Real effects of nominal exchange rate shocks*

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Abstract

This paper develops a flexible price, two-sector model to study the effect and incidence of large nominal exchange rate shocks on sectors and factors of production. We adopt a classical two-sector model of a small open economy and enrich its structure with gradual investment and a preference for real money holdings. A nominal appreciation leads to increased spending (due to the role of money), which pushes nontraded prices up (with gradual capital adjustment, the short-term transformation curve is nonlinear). This translates into changes in factor rewards, capital labor ratios and sector-level employment of capital and labor. Higher nontraded prices lead to extra domestic income, validating some of the initial excess spending. A depreciation would imply exacty the same but reversed effects. This propagation mechanism leads to a persistent real effect (on relative prices, factor rewards, capital accumulation) of nominal shocks, which disappears gradually through money outflow (trade deficit). We also draw parallels with the literature on exchange rate based stabilizations.

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1 Introduction

This paper develops a fully optimizing two-sector model *without price or wage frictions* in which various nominal exchange rate shocks (nominal appreciation or depreciation, a change in the rate of devaluation, the choice of the euro conversion rate) still have a medium-term impact

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on the real exchange rate. Other real variables, like factor rewards, investment and sectorial reallocation are also affected. The model relies on a standard money-in-the-utility specification (an "original nominal rigidity") and gradual investment (capital flows from abroad). Together with a more labor-intensive nontraded sector, this is already sufficient to produce a lasting effect of one-period nominal exchange rate shocks. For example, the model produces an endogenous gradual pass-through of a nominal exchange rate shock into wages and nontradable prices, even with a full and immediate pass-through into tradable prices. It also establishes a link between domestic savings and investment, although capital is fully owned by foreign investors.

The model gives sharp predictions about employment, price and wage dynamics after nominal exchange rate (and income or wealth) shocks. In particular, a nominal appreciation leads to (1) an increase in wages; (2) a reallocation of labor from manufacturing to services; (3) a fall in the rental rate; (4) a halt in investment with a sectorial asymmetry: increase in service sector investments, fall in manufacturing; (5) an increase in the nontraded-traded relative price; (6) an overall consumption boom, accompanied by a deteriorating trade balance; (7) a temporary increase in real GDP. A depreciation would produce exactly the opposite of these effects.

This is in line with the performance of exchange-rate based disinflations,[1] and its reverse conclusions are relevant to price and wage developments after large devaluations.[2] Reinhart and Végh (1995) find the following main stylized facts of exchange rate based stabilization programs: (1) high economic growth, (2) which is dominantly fueled by consumption, (3) slow price adjustment, (4) deteriorating trade balance. Rebelo and Végh (1995) add that (5) there is an increase in the relative price of nontradables,[3] (6) real wages (measured in tradables) increase, (7) the response of real interest rates is ambiguous, (8) there is remonetization of the economy, (9) the real estate market booms.[4] Burstein et al (2002) analyze large *devaluation* episodes, and find that (1) inflation is low relative to the depreciation, (2) the relative price of nontradables fall, (3) export and import prices (goods that are truly traded and not just tradable) track more closely with the exchange rate than the full CPI, (4) real GDP growth declines, and (5) there is

[1] It particularly matches the recent experience of Hungary (2000-2003), showing all the symptomes from above. The policy environment can be summarized as (1) an increase in minimum wage legislation, (2) followed by a large nominal appreciation (monetary restriction), in the form of widening the exchange rate band, (3) followed by a massive fiscal expansion, partly in the form of public sector wage increases. The exact timing of the fiscal expansion is somewhat unclear: the rise in public sector wages unambiguously came after the monetary contraction, but the fiscal stance before and after the monetary developments is subject to heated political debates in Hungary.

[2] In the paper, we mostly focus on the effects of a nominal appreciation, though every conclusion applies one in one to a nominal depreciation, with all the signs reversed. Our choice is motivated by our interest in exchange-rate based disinflations, which essentially involve a restrictive policy shock.

[3] Non-tradable production also seems to increase disproportionally, but it is not well documented.

[4] Hamann (2001) argues that many of these features are not restricted to exchange-rate based stabilizations. Given that we are interested in the response to a nominal exchange rate shock, it is irrelevant whether these features are shared by other types of stabilizations.

a rise in the trade surplus. With the exception of the real interest rate behavior, remonetization and real estate boom results, our model matches all of these features.

One important message thus concerns the adjustment to a nominal exchange rate shock. After a credible revaluation of a fixed exchange rate, traded prices will fall (assuming immediate, and potentially full pass-through of the nominal exchange rate to tradable prices). This increases the value of domestic money holdings in terms of tradables. As a response, wages and nontraded prices show an endogenous and gradual adjustment to the decreased tradable price level, which frequently puzzles central bankers. There is also a marked redistribution of income from capital to labor in the spirit of the Stolper-Samuelson theorem (and vice versa in case of a devaluation), creating a conflict of interest between the two groups. Leamer and Levinsohn (1995) give an overview of the empirical performance of the Stolper-Samuelson theorem with respect to trade openings – our model thus enables a different look, around large exchange rate movements.

A related but inherently fixed exchange rate situation is the choice of the EMU conversion rate for countries that are due to introduce the Euro in the next few years. The model issues the warning that an overvaluation may imply a significant reduction in capital inflows, it may be persistent even with flexible price and wage determination, and it has largely asymmetric effects on different sectors and factors of production. Welfare implications are not clear-cut, since GDP growth may slow down, but consumers experience higher wages and consumption (financed by debt).

The model also has interesting implications about changes in the rate of devaluation vs. the level of the exchange rate. A one-time devaluation of the currency can depress consumption, nontradable production, and boost investment and tradable production as a general equilbrium response to a negative wealth effect. The introduction of a crawling peg, on the other hand, has the opposite effect: an increase in the rate of devaluation decreases steady state money holdings, thus it creates a *positive wealth effect* (money holdings increase relative to the desired level). Consequently, an increase in the rate of devaluation slows down capital accumulation, just like a one-time *revaluation*.

The main mechanism of the model is the following. Consider an appreciation of the nominal exchange rate. It changes the spending behavior of consumers, through influencing their intertemporal (savings) decisions. In particular, domestic (nominal) assets are revalued in terms of tradable goods. If there is a positive link between consumption expenditures and asset values (money holdings), then consumption increases. The money-in-the-utility framework alone implies such a link. This is one "stickiness" in the model. Increased spending must lead to increased production of nontradables, while excess demand in tradables can be satisfied through imports as well. This shift in production leads to an increase in the relative price of nontradables as long as the short-term transformation curve is nonlinear. This is the second and last friction of the model, which is supplied by gradual capital adjustment (q-theory).

The virtue of having only these two dynamic frictions is that one can clearly see the intuitive developments behind all results. It is also evident that both rigidities are necessary for the mechanism to work: without the nominal effect, we could not consider nominal shocks, while without the real friction, excess spending would not alter relative prices (under flexible capital and labor, the transformation curve is linear, and the relative price is fully determined by the supply side). This is why a model with *mobile factors, money-in-the utility and flexible prices* cannot lead to a real effect of nominal exchange rate shocks.

As the economy moves along its transformation curve, factor rewards must also change: if the nontraded sector is more labor-intensive, than r falls and w increases (Stolper-Samuelson theorem). There is a reallocation between the two sectors: both labor and capital migrate from tradables into nontradables. A lower $\frac{r}{w}$ increases capital intensity in both sectors. The decline in r initiates a fall in aggregate capital. Notice that this is compatible with an increase in sectorial capital intensities, since the expanding nontraded sector is less capital intensive than the contracting traded sector. Since we will assume thaat the capital stock is owned by foreigners, rising wages create extra income for domestic consumers ("transfer effect"), which makes the real effect persistent in the medium-term (i.e., more persistent than the trade deficit adjustment process with *exogenous* income). Excess spending slowly returns to equilibrium, through a gradual outflow of domestic money (assets).

The paper is organized as follows. The next section puts the model in context. Section 3 describes the model .Section 4 contains and discusses the qualitative results (loglinearization and impulse responses). Section 5 concludes, and the Appendix contains all the detailed calculations.

2 The context of the model

We consider a dynamic adjustment of a flexible price, two-sector small open economy growth model (the "dependent economy" model[5]). One of the sectors is traded, the other is nontraded. The two sectors differ in pricing: traded prices are set by the law of one price (fixed international prices times the nominal exchange rate), while nontraded prices are determined through domestic market clearing. In traded goods, domestic supply and demand can temporarily deviate from

[5] See, for example, Dornbusch: Open Economy Macroeconomics, chapter 6. Benczúr (2003) also considers a similar framework, but in discrete time. That model also employs certain simplifications of the intertemporal optimization behavior, but the qualitative results are identical.

each other, leading to a trade deficit or surplus. The presence of a traded and a nontraded sector allows to merge trade theory insights with a monetary framework: for example, the presence of nontraded goods means that a redistribution of income between countries will affect their relative wages (the classical transfer problem, like in Krugman (1987)), or the Stolper-Samuelson theorem, linking changes in goods prices with movements in factor rewards. We also allow for exogenous TFP growth in the traded sector (as in the Balassa-Samuelson framework).

There are two dynamic factors in the model. The first one is the intertemporal aspect of consumer behavior: the gradual adjustment of expenditures to income. This can be also viewed as some sort of a nominal rigidity (illusion), which ensures that nominal shocks (nominal exchange rate movements, fiscal policy) have an impact effect on spending. Such a behavior can be microfounded by an explicit intertemporal maximization of a utility function containing real money balances.[6] With respect to nominal exchange rate fluctuations, currency and fixed-income securities are *inherently* sticky, their value changes one in one with exchange rate fluctuations. In this sense, they can be viewed as an "original stickiness".

The nominal effect thus does not come from the rigidity or stickiness of prices or wages, but from the gradual response of consumption expenditures and money holdings. In a more general setting, one can think of the role of money here as a precautionary buffer stock. As the economy grows, consumers want to increase their asset holdings. Moreover, nominal shocks can revalue this stock (as argued by Lane and Milesi-Ferretti (2004), or Gourinchas and Rey (2004)), which in turn changes consumer behavior. Having flexible prices does not imply that real-world prices or wages are flexible, or there is no inflation persistence – all is meant to show that there are systematic effects of nominal shocks on relative prices even under price flexibility. Another gain of having flexible prices is the tractability and intuitiveness of model mechanics.

The other dynamic effect is the accumulation of capital. Due to adjustment costs, this is a gradual process, like in a regular Tobin's q model. For simplicity, we assume that capital is owned 100% by foreigners (alternatively: capital owners consume only tradables, their opportunity cost of funds is the fixed world interest rate, which then makes the nationality of capital owners irrelevant). It implies that changes in capital income will not affect domestic nontraded demand. In reality, there should be such an effect, but one would expect the labor income effect to dominate.

[6] Classical real exchange rate (trade theory) models often use the relationship $E = VH$, nominal expenditures being proportional to money holdings, to allow for nominal shocks. Examples include part 3 of Dornbusch (1980) and Krugman (1987). Dornbusch and Mussa (1975) show that under certain conditions (power-Cobb-Douglas utility and constant inflation), the intertemporal optimization problem with money-in-the-utility implies a saddle path with $E = VH$. This is one simplification adopted by Benczúr (2003).

Money-in-the-utility and gradual investment are already sufficient to produce real effects of a nominal shock: under a nominal appreciation, for example, the value of domestic money holdings (wealth) in terms of tradable goods increases. This leads to more consumption of tradables and nontradables. Since country-level capital is fixed in the short-run, and nontraded consumption must equal production, this implies a change in relative prices between the two sectors, and also influences wages and the rental rate. This latter implies a change in the capital accumulation process, while the former has an impact on consumer income, which may reinforce or counteract the initial excess consumption. If the nontraded sector is more labor intensive than the traded sector, wages rise (the Stolper-Samuelson theorem) and consumer income increases, propagating the real effect of the nominal shock.[7] There is also a positive correlation between domestic savings and investment (like the Feldstein-Horioka (1980) puzzle), although investment is not financed from domestic savings at all. The link comes from a "crowding out" effect of nominal expenditures on investment, which is due to the general equilibrium developments of relative prices.

Classical explanations for nominal shocks having lasting real effects usually build on staggered price or wage contracts. An early example is Taylor (1980). Recently, state- or time-dependent pricing models constitute as the workhorse for analyzing nominal scenarios (see chapter 3 of Woodford (2003) for a general discussion). Instead of pricing problems, we focus on nominal wealth accumulation (captured by money-in-the-utility), which is also influenced by nominal shocks. The pure money-in-the-utility specification might look conceptually less appealing than other models of money (like cash-in-advance, money as a means of facilitating trade, overlapping generations models), but it generally serves to capture nominal wealth accumulation. Thus its role is more general than that of money itself. Moreover, regular sticky-price models also tend to use money-in-the-utility, thus we are not adding any extra effect into those models – we only remove pricing frictions.

Many papers in the literature point to the importance of gradual investment in shaping business cycle properties, inflation or real exchange rate behavior. Eichenbaum and Fisher (2004) argue that the empirical fit of a Calvo-style sticky price model substantially improves with firm-specific capital (and a nonconstant demand elasticity). Christiano et al (2001) present a model in which moderate amounts of nominal rigidities are sufficient to account for observed output and inflation persistence, after introducing variable capital utilization, habit formation and capital adjustment costs. Chapter 4 of the Obstfeld and Rogoff (1996) textbook contains

[7] Benigno (2003) and part 3.2.5 of Woodford (2003) also highlight the role of sectoral asymmetries, though not in the context of traded versus nontraded goods.

an exposition of a two-sector growth model (the standard Balassa-Samuelson framework), with gradual investment in some of the sectors. We depart from these approaches by dropping staggered price setting, but – unlike Obstfeld and Rogoff – still allowing for a nominal side of the economy.

Huffman and Wynne (1999) develop a multisector real model with investment frictions (sector-specific investment goods and costs of adjusting the product mix in the investment sector). Their objective is, however, to match the *closed economy comovements* of real activity across sectors (consumption and investment). In our model, the two sectors have a completely different nature (traded and nontraded). These two sectors do not necessarily move together, as indicated by the countercyclicality or acyclicality of net exports (see Fiorito and Kollintzas (1994) for G7 countries, Aguiar and Gopinath (2004) for emerging economies). Aguiar and Gopinath (2004) also construct a *one-sector real model* to explain the countercyclicality of net exports and the excess volatility of consumption. Balsam and Eckstein (2001) develop a real model with traded and nontraded goods, aimed at explaining the procyclicality of Israel's net exports and excess consumption volatility. None of the existing models, up to our knowledge, share all the distinctive features of our model: a flexible price, nominal, open-economy, two-sector model with investment frictions, giving a lasting real effect of nominal disturbances.[8]

3 The model

3.1 Consumers

Consumers solve the following problem:

$$\begin{aligned}\max U_t &= \int_t^{\infty} e^{-\rho(s-t)} \left[\log C(s) + \gamma \log \frac{H(s)}{P(s)}\right] ds \\ s.t.\ \dot{H} &= WL - PC + T,\end{aligned}$$

where WL is aggregate labor income (supplied inelastically), T is a government transfer,

$$C = C_T^{\lambda} C_{NT}^{1-\lambda},$$

[8] In fact, the general equilibrium tax incidence analysis of Harberger (1962) has very similar features: in his analysis, taxation plays a related role to the nominal exchange rate in our model.

P is the ideal price index associated with C and ρ is the worldwide discount rate and also the rate of interest abroad. The usual intratemporal optimization conditions imply that:

$$
\begin{aligned}
PC &= eC_T + p_{NT}C_{NT} && (1)\\
\frac{eC_T}{p_{NT}C_{NT}} &= \frac{\lambda}{1-\lambda} && (2)\\
P &= \lambda^{-\lambda}\left(1-\lambda\right)^{\lambda-1} e^{\lambda} p_{NT}^{1-\lambda}. && (3)
\end{aligned}
$$

The intertemporal problem is solved by writing down the current-value Hamiltonian:

$$
\mathfrak{H} = \log C + \gamma \log \frac{H}{P} + \theta\left(WL - PC + T\right),
$$

and the first-order conditions are given by

$$
\begin{aligned}
\frac{1}{C} &= \theta P && (4)\\
\dot{\theta} &= \rho\theta - \frac{\gamma}{H} && (5)\\
\dot{H} &= W - PC + T. && (6)
\end{aligned}
$$

Dornbusch and Mussa (1975) use a similar framework to give a microfoundation of the $X = VH$ relationship (nominal spending being proportional to money holdings): with a power Cobb-Douglas aggregate $(C^{\alpha}\left(H/P\right)^{\beta})$ and constant inflation, they show that X/H is indeed constant along the saddle path of the intertemporal optimization, as long as inflation is constant. In our work, however, inflation is changing through time. Given that the proportionality of X and H is no longer true, we decided to use the more standard logarithmic Cobb-Douglas felicity function. This gives a less direct role of money in the consumption decision (the marginal utilities are separable), and it is also the standard choice of new-keynesian intertemporal models (see Woodford (2003), chapter 2.3.4 for consequences of nonseparable utility functions).

3.2 Producers

Production functions are given by

$$
\begin{aligned}
Y_T &= \left(A_T L_T\right)^{\beta} K_T^{1-\beta}\\
Y_{NT} &= L_{NT}^{\alpha} K_{NT}^{1-\alpha},
\end{aligned}
$$

and profit maximization implies

$$\begin{aligned} W &= e\beta A_T^{\beta} L_T^{\beta-1} K_T^{1-\beta} = p_{NT} \alpha L_{NT}^{\alpha-1} K_{NT}^{1-\alpha} \\ r &= e(1-\beta)(A_T L_T)^{\beta} K_T^{-\beta} = p_{NT}(1-\alpha) L_{NT}^{\alpha} K_{NT}^{-\alpha}. \end{aligned}$$

Notice that we assume the indifference of both factors *between the two sectors*, so $w_T = w_{NT} = w$, $r_T = r_{NT}$. This does not automatically imply full international mobility of capital: as we shall see, domestic rental rates can temporarily deviate from the fixed international rate.

We would not argue that the labor mobility assumption is fully realistic, or that the adjustment of labor is fast enough (compared to the adjustment of capital and nominal spending) to validate such an approximation. One could also set up a model with slow labor adjustment. This would, however, excessively complicate the model, while the other two adjustments are vital to our analysis (for a real effect of nominal shocks, we need to have slow adjustment of nominal spending; and slow capital adjustment is necessary to analyze investment behavior).

The other crucial assumption is that capital is indifferent between the *two domestic sectors*, but not necessarily between home and foreign. If the initial difference in sectorial returns of capital is not "too large", their equalization is feasible entirely through new investment. It is possible that a too large shock necessitates disinvestment in one of the sectors. Then one needs to assume that capital is mobile between sectors up to this degree. A further alternative would be to consider two separate q-theories in the two sectors, like Balsam and Eckstein (2001).

We have also assumed that there is an immediate and full pass-through of the nominal exchange rate into tradable prices, but not necessarily into nontradables and factor prices. It is well-documented that the pass-through of exchange rate movements into tradable prices is far from full and immediate. Our focus, however, is essentially on the adjustment of the economy to a change in tradable prices. For this reason, similarly to most of the open economy macro literature, we work with a perfect pass-through into tradable prices.

3.3 Effective variables

Since A_T is growing at a rate g, let us introduce the following effective variables:

$$
\begin{aligned}
k_T &= \frac{K_T}{A_T L_T} \\
k_{NT} &= \frac{K_{NT}}{A_T L_{NT}} \\
p &= \frac{p_{NT}}{e A_T^{\alpha}} \\
w &= \frac{W}{A_T},
\end{aligned}
$$

where p is the adjusted relative price of non-traded goods. The production functions and the profit maximization conditions can then be re-written as

$$
\begin{aligned}
y_T &\equiv \frac{Y_T}{A_T L} = l_T k_T^{1-\beta} && (7) \\
y_{NT} &\equiv \frac{Y_{NT}}{A_T^{1-\alpha} L} = l_{NT} k_{NT}^{1-\alpha} && (8) \\
w/e &= \alpha p k_{NT}^{1-\alpha} = \beta k_T^{1-\beta} && (9) \\
r/e &= (1-\alpha)\, p k_{NT}^{-\alpha} = (1-\beta)\, k_T^{-\beta}, && (10)
\end{aligned}
$$

where $l_i = L_i/L$. These equations describe production equilibrium, and given the relative price p and the total capital stock k, they can be solved for w, r, k_T, k_{NT}, y_T and y_{NT} . In fact, the last four variables are functions of p alone. On the consumption side, let us define

$$
\begin{aligned}
c_T &= \frac{C_T}{A_T L} \\
c_{NT} &= \frac{C_{NT}}{A_T^{1-\alpha} L} \\
c &= c_T^{\lambda} c_{NT}^{1-\lambda} = \frac{C}{A_T^{\lambda+(1-\alpha)(1-\lambda)} L} \\
h &= \frac{H}{A_T L}.
\end{aligned}
$$

3.4 Investment

One of the cornerstones of the "standard", "long-run" Balassa-Samuelson model (the one advocated by chapter 4 of the Obstfeld-Rogoff textbook) is the full mobility of capital. It implies that the rental rate at home equals the international rental rate. However, this implies a very fast and also mechanical capital accumulation and adjustment process. If we add the standard labor flexibility assumption ($w_T = w_{NT}$), the real exchange rate (traded-nontraded relative price) is fully supply-determined. The transformation curve is linear, and nominal variables (or preferences) have no effect on relative prices, only on quantities. For this reason, we assume that investment is subject to adjustment costs, which makes its response only gradual:

$$\begin{aligned} \max V_t &= \int_t^{\infty} e^{-\rho(s-t)} \left[\frac{r(s) K(s)}{e} - I(s) - \frac{\delta}{2} \frac{I(s)^2}{K(s)} \right] ds \\ s.t.\ \dot{K} &= I. \end{aligned}$$

This is the standard q problem, and the first-order conditions are

$$\begin{aligned} q &= 1 + \delta \frac{I}{K} \\ \dot{q} &= \rho q - \frac{r}{e} - \frac{\delta}{2} \left(\frac{I}{K} \right)^2 . \end{aligned}$$

Define $k = K/(A_T L)$ and rearrange to get

$$\frac{\dot{k}}{k} = \frac{q-1}{\delta} - g \tag{11}$$

$$\dot{q} = \rho q - r/e - \frac{(q-1)^2}{2\delta}. \tag{12}$$

3.5 Equilibrium

Let us introduce the term $x = \lambda^{-\lambda} (1-\lambda)^{\lambda-1} ep^{1-\lambda} c = \chi ep^{1-\lambda} c$, which is – as can be seen from (6) – effective nominal expenditures. From (4) we get

$$\dot{x} = \frac{d}{dt} \left(\frac{PC}{A_T L} \right) = \frac{d}{dt} \left(\frac{1}{\theta A_t L} \right) = \frac{-1}{(\theta A_T L)^2} \left(\dot{\theta} A_T L + \theta \dot{A}_T L \right) = -\frac{\dot{\theta}}{\theta} x - gx.$$

Utilizing (5):

$$\frac{\dot{\theta}}{\theta} = \rho - \frac{\gamma}{\theta H} = \rho - \frac{\gamma x}{h},$$

thus

$$\dot{x} = -(\rho + g) x + \gamma \frac{x^2}{h}.$$

The other equilibrium conditions are

$$\begin{aligned}
\frac{\dot{k}}{k} &= \frac{q-1}{\delta} - g \\
\dot{q} &= \rho q - \frac{r}{e} - \frac{(q-1)^2}{2\delta} \\
\dot{h} &= w - gh - x + \tau,
\end{aligned}$$

where $\tau = T/(A_T L)$. The equations for k and q are in foreign currency, which means that the nominal exchange rate e does not directly enter those expressions. Let us transform h and x also into foreign currency be replacing h with h/e (domestic money measured in foreign currency) and x by x/e (nominal spending measured in foreign currency). Then

$$\begin{aligned}
\frac{d}{dt}\left(\frac{x}{e}\right) &= \frac{\dot{x}}{e} - \frac{x}{e}\frac{\dot{e}}{e} = -\left(\rho + g + \frac{\dot{e}}{e}\right)\left(\frac{x}{e}\right) + \gamma\frac{(x/e)^2}{(h/e)} \\
\frac{d}{dt}\left(\frac{h}{e}\right) &= \frac{\dot{h}}{e} - \frac{h}{e}\frac{\dot{e}}{e} = \frac{w}{e} - g\frac{h}{e} - \frac{x}{e} + \frac{\tau}{e} - \frac{h}{e}\frac{\dot{e}}{e}.
\end{aligned}$$

From here on, let us work entirely in foreign currency; the dynamics are summarized by

$$\frac{\dot{k}}{k} = \frac{q-1}{\delta} - g \tag{13}$$

$$\dot{q} = \rho q - r^f - \frac{(q-1)^2}{2\delta} \tag{14}$$

$$\dot{h}^f = w - gh^f - x^f + \tau^f - h^f\frac{\dot{e}}{e} \tag{15}$$

$$\dot{x}^f = -\left(\rho + g + \frac{\dot{e}}{e}\right)x^f + \gamma\frac{\left(x^f\right)^2}{h^f} \tag{16}$$

For a steady state to exist, T has to grow at a rate $g + \frac{\dot{e}}{e}$. (13) - (16) is a system of four equations for five variables: k, q, h^f, x^f and e (the other variables k_T, w^f, r^f and c are functions of these five). A fifth equations is given by monetary policy. One assumption is that the change in the nominal exchange rate is constant, i.e. $\frac{\dot{e}}{e} = \varepsilon$. Now we have four equations with four endogenous variables, and three forcing variables: τ^f, ε and e, which could be viewed as vehicles of monetary and fiscal policy. Any exchange rate level and rate of devaluation are consistent with the long-run steady state, whereas if the long-run outcome is to replicate a real model without money, $\bar{\tau}^f = (g+\varepsilon)\,\bar{h}^f$ must hold. This means that the steady state conditions are

$$\begin{aligned} \bar{q} &= 1+\delta g && (17) \\ \bar{r}^f &= \rho + \frac{\delta g\,(2\rho - g)}{2} && (18) \\ \bar{w}^f &= \bar{x}^f && (19) \\ \bar{h}^f &= \gamma\frac{\bar{w}^f}{\rho+g+\varepsilon}. && (20) \end{aligned}$$

Notice that the exchange rate does not influence $\bar{r}^f$. Consequently, all the technology-determined variables are independent of the path of the nominal exchange rate. Thus $\bar{w}^f$ and $\bar{x}^f$ are also independent from ε, it is only the steady state level of money holdings that depends (inversely) on the rate of devaluation: $\bar{h}^f = \frac{1}{\rho+g+\varepsilon}\gamma\bar{w}^f$.[9] In the current paper, we mostly concentrate on level changes in a fixed exchange rate, thus we set $\varepsilon = 0$ in general. In terms of fiscal policy (τ^f), we assume that the steady state condition $\bar{\tau}^f = (g+\varepsilon)\,\bar{h}^f$ holds in every period, thus for $\varepsilon = 0$, $\tau^f = gh^f$. In other words, the government simply prints a fixed fraction g of the current money stock and distributes it lump-sum (as a helicopter drop) to consumers. For notational comvenience, we will drop the superscript f from here on, meaning that any nominal variable is measured in foreign currency.

3.6 The current account

Recall that the change in money holdings is given by

$$\dot{H} = WL - PC + T = Y_T + p_{NT}Y_{NT} - rK - C_T - p_{NT}C_{NT} + T = (Y_T - C_T) - rK + T. \quad (21)$$

This is purely an accumulation equation (identity): the change in money holdings is equal to GNP minus expenditure, plus government transfers. GNP is the sum of traded and nontraded production (GDP) minus capital rents (that belongs to foreigners). Since the nontraded sector is in equilibrium, the value of nontraded production must equal the value of nontraded consumption. Change in money holdings thus equals the excess production of tradables, minus capital rents, plus the exogenous term T.

This equation implies that the only asset of consumers is money, earning no interest but giving utility. Under fixed exchange rates, any change in money holdings on top of T enters as a money inflow or outflow. Since foreign and domestic interest rates on money are identical (zero),

[9] Notice that $\bar{h}$ must be nonnegative, since it enters the utility of consumers in a logarithmic way. Consequently, there is an upper bound on the rate of devaluation which is still consistent with a steady state.

foreigners are willing to hold any level of domestic money under fixed exchange rates (their demand is perfectly elastic). For this, we in fact need to assume that foreigners accumulate money for the same reason as domestic consumers, and that our economy is infinitesimally small. Under these conditions, equation (21) also postulates the equilibrium of international money markets.

Under flexible exchange rates, the nominal appreciation should come from an increase in interest rates. This appreciation implies the same stimulus on consumption, through the revaluation of money holdings in terms of traded prices. High interest rates, on the other hand, may also depress consumption. This means that the effects in the flexible exchange rate model are smaller than with fixed exchange rates (they may even be reversed). If we assume that, in line with actual experience, the net effect on consumption is stimulating, then the results are only weakened.

Apart from the exogenous term, this expression is closely related to the balance of payments: $e\,(Y_T - C_T)$ is the trade balance, while $-rK$ is investment income paid to foreigners. This determines the dynamics of financial wealth ("net foreign financial assets"). It is important to make the qualification "financial", because K will have its own accumulation equation, and a full balance of payments would reflect capital flows as well.

4 Qualitative solution

4.1 Loglinearization

The Appendix contains the full details of the loglinearization and the evaluation of the signs of impulse responses. Here we only collect the results and show the logic and intuition of their derivation. The following system of differential equations describe the evolution of the four dynamic variables (their log-derivative, or in other words, deviation from steady state):

$$\begin{aligned}
\frac{d}{dt}\hat{q} &= (\rho - g)\,\hat{q} + \frac{\bar{r}}{\bar{q}}\frac{\beta}{\alpha-\beta}\frac{1}{A}\hat{x} - \frac{\bar{r}}{\bar{q}}\frac{\beta}{\alpha-\beta}\frac{B}{A}\hat{k} + 0\cdot\hat{h} \\
\frac{d}{dt}\hat{x} &= 0\cdot\hat{q} + (\rho+g)\,\hat{x} + 0\cdot\hat{k} - (\rho+g)\,\hat{h} \\
\frac{d}{dt}\hat{k} &= \frac{\bar{q}}{\delta}\cdot\hat{q} + 0\cdot\hat{x} + 0\cdot\hat{k} + 0\cdot\hat{h} \\
\frac{d}{dt}\hat{h} &= 0\cdot\hat{q} + \left(\frac{\rho+g}{\gamma}\frac{1-\beta}{\alpha-\beta}\frac{1}{A} - \frac{\rho+g}{\gamma}\right)\hat{x} - \frac{\rho+g}{\gamma}\frac{B}{A}\frac{1-\beta}{\alpha-\beta}\hat{k} + 0\cdot\hat{h},
\end{aligned}$$

where $A = \frac{1-\beta-B}{\alpha-\beta}$ and $B = 1 + \frac{1-\beta}{(1-\lambda)(\beta-\alpha)}$, so $\frac{1-\beta}{\alpha-\beta}\frac{1}{A} - 1 = \frac{B}{1-\beta-B} = \frac{B}{A}\frac{1}{\alpha-\beta}$. The transition matrix can be written as[10]

$$\mathbf{A} = \begin{pmatrix} \rho - g & \frac{\bar{r}}{\bar{q}}\frac{\beta}{\alpha-\beta}\frac{1}{A} & -\frac{\bar{r}}{\bar{q}}\frac{\beta}{\alpha-\beta}\frac{B}{A} & 0 \\ 0 & \rho + g & 0 & -(\rho+g) \\ \frac{\bar{q}}{\delta} & 0 & 0 & 0 \\ 0 & \frac{\rho+g}{\gamma}\frac{B}{A}\frac{1}{\alpha-\beta} & -\frac{\rho+g}{\gamma}\frac{B}{A}\frac{1-\beta}{\alpha-\beta} & 0 \end{pmatrix}. \tag{22}$$

The stability of the system is determined by the signs of (the real part of its) eigenvalues, while general solutions can be obtained as linear combinations of its eigenvectors. Given that the investment and consumption optimization problem is subject to a transversality condition, two initial conditions (on h and k) pin down the system. This means that we must have two stable (with a negative real part) and two unstable eigenvalues.

4.2 Signing impulse responses

4.2.1 Control variables: x and q

The matrix $\mathbf{A}$ must have two convergent and two divergent eigenvalues, since the system is pinned down by two initial conditions (for capital and money) and two terminal conditions (coming from the transversality conditions of consumer and investor optimization). Denote the two eigenvectors corresponding to the convergent roots by $\mathbf{v}_1$ and $\mathbf{v}_2$. Then

$$\left(\hat{k}, \hat{h}, \tilde{q}, \hat{x}\right)_0 = C\mathbf{v}_1 + D\mathbf{v}_2.$$

Coefficients C and D are set by the two initial conditions, so they can be expressed as linear combinations of $\hat{k}_0$ and $\hat{h}_0$. Then $\hat{q}_0$ and $\hat{x}_0$ are also linear combinations, so

$$\hat{x}_0 = c\hat{h}_0 + d\hat{k}_0,$$

where c and d are (known) functions of the two eigenvectors. We need to show that c is positive, which follows easily if one uses that $\mathbf{A}\mathbf{v}_i = \lambda_i \mathbf{v}_i$. As shown by the Appendix in details, this is true regardless of the ranking of α and β, the intensity differential between the two sectors. Since the change in savings ($\frac{d}{dt}\hat{h}$) is negatively proportional to the change in $\hat{x}$, there is a fall in domestic savings.

[10] When $\varepsilon \neq 0$, one needs to replace the terms $\rho + g$ (appearing three times) with $\rho + g + \varepsilon$.

In a similar fashion, one can show that d is always positive. Consequently, an increase in the capital stock leads to more spending. An increase in money holdings (measured in euros, thus it can be either an increase in H or a decrease in e, or both) decreases $\hat{q}_0$ and thus depresses investment as long as the nontraded sector is more labor intensive. This means that there is a gradual decline, then a reversal in $\hat{k}$ after an increase in $\hat{h}$. This already establishes the "Feldstein-Horioka" result of the model: doimestic savings and investment move in the same direction. Finally, an increase in the capital stock unambiguously decreases $\hat{q}$, thus leads to less investment.

4.2.2 Propagation: w and p

The persistence of the effects depend on the size of the stable eigenvalues. Since we cannot solve for their values analytically, we cannot make any explicit statement about the determinants of the speed of adjustment. One observation is key, however: it is easy to see that

$$\hat{w} = \frac{1-\beta}{\alpha-\beta}\hat{p}.$$

It is also true that $\hat{p}$ has the same sign as $\hat{x}$ (see the Appendix for details):

$$\hat{p} = \frac{1}{A}\hat{x} - \frac{B}{A}\hat{k},$$

where A is always positive. Consequently, if there is an increase in money holdings, x and p increase, and w increases if $\alpha > \beta$ – i.e., when the nontraded sector is more labor intensive. This is the propagation mechanism: nominal spending increases, but nominal income also goes up, so the adjustment through the trade balance is slower than without the change in wages. If $\alpha < \beta$, there is an even faster return to equilibrium, since spending and income move in opposite directions. From here on, we concentrate only on the case where the nontraded sector is more labor-intensive ($\alpha > \beta$).

4.2.3 Further variables

The response of capital-labor ratios is described by

$$\hat{k}_T = \hat{k}_{NT} = \frac{1}{\alpha-\beta}\hat{p},$$

which has the same sign as $\alpha - \beta$. If the nontraded sector is more labor-intensive, both sectors substitute away from labor towards capital, since labor is more expensive ($\hat{w} > 0$) and capital is cheaper:

$$\hat{r} = \frac{-\beta}{\alpha - \beta}\hat{p}.$$

An increase in sectorial capital-labor ratios seems to be at odds with a fixed labor force and a slowly declining capital stock, but the two are fully consistent, given that the more labor-intensive nontraded sector expands on the expense of the more capital-intensive traded sector.

There is a reallocation of both capital and labor towards the nontraded sector:

$$\hat{l} = \frac{1 - \alpha + (\alpha - \beta)\lambda}{(\alpha - \beta)(\lambda - 1)}\left(\hat{k} - \frac{1}{\alpha - \beta}\hat{p}\right),$$

which is positive if $\alpha > \beta$, there is an increase in employment in the nontraded sector; and

$$\hat{K}_{NT} = \hat{k}_{NT} + \hat{l} > 0,$$

there is more capital in the nontraded sector, so there must be less capital in the traded sector.

Regarding consumption, it goes up in both sectors, so nontraded production must also increase:

$$\begin{aligned}
\hat{c}_{NT} &= \hat{y}_{NT} = \hat{l} + \frac{1-\alpha}{\alpha - \beta}\hat{p} > 0 \\
\hat{c}_T &= \hat{c}_{NT} + \hat{p} > 0,
\end{aligned}$$

but there is a fall in traded production, given that there is less capital and labor in that sector. This also implies a trade deficit, which must equal the change in money holdings.

The last variable of interest is the total output of the economy (GDP). Given that the two sectors move in opposite directions, and there is also a change in the relative price, one might not get an unambiguous effect. Moreover, one can consider two different measures even for real GDP (i.e., GDP measured in foreign currency), one using the current relative price of the two sectors ("real GDP"), and the other using the steady state relative price $\bar{p}$ ("fixed-price GDP"). Starting with this latter measure, the absolute change equals

$$\hat{k}_T \lambda (1 - \beta)\bar{k}_T^{1-\beta},$$

which is positive. Using the current value of p (instead of $\bar{p}$), one gets an extra term $\hat{p}\bar{y}_{NT}$, which is always positive.

4.3 The effect of the rate of depreciation

Suppose that there is a change in the rate of depreciation $\varepsilon = \frac{\dot{e}}{e}$. It has two effects: one is through changing (16), the law of motion for x, thus altering the eigenvalues λ_1, λ_2 and also the coefficients c, d, e and f. The second effect is through altering the steady state level of money holdings:

$$\bar{h} = \gamma \frac{\bar{w}}{\rho + g + \varepsilon}.$$

As argued before, the steady state wage level is independent from the exchange rate (both from the level and the slope), so an increase in ε brings about a fall in desired money holdings. Keeping the original level of money fixed, this leads to an increase in $\hat{h}_0$.

The overall effect thus depends on the sum of these two. Intuitively, it seems clear that the effect through changing the coefficients c, d, e, f is relatively small, since the transition matrix is linear in ε (see equation (22)). The effect on $\hat{h}_0$, on the other hand, is sizable, given that ρ, g and ε are all comparable small numbers. One would thus expect the increase in $\hat{h}_0$ to dominate. The choice of $\alpha = 0.8$, $\beta = 0.5$, $\lambda = 1/3$, $\rho = r^* = 0.05$,$g = 0.02$, $\gamma = 0.02$, $\delta = 3$, $\varepsilon = 0.03$ illustrates this point:

$$\hat{h}_0' = \frac{h_0 - 0.7\bar{h}}{0.7\bar{h}} = \hat{h}_0 \frac{10}{7} + \frac{3}{7} = -0.34.$$

Calculating c, d, e and f for the two scenarios and then plugging in for $\hat{x}_0$ and $\hat{q}_0$, we get that $\hat{x}_0' - \hat{x}_0 = 0.012$, $\hat{q}_0' - \hat{q}_0 = -0.005$. It means that nominal expenditures increase and the return on capital (thus investment) falls. The fall in steady state money dominates, the economy becomes more overvalued. The dynamic effect might look somewhat different, since λ_1, λ_2 also change – numerically, the results show that after almost 14 years, a depreciation in fact implies a higher q, but the ordering of all other variables remains unchanged. The path of capital, however, remains strictly below the $\varepsilon = 0$ trajectory. Given that the steady state level of capital is unchanged, the reversal in q is hardly surprising:

4.4 Interpretation of the results

The qualitative implications of a nominal appreciation (an increase in $\hat{h}$) are easy to interpret intuitively. A revaluation increases the value of H in terms of traded goods. This leads to an increase in spending on nontraded goods, which increases nontraded production. Given the short-run nonlinearity of the transformation curve, nontraded prices must increase. This is the dominant shock to the economy, all of the other results can be traced to this through the Stolper-Samuelson theorem: if the price of a sector increases, it leads to a more than proportional increase in the price of the factor which is used more intensively by the windfall sector. The price of the other factor of production decreases. In our case, the price of the nontraded sector has increased, and it is more labor-intensive. This leads to a rise in wages and a fall in rental rates. Production becomes more capital-intensive, and the fall in rental rates decreases capital inflows (q falls).

What makes this situation persistent? The explanation is closely related to the phenomenon of the "Dutch disease" (see Krugman (1987), for example): a country receiving a transfer also sees its relative wages (terms of trade) improving. The extra consumption enabled by the transfer falls partly on nontradables, which pushes domestic wages up. In our two factor model, we need some extra conditions for the transfer effect: if the only source of income of domestic consumers is their labor earnings, then the nontraded sector must be more labor intensive than the traded sector. The price of capital falls, but that does not influence domestic spending.

This is the underlying propagation mechanism: the initial shock to consumption increases domestic income, so the excess money stock will flow out only slowly. If some of the capital is domestic, and its income is used for consumption expenditures, then excess spending still creates some of its excess income, but to a smaller degree. In this case, we can get persistence even without the labor intensity assumption.

All these are fully consistent with international trade theory: as long as capital is scarce, it has a high factor price. In the flexible Balassa-Samuelson model, an increase in world interest rates increases the relative price of that sector which uses capital more intensively (inverse Stolper-Samuelson theorem).

It is clear that the parameter γ plays an important role in determining the speed of adjustment through the trade balance: excess spending is proportional to $\gamma\frac{x^2}{h}$, so a small γ leads to a slow outflow of the extra money. Another important determinant of persistence is the weight of nontradables in consumption expenditures, since the larger it is, the more valid the Keynesian thesis that "excess demand creates its supply".

It is important to note that the sectorial labor intensity assumption is not relevant for the increase of the price of nontradables. Its role is to make the price of capital fall and wages increase (through the Stolper-Samuelson theorem). The "wealth effect" of a revaluation hurts or benefits capital (investment), depending on relative factor intensities. We have explored a scenario with the traded sector being more labor intensive. Nontraded prices increased, wages fell, the rental rate increased, and capital accumulation accelerated.

The degree of substitutability between the two goods (by consumers) and the factors of production (by producers) also influences the quantitative behavior of the economy. Starting with the preference side, let us assume that consumption utility is

$$\left((1-\lambda)^{\frac{1}{\theta}} C_T^{\frac{\theta-1}{\theta}} + \lambda^{\frac{1}{\theta}} C_{NT}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}.$$

The choice of $\theta = 1$ corresponds to the original Cobb-Douglas specification. Suppose that $\theta > 1$. An increase in p then implies a larger substitution towards traded goods, so an increase in consumption expenditure must lead to a smaller increase in C_{NT} and p. Keeping the same transformation curve between traded and nontraded goods, a smaller price increase leads to a smaller wage increase and a smaller decrease of the rental rate. This muted impact effect also weakens the endogenous persistence of the shock, since a smaller wage increase leads to a faster outflow of excess money. In summary, a higher degree of substitutability between traded and nontraded goods increases both the impact effect and the persistence of nominal shocks on the real economy. Conversely, $\theta < 1$ increases both the impact effect and the persistence. One can even find parameters such that a nominal appreciation initially *improves* the trade balance[11] (wages increase more than one in one relative to the nominal exchange rate). Later on, the corresponding decline in r and K leads to a fall in w, and excess money flows out in the long-run.

Intuitively, one would expect the opposite impact of substitutability between factors of production: if it is easy to substitute labor with capital, then the same price increase leads to a smaller wage increase. Consequently, the same increase in nontraded expenditure (pC_{NT}) leads to a smaller increase in p and w, thus a smaller impact effect and smaller persistence. The combination of nonunit substitutability both in preferences and technology would produce very complicated general equilibrium cross-effects, as indicated by the early analysis of Harberger (1962).

One comment is in line here, about the large sectorial reallocations showed by the results. These are the consequences of the assumption that only the cross-border adjustment of capital is slow. The price of domestic labor and installed domestic capital is equalized between the two sectors, so there is free sectorial mobility. In reality, all of these adjustments should be gradual, leading to sectorial wage and rental rate differences. This is likely to increase the impact effect of the nominal shock (nontraded prices must increase even more, since near fixed capital and labor imply a larger increase in the cost of production), but its persistence should decrease – by the time labor starts to switch sectors, the price and wage differentials have been nearly

[11] Balsam and Eckstein (2001) also find that by varying three parameters of a CES aggregate and the share of nontradables in government consumption, one can get a procyclical trade balance.

eliminated. Our results still issue a warning about the direction of asymmetries between sectors and factors, and indicate the direction of reallocation.

One can reinterpret the "money effect" as a "wealth effect", or even as a portfolio resizing and rebalancing effect. As shown by Lane and Milesi-Ferretti (2004), nominal exchange rate movements can have large implications on the net external position of a country. The common feature is that a nominal shock will influence the value or the returns of nominal wealth/assets, leading to a change in the intertemporal behavior of consumers. Such a change will translate into a change in consumption expenditures, which is the necessary starting point of the model.

5 Some concluding comments

This paper presents a simple theoretical model that reproduces many stylized aspects of a response to a large nominal exchange rate shock. The model also gives rise to a lasting real effect of nominal shocks without price or wage setting frictions. It is essentially a flexible price, two-sector (traded and nontraded), two-factor small open economy growth model enriched with money-in-the-utility. Due to this latter feature, a nominal appreciation pushes up nominal expenditures. As the growth model features gradual capital accumulation, the short-run transformation curve is nonlinear, thus higher spending on nontradables implies an increase in its relative price. On the other hand, excess tradable consumption can be met from imports, leading to a trade deficit.

This impact effect already highlights the special role of the nontraded sector: any increase in demand must be met from increased supply, which leads to a change in prices. Nontraded goods, however, also play a role in the propagation mechanism: higher nontraded prices create extra domestic income, validating some of the initial increase in expenditures. Overall, the model highlights that real exchange rate developments have deep two-sector, two-factor, open-economy determinants – in particular, adding money-in-the-utility and q-theory to a standard two-sector, two-factor open economy model is enough for short-run non-neutrality. Another notable result is the comovement of investment and savings after a nominal shock, even though investment is financed exclusively form the world capital market. The crucial step is that the nominal exchange rate influences traded prices, while money (or more generally, fixed income securities) are fixed in local currency. In a sense, these assets can be viewed as an "original nominal stickiness".

The results are particularly relevant for understanding the effects of nominal exchange rate movements, or the choice of the euro conversion rates for EMU candidates. The framework can also be utilized in assessing the price level implications of fiscal or income shocks. From a theory point of view, it also embeds a Balassa-Samuelson effect with gradual capital movements, thus a temporary role for demand. Finally, our results show that a multisector model with money-in-the-utility and any real friction that makes the short-run transformation curve nonlinear already implies short- and medium-run non-neutrality of monetary policy. Adding price or wage setting frictions definitely increases the realism, fit and persistence of such a model, but one has to be careful in evaluating the role of price and wage setting in the results.

On the empirical side, there is vast literature on exchange rate based stabilizations (for example, Reinhart and Végh (1995), Rebelo and Végh (1995)), their stylized facts, the sources of success or failure. Burstein et al (2002) also document similar (though naturally of opposite sign) effects about large devaluation episodes. Darvas (2003) identifies two main groups of countries who experienced large (nominal and real) appreciations at the start of disinflations: fixed exchange rate, mostly Latin American countries with a failure in their stabilization programs, and floating countries, mostly industrial, with a success. A similar difference is documented by Detragiache and Hamann (1997), between emerging economies and Greece, Ireland, Italy and Portugal.

According to our results, a nominal appreciation has a negative side effect, through nontraded prices, and under certain factor intensity assumptions, also through wages. If nontraded inflation or wage inflation remains high, then expected inflation may adjust only little, thus making the success of the disinflation less probable. Naturally, there are many additional issues and considerations determining the overall success or failure (credibility, fiscal side, just to name a few), but it would worth exploring whether differences in country experiences can be related to differences in the strength of the mechanisms of our model.

Such a variation can come from the role of money/wealth directly (like the parameter γ), or from different capital adjustment costs, production functions, factor intensities, preferences, intertemporal and intratemporal substitutability of goods. The distance from steady state (industrial country or emerging market), the role of capital income within GNP, or the external portfolio position of the country may also matter. For such an analysis, it is central to have data on wages and nontraded-traded relative prices. Another impediment of such a cross-country comparison is the stance of fiscal policy. Overall, successful countries did not tend to "match" the appreciation with a fiscal expansion, which is much less true for failure countries. As argued earlier, a fiscal expansion has qualitatively similar implications for relative prices and factor rewards as an appreciation; consequently, its effect has to be filtered out. Unfortunately, this might require extensive data on government behavior, and its result might be sensitive to parameters and model specification.

References

[1] Aguiar, M. and G. Gopinath (2004): Emerging Market Business Cycles: The Cycle is the Trend, mimeo, University of Chicago, 2004.

[2] Balsam, A. and Z. Eckstein (2001): Real Business Cycles in a Small Open Economy with Non-Traded Goods, Working Paper 2001/3, Tel Aviv University.

[3] Benczúr, P. (2003): Real Effects of Nominal Shocks: a 2-sector Dynamic Model with Slow Capital Adjustment and Money-in-the-utility, MNB Working Paper 2003/9.

[4] Benigno, P (2003): Optimal Monetary Policy in a Currency Area, Journal of International Economics, 2003.

[5] Burstein, A., Eichenbaum, M., and S. Rebelo (2002): Why is Inflation so Low after Large Devaluations?, NBER Working Paper 8748, 2002.

[6] Christiano, L., M. Eichenbaum and Ch. Evans (2001): Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy, NBER Working Paper 8403, 2001.

[7] Darvas, Zs. (2003): Analysis of large real exchange rate appreciations; MNB Background Studies, 2003/2.

[8] Detragiache, E. and A. J. Hamann (1997): Exchange-Rate-Based Stabilization in Western Europe: Greece, Ireland, Italy and Portugal, IMF Working Paper, 1997.

[9] Dornbusch. R. (1980): Open Economy Macroeconomics, Chapter 6, Basic Books. 1980.

[10] Dornbusch, R. and M. Mussa (1975): Consumption, Real Balances and the Hoarding Function, International Economic Review, 1975.

[11] Eichenbaum, M. and J. D. M. Fisher (2004): Evaluating the Calvo Model of Sticky Prices, NBER Working Paper 10617, 2004.

[12] Feldstein, M. and C. Horioka (1980): Domestic Savings and International Capital Flows, Economic Journal, 1980.

[13] Fiorito, R. and T. Kollintzas (1994): Stylized Facts of Business Cycles in the G7 from a Real Business Cycles Perspective, European Economic Review, 1994.

[14] Gourinchas, P. O. and H. Rey (2004): International Financial Adjustment, mimeo, Berkeley and Princeton.

[15] Hamann, A. J. (2001): Exchange-Rate-Based Stabilization: A Critical Look at the Stylized Facts, IMF Staff Papers, 2001.

[16] Harberger, A. C. (1962): The Incidence of the Corporation Income Tax, Journal of Political Economy, 1962.

[17] Huffman, G. W. and M. A. Wynne (1999): The Role of Intratemporal Adjustment Costs in a Multisector Economy, Journal of Monetary Economics, 1999.

[18] Krugman. P. (1987): The Narrow Moving Band, the Dutch Disease, and the Competitive Consequences of Mrs. Thatcher: Notes on Trade in the Presence of Scale Dynamic Economies, Journal of Development Economics, 1987.

[19] Lane, P. and G. M. Milesi-Ferretti (2004): Financial Globalization and Exchange Rates, mimeo, Trinity College and IMF.

[20] Leamer, E.E. and J. Levinsohn (1995): International Trade Theory: The Evidence, in Handbook of International Economics III, Elsevier, 1995.

[21] Obstfeld, M. and K. Rogoff (1996): Foundations of International Macroeconomics, The MIT Press, 1996.

[22] Rebelo, S. and C. A. Végh (1995): Real Effects of Exchange-Rate Based Stabilization: An Analysis of Competing Theories, in NBER Macroeconomics Annual, 1995.

[23] Reinhart, C. M. and C. A. Végh (1995): Do Exchange Rate-Based Stabilizations Carry the Seeds of Their Own Destruction? Presented at the 1996 AEA meetings; mimeo, IMF and UCLA.

[24] Taylor, J. (1980): Aggregate Dynamics and Staggered Contracts, Journal of Political Economy, 1980.

[25] Woodford, M. (2003): Interest and Prices: Foundations of a Theory of Monetary Policy, Princeton University Press, 2003.

Appendix

Loglinearization

From firm-level profit maximization (9)-(10):

$$\begin{aligned}
r = (1-\beta)\, k_T^{-\beta} &\implies \hat{r} = -\beta \hat{k}_T \\
w = \beta k_T^{1-\beta} &\implies \hat{w} = (1-\beta)\, \hat{k}_T \\
k_{NT} = \frac{1-\alpha}{\alpha}\frac{\beta}{1-\beta} k_T &\implies \hat{k}_{NT} = \hat{k}_T \\
p_{NT} = \frac{\beta}{\alpha}\left(\frac{1-\alpha}{\alpha}\frac{\beta}{1-\beta}\right)^{\alpha-1} k_T^{\alpha-\beta} &\implies \hat{p}_{NT} = (\alpha-\beta)\, \hat{k}_T .
\end{aligned}$$

One can thus express everything in terms of $\hat{p}$:

$$\hat{k}_T = \frac{1}{\alpha-\beta}\hat{p} \tag{23}$$

$$\hat{k}_{NT} = \frac{1}{\alpha-\beta}\hat{p} \tag{24}$$

$$\hat{w} = \frac{(1-\beta)}{\alpha-\beta}\hat{p} \tag{25}$$

$$\hat{r} = \frac{-\beta}{\alpha-\beta}\hat{p}. \tag{26}$$

Notice that $\hat{p}$ can also be interpreted as the misalignment of the real exchange rate (the traded-nontraded relative price). Equations (23)-(26) thus express the current capital intensities and factor prices (the technology side of the economy) as a function of the real exchange rate's misalignment.

Next, capital accumulation is driven by

$$\begin{aligned}
\frac{\dot{k}}{k} &= \frac{q-\bar{q}}{\delta} \\
\frac{d}{dt}\hat{k} &= \hat{q}\frac{\bar{q}}{\delta},
\end{aligned} \tag{27}$$

and the evolution of Tobin's q follows

$$\begin{aligned}
\dot{q} &= \rho q - r - \frac{(q-1)^2}{2\delta} \\
\frac{d}{dt}\hat{q} &= \frac{\dot{q}}{\bar{q}} = \rho\hat{q} - \frac{\bar{r}}{\bar{q}}\hat{r} - \frac{\bar{q}-1}{\delta}\hat{q} = \hat{q}\,(\rho-g) + \frac{\bar{r}}{\bar{q}}\frac{\beta}{\alpha-\beta}\hat{p}.
\end{aligned} \tag{28}$$

Money accumulation is governed by

$$\dot{h} = w - gh - x + \tau = w - x,$$

using that the stance of fiscal policy is described by $\tau = gh$.

Then we can loglinearize the money accumulation equation:

$$\frac{\dot{h}}{\bar{h}} = \frac{w}{\bar{w}}\frac{\bar{w}}{\bar{h}} - \frac{\bar{x}}{\bar{h}}\frac{x}{\bar{x}} = \hat{w}\frac{\bar{w}}{\bar{h}} - \frac{\bar{x}}{\bar{h}}\hat{x} \tag{29}$$

Using the steady state relations (19) and (20), we get

$$\frac{d}{dt}\hat{h} = \frac{\rho + g}{\gamma}\frac{1-\beta}{\alpha-\beta}\hat{p} - \frac{\rho+g}{\gamma}\hat{x}. \tag{30}$$

We still need to obtain an expression for $\frac{d}{dt}\hat{x}$, plus express $\hat{p}$ in terms of the other hat variables (it will be a function of $\hat{x}$ and $\hat{k}$). Then we have the loglinearization of entire system, with 2 state and 2 jumping variables: k and h are state variables (initial conditions), while x and q are jumping variables. The first corresponds to effective nominal expenditures. The reason for changing variables (from $\hat{p}$ to $\hat{x}$) is that the law of motion for $\hat{p}$ is too complicated, while we get an additional zero element in the transition matrix with $\hat{x}$.

Loglinearizing (16):

$$\begin{aligned} \dot{x} &= -(\rho+g)\,x + \gamma\frac{x^2}{h} \\ \frac{\dot{x}}{\bar{x}} &= -(\rho+g)\,\hat{x} + \gamma\frac{\bar{x}}{\bar{h}}\left(2\hat{x} - \hat{h}\right) \end{aligned}$$

Using that in steady state, nominal expenditures equal wages ($\bar{x} = \bar{w}$) and the other steady state conditions, we get

$$\frac{d}{dt}\hat{x} = (\rho+g)\,\hat{x} - (\rho+g)\,\hat{h}.$$

The very last thing is to get $\hat{p}$. Loglinearize the definition of x :

$$\hat{x} = \hat{c} + (1-\lambda)\,\hat{p}.$$

Using the definition of c and the consumption optimality condition (2), we get

$$\begin{aligned} c &= c_T^{\lambda} c_{NT}^{1-\lambda} = \left(\frac{\lambda}{1-\lambda}\right)^{\lambda} p^{\lambda} c_{NT} \\ \hat{x} &= \hat{p} + \hat{c}_{NT}. \end{aligned}$$

From market clearing in nontraded goods:

$$\begin{aligned} c_{NT} &= l k_{NT}^{1-\alpha} \\ \hat{c}_{NT} &= \hat{l} + \frac{1-\alpha}{\alpha-\beta}\hat{p} \\ \hat{x} &= \frac{1-\beta}{\alpha-\beta}\hat{p} + \hat{l}. \end{aligned}$$

Combining capital market clearing with producer maximization, we get

$$l = \frac{k - k_T}{k_{NT} - k_T} = \frac{1}{1 - \frac{\alpha}{1-\alpha}\frac{1-\beta}{\beta}} \left(\frac{k}{k_N} - \frac{\alpha}{1-\alpha}\frac{1-\beta}{\beta} \right).$$

Loglinearization then yields

$$\begin{aligned} \bar{l} + dl &= \frac{1}{1 - \frac{\alpha}{1-\alpha}\frac{1-\beta}{\beta}} \left(\frac{\bar{k}}{\bar{k}_{NT}} \left(1 + \hat{k} - \hat{k}_{NT} \right) - \frac{\alpha}{1-\alpha}\frac{1-\beta}{\beta} \right) \\ \hat{l} &= \frac{\bar{k}/\bar{k}_{NT}}{\bar{k}/\bar{k}_{NT} - \frac{\alpha}{1-\alpha}\frac{1-\beta}{\beta}} \left(\hat{k} - \hat{k}_{NT} \right) = \frac{\bar{k}/\bar{k}_{NT}}{\bar{k}/\bar{k}_{NT} - \frac{\alpha}{1-\alpha}\frac{1-\beta}{\beta}} \left(\hat{k} - \frac{1}{\alpha-\beta}\hat{p} \right), \qquad (31) \end{aligned}$$

and finally,

$$\hat{x} = \frac{1-\beta}{\alpha-\beta}\hat{p} + \frac{\bar{k}/\bar{k}_{NT}}{\bar{k}/\bar{k}_{NT} - \frac{\alpha}{1-\alpha}\frac{1-\beta}{\beta}} \left(\hat{k} - \frac{1}{\alpha-\beta}\hat{p} \right).$$

In steady state:

$$\begin{aligned} \bar{k}_T &= \left(\frac{1-\beta}{\bar{r}} \right)^{1/\beta} \\ \bar{k}_{NT} &= \bar{k}_T \frac{1-\alpha}{\alpha}\frac{\beta}{1-\beta} \\ \bar{p} &= \frac{\beta}{\alpha} \left(\frac{1-\alpha}{\alpha}\frac{\beta}{1-\beta} \right)^{\alpha-1} \left(\frac{1-\beta}{\bar{r}} \right)^{\frac{\alpha-\beta}{\beta}} \\ \bar{c} &= \left(\frac{\lambda}{1-\lambda} \right)^{\lambda} (\bar{p})^{\lambda} \, \bar{l}\bar{k}_{NT}^{1-\alpha} \\ \bar{w} &= \beta \bar{k}_T^{1-\beta} = \chi \bar{p}^{1-\lambda}\bar{c}/\gamma \end{aligned}$$

Plugging everything into this last expression yields

$$\alpha(1-\lambda) = \bar{l},$$

which implies

$$\frac{k}{k_{NT}} = \frac{\alpha(1-\beta)}{\beta(1-\alpha)} \frac{1+\alpha\lambda-\beta\lambda-\alpha}{1-\beta}, \tag{32}$$

so the expression for $\hat{x}$ is

$$\begin{aligned}
\hat{x} &= \frac{1-\beta}{\alpha-\beta}\hat{p} + \left(1 + \frac{1-\beta}{(1-\lambda)(\beta-\alpha)}\right)\left(\hat{k} - \frac{1}{\alpha-\beta}\hat{p}\right) \\
\hat{x} &= A\hat{p} + B\hat{k}
\end{aligned}$$

which can be inverted to

$$\hat{p} = \frac{1}{A}\hat{x} - \frac{B}{A}\hat{k}.$$

The log-linearized dynamic system is therefore

$$\begin{aligned}
\frac{d}{dt}\hat{q} &= (\rho-g)\,\hat{q} + \frac{\bar{r}}{\bar{q}}\frac{\beta}{\alpha-\beta}\frac{1}{A}\hat{x} - \frac{\bar{r}}{\bar{q}}\frac{\beta}{\alpha-\beta}\frac{B}{A}\hat{k} + 0\cdot\hat{h} \\
\frac{d}{dt}\hat{x} &= 0\cdot\hat{q} + (\rho+g)\,\hat{x} + 0\cdot\hat{k} - (\rho+g)\,\hat{h} \\
\frac{d}{dt}\hat{k} &= \frac{\bar{q}}{\delta}\cdot\hat{q} + 0\cdot\hat{x} + 0\cdot\hat{k} + 0\cdot\hat{h} \\
\frac{d}{dt}\hat{h} &= 0\cdot\hat{q} + \left(\frac{\rho+g}{\gamma}\frac{1-\beta}{\alpha-\beta}\frac{1}{A} - \frac{\rho+g}{\gamma}\right)\hat{x} - \frac{\rho+g}{\gamma}\frac{B}{A}\frac{1-\beta}{\alpha-\beta}\hat{k} + 0\cdot\hat{h}.
\end{aligned}$$

Now $A = \frac{1-\beta-B}{\alpha-\beta}$, so $\frac{1-\beta}{\alpha-\beta}\frac{1}{A} - 1 = \frac{B}{1-\beta-B} = \frac{B}{A}\frac{1}{\alpha-\beta}$, thus the transition matrix is

$$\begin{pmatrix}
\rho-g & \frac{\bar{r}}{\bar{q}}\frac{\beta}{\alpha-\beta}\frac{1}{A} & -\frac{\bar{r}}{\bar{q}}\frac{\beta}{\alpha-\beta}\frac{B}{A} & 0 \\
0 & \rho+g & 0 & -(\rho+g) \\
\frac{\bar{q}}{\delta} & 0 & 0 & 0 \\
0 & \frac{\rho+g}{\gamma}\frac{B}{A}\frac{1}{\alpha-\beta} & -\frac{\rho+g}{\gamma}\frac{B}{A}\frac{1-\beta}{\alpha-\beta} & 0
\end{pmatrix}.$$

The stability of the system is determined by the signs of (the real part of its) eigenvalues, while general solutions can be obtained as linear combinations of its eigenvectors. Given that the investment and consumption optimization problem is also subject to a transversality condition, two initial conditions (on h and k) pin down the system. This means that we must have two stable (with a positive real part) and two unstable eigenvalues.[12]

[12] When $\varepsilon \neq 0$, one needs to replace the terms $\rho+g$ (appearing three times) with $\rho+g+\varepsilon$.

Signing impulse responses

With two stable and two divergent roots, the general solution can be written as the linear combinations of the two basic solutions $\boldsymbol{v}_1 e^{\lambda_1 t}$ and $\boldsymbol{v}_2 e^{\lambda_2 t}$ (the eigenvectors):

$$\begin{aligned}
\hat{q} &= C v_{11} e^{\lambda_1 t} + D v_{12} e^{\lambda_2 t} \\
\hat{x} &= C v_{21} e^{\lambda_1 t} + D v_{22} e^{\lambda_2 t} \\
\hat{k} &= C v_{31} e^{\lambda_1 t} + D v_{32} e^{\lambda_2 t} \\
\hat{h} &= C v_{41} e^{\lambda_1 t} + D v_{42} e^{\lambda_2 t}.
\end{aligned}$$

We have initial conditions on $\hat{k}$ and $\hat{h}$, which pin down C and D. It is immediate to see that

$$\begin{aligned}
C &= \frac{-\hat{k}_0 v_{42} + v_{32}\hat{h}_0}{v_{41}v_{32} - v_{31}v_{42}} \\
D &= \frac{-\hat{h}_0 v_{31} + v_{41}\hat{k}_0}{v_{41}v_{32} - v_{31}v_{42}}.
\end{aligned}$$

Then the first two equations describe how the initial values of the jumping variables depend on initial conditions:

$$\begin{aligned}
\hat{q} &= \frac{v_{32}v_{11} - v_{31}v_{12}}{v_{41}v_{32} - v_{31}v_{42}}\hat{h}_0 + \frac{v_{41}v_{12} - v_{42}v_{11}}{v_{41}v_{32} - v_{31}v_{42}}\hat{k}_0 \\
\hat{x} &= \frac{v_{32}v_{21} - v_{31}v_{22}}{v_{41}v_{32} - v_{31}v_{42}}\hat{h}_0 + \frac{v_{22}v_{41} - v_{21}v_{42}}{v_{41}v_{32} - v_{31}v_{42}}\hat{k}_0.
\end{aligned}$$

Each coefficient represents the impact effect of changes in the state variables on q and x (the jumping variables). In particular, $\frac{v_{32}v_{21} - v_{31}v_{22}}{v_{41}v_{32} - v_{31}v_{42}} > 0$ means that an increase in the money stock (which is equivalent to a revaluation of the fixed exchange rate) increases x, which is nominal spending. It is easy to check that an increase in nominal spending increases prices ($A > 0$ regardless of the ranking of α and β). This is the crucial step for the intuitive story: excess money increases nominal spending. After this, the model's entire intuition applies.

So let us examine $\frac{v_{32}v_{21} - v_{31}v_{22}}{v_{41}v_{32} - v_{31}v_{42}}$ first. One cannot solve for the negative eigenvalues of the transition matrix analytically, but for a given eigenvalue λ_i, one can establish links among the coordinates of the corresponding eigenvector $(v_{1i}, v_{2i}, v_{3i}, v_{4i})$:

$$\begin{aligned}
(\rho - g) v_{1i} + \frac{\bar{r}}{\bar{q}} \frac{\beta}{\beta - \alpha} \frac{1}{A} v_{2i} - \frac{\bar{r}}{\bar{q}} \frac{\beta}{\beta - \alpha} \frac{B}{A} v_{3i} &= \lambda_i v_{1i} \\
(\rho + g)(v_{2i} - v_{4i}) &= \lambda_i v_{2i} \\
\frac{\bar{q}}{\delta} v_{1i} &= \lambda_i v_{3i} \\
\frac{\rho + g}{\gamma} \frac{B}{A} \frac{1}{\alpha - \beta} (v_{2i} - (1 - \beta) v_{3i}) &= \lambda_i v_{4i}.
\end{aligned}$$

From the second:

$$v_{4i} = \frac{\rho + g - \lambda_i}{\rho + g} v_{2i},$$

from the fourth:

$$v_{3i} = \underbrace{\left(\frac{1}{1 - \beta} - \lambda_i \frac{\alpha - \beta}{1 - \beta} \frac{A}{B} \gamma \frac{\rho + g - \lambda_i}{\rho + g} \right)}_{C_i} v_{2i}.$$

Since $v_{41} v_{32} - v_{31} v_{42}$ shows up in all four cases, let us factor this term first:

$$v_{41} v_{32} - v_{31} v_{42} = v_{21} v_{22} \left(\frac{\rho + g - \lambda_1}{\rho + g} C_2 - \frac{\rho + g - \lambda_2}{\rho + g} C_1 \right).$$

Expanding the bracket term:

$$\begin{aligned}
& \left(1 - \frac{\lambda_1}{\rho + g}\right) \left(\frac{1}{1 - \beta} - \frac{A}{B} \frac{\alpha - \beta}{1 - \beta} \lambda_2 + \frac{(\alpha - \beta) A}{B (1 - \beta)(\rho + g)} \lambda_2^2 \right) \\
& - \left(1 - \frac{\lambda_2}{\rho + g}\right) \left(\frac{1}{1 - \beta} - \frac{A}{B} \frac{\alpha - \beta}{1 - \beta} \lambda_1 + \frac{(\alpha - \beta) A}{B (1 - \beta)(\rho + g)} \lambda_1^2 \right) \\
= \; & (\lambda_2 - \lambda_1) \frac{1}{(\rho + g)(1 - \beta)} \\
& - (\lambda_2 - \lambda_1) \frac{A}{B} \frac{\alpha - \beta}{1 - \beta} + \frac{(\alpha - \beta) A}{B (1 - \beta)(\rho + g)} \left(\lambda_2^2 - \lambda_1^2\right) - \lambda_1 \lambda_2 (\lambda_2 - \lambda_1) \frac{(\alpha - \beta) A}{B \; (1 - \beta)(\rho + g)} \\
= \; & (\lambda_2 - \lambda_1) \left(\frac{1}{(\rho + g)(1 - \beta)} - \frac{A}{B} \frac{\alpha - \beta}{1 - \beta} + (\lambda_1 + \lambda_2) \frac{(\alpha - \beta) A}{B (1 - \beta)(\rho + g)} - \lambda_1 \lambda_2 \frac{(\alpha - \beta) A}{B \; (1 - \beta)(\rho + g)} \right).
\end{aligned}$$

Now it is easy to see that B has the opposite sign as $\alpha - \beta$,[13] and from the negativity of the real part of λ_1 and λ_2, their sum is negative and their product is positive. Consequently, $v_{41} v_{32} - v_{31} v_{42}$ has the same sign as $v_{21} v_{22} (\lambda_2 - \lambda_1)$.

[13] $1 > \beta\lambda + \alpha(1 - \lambda)$ clearly holds, which implies $(1 - \lambda)(\beta - \alpha) + 1 - \beta > 0$. Dividing by $(1 - \lambda)(\beta - \alpha)$, we get that $B = 1 + \frac{1 - \beta}{(1 - \lambda)(\beta - \alpha)}$ is negative if $\alpha > \beta$, and it is negative if $\alpha < \beta$.

Look at $v_{32}v_{21} - v_{31}v_{22}$ now:

$$\begin{aligned}
v_{32}v_{21} - v_{31}v_{22} &= v_{22}C_2v_{21} - v_{21}C_1v_{22} \\
&= v_{22}v_{21}\left(-\frac{A}{B}\frac{\alpha-\beta}{1-\beta}\lambda_2 + \frac{A}{B}\frac{\alpha-\beta}{1-\beta}\lambda_1 + \frac{(\alpha-\beta)A}{B(1-\beta)(\rho+g)}\lambda_2^2 - \frac{(\alpha-\beta)A}{B(1-\beta)(\rho+g)}\lambda_1^2\right) \\
&= v_{22}v_{21}(\lambda_2-\lambda_1)\left(-\frac{A}{B}\frac{\alpha-\beta}{1-\beta} + (\lambda_1+\lambda_2)\frac{(\alpha-\beta)A}{B(1-\beta)(\rho+g)}\right).
\end{aligned}$$

The bracket term is again positive. Using our earlier result for $v_{41}v_{32} - v_{31}v_{42}$, it is immediate that c is always positive, so $\frac{\partial x_0}{\partial h_0} > 0$. An increase in money holdings leads to an increase in nominal spending and nontradable prices.[14]

Now look at the numerator of $\frac{\partial x_0}{\partial k_0}$:

$$v_{22}v_{41} - v_{21}v_{42} = v_{22}v_{21}(\lambda_2-\lambda_1),$$

so d is unambiguously positive. An increase in the capital stock increases nominal spending.

The impulse response of investment is given by the partial derivatives of q_0. It turns out that we can sign these derivatives easier if we switch back from $\hat{x}$ to $\hat{p} = \frac{1}{A}\hat{x} - \frac{B}{A}\hat{k}$. The laws of motion can be rewritten as (we do not need the equation for $\hat{p}$):

$$\begin{aligned}
\frac{d}{dt}\hat{q} &= (\rho-g)\hat{q} + \frac{\bar{r}}{\bar{q}}\frac{\beta}{\alpha-\beta}\hat{p}u \\
\frac{d}{dt}\hat{k} &= \frac{\bar{q}}{\delta}\hat{q} \\
\frac{d}{dt}\hat{h} &= \frac{\rho+g}{\gamma}\left(\frac{1-\beta}{\alpha-\beta} - A\right)\hat{p} - \frac{\rho+g}{\gamma}B\hat{k}.
\end{aligned}$$

Then the eigenvectors and eigenvalue satisfy the following conditions:

$$\begin{aligned}
(\rho-g)v_{1i} + \frac{\bar{r}}{\bar{q}}\frac{\beta}{\alpha-\beta}v_{2i} &= \lambda_i v_{1i} \\
\frac{\bar{q}}{\delta}v_{1i} &= \lambda_i v_{3i} \\
\left(\frac{\rho+g}{\gamma}\right)\left(\left(\frac{1-\beta}{\alpha-\beta} - A\right)v_{2i} - Bv_{3i}\right) &= \lambda_i v_{4i}.
\end{aligned}$$

Starting with the numerator of $\frac{\partial q_0}{\partial h_0}$:

$$v_{32}v_{11} - v_{31}v_{12} = \frac{\bar{q}}{\delta}\frac{1}{\lambda_2}v_{12}v_{11} - \frac{\bar{q}}{\delta}\frac{1}{\lambda_1}v_{11}v_{12} = v_{11}v_{12}\frac{\bar{q}}{\delta}\frac{\lambda_1-\lambda_2}{\lambda_1\lambda_2}.$$

[14] This argument and all the following remain unchanged when $\varepsilon \neq 0$: all the terms $\rho+g$ are replaced by $\rho+g+\varepsilon$, but that has the same sign (remember that $\rho+g+\varepsilon > 0$ must hold in order for a steady state to exist).

The denominator is

$$
\begin{aligned}
& v_{41}v_{32} - v_{31}v_{42} \\
= \ & \frac{\rho+g}{\gamma\lambda_1}\left(\left(\frac{1-\beta}{\alpha-\beta} - A\right)v_{21} - B\frac{\bar{q}}{\delta}\frac{1}{\lambda_1}v_{11}\right)\frac{\bar{q}}{\delta}\frac{1}{\lambda_2}v_{12} - \frac{\rho+g}{\gamma\lambda_2}\left(\left(\frac{1-\beta}{\alpha-\beta} - A\right)v_{22} - B\frac{\bar{q}}{\delta}\frac{1}{\lambda_2}v_{21}\right)\frac{\bar{q}}{\delta}\frac{1}{\lambda_1}v_{11} \\
= \ & \frac{(\rho+g)\,\bar{q}v_{11}v_{12}}{\gamma\delta\lambda_1\lambda_2} \\
& * \left(\underbrace{\left(\frac{1-\beta}{\alpha-\beta} - A\right)}_{\frac{B}{\alpha-\beta}}\frac{\bar{q}}{\bar{r}}\frac{\alpha-\beta}{\beta}(\lambda_1 - \rho + g) - B\frac{\bar{q}}{\delta}\frac{1}{\lambda_1} - \left(\frac{1-\beta}{\alpha-\beta} - A\right)\frac{\bar{q}}{\bar{r}}\frac{\alpha-\beta}{\beta}(\lambda_2 - \rho + g) + B\frac{\bar{q}}{\delta}\frac{1}{\lambda_2}\right) \\
= \ & \frac{(\rho+g)\,\bar{q}v_{11}v_{12}}{\gamma\delta\lambda_1\lambda_2}B\left(\frac{\bar{q}}{\bar{r}\beta}(\lambda_1 - \lambda_2) - \frac{\bar{q}}{\delta}\frac{\lambda_2 - \lambda_1}{\lambda_1\lambda_2}\right) = \frac{(\rho+g)\,\bar{q}v_{11}v_{12}}{\gamma\delta\lambda_1\lambda_2}B(\lambda_1 - \lambda_2)\left(\frac{\bar{q}}{\bar{r}\beta} + \frac{\bar{q}}{\delta\lambda_1\lambda_2}\right).
\end{aligned}
$$

Using the positivity of $\bar{q}$, δ, γ, $\rho+g$, $\bar{r}$, β, and the negativity of λ_1 and λ_2, we see that the sign of $\frac{\partial q_0}{\partial h_0}$ is determined by B. This means that $\frac{\partial q_0}{\partial h_0} > 0$ is $\alpha < \beta$, and it is negative otherwise. An increase in money holdings hurts investment if the nontraded sector is more labor-intensive.

The calculation of $\partial q/\partial k$ is more problematic. As we have just seen, the denominator has the same sign as $B(\lambda_1 - \lambda_2)\,v_{11}v_{12}$. The numerator is $v_{41}v_{12} - v_{42}v_{11}$. Using the relationships among the coordinates of the eigenvectors:

$$
\begin{aligned}
v_{4i} &= \frac{\rho+g}{\gamma\lambda_i}\left(\left(\frac{1-\beta}{\alpha-\beta} - A\right)v_{2i} - Bv_{3i}\right) = \frac{\rho+g}{\gamma\lambda_i}B\left(\frac{v_{2i}}{\alpha-\beta} - v_{3i}\right) \\
&= \frac{\rho+g}{\gamma\lambda_i}B\left(\frac{\bar{q}}{\beta\bar{r}}(\lambda_i - \rho + g) - \frac{\bar{q}}{\delta\lambda_i}\right)v_{1i}.
\end{aligned}
$$

Consequently,

$$
\begin{aligned}
& v_{41}v_{12} - v_{42}v_{11} \\
= \ & \frac{\rho+g}{\gamma}\bar{q}Bv_{11}v_{12}\left(\frac{\lambda_1 - \rho + g}{\lambda_1\beta\bar{r}} - \frac{1}{\delta\lambda_1^2} - \frac{\lambda_2 - \rho + g}{\lambda_2\beta\bar{r}} + \frac{1}{\delta\lambda_2^2}\right) \\
= \ & \frac{\rho+g}{\gamma}\bar{q}Bv_{11}v_{12}\frac{\lambda_1 - \lambda_2}{\lambda_1\lambda_2}\left(\frac{\rho-g}{\beta\bar{r}} + \frac{1}{\delta}\frac{\lambda_1 + \lambda_2}{\lambda_1\lambda_2}\right).
\end{aligned}
$$

The sign of the ratio is determined by

$$
\frac{\delta(\rho-g)}{\beta\bar{r}} + \frac{\lambda_1 + \lambda_2}{\lambda_1\lambda_2}.
$$

It is negative if

$$
\frac{\delta(\rho-g)}{\beta\bar{r}} < -\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right). \tag{33}
$$

This means that the convergent eigenvalues should be "sufficiently close" to zero. Calculating the characteristic polynomial of the transition matrix yields

$$\gamma Z\left(Z^3-2\rho Z^2+\left(\rho^2-g^2+\frac{\bar{r}\beta B}{\delta\left(1-\beta-B\right)}\right)Z-\frac{\bar{r}\beta B\left(\rho+g\right)}{\delta\left(1-\beta-B\right)}\right)$$
$$+\left(\rho+g\right)^2\frac{B}{A\left(\alpha-\beta\right)}\left(Z^2-\left(\rho-g\right)Z-\frac{\bar{r}}{\delta}\beta\right).$$

The term not multiplied by γ is a downward-looking parabole, with a negative and a positive root. Around its convergent root $\left(\lambda_-=\frac{\rho-g}{2}-\sqrt{\left(\frac{\rho-g}{2}\right)^2+\frac{\bar{r}\beta}{\delta}}\right)$, the value of the term multiplied by γ can be shown to be positive if $\alpha>\beta$. To see this, replace Z^3 by $\left(\rho-g\right)Z^2+\frac{\bar{r}\beta}{\delta}Z$, and then replace Z^2 by $\left(\rho-g\right)Z+\frac{\bar{r}\beta}{\delta}$ to get

$$\frac{\bar{r}\beta}{\delta}\frac{1-\beta}{1-\beta-B}\left(Z-\left(\rho+g\right)\right).$$

Now $Z<0$ and $1-\beta-B=A\left(\alpha-\beta\right)$, which has the same sign as $\alpha-\beta$. Consequently, this part of the characteritsic polynomial is positive if $\alpha>\beta$ and negative otherwise. For small values of γ, this implies that there is a convergent root slightly *below* λ_- if $\alpha>\beta$.

To complete the argument, we show that

$$\frac{\delta\left(\rho-g\right)}{\beta\bar{r}}<\frac{-1}{\lambda_-}. \tag{34}$$

This means that as long as γ is small enough, already one root is sufficient to ensure (33). Consequently, $\partial q/\partial k<0$ for small levels of γ. Intuitively it seems plausible that this feature is also true in general, but we cannot show it for arbitrary values of γ.

Now, (34) is equivalent to (assuming $\rho>g$)

$$\begin{aligned}-\lambda_- &< \frac{\beta\bar{r}}{\delta\left(\rho-g\right)}\\ \sqrt{\left(\frac{\rho-g}{2}\right)^2+\frac{\bar{r}\beta}{\delta}} &< \frac{\beta\bar{r}}{\delta\left(\rho-g\right)}+\frac{\rho-g}{2}\\ \left(\frac{\rho-g}{2}\right)^2+\frac{\bar{r}\beta}{\delta} &< \left(\frac{\rho-g}{2}\right)^2+\frac{\bar{r}\beta}{\delta}+\left(\frac{\beta\bar{r}}{\delta\left(\rho-g\right)}\right),\end{aligned}$$

which obviously holds.

The last thing to check is the evolution of fixed-price GDP:

$$\begin{aligned} y^* &= \bar{p}y_{NT} + y_T = \bar{p}\bar{y}_{NT} + \bar{p}\hat{l}\bar{y}_{NT} + \bar{p}\left(1-\alpha\right)\bar{y}_{NT}\hat{k}_{NT} + \bar{y}_T + \bar{y}_T\left(1-\beta\right)\hat{k}_T - \hat{l}\frac{\bar{l}}{1-\bar{l}}\bar{y}_T \\ &= \bar{y} + \left(\bar{p}\bar{y}_{NT}\left(1-\alpha\right) + \bar{y}_T\left(1-\beta\right)\right)\hat{k}_T + \left(\bar{p}\bar{y}_{NT} - \bar{y}_T\frac{\bar{l}}{1-\bar{l}}\right)\hat{l}. \end{aligned}$$

Using

$$\hat{k}_T = \frac{1}{\alpha-\beta}\hat{p}$$

and combining (31) and (32) into

$$\hat{l} = \frac{1-\alpha+\left(\alpha-\beta\right)\lambda}{\left(\beta-a\right)\left(1-\lambda\right)}\left(\hat{k}-\hat{k}_T\right),$$

one obtains

$$\begin{aligned} y^* - \bar{y} &= \hat{k}_T\left(\bar{p}\bar{y}_{NT}\left(1-\alpha\right) + \bar{y}_T\left(1-\beta\right) - \bar{p}\bar{y}_{NT}\frac{1-\alpha+\left(\alpha-\beta\right)}{\left(\beta-\alpha\right)\left(1-\lambda\right)} + \bar{y}_T\frac{\bar{l}}{1-\bar{\lambda}}\frac{1-\alpha+\left(\alpha-\beta\right)\lambda}{\left(\beta-\alpha\right)\left(1-\lambda\right)}\right) \\ &\quad + \hat{k}\left(\bar{p}\bar{y}_{NT} - \frac{\bar{l}}{1-\bar{l}}\bar{y}_T\right)\frac{1-\alpha+\left(\alpha-\beta\right)\lambda}{\left(\beta-\alpha\right)\left(1-\lambda\right)}. \end{aligned}$$

Using steady state relationships, this reduces to

$$\hat{k}_T\lambda\left(1-\beta\right)\bar{k}_T^{1-\beta} + \hat{k}\left(\beta-\alpha\right)\left(1-\lambda\right)\bar{k}_T^{1-\beta}.$$