Embodied Technical Change and the Persistence of Vacancies*

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Abstract

This paper adresses earlier critiques according to which a prototypical search and matching model with wage setting under Nash bargaining fails to generate the observed strong negative correlation between unemployment and job vacancies at business cycle frequencies and the persistent response of vacancies to aggregate shocks. The setting is a general equilibrium model with capital accumulation and endogenous job destruction.

After a neutral technology shock, vacancies return close to steady state with no delay and are not persistent. Following the shock there is no near perfect negative correlation between vacancies and unemployment.

An investment-specific technology shock has very different implications for the labor market. Such a shock does not improve the productivity of labor on impact. Only new capital goods improve labor productivity over time. The reponse of vacancies to such a shock is persistent. As a result unemployment and job vacancies are strongly negatively correlated.

JEL Codes: E24, E32, J64

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1 Introduction

Over the past decade dynamic general equilibrium models that include unemployment and the time-consuming progress of matching unemployed workers in new jobs as an extra use of time, have booked significant success in understanding the behavior of labor market variables over the business cycle. Merz (1995) and Andalfatto (1996) were among the first to study search in a traditional RBC model with risk aversion and capital accumulation. Their models provide additional amplification by allowing for more elastic labor as workers move between work and job search. Including endogenous job destruction in such models, as is done in den Haan, Ramey and Watson (2000), leads to substantially more amplification.

In a recent paper Shimer (2004) points out that the plain vanilla search model fails to match the Beveridge curve, the observed strong negative correlation between unemployment and job vacancies at business cycle frequencies. This is problematic as the Beveridge curve is one of the key business cycle facts of modern Western economies. According to Shimer there is a lack of propagation in the model as the vacancy-unemployment ratio in the search model fails to respond sufficiently to shocks in labor productivity and job destruction. In the model these shocks are only slightly amplified. In reality, vacancies are significantly more procyclical than the small improvements observed in labor productivity during a boom. Shimer's argument reads as a critique against the Nash bargaining assumption in search models. Nash bargaining means that the match surplus in new jobs is divided proportionally between the worker and the firm. Consider the effects of a labor productivity increase in the case of Nash bargaining. As the supply of jobs is perfectly elastic, firms respond to an increase in profits by creating more vacancies. This raises unemployed workers' meeting rate and hence the value of unemployment, which is also workers' threat point in the bargain. Firms anticipate having to pay higher wages and are reluctant to create new jobs. Nash bargaining implies that workers receive the gains of productivity increases via higher wages and little is left to shift the economy along the Beveridge curve. The volatility of the productivity shock necessary to generate realistic vacancy and unemployment dynamics is implausibly large.

In related work, Fujita (2003) argues that the reason why the basic search and matching model fails to replicate a realistic Beveridge curve lies in the free entry condition in vacancies and the associated "echo effect". With a free entry condition, the expected returns to posting a vacancy are equal to the vacancy cost and firms post vacancies as long as this is expected to be profitable. How does the echo effect work? Consider again the case of a positive productivity shock. Such a shock increases the expected returns of posting. The number of vacancies goes up to satisfy the free-entry condition. However, the decrease in unemployment following the positive productivity shock pushes up employment. The odds for a firm to find an unemployed worker worsen. So there is a clear incentive for a firm to post less vacancies. This is the echo and in the standard Mortensen-Pissarides model, vacancies converge far too fast to the steady state after a shock. Fujita (2004) emphasizes how the echo implies a lack of vacancy persistence in the standard model compared to an estimated trivariate vector autoregression with only sign restrictions on the responses of gross worker flows and employment growth and no restrictions on the behavior of vacancies.

In this paper I extend the canonical search model such that another mechanism than altering the Nash bargaining assumption affects the location of the vacancy-unemployment locus. The setting is a DSGE model with capital accumulation and endogenous job destruction of employment opportunities. Cyclical variation in the job destruction rate affects the capital demand of firms, and leads to strong internal propagation. Following a neutral technology shock, vacancies return to steady state with no delay and are not persistent. There is no strong negative correlation between vacancies and unemployment.

I then study the effects of improvements in investment-specific technology over the business cycle on the labor and capital market. The notion that innovations in the productivity of capital goods have implications for the aggregate economy is hardly a new one. In the General Theory, Keynes (1936) already argues that shocks to the marginal productivity (or efficiency) of investment goods is one of the main propagation mechanism for output fluctuations. Such shocks affect the cost of investment goods relative to consumption goods. The formation of new capital then generates output and employment fluctuations. This mechanism is in sharp contrast from the earlier RBC work where investment reacts to an exogenous shock to the production function (see Greenwood, Hercowitz and Huffman (1988) for an early contribution to the contrary). In the search setting an investment-specific technology shock has, compared to a neutral technology shock, very different implications for key labor market variables. Such a shock does not improve the productivity of labor in impact. Only new capital goods will improve that productivity in the long-run. The response of vacancies to such a shock is persistent and unemployment and job vacancies are strongly negatively correlated.

Section 2 briefly describes some well known stylized facts on labor markets and capital embodied technical change. Section 3 lays out the model. I introduce disembodied and investment-specific technical change in a RBC model with a Pissarides and Mortensen style search-and-matching labor market with endogenous job separation. Section 4 discusses the simulation exercise. Section 5 concludes.

2 Stylized Facts

This section documents some empirical regularities on the price of investment and labor market variables at business cycle frequencies.

2.1 Investment-Specific Technology Shocks

Recent empirical work does not find much support for two commonly stated assumptions of the earlier business cycle literature literature, namely that (i) technological change is homogenous in nature, allowing an economy to produce consumption and investment goods symetrically, and (ii) shocks to aggregate demand drive the business cycles and affect the demand for investment goods. The culprit is the trend and cyclical behavior of the relative price of business equipment since the late 1950s. Ceteris paribus an improvement in investment-specific technology (e.g. a processor's speed doubling every 18 months) lowers the cost of investment goods relative to other goods. As measures for this real price (like Gordon(1990)'s) fell about 200% between 1955 and 2000, economists have a proxy for the substantial investment-specific technological progress over this period.[1] With respect to the trend behavior of this series, Greenwood, Hercowitz and Krusell (1997) examine the role of

[1] Gordon (1990)'s goal is to correct the mismeasurement in equipment price indices due to quality change. He builds quality-adjusted indices for 22 types of equipment and their components. Taking the ratio of his single aggregate index and the corresponding BLS index, he calculates the rate of embodied technical change to be 3.44% for US manufacturing during 1949-1983. Another approach is Sakellaris and Wilson (2004) who estimate the rate of embodied technical change directly from US plant-level manufacturing data for the period 1972-1996. Their average plant estimates for the rate of embodied technical change range between 8 and 17 percent.

capital-embodied technological changes as a source for long term economic growth. They find investment-specific technology shocks to be the unique source of trend in the real price of investment goods. In almost every year since the end of the 1950s business equipment had become cheaper in terms of its value in consumption goods.

More importantly for understanding business cycles, detrending real equipment investment and its real price suggests this is in fact an important business-cycle supply shock, because of the negative comovement between both series.[2] The negative correlation also applies to other measures of investment. For the time period 1967:II-1999:I over which the CPS worker flows are available, the unconditional correlation between the ratio of the detrended quantity of total investment and consumption durables to output (with the latter measured in consumption units using Fisher's consumption deflator) and the detrended Fisher's price of investment is -0.18. Greenwood et al. (2000) quantitively investigate a model with investment-specific technological change as sole source for output fluctuations, finding it could account about 30% of output fluctuations.

In line of these papers, Fisher (2003) shows the importance of investment-specific technology shocks for understanding business cycle frequencies. The real price he considers is a producer durable equipment price deflator divided by a deflator for consumption. According to growth theory, this real price measures investment-specific technological change. Fisher finds that these shocks can account for over 50% of business cycle variation in hours worked, compared to only 6% accounted for by shocks to total factor productivity. Such results (see Gali and Rabanal (2004) for a deeper motivation) suggest that this is the appropriate framework to think about technical change as a source of business cycle fluctuations.

2.2 Unemployment and Vacancies

The unemployment rate is the traditional indicator of job search activity. There is an open debate with respect to the use of the unemployment rate versus the employment-population ratio as the most appropriate measure of job search activity. Blanchard and Diamond (1990) show evidence that the flow of workers moving directly into employment from out-of-the-labor force is at least as large as the number who move from unemployment to employment (from the Abowd-Zellner adjusted gross flows). Regardless of such objections labor market participation is procyclical and, as Shimer (2004) points out, the employment-population ratio is as such the more cyclical measure of job search activity. Its use would only serve to worsen the above mentioned problem with the search model. The upper panel of figure (2) shows the unemployment rate and its trend. The upper panel of figure (2) shows its cyclical component.

As a measure for the time series evolution of vacancies I use the Conference Board's Index of Help Wanted Advertising in Newspapers. The index, shown in figure (2), measures

[2] This is of course not the only cost associated with capital over the business cycle. Eisfeldt and Rampini (2003) take a closer look at the empirical properties of capital reallocation over the business cycle. They establish two basic quantitative facts: (i) capital reallocation is procyclical and (ii) the cross-sectional standard deviation of capital productivity is countercyclical. The countercyclical standard deviation shows up regardless of the measured used for capital productivity (such as Tobin's q, TFP growth rates, capacity utilization rates). This standard deviation implies that capital reallocation does not take place when potential measured benefits are highest and suggests that the empirical cost or frictions in reallocating capital are countercyclical. In contrast to most models of illiquidity where quantities like the amount of adverse selection and level of agency costs are not measured, Eisfeld and Rampini (2003) impute this liquidity cost from the Compustat data and find it to be countercyclical.

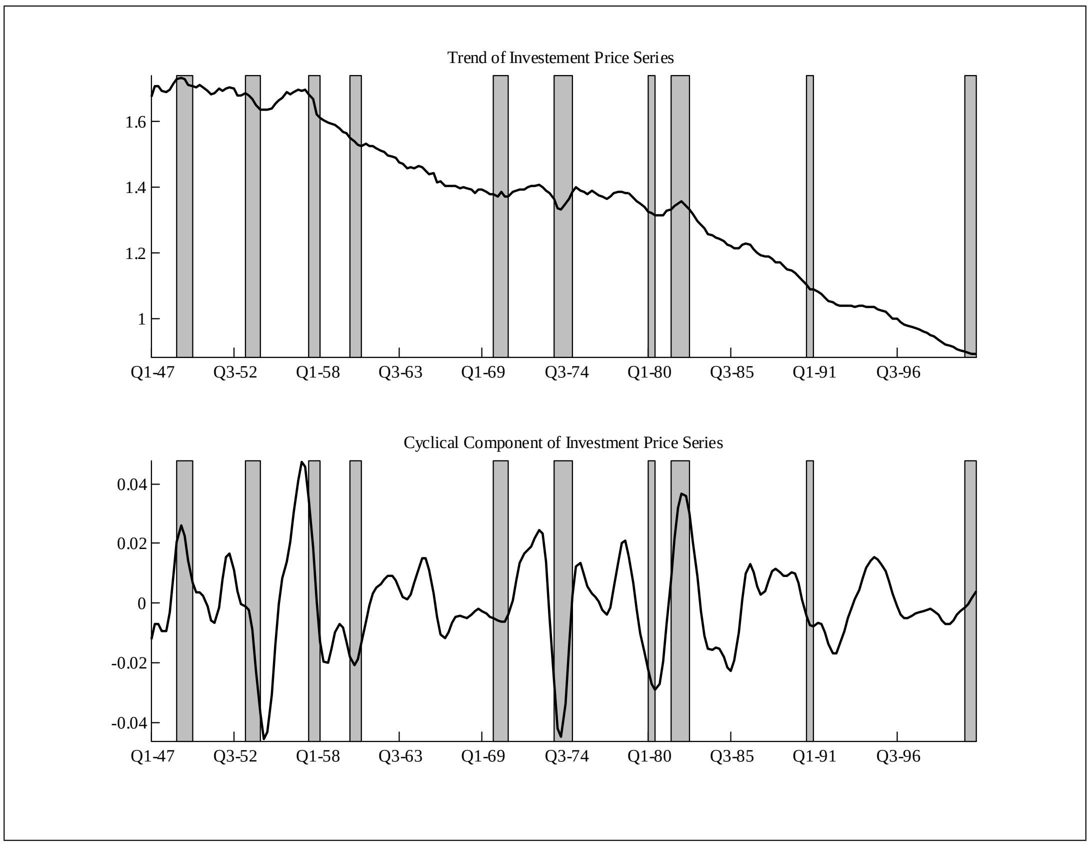

Figure 1: Real Investment Price and its Cyclical Component 1947:I-2001:IV.

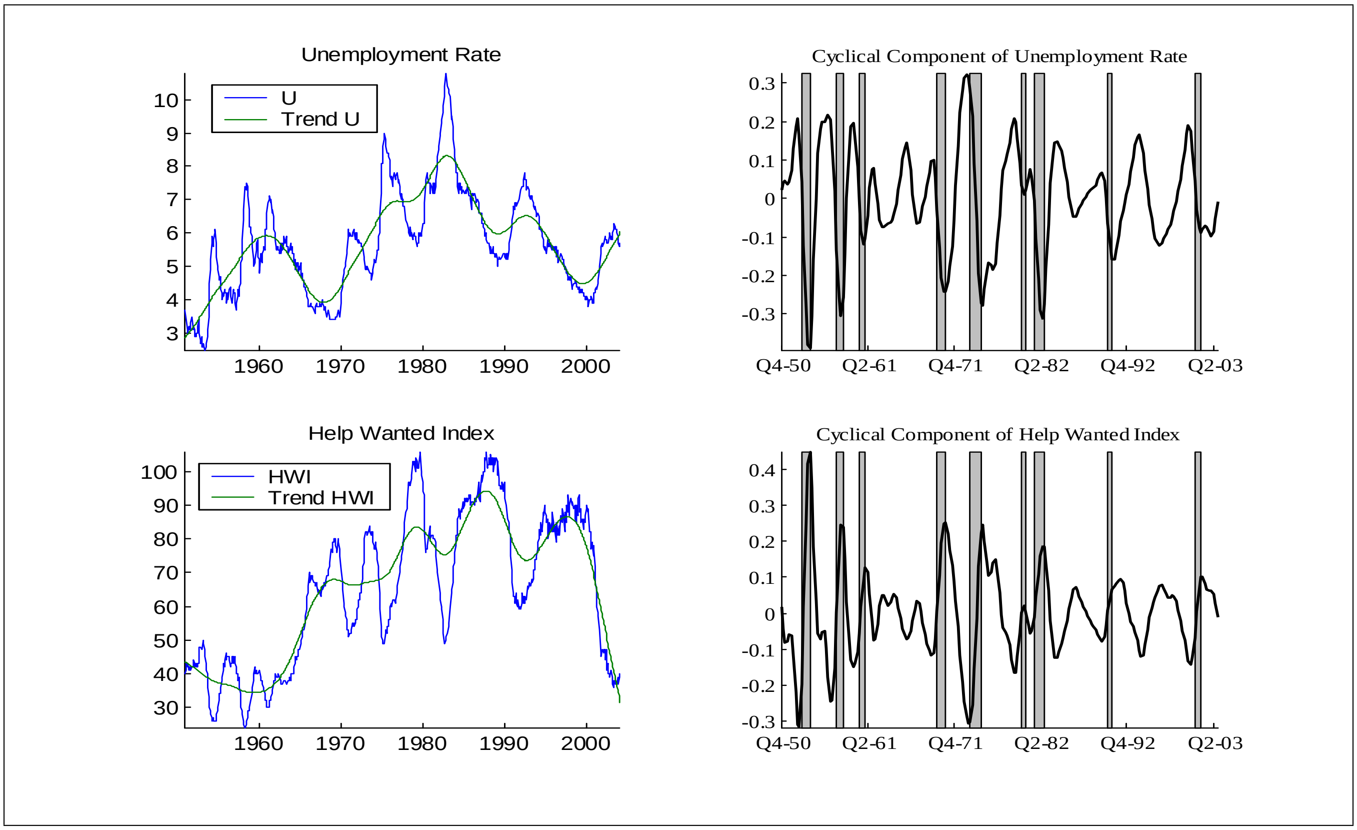

Figure 2: Unemployment Rate and Help Wanted Index 1951:I-2004:I. The unemployment rate is the seasonally adjusted monthly series of the Bureau of Labor Statistics. The help-wanted advertising index is the Conference Board's seasonally adjusted monthly series. The trend is an HP filter with the smoothing parameter $\lambda = 129600$, the value suggested by Ravn and Uhlig (2002) to adjust the HP filter to monthly data. The shades are the NBER's business cycle dating committee peak to through recessions.

Industry	Ratio of Vacancy Rates in 12/02 and 12/00
Mining	0.36
Construction	0.38
Durables	0.45
Nondurables	0.48
Transportation and utilities	0.80
Wholesale trade	0.52
Retail trade	0.60
Finance, insurances and real estate	0.79
Services	0.68
Federal government	0.54
State and local government	0.70

Table 1: Change in Vacancy Rates by Industry. Source: Hall (2004a) from JOLTS.

the number of help-wanted advertisements in 51 major newspapers. There is some critique on the relevance of this series for labor markets as its low frequency component is not necessarily affected by evolutions on the labor market but by technological advances such as computerization of employment offices, advertising on the internet, ... I therefore detrend the time series using a band-pass filter in figure (2).

Since December 2000 a new data series of the Bureau of Labor Statistics, the Job Openings and Labor Turnover Survey (JOLTS), provides more accurate information on variations in recruiting effort over time.[3] The JOLTS calculates the number of job openings and separations for 16000 business establishments from a universe frame of approximately 8 million establishments (including all employers subject to State Unemployment Insurance laws and Federal agencies subject to the Unemployment Compensation for Federal Employees program). Table (1) is taken from Hall (2004a) and shows ratios of changes in the JOLTS vacancy rate in 2002 and 2000 by industry. Vacancies are very volatile and there is strong comovement in recruiting effort across all industries.

3 A Search and Matching Model with Capital Accumulation

3.1 Households

Each household is endowed with a unit of labor which is supplied inelastically to the labor market. A representative household maximizes the following expected-utility function:

$$\max_{\{C_t\}} \mathbf{E}_0 \sum\nolimits_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} + (1-\kappa_t)b - \kappa_t h \right], \tag{1}$$

where C_t is consumption. If the agent works ($\kappa_t = 1$), he suffers a disutility h. If unemployed ($\kappa_t = 0$), he enjoys b, the value of leisure or household production. Households

[3] The definition for a job opening of the Bureau of Labor Statistics speaks for itself: "A job opening requires that 1) a specific position exists, 2) work could start within 30 days, and 3) the employer is actively recruiting from outside of the establishment to fill the position. Included are full-time, part-time, permanent, temporary, and short-term openings. Active recruiting means that the establishment is engaged in current efforts to fill the opening, such as advertising in newspapers or on the internet, posting help-wanted signs, accepting applications, or using similar methods." See www.bls.gov/jlt for more information on this new data source.

pool their incomes at the end of the period. Merz (1995) and Andolfatto (1996) show how perfect income insurance of heterogenous households leads to the representative agent form. Their reasoning does not change under endogenous separation decisions. As den Haan et al. (2000) point out exact aggregation requires a set of assets that spans the space of idiosyncratic shocks without conditioning on employment status. As is pointed out in the finance literature, a limited set of assets provides already fairly good consumption insurance.

The household's budget constraint is:

$$C_t + I_t \leq \Pi_t + R_t^k K_t + w_t n_t. \tag{2}$$

Household own and rent out capital to the firms. For this they receive profits Π_t and capital income $R_t^k K_t$. Labor income is $w_t n_t$.

The evolution of the capital stock is given by:

$$K_{t+1} = (1-\delta)K_t + z_t I_t \tag{3}$$

$$z_t = z_{t-1}^{\rho_v} \exp(\varepsilon_{vt}),$$

where z_t is a technology shock affecting the productivity of new capital goods. The productivity of the already installed capital stock is not directly affected by the new technology. The number of consumption units that must be given up to get an additional efficiency unit of new capital is $\frac{1}{z_t}$ (I_t is expressed in consumption units in (2)). In the competitive equilibrium the real price of investment $P_{I,t}$ is then the inverse of the investment-specific technology shock z_t. Only innovations to z_t have a permanent effect on $P_{I,t}$.

In this paper I take a private sector equilibrium approach. Hence the equilibrium allocations for the households are found by maximizing (1) subject to (2) and (3). Taking the first-order conditions and eliminating multipliers leads to:

$$C_t^{-\sigma} P_{I,t} = \beta \mathbf{E}_t C_{t+1}^{-\sigma} \left[(1-\delta) P_{I,t+1} + R_{t+1}^k \right]. \tag{4}$$

Equation (4) describes each household's intertemporal decision to optimally allocate investment into capital. It implies that the current utility cost of investing $P_{I,t}$ equals the present value, in utility terms, of the future product of that capital investment.

The rate of return on a risk-free asset in this economy is:

$$r_t^f = \frac{C_t^{-\sigma}}{\beta E_t \left(C_{t+1}^{-\sigma} \right)} - 1.$$

The rate of return on equity is:

$$r_{t+1}^e = \frac{MP_{t+1} + P_{k,t+1}}{P_{k',t}} - 1. \tag{5}$$

$P_{k',t}$ is the consumption good value of a newly installed unit of capital, to be used in the production process at $t+1$. $P_{k,t+1}$ is the value of this unit of capital at the end of period $t+1$. Boldrin, Christiano and Fisher (2001) refer to $P_{k',t}$ as the date t price of equity and to the ratio $\frac{P_{k,t+1}}{P_{k',t}}$ as the capital gain. The mean equity premium is then $E(r_{t+1}^e - r_t^f)$. In equation (5) $P_{k',t} = P_{I,t}$ and $P_{k,t+1} = (1-\delta)P_{I,t+1}$.

3.2 Firms and Labor Market Frictions

Each good Y_{it} is produced by a firm i at time t using capital and labor as inputs. The output for a job at firm i is:

$$A_t a_{ijt} k_{it}^{\alpha},$$

where A_t is a random aggregate disturbance. a_{ijt} is a match-specific disturbance for job j at firm i. This idiosyncratic, job-specific productivity is drawn from a distribution with c.d.f. $F(a)$ and support $[\underline{a}, \overline{a}]$. A job is profitable for the firm if $a_{ijt} \geq \tilde{a}_{it}$, where $\tilde{a}_{it}$ is the cut-off productivity level. If $a_{it} < \tilde{a}_{it}$, a job is not profitable and the relationship is severed. Workers are identical so to the firm it does not matter which worker keeps a job with productivity a_{ij} open. Firms rent a unit of capital k_{it} in a prefectly competitive market from the households.

Total output at firm i is then given by a production technology including the mass n_{it} of employment relationships, average per-job capital level k_{it}, the aggregate disturbance A_t, and the job mass of idiosyncratic productivities a at which firm i is producing:

$$Y_{it} = A_t k_{it}^{\alpha} \int_0^{n_{it}} \int_{\tilde{a}_{it}}^{\bar{a}} a f(a, n) da dn.$$

The identical worker assumption implies $f(a, n) = f(a)f(n)$. Because the mass of workers is uniformly distributed over $[0, 1]$, $f(n) = 1$, the above expression simplifies to:

$$Y_{it} = A_t n_{it} k_{it}^{\alpha} \int_{\tilde{a}_{it}}^{\bar{a}} a f(a) da.$$

The assumptions on the labor market are standard in the search and matching literature. There is a continuum of identical consumer-workers with total mass equal to one. The function matching unemployed workers u and firms with vacant jobs v is $M : [0, 1] \times R^{+} \rightarrow [0, 1]$. This function represents meeting frictions and determines the instanteneous number of meetings as a function of the number of searchers on each side of the market. M is Constant Returns to Scale and bounded above by $min\{u, v\}$. The functional form for M is $M(u_t, v_t) = \mu u_t^{\xi} v_t^{1-\xi}$. ξ is the elasticity to unemployment and μ is an efficiency parameter. For notational convenience, I define $m(\theta_t) \equiv M(1, \theta_t)$ where $\theta = \frac{v_t}{u_t}$ measures the degree of labor market tightness. Every unemployed worker meets an employee with probability:

$$m(\theta_t) = \frac{M(u_t, v_t)}{u_t} = \mu {\theta_t}^{1-\xi}.$$

Likewise, the probability that a vacancy contacts a worker is:

$$q(\theta_t) = \frac{m(\theta_t)}{\theta_t} = \mu {\theta_t}^{-\xi}.$$

The total flow of new hires for an individual firm in t+1 is $v_{it} q(\theta_t)$.

Job destruction ρ_{it} at firm i is given by the probability ρ^x of exogenous and constant job separation and the endogenous component ρ_{it}^n:

$$\rho_{it}^n = \int_{\overline{a}}^{\tilde{a}_{it}} f(a) da = F(\tilde{a}_{it}).$$

The separation rate ρ_{it} at firm i is then the sum of the exogenous separation rate ρ^x and an endogenously destroyed fraction ρ_{it}^n of the remaining jobs:

$$\rho_{it} = \rho^x + (1-\rho^x)F(\tilde{a}_{it}), \tag{6}$$

implying a survival rate $\varphi_t = (1-\rho^x)(1-F(\tilde{a}_{it}))$. The evolution of employment at firm i is described by:

$$n_{it+1} = (1-\rho_{it})n_{it} + v_{it}q(\theta_t)$$

The total wage bill for a firm i can be calculated as the product of the mass of employment relationships at time t and all wages in jobs with an idiosyncratic productivity level above the cutoff $\tilde{a}_{it}$:

$$W_{it} = n_{it}\int_{\tilde{a}_{it}}^{\bar{a}} w_t(a)f(a)da.$$

Firms chooses capital, employment and numbers of job posted and destroyed so as to maximise profits. An individual firm i's expected revenues net of expenses at time t are:

$$\Pi_{it} = \mathbf{E}_0\sum_{t=0}^{\infty}\beta^t\frac{\lambda_t}{\lambda_0}\left[n_{it}\left(A_t k_t^{\alpha}\int_{\tilde{a}_{it}}^{\bar{a}} af(a)da - r_t^k k_t\int_{\tilde{a}_{it}}^{\bar{a}} f(a)da\right) - W_{it} - cv_{it}\right], \tag{7}$$

The discount term $\beta^t\frac{\lambda_t}{\lambda_0}$ is the marginal rate of substitution of the households. Profits are only evaluated in terms of value attached to them by the households, who own the firms. The employment flow equation for an individual firm is:

$$n_{it+1} = (1-\rho_{it})n_{it} + v_{it}q(\theta_t), \tag{8}$$

where multiplier μ_{t+1} will be the shadow value of employment. Comparing equations (7) and (8) to the classic theory of factor demands, it is clear search theory is a natural way of introducing adjustment costs on labor. More specifically, the cost of adjustment for employment is $\frac{c}{q(\theta_t)}(n_{it+1} - (1-\rho_{it})n_{it})$. This cost of adjustment is linear in n_{it+1} and the vacancy cost c. It has an intuitive relation with the other key labor market variables θ_t and ρ_{it+1}. It is harder for a firm to adjust labor when the labor market is tight (high θ). A higher rate of job destruction also pushes up adjustment costs: attaining a certain level of employment is harder when more jobs are destroyed in any given period.

The first order condition for capital k_t is (from now on, I drop index i because of representative firm assumption):

$$A_t\alpha k_t^{\alpha-1}\int_{\tilde{a}_t}^{\bar{a}} af(a)da = r_t^k\int_{\tilde{a}_{it}}^{\bar{a}} f(a)da. \tag{9}$$

Equation (9) states that firms rent capital up to the point where the marginal benefit of an additional unit of capital in every job equals the rental cost.

The efficiency equations for $\{n_t, v_t, \tilde{a}_t\}$ are:

$$\mu_t = A_t k_t^{\alpha}\int_{\tilde{a}_t}^{\bar{a}} af(a)da - r_t^k k_t\int_{\tilde{a}_{it}}^{\bar{a}} f(a)da - \frac{\partial W_t}{\partial n_t} + \beta\mathbf{E}_t\left[(\frac{\lambda_{t+1}}{\lambda_t})\mu_{t+1}(1-\rho_t)\right] \tag{10}$$

$$\frac{c}{q(\theta_t)} = \beta\mathbf{E}_t(\frac{\lambda_{t+1}}{\lambda_t})\mu_{t+1} \tag{11}$$

$$n_t\left(A_t k_t^{\alpha}\frac{\partial\left(\int_{\tilde{a}_t}^{\bar{a}} af(a)da\right)}{\partial\tilde{a}_t}+f(\tilde{a}_t)r_t^k k_t\right)-\frac{\partial W_t}{\partial\tilde{a}_t}=\beta\mathbf{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\mu_{t+1}(1-\rho_x)f(\tilde{a}_t)n_t\right] \quad (12)$$

To get a more familiar expression for the job creation condition, substitute (10) into (11):

$$\frac{c}{q(\theta_t)}=\beta\mathbf{E}_t(\frac{\lambda_{t+1}}{\lambda_t})\left[A_{t+1}k_{t+1}^{\alpha}\int_{\tilde{a}_t}^{\bar{a}} af(a)da-r_{t+1}^k k_{t+1}\int_{\tilde{a}_{it+1}}^{\bar{a}} f(a)da-\frac{\partial W_{t+1}}{\partial n_{t+1}}+(1-\rho_{t+1})\frac{c}{q(\theta_{t+1})}\right]. \quad (13)$$

Expression (13) equates the cost of vacancy creation, that is flow cost c times the expected duration it takes to fill the job, to the expected return of creating a new job.

To obtain a similar expression for the job destruction condition consider the first order condition for the treshold $\tilde{a}_t$:

$$n_t\left(A_t k_t^{\alpha}\frac{\partial\left(\int_{\tilde{a}_t}^{\bar{a}} af(a)da\right)}{\partial\tilde{a}_t}+f(\tilde{a}_t)r_t^k k_t\right)-\frac{\partial W_t}{\partial\tilde{a}_t}=(1-\rho_x)f(\tilde{a}_t)n_t\beta\mathbf{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\mu_{t+1}\right]. \quad (14)$$

In equilibrium those jobs are destroyed where the payoff of hiring new workers is higher. To see this more clearly, I use the equation for the evolution of employment, the job destruction relationship $\rho'(\tilde{a}_t)=(1-\rho^x)F'(\tilde{a}_t)=(1-\rho^x)f(\tilde{a}_t)$ and $\frac{\partial\left(\int_{\tilde{a}_t}^{\bar{a}} af(a)da\right)}{\partial\tilde{a}_t}=-\tilde{a}_t f(\tilde{a}_t)$ (from the Leibnitz'rule) to get:[4]

$$f(\tilde{a}_t)n_t(-A_t k_t^{\alpha}\tilde{a}_t+r_t^k k_t+w(\tilde{a}_t))=(1-\rho_x)f(\tilde{a}_t)n_t\frac{c}{q(\theta_t)}.$$

The simplified expression for the job destruction condition (14) is then:

$$(r_t^k k_t+w_t(\tilde{a}_t)-A_t k_t^{\alpha}\tilde{a}_t)=(1-\rho_x)\frac{c}{q(\theta_t)}. \quad (15)$$

Equation (15) has an intuitive enough interpretation. The left-hand side is the cost of keeping a job with productivity $\tilde{a}_t$ open. In equilibrium this equals the expected gain of a new job in the next period net of exogenous destruction. This implies there is some labor hoarding in the model as firms pay a cost for keeping jobs open that might be profitable in the future.

To solve for the treshold $\tilde{a}_t$, I need to make an assumption on $w_t(\tilde{a})$, beyond assuming it is increasing in $\tilde{a}_t$. However, using the implicit function theorem, it's already possible to get some good comparative statics. Ceteris paribus, $\tilde{a}_t$ is higher for lower realizations of A_t and for a higher rental rate on capital r_t^k.

[4] The derivatives for the wage bill are:

$$\begin{aligned}\frac{\partial W_t}{\partial\tilde{a}_t} &= \frac{\partial}{\partial\tilde{a}_t}\left[n_t\int_{\tilde{a}_t}^{\bar{a}} w_t(a)f(a)da\right]\\ &= n_t\left[-f(\tilde{a}_t)w_t(\tilde{a}_t)\right]\end{aligned}$$

$$\frac{\partial W_t}{\partial n_t}=\int_{\tilde{a}_t}^{\bar{a}} w_t(a)f(a)da.$$

Imposing symmetry in equilibrium leads to following flow equations for the behavior of the aggregate labor market:

$$n_{t+1} = (1-\rho_{t+1})n_t + v_t q(\theta_t) \tag{16}$$

$$u_t = 1 - n_t \tag{17}$$

3.3 Job Creation and Destruction

I define job creation at time t as net employment gains via the matching process. In what follows I will always express job creation and destruction in their rate form. Job creation at time t is :

$$jcr_t = \frac{M(u_t, v_t)}{n_t}. \tag{18}$$

Total job destruction amounts to little more than the job separation rate:

$$jdr_t = \rho_t. \tag{19}$$

The net change in employment is then the difference between job creation and separation:

$$\frac{n_t - n_{t-1}}{n_{t-1}} = jcr_t - jdr_t = \frac{M(u_t, v_t)}{n_t} - \rho_t. \tag{20}$$

The literature often presents definitions for job creation and separation where exogenous worker turnover (taken to be ρ^x) is substracted from (18) and (19). The reason is that exogenous worker turnover can hardly be considered as firm induced changes in employment. This is clearly a definitional issue and does not change the definition for the net change in employment.

3.4 Wage Determination

The matching friction gives rise to a bilateral monopoly context. As is well known a multiplicity of allocations will satisfy the individual rationality constraint. I follow the literature by applying the Nash Bargaining Solution, where the wage w_t is determined as the outcome of a Nash bargain between firms and workers. Since both firms and workers participate as long as the surplus $S(a) \geq 0$ and the individual rationality conditions $[J_t \geq 0] \cap [W_t \geq U]$ are satisfied. Unemployed workers work if they can $W_t(a^u) \geq U$, so in this model filled jobs are destroyed when $J(\tilde{a}) = (1-\beta)S(\tilde{a}) = 0$.

3.4.1 Bellman equations

The workers' value functions in the unemployment and employment states satisfy:

$$U_t = b + \beta \mathbf{E}_t \frac{\lambda_{t+1}}{\lambda_t}\left[m(\theta_{t+1}) \frac{\int_{\tilde{a}_{t+1}}^{\bar{a}} E_{t+1}(a) f(a) da}{1 - F(\tilde{a}_{t+1})} + (1 - m(\theta_{t+1}))U_{t+1} \right], \tag{21}$$

$$E_t(a) = w_t(a) + \beta \mathbf{E}_t \frac{\lambda_{t+1}}{\lambda_t}\left[(1-\rho_{t+1}) \frac{\int_{\tilde{a}_{t+1}}^{\bar{a}} E_{t+1}(a) f(a) da}{1 - F(\tilde{a}_{t+1})} + \rho_{t+1} U_{t+1} \right], \tag{22}$$

where only the value function for employment is a function of time-t idiosyncratic productivity.

The firms' value functions for a job is:

$$J_t(a) = A_t k_t^{\alpha} a_t - r_t^k k_t - w_t(a) + \beta \mathbf{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left[(1-\rho_{t+1}) \frac{\int_{\tilde{a}_{t+1}}^{\bar{a}} J_{t+1}(a) f(a) da}{1 - F(\tilde{a}_{t+1})} \right] \tag{23}$$

The free-entry condition results in:

$$\frac{c}{q(\theta_t)} = \beta \mathbf{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left[(1-\rho_{t+1}) \frac{\int_{\tilde{a}_{t+1}}^{\bar{a}} J_{t+1}(a) f(a) da}{1 - F(\tilde{a}_{t+1})} \right]. \tag{24}$$

3.4.2 Wage Setting

Wage in equilibrium is derived from the maximization of the Nash product:

$$w = \arg\max_w (E(a) - U)^{\eta} J(a)^{1-\eta}, \tag{25}$$

where $\eta \ \epsilon(0,1)$ measures the bargaining power in a relationship. As in Mortensen and Pissarides (1994) I can then derive the solution to (25).

Proposition 1 *The wage schedule solving (25) is given by*

$$w(a) = \eta(Ak^{\alpha}a - r^k k + \theta c) + (1-\eta)b \tag{26}$$

Proof. The first-order condition for (25) is: $(1-\eta)(E(a) - U) = \eta J(a)$. In terms of equations of the previous section, this can be rewritten as:

$$w(a) - b + \beta \left[((1 - \rho - m(\theta))(E(a) - U) \right] = \frac{\eta}{1-\eta} \left(Ak^{\alpha}a - r^k k - w(a) + \beta(1-\rho)J(a) \right).$$

Substituting $\beta J(a) = \frac{c}{q(\theta)}$ on the right hand side and the first order condition for the Nash product (25), $(E(a) - U) = \frac{\eta}{(1-\eta)} J(a)$, on the left hand side leads to (26). ■

As discussed in Pissarides (2000), there is some nice intuition behind the appearance of θc in this wage equation. After all $\theta c = \frac{cv}{u}$ is the average hiring cost for each unemployed worker. So under Nash bargaining workers are rewarded with a higher wage when hiring is more costly (workers have more bargaining strength). It is also possible to get a cross-sectional distribution of wages from (26). This will depend on the distribution of the idiosyncratic productivities. I will assume a follows a lognormal distribution. The upper panel of figure (3) displays the distribution of a (the figure is drawn using parameter values described in the calibration section). The cut-off productivity level is $\tilde{a}$. The lower panel traces out the cross-sectional distribution of the wages given a.

Given the wage schedule (26), the wage bill at a given moment t will be:

$$\begin{aligned} W_t &= n_t \left(\int_{\tilde{a}_t}^{\bar{a}} w_t(a_t) f(a) da \right) & (27) \\ &= n_t \left(\eta (A_t k_t^{\alpha} \int_{\tilde{a}_t}^{\bar{a}} a_t f(a) da - r_t^k k_t \int_{\tilde{a}_t}^{\bar{a}} f(a) da) + \eta \theta_t c \int_{\tilde{a}_t}^{\bar{a}} f(a) da + (1-\eta) b \int_{\tilde{a}_t}^{\bar{a}} f(a) da \right) \end{aligned}$$

Following proposition shows how to get an analytic expression for the job separation treshold $\tilde{a}_t$.

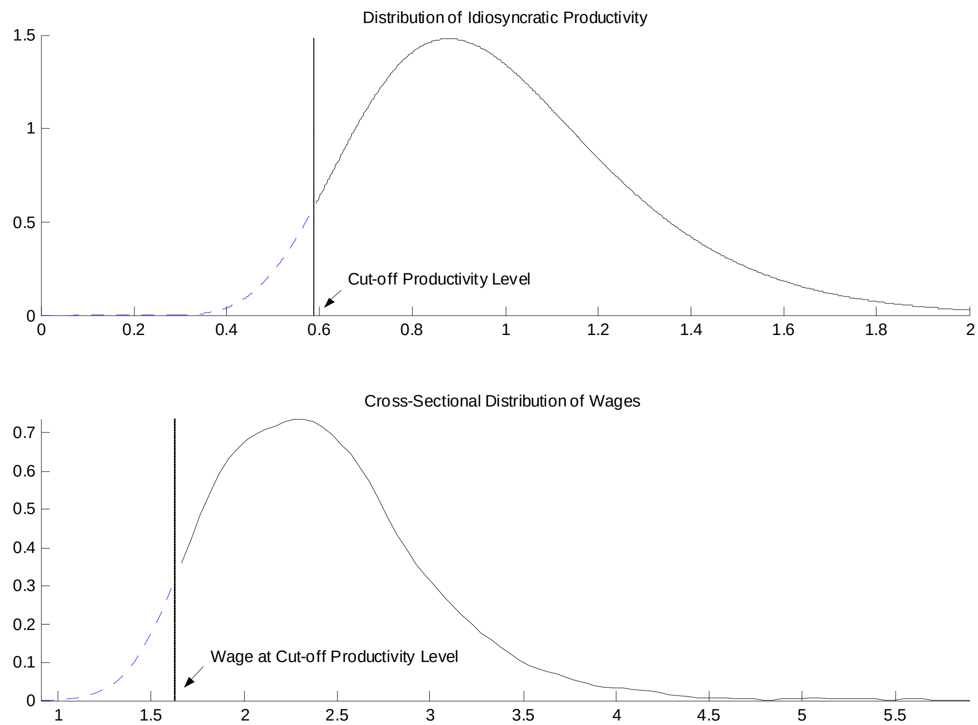

Figure 3: Distribution of Idiosyncratic Productivity a and Cross-Sectional Distribution of Wages $w(a)$ in Steady State. I generated 3000 values for the productivity distribution. Using this simulation, the cross-section of the wages is then a nonparametric density estimation with the normal as kernel. For the individual parameter values, see the calibration section below.

Proposition 2 *In equilibrium, the job separation treshold is:*

$$(1-\eta)r_t^k k_t + \eta\theta c + (1-\eta)b - (1-\eta)A_t k_t^{\alpha}\tilde{a}_t = (1-\rho_x)\frac{c}{q(\theta_t)}.$$

Proof. Take (15) and plug in $w(a)$ evaluated at $\tilde{a}$ and $\int_{\tilde{a}}^{\bar{a}} w(a)f(a)da$. ■

There are two effects of θ on the job separation threshold $\tilde{a}$. On the one hand, workers find it easier to find new jobs when the labor market is tight. They require a higher share of the pie in the bargain. This will tend to increase wages and will push up the job separation treshold. On the other hand, more vacancies for a given value of unemployment decrease the job-filling rate and firms will destroy a job less easily.

Using the expression for the wage bill (27), it is possible to get a Pissarides style job creation condition.

Proposition 3 *In equilibrium, the job creation condition is:*

$$\frac{c}{q(\theta_t)} = \beta \mathbf{E}_t(\frac{\lambda_{t+1}}{\lambda_t}) \left[\begin{array}{c} (1-\eta)\left(A_{t+1}k_{t+1}^{\alpha}H(\tilde{a}_{t+1}) - r_{t+1}^k k_{t+1}\int_{\tilde{a}_{it+1}}^{\bar{a}} f(a)da - b\int_{\tilde{a}_t}^{\bar{a}} f(a)da\right) \\ -\eta\theta_t c\int_{\tilde{a}_t}^{\bar{a}} f(a)da + (1-\rho_{t+1})\frac{c}{q(\theta_{t+1})} \end{array} \right]$$

3.4.3 Aggregate Wage

What is the aggregate wage paid in this economy at time t? It can be calculated as the weighted average of individual wages paid:

$$\begin{aligned} E(w(a) \quad | \quad a > \tilde{a}) &= \frac{1}{1-F(\tilde{a})}\int_{\tilde{a}_t}^{\bar{a}} w_t(a_t)f(a)da \\ &= \eta(A_t k_t^{\alpha}\frac{\int_{\tilde{a}_t}^{\bar{a}} a_t f(a)da}{1-F(\tilde{a})} - r_t^k k_t) + \eta\theta_t c + (1-\eta)b. \end{aligned}$$

The conditional expectation is increasing in $\tilde{a}_t$. When firms decide to destroy jobs, those jobs with a lower idiosyncratic productivity go out first. This implies that the expected productivity of the remaining jobs increases. Hence, the aggregate wage in this economy is also increasing in $\tilde{a}_t$.

3.5 Beveridge Curve

At this point, it is also worthwile to pause for a moment to derive the expression for the Beveridge curve in this economy. The evolution of unemployment in this economy is given by:

$$u_{t+1} = u_t + \rho_t n_t - \theta_t q(\theta_t)u_t$$

The Beveridge curve under endogenous job separation is then:

$$u = \frac{\rho}{\rho + \theta q(\theta)}.$$

Contrary to the exogenous job separation case, where the Beveridge curve only depends on u and v, the Beveridge curve now also depends on $\tilde{a}$.

3.6 Closing the Model

The expression relating aggregate consumption and investment to aggregate employment and the aggregate stock of capital is given by:

$$Y_t = A_t n_t k_t^{\alpha} \int_{\tilde{a}_t}^{\bar{a}} a f(a) da = I_t + C_t + cv_t - bu_t. \tag{28}$$

In equilibrium I also assume that the rental rate of capital is determined such that aggregate capital demanded in period t is equal to the aggregate supply of capital:

$$n_t k_t \int_{\tilde{a}_t}^{\bar{a}} f(a) da = K_t. \tag{29}$$

The left-hand side of (29) is the total demand for capital, given by the number of employment relationships times the amount of capital chosen by the firm in each relationship. K_t is the total amount of capital supplied by the households.

The shocks follow these processes:

$$\begin{aligned} A_t &= A_{t-1}^{\rho_a} \exp(\varepsilon_{at}) \\ z_t &= z_{t-1}^{\rho_z} \exp(\varepsilon_{vt}) \\ & \quad [\varepsilon_{at}, \varepsilon_{vt}] \,\tilde{}\, N(0, D) \end{aligned} \tag{30}$$

3.7 Equilibrium

A private sector equilibrium is a set of allocations $\{C_t, K_{t+1}, k_t, n_t, v_t, \tilde{a}_t\}$ and prices $\{p_{I,t}, r_t^k, w_t(a)\}$ such that:

- $\{C_t, K_{t+1}\}$ solves the household's problem (1) subject to the budget constraint (2) and the capital accumulation technology.
- Firms optimize, they choose $\{k_t, n_t, v_t, \tilde{a}_t\}$ to maximize profits (7) subject to the employment flow equation (8).
- Markets clear. Capital demand in period t is equal to the supply of capital (29) and the resource constraint (28) holds.
- Laws of motion for the number of relationships and the number of unemployed workers, are given by (16) and (17).
- Wages $w_t(a)$ are determined by Nash bargaining after matching.

4 Calibration

4.1 Steady State Parameters

Preferences, Capital Share and Depreciation

Table (2) displays the parameter values for β, α, δ and σ. In this model α corresponds to the factor share and a output/capital ratio of about ten percent.[5] The coefficient of relative risk aversion σ ensures the logarithmic utility function.

Parameter	Value	Interpretation	Source
β	$(1.03)^{-1/4}$	Discount factor	Annual interest rate of about 3%
σ	1	Coefficient of relative risk aversion	
α	.36	Factor share	US
δ	0.025	Depreciation rate	US Data

Table 2: Simulation Parameters

Labor Market Variables

Key labor market parameters are obtained from empirical studies. Unemployment covers both those who are not in the labor force but "want a job" and the officially unemployed,. Its data estimation for 1968-1986 is $u = 0.11$ (see Blanchard and Diamond (1990)).

I follow Cole and Rogerson (1999) and den Haan et al. (2000) and set the job-filling probability $q(\theta)$ to be equal to 0.7. In a quarterly setting, this implies that the average time it takes to fill a vacancy is about a quarter and a half, as is found in empirical studies. I pin down a value for the job-finding rate $m(\theta)$ from the steady state relationships. Clark and Summers (1979) correct for the downward biases in measured unemployment spells and estimate average unemployment duration at 19.9 weeks in 1974, which corresponds to a value of $m(\theta) = 0.60$.

Petrongolo and Pissarides (2001) survey the evidence on the matching function. Most studies find that a loglinear approximation fitts the data well. The estimated functions typically satisfy constant returns to scale (some manufacturing data suggest mildly increasing returns). When the dependent variable is the total outflow from unemployment, an estimation for the elasticity on unemployment ξ is about 0.7, implying an elasticity on vacancies of about 0.3. More generally, Petrongolo and Pissarides consider 0.5 to 0.7 a plausible range for the estimated unemployment elasticity. Hall (2004) estimates the elasticity of the matching function to unemployment using the new aggregate JOLTS data. He proceeds by dividing the change in the log of the job-finding rate by the change in the log of the vacancy-unemployment rate. The value for the estimate is 0.765. Hence, I set ξ equal to 0.7.

There exist several measures for the job separation parameter ρ. In a survey paper Hall (1995) finds that quarterly US separation rates lie in the range of 8 to 10%. Davis et al. (1996) get an annual separation rate of 36.8% from the Current Population Survey. The average job separation rate in the Davis et al. quarterly manufacturing plant-level data for 1972:2-1993:4, is 0.055. Den Haan et al. (2000) set the steady state separation rate ρ equal to 0.1. This corresponds to the value reported in the Current Population Survey where workers are asked how long ago they began their current jobs (the shortest category, however, is 6 months). Merz (1995) uses 0.07 and Andolfatto (1996) 0.15.[6]

[5] To clarify the factor share interpretation. The production function is $Y_t = A_t n_t k_t^{\alpha} \int_{\tilde{a}_t}^{\bar{a}} af(a)da$. Given $k = K/n$, Y_t can be rewritten as $\left[A_t \int_{\tilde{a}_t}^{\bar{a}} af(a)da\right] n_t^{1-\alpha} K_t^{\alpha}$.

[6] It is worth emphasizing that the average job flow rates in Europe are significant lower then these values. According to Bureau Van Dijk's annual firm level observations over 1992-2001 the job destruction rate ρ is about 3.7% in the Euro area. Within the Euro area, Gomez-Salvador, Messina, and Vallanti (2004) find that employment protection, unemployment benefits and more coordinated wage bargaining reduces the magnitude of job flows.

Parameter	Value	Interpretation	Source
ρ_n	0.018	Endogenous job separation	US data (PSID)
ρ_x	0.0835	Exogenous job separation	US data (CPS)
$q(\theta)$	0.71	Job-filling rate	US data
$m(\theta)$		Job-finding rate	Calibration
ξ	0.7	Elasticity of matching function w.r.t. u	US data
μ		Efficiency parameter	Calibration
μ_a		Mean job distribution	Match volatility of separation
σ_a		Volatility job distribution	Match volatility of separation
c		Vacancy cost	Calibration
b		Value of leisure or unemployment compensation	Calibration

Table 3: Labor Market Parameters

The next question is how we can distinguish between the endogenous and exogenous component of the separation rate. Den Haan et al. assume that exogenous separations are worker-initiated (so worker turnover) and that the endogenous job separation rate corresponds to the permanent lay-off rate. Topel (1990) estimates the quarterly permanent layoff rate from the PSID at 0.018. In Hall (1995) this is the upper value for estimates on separations initiated by employers where workers had held long-term jobs.

From ρ_n, I can get the treshold $\tilde{a}$ by taking the inverse of the cumulative distribution function $F^{-1}(\rho_n)$. A natural distributional assumption for a_t is the lognormal one:

$$\log a_t \sim N(\mu_a, \sigma_a^2),$$

The standard deviation of a_t is chosen to match the ratio of the standard deviation of job separation to output. Given the distributional assumption of lognormality, I can then easily obtain an expression for $\int_{\tilde{a}}^{\bar{a}} af(a)da$ and the conditional expectation in the aggregate wage.[7]

The vacancy cost c and the value of leisure/ unemployment insurance are calibrated such that the system of stationary variables is solvable. The obtained value for b yields a replacement ratio while searching of about 75 percent of the average aggregate wage. The value for the vacancy cost implies cv_t is about a percent of total output.

Shocks

The logarithm of the aggregate productivity shock follows an AR(1) process with coefficient $\rho_a = 0.95$ and a standard deviation $\sigma_a = .004$ to match the standard deviation of US GDP.

To simulate the model's response to innovations in investment-specific technology, a process for the investment specific shocks z_t in (30) must be specified. Greenwood et al.

[7] Remember following asssumptions on a lognormal distribution with parameters μ and σ^2 (see Johnson, Kotz, and Balakrishnan (1994)). Its mean θ and variance λ^2 are given by $\theta = \exp(\mu + \frac{1}{2}\sigma^2)$ and $\lambda^2 = \exp(2\mu+\sigma^2)(\exp(\sigma^2)-1)$. Alternatively, If you have the mean and variance of a lognormal distribution, you can then calculate μ and σ^2 of the associated normal where $\mu = 2\ln\theta - \frac{1}{2}\ln(\theta^2+\lambda^2)$ and $\sigma^2 = \ln(1+\lambda^2/\theta^2)$.
The mean of the truncated distribution is then:

$$E(a \mid a > \tilde{a}) = E(a)\frac{1-\Phi(\gamma-\sigma)}{1-\Phi(\gamma)},$$

where $\gamma = [\log(\tilde{a}) - \mu]/\sigma$ and $\Phi(\cdot)$ denotes the cumulative distribution function of the standard normal.

(2000) suggest to estimate the process as the AR(1) on the inverse of the relative price of equipment. They estimate:

$$\ln(1/PI_t) = \mathrm{cons}\tan\mathrm{t} + t\ln(\gamma_q) + z_t,$$

where

$$z_t = \rho_z z_{t-1} + \xi_t \text{ and } \xi_t \sim N(0,\sigma) \tag{31}$$

For Fisher (2003)'s quarterly sample over the period 1959:I-2001:IV, the estimated equation (31) for transitory shocks in z_t is:

$$z_t = \underset{(103)}{0.98} z_{t-1} + \xi_t \text{ and } \xi_t \sim N(0, 0.0061) \text{ with DW} = 1.30,$$

which I will use for the simulation.[8] *(I'm working on better identification schemes for the investment-specific shock)*

5 Simulation Analysis

If the solution to the model is unique, it is possible to eliminate all the static variables and write the system as $x_t = Ax_{t-1} + h_t$. Blanchard-Kahn then state the necessary and sufficient condition for this equation to determine a unique and stable path, i.e. that matrix A has as many eigenvalues of absolute values smaller than one as there are predetermined endogenous variables and as many eigenvalues or absolute values larger than one as there are anticipated variables. Indeterminacy and non-existence of equilibrium are possible in a business cycle model with search and matching functions (see Krause and Lubik (2004b)), but do not arise for the above parametrizations.

The above model is non-linear. I approximate the solution to this model using a second-order approximation to the policy functions as described in Schmitt-Grohé and Uribe (2004).

5.1 Neutral Technology Shock

Figure (4) presents impulse responses in steady-state percentage deviations to a one standard deviation neutral technology shock on output and key labor market variables. The upper left panel has both the responses of output and the shock in it. After a positive productivity shock, adjustment of output towards the steady state is much slower than the adjustment of the productivity shock. As in den Haan et al. (2000) interactions between capital adjustment and job-separation render the output effects of the aggregate productivity shock more persistent. Output dynamics no longer track the dynamics of the productivity shock closely. Den Haan et al. (2000) emphasize that their model, because of the interaction between capital adjustment and the job-separation rate, generates considerable amplification of shocks. They distinguish between the impact magnification and total magnification of a shock, where the latter is a measure for the persistency of a shock on output. Impact magnification is the ratio of the change in output over the standard deviation of the productivity shock. Total magnification is the ratio of the standard deviation of output to the standard deviation of the productivity shock. Table (7) reports these ratios for a variety of models. All models have some impact and/or total magnification. The last

[8] An issue might be that Fisher interpolates the quarterly data. How does this estimate compare with the estimate in a full macro model? Using Bayesian estimation techniques, Smets and Wouters (2003) find $\rho_z = 0.72$ and $\sigma_\xi = 0.00357$.

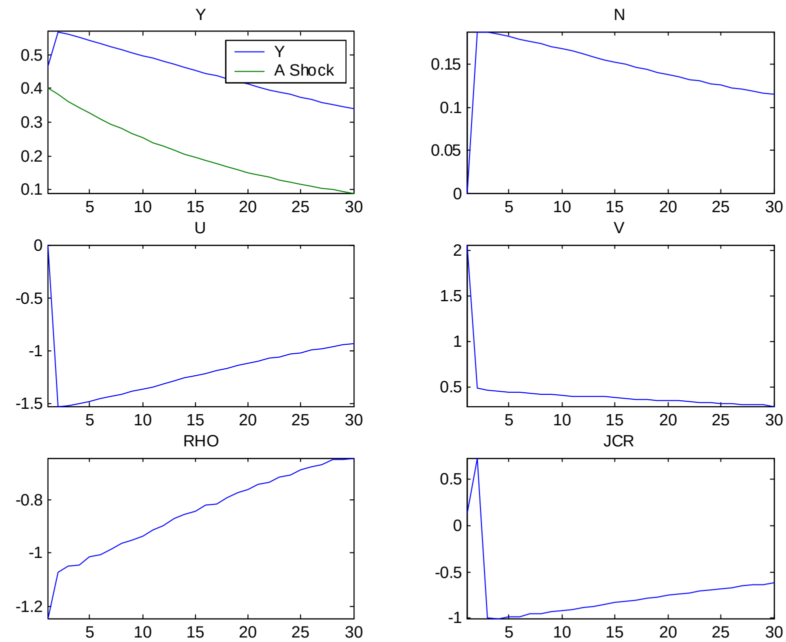

Figure 4: Impulse responses in percentage deviations to a one standard deviation neutral technology shock

$\text{corr}(v_{t+k}, u_t)$	-3	-2	-1	0	1	2	3
US data	-0.60	-0.80	-0.94	-0.95	-0.81	-0.59	-0.35
Model under Neutral Shocks	-0.42	-0.44	-0.48	-0.49	-0.73	-0.68	-0.67
Model under Embodied Shocks	-0.98	-0.98	-0.98	-0.97	-0.95	-0.94	-0.92

Table 4: Beveridge Curve.

column reports the results for my model. Impact and total magnification is significantly higher than in the other models. Again, this suggests persistent output dynamics different from that of the underlying shock. *(Have to write more on why this is...)*

There is a sharp and immediate response of vacancies and unemployment to the shock. The positive shock increases the return of a job and posting a vacancy to the firm. Firms destroy less jobs and post more vacancies. Unemployment and the job-filling rate go down. There is an echo in the vacancy series in the sense that this variable returns to steady state at a much faster rate then the other variables. This lack of vacancy persistence leads to a number of anomalies compared to the data.

Table (4) presents dynamic correlations between unemployment and vacancies. The second row presents correlations from the model under neutral technology shocks. As recently emphasized in the literature, such a model cannot mimic the near perfect negative correlation between unemployment and vacancies. Furthermore, the model fails to capture the slightly leading character of vacancies we observe in the data.

The autocorrelation function of the generated vacancy series on the third row in table

Lags	1	2	3	4
US data (HWI)	0.99	0.96	0.92	0.86
Model under Neutral Shocks	0.50	0.49	0.50	0.46
Model under Embodied Shocks	0.98	0.97	0.95	0.93

Table 5: Autocorrelation Function of Logged Vacancy Series

Statistic	US Data	EJD by Firms with Neutral Shocks
σ_Y	1.81	2.18
σ_C/σ_Y	0.74	0.64
σ_I/σ_Y	2.93	2.29
σ_N/σ_Y	0.80	0.35
$\sigma_{Y/N}/\sigma_Y$	0.56	0.66
σ_W/σ_Y	0.38	0.39
σ_r/σ_Y	0.16	0.03
$\rho(Y,C)$	0.88	0.90
$\rho(Y,I)$	0.80	0.93

Table 6: Comparison of Basic Business Cycle Statistics US and Model Data. All variables have been logged (with the exception of the real interest rate). Source: King and Rebelo (1999) handbook chapter.

(5) is also highly symptomatic of the problems in the model under neutral shocks. Vacancies are highly autocorrelated in the data and this is not the case in the model.

Table (**??**) reports a classic evaluation of the model's basic business cycle statistics. This plethora of statistics suggest the model does well on mimicking some important features of the US economy. Unlike the RBC model, which predicts a strongly procyclical real wage, the volatility of the search model's real aggregate wage is of the same magnitude as the one in the data. The reason is that the margin of labor adjustment for the firms is on the jobs with relatively low idiosyncratic productivities. This dampens the procyclicality of the observed real wage compared with a RBC model or a search model without endogenous job separation.

5.2 Capital Embodied Technology Shocks

Figure (5) shows that an investment-specific technology shock has very different implications for the behavior of vacancies. Such a shock does not improve the productivity of labor on impact. Only new capital goods enhance that productivity over time. Following the shock, there is a hump-shaped increase in vacancies and a hump-shaped decline in unemployment. There is no echo. In response to an embodied shock there is an investment boom as

	RBC Model	Indivisible Labor	Exogenous Separations	den Haan, Ramey, and Watson	Model under Neutral Shocks
Impact Magnification	1.57	1.86	1.00	1.28	2.05
Total Magnification	1.55	1.86	1.25	2.45	2.88

Table 7: Impact and Total Magnification

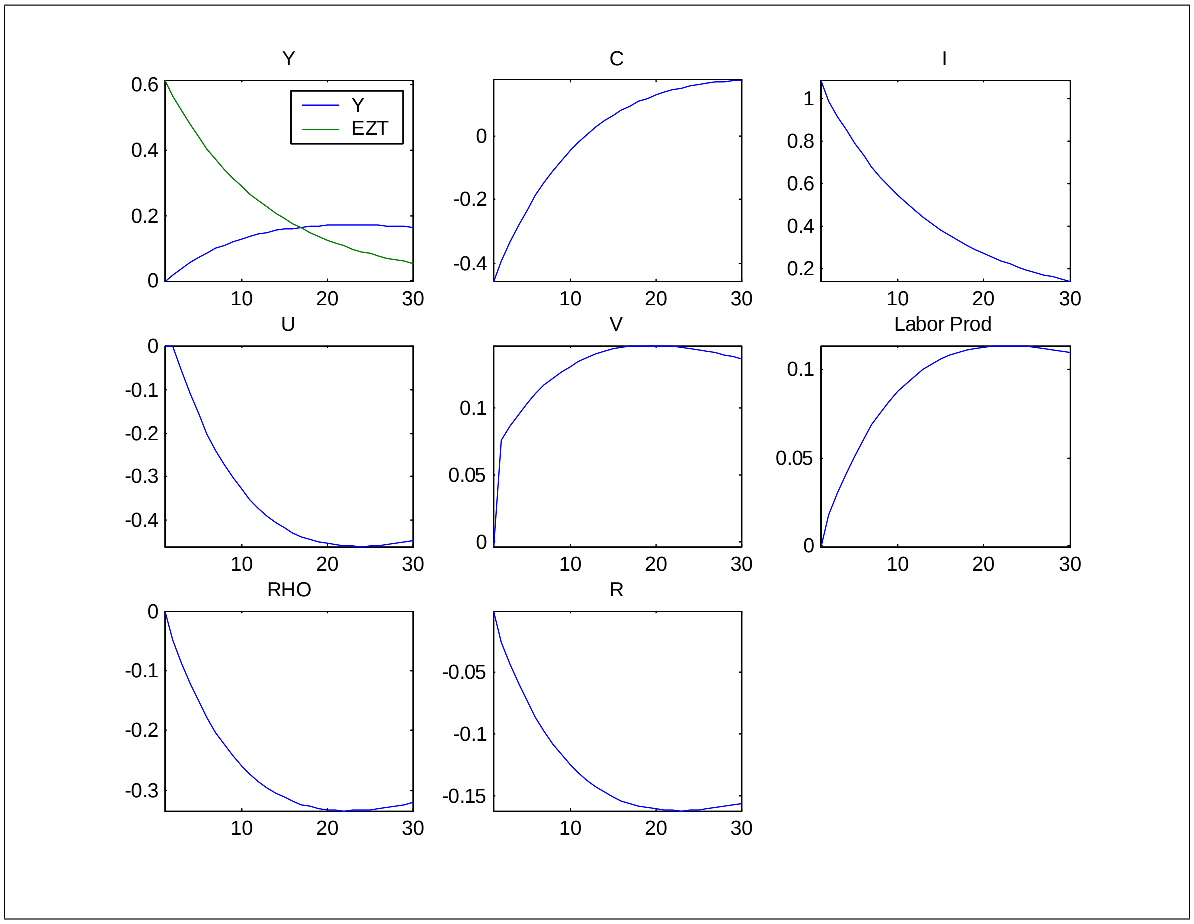

Figure 5: Impulse Responses in percentage deviations to a one standard deviation positive capital embodied shock.

Statistic (%)	Data	Model with I-shocks
σ_Y	1.81	0.58
σ_C	1.34	1.01
σ_I	5.30	3.64
σ_N	1.14	0.65

Table 8: Fraction of Output Volatility Explained by (I)-shocks.

households take maximum advantage from the temporary improvement in the production of capital goods and substitute current consumption for future consumption (remember, there is no endogenous hours-worked decision in the model).[9] Table (**??**) documents that both variables are strongly negatively correlated. Table (5) illustrates the generated vacancy series by itself is also highly autocorrelated.

In his critique on the deterministic Mortensen-Pissarides model, Shimer (2004) initially focuses on shocks in the average labor productivity as driving force. As average labor productivity is only weakly procyclical, it seems impossible to square with a very volatile vacancy-unemployment ratio in a search and matching model. However, in the above model following relationship holds:

$$\mathrm{std}(\log(\theta))/\mathrm{std}(\log(\text{Labor Productivity})) = 5.34,$$

suggesting considerable amplification.

The volatility of output in the model with embodied shocks is 0.58 (see table 8). So 30% of the cyclical variation in output could be explained by the embodied shocks.

6 Related Literature (Incomplete)

Fujita (2003) introduces three different modifications to the Mortensen and Pissarides model to kill the above-mentioned "echo effect". His model does not have capital accumulation. First of all, he assumes there is costly planning of posting vacancies. Firms can only post vacancies after completing the planning stage and completion occurs with some probability each period. A second assumption is the mothballing of jobs by firms. This means that a firm has the option to render a job temporarily inactive and repost it later on by paying a one-time retooling cost. A third modification is the introduction of a non-trivial job rejection decision. The immediate implication of this assumption is that not all meetings result in job creation. Newly-formed meetings draw idiosyncratic productivity shocks and a bad shock could lead to job rejection. In Fujita (2003), job rejection and job separation take place at different rates. Quantitatively evaluating this model, Fujita finds that the Mortensen and Pissarides model can replicate the Beveridge curve and the observed correlations between vacancies and job creation.

Shimer (2004) takes an altogether different view. He argues that the source of the standard search model is its lack of wage rigidity due to the Nash bargaining. If unemployment and vacancies are more substitutable then implied by the Cobb-Douglas matching function, fluctuations are amplified in a centralized economy because the shadow wage is less procyclical. Menzio (2004) offers a new theory of high-frequency wage rigidity.

[9] Comparing the figure with empirical impulse response functions (see for example figure 10 in Smets and Wouters (2003)), the data seem to suggest an initial decline in consumption and a prolonged decline in the rental rate of capital.

7 Conclusion

This paper argues that, in the context of a frictional model of the labor market, investment-specific technological change is a more promising avenue for the study of technology shocks as a source of business cycle fluctuations. The model with neutral technogy shocks fails to match stylized facts on labor and capital markets. Following such a technology shock, vacancies return to steady state without delay and are not persistent. In the model with neutral shocks there is no near perfect negative correlation between vacancies and unemployment.

An investment-specific technology shock has very different implications for the labor market. Such a shock does not improve the productivity of labor on impact. Only new capital goods improve labor productivity but building these goods takes time. The reponse of vacancies to such a shock is persistent. Unemployment and job vacancies are strongly negatively correlated. As in the data the vacancy series is highly autocorrelated.

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