Real Exchange Rate Fluctuations and Endogenous Tradability

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Abstract

This paper examines, empirically and theoretically, the sectoral decomposition of the volatility of real exchange rates. For the purpose of this decomposition, goods are classified as being traded or nontraded in international markets. The first part performs an empirical analysis for a broad cross section of countries. The relative price of nontraded goods to traded goods is found to be relatively more important in movements of real exchange rates of the country pairs that maintain stable nominal exchange rates. The paper goes on to construct a model with endogenous tradability to suggest an explanation for the evidence. The key features of the model are heterogeneous productivity, transport costs, and sticky wages. The nontraded sector arises from non-zero trade costs. The relative price of goods depends on productivity, transport costs, and in the short run, on the exchange rate regime. The calibration shows that the relative price of nontraded goods makes a much greater contribution to overall real exchange rate volatility under a fixed exchange rate regime than a flexible regime, as in the data.

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1 Introduction

This paper studies the sectoral decomposition of the volatility of real exchange rates. The real exchange rate between two countries is the relative price of a representative goods basket. The sectoral decomposition of real exchange rate fluctuations is important, because it has important implications for the dynamic adjustment of an open economy to exogenous shocks. For some countries, the movements in relative price levels come from the relative prices of internationally nontraded goods such as housing or construction, while for others from those of traded goods such as manufactures. The empirical part in the next section finds that, while in general the relative prices of traded goods are the most important in driving real exchange rate movements, the relative prices of nontraded goods are relatively more important for the country pairs that maintain stable nominal exchange rates.

To explain the empirical evidence, I construct a general equilibrium monetary model with heterogeneous productivity and endogenous tradability. The model shows how real exchange rate dynamics are connected to shifts in the tradability of goods through firms' price setting behavior. In some cases, the shifts and the simultaneous movements in nominal exchange rates lead to strong substitution effects effects among traded goods. In these cases, the relative prices of traded goods dictate the movements in real exchange rates. Therefore, limiting flexibility in nominal exchange rates can delay the adjustment in the relative prices of traded goods and raise the contribution to the volatility of the relative prices of nontraded goods, as observed in the data. My model offers a set of reasons why the relative price of nontraded to traded goods may be the major source of fluctuations in real exchange rates. The model can be extended to perform welfare analysis under various trade structures. Clearly, it also has strong implications for a design of exchange rate policy. These are the contributions of my paper.

In principle, we can decompose the fluctuations in real exchange rates into their traded and their nontraded goods components. The traded component is deviations from the law of one price for traded goods, while the nontraded component is fluctuations in the relative prices of nontraded to traded goods across countries. The traded component is by far dominant for the real exchange rates among the OECD countries that allow their currency to move freely (Engel 1999; Obstfeld 2001). To the contrary, the nontraded component is significantly more important in international data (Betts and Kehoe 2001a). The relative volatility of the nontraded real exchange rates and the overall volatility is 30 percent on average. The number becomes much higher as the volatility of real and nominal exchange rates falls. Such a pattern is consistent with what was found in the study on Mexico's real exchange rates by Mendoza (2000). As I present in the next section, the volatility of the nontraded real exchange rates even exceeds its traded counterpart and the overall volatility in some cases. In particular, the nontraded component of the real exchange rates between several member countries of the European Monetary System has displayed substantial volatility. In light of these empirical features, there seem to be linkages between the exchange rate regime and the sectoral decomposition of real exchange rate fluctuations. Although one may perceive the linkage as a result of local-currency pricing, the evidence for local-currency pricing does not apply to emerging market economies, which also display the same pattern in real exchange rate volatility (Goldberg and Knetter, 1997; Campa and Goldberg, 2002).

In fact, the studies by Mussa (1986) and Baxter and Stockman (1989) have confirmed that the real exchange rate volatility is very different under different exchange rate regimes. The literature on real exchange rate fluctuations is precisely divided by the views regarding the source of these fluctuations. One strand of the literature puts an emphasis on the nontraded component by assuming nominal rigidities in the nontraded sector or in factor prices (see Dornbusch (1983) and Hau (2000), for example). The other asserts the importance of the traded component or deviations from the law of one price. (see Betts and Devereux (1996; 2000; 2001), Chari et al. (2002), Obstfeld (2000), and Rogoff (1996)). Few researchers have attempted to reconcile the literature using a theory of endogenous tradability. Betts and Kehoe (2001b) model endogenous tradability in a flexible price two-country framework. Bergin and Glick (2003) use a two-period small country model where firms take world prices as given. In both studies, the source of heterogeneity is product-specific transport costs. The mechanism therein is essentially the tradeoff between quantity and price adjustments in the nontraded sector. Once the nontraded sector can shrink or expand in response to shocks, the volatility of the relative price of nontraded to traded falls. The approach provides a rationale for cases where the nontraded component exhibits moderate to low volatility. Unfortunately, it fails to explain why we observe much higher volatility in the nontraded component for a significant number of country pairs.

In my view, the more important source of heterogeneity than product-specific transport costs is product-specific productivity, because it drives firm-specific pricing strategies and international trade, giving rise to fundamental differences in prices across countries. (See Baier and Bergstrand (2001) for evidence against the role of transport costs in the expansion of world trade.) Several empirical studies have documented differences in the frequency and the magnitude of price changes across product categories. (See the recent studies by Bils and Klenow (2002) and Campa and Goldberg (2002), for example. Taylor (1999) provides an excellent survey of the topic.) With the sectoral heterogeneity in price dynamics, shifts in the composition of import bundles in response to shocks must play a major role in the international shock transmission mechanism. I use this mechanism to study the volatility decomposition of real exchange rate between their traded and nontraded components. It should be noted that the heterogeneity in price dynamics has also attracted some researchers to explore its potential as an explanation for the persistence in real exchange rates (Imbs et al. 2002a; 2002b).

In order to address the issue of heterogeneous price dynamics in an open-economy context, production and trade patterns must be endogenous. The model is a dynamic and monetary version of the trade model with a continuum of goods by Dornbusch et al. (1977). There is a continuum of differentiated goods and goods markets are monopolistically competitive. Firms are differentiated by productivity and they set prices in their own currency. I abstract from local currency pricing because the main predictions of my model carry through regardless of the currency of denomination. I assume that prices are flexible and the source of nominal rigidities is households' wage-setting behavior. In fact, Huang and Liu (2002) find that wage stickiness is more powerful than price stickiness in explaining persistence of output. Although wage-stickiness can potentially give rise to weakly countercyclical movements in real wages, recent empirical studies do not suggest that real wages are systematically procyclical or countercyclical (Abraham and Haltiwanger 1995; Christiano, Eichenbaum, and Evans 1999). Hence, assuming away price stickiness helps simplify a framework where an analysis of monetary policy is possible (see Collard and Dellas (2002), for example). In my model, wage stickiness arises from convex adjustment costs *à la* Rotemberg (1982). Deviations from the law of one price take a form of iceberg-type transport costs and whether a good is traded is therefore endogenous.

In a free trade world, patterns of trade and production mostly follow the principle of comparative advantage. Trade is, however, not solely driven by comparative advantage, because of monopolistic competition in product and labor markets. Both countries produce exportables and nontradables, and import a range of goods from each other. By comparative advantage, both countries produce and export goods produced by relatively productive firms. The exportable firms of a country are therefore more productive than its nontradable ones. Heterogeneous productivity also leads to heterogeneous price dynamics. However, there is no clear relationship between the sectoral productivity and price dynamics, because a shock can affect the productivity of different sector differently depending on how it changes the size of each sector. As a result, the relative volatility of prices of imports, exports and nontraded goods is ambiguous and depend on the type of shocks. The sectoral composition of real exchange rate volatility mainly depends on the size of exchange rate expenditure switching effects.

Given a flexible exchange rate regime, the nominal exchange rate endogenously responds to shocks. In a model with nominal rigidities, the exchange rate movement is the central adjustment mechanism, because it alters the relative prices of traded goods and causes the consumers to substitute between imports and domestically produced good. The expenditure switching effect can be captured by changes in expenditure share of export goods, import goods and nontraded goods, comparing to those under fixed exchange rate regime. When the expenditure switching effects is large, shutting down the nominal exchange rate channel can create large swings in the relative price of nontraded goods, which is an alternative adjustment mechanism. In that case, the relative price of nontraded to traded goods can partly account for the volatility of real exchange rates. So, the question here is, when do we observe a large expenditure switching effect?

My paper explores several answers to this question based on endogenous tradability. In my model, tradability or the size of traded goods sector is the share of consumption of all traded goods in a consumption basket. It partially determines the expenditure share of goods, because the expenditure share consisted of two components: The share of goods in a consumption basket and their price. Intuitively, the substitution among traded goods becomes an important adjustment channel when their consumption share is large. One key factor that influences the consumption share of traded goods is transport costs. Transport costs raise the relative price of imports and therefore lower the consumption share of traded goods and expand the size of the nontraded sector. The decrease in tradability is the mechanism through which a rise in transport costs decreases the expenditure switching effect. Besides transport costs, the intratemporal elasticity of substitution is also relevant. With a higher value of the elasticity of substitution, quantity becomes very responsive to a shock and an adjustment process can take place through small price changes. Therefore, the effect of the elasticity on consumption expenditure is ambiguous.

To investigate the relationships between the expenditure switching effect and transport costs, and the intratemporal elasticity of substitution, I calibrate the model based on a productivity shock and an interest rate shock, with various transport costs and elasticity parameters, under two different exchange rate regimes. The impulse responses confirm that the nontraded component of real depreciation becomes more volatile under a fixed exchange rate regime as transport costs fall, regardless of type of shocks. However, there is no clear relationship between the elasticity of substitution and the sectoral decomposition of the real exchange rate volatility.

The gist of a theory of endogenous tradability lies in a temporary shift in patterns of trade as a result of an exogenous shock. In the short run, a positive productivity shock in the home country raises real wage in the home country and lowers that in the foreign country. Such a change in relative wage alter the pattern of trade through comparative advantage. To be specific, some of the home exporters exit export markets and become nontraded goods producers, while some of the foreign nontraded firms become new exporters. The fall in real wage and contraction in the foreign country drives down the foreign inflation and hence we observe real exchange rate appreciation. The impulse responses comfirm that the nontraded component of real appreciation becomes more volatile under a fixed exchange rate regime.

The key feature arising from endogenous tradability is the shift in the aggregate productivity and the relative price of goods in the short run. When some of the home export firms exit export markets, and some low productivity nontraded firms disappear, they increase the average productivity in the nontraded sector. As a result, the price of nontraded goods relative to export goods falls. Combining with an improvement in the terms of trade, the home residents substitute the export goods consumption with the nontraded goods and import goods consumption. The degree of substitution depends on changes in the relative prices of goods which in turn are influenced by exchange rates. Sluggish price adjustment due to a fixed exchange rate policy increases an incentive for the home residents to substitute towards nontraded goods, and that expands the nontraded sector and shrinks the export sectors. Such a shift in turn raises productivity of the nontraded sector even more under a fixed exchange rate regime. That is why we observe a larger fall in the relative price of nontraded goods to export goods with a fixed exchange rate, which contributes to a larger depreciation of the nontraded component of real exchange rate.

Interestingly, the shift of patterns of trade and of the aggregate productivity in response to a positive foreign interest rate shock is qualitatively identical to that with a productivity shock, although it is from a different mechanism. A rise in foreign interest rate reduces foreign demand and output, and that raises terms of trade and real wage in the home country, and lowers real wage in the foreign country. The contraction in demand in the foreign country puts downward pressure on its inflation. Consequently, we observe real appreciation in the short run. The nontraded component of real appreciation also depicts higher volatility under a fixed exchange rate regime.

The expenditure switching effect of exchange rates, which is measured by the differences in impulse responses of expenditure share, is also found to be decreasing in transport costs, regardless of type of shocks. Its relationship with the intratemporal elasticity of substitution is ambiguous. Such findings support the argument that removing nominal exchange rate flexibility in an environment where its expenditure switching effect is large will result in volatile movements in the relative price of nontraded to traded goods.

To summarize, my model predicts that the nontraded component of the real exchange rate of a pair of countries that are not perfectly but highly integrated is volatile in response to a productivity and an interest rate shock. My model illustrates that the movements in real exchange rate can be understood better when monetary nonneutrality exists in a trade model. It offers a different and more comprehensive explanation than that based on local-currency pricing. In addition, the finding that non-productivity shocks change the aggregate productivity through international trade emphasizes the importance of further research on the role of heterogeneous economies in shock transmission mechanism.

The rest of the paper is organized as follows. The next section gives stylized facts of real exchange rate volatility. The model is developed in Section 3. Section 4 explains the equilibrium dynamics and the simulation results. I conclude the paper in Section 5.

2 Stylized Facts

This section gives empirical regularities about the volatility of real exchange rates, their sectoral composition and their connection with exchange rate policy. It illustrates that the volatility of real exchange rates of country pairs that maintain stable nominal exchange rates tends to come from the nontraded component.

The data are quarterly and cover the period from 1980:1 to 1998:4. The data set covers 35 countries and produces 595 pairs of bilateral real exchange rates. [1] The price data are

[1]The sample countries are Australia, Austria, Belgium, Brazil, Canada, Chile, Colombia, Denmark, Egypt, El Salvador, Finland, Germany, Greece, India, Indonesia, Ireland, Israel, Japan, Korea, Mexico,

from the International Financial Statistics (IFS). The exchange rate data are originally from the World Currency Report and provided by Reinhart and Rogoff (2004). They are viewed as market-determined exchange rates, unlike the conventional official exchange rates in the IFS database. In the case of emerging markets, there are large discrepancies between the exchange rate data from the two databases. I use the market exchange rates because they reflect the stance of monetary policy bettter and they are relevant to the equilibrium allocations.

I construct series of real exchange rates and their components using one of the methods in Engel (1999) and Betts and Kehoe (2001a). Define the real exchange rate as $Q_t = S_t P_t^\star / P_t$, where S_t is the nominal exchange rate, P_t and $P_t^\star$ are the home and foreign price level. Define the traded component as $Q_{t,T} = S_t P_{t,T}^\star / P_{t,T}$, where $P_{t,T}$ and $P_{t,T}^\star$ are traded goods price indices in the two countries. Then the nontraded component can be defined as

$$Q_{t,N} = Q_t / Q_{t,T}.$$

For instance, if we assume geometric price indices in both countries $P_t = P_{t,T}^{1-\delta} P_{t,N}^{\delta}$ and $P_t^\star = P_{t,T}^{\star 1-\delta^\star} P_{t,N}^{\star \delta^\star}$, then $Q_{t,N} = (P_{t,N}^\star / P_{t,T}^\star)^{\delta\star} / (P_{t,N}/P_{t,T})^{\delta}$. The weights δ and $\delta^\star$ are the consumption shares of nontraded goods in the two countries.

I use the consumer price index (CPI) as the measure of overall good prices and the producer price index (PPI) as the measure of traded goods prices. In logarithms,

$$q_t = s_t + \ln(CPI_t^\star) - \ln(CPI_t),$$

$$q_{t,T} = s_t + \ln(PPI_t^\star) - \ln(PPI_t),$$

$$q_{t,N} = \ln(CPI_t^\star) - \ln(PPI_t^\star) - (\ln(CPI_t) - \ln(PPI_t)).$$

The current method of decomposition has several problems and is better viewed as an imperfect approximation. First, the PPI does contain a large nontraded intermediate input (see Calvo and Kumhof (2003) who emphasize this issue). Second, the CPI and the PPI are constructed with different methodologies. Third, the traded and the nontraded components are negatively correlated by construction. Despite these drawbacks, the decomposition allows us to approximate both components of real exchange rates in a large sample, since the disaggregated price data are not available for most of the emerging markets in the sample. I apply the two methodologies to the real exchange rate series. First, I investigate their volatility based on relative standard deviations. Second, I decompose the variance of real exchange rates following the mothod used by Engel (1999).

Netherlands, New Zealand, Norway, Pakistan, Peru, Philippines, Singapore, Spain, Sri Lanka, Sweden, Switzerland, Thailand, United Kingdom, United States and Venezuela.

2.1 Relative Standard Deviations

I detrend the series using a Baxter-King (1999) band-pass filter, with 8 leads and lags and a pass-band to 6 and 32 quarters. Table 1 reports the summary statistics.

The first three columns in Table 1 describe the volatility of the nontraded component relative to the overall volatility, as measured by the relative standard deviation $\sigma(q_{t,N})/\sigma(q_t)$. The volatility of nontraded real exchange rates varies from 7 to 123 percent of the overall volatility. The average is 29 percent and slightly lower than in Betts and Kehoe (2001a). The nontraded real exchange rate is more volatile in 1980s than 1990-98. Although I do not report the calculation based on official exchange rates, it should be noted that the standard deviation of the ratios approximately doubles.

Table 1. Summary Statistics of Volatility of Nontraded Real Exchange Rates

	$\sigma(q_{t,N})/\sigma(q_t)$			$\sigma(q_{t,N})/\sigma(q_{t,T})$		
Period	1980-98	1980s	1990-98	1980-98	1980s	1990-98
Average	0.29	0.30	0.31	0.30	0.30	0.31
Standard deviation	0.15	0.20	0.18	0.14	0.17	0.16
Maximum	1.23	2.27	1.46	0.90	1.12	1.07
Minimum	0.07	0.06	0.05	0.07	0.05	0.05
Sample size	595	595	595	595	595	595

The last three columns correspond to the relative volatility of the nontraded and the traded component, as measured by the relative standard deviation $\sigma(q_{t,N})/\sigma(q_{t,T})$. The volatility of the nontraded real exchange rates varies from 7 to 90 percent of its traded counterpart in the overall period. In both subperiods, there are cases where the volatility of the nontraded real exchange rate exceeds its traded counterpart. As the primary interest of this paper is the volatility of the nontraded real exchange rates relative to its traded counterpart, I focus on their relative variability and investigate their relationship with the volatility of nominal exchange rates.

Table 2. Volatility of Nontraded Real Exchange Rates and Volatility of Nominal Exchange Rates

	Average of $\sigma(s_t - s_{t-1})$		
Period	1980-98	1980s	1990-98
Group 1: Small nontraded component	0.12	0.15	0.09
Sample size	60	40	60
Group 2: Large nontraded component	0.06	0.07	0.05
Sample size	72	73	73

I divide the sample into two groups according to $\sigma(q_{t,N})/\sigma(q_{t,T})$. When the volatility measure is lower than its "average - standard deviation," I classify it as "small nontraded component." When the volatility measure exceeds its "average + standard deviation," I classify it as "large nontraded component." Table 2 reports the volatility of nominal exchange rates in each subgroups. Since the nominal exchange rates are volatile with high frequencies, it is appropriate to use a high-frequency filtering technique. Here, I use the log-differenced series, or the percentage changes. The standard deviation of percentage changes in nominal exchange rates of the small-nontraded-component group is approximately twice that of the large-nontraded-component group in all periods. In other words, a lower volatility of nominal exchange rate depreciation tend to accompany a higher contribution of the nontraded component of real exchange rate volatility. In fact, the European Monetary System (EMS) country pairs account for 21 percent of the sample of the large-nontraded-component group in 1980-98. The corresponding numbers for the subperiods 1980s and 1990-98 are 26 and 15 percent. On the other hand, they do not appear in the small-nontraded-component group at all. This clearly suggests a connection between exchange rate policy and the volatility of real exchange rates.

Table 3 divides the sample into two groups using the standard deviation of nominal exchange rate depreciations. When the standard deviation is less than 10 percent, I classify it as relatively "fixed" nominal exchange rate regime. Otherwise, I classify it as relatively "flexible" nominal exchange rate regime. The relative volatility of the nontraded real exchange rates is 1.5 times the relative volatility in the flexible rates pairs in the fixed-exchange-rate pairs in the overall period and the 1980s, and it is 14 percent higher in 1990-98. Similar to Table 2, Table 3 emphasizes the influence of exchange rate policy on real exchange rates.

Table 3. Exchange Rate Regime and Volatility of Nontraded Real Exchange Rates

	Average of $\sigma(q_{t,N})/\sigma(q_{t,T})$		
Period	1980-98	1980s	1990-98
Fixed exchange rate regime	0.33	0.36	0.32
Sample size	342	296	450
Flexible exchange rate regime	0.22	0.24	0.28
Sample size	253	299	145

2.2 Variance Decomposition

This section reports the variance decomposition following the methodology of Engel (1999). The results again confirm that the volatility of the nontraded component is high when nominal exchange rates are stable.

The movement of real exchange rates is measured by the mean-squared (MSE) error of their changes, or the sum of the variance and the squared drift,

$$MSE(q_t - q_{t-n}) = var(q_t - q_{t-n}) + [mean(q_t - q_{t-n})]^2.$$

The series are log-differenced at two horizons. One is one-quarter and the other is one year.

The fraction of the mean-squared error of $q_t - q_{t-n}$ accounted for by the mean-squared error of the nontraded component $q_{t,N} - q_{t-n,N}$, can be computed by [2]

$$\frac{MSE(q_{t,N} - q_{t-n,N})}{MSE(q_{t,N} - q_{t-n,N}) + MSE(q_{t,T} - q_{t-n,T})}.$$

The results are in Table 4.

On average the nontraded component accounts for approximately 10 percent of the overall variance. However, there are cases where the variance exceeds that of the traded component. I divide samples into two groups and examine their nominal exchange rate volatility as done in the previous section. When the nontraded variance is smaller than 0.10, I classify it as "small nontraded component." When it exceeds 0.30, I classify it as "large nontraded component." The corresponding volatility of nominal exchange rates are reported in Table 5.

Table 4. Summary Statistics of Nontraded Component in Variance Decomposition

	Nontraded Component					
Period	1980-1998		1980s		1990-98	
Horizon	n=1	n=4	n=1	n=4	n=1	n=4
Average	0.08	0.10	0.07	0.10	0.09	0.11
Standard deviation	0.05	0.06	0.05	0.07	0.08	0.08
Maximum	0.38	0.42	0.37	0.46	0.48	0.57
Minimum	0.01	0.01	0.01	0.01	0.00	0.01

In Table 5, the standard deviation of exchange rate depreciations in the small-nontraded-component group is 1 to 7 times of that in the large-nontraded-component group, depending on the time horizon. Table 5 presents similar facts to Tables 2 and 3. The volatility of real exchange rates of country pairs that experience low volatility in nominal exchange rates originates to a relatively larger extent in the nontraded real exchange rates. This finding is indeed consistent with what is found in the study on Mexico's real exchange rates by Mendoza (2000). He finds that during the fixed exchange rate period the nontraded component of U.S.- Mexico real exchange rates accounts for 29-71 percent of the overall variance, depending on the time horizon.

Table 5. Nontraded Component in Variance Decomposition and Volatility of Nominal Exchange Rates

[2] I do not correct for the negative correlation between the traded and the nontraded real exchange rates, as the correlation is an artifact of the data.

	Average of $\sigma(s_t - s_{t-n})$					
Period	1980-1998		1980s		1990-98	
Horizon	n=1	n=4	n=1	n=4	n=1	n=4
Group 1: Small nontraded component	0.14	0.12	0.14	0.13	0.12	0.12
Sample size	470	372	483	353	394	291
Group 2: Large nontraded component	0.02	0.10	0.02	0.13	0.03	0.08
Sample size	4	6	2	13	18	20

To summarize, these stylized facts show that the traded-nontraded decomposition of volatility of real exchange rates is closely connected to exchange rate policy. When exchange rate is freely floating, the traded component dominates the nontraded component in almost all cases but signifiantly less so when the exchange rate is fixed. The model in the next section suggests a mechanism through which exchange rate policy may influence the movement in real exchange rates.

3 The Model

The basic setup follows a two-country model in the new open-economy macroeconomics literature. It is an extension of the trade model with a continuum of goods by Dornbusch et al. (1977) and the sticky wage model along the same line as Rotemberg (1982). This combination gives one new key feature: Heterogeneous and multi-period price dynamics. The other main element is the explicit treatment of deviations from the law of one price in the form of ice-berg type transport costs, where a fraction τ $(0 < \tau < 1)$ of shipped goods is lost in transit. [3] The world economy consists of two open economies called home and foreign country. Let i denote the type of households and $i \in [0, 1]$. The home households are located in the range $[0, \alpha]$, where $0 < \alpha < 1$, and the rest are in foreign country. Each household i monopolistically supplies her labor to a competitive employment agency by setting a nominal wage, W_t^i. The employment agency sells aggregate labor services to all domestic firms taking the aggregate wage W_t as given. There is a continuum of differentiated goods indexed by z, where $z \in [0, 1]$. The goods markets are also monopolistically competitive. The home goods are located in $[0, z_t^h]$ while the foreign ones are in $[z_t^l, 1]$. The preference specification requires consumers to consume all goods. Hence the range of home imports and exports are $(z_t^h, 1]$ and $[0, z_t^l)$. Note that z_t^h and z_t^l are endogenous and the size of the nontraded sector $\delta_t = z_t^h - z_t^l$, $z_t^l < z_t^h$. I verify in a subsequent section that the condition $z_t^l < z_t^h$ always holds in equilibrium with $0 < \tau < 1$.

[3] See Brunner and Naknoi (2003) for modeling trade frictions in a framework of new open-economy macroeconomics.

3.1 Employment Agency

A competitive employment agency buys labor services from all households at the wage W_t^i, $i \in [0, \alpha]$, and sells the aggregate labor to all domestic firms at the wage W_t. Let l_t^i denote labor services supplied by the household i. The aggregate labor L_t is defined by

$$L_t = \left[\left(\frac{1}{\alpha} \right)^{\frac{1}{\eta}} \int_0^\alpha l_t^{i\,\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}.$$

The agency faces the following cost minization problem.

$$min_{l_t^i} \int_0^\alpha W_t^i l_t^i di \qquad s.t. \qquad L_t = 1.$$

It chooses the stochastic processes $\{l_t^i\}_{t=0}^\infty$, $i \in [0, \alpha]$, that solve the minimization problem taking wages W_t^i and W_t as given. The technical appendix shows that the optimal demand for household i's labor is as follows.

$$l_t^i = \frac{1}{\alpha} \left[\frac{W_t^i}{W_t} \right]^{-\eta} L_t, \tag{1}$$

where

$$W_t = \left[\frac{1}{\alpha} \int_0^\alpha W_t^{i\,1-\eta} di \right]^{\frac{1}{1-\eta}}.$$

3.2 Households

Each household i supplies her labor service to a competitive employment agency, which sells the aggregate labor service to all domestic firms, and set nominal wage W_t^i. The household i's utility function in the home country depends on a basket of all goods C_t^i, real money balances $m_t^i = M_t^i / P_t$ and labor supply l_t^i.

$$U_t^i = E_t \sum_{t=0}^\infty \beta^t \left[\frac{\sigma}{\sigma - 1} C_t^{i\,\frac{\sigma-1}{\sigma}} + \frac{\chi}{1-\epsilon} m_t^{i\,1-\epsilon} - \frac{1}{\mu} l_t^{i\,\mu} \right] \tag{2}$$

where $0 < \beta < 1$, $\mu < 1$, $\sigma > 0$, $\epsilon > 0$. The consumption aggregate C_t^i is defined as

$$C_t^i = \left[\int_0^1 c_t^i(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}.$$

$c_t^i(z)$ is the consumption of good z, and θ ($\theta > 1$) is the elasticity of substitution between goods. The price index P_t is [4]

$$P_t = \left[\int_0^1 p_t(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}}.$$

Suppose there are complete domestic asset markets to insure idiosyncratic income risk from sticky wage-setting and incomplete international asset markets. There are two noncontingent bonds traded internationally. One is issued by the home households and denominated in the home currency, while the other is by the foreign households and in the foreign currency. The households have to incur a quadractic portfolio adjustment cost. The quadratic portfolio adjustment cost, which is first suggested by Neumeyer and Perri (2001), assures that the model has a unique steady state and stationary bond holdings. Let F_t^i and $F_t^{f,i}$ denote the stock of the home bond and the foreign bond owned by the household i. The functional form of the quadratic adjustment cost associated with the home bond is

$$\Phi(f_t^i) = \frac{1}{2}\phi(f_t^i - f_{ss}^i)^2,$$

where f_t is the real value of the home bond and defined as F_t^i/P_t. ϕ is a parameter. The cost is quadratic in deviation from the steady state level of bond holdings. The portfolio adjustment cost associated with the foreign bond is defined in a similar way.

$$\Phi^f(f_t^{f,i}) = \frac{1}{2}\phi^\star(f_t^{f,i} - f_{ss}^{f,i})^2$$

where $f_t^{f,i} = F_t^{f,i} S_t/P_t$ and S_t is nominal exchange rate. In general, $\phi^\star \neq \phi$.

The period-t budget constraint for households in the home country are:

$$\begin{aligned} P_t C_t^i + (M_t^i - M_{t-1}^i) + (F_t^i - F_{t-1}^i) + S_t(F_t^{f,i} - F_{t-1}^{f,i}) + P_t\Phi(F_t^i/P_t) + P_t\Phi^f(F_t^{f,i}S_t/P_t) \\ = P_t T_t^i + (1+\tau_w)W_t^i l_t^i - P_t h(\pi_t^{w^i}) + \Pi_t^i + i_{t-1}F_{t-1}^i + i_{t-1}^\star S_t F_{t-1}^{f,i} \end{aligned} \quad (3)$$

where T_t^i is the transfer from the government. Π_t^i is the nominal dividends distributed to the household

$$\Pi_t^i = \Pi_t = \int_0^{z_t^h} \frac{\Pi_t(z)}{\alpha} dz, \quad (4)$$

where $\pi_t(z)$ is the firm z's dividend. It is distributed evenly to all households through

[4] P_t is the cost associated with the solution of the cost minimization problem

$$min_{c_t^i(z)} \int_0^1 p_t(z) c_t^i(z) dz \qquad s.t. \qquad C_t^i = 1.$$

the complete domestic asset markets. τ_w is the rate of subsidy paid to the households to remove the steady state markup distortion and $\tau_w = 1/(\eta - 1)$. $h(\pi_t^{w^i})$ is the cost of changing nominal wages and it is a convex function of wage (gross) inflation $\pi_t^{w^i} = W_t^i/W_{t-1}^i$. The adjustment cost induces wage stickiness analogous to what generates price stickiness as pioneered by Rotemberg (1982). I assume that $h(\pi_t^{w^i})$ is quadratic in deviations from the deterministic steady state level of wage inflation $\pi_t^{w^i} - \pi_{ss}^{w^i}$, $h(\pi_t^{w^i}) = \phi^w\left(\pi_t^{w^i} - \pi_{ss}^{w^i}\right)^2/2$. ϕ^w is the wage adjustment cost parameter. i_{t-1} is the home nominal interest rate set in the period $t-1$. Let i_t and r_t denote the home nominal and real interest rate.

The household i chooses the set of stochastic processes $\{c_t^i(z), C_t^i, m_t^i, F_t^i, F_t^{f,i}, W_t^i\}_{t=0}^{\infty}$ to maximize (2) subject to (1), (3) and (4) and the transversality conditions

$$lim_{j\to\infty} E_t\left[F_{t+j}^i/\Pi_{s=0}^{j-1}(1+i_{t+s})\right] \geqslant 0, \quad lim_{j\to\infty} E_t\left[S_{t+j}F_{t+j}^{f,i}/\Pi_{s=0}^{j-1}(1+i_{t+s}^{\star})\right] \geqslant 0,$$

taking as given the sequences $\{p_t(z), P_t, \Pi_t\}_{t=0}^{\infty}$ and the initial conditions $(M_{-1}^i, F_{-1}^i, F_{-1}^{f,i}, W_{-1}^i)$. By the assumption of complete domestic equity markets and identical preferences, all household choices are symmetric, given by $\{c_t(z), C_t, m_t, F_t, F_t^f\, W_t\}$. The relevant first order conditions are as follows.

$$c_t(z) = \left[\frac{p_t(z)}{P_t}\right]^{-\theta} C_t \tag{5}$$

$$1 + \Phi(f_t - f_{ss}) = \beta(1+i_t)E_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{1/\sigma}\frac{P_t}{P_{t+1}}\right] \tag{6}$$

$$1 + \Phi^f(f_t^f - f_{ss}^f) = \beta(1+i_t^{\star})E_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{1/\sigma}\frac{P_t}{P_{t+1}}\frac{S_{t+1}}{S_t}\right] \tag{7}$$

$$m_t^{\epsilon} = \chi {C_t}^{1/\sigma}\left(\frac{1+i_t}{i_t}\right) \tag{8}$$

$$C_t^{-1/\sigma}\left(\eta\frac{l_t}{P_t} + \phi^w\frac{\pi_t^w - \pi_{ss}^w}{W_{t-1}} - \phi^w E_t\left[\frac{1}{1+r_t}\frac{(\pi_{t+1}^w - \pi_{ss}^w)W_{t+1}}{W_t^2}\right]\right) = \eta\frac{l_t^{\mu}}{W_t} \tag{9}$$

where

$$\frac{1}{1+r_t} = \beta E_t\left(\frac{C_{t+1}}{C_t}\right)^{-1/\sigma}.$$

(5) is the intratemporal consumption decision. (6) and (7) are the intertemporal consumption decision. (8) is the optimal money demand function. (9) gives the optimal

wage setting rule. Define the real wage $w_t = W_t/P_t$ and rewrite the wage setting rule.

$$C_t^{-1/\sigma}\left(\eta l_t w_t + \phi^w(\pi_t^w - \pi_{ss}^w)\pi_t^w - \phi^w E_t\left[\frac{(\pi_{t+1}^w - \pi_{ss}^w)}{(1+r_t)}\pi_{t+1}^w\right]\right) = \eta l_t^{\mu}. \tag{10}$$

The households stabilize their wage inflation at the inflation level in the steady state, $\pi_{ss}^w = \pi_{ss}$ where π_{ss} is the steady state rate of inflation of the nominal anchor. I assume identical preferences in the foreign country, and define the foreign variables in a similar way. As usual, the superscript star denotes the foreign variables. The analogous equations hold for the foreign households.

3.3 Interest Parity

I derive the relationship between the two interest rates from the two Euler equations.

$$1 + i_t = (1 + i_t^{\star})\frac{1 + \Phi(f_t - f_{ss})}{1 + \Phi^f(f_t^f - f_{ss}^f)}\left[E_t\left[\frac{S_{t+1}}{S_t}\right] + \frac{Cov_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{1/\sigma}\frac{P_t}{P_{t+1}}\frac{S_{t+1}}{S_t}\right]}{E_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{1/\sigma}\frac{P_t}{P_{t+1}}\right]}\right]. \tag{11}$$

With the portfolio adjustment cost, the uncovered interest rate parity does not hold. The deviation from the uncovered interest parity clearly depends on the portfolio adjustment cost parameters ϕ and $\phi^{\star}$. I assume $F_{ss} = F_{ss}^{\star} = 0$, thus the uncovered interest parity holds in the steady state.

3.4 International Trade and Aggregate Prices

International trade is costly. To be specific, I assume the iceberg-type transport costs, where a fraction τ $(0 < \tau < 1)$ of shipped goodss is lost in transit. An increase in τ implies more deviations from the law of one price. Let $p_t(z)$ denote the consumer price of good z in the home country charged by the home firm z. Similarly, $p_t^{\star}(z)$ denotes the consumer price in the foreign country charged by a foreign firm. The domestic price of home imports therefore becomes

$$p_t(z) = \frac{S_t p_t^{\star}(z)}{1-\tau} \qquad for \quad z \in (z_t^h, 1].$$

Similarly, the foreign consumer price of home exports is

$$p_t^{\star}(z) = \frac{p_t(z)}{S_t(1-\tau)} \qquad for \quad z \in [0, z_t^l).$$

Using the above relationships, I can rewrite the home CPI

$$P_t = \left[z_t^l P_{t,H}^{1-\theta} + \delta_t P_{t,N}^{1-\theta} + (1-z_t^h) P_{t,F}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{12}$$

where

$$P_{t,H} = \left[\frac{1}{z_t^l}\int_0^{z_t^l} p_t(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}},$$

$$P_{t,N} = \left[\frac{1}{\delta_t}\int_{z_t^l}^{z_t^h} p_t(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}},$$

$$P_{t,F} = \left[\frac{1}{1-z_t^h}\int_{z_t^h}^{1} \left(\frac{S_t p_t^\star(z)}{1-\tau}\right)^{1-\theta} dz\right]^{\frac{1}{1-\theta}}.$$

The price subindex $P_{t,j}$ ($j \in (F, H, N)$) is defined as the minimum expenditure required to obtain one unit of $C_{t,j}$ where C_j is the consumption subindex implicitly defined by

$$C_t = \left[{z_t^l}^{\frac{1}{\theta}} C_{t,H}^{\frac{\theta-1}{\theta}} + \delta_t^{\frac{1}{\theta}} C_{t,N}^{\frac{\theta-1}{\theta}} + (1-z_t^h)^{\frac{1}{\theta}} C_{t,F}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$$

where

$$C_{t,H} = \left[\left(\frac{1}{z_t^l}\right)^{\frac{1}{\theta}} \int_0^{z_t^l} c_t(z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}},$$

$$C_{t,N} = \left[\left(\frac{1}{\delta_t}\right)^{\frac{1}{\theta}} \int_{z_t^l}^{z_t^h} c_t(z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}},$$

$$C_{t,F} = \left[\left(\frac{1}{1-z_t^h}\right)^{\frac{1}{\theta}} \int_{z_t^h}^{1} c_t(z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}.$$

Note that the traded price index $P_{t,T}$ is implicitly defined as

$$(1-\delta_t) P_{t,T}^{1-\theta} = z_t^l P_{t,H}^{1-\theta} + (1-z_t^h) P_{t,F}^{1-\theta}.$$

The CPI can be expressed in terms of the traded and nontraded prices as

$$P_t = \left[(1-\delta_t) P_{t,T}^{1-\theta} + \delta_t P_{t,N}^{1-\theta}\right]^{\frac{1}{1-\theta}}.$$

The foreign CPI can be obtained in a similar fashion.

$$P_t^\star = \left[z_t^l P_{t,H}^{\star 1-\theta} + \delta_t P_{t,N}^{\star 1-\theta} + (1-z_t^h) P_{t,F}^{\star 1-\theta}\right]^{\frac{1}{1-\theta}} \tag{13}$$

where

$$P^\star_{t,H} = \left[\frac{1}{z^l_t} \int_0^{z^l_t} \left(\frac{p_t(z)}{S_t(1-\tau)} \right)^{1-\theta} dz \right]^{\frac{1}{1-\theta}},$$

$$P^\star_{t,N} = \left[\frac{1}{\delta_t} \int_{z^l_t}^{z^h_t} p^\star_t(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}},$$

$$P^\star_{t,F} = \left[\frac{1}{1-z^h_t} \int_{z^h_t}^{1} p^\star_t(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}.$$

Multiply the foreign price subindices $P^\star_{t,H}$, $P^\star_{t,N}$ and $P^\star_{t,F}$ by S_t.

$$S_t P^\star_{t,H} = \left[\frac{1}{z^l_t} \int_0^{z^l_t} \left(\frac{p_t(z)}{1-\tau} \right)^{1-\theta} dz \right]^{\frac{1}{1-\theta}},$$

$$S_t P^\star_{t,N} = \left[\frac{1}{\delta_t} \int_{z^l_t}^{z^h_t} S_t p^\star_t(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}},$$

$$S_t P^\star_{t,F} = \left[\frac{1}{1-z^h_t} \int_{z^h_t}^{1} S_t p^\star_t(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}.$$

It is evident from the above equations that $P_{t,j} = S_t P^\star_{t,j}$ $(j = H, F)$ fails unless $\tau = 0$. I show in a subsequent section that when $\tau = 0$, $z^h_t = z^l_t$ and the nontraded do not exist in the equilibrium. In short, the absolute purchasing power parity (PPP), $P_t = S_t P^\star_t$, or $Q_t = 1$, breaks down because of the presence of trade frictions.

3.5 Firms

Firms are price setters and prices are flexible. The technology is heterogeneous across firms and the production function of firm z is characterized by

$$y_{t,j}(z) = X_t a(z) l_{t,j}(z) \qquad j = H, N. \tag{14}$$

$y_{t,j}(z)$ $(j = H, N)$ is the output of the good z. X_t is the stochastic component of productivity. $a(z)$ is the firm-specific productivity parameter, and $l_{t,j}(z)$ is the aggregate labor input used by the firm. Whether a good z is produced in the home country at all $(z < z^h_t)$, and whether it is an exportable $(z < z^l_t)$ or not is determined in equilibrium.

3.5.1 Exportable Firms

A firm $z \in [0, z_t^l]$ sells its products at the price $p_t^h(z)$ to the home demand $c_t^h(z)$ and to the foreign demand $c_t^{h\star}(z)$. Let $y_{t,H}(z)$ be total output of the home exportable z.

$$y_{t,H}(z) = c_t^h(z) + \frac{c_t^{h\star}(z)}{1-\tau} \tag{15}$$

As indicated by the denominator $1-\tau$ in the second term, a fraction τ of the exportable is lost in transit and the foreign buyers incur the loss, by having to pay a higher price for an effective unit of the good. As I do not analyze the effects of fiscal shocks, I assume zero government consumption throughout. The aggregate demand in each country therefore equals total private consumption.

$$\begin{aligned} D_t &= \alpha C_t, \\ D_t^\star &= (1-\alpha) C_t^\star. \end{aligned}$$

Demand for products are given by the intratemporal consumption decision.

$$c_t^h(z) = \left[\frac{p_t^h(z)}{P_t}\right]^{-\theta} D_t, \tag{16}$$

$$c_t^{h\star}(z) = \left[\frac{p_t^h(z)}{S_t P_t^\star (1-\tau)}\right]^{-\theta} D_t^\star \tag{17}$$

The government subsidizes the production with the rate $\tau_y = 1/(\theta - 1)$ to eliminate the distortions arising from monopolistic competition. The profit function $\Pi_t^h(z)$ of the tradable firm z becomes

$$\Pi_t^h(z) = \left[(1+\tau_y)p_t^h(z) - MC_t(z)\right]\left(c_t^h(z) + \frac{c_t^{h\star}(z)}{1-\tau}\right) \tag{18}$$

The firm maximizes the present discounted value of its real profit stream

$$V_t = E_t \Pi_{s=0}^{\infty} R_{t,t+s} \frac{\Pi_{t+s}^h(z)}{P_{t+s}} \tag{19}$$

where $R_{t,t+s}$ is the s period ahead real discount factor,

$$R_{t,t+s} = \Pi_{j=1}^{s} \frac{1}{1 + r_{t+j}}.$$

An exportable firm chooses the stochastic processes $\{p_t^h(z)\}_{t=0}^{\infty}$ to maximize (19) subject to (14)-(18) taking the sequences $\{P_t, P_t^\star, W_t, D_t, D_t^\star\}_{t=0}^{\infty}$ as given. The optimal price setting rule is

$$p_t^h(z) = MC_t(z) = \frac{W_t}{X_t a(z)}. \tag{20}$$

Deflate the nominal variables with P_t, $mc_t = MC_t/P_t$ and $\tilde{p}_t^h(z) = p_t^h(z)/P_t$. Rewrite the price setting rule.

$$\tilde{p}_t^h(z) = mc_t(z) = \frac{w_t}{X_t a(z)} \tag{21}$$

All exportable firms stabilize their prices at the marginal cost level, and in the steady state, $\tilde{p}_{ss}^h(z)a(z) = w_{ss}$.

3.5.2 Nonexportable Firms

For any $z \in (z_t^l, z_t^h)$, the firm z sells output $y_{t,N}(z)$ in the domestic market at the price $p_t^n(z)$.

$$y_{t,N}(z) = c_t(z) = \left[\frac{p_t^n(z)}{P_t}\right]^{-\theta} D_t. \tag{22}$$

They receive the production subsidy with the rate $1 + \tau_y$ as well. The profit function for firm z becomes

$$\Pi_t^n(z) = \left[(1 + \tau_y)p_t^n(z) - MC_t(z)\right] y_{t,N}(z). \tag{23}$$

The firm's objective function becomes

$$V_t = E_t \Pi_{s=0}^{\infty} R_{t,t+s} \frac{\Pi_{t+s}^n(z)}{P_{t+s}} \tag{24}$$

A nonexportable firm chooses the stochastic process $\{p_t^n(z)\}_{t=0}^{\infty}$ to maximize (24) subject to (14), (22) and (23) taking the sequences $\{P_t, W_t, D_t\}_{t=0}^{\infty}$ as given. The price setting rule is

$$p_t^n(z) = MC_t(z) = \frac{W_t}{X_t a(z)}. \tag{25}$$

We can normalize the nominal variables as in the previous subsection, $\tilde{p}_t(z, n) = p_t^n(z)/P_t$. The price setting rule for the nonexportable firm z becomes

$$\tilde{p}_t^n(z) = mc_t(z) = \frac{w_t}{X_t a(z)} \tag{26}$$

All nonexportable firms also stabilize their prices at the marginal cost level. In the steady state $\tilde{p}^n_{ss}(z)a(z) = w_{ss}$.

The foreign firms face the similar decision problems and the analogous equations hold. Also, I use the superscripts $f^\star$ and $n^\star$ to denote variables associated with the foreign exportable and nonexportable firms, respectively.

3.6 National Account

It is useful to define the gross output or GDP as Y_t and the net exports N_t.

$$P_t Y_t = \int_0^{z_t^l} p_t^h(z) y_{t,H}(z) dz + \int_{z_t^l}^{z_t^h} p_t^n(z) y_{t,N}(z) dz$$

$$P_t N_t = \int_0^{z_t^l} p_t^h(z) y_t^{h\star}(z) dz - \int_{z_t^h}^{1} \frac{S_t p_t^{f\star}(z)}{1-\tau} y_t^f(z) dz$$

The foreign real GDP and net exports can be defined in a similar way.

3.7 Fiscal Policy

For simplicity, I assume that the government rebates the seigniorage revenues net of the subsidy expense to the consumers in a lump-sum fashion. The home government budget constraint is

$$\alpha(M_t - M_{t-1}) = \alpha P_t T_t + \alpha \tau_w W_t L_t + \tau_y P_t Y_t \tag{27}$$

The analogous equation holds for the foreign government.

3.8 Monetary Policy

3.8.1 Fixed Exchange Rate Regime

A fixed exchange rate regime is equivalent to targeting the rate of depreciation. Define exchange rate depreciation as $d_t = E_t[S_{t+1}/S_t]$. I define a fixed exchange rate regime as a policy that targets the path of d_t at the constant level d and that rules out any discrete jump in the path of S_t. The interest rate parity is therefore constrained by

$$d_t = d. \tag{28}$$

The monetary authority in the foreign country conducts an independent monetary policy. The foreign country follows a Taylor rule of the following form.

$$log(1+i_t^\star) = \lambda_i log(1+i_{t-1}^\star) + (1-\lambda_i)\left(\lambda_\pi E_t log(\frac{\pi_{t+1}^\star}{\pi_{ss}^\star}) + \lambda_y log(\frac{Y_t^\star}{Y_{ss}^\star})\right) + (1-\lambda_i) log(1+i_{ss}^\star) + H_t$$

$$ \tag{29} $$

H_t is the foreign interest rate shock.

3.8.2 Flexible Exchange Rate Regime

A flexible exchange rate regime is consistent with a monetary target rule, an interest rule, an inflation target rule or a price target rule. I define the flexible exchange regime using the following price target rule in the home country.

$$ \pi_t = \pi_{ss} \tag{30} $$

I also rule out any discrete jumps in the path of P_t. The foreign monetary authority implements the Taylor rule specified by Equation (29). The path of S_t is implied by the household optimality conditions.

3.9 Stochastic Processes of Shocks

The two countries are subject to two types of shocks, namely, productivity shocks and foreign interest rate shocks. The productivity shocks follow the following stochastic processes.

$$ \begin{aligned} log(X_t) &= \rho_x log(X_{t-1}) + u_t, && (31) \\ log(X_t^\star) &= \rho_x log(X_{t-1}^\star) + u_t^\star. && (32) \end{aligned} $$

where u_t and $u_t^\star$ is a normally distributed shock with zero mean.

The foreign interest rate shock process is,

$$ log(H_t) = \rho_i log(H_{t-1}) + v_t, \tag{33} $$

where v_t is a normally distributed shock with zero mean.

3.10 Equilibrium

Before I define an equilibrium, I first outline the equilibrium pattern oftrade as it is required for solving for the solution.

3.10.1 Equilibrium Pattern of Trade

The determination of the equilibrium pattern of trade follows the principle of comparative advantage as suggested by Dornbusch et al. (1977). In the short-run, trade is driven not only by comparative advantage but also by the effect of imperfect competition in the labor and goods markets and nominal rigidities. Define the deterministic and the stochastic components of *relative productivity* of a home firm, respectively, as $A(z) = a(z)/a^\star(z)$ and $\chi_t = X_t/X_t^\star$. We can rank z such that $A'(z) < 0$. In short, the home country has comparative advantage in the low end of z. To be specific, I assume $a(z) = 2(1-z)$ and $a^\star(z) = 1$. Let ω_t denote the equilibrium relative wage $W_t/S_tW_t^\star$.

The world trade pattern depends on the relative price of imports and domestically produced products. A home firm will produce any variety z if and only if its price does not exceed the import price of the foreign product of the same variety,

$$\frac{S_t p_t^\star(z)}{1-\tau} \geqslant p_t(z).$$

The price-setting rules yield

$$\omega_t \leqslant \frac{\chi_t A(z)}{1-\tau}.$$

Otherwise a foreign firm will export the variety z to the home country. Similarly, a foreign firm will produce any variety z if and only if its price does not exceed the import price of the identical product.

$$\frac{p_t(z)}{S_t(1-\tau)} \geqslant p_t^\star(z)$$

The price-setting rules give

$$\omega_t \geqslant \chi_t A(z)(1-\tau).$$

Otherwise a home firm will export the variety z to the foreign country.

I can summarize the equilibrium pattern of production and trade as follows.

1. The home country produces any variety $z \in [0, z_t^h]$ and imports $z \in (z_t^h, 1]$ from the foreign country where z_t^h satisfies

$$\omega_t = \frac{\chi_t A(z_t^h)}{1-\tau}. \tag{34}$$

2. The foreign country produces any variety $z \in [z_t^l, 1]$ and imports $z \in [0, z_t^l]$ from the home country where z_t^l satisfies

$$\omega_t = \chi_t A(z_t^l)(1-\tau). \tag{35}$$

3. $z_t^l \leq z_t^h$ and any variety $z \in (z_t^l, z_t^h)$ is produced in both countries but not traded internationally. It is easy to verify that (1) $z_t^l < z_t^h$ in the equilibrium with $0 < \tau < 1$, and (2) $z_t^l = z_t^h$ when $\tau = 0$. Suppose $z_t^l > z_t^h$, then by (34) there is a variety $z' \in (z_t^h, z_t^l)$. Then by $z' > z_t^h$, there must be a variety that a foreign tradable firm is willing to export to home country. But for z' to be produced for an exporting purpose in the foreign country, by (35) $z_t'^l$ must hold. This contradicts the definition

of z'. Therefore $z_t^l < z_t^h$ must hold in any equilibrium with non-zero transport costs. If $\tau = 0$, then $A(z)/(1-\tau) = A(z)(1-\tau)$ for all z. In that case, the conditions (34) and (35) are equivalent and $z_t^l = z_t^h$.

Because of the monotonicity of $A(z)$ in z, z_t^h and z_t^l are unique.

3.10.2 The Definition of Equilibrium

An equilibrium of the world economy is defined as the stochastic processes of allocation $\{c_t(z), C_t, c_t^\star(z), C_t^\star, m_t, m_t^\star, l_{t,H}(z), l_{t,N}(z), L_t, l_{t,F}(z), l_{t,N}^\star(z), L_t^\star, \Pi_t, \Pi_t^\star, T_t, T_t^\star, F_t, F_t^\star, F_t^f, F_t^{f\star}, y_{t,H}(z), y_t^h(z), y_t^{h\star}(z), y_{t,N}(z), y_{t,F}(z), y_t^f(z), y_t^{f\star}(z), y_{t,N}^\star(z), \Pi_t(z), \Pi_t^\star(z)_{z\in[0,1]}\}_{t=0}^\infty$, the price system $\{p_t^h(z), p_t^n(z), P_t, p_t^{f\star}(z), p_t^{n\star}(z), P_t^\star, W_t, W_t^\star, i_t, r_t\}_{t=0}^\infty$ for a fixed exchange rate regime, the price system $\{p_t^h(z), p_t^n(z), P_t, p_t^{f\star}(z), p_t^{n\star}(z), P_t^\star, W_t, W_t^\star, i_t, r_t, S_t\}_{t=0}^\infty$ for a flexible exchange rate regime, the world production pattern $\{z_t^h, z_t^l\}_{t=0}^\infty$, a pair of government policy (28), (29) under a fixed exchange rate regime or (29), (30) under a flexible exchange rate regime, and the exogenous shocks $\{X_t, X_t^\star, H_t\}_{t=0}^\infty$ that satisfy the following.

1. The household's maximization problem: Equations (2)-(9) and their foreign analogues.

2. The firm's profit maximization problem: Equations (14), (18), (20), (23), (25) and their foreign analogues.

3. The employment agency's cost minimization problem: Equation (1), and its foreign analogue.

4. The labor market clearing condition.

$$\int_0^{z_t^l} l_{t,H}(z)dz + \int_{z_t^l}^{z_t^h} l_{t,N}(z)dz = L_t. \tag{36}$$

$$\int_{z_t^l}^{z_t^h} l_{t,N}^\star(z)dz + \int_{z_t^h}^{1} l_{t,F}(z)dz = L_t^\star. \tag{37}$$

5. The goods market clearing conditions: Equations (15)-(17), (22) and their foreign analogues.

6. The international bond markets clear.

$$\alpha F_t + (1-\alpha)F_t^\star = 0 \tag{38}$$

$$\alpha F_t^f + (1-\alpha)F_t^{f\star} = 0 \tag{39}$$

7. The world production pattern follows Equations (34) and (35).

8. The exogenous shocks follow the stochastic processes (31)-(33).

The total number of variables excluding the shock variables is 44.

4 Equilibrium Dynamics

This section presents the key dynamic equations which describe the adjustment mechanism in the model. [5] First, I outline the wage inflation dynamics. Next, I derive the aggregate price dynamics. Finally, I close the section with the discussion on the real exchange rate dynamics.

Let $\hat{x}_t = dx_t/x_{ss}$ denote the deviation of x_t from its deterministic steady state level x_{ss}. In the steady state, $r_{ss} = (1-\beta)/\beta$. The CPI inflation rates $\pi_{ss}, \pi^\star_{ss}$ and the depreciation rate d_{ss} depend on the monetary policy rules. Relative purchasing power parity holds in steady state, therefore $d_{ss} = \pi_{ss} - \pi^\star_{ss}$. I assume $F_{ss} = F^\star_{ss} = F^f_{ss} = F^{f\star}_{ss} = 0$.

4.1 Wage Inflation Dynamics

The linear approximation of the wage equation gives the following wage inflation dynamics.

$$\hat{\pi}^w_{t+1} = \frac{1}{\beta}\hat{\pi}^w_t - B_w\left[(\mu-1)\hat{l}_t + \frac{1}{\sigma}\hat{C}_t - \hat{w}_t\right], \tag{40}$$

$B_w = \eta l_{ss} w_{ss}(\phi^w \beta \pi^2_{ss})^{-1}$. Note that the definition of w_t implies

$$\hat{w}_t = \hat{w}_{t-1} + \hat{\pi}^w_t - \hat{\pi}_t. \tag{41}$$

The corresponding equation in the foreign country is

$$\hat{\pi}^{w\star}_{t+1} = \frac{1}{\beta}\hat{\pi}^{w\star}_t - B^\star_w\left[(\mu-1)\hat{l}^\star_t + \frac{1}{\sigma}\hat{C}^\star_t - \hat{w}^\star_t\right], \tag{42}$$

where $B^\star_w = \eta l^\star_{ss} w^\star_{ss}(\phi^{w\star}\beta {\pi^\star_{ss}}^2)^{-1}$ and

$$\hat{w}^\star_t = \hat{w}^\star_{t-1} + \hat{\pi}^{w\star}_t - \hat{\pi}^\star_t. \tag{43}$$

[5] The detailed derivation of the linear approximation of the model can be found in the technical appendix, which is available upon request.

4.2 Price Dynamics of Export Sector

Define the deterministic component of the average productivity of the home export sector as

$$a_{t,H} = \left(\frac{1}{z_t^l} \int_0^{z_t^l} a(z)^{\theta-1} dz \right)^{\frac{1}{\theta-1}}.$$

The price dynamics corresponding to the home export sector are summarized by two variables, $\hat{\tilde{P}}_{t,H}$, $\hat{a}_{t,H}$ and $\hat{\pi}_{t,H}$. From the optimal price setting rule and the definition of $P_{t,H}$,

$$\hat{\tilde{P}}_{t,H} = \hat{w}_t - \hat{X}_t - \hat{a}_{t,H}. \tag{44}$$

$\hat{a}_{t,H}$ can be derived from its definition above.

$$\hat{a}_{t,H} = \frac{(a(z_{ss}^l)/a_H)^{\theta-1} - 1}{\theta - 1} \hat{z}_t^l, \tag{45}$$

where a_H denote the steady state level productivity. Evidently, the equation above indicates that endogenous tradability influences the average productivity of the export sector. In other words, the aggregate productivity of the economy is affected by not only productivity shocks but also other kinds of shocks such as foreign interest rate shocks through its deterministic component $\hat{a}_{t,H}$. In fact, $a(z_{ss}^l) < a_H$, because $A'(z) < 0$. Hence $d\hat{a}_{t,H}/d\hat{z}_t^l < 0$. When home nontraded firms enter export markets, they lowers the sectoral productivity due to their cost disadvantage.

The dynamics of sectoral inflations can be derived from their definition. By definition of $\pi_{t,H}$,

$$\pi_{t,H} = \frac{\tilde{P}_{t,H}}{\tilde{P}_{t-1,H}} \pi_t.$$

Therefore,

$$\hat{\pi}_{t,H} = \hat{\tilde{P}}_{t,H} - \hat{\tilde{P}}_{t-1,H} + \hat{\pi}_t. \tag{46}$$

In the foreign country, the export sector price dynamics are characterized by two variables, $\hat{\tilde{P}}^\star_{t,F}$ and $\hat{\pi}^\star_{t,F}$. The dynamics of these variables are similar to those in the home country.

$$\hat{\tilde{P}}^\star_{t,F} = \hat{w}^\star_t - \hat{X}^\star_t \tag{47}$$

$$\hat{\pi}^\star_{t,F} = \hat{\tilde{P}}^\star_{t,F} - \hat{\tilde{P}}^\star_{t-1,F} + \hat{\pi}^\star_t \tag{48}$$

4.3 Price Dynamics of Nontraded Sector

Define the deterministic component of the average productivity of the home nontraded sector as

$$a_{t,N} = \left(\frac{1}{\delta_t} \int_{z_t^l}^{z_t^h} a(z)^{\theta-1} dz \right)^{\frac{1}{\theta-1}}.$$

Let a_N denote the steady state level productivity. The dynamics of the nontraded sector price are governed by the following equations.

$$\hat{\hat{P}}_{t,N} = \hat{w}_t - \hat{X}_t - \hat{a}_{t,N} \tag{49}$$

$$\hat{a}_{t,N} = \frac{z_{ss}^h}{\delta_{ss}} \frac{(a(z_{ss}^h)/a_N)^{\theta-1} - 1}{\theta - 1} \hat{z}_t^h - \frac{z_{ss}^l}{\delta_{ss}} \frac{(a(z_{ss}^l)/a_N)^{\theta-1} - 1}{\theta - 1} \hat{z}_t^l \tag{50}$$

$$\hat{\pi}_{t,N} = \hat{\hat{P}}_{t,N} - \hat{\hat{P}}_{t-1,N} + \hat{\pi}_t \tag{51}$$

Note that $a_N < a(z_{ss}^l)$ and $a(z_{ss}^h) < a_N$, since $A'(z) < 0$. Then $d\hat{a}_{t,N}/d\hat{z}_t^l < 0$ and $d\hat{a}_{t,N}/d\hat{z}_t^h < 0$. This implies that an entry of home nontraded firms to export markets lowers the aggregate productivity of the home nontraded sector. It is so because such nontraded firms have cost advantage over those remaining in the industry. The aggregate productivity of the home nontraded sector also falls when foreign export firms stop exporting and some lowly productive home firms have to produced those goods to serve domestic demand.

Similar equations hold for the foreign nontraded sector.

$$\hat{\hat{P}}_{t,N}^{\star} = \hat{w}_t^{\star} - \hat{X}_t^{\star} \tag{52}$$

$$\hat{\pi}_{t,N}^{\star} = \hat{\hat{P}}_{t,N}^{\star} - \hat{\hat{P}}_{t-1,N}^{\star} + \hat{\pi}_t^{\star} \tag{53}$$

4.4 Price Dynamics of Import Sector

For the home import sector,

$$\hat{\hat{P}}_{t,F} = \hat{Q}_t + \hat{w}_t^{\star} - \hat{X}_t^{\star} \tag{54}$$

$$\hat{\pi}_{t,F} = \hat{\hat{P}}_{t,F} - \hat{\hat{P}}_{t-1,F} + \hat{\pi}_t. \tag{55}$$

For the foreign import sector,

$$\hat{\pi}_{t,H}^{\star} = \hat{\hat{P}}_{t,H}^{\star} - \hat{\hat{P}}_{t-1,H}^{\star} + \hat{\pi}_t^{\star} \tag{56}$$

$$\hat{\hat{P}}_{t,H}^{\star} = \hat{w}_t - \hat{Q}_t - \hat{X}_t - \hat{a}_{t,H}. \tag{57}$$

4.5 CPI Inflation

The path of CPI inflation follows its definition. In the home country,

$$
\begin{aligned}
\hat{\pi}_t \;=\;& z^l_{ss}(\pi_{ss}\frac{w_{ss}}{a_H})^{1-\theta}(\frac{1}{1-\theta}\hat{z}^l_t + \hat{\pi}_{t,H} + \hat{\tilde{P}}_{t-1,H}) \\
&+\delta_{ss}(\pi_{ss}\frac{w_{ss}}{a_N})^{1-\theta}(\frac{1}{1-\theta}(\frac{z^h_{ss}}{\delta_{ss}}\hat{z}^h_t - \frac{z^l_{ss}}{\delta_{ss}}\hat{z}^l_t) + \hat{\pi}_{t,N} + \hat{\tilde{P}}_{t-1,N}) \\
&+(1-z^h_{ss})(\pi_{ss}\frac{w^\star_{ss}Q_{ss}}{1-\tau})^{1-\theta}(\frac{-z^h_{ss}}{1-z^h_{ss}}\frac{1}{1-\theta}\hat{z}^h_t + \hat{\pi}_{t,F} + \hat{\tilde{P}}_{t-1,F}) \qquad (58)
\end{aligned}
$$

The foreign CPI inflation is derived in a similar fashion.

$$
\begin{aligned}
\hat{\pi}^\star_t \;=\;& z^l_{ss}(\pi^\star_{ss}\frac{w_{ss}}{a_H Q_{ss}(1-\tau)})^{1-\theta}(\frac{1}{1-\theta}\hat{z}^l_t + \hat{\pi}^\star_{t,H} + \hat{\tilde{P}}^\star_{t-1,H}) \\
&+\delta_{ss}(\pi^\star_{ss}{w_{ss}}^\star)^{1-\theta}(\frac{1}{1-\theta}(\frac{z^h_{ss}}{\delta_{ss}}\hat{z}^h_t - \frac{z^l_{ss}}{\delta_{ss}}\hat{z}^l_t) + \hat{\pi}^\star_{t,N} + \hat{\tilde{P}}^\star_{t-1,N}) \\
&+(1-z^h_{ss})(\pi^\star_{ss}w^\star_{ss})^{1-\theta}(\frac{-z^h_{ss}}{1-z^h_{ss}}\frac{1}{1-\theta}\hat{z}^h_t + \hat{\pi}^\star_{t,F} + \hat{\tilde{P}}^\star_{t-1,F}) \qquad (59)
\end{aligned}
$$

4.6 Real Exchange Rate Dynamics

In this subsection, I decompose the movements in real exchange rates and show the linkages with endogenous tradability. Using the price indices in Equation (12) and (13), I can decompose the deviation of the real exchange rate from its steady-state level as follows.

$$
\begin{aligned}
\hat{Q}_t - \hat{Q}_{t-1} &= \hat{q}_{t,T} + \hat{q}_{t,N} && (60)\\
\hat{q}_{t,T} &= \left[\hat{d}_t + \pi_{ss}^{\star 2(1-\theta)}(\hat{\pi}^{\star}_{t,F} + \hat{\tilde{P}}^{\star}_{t-1,F}) - \pi_{ss}^{2(1-\theta)}(\hat{\pi}_{t,F} + \hat{\tilde{P}}_{t-1,F})\right]\\
&+\pi_{ss}^{\star 1-\theta} z_{ss}^{l}(\pi^{\star}_{ss}\frac{w_{ss}}{Q_{ss}(1-\tau)a_H})^{1-\theta}\left[\hat{\pi}^{\star}_{t,H} - \hat{\pi}^{\star}_{t,F} + \hat{\tilde{P}}^{\star}_{t-1,H} - \hat{\tilde{P}}^{\star}_{t-1,F}\right]\\
&-\pi_{ss}^{1-\theta} z_{ss}^{l}(\pi_{ss}\frac{w_{ss,H}}{a_H})^{1-\theta}\left[\hat{\pi}_{t,H} - \hat{\pi}_{t,F} + \hat{\tilde{P}}_{t-1,H} - \hat{\tilde{P}}_{t-1,F}\right]\\
&+\frac{1}{1-\theta}\pi_{ss}^{\star 2(1-\theta)}\left[z_{ss}^{l} w_{ss}^{\star 1-\theta}\hat{z}_t^{l} - z_{ss}^{h} w_{ss}^{\star 1-\theta}\hat{z}_t^{h}\right]\\
&-\frac{1}{1-\theta}\pi_{ss}^{2(1-\theta)}\left[z_{ss}^{l}\left(\frac{w_{ss}}{a_H}\right)^{1-\theta}\hat{z}_t^{l} - z_{ss}^{h}\left(\frac{Q_{ss}w^{\star}_{ss}}{1-\tau}\right)^{1-\theta}\hat{z}_t^{h}\right]\\
&+0.5\frac{(\pi^{\star}_{ss}w^{\star}_{ss})^{1-\theta} - (\pi_{ss}w_{ss}/a_N)^{1-\theta}}{(1-\theta)(1-\delta_{ss})}\left[z_{ss}^{h}\hat{z}_t^{h} - z_{ss}^{l}\hat{z}_t^{l}\right] && (61)\\
\hat{q}_{t,N} &= \delta_{ss}\left[(\pi^{\star}_{ss}w^{\star}_{ss})^{1-\theta}(\hat{\pi}^{\star}_{t,N} - \hat{\pi}^{\star}_{t,T} + \hat{\tilde{P}}^{\star}_{t-t,N} - \hat{\tilde{P}}^{\star}_{t-1,T})\right]\\
&-\delta_{ss}\left[(\pi_{ss}\frac{w_{ss}}{a_N})^{1-\theta}(\pi_{t,N} - \pi_{t,T} + \hat{\tilde{P}}_{t-1,T} - \hat{\tilde{P}}_{t-1,N})\right]\\
&+0.5\frac{(\pi^{\star}_{ss}w^{\star}_{ss})^{1-\theta} - (\pi_{ss}w_{ss,N}/a_N)^{1-\theta}}{(1-\theta)(1-\delta_{ss})}\left[z_{ss}^{h}\hat{z}_t^{h} - z_{ss}^{l}\hat{z}_t^{l}\right] && (62)
\end{aligned}
$$

$\hat{q}_{t,T}$ and $\hat{q}_{t,N}$ are the traded and nontraded component of the deviation of the real depreciation from its steady state level, respectively. The traded component comes from two channels: the substitution between the imports and export goods in the two countries (the first, second and third terms) and endogenous tradability (the last three terms). For the nontraded component, its deviation from the steady state level depends on the international difference in inflation differentials of the nontraded and the traded sector (the first and second terms), and endogenous tradability (the last term). The first and second terms actually capture the substituation between nontraded and traded goods consumption. The home bias in consumption is essentially represented by the first, second and third terms in $\hat{q}_{t,T}$ and the first and second terms in $\hat{q}_{t,N}$.

The transport costs in my model lead to two important characteristics of the real exchange rate dynamics. First, they create the wedge between domestic price and import price of import goods. It is precisely what drives the substitution between domestically produced and import goods. It is precisely what Rogoff (1996) refers as the absence of arbitrage and claimed to be an explanation for the persistent deviations of real exchange rates from the purchasing power parity. Second, it affects the real exchange rate dynamics by determining the scope of intratemporal substitution between goods. Since the size of traded and nontraded sector also captures its share in the overall consumption basket and determines its share in the total consumption expenditure, the endogenous tradability terms represent the effect of the expenditure share on real exchange rates.

Exchange rate policy plays a role on the determination of real exchange rate through the expenditure share channel. Under a flexible exchange rate regime, movements in nominal exchange rates alter the relative prices of traded goods and cause consumers to switch their expenditure from one good to another. However, under fixed exchange rate regime, the expenditure switching effect of exchange rates entirely disappears. Hence the domestic price channel or the relative price of nontraded to traded goods becomes a more important source of adjustment than under a flexible regime. For this reason, the nontraded component of real exchange rate should display higher volatility under a fixed exchange rate regime. The definition of the expenditure switching effect of exchange rate is given in the next subsection.

4.7 Expenditure Switching Effect of Exchange Rates

By the definition of price indices and consumption sub-baskets, variation in their share in total consumption expenditure in the home country can be written as follow.

$$\xi_{t,H} = \hat{z}_t^l + (1-\theta)\hat{\tilde{P}}_{t,H} \quad (63)$$

$$\xi_{t,N} = \hat{\delta}_t + (1-\theta)\hat{\tilde{P}}_{t,N} \quad (64)$$

$$\xi_{t,F} = -\frac{z_{ss}^h}{1-z_{ss}^h}\hat{z}_t^h + (1-\theta)\hat{\tilde{P}}_{t,F} \quad (65)$$

Intuitively, the changes in expenditure share of a particular sub-basket come from the changes in two components: its share in the overall consumption basket and its share in one unit of consumption expenditure. Let the superscripts $flex$ and fix denote the variables under a flexible and a fixed exchange rate regime, respectively. Then we can define the expenditure switching effect of exchange rates at the sectoral level as

$$\Delta\xi_{t,j} = \xi_{t,j}^{flex} - \xi_{t,j}^{fix}, j \in (F, H, N).$$

5 Calibration

I calibrate the model under two exchange rate regimes as described by the two different monetary rules. I first calibrate the benchmark model and then vary some parameters of interest, namely the transport cost parameter and the intratemporal elasticity of substitution. Table 6 summarizes the parameter values.

In the benchmark model, the transport costs parameter is 0.25. The transport cost parameters are from the study by Brunner and Naknoi (2003). Transport costs are found to vary from zero to 0.30, taking into account other trade costs that cannot be easily measured such as language and other information barriers. For the purpose of sensitivity analysis, the transport costs are varied from 0.15 to 0.30. The resulting output share of the nontraded goods in the home country in the steady state is 60 percent and 30 percent in the foreign country. Country are assume to have symmetric size. The rate of time preferences gives 4 percent of the real interest rate. The intratemporal elasticity of substitution is 6, as suggested by Obstfeld (2000). In the sensitivity analysis, the value is varied from 3 to 6. The portfolio adjustment coefficient is 0.02 for both type of bonds.

According to the study by Huang and Liu (2002), the elasticity of substitution of labors can vary from 2 to 4. I set the parameter at 2. The wage adjustment coefficients are assumed to be identical in both countries. It is set so that the resulting elasticity of wage inflation with respect to the marginal rate of substitution between leisure and consumption are 0.25 in the home country, and 0.20 in the foreign country. The first order-correlation of productivity shock and foreign interest rate shock is assumed to be 0.5. The Taylor rule specification follows Clarida et al. (2000), except for that the coefficient of the inflation term is changed to lower value due to a problem of indeterminancy. The steady-state inflation rate in the two country is 1.02.

Table 6. Parameter Values

	Parameters
Transport costs	$\tau = 0.15 - 0.30$
Country size	$\alpha = 0.5$
Preferences	$\theta = 6 - 3$, $\sigma = 0.5$
	$\beta = 0.99$, $\mu = 3$, $\epsilon = 9$
Portfolio adjustment cost	$\phi = \phi^\star = 0.2$
Labor demand	$\eta = 2$
Nominal rigidities	
Wage stickiness	$\phi^w = \phi^{w\star} = 50$
Technology	$\rho_x = 0.5$
Monetary rules	
Home country	$d_{ss} = 1$ or $\pi_{ss} = 1.02$
Foreign country	$\pi^\star_{ss} = 1.02$ $\lambda_i = 0.79$,
	$\lambda_\pi = 0.5/(1 - 0.79)$,
	$\lambda_y = 0.93/4$, $\rho_v = 0.5$

I discuss the steady state relationship in the next subsection. Then, I solve the log-linearized system using the algorithm developed by Uhlig (1999). The later subsection reports the impulse responses.

5.1 Steady State Equilibrium

This subsection summarizes the key features in the steady state equilibrium. In the benchmark case, $z^l_{ss} = 0.07$ and $z^h_{ss} = 0.48$. Hence, the size of nontraded sector is 0.41.

Figure 1 reports the relationship between the size of nontraded sector and transport costs. As transport costs increase, more goods become nontraded. It is so because high transport costs cause consumers to substitute towards the nontraded goods and the exportables. Figure 2 confirms this intuition. The home bias in consumption, which is defined as the share of consumption expenditure on the domestically produced goods is found to rise when transport costs increases. These steady state relationships provide some intuitions for the expenditure switching effect of exchange rates and the short-run dynamics in the next subsection.

5.2 Short-run Dynamics

First, this subsection discusses the impulse responses with respect to a one percent temporary increase in the home productivity shock and the foreign interest rate under the two exchange rate regimes. The impulse responses should be interpreted as qualitative predictions of the model, since the underlying variance of the noise term is not explicitly addressed. Next, I explore the sensitivity of the nontraded component of real exchange rate volatility to the choices of transport costs and the intratemporal elasticity of substitution. Finally, I investigate the relationships of the expenditure switching effect of exchange rates with the two parameters.

5.2.1 Impulse Responses

The impulse responses corresponding to the productivity shock under the flexible exchange rate regime is in Figure 3. The essence of a theory of endogenous tradability lies in a temporary shift in patterns of trade in response to an exogenous shock. In the short run, a positive productivity shock in the home country raises real wage in the home country, and cause some of the home exporters to lose competitiveness. At the same time, the rise in real wage also cause some of the home nontraded producers to be unable to compete with some import goods producers. In other words, some of the home exporters become nontraded goods producers, while some of the foreign nontraded firms become new exporters. As a result, we observe real exchange rate appreciation, contraction of export sector, and expansion of nontraded and import sector.

In addition, the rises in real wage raise the relative price of export goods and terms of trade of the home country. The home residents therefore substitution away from the home export goods consumption. That results in rise in consumption of nontraded and import goods in the home country. For the foreign country, the deterioration of its terms of trade reduces income, consumption demand, and output of all goods in the foreign country. In aggregate, consumption and output expands in the home country, while they contract in the foreign country. The home country initially runs trade deficit and current account deficit, which soon becomes surplus subsequently due to terms of trade effect and nominal exchange rate depreciation. Such balance of payments and consumption pattern are consistent with the the initial fall of the home real interest rate.

The most important variable of interest here is the real exchange rate. Since the contraction in the foreign production puts downward pressure on the foreign real wages and inflation, we observe real exchange rate appreciation in response to a productivity shock. When I decompose the real appreciation into the traded and nontraded component, the traded component dominates its nontraded counterpart. It explains all the movements and volatility of real appreciation.

Figure 4 reports the impulse responses to the same productivity shock under a fixed exchange rate regime. Except for the policy variables, they are almost identical to those under the flexible regime. However, the nontraded component of real exchange rate appreciation becomes slightly more volatile under a fixed exchange rate regime.

Figure 5 summarizes the impulse responses to one percent increase in the foreign interest rate under the flexible exchange rate regime. The rise in the foreign interest rate reduces the foreign consumption demand for all goods. That leads to a fall in foreign output, labor demand and real wage. Such a fall in the foreign real wage drives foreign export goods price down and hence their demand up. In addition, it turns some nontraded firms in the foreign country to be competitive in export markets and become new exporters. At the same time, some nontraded firms in the foreign country can compete with some import goods producers in the home country. Overall, we see a contraction in the home export sector, but an expansion in the nontraded sector and the foreign export sector.

As some of the home export firms exit the export market, and some less productive nontraded firms disappear, they increases the average productivity in the nontraded sector. As a result, the price of nontraded goods to export goods fall. This is the reason why the consumption and output in the home country rise in the nontraded sector but fall in the export sector. The expansion in the nontraded sector puts upward pressure on the real wage. The rise in the home real wage combining with the improvement in the terms of trade dampen the demand for home export goods so much that the home output falls, despite the expansion in the nontraded sector. In the short-run, the home country faces trade deficits and current deficits, but the deficits subsequently turn to surpluses due to the improvement of its terms of trade. Such consumption and balance of payments pattern are consistent with the initial fall in the home real interest and the initial rise in the foreign real interest rate.

For the real exchange rate, we observe real appreciation, as the contraction in the foreign country puts downward pressure on its inflation rate. The traded component of real exchange rate by far dominates its nontraded counterpart. The nontraded component of real exchange rate barely moves at all.

The impulse responses to the interest rate shock under the fixed exchange rate regime are depicted in Figure 6. Evidently, they are identical in direction but have a larger scale than those under the flexible exchange rate regime for almost all variables including real exchange rate. Most importantly, the nontraded component of real exchange rate does deviates from its long-run level in the short run. Although the depreciation of the nontradable component is approximately 0.2 percent at its peak, it confirms that the nontraded component of real exchange rate has higher volatility under a fixed exchange rate regime.

The movements of the nontraded component can be understood as follow. As some of the home export firms exit the export market, and some less productive nontraded firms disappear, they increases the average productivity in the nontraded sector. As a result, the price of nontraded goods to export goods fall. Combining with an improvement in the terms of trade, the home residents substitute the export goods consumption with the nontraded goods and import goods. The degree of substitution depends on changes in the relative prices of goods which in turn are influened by exchange rates. Sluggish price adjustment due to a fixed exchange rate policy increases an incentive for the home residents to substitute towards nontraded goods, and that expands the nontraded sector and shrinks the export sectors. Such a shift in turn raises productivity of the nontraded sector even more under a fixed exchange rate regime. That is why we observe a larger fall in the relative price of nontraded goods to export goods with a fixed exchange rate, which contributes to a larger depreciation of the nontraded component of real exchange rate.

Interestingly, the shift of patterns of trade in response to the foreign interest rate shock is qualitatively identical that with a productivity shock, although it is from a different mechanism. The fundamental forces that drive such a shift is the rise in the real wage in the home country relative that in the foreign country. A rise in the relative real wage always reduces comparative advantage in the home export sector, and raises comparative advantage in the foreign export sector. The mechanism that drives the depreciation of the nontraded component of real exchange rate is the same as that with a productivity shock, as the dynamics of real wages and endogenous tradability is the same. Clearly, this implies that non-productivity shocks such as the interest rate shock can also alter the productivity structure of the economy at the sectoral and aggregate level through the deterministic component of productivity in the manner described above.

Another interesting features in all cases is the large scale of the short-run deviation of the home consumption sub-baskets, the home consumption share and the size of home export sector. They are mainly artifacts of the assumed productivity structure. In the steady state, the size of the home export sector is only 0.07. When a small number of home firms enter or exits export markets, it translate into a large percentage. That also translates into a large variation in the consumption share of home export goods, which in turn leads to a large offsetting movement in the consumption share of nontraded and import goods. Such changes eventually lead to a large swing in the impulse responses of the home consumption sub-baskets.

5.2.2 Sensitivity Analysis

In this section, I investigate the relationships between the nontraded component of real exchange rate volatility and transport costs and the intratemporal elasticity of substitution. Figures 7.1 and 7.2 compares the movements in real exchange rate depreciation and its components under two parameter values: 0.25 and 0.15. As transport costs fall, the nontraded component is found to be more volatile, regardless of the type of shocks. Such a pattern also applies to other cases of comparison based on the parameter values 0.20 and 0.30, although they are not reported here.

What is an explanation for the finding? In fact, the steady state equilibrium suggests that the size of traded sector rises when transport costs fall. Intuitively, the expenditure switching effect of exchange rates should play a more important role in an economy where the traded sector is large, since exchange rate movements alter the relative price of traded goods. Removing nominal exchange rate flexibility will force such an economy to adjust through the relative price of nontraded goods in a relatively large scale. In other words, the volatility of the relative price of nontraded to traded goods can account for the volatility of real exchange rate of a highly but imperfectly integrated pair of countries better under a fixed exchange rate regime. Note that the volatility of the overall real depreciation becomes less volatile when transport costs fall. This finding is consistent with other studies, for example Kraay and Ventura (2002) and Naknoi and Brunner (2003).

Figures 8.1 and 8.2 illustrates the effects of the intratemporal elasticity of substitution on the volatility of the nontraded component. The parameters compared are 6 and 3. There is no clear connection from the elasticity to the volatility. Intuitively, a high value of the elasticity generates a large change in consumption quantity and requires only a small change in price. Since a change in consumption expenditure depends on a change in both price and quantity, theoretically the effect of the intratemporal elasticity of substitution on the variation on consumption expenditure and hence the composition of real exchange rate volatility is ambiguous.

In addition, I also directly explore the properties of the expenditure switching effect of exchange rates. Figures 9.1 and 9.2 indicate that the effect rises as transport costs fall, regardless of the type of shocks. Its relationship with the intratemporal elasticity of substitution is ambiguous, as in Figures 10.1 and 10.2.

To summarize, the nontraded component of real exchange rate volatility is found to increase when we switch exchange policy for a flexible to a fixed regime, with a productivity and an interest rate shock. The fundamental forces driving the finding is the expenditure switching effect of exchange rates, which is found to increase as transport costs fall. It should be noted that the volatility is higher with an interest shock in all cases. These findings emphasize the role of exchange rate and monetary policy in the short-run determination of real exchange rate volatility.

6 Concluding Remarks

This paper takes a new look at an old issue: Is the relative price of nontraded to traded goods a source of real exchange rate fluctuations? The first part of the paper establishes stylized facts about the volatility of real exchange rates. The answer to the question is yes, for some countries. These countries are found to share one common characteristic, namely, stable bilateral nominal exchange rates.

Taking the evidence into account, I construct a model that makes an analysis of exchange rate regime and pattern of trade possible. The model highlights the role of the expenditure switching effect of exchange rates as the central price adjustment mechanism when tradability is endogenous. The effect increases as transport cost falls or as the two economies are more integrated. When the effect is large, limiting the movements in nominal exchange rate can increase the volatility of the relative price of nontraded to traded goods. Such a linkage between exchange rate policy and the sectoral decomposition of real exchange rate is supported by the impulse responses in a calibration exercise. In all cases, the relative price of nontraded to traded goods is more volatile under a fixed exchange rate regime. Interestingly, an interest rate shock generates a higher volatility in the relative price of nontraded goods to traded goods than a productivity shock. The calibration results are, however, preliminary and should be interpreted as qualitative predictions.

An immediate extension of this piece of research is to extend it to a stochastic calibration. The framework is directly applicable to an optimal policy analysis. In addition, incorporating nontraded intermediate inputs or local-currency pricing in a sticky-price setup is an interesting avenue that might further explains the short-run dynamics of real exchange rate.

References

Abraham, K. G. and J. C. Haltiwanger (1995). Real wages and the business cycle. *Journal of Economic Literature 33*, 1215–1264.

Baier, S. L. and J. H. Bergstrand (2001). The growth of world trade: Tariffs, transport costs, and income similarity. *Journal of International Economics 53*, 1–27.

Baxter, M. and R. G. King (1999). Measuring business cycles: Approximate band-pass filters for economic time series. *Review of Economics and Statistics 81*, 575–593.

Baxter, M. and A. Stockman (1989). Business cycles and the exchange rate regime: Some international evidence. *Journal of Monetary Economics 23*, 377–400.

Bergin, P. R. and R. Glick (2003). Endogenous nontradability and macroeconomic implications. *NBER Working Paper 9739*.

Betts, C. and M. B. Devereux (1996). The exchange rate in a model of pricing-to-market. *European Economic Review 40*, 1007–1021.

Betts, C. and M. B. Devereux (2000). Exchange rate dynamics in a model of pricing-to-market. *Journal of International Economics 50*, 215–244.

Betts, C. and M. B. Devereux (2001). The international effects of monetary and fiscal policy in a two-country model. In G. Calvo, R. G. Dornbusch, and M. R. Obstfeld (Eds.), *Money, Capital Mobility, and Trade: Essays in Honor of Robert Mundell*, pp. 9–52. MIT Press, Cambridge, USA.

Betts, C. M. and T. J. Kehoe (2001a). Real exchange rate movements and the relative price of nontraded goods. University of Minnesota and University of Southern California, Working Paper.

Betts, C. M. and T. J. Kehoe (2001b). Tradability of goods and real exchange rate fluctuations. University of Minnesota and University of Southern California, Working Paper.

Betts, C. M. and T. J. Kehoe (2003). U.s. real exchange rate movements. University of Minnesota and University of Southern California, Working Paper.

Bils, M. and P. J. Klenow (2002). Some evidence on the importance of sticky prices. *NBER Working Paper* (9069).

Brunner, A. D. and K. Naknoi (2003). Trade costs, market integration, and macroeconomic volatility. *IMF Working Paper* (0354).

Calvo, G. and M. Kumhof (2003). Trade openness and exchange rate misalignments. University of Maryland and Stanford University.

Campa, J. M. and L. Goldberg (2002). Exchange rate pass-through into imports prices. An earlier version was published as NBER Working Paper 8934.

Chari, V. V., P. J. Kehoe, and E. R. McGrattan (2002). Can sticky price models generate volatile and persistent real exchange rates? *The Review of Economic Studies 69*, 533–563.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (1999). Monetary policy shocks: What have we learned and to what end? In J. B. Taylor and M. Woodford (Eds.), *Handbook of Macroeconomics*, pp. 65–148. Elsevier.

Clarida, R., J. Gali, and M. Gertler (2000). Monetary policy rules and macroeconomic stability: Evidence and some theory. *Quarterly Journal of Economics 115*, 147–180.

Collard, F. and H. Dellas (2002). Exchange rate systems and macroeconomic stability. *Journal of Monetary Economics 49*, 571–599.

Dornbusch, R., S. Fischer, and P. A. Samuelson (1977). Comparative advantage, trade, and payments in a ricardian model with a continuum of goods. *American Economic Review 67*(5), 823–839.

Engel, C. (1999). Accounting for u.s. real exchange rate. *Journal of Political Economy 107*(3), 507–538.

Huang, K. X. and Z. Liu (2002). Staggered price-setting, staggered wage-setting and business cycle persistence. *Journal of Monetary Economics 49*, 405–433.

Imbs, J., H. Mumtaz, M. O. Ravn, and H. Rey (2002a). Non linearities and real exchange rate dynamics. *Journal of European Economic Association 3*, 639–649.

Imbs, J., H. Mumtaz, M. O. Ravn, and H. Rey (2002b). Ppp strikes back: Aggregation and the real exchange rate. *NBER Working Paper 9372*.

Mendoza, E. G. (2000). On the stability of variance decompositions of the real exchange rate across exchange-rate regimes: Evidence from mexico and the united states. *NBER Working Paper 7768*.

Mussa, M. (1986). Nominal exchange rate regimes and the behavior of real exchange rates: Evidence and implications. *Carnegie-Rochester Conferences Series on Public Policy 25*, 117–214.

Neumeyer, P. A. and F. Perri (2001). Business cycles in emerging economies: The role of interest rates. New York University,Working Paper.

Obstfeld, M. (2000). The six major puzzles in international macroeconomics: Is there a common cause? In B. S. Bernanke and K. Rogoff (Eds.), *Macroeconomic Annual*, pp. 339–412. NBER.

Obstfeld, M. (2001). International macroeconomics: Beyond the mundell-fleming model. *IMF Staff Papers 47*, 1–39.

Reinhart, C. and K. Rogoff (2004). The modern history of exchange rate arrangements: A reinterpretation. *Quarterly Journal of Economics (forthcoming)*.

Rogoff, K. (1996). The purchasing power parity puzzle. *Journal of Economic Literature 34*, 647–668.

Rotemberg, J. J. (1982). Sticky prices in the united states. *Journal of Political Economy 90*, 1187–1211.

Taylor, J. B. (1999). Staggered price and wage setting in macroeconomics. In J. B. Taylor and M. Woodford (Eds.), *Handbook of Macroeconomics*, pp. 1009–1050. Elsevier.

Uhlig, H. (1999). A toolkit for analyzing nonlinear dynamic stochastic model easily. In R. Marimon and A. Scott (Eds.), *Computational Methods for the Study of Dynamic Economies*. Oxford University Press.

Figure 1 Size of nontraded sector in the steady state

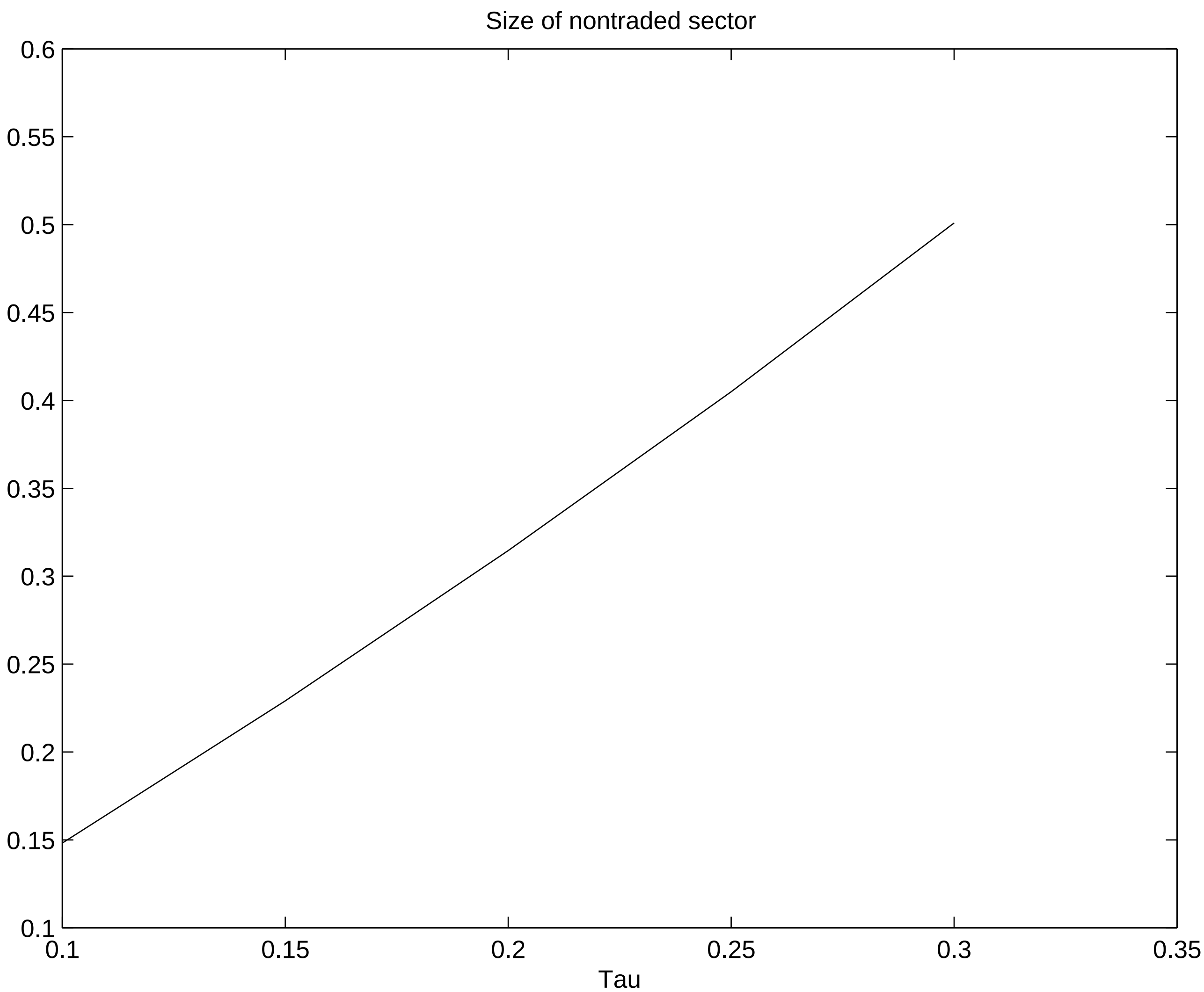

Figure 2 Home bias in consumption

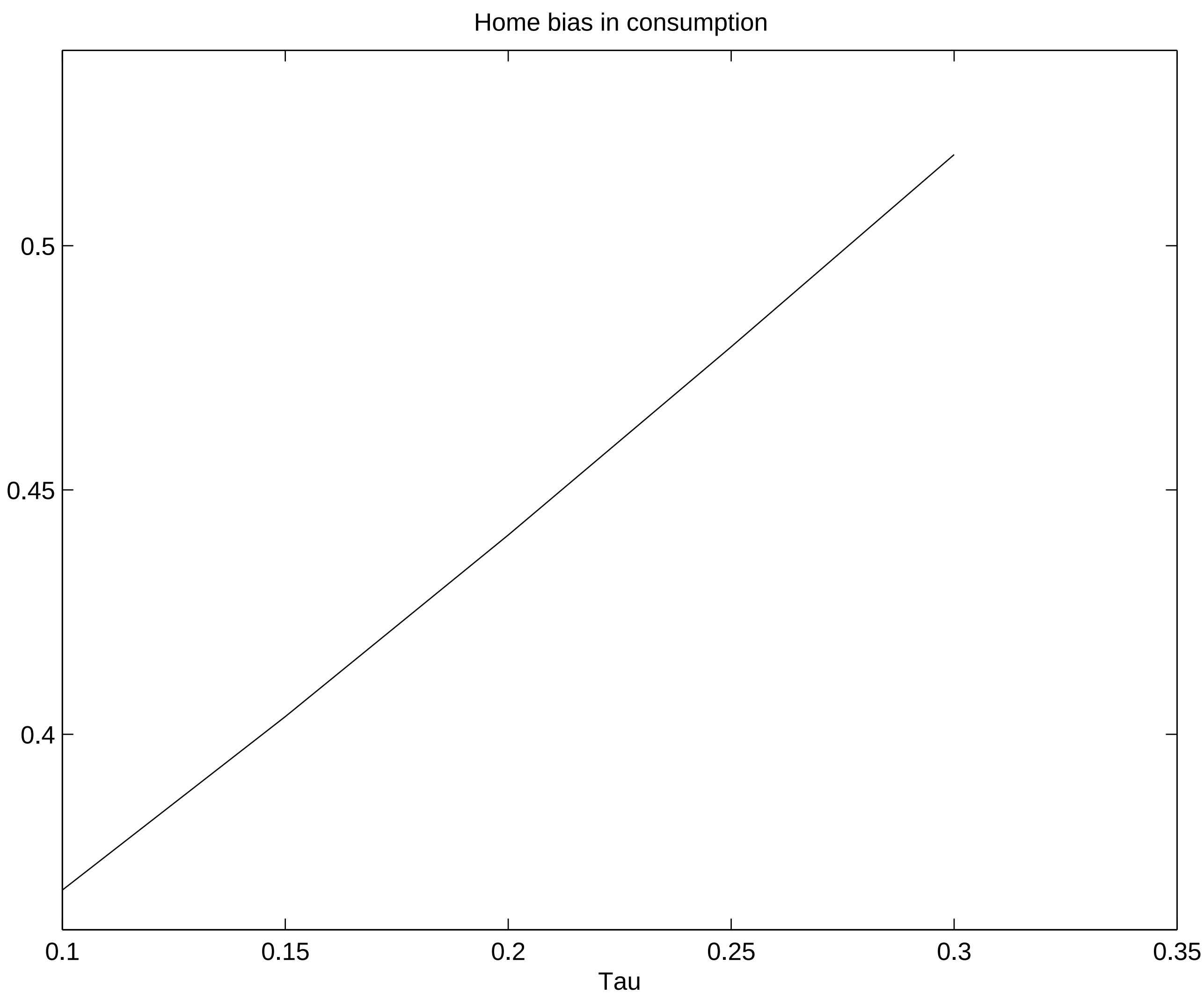

Figure 3 Productivity shock - Flexible exchange rate

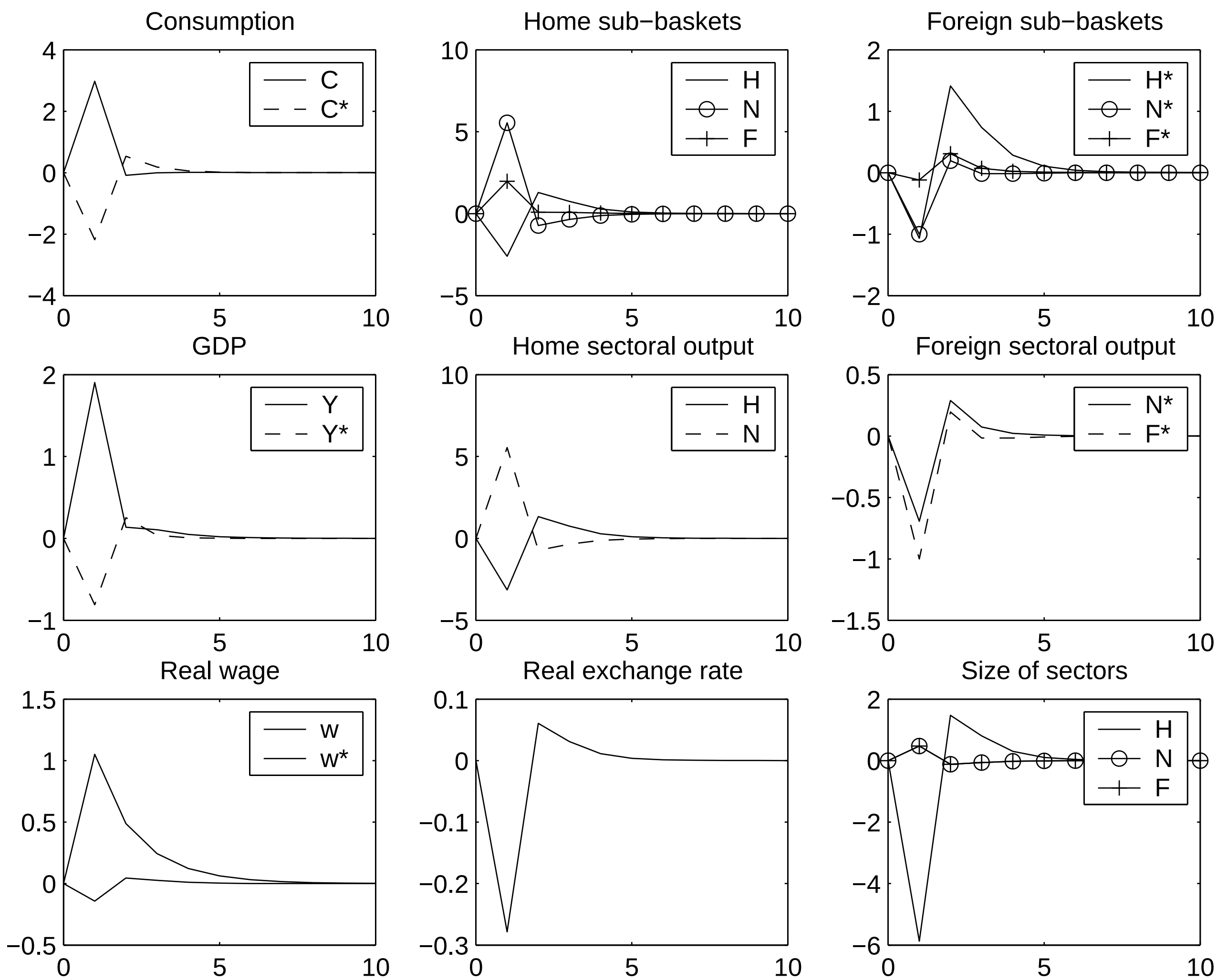

Figure 3(continued) Productivity shock - Flexible exchange rate

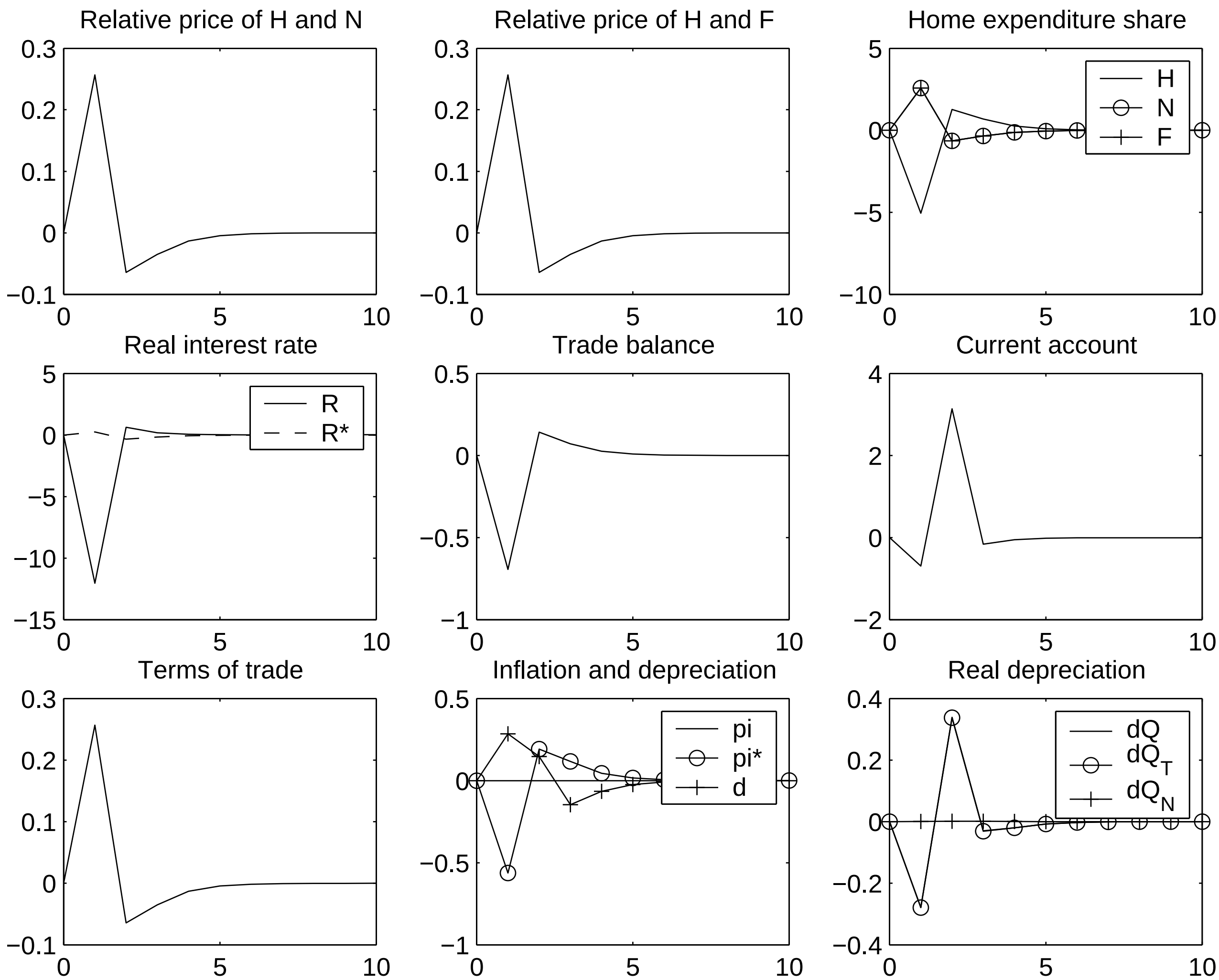

Figure 4 Productivity shock - Fixed exchange rate

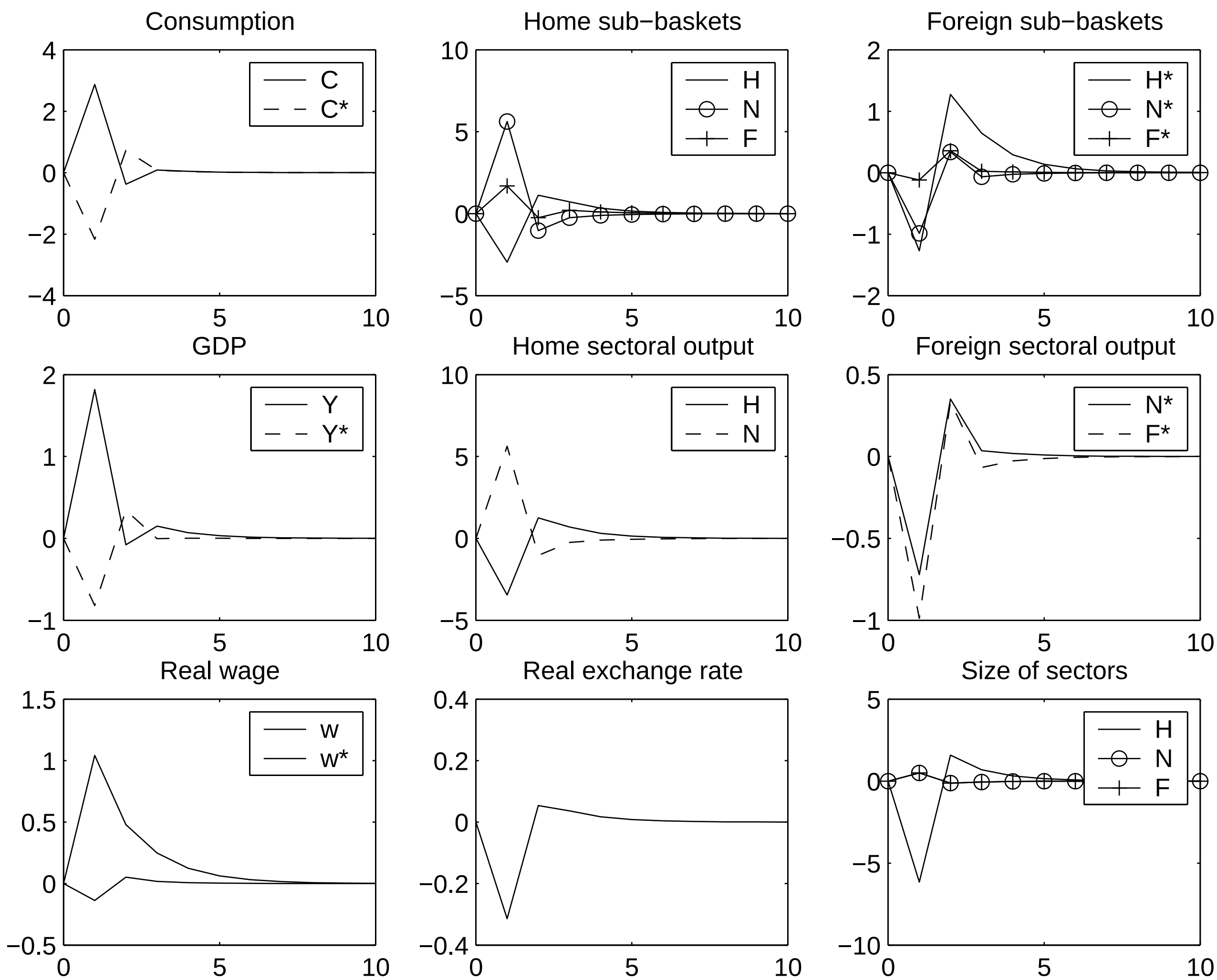

Figure 4(continued) Productivity shock - Fixed exchange rate

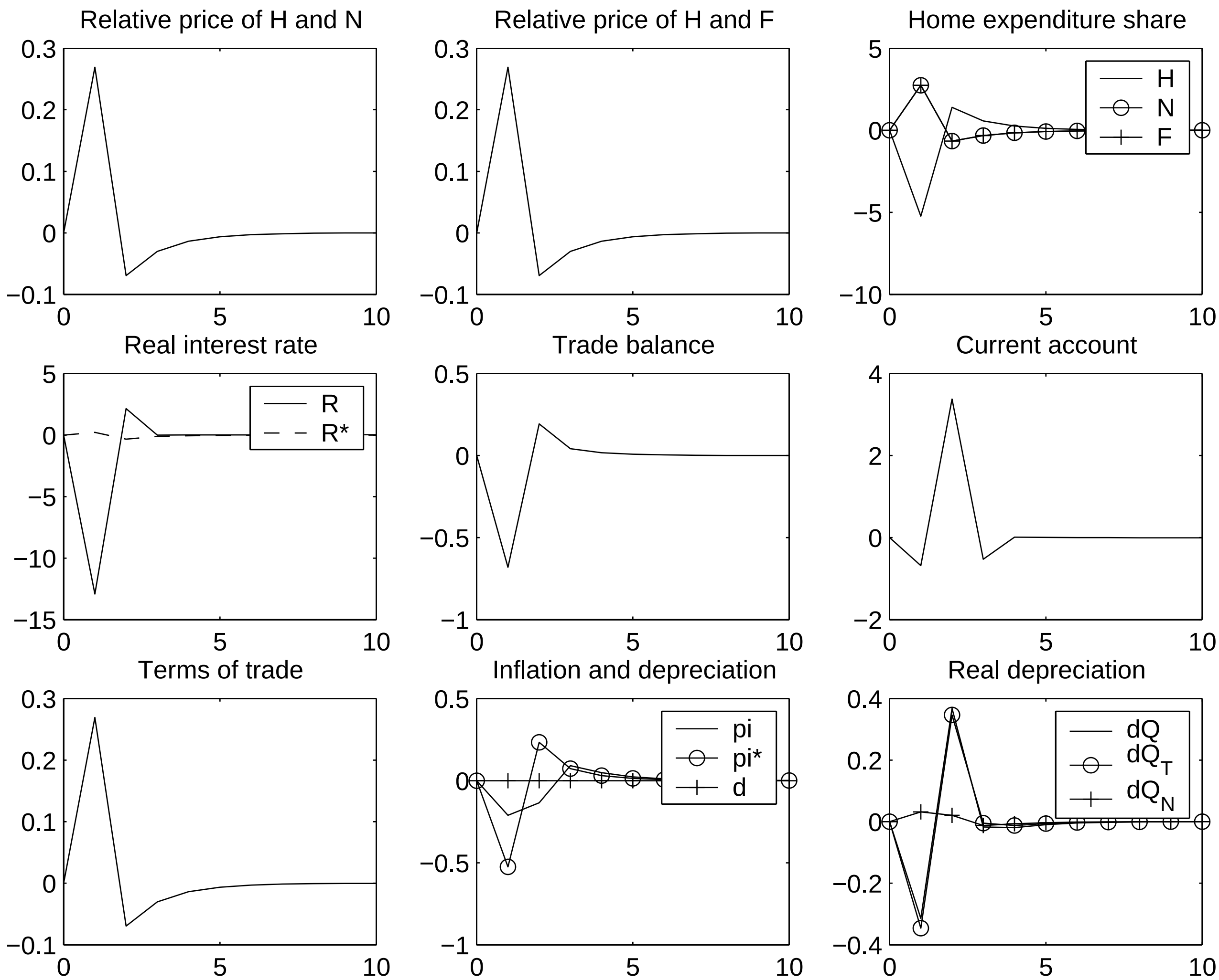

Figure 5 Interest rate shock - Flexible exchange rate

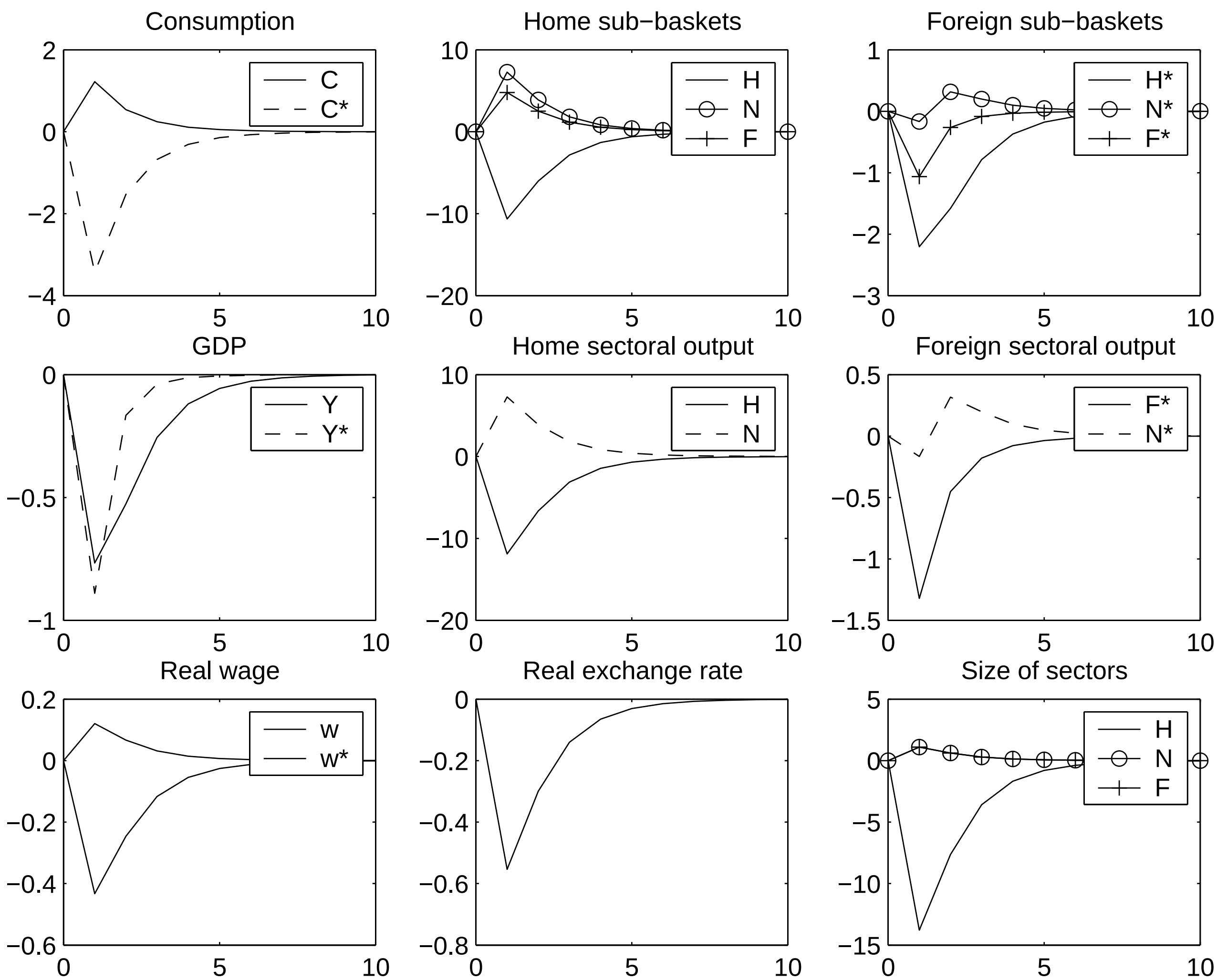

Figure 5(continued) Interest rate shock - Flexible exchange rate

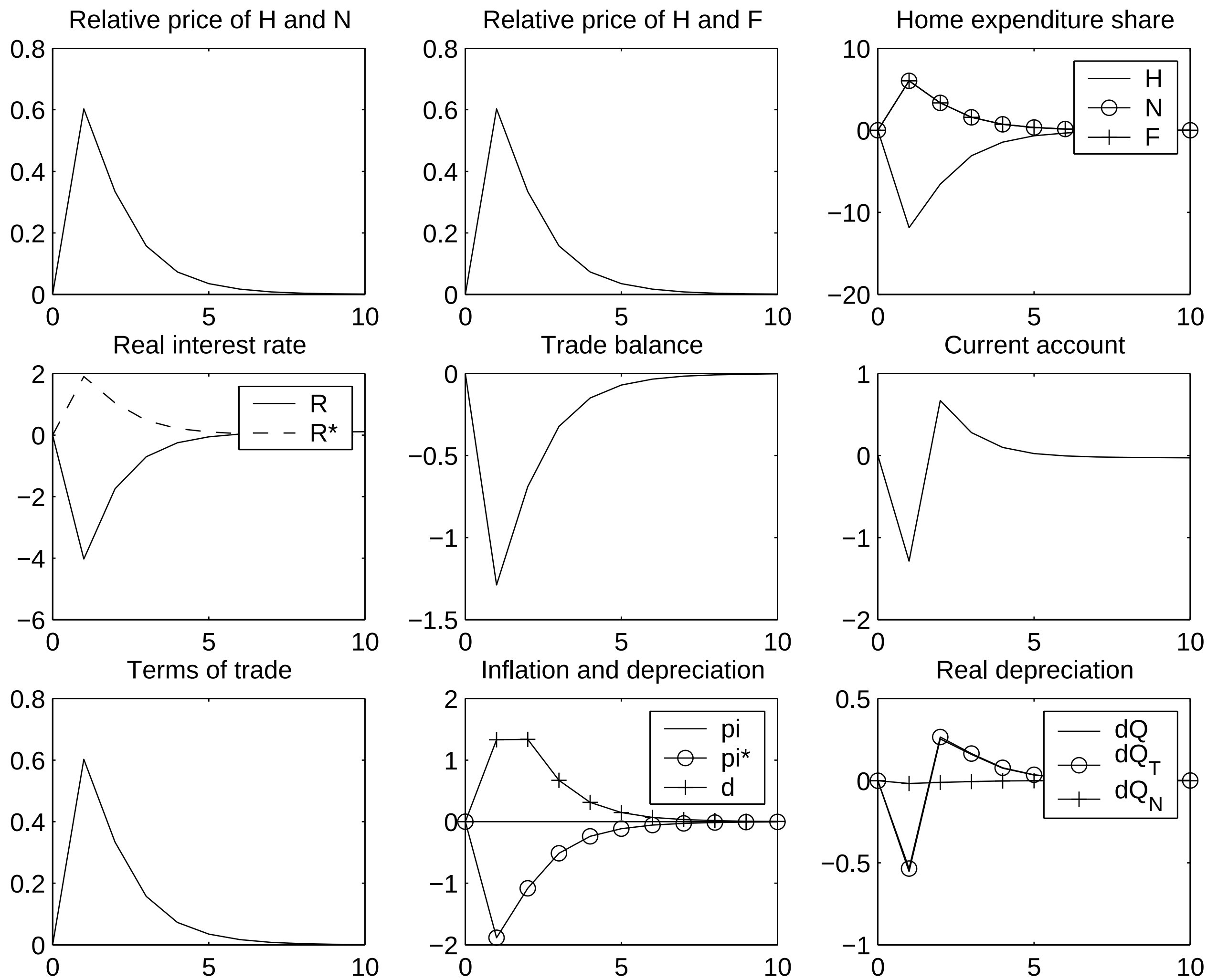

Figure 6 Interest rate shock - Fixed exchange rate

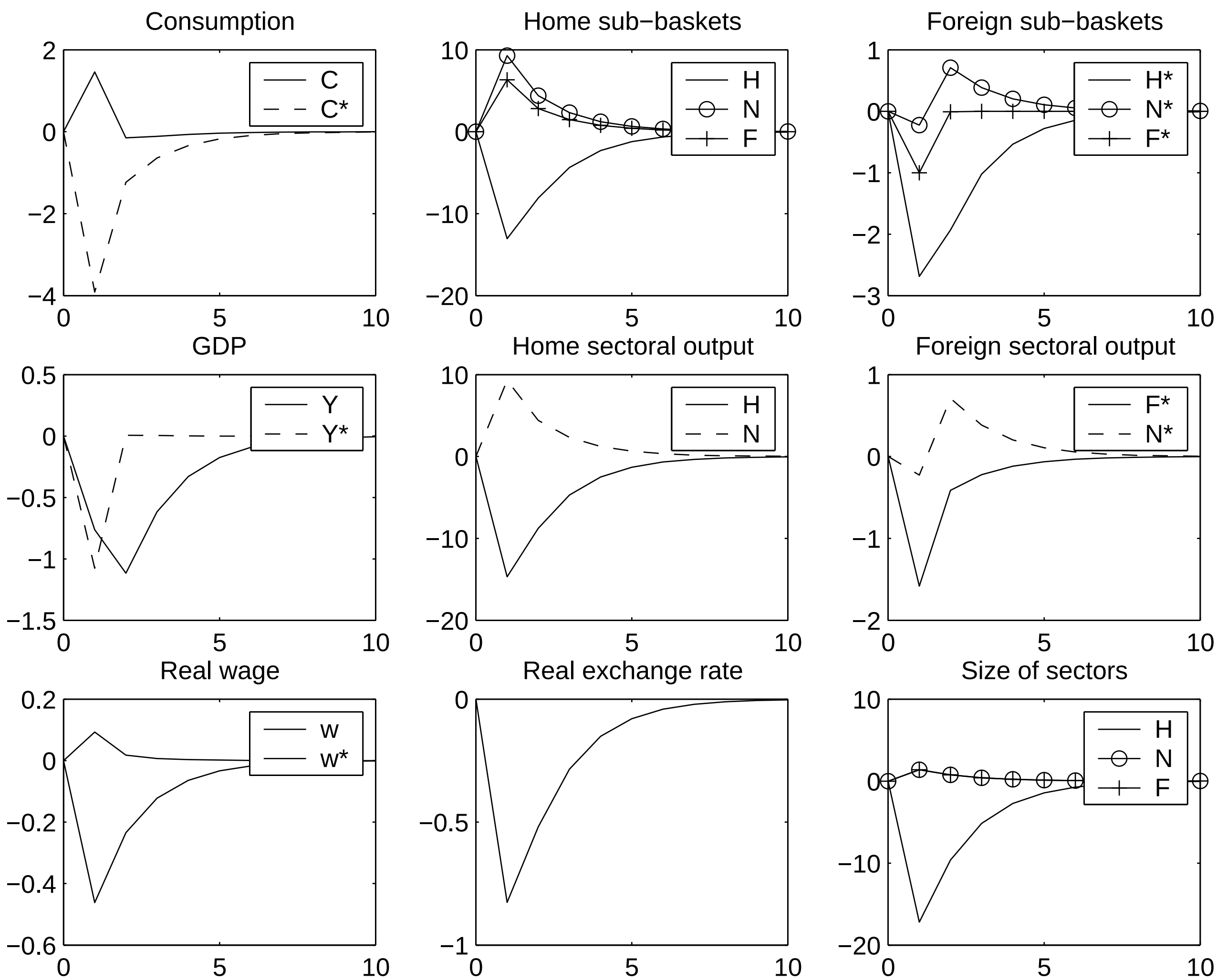

Figure 6(continued) Interest rate shock - Fixed exchange rate

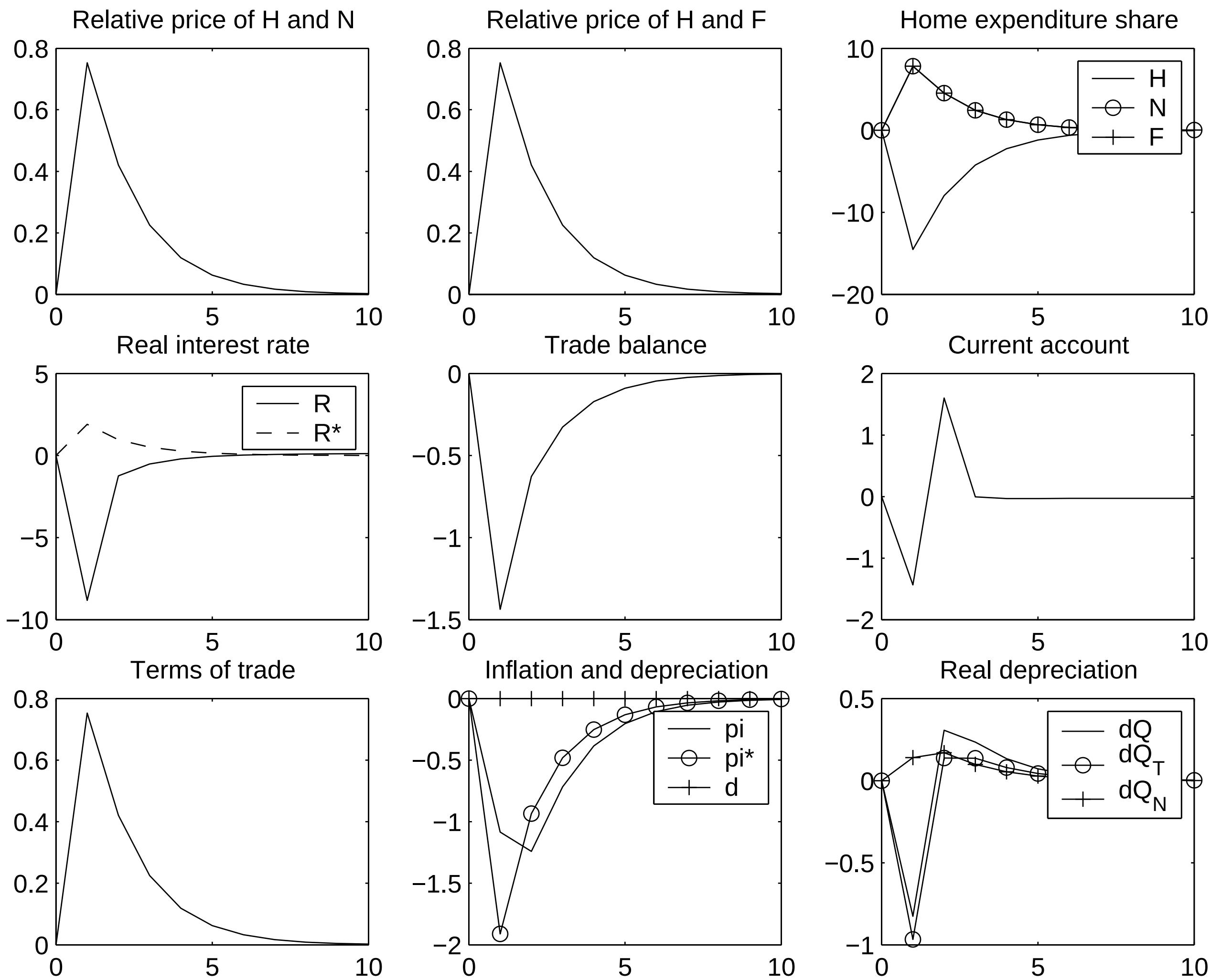

Figure 7.1 Real exchange rate decomposition and transport costs: Productivity shock

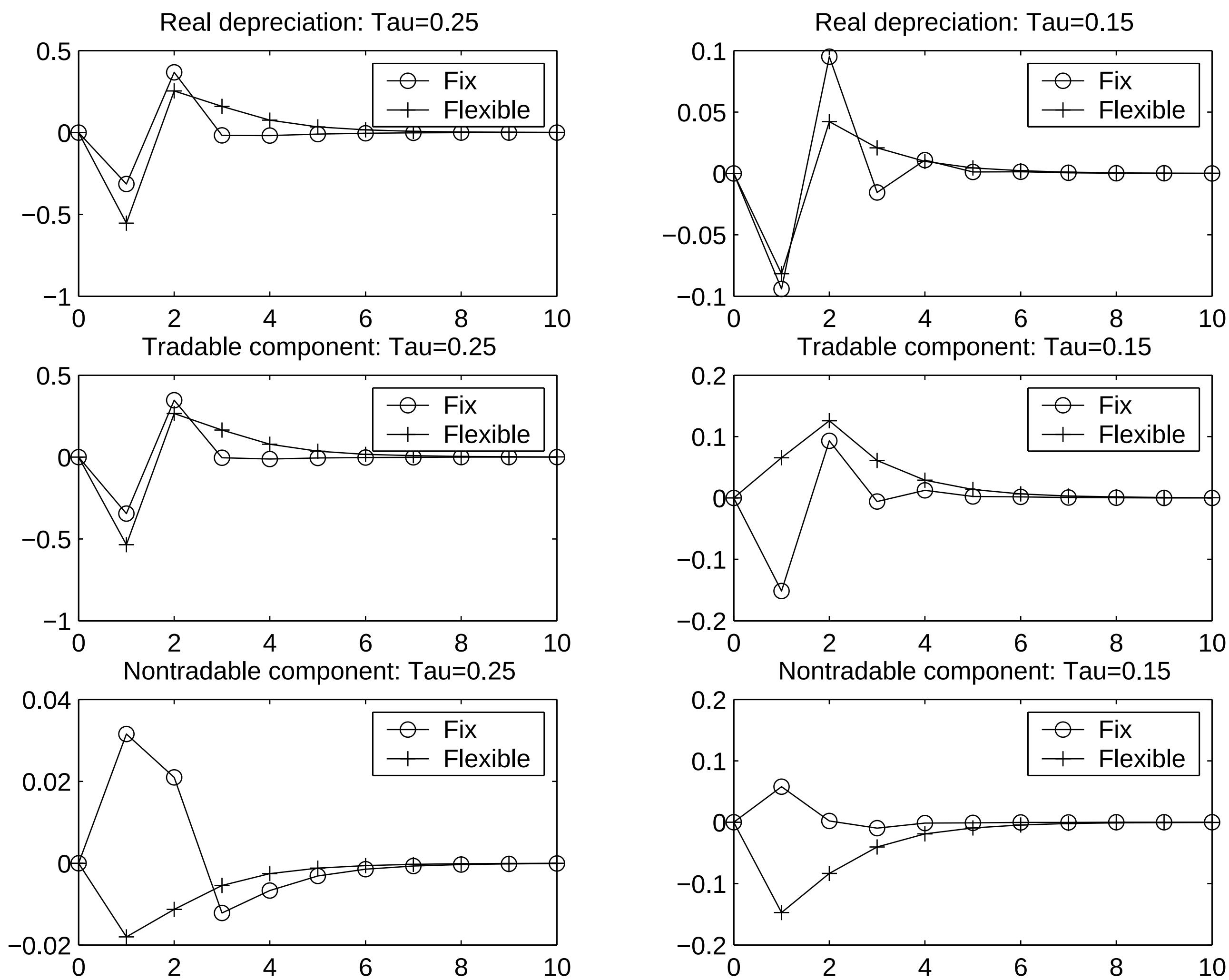

Figure 7.2 Real exchange rate decomposition and transport costs: Interest rate shock

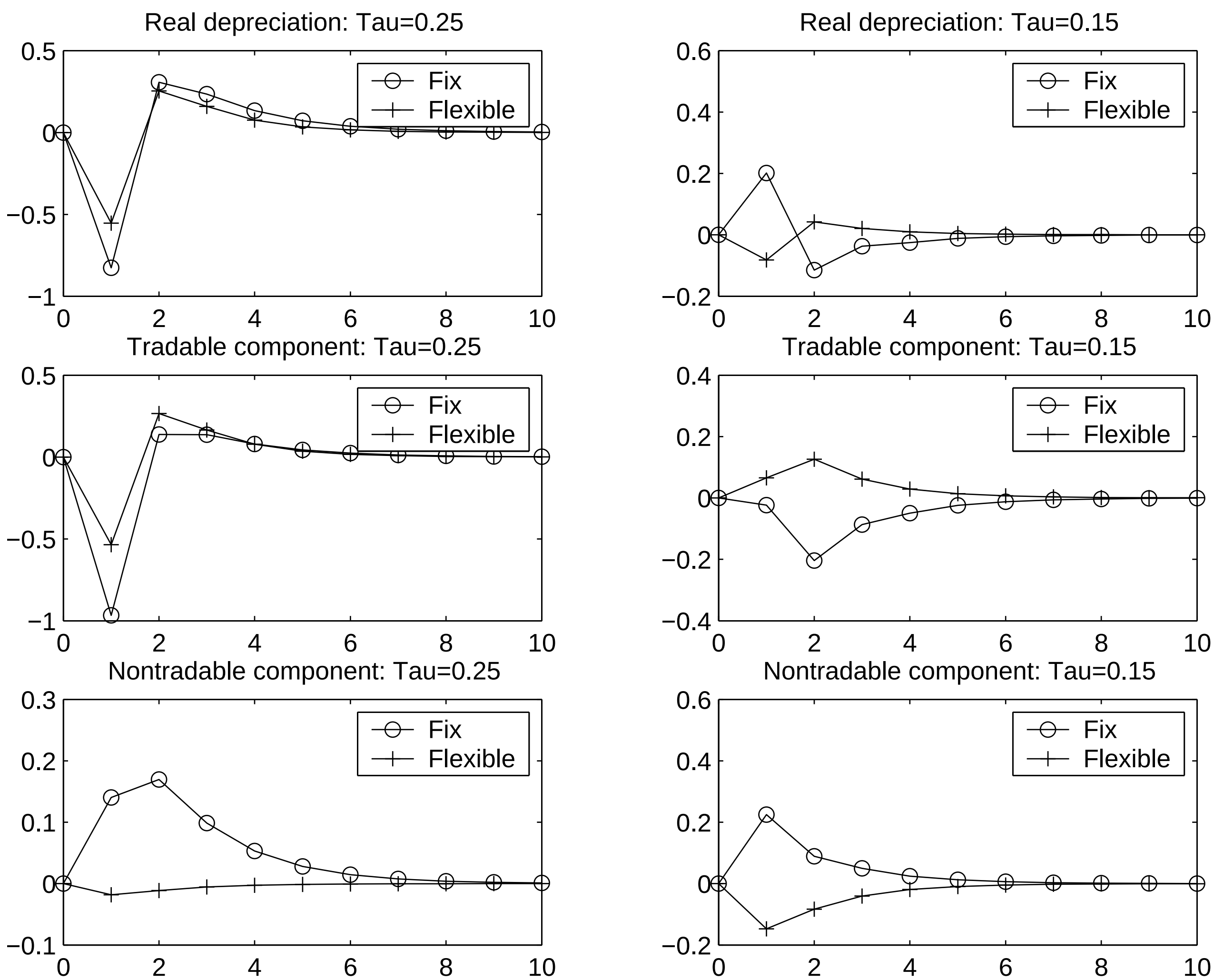

Figure 8.1 Real exchange rate decomposition and intratemporal elasticity of substitution: Productivity shock

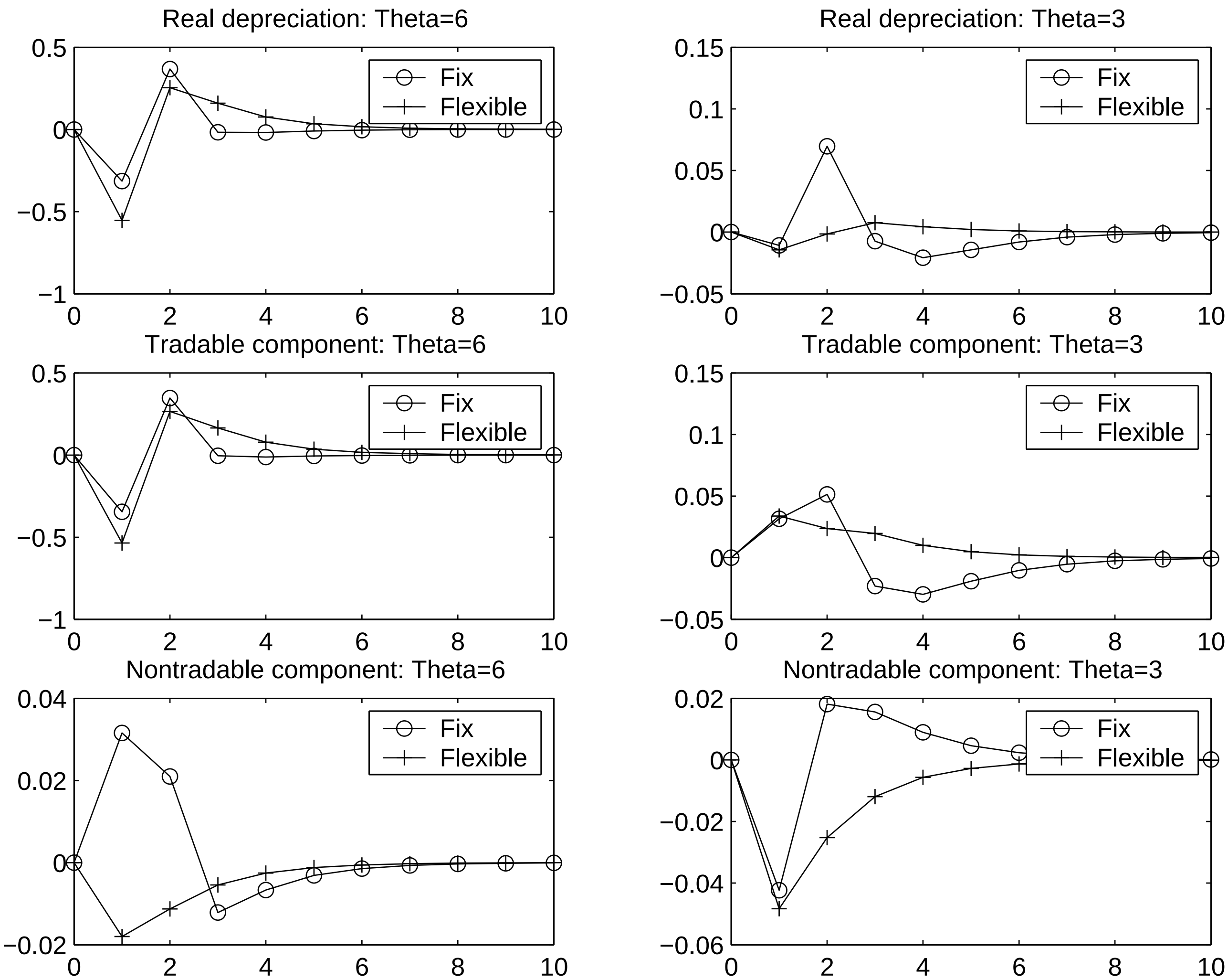

Figure 8.2 Real exchange rate decomposition and intratemporal elasticity of substitution: Interest rate shock

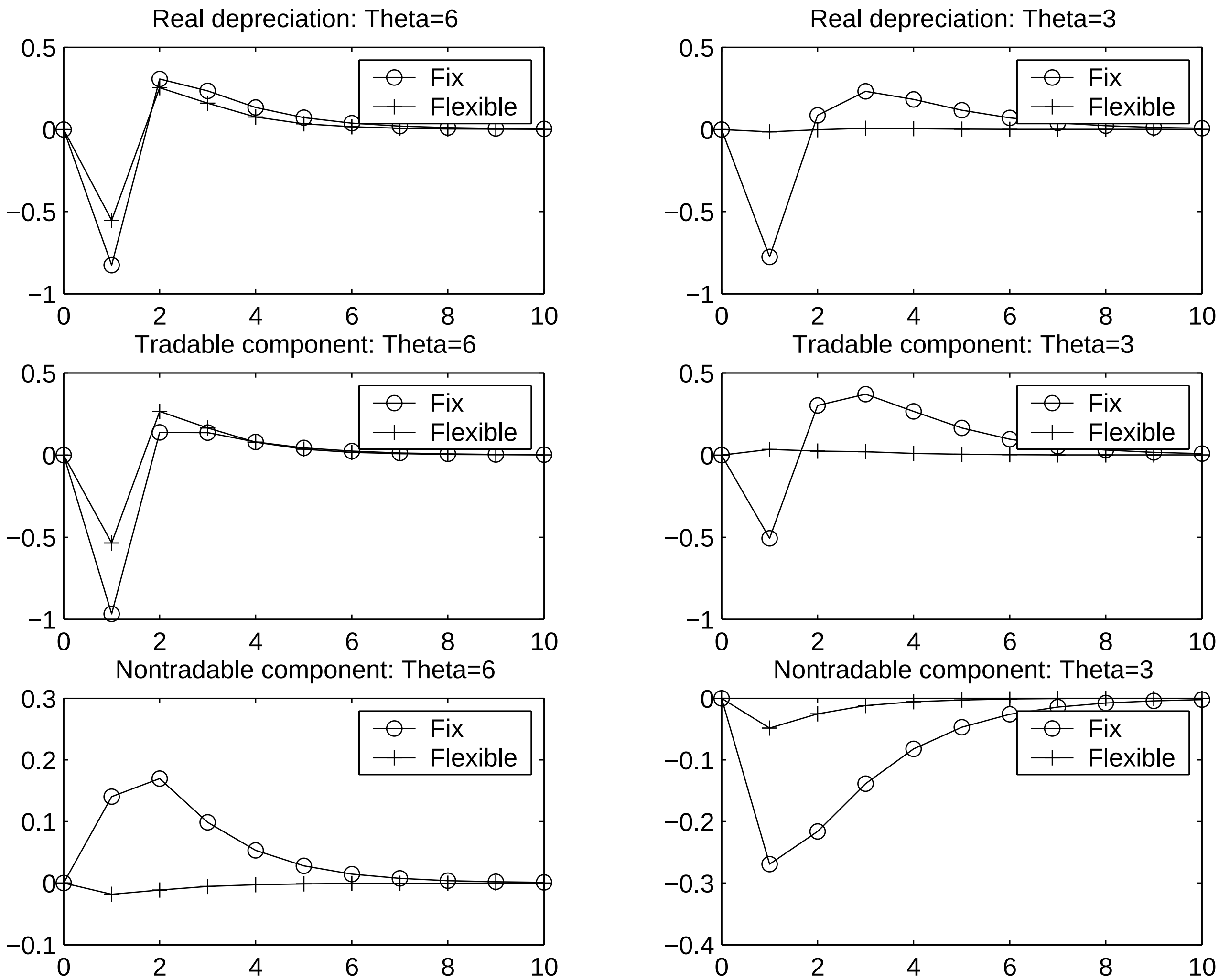

Figure 9.1 Expenditure switching effect of exchange rates and transport costs: Productivity shock

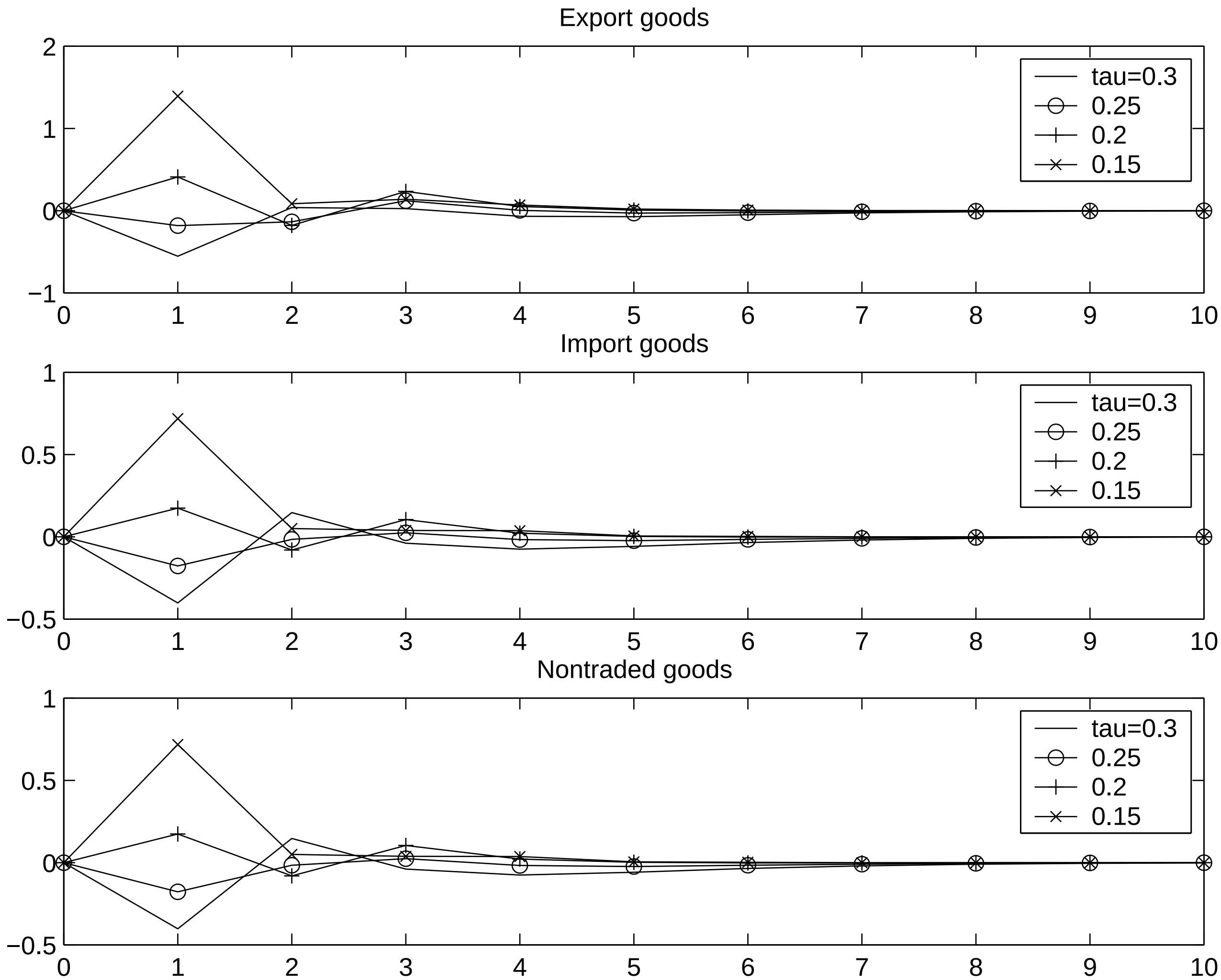

Figure 9.2 Expenditure switching effect of exchange rates and transport costs: Interest rate shock

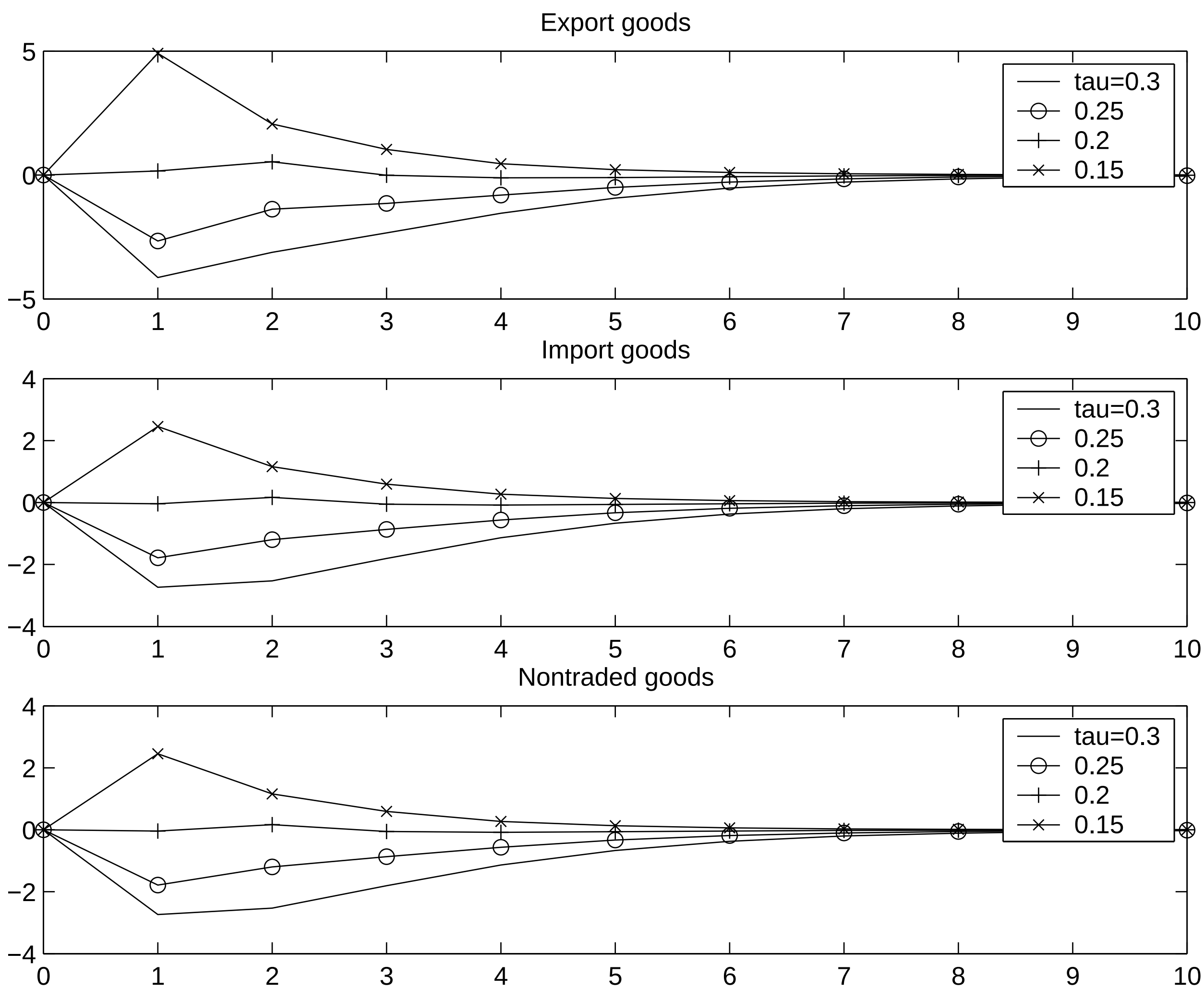

Figure 10.1 Expenditure switching effect of exchange rates and intratemporal elasticity of substitution: Productivity shock

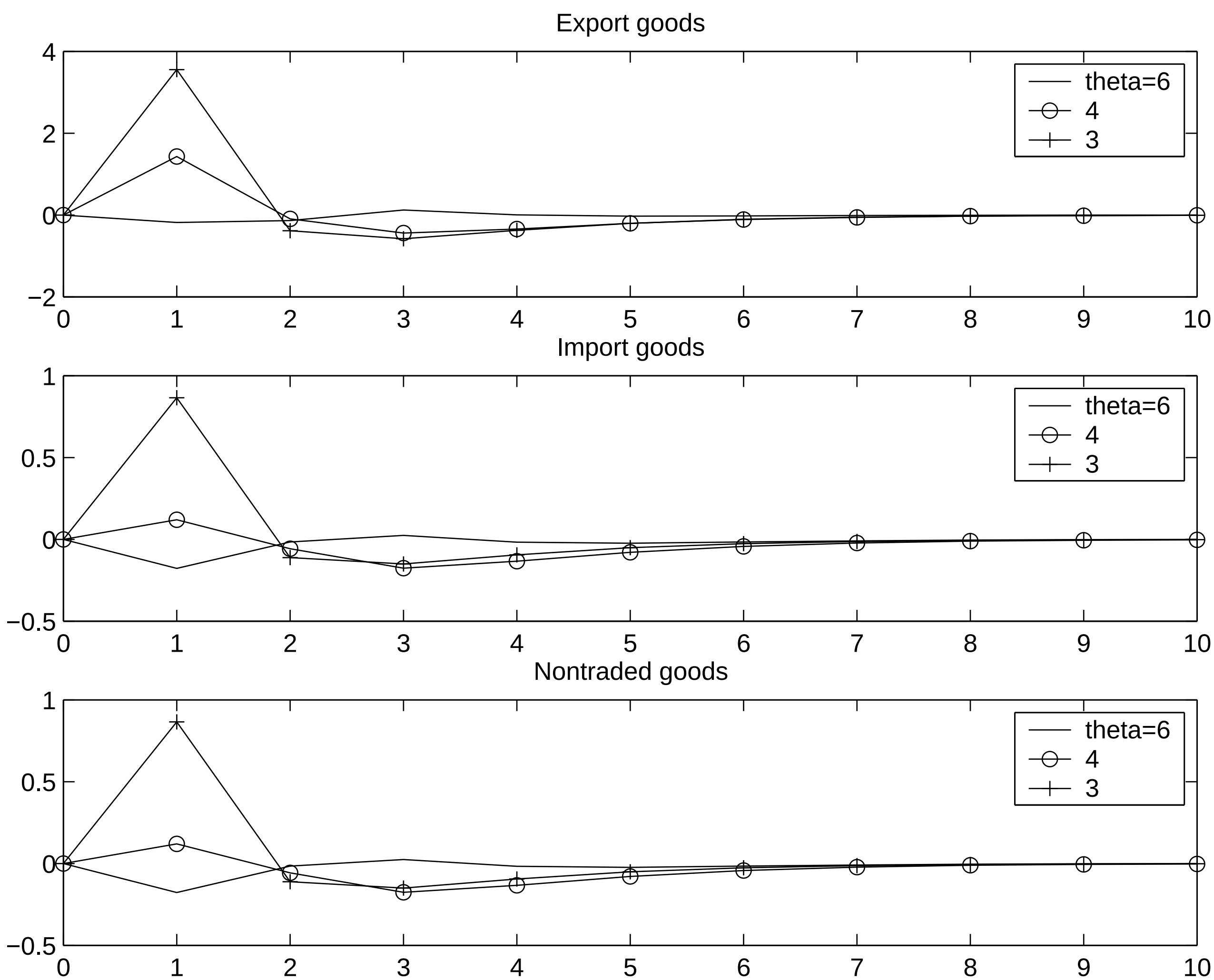

Figure 10.2 Expenditure switching effect of exchange rates and intratemporal elasticity of substitution: Interest rate shock