_t^y = \left[ \int_0^1 P_t^y(i)^{1-\theta_t^y} di \right]^{\frac{1}{1-\theta_t^y}}$	$Y_t^{dq}(i) = \left( \frac{P_t^y(i)}{P_t^y} \right)^{-\theta_t^y} Q_t^d$
$Q_t^m = \left[ \int_0^1 Y_t^{mq}(m) \frac{\theta_t^m - 1}{\theta_t^m} dm \right]^{\frac{\theta_t^m}{\theta_t^m - 1}}$	$P_t^m = \left[ \int_0^1 P_t^m(m)^{1-\theta_t^m} dm \right]^{\frac{1}{1-\theta_t^m}}$	$Y_t^{mq}(m) = \left( \frac{P_t^m(m)}{P_t^m} \right)^{-\theta_t^m} Q_t^m$
$Z_t = \left[ \alpha \frac{1}{\nu} (Z_t^d)^{\frac{\nu-1}{\nu}} + (1-\alpha) \frac{1}{\nu} (Z_t^m)^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}}$	$P_t^z = \left[ \alpha (P_t^y)^{1-\nu} + (1-\alpha) (P_t^m)^{1-\nu} \right]^{\frac{1}{1-\nu}}$	$Z_t^d = \alpha \left( \frac{P_t^y}{P_t^z} \right)^{-\nu} Z_t, \quad Z_t^m = (1-\alpha) \left( \frac{P_t^m}{P_t^z} \right)^{-\nu} Z_t$
$Z_t^d = \left[ \int_0^1 Y_t^{dz}(i) \frac{\theta_t^y - 1}{\theta_t^y} di \right]^{\frac{\theta_t^y}{\theta_t^y - 1}}$	$P_t^y = \left[ \int_0^1 P_t^y(i)^{1-\theta_t^y} di \right]^{\frac{1}{1-\theta_t^y}}$	$Y_t^{dz}(i) = \left( \frac{P_t^y(i)}{P_t^y} \right)^{-\theta_t^y} Z_t^d$
$Z_t^m = \left[ \int_0^1 Y_t^{mz}(m) \frac{\theta_t^m - 1}{\theta_t^m} dm \right]^{\frac{\theta_t^m}{\theta_t^m - 1}}$	$P_t^m = \left[ \int_0^1 P_t^m(m)^{1-\theta_t^m} dm \right]^{\frac{1}{1-\theta_t^m}}$	$Y_t^{mz}(m) = \left( \frac{P_t^m(m)}{P_t^m} \right)^{-\theta_t^m} Z_t^m$
$Y_t^x = \left[ \frac{1}{\varpi} \frac{\theta_t^x - 1}{\theta_t^x} (Y_t^{xp}) \frac{\theta_t^x - 1}{\theta_t^x} + (1-\varpi) \frac{1}{\theta_t^x} (Y_t^x) \frac{\theta_t^x - 1}{\theta_t^x} \right]^{\frac{\theta_t^x}{\theta_t^x - 1}}$	$P_t^x = \left[ \varpi (P_t^{xp})^{1-\theta_t^x} + (1-\varpi) (e_t P_t^{xl})^{1-\theta_t^x} \right]^{\frac{1}{1-\theta_t^x}}$	$Y_t^{xp} = \varpi \left( \frac{P_t^{xp}}{P_t^x} \right)^{-\theta_t^x} Y_t^x, \quad Y_t^{xl} = (1-\varpi) \left( \frac{e_t P_t^{xl}}{P_t^x} \right)^{-\theta_t^x} Y_t^x$
$Y_t^{xp} = \left[ \left( \frac{1}{\varpi} \right) \frac{\theta_t^x - 1}{\theta_t^x} \int_0^1 Y_t^{xp}(i) \frac{\theta_t^x - 1}{\theta_t^x} di \right]^{\frac{\theta_t^x}{\theta_t^x - 1}}$	$P_t^{xp} = \left[ \frac{1}{\varpi} \int_0^1 P_t^{xp}(i)^{1-\theta_t^x} di \right]^{\frac{1}{1-\theta_t^x}}$	$Y_t^{xp}(i) = \frac{1}{\varpi} \left( \frac{P_t^{xp}(i)}{P_t^{xp}} \right)^{-\theta_t^x} Y_t^{xp}$
$Y_t^{xl} = \left[ \left( \frac{1}{1-\varpi} \right) \frac{\theta_t^x - 1}{\theta_t^x} \int_0^1 Y_t^{xl}(i) \frac{\theta_t^x - 1}{\theta_t^x} di \right]^{\frac{\theta_t^x}{\theta_t^x - 1}}$	$P_t^{xl} = \left[ \frac{1}{1-\varpi} \int_0^1 P_t^{xl}(i)^{1-\theta_t^x} di \right]^{\frac{1}{1-\theta_t^x}}$	$Y_t^{xl}(i) = \frac{1}{1-\varpi} \left( \frac{P_t^{xl}(i)}{P_t^{xl}} \right)^{-\theta_t^x} Y_t^{xl}$
$Y_t^m = \left[ \left( \varpi_f \right) \frac{\theta_t^m - 1}{\theta_t^m} (Y_t^{mp}) \frac{\theta_t^m - 1}{\theta_t^m} + (1-\varpi_f) \frac{\theta_t^m - 1}{\theta_t^m} (Y_t^{ml}) \frac{\theta_t^m - 1}{\theta_t^m} \right]^{\frac{\theta_t^m}{\theta_t^m - 1}}$	$P_t^m = \left[ \varpi_f (e_t P_t^{mp})^{1-\theta_t^m} + (1-\varpi_f) (P_t^{ml})^{1-\theta_t^m} \right]^{\frac{1}{1-\theta_t^m}}$	$Y_t^{mp} = \varpi_f \left( \frac{e_t P_t^{mp}}{P_t^m} \right)^{-\theta_t^m} Y_t^m, \quad Y_t^{ml} = (1-\varpi_f) \left( \frac{P_t^{ml}}{P_t^m} \right)^{-\theta_t^m} Y_t^m$
$Y_t^{mp} = \left[ \left( \frac{1}{\varpi_f} \right) \frac{\theta_t^m - 1}{\theta_t^m} \int_0^1 Y_t^{mp}(m) \frac{\theta_t^m - 1}{\theta_t^m} dm \right]^{\frac{\theta_t^m}{\theta_t^m - 1}}$	$P_t^{mp} = \left[ \frac{1}{\varpi_f} \int_0^1 P_t^{mp}(m)^{1-\theta_t^m} dm \right]^{\frac{1}{1-\theta_t^m}}$	$Y_t^{mp}(m) = \frac{1}{\varpi_f} \left( \frac{P_t^{mp}(m)}{P_t^{mp}} \right)^{-\theta_t^m} Y_t^{mp}$
$Y_t^{ml} = \left[ \left( \frac{1}{1-\varpi_f} \right) \frac{\theta_t^m - 1}{\theta_t^m} \int_0^1 Y_t^{ml}(m) \frac{\theta_t^m - 1}{\theta_t^m} dm \right]^{\frac{\theta_t^m}{\theta_t^m - 1}}$	$P_t^{ml} = \left[ \frac{1}{1-\varpi_f} \int_0^1 P_t^{ml}(m)^{1-\theta_t^m} dm \right]^{\frac{1}{1-\theta_t^m}}$	$Y_t^{ml}(m) = \frac{1}{1-\varpi_f} \left( \frac{P_t^{ml}(m)}{P_t^{ml}} \right)^{-\theta_t^m} Y_t^{ml}$
$C_t = \left[ \int_0^1 C_t(c) \frac{\theta_t^c - 1}{\theta_t^c} dc \right]^{\frac{\theta_t^c}{\theta_t^c - 1}}$	$P_t^c = \left[ \int_0^1 P_t^c(c)^{1-\theta_t^c} dc \right]^{\frac{1}{1-\theta_t^c}}$	$C_t(c) = \left( \frac{P_t^c(c)}{P_t^c} \right)^{-\theta_t^c} C_t$
$H_t = \left[ \int_0^1 H_t(j) \frac{\theta_t^h - 1}{\theta_t^h} dj \right]^{\frac{\theta_t^h}{\theta_t^h - 1}}$	$W_t = \left[ \int_0^1 W_t(j)^{1-\theta_t^h} dj \right]^{\frac{1}{1-\theta_t^h}}$	$H_t(j) = \left( \frac{W_t(j)}{W_t} \right)^{-\theta_t^h} H_t$

## B Variable definitions and sources

$P^y$  PPI: All manufacturing excl. duty (SA). Source: Office of National Statistics (ONS) [PVNQ]

$e$  Nominal effective exchange rate index (qtr ave). Source: Bank of England Monetary & Financial Statistics Interactive Database (BoE IADB) [XUQAGBG]

$P^c$  RPIX: Retail price index excl mortgage interest payments (linked back to 1975)<sup>24</sup>. Source: ONS [CHMK]

$P^x$  Deflator exports of manufactures SITC 5–8 (SA). Source: ONS [BPAN/BOXS]

$P^m$  Deflator imports of manufactures SITC 5–8 (SA). Source: ONS [BQBD/BPIS]

---

<sup>24</sup> As no official seasonally adjusted RPIX exists this series was seasonally adjusted using the X12 method as implemented in EVIEWS.

Table 2: Baseline calibration

Parameter	Value
Share of intermediate goods production of final goods $\phi_c$	0.42
Share of intermediate goods production of intermediate goods $\phi_y$	0.77
Elasticity of substitution varieties of domestic intermediate goods domestic market $\theta^y$	6
Elasticity of substitution varieties of domestic intermediate goods foreign market $\theta^x$	15
Elasticity of substitution varieties of domestic final goods $\theta^c$	6
Elasticity of substitution varieties of imported intermediate goods $\theta^m$	6
Elasticity of substitution differentiated labour services $\theta^h$	6
Share of domestic intermediate goods production of domestic goods $\alpha$	0.85
Share of domestic intermediate goods production of goods in foreign economy $\alpha_f$	0.064
Elasticity of substitution domestic and foreign goods domestic economy $\nu$	1.5
Elasticity of substitution between domestic and foreign goods foreign economy $\nu_f$	1.5
Habit persistence parameter $\zeta$	0.85
Inverse of Frisch elasticity of labour supply $\chi$	3
Weight on labour in utility function $\eta$	0.570
Discount factor $\beta$	1.03 <sup>0.25</sup>
Inflation target $\pi^c$	1.005
Units of labour required to distribute one unit of imported intermediate good $\delta$	0.4
Units of labour required to distribute one unit of imported intermediate good foreign economy $\delta_f$	0.4
Adjustment cost parameter domestic final goods prices $\phi_{c1}$	0
Adjustment cost parameter domestic final goods prices $\phi_{c2}$	400
Adjustment cost parameter domestic intermediate goods prices $\phi_{y1}$	0
Adjustment cost parameter domestic intermediate goods prices $\phi_{y2}$	400
Adjustment cost parameter export prices $\phi_{x1}$	0
Adjustment cost parameter export prices $\phi_{x2}$	400
Adjustment cost parameter import prices $\phi_{m1}$	0
Adjustment cost parameter import prices $\phi_{m2}$	400
Adjustment cost parameter wages $\phi_{w1}$	0
Adjustment cost parameter wages $\phi_{w2}$	400
Proportion of PCP firms domestic economy $\varpi$	0.6
Proportion of PCP firms foreign economy $\varpi_f$	0.4
Sensitivity of premium on foreign bond holdings w.r.t. net foreign assets $\psi$	0.02
Coefficient on lagged interest rates in interest rate rule $\rho_R$	0.65
Coefficient on inflation in interest rate rule $\rho_\pi$	1.8
AR coefficient in process for $\theta_t^y, \rho_{\theta^y}$	0.3
AR coefficient in process for $\theta_t^x, \rho_{\theta^x}$	0.75
AR coefficient in process for $\theta_t^c, \rho_{\theta^c}$	0.5
AR coefficient in process for $\theta_t^m, \rho_{\theta^m}$	0.5
AR coefficient in process for risk premium shock $\rho_u$	0.9
Standard deviation shock to $\theta_t^y, \varepsilon_{\theta^y,t}$	0.2
Standard deviation shock to $\theta_t^x, \varepsilon_{\theta^x,t}$	0.35
Standard deviation shock to $\theta_t^c, \varepsilon_{\theta^c,t}$	0.2
Standard deviation shock to $\theta_t^m, \varepsilon_{\theta^m,t}$	0.35
Standard deviation risk premium shock $\varepsilon_{u,t}$	0.005
Foreign inflation target $\pi_f$	1.005

Table 3: Second order moments: Model and UK data 1980Q1–2003Q4

Standard deviation	Data	Model
$\Delta \ln e_t$	0.031	0.035
$\Delta \ln P_t^m$	0.017	0.019
$\Delta \ln P_t^x$	0.013	0.018
$\Delta \ln P_t^y$	0.004	0.006
$\Delta \ln P_t^c$	0.003	0.006

First-order autocorrelation	Data	Model
$\Delta \ln e_t$	0.21	-0.07
$\Delta \ln P_t^m$	0.36	0.29
$\Delta \ln P_t^x$	0.29	0.30
$\Delta \ln P_t^y$	0.76	0.86
$\Delta \ln P_t^c$	0.79	0.81

Table 4: Distribution of chosen lag length for different lag order selection criteria. VAR in first differences. In per cent

$T = 100$					
	$L = 1$	$L = 2$	$L = 3$	$L = 4$	$L = 5$
<i>LR</i>	0.00	67.34	11.86	11.50	9.30
<i>AIC</i>	0.16	90.56	6.72	1.78	0.78
<i>HQ</i>	8.82	91.16	0.02	0.00	0.00
<i>SC</i>	71.52	28.48	0.00	0.00	0.00

$T = 200$					
	$L = 1$	$L = 2$	$L = 3$	$L = 4$	$L = 5$
<i>LR</i>	0.00	43.08	19.34	20.28	17.30
<i>AIC</i>	0.00	89.42	8.32	1.92	0.34
<i>HQ</i>	0.00	100.00	0.00	0.00	0.00
<i>SC</i>	2.44	97.56	0.00	0.00	0.00

Table 5: Distribution of chosen lag length for different lag-order selection criteria. VAR in relative prices and first differences of consumer prices. In per cent

$T = 100$						
	$L = 1$	$L = 2$	$L = 3$	$L = 4$	$L = 5$	$L = 6$
<i>LR</i>	0.0	27.1	49.9	7.3	7.1	8.5
<i>AIC</i>	0.0	47.4	46.2	3.7	1.4	1.2
<i>HQ</i>	0.0	95.3	4.7	0.0	0.0	0.0
<i>SC</i>	0.0	100.0	0.0	0.0	0.0	0.0

$T = 200$						
	$L = 1$	$L = 2$	$L = 3$	$L = 4$	$L = 5$	$L = 6$
<i>LR</i>	0.0	0.4	80.7	7.0	5.5	6.4
<i>AIC</i>	0.0	3.4	95.0	1.6	0.1	0.0
<i>HQ</i>	0.0	62.1	37.9	0.0	0.0	0.0
<i>SC</i>	0.0	99.7	0.3	0.0	0.0	0.0



Table 6: Absolute value of per cent difference between pointwise mean of estimated accumulated responses and DSGE model's responses over first ten quarters

$T = 100$					
	$\ln P_t^m$	$\ln P_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln e_t$
<i>LR</i>	5.6	8.8	25.6	24.1	30.6
<i>AIC</i>	6.0	10.7	29.7	24.8	34.6
<i>HQ</i>	6.6	11.2	30.4	25.1	36.8
<i>SC</i>	11.0	11.9	33.3	24.8	42.9
$L = 2$	6.2	11.5	30.3	25.2	36.2
$L = 4$	6.0	5.4	25.5	22.2	21.7

$T = 200$					
	$\ln P_t^m$	$\ln P_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln e_t$
<i>LR</i>	8.9	10.2	24.3	20.9	34.8
<i>AIC</i>	10.3	13.4	27.1	23.5	42.4
<i>HQ</i>	10.5	14.1	27.7	24.1	43.8
<i>SC</i>	10.7	14.0	27.7	24.1	43.9
$L = 2$	10.5	14.1	27.7	24.1	43.8
$L = 4$	8.0	8.4	22.0	18.7	29.4

Table 7: Absolute value of per cent difference between pointwise mean of estimated responses and DSGE model's responses over first twenty quarters

$T = 100$					
	$\ln P_t^m$	$\ln P_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln e_t$
<i>LR</i>	39.4	6.6	38.7	29.4	81.1
<i>AIC</i>	42.6	7.7	40.0	30.5	88.6
<i>HQ</i>	44.5	8.0	40.2	30.5	92.2
<i>SC</i>	54.3	9.1	38.6	26.7	100.6
$L = 2$	43.6	8.1	40.5	31.0	91.3
$L = 4$	33.0	5.1	35.4	26.9	63.8

$T = 200$					
	$\ln P_t^m$	$\ln P_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln e_t$
<i>LR</i>	45.9	7.3	34.7	25.4	88.9
<i>AIC</i>	51.0	9.5	38.1	28.5	101.4
<i>HQ</i>	51.7	9.9	38.7	29.1	103.6
<i>SC</i>	52.1	9.9	38.6	29.0	103.8
$L = 2$	51.7	9.9	38.7	29.1	103.6
$L = 4$	42.5	6.3	33.2	23.0	80.0

Table 8: Absolute value of per cent difference between pointwise mean of estimated accumulated responses from VEQM and DSGE model's responses over first ten quarters

$T = 100$					
	$\ln P_t^m$	$\ln P_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln e_t$
<i>LR</i>	21.7	26.3	27.7	22.0	25.0
<i>AIC</i>	19.2	24.2	25.2	19.3	22.7
<i>HQ</i>	15.9	22.4	22.4	16.2	19.9
<i>SC</i>	15.5	22.2	22.0	15.9	19.6
$L = 3$	20.7	24.9	26.2	20.6	23.8
$L = 5$	27.9	31.9	33.2	27.8	30.8

$T = 200$					
	$\ln P_t^m$	$\ln P_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln e_t$
<i>LR</i>	7.7	12.6	16.1	11.4	8.7
<i>AIC</i>	7.0	12.0	15.5	10.8	8.1
<i>HQ</i>	4.8	12.3	14.3	9.1	7.0
<i>SC</i>	3.0	12.7	13.4	7.8	6.1
$L = 3$	7.0	11.9	15.5	10.8	8.1
$L = 5$	9.8	14.5	17.6	13.3	10.8

Table 9: Absolute value of per cent difference between pointwise mean of estimated accumulated responses from VEQM and DSGE model's responses over first twenty quarters

$T = 100$					
	$\ln P_t^m$	$\ln P_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln e_t$
<i>LR</i>	20.0	25.6	27.3	23.8	20.2
<i>AIC</i>	17.7	22.8	24.5	21.0	17.8
<i>HQ</i>	15.3	18.9	20.9	17.4	14.9
<i>SC</i>	15.1	18.5	20.5	17.0	14.7
$L = 3$	18.4	24.6	25.9	22.5	19.3
$L = 5$	26.7	32.5	33.9	30.5	27.4

$T = 200$					
	$\ln P_t^m$	$\ln P_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln e_t$
<i>LR</i>	8.4	10.7	14.2	10.4	9.2
<i>AIC</i>	8.3	9.8	13.2	9.5	9.1
<i>HQ</i>	6.2	7.3	11.1	7.3	9.3
<i>SC</i>	4.9	7.5	9.6	5.7	9.7
$L = 3$	8.4	9.8	13.2	9.5	9.1
$L = 5$	8.5	13.5	17.0	13.3	9.3

Table 10: Distribution of chosen lag length for different lag-order selection criteria. 5% significance level in individual LR tests. Variables in (log) levels. In per cent

$T = 100$						
	$L = 1$	$L = 2$	$L = 3$	$L = 4$	$L = 5$	$L = 6$
<i>LR</i>	0.0	13.8	55.4	8.9	9.4	12.4
<i>AIC</i>	0.0	27.7	59.3	6.3	3.1	3.6
<i>HQ</i>	0.0	86.5	13.4	0.0	0.0	0.0
<i>SC</i>	0.0	99.9	0.1	0.0	0.0	0.0

$T = 200$						
	$L = 1$	$L = 2$	$L = 3$	$L = 4$	$L = 5$	$L = 6$
<i>LR</i>	0.0	0.0	78.1	8.2	6.4	7.2
<i>AIC</i>	0.0	0.1	97.6	2.2	0.1	0.0
<i>HQ</i>	0.0	14.2	85.8	0.0	0.0	0.0
<i>SC</i>	0.0	90.5	9.5	0.0	0.0	0.0

Table 11: Frequencies of chosen cointegration rank using Johansen's trace test for different lag-order selection criteria. Numbers in parentheses denote the preferred rank when using a small sample correction to the trace test

$T = 100$						
	$r = 0$	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$
<i>LR</i>	10.7 (51.7)	31.4 (28.6)	33.6 (11.8)	17.6 (6.1)	5.5 (1.5)	1.2 (0.4)
<i>AIC</i>	10.8 (42.2)	28.4 (26.1)	30.2 (16.9)	21.1 (11.3)	8.2 (3.2)	1.3 (0.5)
<i>HQ</i>	2.5 (8.1)	8.4 (14.5)	28.6 (36.5)	39.7 (30.9)	17.6 (8.7)	3.3 (1.3)
<i>SC</i>	0.0 (0.6)	3.8 (12.2)	29.0 (41.2)	43.8 (34.8)	19.8 (9.8)	3.6 (1.5)
$L = 3$	17.5 (61.2)	41.3 (29.7)	28.0 (7.2)	10.0 (1.6)	2.7 (0.3)	0.5 (0.0)
$L = 5$	13.7 (84.4)	40.1 (13.9)	32.2 (1.7)	10.5 (0.2)	2.5 (0.0)	0.5 (0.0)

$T = 200$						
	$r = 0$	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$
<i>LR</i>	0.3 (2.1)	3.9 (11.0)	19.0 (29.5)	43.5 (38.4)	28.1 (16.3)	5.2 (2.6)
<i>AIC</i>	0.1 (0.5)	2.1 (6.9)	16.2 (28.7)	45.5 (41.8)	30.5 (19.1)	5.7 (3.0)
<i>HQ</i>	0.0 (0.5)	1.9 (6.3)	14.4 (25.4)	42.4 (39.4)	33.6 (23.4)	7.6 (5.0)
<i>SC</i>	0.0 (0.1)	0.3 (1.0)	2.4 (4.2)	25.6 (31.0)	53.2 (48.7)	18.4 (15.1)
$L = 3$	0.1 (0.5)	2.0 (6.8)	16.1 (28.5)	45.5 (42.0)	30.6 (19.2)	5.7 (3.0)
$L = 5$	1.5 (12.3)	11.9 (32.9)	32.0 (32.7)	37.2 (16.9)	14.7 (4.3)	2.6 (0.1)

Table 12: Rejection frequencies for LR tests of restrictions on cointegration space conditional on  $r=4$  for different lag-order selection criteria. 5% significance level. Numbers in parentheses are rejection frequencies based on only the datasets for which the correct cointegration rank is chosen.

$T = 100$						
	$\ln(P_t^m/P_t^c) \sim I(0)$	$\ln(P_t^x/P_t^c) \sim I(0)$	$\ln(P_t^y/P_t^c) \sim I(0)$	$\ln(e_t/P_t^c) + 0.005t \sim I(0)$	Joint	
LR	27.9	24.8	24.4	26.5	81.6 (94.6)	
AIC	29.1	24.2	23.9	27.3	79.2 (91.7)	
HQ	38.8	29.4	28.4	35.8	83.3 (92.1)	
SC	41.2	30.3	29.6	38.1	84.2 (91.0)	
$L = 3$	22.4	20.2	19.7	21.1	73.4 (81.5)	
$L = 5$	26.8	25.9	25.7	26.3	88.1 (94.4)	

$T = 200$						
	$\ln(P_t^m/P_t^c) \sim I(0)$	$\ln(P_t^x/P_t^c) \sim I(0)$	$\ln(P_t^y/P_t^c) \sim I(0)$	$\ln(e_t/P_t^c) + 0.005t \sim I(0)$	Joint	
LR	19.4	13.5	15.4	15.4	40.6 (45.7)	
AIC	18.2	13.4	14.7	14.5	37.2 (43.2)	
HQ	20.0	15.1	16.4	16.6	40.3 (49.0)	
SC	29.3	25.5	25.0	26.0	60.5 (64.5)	
$L = 3$	18.2	13.2	14.5	14.3	36.7 (42.8)	
$L = 5$	22.0	14.3	16.7	17.9	48.0 (57.9)	

Table 13: Rejection frequencies for single-equation and vector tests for non-normality for different lag-order selection criteria. 5% significance level

$T = 100$						
	$\ln P_t^m$	$\ln P_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln e_t$	Vector test
LR	5.0	5.1	4.8	4.8	5.0	5.2
AIC	5.2	5.0	4.9	4.8	5.2	5.2
HQ	4.7	4.9	5.5	4.8	4.9	5.4
SC	4.8	5.0	5.1	4.8	5.1	5.4
$L = 3$	5.1	4.9	4.9	4.6	4.6	5.0
$L = 5$	5.4	4.8	5.1	5.5	5.4	5.2

$T = 200$						
	$\ln P_t^m$	$\ln P_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln e_t$	Vector test
LR	5.2	4.9	5.3	4.9	5.3	5.6
AIC	5.0	4.9	5.4	5.1	5.2	5.5
HQ	5.4	4.9	5.5	5.0	5.3	5.5
SC	5.0	5.1	4.9	5.4	5.3	5.7
$L = 3$	5.0	5.0	5.5	5.2	5.2	5.5
$L = 5$	5.1	4.9	5.1	5.3	5.4	6.0

Table 14: Rejection frequencies for single-equation and vector tests for residual autocorrelation. 5% significance level

$T = 100$						
	$\ln P_t^m$	$\ln P_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln e_t$	Vector test
<i>LR</i>	10.2	9.5	13.0	11.6	9.4	19.5
<i>AIC</i>	9.5	9.0	16.6	11.7	8.9	22.5
<i>HQ</i>	10.5	14.5	46.5	21.0	8.8	50.0
<i>SC</i>	11.5	16.9	55.5	24.1	9.1	59.1
$L = 3$	11.0	9.6	10.5	11.4	10.3	23.2
$L = 5$	12.8	12.0	11.8	13.7	12.4	32.6

$T = 200$						
	$\ln P_t^m$	$\ln P_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln e_t$	Vector test
<i>LR</i>	5.9	4.8	7.1	6.2	5.5	5.5
<i>AIC</i>	7.1	5.6	8.4	7.1	6.6	9.7
<i>HQ</i>	7.4	8.3	20.5	12.3	6.7	22.1
<i>SC</i>	11.1	33.7	89.5	53.2	7.6	89.6
$L = 3$	7.4	5.8	8.6	7.3	6.8	10.7
$L = 5$	6.1	5.3	7.1	6.9	6.1	8.6

Table 15: Distribution of chosen lag length for different lag order selection criteria. Data generated from VAR(5) [VAR(3)]. 5% significance level in individual LR tests. Variables in (log) levels. In per cent

$T = 100$						
	$L = 1$	$L = 2$	$L = 3$	$L = 4$	$L = 5$	$L = 6$
<i>LR</i>	0.0 [0.0]	14.1 [36.3]	54.4 [32.8]	9.0 [8.6]	10.7 [10.0]	11.8 [12.3]
<i>AIC</i>	0.0 [0.0]	27.5 [61.8]	59.5 [29.5]	6.6 [3.9]	3.0 [2.0]	3.3 [2.8]
<i>HQ</i>	0.0 [0.0]	87.8 [98.4]	12.2 [1.5]	0.0 [0.0]	0.0 [0.0]	0.0 [0.0]
<i>SC</i>	0.0 [0.0]	99.9 [100.0]	0.1 [0.0]	0.0 [0.0]	0.0 [0.0]	0.0 [0.0]

$T = 200$						
	$L = 1$	$L = 2$	$L = 3$	$L = 4$	$L = 5$	$L = 6$
<i>LR</i>	0.0 [0.0]	0.0 [6.9]	79.2 [75.3]	7.1 [5.8]	7.0 [5.3]	6.7 [6.7]
<i>AIC</i>	0.0 [0.0]	0.1 [24.5]	97.7 [74.3]	2.1 [1.1]	0.1 [0.0]	0.0 [0.0]
<i>HQ</i>	0.0 [0.0]	16.1 [93.0]	83.9 [7.0]	0.0 [0.0]	0.0 [0.0]	0.0 [0.0]
<i>SC</i>	0.0 [0.0]	92.0 [100.0]	8.0 [0.0]	0.0 [0.0]	0.0 [0.0]	0.0 [0.0]

Table 16: Frequencies of chosen cointegration rank using Johansen's trace test for different lag-order selection criteria. 5% significance level. Data generated from VAR(5) [VAR(3)].

$T = 100$						
	$r = 0$	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$
<i>LR</i>	11.5 [0.6]	33.2 [14.5]	31.6 [39.7]	17.5 [33.6]	5.3 [9.9]	0.8 [1.6]
<i>AIC</i>	11.5 [0.4]	30.6 [10.7]	28.6 [38.0]	20.5 [36.1]	7.6 [12.8]	1.2 [2.0]
<i>HQ</i>	2.1 [0.3]	8.7 [6.8]	29.0 [35.0]	39.8 [40.2]	18.1 [15.4]	2.5 [2.3]
<i>SC</i>	0.0 [0.3]	3.5 [6.7]	29.6 [34.9]	44.0 [40.3]	19.9 [15.5]	2.9 [2.3]
$L = 3$	18.9 [2.5]	42.1 [23.1]	27.2 [41.9]	9.5 [25.7]	1.9 [5.9]	0.3 [0.1]
$L = 5$	14.6 [5.2]	41.6 [30.1]	31.8 [40.3]	9.7 [19.4]	2.0 [4.4]	0.2 [0.6]

$T = 200$						
	$r = 0$	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$
<i>LR</i>	0.2 [0.0]	5.1 [0.3]	25.1 [6.8]	44.8 [52.3]	21.8 [37.1]	3.0 [3.5]
<i>AIC</i>	0.0 [0.0]	3.1 [0.0]	22.6 [3.3]	46.6 [48.5]	24.5 [43.5]	3.2 [4.7]
<i>HQ</i>	0.0 [0.0]	2.9 [0.0]	20.0 [1.4]	42.8 [37.2]	29.9 [54.5]	4.3 [6.9]
<i>SC</i>	0.0 [0.0]	0.3 [0.0]	2.5 [0.3]	27.3 [27.6]	57.5 [62.8]	12.4 [9.2]
$L = 3$	0.0 [0.0]	3.1 [0.0]	22.5 [4.1]	46.7 [54.0]	24.5 [38.6]	3.1 [3.3]
$L = 5$	1.5 [0.0]	16.2 [1.7]	37.1 [22.6]	33.2 [52.0]	10.2 [21.5]	1.7 [2.3]

Table 17: Rejection frequencies for single-equation and vector tests for residual autocorrelation for different lag-order selection criteria. Data generated by VAR(5) [VAR(3)]. 5% significance level

$T = 100$						
	$\ln P_t^m$	$\ln P_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln e_t$	Vector test
<i>LR</i>	10.1 [9.2]	9.8 [9.5]	13.1 [25.6]	11.6 [12.3]	9.7 [9.7]	18.5 [18.6]
<i>AIC</i>	9.3 [8.1]	8.9 [8.3]	16.5 [39.0]	12.5 [12.8]	8.6 [8.2]	21.7 [22.8]
<i>HQ</i>	9.7 [9.1]	14.4 [9.1]	48.2 [66.2]	22.3 [14.5]	7.8 [8.5]	50.2 [38.3]
<i>SC</i>	10.2 [9.1]	16.6 [9.1]	56.9 [67.6]	25.5 [14.4]	8.1 [8.4]	58.4 [39.1]
$L = 3$	10.5 [11.0]	8.7 [9.5]	9.8 [10.0]	11.3 [13.3]	10.1 [10.9]	23.3 [23.6]
$L = 5$	12.3 [13.0]	12.1 [13.2]	12.4 [13.2]	12.7 [14.4]	12.0 [12.5]	33.0 [32.8]

$T = 200$						
	$\ln P_t^m$	$\ln P_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln e_t$	Vector test
<i>LR</i>	5.4 [4.9]	4.7 [4.8]	6.7 [11.0]	6.3 [6.2]	5.3 [5.1]	5.1 [4.6]
<i>AIC</i>	5.9 [5.2]	4.8 [4.9]	8.5 [27.7]	7.1 [6.6]	5.9 [5.6]	8.8 [11.7]
<i>HQ</i>	6.3 [5.8]	8.5 [5.4]	22.6 [92.2]	13.2 [7.9]	5.8 [7.1]	22.1 [51.2]
<i>SC</i>	10.9 [5.6]	36.3 [5.6]	90.9 [98.9]	55.8 [8.3]	6.8 [7.2]	90.2 [58.2]
$L = 3$	6.1 [5.7]	5.0 [5.7]	8.7 [6.1]	7.2 [6.8]	6.1 [5.9]	9.7 [7.9]
$L = 5$	5.5 [6.0]	5.7 [5.6]	5.1 [5.4]	6.2 [6.6]	6.0 [6.1]	7.3 [7.4]

Table 18: Rejection frequencies for single-equation and vector tests for non-normality for different lag-order selection criteria. Data generated from VAR(5) [VAR(3)]. 5% significance level

$T = 100$						
	$\ln P_t^m$	$\ln P_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln e_t$	Vector test
<i>LR</i>	4.9 [4.9]	5.0 [4.7]	5.0 [4.5]	5.0 [5.0]	4.9 [4.7]	5.4 [5.4]
<i>AIC</i>	5.1 [4.9]	5.0 [4.5]	4.9 [4.9]	4.7 [4.9]	5.0 [4.6]	5.2 [5.3]
<i>HQ</i>	5.0 [5.0]	4.9 [4.7]	5.1 [5.1]	5.0 [4.9]	4.8 [4.7]	5.2 [5.2]
<i>SC</i>	5.3 [5.1]	5.0 [4.8]	5.1 [5.0]	5.1 [4.9]	5.0 [4.8]	5.2 [5.3]
$L = 3$	5.0 [5.0]	4.9 [4.9]	4.7 [4.6]	4.9 [4.8]	4.9 [4.8]	5.3 [5.8]
$L = 5$	4.8 [4.7]	4.5 [4.5]	4.5 [4.6]	5.3 [5.1]	5.1 [5.2]	5.2 [5.4]

$T = 200$						
	$\ln P_t^m$	$\ln P_t^x$	$\ln P_t^c$	$\ln P_t^y$	$\ln e_t$	Vector test
<i>LR</i>	5.4 [5.5]	4.9 [4.8]	5.1 [4.5]	5.0 [4.9]	5.3 [5.0]	5.6 [5.2]
<i>AIC</i>	5.5 [5.5]	5.0 [4.9]	5.0 [4.6]	5.1 [4.8]	5.1 [4.9]	5.3 [5.2]
<i>HQ</i>	5.6 [5.5]	5.0 [5.1]	4.8 [5.3]	5.0 [5.4]	5.2 [5.1]	5.4 [5.7]
<i>SC</i>	5.5 [5.6]	4.5 [5.2]	5.4 [5.3]	5.1 [5.4]	5.3 [5.2]	5.8 [5.7]
$L = 3$	5.5 [5.4]	5.0 [4.9]	5.0 [4.7]	5.0 [4.9]	5.3 [4.9]	5.4 [5.4]
$L = 5$	5.2 [5.3]	5.2 [4.9]	5.1 [5.1]	5.0 [5.1]	5.4 [5.5]	5.8 [5.6]

Table 19: Rejection frequencies for LR tests of restrictions on cointegration space conditional on  $r=4$  for different lag-order selection criteria. 5% significance level. Data generated by VAR(5) [VAR(3)]

$T = 100$					
	$\ln(P_t^m/P_t^c) \sim I(0)$	$\ln(P_t^x/P_t^c) \sim I(0)$	$\ln(P_t^y/P_t^c) \sim I(0)$	$\ln(e_t/P_t^c) + 0.005t \sim I(0)$	Joint
<i>LR</i>	29.0 [34.4]	23.3 [22.3]	23.2 [25.3]	26.9 [35.5]	81.3 [74.6]
<i>AIC</i>	29.7 [33.7]	23.0 [20.5]	22.4 [23.8]	27.4 [8.2]	79.1 [69.7]
<i>HQ</i>	38.9 [33.8]	28.0 [18.9]	27.0 [23.5]	35.9 [35.5]	83.2 [67.1]
<i>SC</i>	17.1 [33.9]	30.0 [18.9]	8.4 [23.4]	26.5 [35.6]	58.4 [66.8]
$L = 3$	23.1 [29.3]	18.7 [18.3]	18.7 [20.1]	21.9 [29.9]	73.7 [65.9]
$L = 5$	28.1 [34.9]	24.9 [25.2]	25.1 [26.1]	27.6 [35.9]	87.8 [85.1]

$T = 200$					
	$\ln(P_t^m/P_t^c) \sim I(0)$	$\ln(P_t^x/P_t^c) \sim I(0)$	$\ln(P_t^y/P_t^c) \sim I(0)$	$\ln(e_t/P_t^c) + 0.005t \sim I(0)$	Joint
<i>LR</i>	14.5 [16.4]	12.2 [11.8]	12.6 [12.6]	14.2 [16.8]	35.1 [28.6]
<i>AIC</i>	14.3 [18.0]	11.7 [12.6]	12.3 [14.2]	13.8 [18.0]	31.8 [28.1]
<i>HQ</i>	17.5 [24.0]	14.0 [16.2]	14.6 [20.9]	16.4 [24.3]	36.1 [34.0]
<i>SC</i>	31.4 [24.5]	25.8 [16.5]	26.4 [21.8]	27.7 [24.9]	60.7 [34.6]
$L = 3$	14.4 [15.3]	11.5 [10.9]	12.2 [11.4]	13.7 [15.3]	31.4 [25.2]
$L = 5$	15.4 [17.5]	12.4 [13.0]	13.3 [13.3]	15.0 [18.1]	40.9 [35.3]

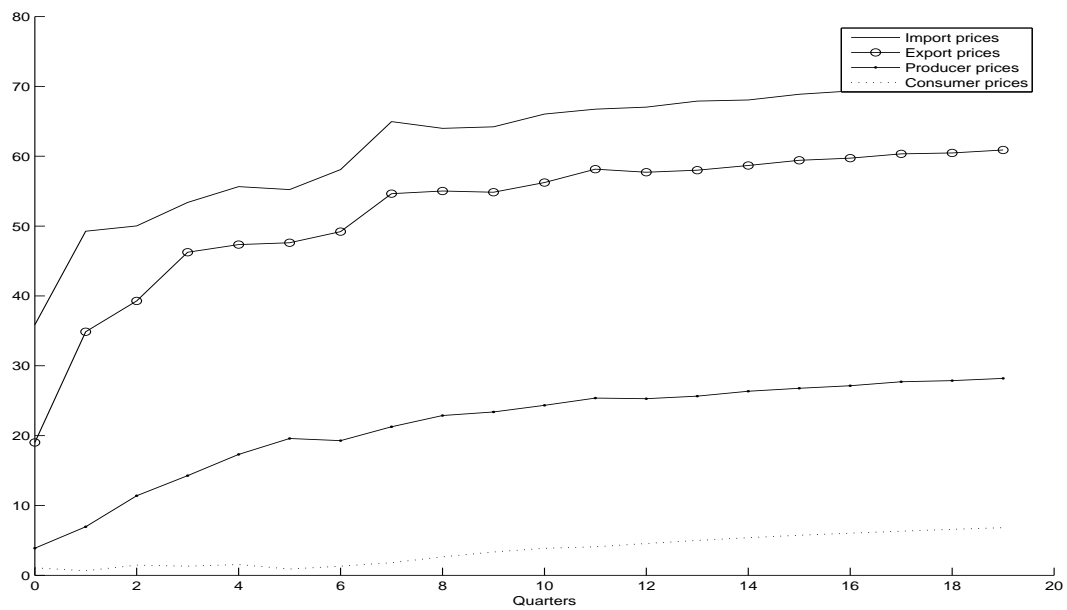


Figure 1: Normalised impulse responses to exchange rate shock. UK data. In per cent

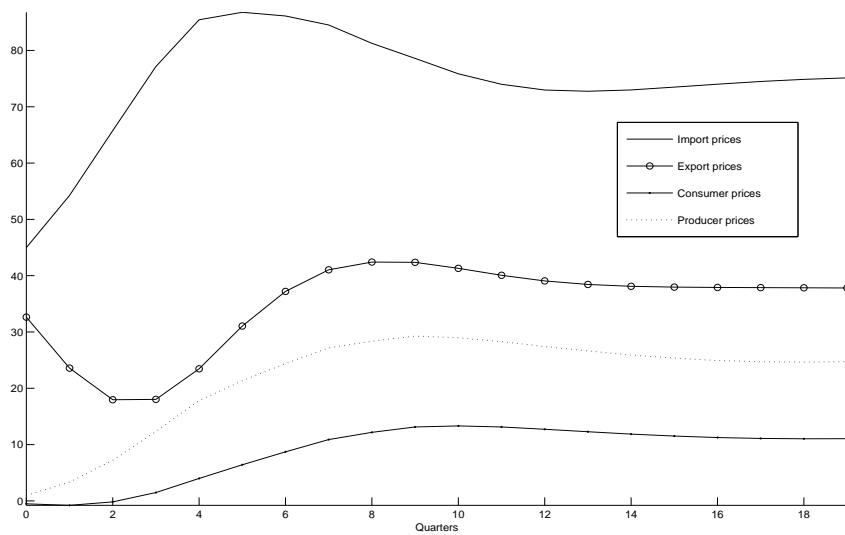


Figure 2: Normalised responses to exchange rate shock. Mean of 5000 datasets from DSGE model using recursive identification scheme.  $T = 100$ . In per cent



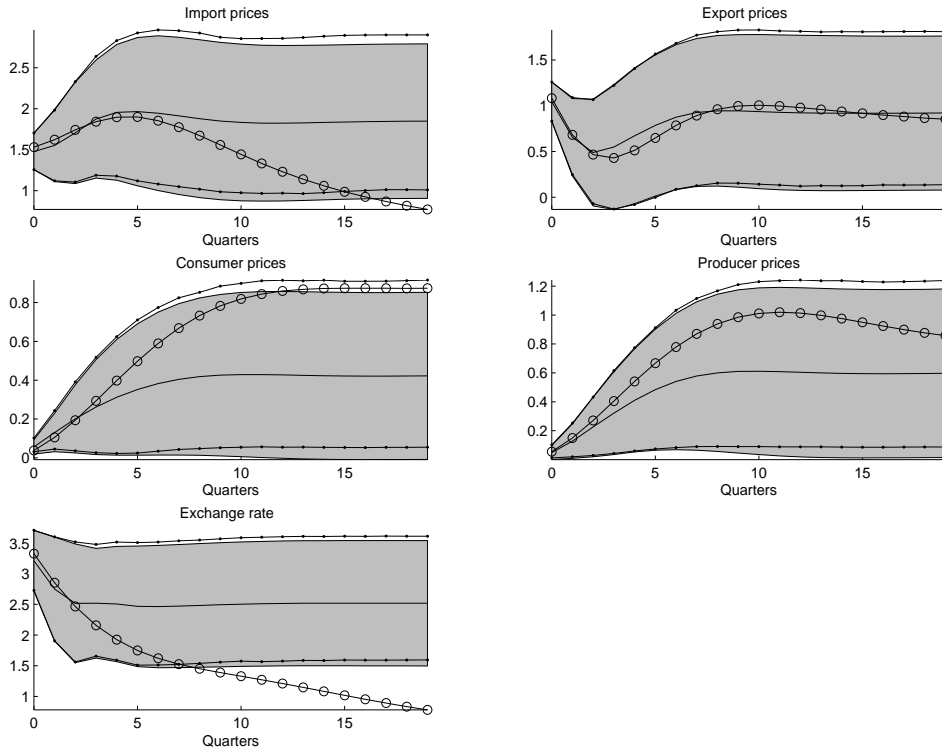


Figure 3: Responses to a one standard deviation UIP shock. In per cent.  $T = 100, L = 2$ .

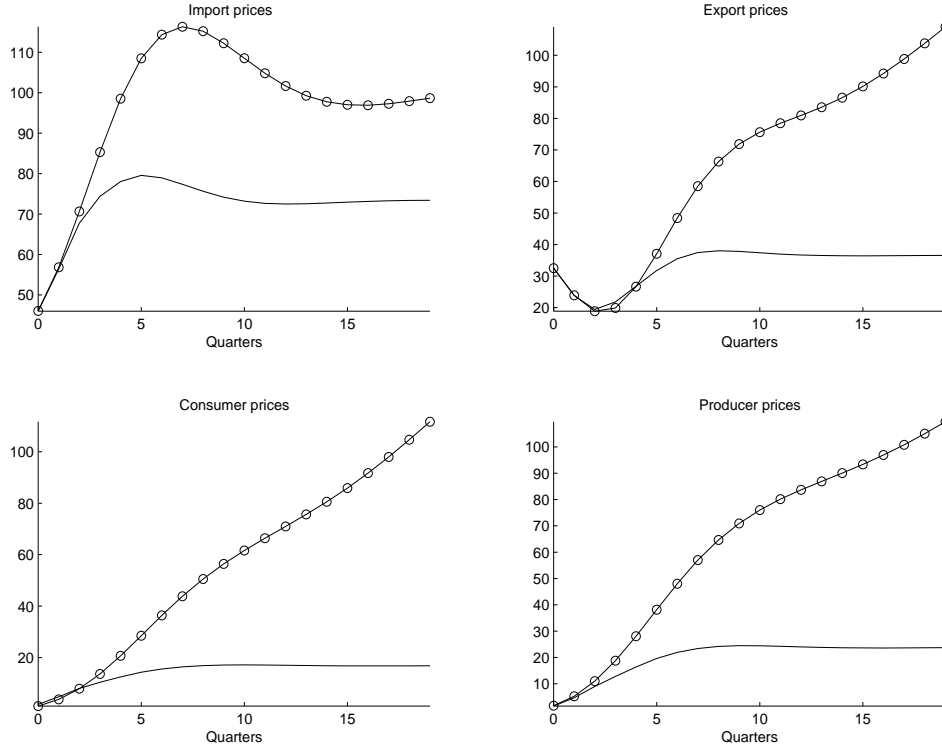


Figure 4: Responses to UIP shock normalised on exchange rate response. In per cent.  $T = 100, L = 2$ .

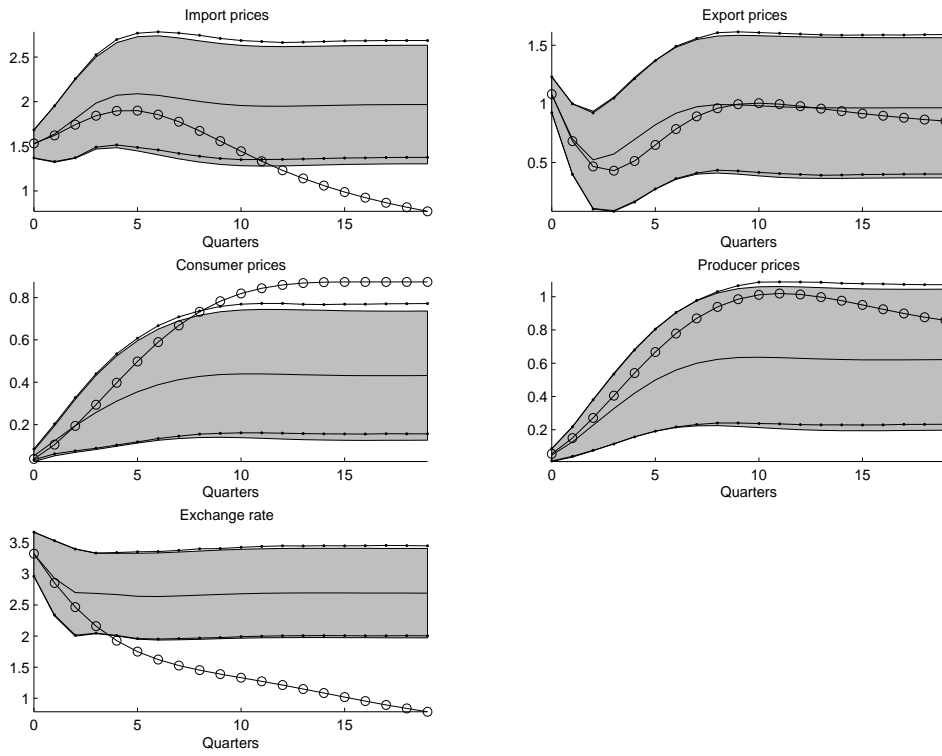


Figure 5: Accumulated responses to one standard deviation UIP shock. In per cent.  $T = 200, L = 2$ .

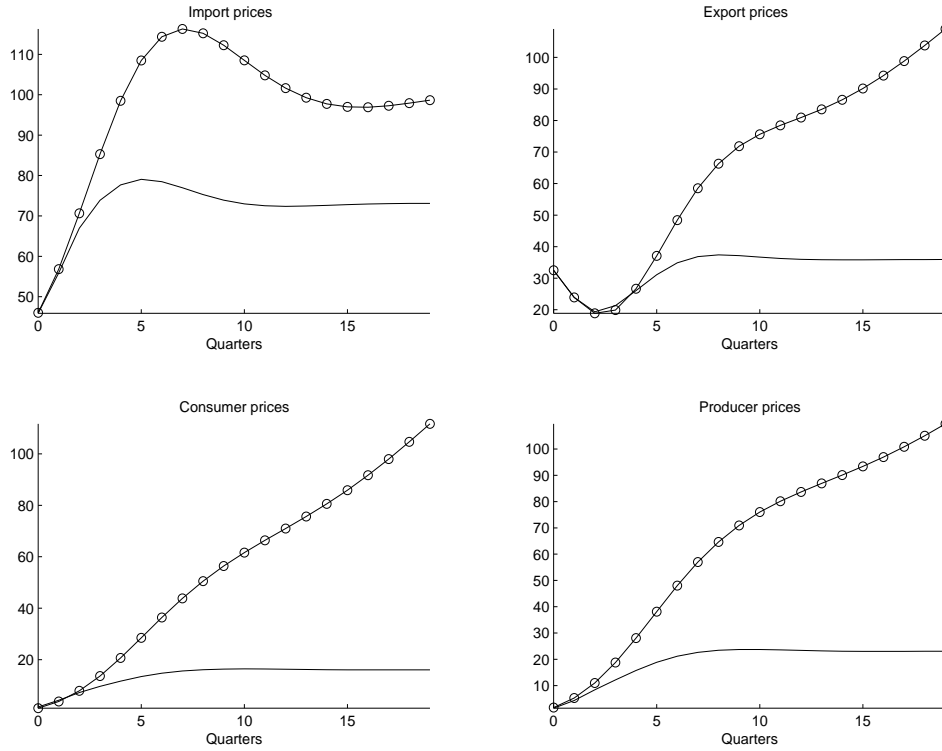


Figure 6: Responses to UIP shock normalised on exchange rate response. In per cent.  $T = 200, L = 2$ .

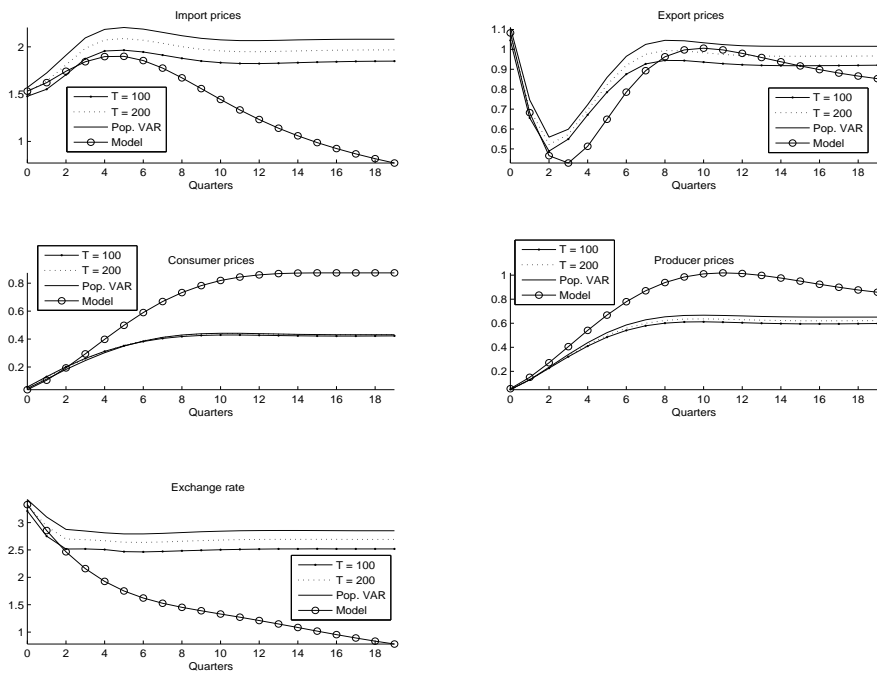


Figure 7: Accumulated responses to one standard deviation UIP shock. In per cent.  $L = 2$ .

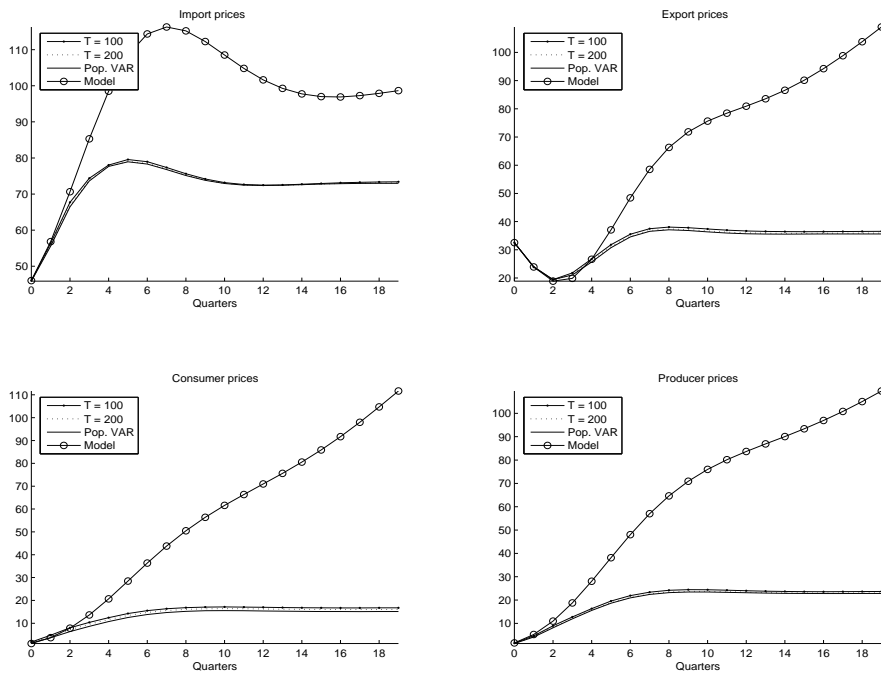


Figure 8: Normalised responses to one standard deviation UIP shock. In per cent.  $L = 2$ .

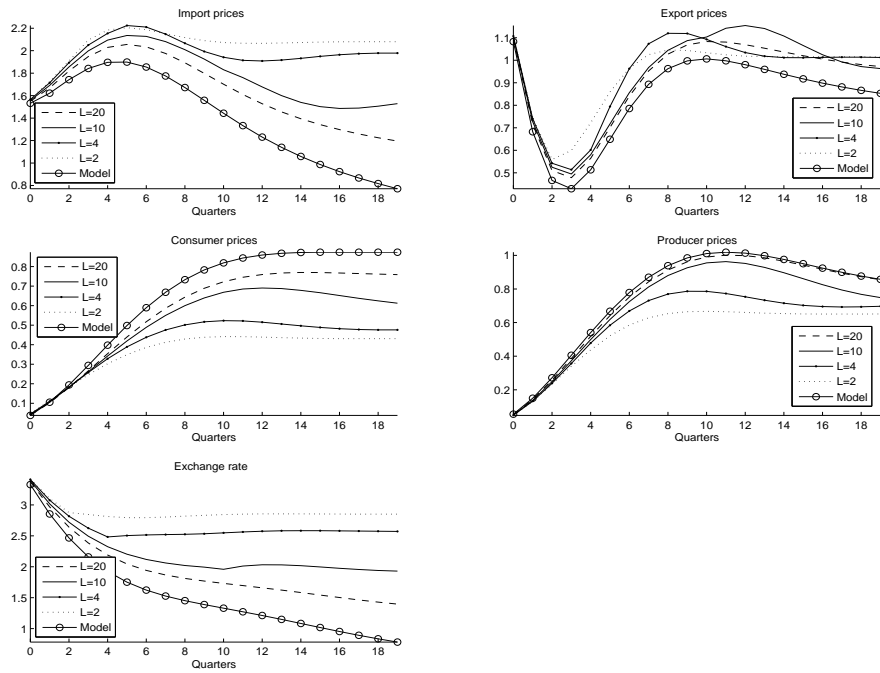


Figure 9: Accumulated responses to one standard deviation UIP shock in population version of VAR for different lag orders. In per cent.

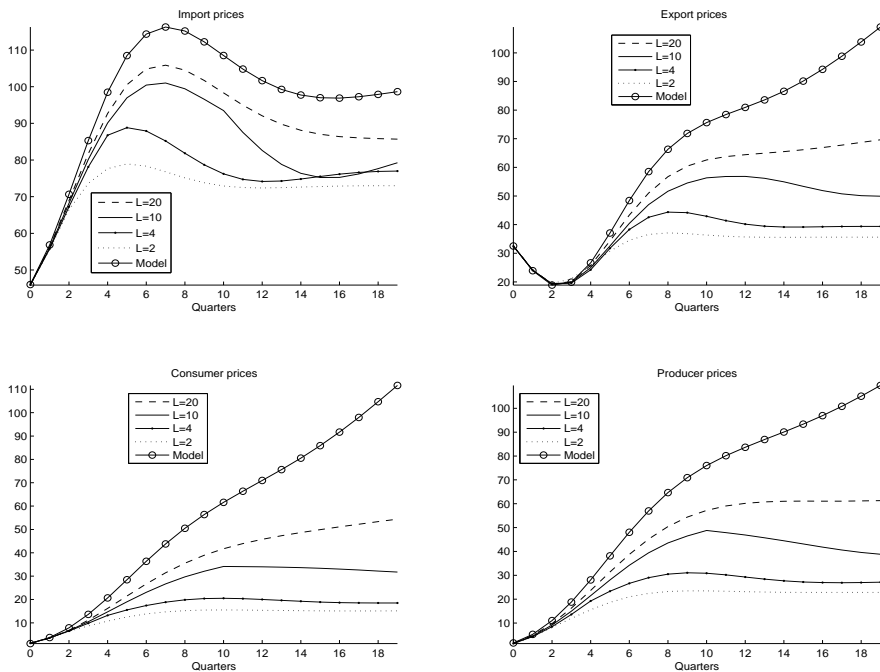


Figure 10: Normalised impulse responses to one standard deviation UIP shock in population version of VAR for different lag orders. In per cent.

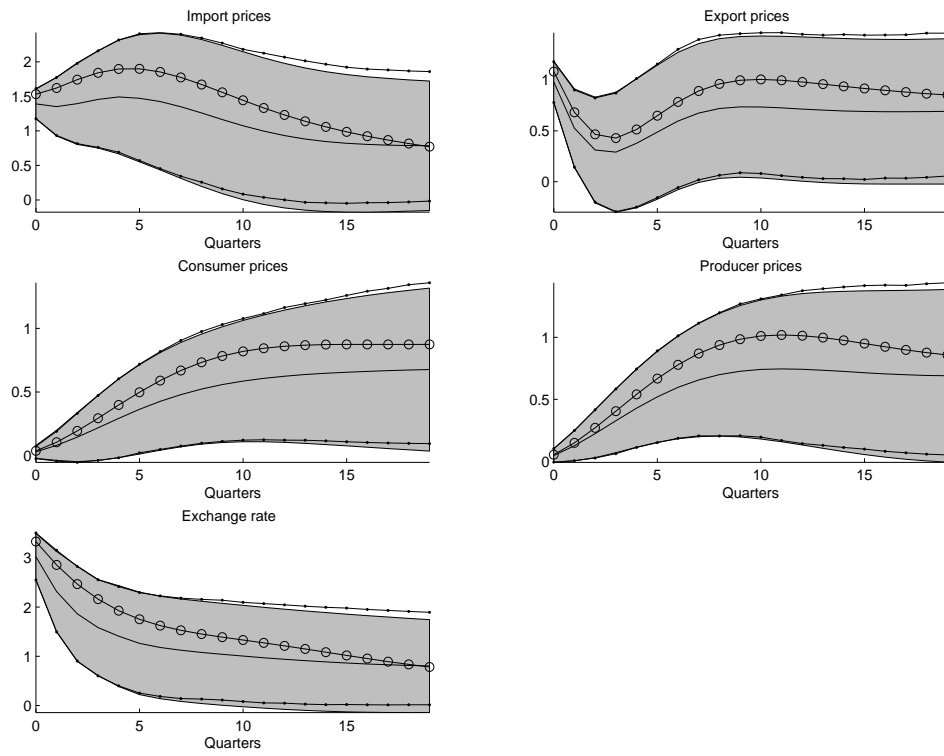


Figure 11: Accumulated responses to one standard deviation UIP shock. In per cent. VEqCM.  $T = 100, L = 3$ .

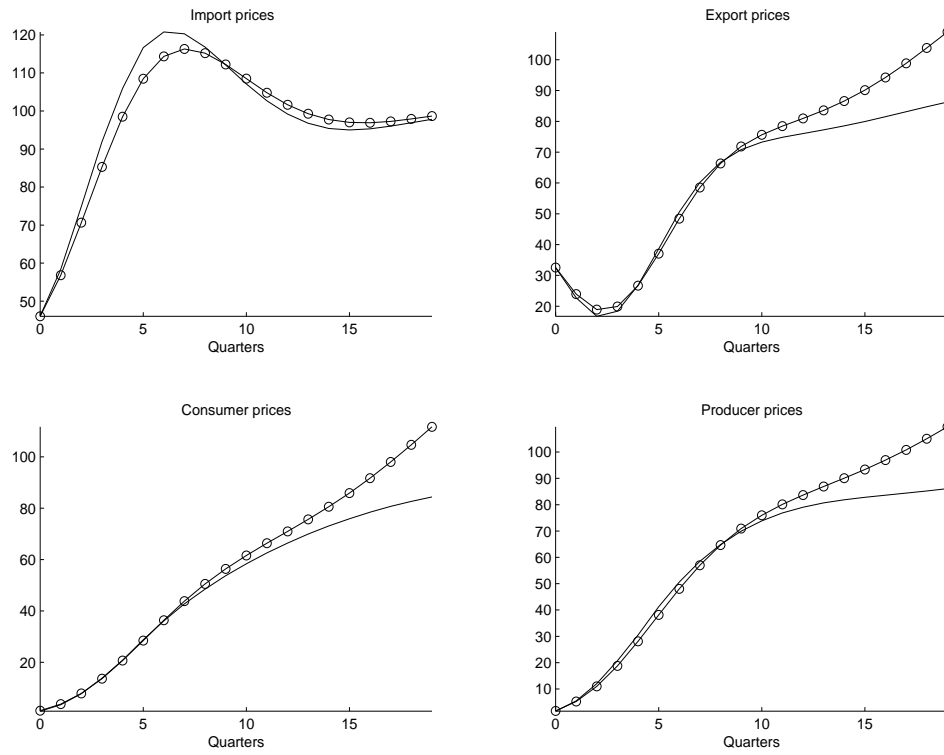


Figure 12: Normalised responses to one standard deviation UIP shock. In per cent. VEqCM.  $T = 100, L = 3$ .

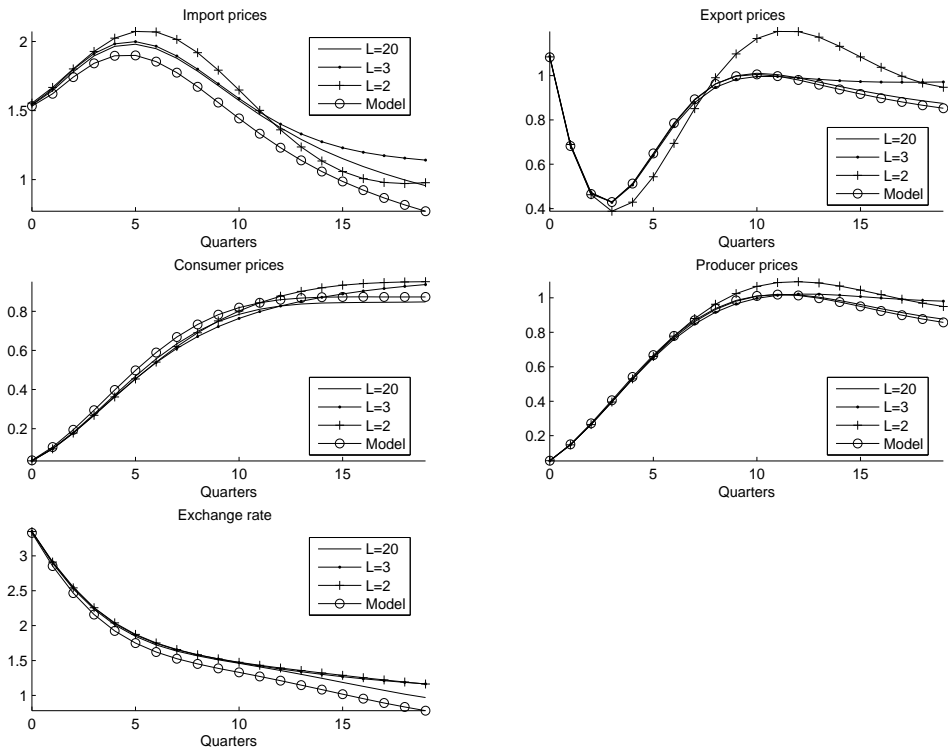


Figure 13: Accumulated responses to one standard deviation UIP shock from population version of VEqCM for different lag orders. In per cent.

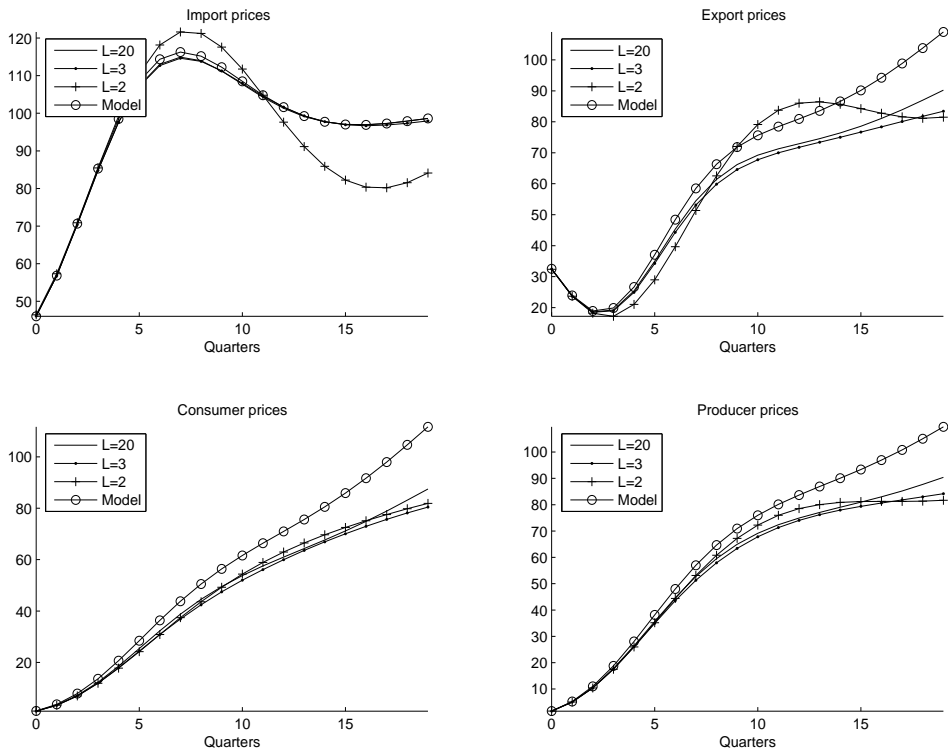


Figure 14: Normalised responses to one standard deviation UIP shock from population version of VEqCM for different lag orders. In per cent.

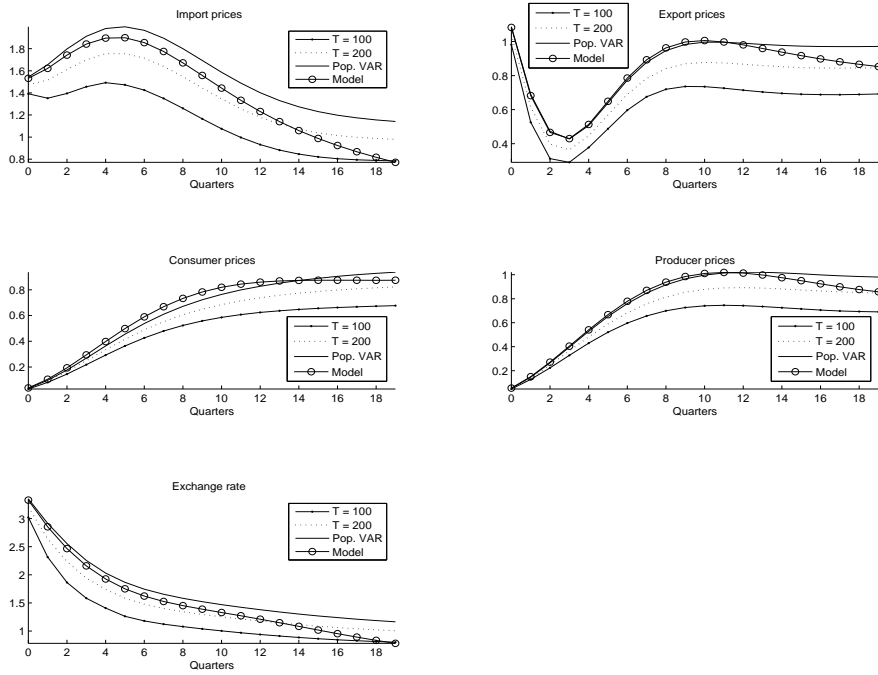


Figure 15: Accumulated responses to one standard deviation UIP shock from population version  $VEqCM(3)$  and mean responses from  $VEqCM(3)$  for  $T = 100$  and  $T = 200$ . In per cent.

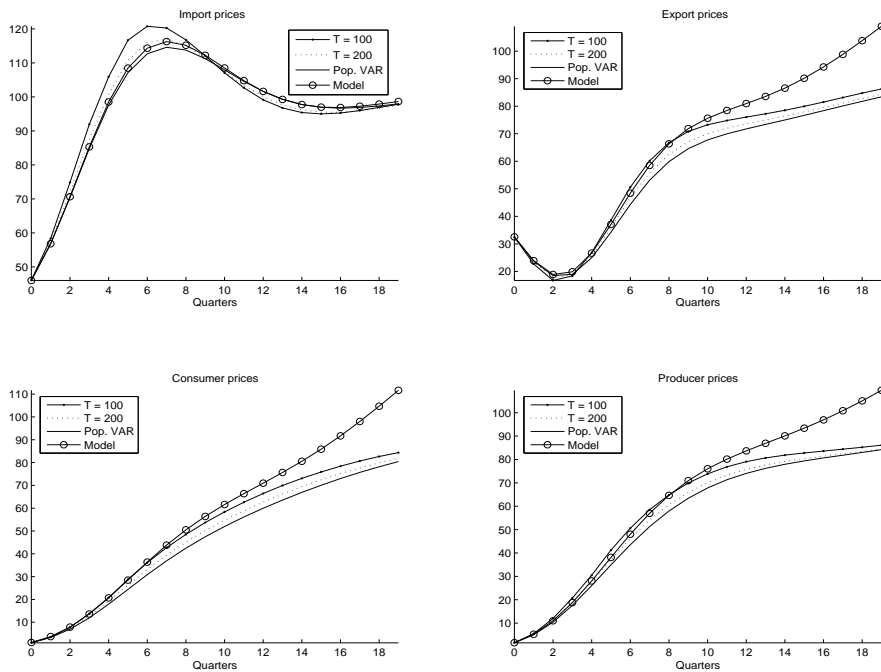


Figure 16: Normalised responses to one standard deviation UIP shock from population version  $VEqCM(3)$  and mean responses from  $VEqCM(3)$  for  $T = 100$  and  $T = 200$ . In per cent.

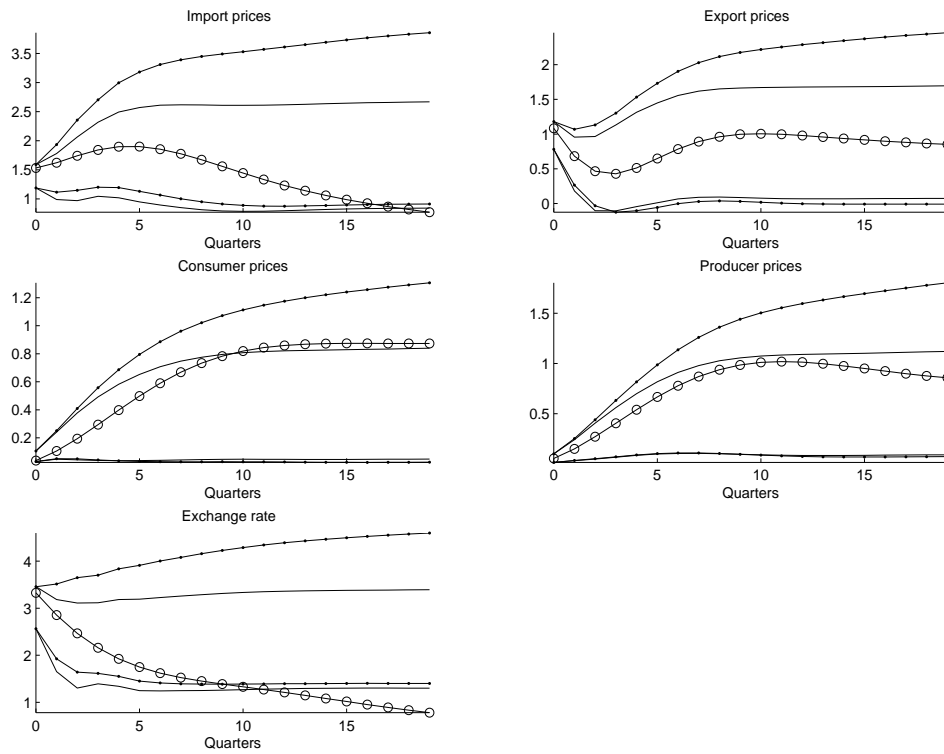


Figure 17: Accumulated responses to UIP shock from DSGE model (circled line) and mean of 95% standard bootstrap interval (solid lines) and 95% bias-corrected bootstrap interval (pointed lines). VAR in first differences.

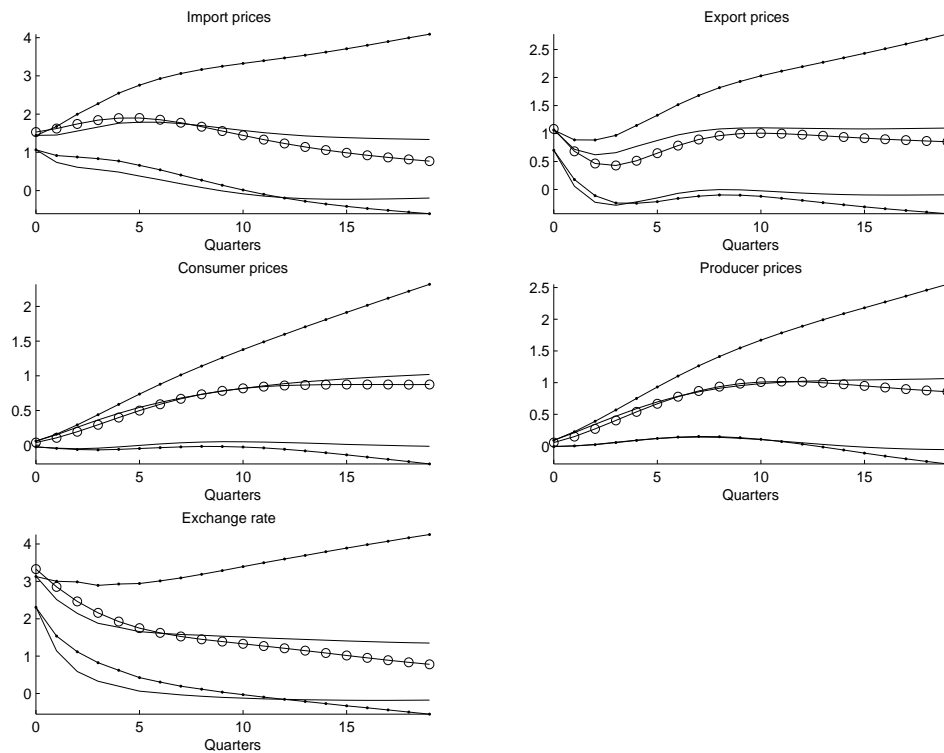


Figure 18: Accumulated responses to UIP shock from DSGE model (circled line) and mean of 95% standard bootstrap interval (solid lines) and 95% bias-corrected bootstrap interval (pointed lines). VEqCM