## Mechanism Design and Payments<sup>\*</sup>

Thorsten Koeppl Queen's University Ontario, Canada thor@qed.econ.queensu.ca Cyril Monnet European Central Bank Frankfurt, Germany cyril.monnet@ecb.int

Ted Temzelides University of Pittsburgh Pittsburgh, PA 15260 tedt@pitt.edu

#### PRELIMINARY AND INCOMPLETE

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#### Abstract

We use *mechanism design* to study the framework through which transactions among financial institutions take place. We find that the existence of an equilibrium in which banks transact with each other through a payment system requires certain caps on banks' short-term borrowing. Networks that have knowledge of the member banks' histories support efficient transactions among their members. If banks transact frequently outside their network, incentives constraints imply that inter-network transactions can take place only at the cost of fewer intra-network transactions. Whether the efficient arrangement involves a centralized payment system or several local ones depends on the relative frequency of transactions between different networks. Banks are penalized for borrowing within the network and are rewarded for lending outside the network.

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## 1 Introduction

The Walrasian competitive equilibrium model is one of the cornerstones of economic theory. It has also proved useful as a framework within which questions about actual economies are formalized and studied. One of the features of the economy that the Walrasian model abstracts from is the mechanism through which payments for goods and services take place. While for the study of certain questions this abstraction is indeed one of the main strengths of the Walrasian model, this feature also makes it an inappropriate tool for the study of questions related to transactions. Thus, new models are needed to study payments, and our goal is to develop such models using mechanism design theory.<sup>1</sup>

Some open questions in payment systems that we want to shed some light on include:

- Should access to a centralized payment system (say TARGET, or FED-WIRE) be directly available to everyone (say any bank, anywhere in Europe or in the US, respectively) or should it be restricted to a small network of intermediaries, with the other banks transferring payments through such "correspondent" banks?

- Should there be binding limits or "caps" imposed by the payment system on the short-term borrowing by banks? How should these caps be determined? In particular, should they be fixed, or should they depend on the individual banks' histories of transactions?

- What are the effects of a bank's reputation through its repeated interaction with the payment system? Can reputation be used together with other instruments, such as interest rate penalties, in order to induce socially efficient behavior by banks? This question is of particular policy relevance since TARGET and FEDWIRE use different ways to penalize banks for excessive short term borrowing. One would like to know what the respective merits of these policies are.

- What is the role of private information and imperfect monitoring in answering the above questions? Can local bank networks or "correspondent banks" lead to improved allocations through their improved information on

<sup>&</sup>lt;sup>1</sup>To get some idea of the magnitudes involved in payment systems, the value of the transactions processed through TARGET, the main public payment system in Europe, during March 2004 was over 40 Euro billions, with a daily average of about 1.7 Euro billions. In the United States, the average daily value of transfers through FEDWIRE, the US equivalent of TARGET, during the first quarter of 2004 was 1,683,265 \$ millions.

#### local banks?

Our approach is motivated by the work of Kocherlakota (1998) and Kocherlakota and Wallace (1998), who use results from mechanism design to understand *monetary exchange*. More recently, and more related to our modelling strategy, extending the model of Mirrlees (1971), Kocherlakota and others have used dynamic mechanism design to study *optimal taxation* under private information.<sup>2</sup> We wish to emphasize the following methodological innovation in our approach. Some of the questions that optimal payment system design poses are inherently *dynamic* and, therefore, very hard or impossible to study within the existing payment systems literature, which is almost exclusively static.<sup>3</sup>

In order to concentrate on the role of banks as players within a payment system, we abstract from modeling explicitly the traditional role of banks as deposit/loan contract providers. We model banks as abstract economic agents that face random needs for liquidity as well as random opportunities to build reserves over time in order to meet these needs. To perform either of these activities, they need to interact through a network of other banks. The model that we employ in our analysis is a model of decentralized exchange as in Kiyotaki and Wright (1989,1993).<sup>4</sup> This model is appropriate for our study for several reasons. First, it is a game-theoretic model in which transactions are explicit, and where it is natural to introduce and study the implications of lack of commitment, incomplete information, and reputation. Second, the abstract random matching shocks that agents are subject to in this model are equivalent, under certain conditions, to random taste shocks. These shocks are simply a tractable way of modelling random needs for liquidity during certain periods. Finally, this setup naturally lends itself to mechanism design analysis. It is important to note that, unlike the standard random matching monetary approach, our model involves credit arrangements only, in an otherwise non-monetary economy.<sup>5</sup>

<sup>&</sup>lt;sup>2</sup>See Kocherlakota's (2004) plenary talk at the *Society of Economic Dynamics* meetings in Florence for an excellent introduction to this literature.

<sup>&</sup>lt;sup>3</sup>See Kahn and Roberds (1998) for a well cited paper in this literature.

<sup>&</sup>lt;sup>4</sup>Durrell Duffie et. al. (2004) use a similar model to study illiquidity in over-the-counter financial markets.

<sup>&</sup>lt;sup>5</sup>To be precise, there is no currency in our model. Abstracting from currency trades, on which the Kiyotaki-Wright literature concentrates, seems appropriate for the issues we study. Atkeson (2004), for example, criticizes this literature for concentrating on currency exchanges.

Our findings suggest that in a variety of environments, the existence of an equilibrium in which banks transact with each other through a payment system requires the existence of certain caps on the banks' short-term borrowing. Put differently, in order for liquidity to be of value, it has to be scarce. Importantly, the introduction of caps implies a welfare loss, as it rules out certain welfare improving transactions that would take place in the absence of the information friction. This is an example of the common tradeoff between efficiency and incentives, since truthful revelation comes at some cost. For optimality, caps should be set at the maximum sustainable level; i.e., at the maximum value consistent with incentive compatibility.

Coming to the question of who should have access to the payment system, we find that if banks are divided into local networks that have intimate knowledge of the member banks' histories of transactions, a standard reputation argument implies that these networks will operate as local payment systems, supporting efficient patterns of transactions among their members. However, if banks need to transact frequently with other banks outside their own network, incentives constraints have to be taken into consideration. We find that this implies that inter-network transactions can take place only at the cost of sacrificing some intra-network transactions. Thus, whether the efficient arrangement involves a centralized payment system or several local networks of banks depends on the relative frequency of transactions between banks belonging to different networks. The implementation of the full-blown mechanism design problem features bank-specific caps that depend on individual banks' histories of transactions as summarized by their current reserve holdings. In addition, banks are penalized for borrowing within the network, and are rewarded for lending to banks outside the network.

## 2 The Model and Preliminaries

We first introduce the basic economic setup and discuss some benchmark cases. Then, we proceed by studying a variety of environments, involving different degrees of private information by the payment system participants, and we discuss payment system design in these environments. This section derives some results that are of use later, when the full-blown mechanism design problem is studied.

Time is discrete, t, measured over the positive integers. There is a  $[0, \frac{1}{k}]$  continuum of each of k types of infinitely lived agents (the banks), and there

are  $k \geq 3$  indivisible perishable goods.<sup>6</sup> The total measure of banks in the economy is 1. To generate transactions between banks, we assume that they are specialized in terms of the goods they can offer as well as those they need to be supplied by other banks. More precisely, banks of type *i* need good *i* only and offer good i + 1 only (mod k). Banks' discount factor is  $\beta \in (0, 1)$ . Randomness in payments, and the corresponding need for liquidity by banks, is captured by assuming that banks are randomly matched pairwise, once in every period. Note that the assumptions on specialization rule out double coincidence meetings. Henceforth, we will refer to meetings in which there is a single coincidence of wants as *trade meetings*. Effectively, a bank will need to use liquidity in order to consume, while a bank will create liquidity when it produces. To keep track of production opportunities for banks, we introduce a random variable  $s \in \{0, 1\}$ , which equals 1 if a meeting is a production meeting and 0 otherwise.

Consumption of one unit of good gives utility u, and production of one unit gives disutility e. We assume that u > e. We let  $p \in [0, 1]$  denote the probability with which a bank agrees to produce. Therefore  $p(s) \in \{0, 1\}$ denotes the outcome in a meeting of type s. If s = 1 and p(s) = 1, we will assume that automatically, the consumer bank receives utility u, and the producer receives disutility e. Thus, liquidity is both destroyed and created within a production meeting. An *allocation* within a match is a function  $p: s \to \{0, 1\}$ .

To familiarize the reader with the setup we first consider two benchmarks. First, assume that banks are anonymous and that there is no fiat money or other assets in the economy. An additional important feature of this environment is that there is *no commitment*. The above assumptions rule out reputation effects, as well as any type of trade using currency or any other asset. We discuss symmetric stationary allocations that can be supported by strategies that constitute perfect equilibria. We will refer to these allocations as *Incentive Feasible Allocations (IFAs)*.

**Proposition 1** The only IFA in the above economy is autarky, i.e., p(s) = 0 for all s.

For a second benchmark we go to the other extreme. Assume the existence of a perfect monitoring and record-keeping technology that allows for

 $<sup>^{6}\</sup>mathrm{We}$  start with the case where goods are indivisible for expository purposes. Later we will assume that goods are perfectly divisible.

the types and actions of all banks to be perfectly observed and recorded in every period. In this case, a "credit" equilibrium can be sustained through a standard reputation argument. We have the following.

**Proposition 2** If  $\beta$  is sufficiently high, p(s) = 1, for all s = 1, is an IFA, i.e., trade can be sustained in each production meeting.

**Proof.** Trade is sustained under the threat that if a bank deviates it is punished to permanent autarky. Let  $v^c$  stand for the value of a bank along the equilibrium path and  $v^d$  stand for that of a deviating bank. Also, note that the binding constraint comes from the bank that has to produce during the period. We then have that p(s) = 1 iff

$$-e + \frac{\beta(u-e)}{k(1-\beta)} > 0, \tag{1}$$

or iff

$$u - e > \left(\frac{1 - \beta}{\beta}\right) ke. \tag{2}$$

The interesting cases, of course concern situations in between these two extremes. To begin studying such cases, suppose that a perfect monitoring technology is not available. Assume instead that each bank is endowed with the ability to costlessly record the situations in which it demanded production, as well as the situations in which it produced for another bank, as entries with a clearinghouse. The clearinghouse does not have a monitoring technology to verify whether banks have a production opportunity within a given period. However, production can be verified. In that case, if, say, bank i produces for bank j, the "account" of bank i with the clearinghouse is credited by "+1," while the one of bank j receives an entry of "-1." Thus, banks now have the technology to record-keep their transactions by building reserves with the clearinghouse. We let d denote the (integer, not restricted in sign, no upper or lower bound) balance of a bank with the clearinghouse. The fact that there is no bound on d can be interpreted as a clearinghouse policy that imposes no caps on individual banks' borrowing. We now index the allocation in a match involving a bank with balance d by  $p_d(s)$ . We have the following.

**Proposition 3** For any  $\beta$ , the only IFA in this economy is autarky, i.e.,  $p_d(s) = 0$  for all s and d.

**Proof.** Clearly, no bank has an incentive to build reserves as it can always claim that it did not have a production opportunity. At the same time, since there is unlimited borrowing offered by the clearinghouse, declining to increase ones reserve balance does not by itself decrease the probability of consuming in the future.  $\blacksquare$ 

The above Proposition is interesting because it establishes the need for caps in order for any non-autarky arrangement to be viable. Put differently, in order for liquidity to be valuable it has to be scarce. Thus, the need for caps in banks' borrowing is a necessary condition for banks to provide liquidity. Later, we will derive a related but different argument for the existence of caps, one that relates to the restrictions imposed by certain incentive compatibility constraints.

In the next section we introduce borrowing caps by assuming that a clearinghouse can limit borrowing by imposing a cap of  $C \leq 0$ . In this environment, no penalty is imposed on the banks that hit the cap other than that they cannot borrow further unless they suffer the cost of production in order to improve their reserve balance. As we show below, this policy can implement trade, provided that the discount factor is sufficiently high. Intuitively, the existence of C implies that liquidity is now sufficiently scarce to be of value. In addition, sufficiently patient banks will produce in order to avoid being in a meeting in which the lack of liquidity prevents them from enjoying consumption.

This discussion identifies an interesting trade-off created by a liquidity providing clearinghouse. If the clearinghouse provides little or no liquidity, by setting the cap close to or equal to zero, some welfare improving trades will not be realized. In the context of the model, this occurs if a bank faced with a consumption opportunity has run out of reserves. To minimize the frequency of such inefficient meetings, the clearinghouse should set the borrowing cap as high as possible. This, however, might result in the non-existence of an equilibrium with trade. An *optimal cap policy* will involve the best way to balance between these two effects.

In order to implement an allocation  $p_d(s)$ , we only consider the game where the bank's choice set is  $\{0, 1\}$ . If one bank chooses 0, both banks remain in autarky in that match. If both banks choose 1, the allocation is implemented. As we assumed before, if a bank produces, its balance is credited by "+1," while if it consumes it receives a "-1."

#### 2.1 Stationary Incentive Feasible Allocations

We now turn to a characterization of IFA for the environment of the previous section. We will restrict our attention to allocations in trade meetings (thus, we drop the index s in what follows), assuming that no trade takes place in all other meetings.

Let  $\mathbf{p} = [p_{-C}, ..., p_0, p_1, ...]$  denote the vector of allocations and let  $\mathbf{x} = [x_{-C}, ..., x_0, x_1, ...]$  denote the distribution of banks (both in the population and per type) across states. In an IFA, we have the following value functions for a bank in state d:

$$v_{-C} = \frac{1}{k} (1 - x_{-C}) [p_{-C}(-e + \beta v_1) + (1 - p_{-C})\beta v_0] + [1 - \frac{1}{k} (1 - x_{-C})]\beta v_0, \quad (3)$$

$$v_{d\geq -C} = \frac{1}{k} px[u + \beta v_{d-1}] + \frac{1}{k} (1 - x_{-C}) [p_d(-e + \beta v_{d+1}) + (1 - p_d)\beta v_d] + [1 - \frac{1}{k} px - \frac{1}{k} (1 - x_{-C})]\beta v_d.$$
(4)

For the allocation to be incentive feasible, it must be the case that banks are better off when they choose strategies which result in this allocation. That is, it must be that, for all d such that  $p_d = 1$ , we have

$$-e + \beta v_{d+1} \ge \beta v_d, \quad and$$

$$\tag{5}$$

$$u + \beta v_{d'-1} \ge \beta v_{d'}, \quad \forall d' \ge -C.$$
(6)

In addition, it must be the case that, for all d such that  $p_d = 0$ , we have

$$-e + \beta v_{d+1} \le \beta v_d. \tag{7}$$

Note that discounting implies that consuming today is always better than consuming at a later date. Therefore, a bank with a consumption opportunity will always prefer to consume. Hence, when  $p_d = 0$ , only the producer can be better off by not producing. Finally, note that the allocation **p** also affects the law of motion of the distribution of banks **x**. We will only be interested in IFA,  $p_d$ , for which  $x_d > 0$ .

We first show that the set of IFA where  $p_d = 1$ , for some d, is non-empty.

**Lemma 4** The set of stationary IFA in which  $p_d = 1$ , for some d, is nonempty. **Proof.** We demonstrate the existence of an incentive feasible allocation with the desired properties for the special case where banks' reserves are restricted to  $\{0, 1\}$ . The value functions of the banks, imposing that  $p_0 = 1$  and  $p_1 = 0$ , are:

$$v_{0} = \frac{1}{k}x_{1}(-e+\beta v_{1}) + [1-\frac{1}{k}x_{1}]\beta v_{0},$$
  

$$v_{1} = \frac{1}{k}x_{0}(u+\beta v_{0}) + [1-\frac{1}{k}x_{0}]\beta v_{1}.$$
(8)

For the conjectured equilibrium to exist we need that  $-e + \beta v_1 \ge \beta v_0$ , or,

$$\frac{(k+\beta k-2\beta x_1)[-ek+\beta(x_0u+e(x_0-k))]}{(\beta+1)k[\beta(k-1)+k]} \ge 0,$$
(9)

or,

$$\beta \ge \frac{ek}{(u+e)x_0 - ek} > 0. \tag{10}$$

In addition, we need that  $-e + \beta v_2 < \beta v_1$ . Since  $v_2 \leq u + \beta v_1$ , this inequality holds if  $-e + \beta u < \beta (1 - \beta) v_1$ , or if

$$e + \frac{\beta(1-\beta)x_0[(1+\beta)ku - \beta(e+u)x_1]}{(1+\beta)k[\beta(k-1)+k]} \ge \beta u.$$
 (11)

It can be shown that both of these constraints are satisfied in an open subset of the parameter space.  $\blacksquare$ 

We now can provide a partial characterization of the set of IFA. Autarky is always an IFA. The next proposition asserts that all IFA in which there is trade have the property that banks wish to increase their reserves up to a point. If their reserves become sufficiently high, banks will find it individually optimal to decline opportunities to increase their reserves further. This results in some welfare loss, since consumption does not occur in some production meetings.<sup>7</sup>

<sup>&</sup>lt;sup>7</sup>The clearinghouse could induce such banks to produce by recording R(d) > 1 units of reserves to the producer bank. We ignore this possibility for now but will discuss it later, when we study the full-blown mechanism design problem.

**Proposition 5** Assume  $\beta$  is sufficiently high. All stationary IFAs have the property that either  $p_d = 0$  for all d, or there exists  $\overline{D} \ge 0$  such that (a)  $p_d = 1, \forall d \le \overline{D}, \text{ and } (b) \ p_d = 0, \forall d > \overline{D}.$ 

It should be clear that as  $\beta$  becomes arbitrarily small, it is not possible to implement production even if there are caps. For part (a), we assume that the cap set by the clearinghouse is C = 0. Let  $p^*$  denote a stationary allocation. Then

$$v_{d+1} - v_d = \frac{1}{k} \mathbf{x} \mathbf{p} \beta(v_d - v_{d-1}) + (1 - \frac{\mathbf{x} \mathbf{p}}{k}) \beta(v_{d+1} - v_d) + \frac{1}{k} (1 - x_0) p_{d+1}^* [\beta(v_{d+2} - v_{d+1}) - e] - \frac{1}{k} (1 - x_0) p_d^* [\beta(v_{d+1} - v_d) - e].$$
(12)

The proof follows the earlier existence proof since  $v_d$  is strictly increasing in d. For the proof of part (b) we use the following Lemmas.

**Lemma 6** Suppose that d' > d. There is no IFA with  $d \ge C$  such that  $p_d^* = 0$  and  $p_{d'}^* = 1$ .

**Proof.** We proceed by contradiction. Without loss of generality, let d' = d + 1. As  $p_d^* = 0$  and  $p_{d+1}^* = 1$ , we can use the expression for  $v_{d+2} - v_{d+1}$  to get

$$\beta(v_{d+1} - v_d) = \beta(v_{d+2} - v_{d+1}) + \frac{1 - \beta}{\mathbf{xp}/k}(v_{d+2} - v_{d+1}) + \frac{1 - x_0}{k}\beta[(v_{d+2} - v_{d+1}) - (v_{d+3} - v_{d+2})].$$
(13)

Since  $\beta(v_{d+2} - v_{d+1}) + \frac{1-\beta}{\mathbf{xp}/k}(v_{d+2} - v_{d+1}) > e$  and  $p_d^* = 0$ , it must be that  $(v_{d+3} - v_{d+2}) > (v_{d+2} - v_{d+1})$ . For  $p^*$  to be incentive feasible it must then be that  $p_{d+2}^* = 1$ . In addition, re-writing the expression for  $v_{d+3} - v_{d+2}$ , we have that  $v_{d+4} - v_{d+3} > v_{d+3} - v_{d+2}$ . Proceeding by induction, we obtain that  $v_{d+n+1} - v_{d+n} > v_{d+n} - v_{d+n-1}$ , for all n > 0, and  $p_{d+n+1}^* = 1$ , for all integers n. Hence, for some n large enough, we must have  $\beta(v_{d+n+1} - v_{d+n}) > u$ , which contradicts the incentive feasibility condition (6).

**Lemma 7** There is no  $d \ge C$  and d' > d such that  $x_d = 0$  and  $x_{d'} > 0$ .

**Proof.** Entries with the clearinghouse only increase by increments of one. Therefore to get to d', it must be that some banks have d entries with the clearinghouse.

The lemma above is useful as it limits the number of cases we need to consider for the following lemma.

**Lemma 8** For  $\beta$  sufficiently large, in all IFAs  $v_{d+1} - v_d$  is strictly decreasing in d.

**Proof.** The proof is by backward induction. We first show that  $v_{d+2} - v_{d+1} < v_{d+1} - v_d$  whenever  $v_{d+1} - v_d < v_d - v_{d-1}$ , for d such that  $p_{d+1}^* = 1$ . Then, we show that  $v_{d+2} - v_{d+1} < v_{d+1} - v_d$  for d such that  $p_{d+1}^* = 0$ . Finally, we show that  $v_2 - v_1 < v_1 - v_0$ . Consider the following.

$$v_{d+1} - v_d = \frac{1}{k} \mathbf{x} \mathbf{p} \beta (v_d - v_{d-1}) + (1 - \frac{\mathbf{x} \mathbf{p}}{k}) \beta (v_{d+1} - v_d) + \frac{1}{k} (1 - x_0) p_{d+1}^* [\beta (v_{d+2} - v_{d+1}) - e] - \frac{1}{k} (1 - x_0) p_d^* [\beta (v_{d+1} - v_d) - e].$$
(14)

Let  $p_{d+1}^* = 1$ . Then by lemma above, we have that  $p_d^* = 1$ . Rearranging terms in order to obtain an expression for  $v_{d+2} - v_{d+1}$ , we have

$$\frac{1-x_0}{k}\beta[(v_{d+2}-v_{d+1})-(v_{d+1}-v_d)]$$
  
=  $(1-\beta)(v_{d+1}-v_d) - \frac{\mathbf{x}\mathbf{p}}{k}\beta[(v_d-v_{d-1})-(v_{d+1}-v_d)].$  (15)

As  $\beta$  becomes large, the first term on the right hand side becomes arbitrarily small and as  $v_d - v_{d-1} > v_{d+1} - v_d$ , the result follows. Now, let  $p_{d+1}^* = 0$ . Then

$$= \frac{1}{k} \mathbf{x} \mathbf{p} \beta (v_d - v_{d-1}) + (1 - \frac{\mathbf{x} \mathbf{p}}{k}) \beta (v_{d+1} - v_d) - \frac{1}{k} (1 - x_0) [\beta (v_{d+1} - v_d) - e]$$
  
$$\leq \frac{1}{k} \mathbf{x} \mathbf{p} \beta (v_d - v_{d-1}) + (1 - \frac{\mathbf{x} \mathbf{p}}{k}) \beta (v_{d+1} - v_d).$$
(16)

Rearranging the terms and factoring out  $v_{d+1} - v_d$ , we obtain

$$v_{d+1} - v_d \le \frac{(1/k)\mathbf{x}\mathbf{p}\beta}{1 - \beta(1 - \mathbf{x}\mathbf{p}/k)} (v_d - v_{d-1}) < v_d - v_{d-1}.$$
 (17)

This proves the second claim. Finally, in the special case where d = 0, we have slightly different expression for  $v_0$  which gives

$$v_{1} - v_{0} = \frac{\mathbf{x}\mathbf{p}}{k}[u + \beta(v_{0} - v_{1})] + \beta(v_{1} - v_{0}) + \frac{1}{k}(1 - x_{0})p_{1}^{*}[\beta(v_{2} - v_{1}) - e] - \frac{1}{k}(1 - x_{0})p_{0}^{*}[\beta(v_{1} - v_{0}) - e].$$
(18)

If  $p_1^* = 0$ , we are done. If  $p_1^* = 1$ , rewrite the difference of the value functions as

$$(1-\beta)(v_1-v_0) = \frac{\mathbf{x}\mathbf{p}}{k} [u-\beta(v_1-v_0)] + \frac{1}{k}(1-x_0)\beta[(v_2-v_1)-(v_1-v_2)].$$
(19)

We know that  $v_1 > v_0$  and  $\beta(v_1 - v_0) < u$ . Therefore as  $\beta$  becomes large, it must be that  $v_2 - v_1 < v_1 - v_0$ , as the left-hand side becomes arbitrarily close to zero. This proves the third and last claim.

We have the following.

**Lemma 9** If  $\beta$  is sufficiently high, all stationary IFAs have the property that  $p_d = 0, \forall d > \overline{D}$ , for some  $\overline{D} \ge 0$ .

**Proof.** Given that  $v_{d+1} - v_d$  is monotonically decreasing, it is easy to see that  $v_{d+1} - v_d < \beta(v_d - v_{d-1})$  so that  $v_{d+1} - v_d < \beta^d(v_1 - v_0) \leq \beta^{d-1}u$ , where the last inequality follows from the fact that  $\beta(v_1 - v_0) < u$ . Hence, as  $\beta < 1$ , there is  $\overline{D}$  sufficiently large so that  $\beta(v_{\overline{D}+1} - v_{\overline{D}}) < e$  and  $p_d^* = 0$  for all  $d \geq \overline{D}$ .

We now turn to the choice of the borrowing cap, C, by the clearinghouse. Recall that, unlike C, the variable  $\overline{D}$  is not chosen by the clearinghouse. Rather it is the endogenously determined level of reserves beyond which a bank stops further improving its reserve position. The next proposition asserts that for bank transactions to take place it must be that the cap is set sufficiently low. **Proposition 10** Assume that  $\beta$  is sufficiently high. There exist IFA with  $p_d = 1$  for some d, iff  $C > \underline{C}$ .

**Proof.** The first part of the assertion follows from the last Lemma if we set  $\underline{C} = -\overline{D}$ . The second part follows as in the first Proposition.

We turn now to the question of existence of a stationary distribution,  $x^*$ , for banks across states. For any given distribution x, we have shown that there exists  $\overline{D}$  such that  $p_d^* = 1$ , for all  $d < \overline{D}$ , and  $p_d^* = 0$ , otherwise. We have the following.

**Proposition 11** If  $\beta$  is sufficiently high, there exists a stationary IFA in which  $p_d = 1$ , for some d. The IFA gives rise to a uniform stationary distribution of banks  $x^*$ . Furthermore, if  $\overline{D} = 1$ , any distribution is a stationary distribution.<sup>8</sup>

**Proof.** Fix  $\overline{D}$  as the number of states where  $p_d^* = 1$ , for all  $d < \overline{D}$ . With a stationary distribution, this set cannot change. The law of motion for x given the banks' decision rules is then characterized by

$$\begin{aligned} x'_{0} &= x_{0} \left(1 - \frac{1}{k} (1 - x_{0})\right) + x_{1} \frac{1}{k} (1 - x_{\overline{D}}) \\ x'_{i} &= x_{i-1} \frac{1}{k} (1 - x_{k}) + x_{i} \left(1 - \frac{1}{k} (2 - x_{0} - x_{\overline{D}})\right), \\ &+ x_{i+1} \frac{1}{k} (1 - x_{0}), \quad \forall 1 < i < \overline{D}, \\ x'_{\overline{D}} &= x_{\overline{D} - 1} \frac{1}{k} (1 - x_{0}) + x_{\overline{D}} \left[1 - \frac{1}{k} (1 - x_{\overline{D}})\right]. \end{aligned}$$
(20)

For a stationary distribution we have  $x'_i = x_i$  for all *i*. Suppose  $x_0 = x_{\overline{D}}$ . Then, the law of motion for *x* implies that  $x_0 = x_1$  and  $2x_i = x_{i+1} + x_{i-1}$ . Hence,  $x_i = x_j$  for all  $0 < i, j \leq \overline{D}$  is a solution to the set of equations describing the law of motion. In other words, the uniform distribution is stationary.

Finally, suppose  $\overline{D} = 1$ . Then, the conditions for a stationary distribution collapse to  $x_0 = 1 - x_1$ .

We remark that there is a connection between the pure "credit" economy we have described so far and a monetary economy. The economy with a

<sup>&</sup>lt;sup>8</sup>We cannot prove that the stationary distribution is either unique or stable since standard results on Markov chains cannot be directly applied as the transition matrix  $\Pi$  is state dependent.

clearinghouse and a cap of -C on banks' borrowing, is equivalent to one in which banks have no access to a clearinghouse but in which each has Cunits of fiat money (that they can spend one at a time). Therefore, the question of what is the optimal cap in the first economy is equivalent to the question of what is the optimal quantity of money in the second. The following proposition suggests that the clearinghouse should set the cap at the maximum level that is consistent with the existence of an equilibrium with trade.

**Proposition 12** Consider two IFAs that are supported by a uniform distribution of reserves and respective caps C and C', with C > C'. Welfare is higher in the allocation resulting from the greater cap, C.

**Proof.** Given the policy rules for banks, and given that the distribution of money holdings is uniform, each individual bank will set  $p_d^* = 1$  for d < C(C'), and  $p_d^* = 0$ , otherwise. In a cap y-allocation supported by a uniform distribution of reserves, the probability of consuming is y/(y + 1), and the probability of producing is y/(y + 1). Welfare is given by  $\frac{1}{k}(y/(y + 1))^2$ , which is clearly increasing in y. Hence, welfare is higher in the C-allocation.

We end this section by briefly discussing the case in which the CH can condition its policy on reports by *both banks* involved in any given transaction. More precisely, suppose that the clearinghouse can observe that banks i and j are in a match during a given period and consider the following rule. If both banks report that one produced for the other, say i for j, then the CH assigns a "+1" and a "-1" to the producer and the consumer, respectively. If the consumer reports that he did *not* receive production, the producer is punished to permanent autarky. We have the following.

**Proposition 13** Under the above policy, the allocation in which production takes place in each trade meeting is incentive feasible. Furthermore, there is no need for caps.

While the above Proposition asserts that the first best allocation can be supported, its conclusion relies on rather strong informational requirements. For example, it is necessary that the CH can verify that i and j are in a trade meeting in which i is the potential producer. If this assumption is withdrawn then i can claim (falsely) that it was j that did not produce for him, etc. In what follows, we shall restrict ourselves to environments in which the CH does not possess such information and in which certain transactions are subject to a form of imperfect monitoring.<sup>9</sup>

#### 2.2 Local Clearinghouses

Throughout this paper, we wish to study the effects of various kinds of interactions between banks and the payment system. Indirect interactions are of particular interest and could occur, for example, if small banks interact with the central clearinghouse through a small network of larger, correspondent banks. Such banks might have specific information about the local network of banks and, indeed, might act themselves as local payment systems.

To this end, we now assume that the continuum of banks is divided equally into two symmetric locations or *networks*: I and II. The distribution of banks across types is also symmetric across the two networks. We assume that a bank needs to transact with another bank from the same network with probability  $\alpha$  and with a bank outside the network with probability  $1 - \alpha$ . In order to isolate the effects of local information on banks within a network from the effects related to the frequency of meetings, we will sometimes assume that any two banks are matched with equal probability regardless of their respective networks, i.e.,  $\alpha = 1/2$ . In that case, the only asymmetry between the two networks concerns the flow of information of the banks' histories of interactions with the payment system. We first consider two benchmark cases regarding the information structure. In one case, banks in each network are connected to a Local Clearinghouse (LCH) that can monitor the types and actions of all banks in every meeting in which both banks belong to the network. There is no record-keeping regarding meetings among banks belonging to different networks. In this environment, a "local credit" equilibrium can be sustained within each network through a standard reputation argument. On the other hand, no inter-network trade is possible. The following Proposition characterizes IFAs for the first benchmark. It can easily be demonstrated using arguments from the previous section.

#### **Proposition 14** If $\beta$ is sufficiently high, the allocation where production oc-

<sup>&</sup>lt;sup>9</sup>Even in such cases, the CH can accomplish more by reverting to collective punishments. For example, even if a trade meeting is not verifiable, the CH could punish *both i* and j to permanent autarky unless their reports were mutually consistent. We shall ignore collective punishments in what follows.

curs in each trade meeting between banks belonging to the same network is IF, under the threat that if a bank deviates it is punished to permanent autarky. All IFAs imply autarky in meetings between banks belonging to different networks.

The second benchmark concerns the case where such LCHs do not exist. Instead, each bank is endowed with the ability to costlessly record its borrowing and its reserves in each period with a *Central Clearinghouse (CH)*. As before, unlike local networks, the CH does not have a monitoring technology which would allow it to verify whether banks have a production opportunity within a given period. However, if bank i produces for bank j then, like before, bank i's account is credited and bank j's account is debited. Thus, banks now have a capacity to record-keep their transactions, independent of the network of their trading partner, by building reserves with the CH. The second benchmark gives rise to the same environment as that of the previous section. Therefore, we have the following.

**Proposition 15** The IFA are characterized by the same conditions as in the previous section. The optimal cap policy is also the same.

Now consider the case in which the probability with which two banks need to transact,  $\alpha \in [0, 1]$ , depends on whether or not they belong to the same network. We have the following.

**Proposition 16** The first benchmark arrangement, when it exists, dominates the second if  $\alpha$  is sufficiently high. The second benchmark arrangement, when it exists, dominates the first if  $\alpha$  is sufficiently low.

The first assertion is true since the first arrangement involves no caps. Therefore, consumption takes place in each production meeting between two banks belonging to the same network. The second assertion is true since the first benchmark will involve very little or no trade. This is because meetings between banks belonging to different networks always result in autarky under this benchmark. We now turn to the main part of this project, which involves dynamic clearing arrangements under private information.

## 3 Dynamic Clearing under Private Information

Here we build on the first benchmark of the previous section. We apply mechanism design to study optimal transactions under private information, no commitment, and imperfect monitoring. We still assume that the population of banks is divided into two symmetric networks, I and II, and that banks are matched with other banks belonging to the same network with probability  $\alpha$ . Banks in each network are connected to a local clearinghouse (LCH). Currently, we shall think of each LCH as a local planner and will rule out communication between clearinghouses.<sup>10</sup>

We want to capture the feature that sometimes a is involved in transactions that are observed by the LCH, while in other cases, a bank's transactions are its own private information and the LCH will have to rely on incentives that will induce truthful revelation. To this end, we assume that the information technology is such that the LCH can observe the types and actions of banks only in meetings in which *both* banks belong to the network, while there is no record-keeping regarding meetings among banks belonging to different networks. The first Proposition in the previous section establishes that if  $\beta$  is sufficiently high, an equilibrium can be sustained in which trade takes place in each production meeting between banks from the same network, under the threat that if a bank deviates it is punished to autarky.

While the above mechanism gives rise to an efficient exchange within each network, it implies no transactions across networks, with the resulting efficiency loss. Thus, an obvious question is whether the above allocation can be improved upon via the use of a different mechanism. The difficulty lies in that LCHs cannot verify whether a trade meeting has taken place in situations involving banks belonging to different networks. To see this, consider a distinguished bank from location I which, say, for the *n*-th time in a row reports to the LCH that it could not produce since it did not have the necessary trade meeting with a bank outside the network. Given the information structure, the LCH in network I can verify that the bank had n consecutive meetings with banks belonging to network II (this is an event

<sup>&</sup>lt;sup>10</sup>This is consistent with viewing the LC as acting on behalf of the grand coalition of banks prior to each bank learning which network it will belong to. However, there are incentive issues that will arise if we assume that the LC acts as a representative of the local coalition of banks. We intend to study these issues in the future.

of probability  $(1 - \alpha)^n$ ). It can also verify that the bank did not produce in any of these meetings. What the LCH cannot verify, however, is whether that bank had an opportunity to produce and simply declined or whether it did not have any meetings in which it could produce (an event of probability  $(1 - \frac{1}{k})^n)$ .<sup>11</sup>

To see that the above difficulty could be overcome, consider the following candidate policy. Recall that the LCHs observe banks' trading histories in meetings within their network. Assume that the local banks are asked to make reports about their transactions in meetings with banks outside the network. Further, assume that each LCH triggers a penalty on banks that report histories that are "unlikely fortunate." For example, the penalty of "no consumption unless they produce for l periods for banks within their network" could be imposed on any bank who reports no production for banks outside the network for, say, n periods in a row. Such a policy, however, will typically imply a social cost. This is because a bank might simply reach the punishment threshold, n, due to bad luck. On the other hand, the benefits come from the fact that, for certain parameters, such a policy induces trade among banks in different networks. To analyze this issue formally, one could employ techniques in Green and Porter (1984) and Abreu, Pearce, and Stacchetti (1986, 1990).

Throughout this section we assume that the goods are perfectly divisible. Let u(q) stand for the utility that a consumer receives if he consumes q units of his favorite good. Similarly, let e(q) be the cost to the producer. We assume that e'(q) > 0,  $e^{''}(q) > 0$ , and  $\exists \overline{q}$  s.t. e(q) < q, if  $q < \overline{q}$ , and e(q) > q, if  $q \ge \overline{q}$ . It is then without loss of generality to assume that u(q) = q. We proceed to setup the problem as a mechanism design problem.

In each period, a bank may have a meeting as a consumer, as a producer, or a no trade meeting within its network. In that case, the LCH observes the type of the meeting, as well as the quantity consumed or produced. It is only when the bank has a meeting with a bank outside the network that its state is private information. In that case,  $q_t^{-i} < 0$  indicates that bank *i* reports production of *q* units, while  $q_t^{-i} > 0$  indicates that bank *i* reports consumption of *q* units. The bank reports a no trade meeting if it sets  $n_t^{-i} = 1$ . Otherwise, it sets  $n_t^{-i} = 0$ . Summarizing, the reported

<sup>&</sup>lt;sup>11</sup>Since meetings between banks from different networks are anonymous, the distinguished bank cannot be identified outside its network. As mentioned before, we will rule out collective penalties throughout.

experience of a bank belonging to network i in any period t, is given by the following array of functions:  $\theta_t = \{q_t^i, n_t^i, q_t^{-i}, n_t^{-i}\} \in \mathbf{R}^2 = \Theta_t$ . We want to design a mechanism that induces truthful revelation in every period. Let  $\theta^T = (\theta_0, \theta_1, ..., \theta_T) \in \Theta^T = \times_T \Theta_t$  denote a history of experiences of length  $T, T = 0, 1, ..., \infty$ . Let  $\mu$  be a probability measure over the Borel subsets of  $\Theta^{\infty}$ . Risk is modelled as follows. At the beginning of period 0, an element of  $\Theta^{\infty}$  is realized for each bank, consistent with the probability measure  $\mu$ . In each period T, a bank's choice in each period T has to be a  $\theta^T$ -measurable function.

In any given period, a banks's allocation depends on its reported history. In meetings between banks belonging to the same network the allocation may depend on the reported history of both banks. We wish to study resulting allocations that are *incentive compatible*. To this end, we will use the revelation principle and restrict ourselves to direct revelation mechanisms. Allocations that are both incentive and resource feasible are, like before, referred to as *Incentive Feasible (IFAs)*. Incentive feasible allocations that maximize ex-ante expected utility are called *Incentive Efficient Allocations* or, simply, *Optimal*. The problem can be divided into two parts. The first part involves characterizing the set of optimal allocations. The second part involves finding the clearinghouse policy, the payment system, that *implements* an optimal allocation, i.e., that supports an optimal allocation as a perfect equilibrium of the resulting game.

#### 3.1 Optimal Allocations

The planner's problem involves maximization of period 0 expected utility subject to resource and incentive constraints. The objective is given by

$$\max E \sum_{t=0}^{\infty} \beta^t I_t(q), \tag{21}$$

where  $I_t(q) \in \mathbf{R}$ , for all t, and it is positive or negative depending on whether there is consumption or production. Let  $H^t$  denote the entire history of length t. Let  $I = \{I_t\}_{t=0}^{\infty}$  denote an *allocation*, where  $I : H^{\infty} \to I(q)^{\infty}$ . The expectation in the above expression is taken over all possible future histories of random matching shocks. The allocation has to be resource feasible. This implies that consumption has to equal production in each meeting, a constraint that is automatically satisfied in our setup. Finally, the allocation has to be incentive compatible, which means that, in each period, each bank has to be (weakly) better off by reporting its true state regarding meetings with banks outside its network. More formally, Define a *reporting strategy* for a bank to be a function  $\sigma : \Theta^{\infty} \to \Theta^{\infty}$ , where  $\sigma_T$  is a  $\theta^T$ measurable function, for all  $T \geq 1$ . Let  $\Sigma$  denote the set of all such strategies. Define a function  $W(\cdot, I) : \Sigma \to \mathbf{R}$  by  $W(\sigma, I) = \sum_{t=0}^{\infty} \beta^t \int_{\Theta^{\infty}} I_t(\sigma) d\mu$ . The function W gives the expected utility from reporting strategy  $\sigma$  given I. Let  $\hat{\sigma} \in \Sigma$  be the truth-telling strategy, i.e.,  $\hat{\sigma}(\theta^{\infty}) = \theta^{\infty}$ , for all  $\theta^{\infty} \in \Theta^{\infty}$ . An allocation I is incentive compatible (IC) iff

$$W(\hat{\sigma}, I) \ge W(\sigma, I), \quad \forall \sigma \in \Sigma.$$
 (22)

An optimal allocation is one that maximizes

$$\sum_{t=0}^{\infty} \beta^t \int_{\Theta^{\infty}} I_t(\sigma) d\mu, \qquad (23)$$

subject to IC as well and resource feasibility constraints. Next we discuss some properties of the optimal arrangement for this economy. In what follows, unless we specify otherwise, we will assume that  $\alpha$ , the probability of interacting with banks within the network, is bounded away from 0 and 1, so that the optimum will involve some transactions between networks.

First, note that an allocation in which all production meetings result in trade is not IF. The reason is simple. In order to induce a bank to report truthfully and produce for a bank belonging to a different network, the bank's future expected utility has to increase. But if it already receives the maximum utility within its network, any such increase is impossible. Thus, interestingly, the existence of incentive compatible transactions with "outsiders" can only come at a cost of a reduction in the frequency of transactions with "insiders." We summarize this in the following.

# **Proposition 17** There is no IFA in which production occurs in all trade meetings within a network as well as in at least some trade meetings between networks.

Yet, the following Proposition suggests that transactions among banks belonging to different networks are generally desirable even though this comes at the cost of giving up some intra-network transactions. **Proposition 18** If  $\alpha = 0$ , the only IFA involves autarky. If  $\alpha = 1$ , IFAs involve no transactions between networks. If  $\alpha$  is bounded way from 0 and 1, and  $\beta$  is sufficiently high, the optimal allocation involves some transactions between networks.

Let  $W_{T+1}^{p^i(c^i)}(\sigma, I)$  denote the continuation expected utility of a bank belonging to network *i*, evaluated in period *T*, i.e.,

$$W_{T+1}^{p^{i}(c^{i})} = E \sum_{t=1}^{\infty} \beta^{t-1} I_{T+t}, \qquad (24)$$

where the superscript denotes whether the bank has a meeting in period T as a producer (consumer) of q units of the good with a bank belonging to the same network, i, and similarly for a meeting with a bank outside the network, -i. Here, the function  $I_T$  indicates, as before, an allocation for a bank in period T. We will refer to a bank's W as the state of the bank. In what follows, we find it is convenient to write  $W_T$  (the expected lifetime utility evaluated in period T-1) in the following form.

$$W_{T} = \frac{\alpha}{k} [EI_{T}(\theta^{T}, \theta^{T'}) + \beta EW_{T+1}^{c^{i}}] + \frac{\alpha}{k} [EI_{T}(\theta^{T}, \theta^{T'}) + \beta EW_{T+1}^{p^{i}}] + \frac{(1-\alpha)}{k} [EI_{T}(\theta^{T}) + \beta EW_{T+1}^{c_{-i}}] + \frac{(1-\alpha)}{k} [EI_{T}(\theta^{T}) + \beta EW_{T+1}^{p^{-i}}] + \alpha(1-\frac{1}{k}-\frac{1}{k}) [EI_{T}(\theta^{T}, \theta^{T'}) + \beta EW_{T+1}^{n^{i}}] + (1-\alpha)(1-\frac{1}{k}-\frac{1}{k}) [EI_{T}(\theta^{T}) + \beta EW_{T+1}^{n^{-i}}].$$

where  $EW_{T+1} = \int_{(\theta^T, \theta'^T)} W_{T+1} d\mu$ .

The next proposition asserts when it is incentive compatible for banks to improve their future expected utility by producing for banks belonging to a different network. The rewards come from an increased frequency of consumption within the network. If a bank's promised utility falls too low, it is declined consumption within its own network. This is the planner's way of supporting incentive compatible trade across banking networks.

**Proposition 19** Fix a trade meeting in period T between two banks belonging to the same network. In an optimal allocation, there exists a  $\underline{W}_T \ge 0$ such that  $I_T > 0$  if and only if  $W_T > \underline{W}_T$ . We will denote by  $x_{\underline{W}_T}$  the fraction of banks in states  $\underline{W}_T$ . The state space for a bank is  $\Lambda \subseteq [\underline{W}_T, \infty]$ , for all T.

The next Proposition further describes properties of the optimal arrangement in meetings between banks belonging to different networks. A bank is not punished when it consumes in such a meeting, otherwise it will not report truthfully. In order to report truthfully when a bank produces, it needs to be compensated for the disutility of production.

**Proposition 20** Fix a trade meeting in period T between two banks belonging to different networks. Any IFA such that  $I_T > 0$  satisfies  $EW_{T+1}^{c^{-i}} \ge W_T$ . Any IFA such that  $I_T < 0$  satisfies  $EW_{T+1}^{p^{-i}} - e/\beta \ge EW_{T+1}^{n^{-i}}$ , and  $EW_{T+1}^{c^{-i}} = EW_{T+1}^{n^{-i}}$ .

**Proof.** Suppose that  $EW_{T+1}^{c^{-i}} < EW_{T+1}^{n^{-i}} = W_T$ . Then, all banks would report a no coincidence meeting in situations where they actually consume. On the other hand, if  $EW_{T+1}^{c^{-i}} > EW_{T+1}^{n^{-i}} = W_T$ , banks in no coincidence meetings would falsely report that they consumed.

The future expected utility for banks in state  $\underline{W}_T$ , as well as the law of motion of banks across x, can be described as in the previous section. Next, we turn to the question of implementation of optimal allocations through a payment system.

#### 3.2 Implementation

The candidate mechanism for implementing an optimal allocation can be interpreted as follows. In each period, a bank's position is summarized by  $d \in \mathbf{R}$ . Each LCH imposes a *cap* of  $C(d) \in \mathbf{R}$  on the borrowing of banks. In addition, each bank receives an entry of  $R(d,q)(K(d,q)) \in \mathbf{R}$  every time it trades with a bank outside (inside) the network. If a bank does not produce for a bank within its network, it is punished to permanent autarky, as before, unless the potential consumer has a balance of C(d). In that case, the bank is instructed by the LCH to not produce. We are now ready to formally define a payment system.

**Definition 21** A Payment System is defined to be an array of three functions  $\{C(d), R(d,q), K(d,q)\}$ . A payment system is optimal if the array is chosen so as to implement an optimal allocation.<sup>12</sup>

 $<sup>^{12}</sup>$ Notice that in the spirit of mechanism design, unlike the existing literature, we do

The value function of a bank with reserves  $d \in (-C, \infty)$  is given by

$$v_{d} = \frac{\alpha}{k} (q + \beta v_{d+R(d,q)}) + \frac{(1-\alpha)}{k} (q + \beta v_{d+K(d,q)}) + \frac{\alpha}{k} (1-x_{-C}) (-q + \beta v_{d+R(d,q)}) + \frac{(1-\alpha)}{k} (-q + \beta v_{d+K(d,q)}) + [1-\frac{\alpha}{k} - \frac{(1-\alpha)}{k} - \frac{\alpha}{k} (1-x_{-C})] \beta v_{d},$$
(25)

while, if d = -C, it is given by

$$v_{-C} = \frac{(1-\alpha)}{k} (q+\beta v_{-C}) + \frac{\alpha}{k} (1-x_{-C})(-q+\beta v_{-C+R(-C,q)}) + \frac{(1-\alpha)}{k} (-q+\beta v_{-C+K(-C,q)}) + [1-\frac{(1-\alpha)}{k} - \frac{\alpha}{k} (1-x_{-C}) - \frac{(1-\alpha)}{k}] \beta v_{-C}.$$
 (26)

The Incentive Compatibility conditions require

$$-q + \beta v_{d+R(d,q)} > 0, \text{ and} -q + \beta v_{d+K(d,q)} > \beta v_d.$$
(27)

To define the law of motion of x, we need to first introduce some notation. Let

$$J^{R}(j) = \{d: d + R(d,q) = j\},\$$
  
$$J^{K}(j) = \{d: d + K(d,q) = j\}.$$

The law of motion of x is given by

not impose any properties on the payment system (such as whether it operates under gross or net settlement rules). However, our specification is general enough so that payment systems exhibiting gross, net, or other settlement rules, are implicitly considered as candidates for the optimum.

$$x'_{-C} = x_{-C} \left[1 - \frac{(1-\alpha)}{k}\right] + \int_{j \in J^{R}(-C)} \frac{\alpha}{k} x_{R(d,q)-C} d\mu + \int_{j \in J^{K}(-C)} \frac{(1-\alpha)}{k} x_{K(d,q)-C} d\mu, \quad (28)$$

while, for all  $i \in (-C, \infty)$ , we have

$$\begin{aligned} x'_{i} &= \int_{j \in J^{K}(i)} \frac{(1-\alpha)}{k} x_{i-K(d,q)} d\mu + x_{i} [1 - \frac{\alpha}{k} - \frac{(1-\alpha)}{k}] \\ &+ \int_{j \in J^{R}(i)} \frac{\alpha}{k} x_{i+R(d,q)} d\mu + \int_{j \in J^{R}(i)} \frac{\alpha}{k} x_{i-R(d,q)} d\mu \\ &+ \int_{j \in J^{K}(i)} \frac{(1-\alpha)}{k} x_{i+K(d,q)} d\mu. \end{aligned}$$
(29)

If banks trade with other banks in their network with probability 1, then it is optimal to impose no caps on these transactions. If, on the other hand, banks need to transact with banks outside their network with probability 1, the only IFA implies autarky. We, thus, have the following.

**Proposition 22** If  $\alpha = 1$  then  $\{C(d) = -\infty, R(d, q) \in \mathbf{R}, K(d, q) \in \mathbf{R}\}$ ,  $\forall d$ , constitutes an optimal payment system under the threat of exclusion if a bank does not produce within its network. If  $\alpha = 0$ , the optimal payment system implies autarky.

If  $\alpha$  is bounded away from 0 and 1, the optimal payment system will feature *bank specific caps* that will depend on individual banks' histories of transactions as summarized by their current reserve holdings. In addition, banks are penalized for using liquidity and are rewarded for creating liquidity when they produce. Further characterization of the optimal payment system is work in progress.

## 4 Conclusion

The main insight that comes out of our analysis so far is that an optimal payment system needs to explore intertemporal incentives. In order to accomplish this, certain transactions that are desirable under complete information are no longer feasible. Thus, there is a trade-off between efficiency and truthful revelation of banks' histories. Other issues that we want to consider as part of this proposal include:

- The possibility of default by either a single bank or an entire network of banks. Indeed, the existence of local banking networks implies that the resulting allocation has to be robust not only to deviations by individual banks, but also to coalitional deviations. In the presence of aggregate risk, an entire local network might find it profitable to exit the system if its position within the system becomes sufficiently unfavorable. This, in turn, could create certain contagion effects. Avoiding this problem will require the study of stronger forms of implementation, say in *coalition proof perfect equilibria*.

- To the extent that payment system provision has certain characteristics of a public good, it is interesting to study efficient pricing of this service given dynamic incentive constraints. This is particularly relevant as there is an ongoing policy debate on whether public payment systems that coexist with private ones should be subsidized.

- Given that we deal with dynamic incentives, we need to study imperfect commitment and the issue of *time consistency* of various clearinghouse policies, a problem that the current analysis abstracts from. This, once again, relates to the debate of public versus private provision of payment system services since, typically, optimal dynamic schemes require a high degree of commitment. Related issues will arise immediately in our model once we endogenize the interactions between LCHs, which we currently model as passive tools of a central planner.

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