Financial Crises and Interest Rates*

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Abstract

We consider a stylized currency crises model with heterogeneous information among investors, with endogenous determination of interest rates in a noisy rational expectations equilibrium. We explore the payoff and informational channels through which interest rates determine devaluation outcomes, and examine the implications for equilibrium selection by global games methods. When we take into account the role of domestic interest rates in determining devaluation outcomes, as discussed by Obstfeld (1986, 1996), multiplicity of equilibria reemerges robustly, even when we allow for heterogeneous information as suggested by Morris and Shin (1998). Multiplicity is not so much due to informational assumptions, but results from the role of domestic interest rates in coordinating individual investment decisions, along with determining the ultimate devaluation outcome.

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1 Introduction

It is a commonly held view that financial crises, such as speculative attacks against a fixed exchange rate regime, bank runs, debt crises or asset price crashes may be the result of self-fulfilling expectations and coordination failures in environments that are inherently unstable and admit multiple equilibria.[1] Building on game-theoretic advances by Carlsson and van Damme (1993), this view has recently been challenged by Morris and Shin (1998), who argue that multiplicity may be the unintended consequence of assuming that fundamentals are common knowledge among market participants. Morris and Shin (1998) illustrate their argument with a currency crises model, in which traders observe the fundamentals with small idiosyncratic noise, showing that this leads to the selection of a unique equilibrium, whose outcome is uniquely determined by economic fundamentals.

In this paper, we reexamine the forces underlying uniqueness vs. multiplicity in models of financial crises, with a particular focus on the role of domestic interest rates. We start from the observation that Morris and Shin's (1998) selection argument requires the game's payoffs, in particular the spread between domestic and foreign interest rates, to be exogenously fixed. Therefore, their model departs from the multiple equilibrium models not only by introducing a lack of common knowledge, but also by making very specific assumptions about the market environment.

We consider a stylized currency crises game with heterogeneous information among traders, allowing for the endogenous determination of domestic interest rates in the domestic bond market. Our main technical contribution is to bring asset markets into a global games model of currency crises, using a noisy rational expectations equilibrium approach along the lines of Grossman and Stiglitz (1976, 1980) and Hellwig (1980). Our model captures three key features of domestic interest rates: First, the opportunity cost of attacking the currency responds to the investors' behavior; a lower demand for domestic assets will increase their rate of return. Second, the domestic interest rate may influence the central bank's preferences for a fixed exchange rate: as suggested by Obstfeld (1996), a central bank may abandon a fixed exchange rate, if the political and economic costs of rising interest rates become too high. Finally, when traders are heterogeneously informed, the domestic interest rate serves as a public signal which aggregates private information about fundamentals.

[1] This view has been formalized for currency crises by Obstfeld (1986, 1996), for bank runs, by Diamond and Dybvig (1983), for debt crises by Calvo (1988) and Cole and Kehoe (2000) and for asset price crashes by Gennotte and Leland (1990) and Barlevy and Veronesi (2003), among others.

We analyze two different versions of our model, first with common knowledge, then with incomplete, heterogeneous information about fundamentals. To prevent the domestic interest rate from perfectly revealing the state, we introduce a shock to the domestic supply of bonds. Our main conclusion is that multiplicity is not an artifact of common knowledge, but is due to other factors. Both versions of our model feature multiple equilibria, either, when the central bank cares sufficiently strongly about the cost of high domestic interest rates, or when the devaluation outcome is determined by the central bank's loss of foreign reserves, provided that the domestic supply of bonds is sufficiently elastic in the interest rate and/or shocks to the domestic bond supply sufficiently small. The first case corresponds to Obstfeld (1996), the second case to Obstfeld (1986), among others.

These multiplicity results are based on a comparison between the effects of domestic interest rates for the return on domestic and foreign assets. On the one hand, an increase in the domestic interest rate raises the return on domestic assets, which reduces the traders' willingness to attack the currency, all else equal. On the other hand, if an increase in domestic interest rates raises the likelihood of a devaluation, this also increases the net return on foreign assets. The demand for domestic assets trades off these two effects: If the effect on expectations about a devaluation dominates the direct payoff effect on domestic assets, the asset demand schedule may become locally decreasing in the interest rate, i.e. as the return on domestic assets increases, the demand for these assets goes down. This may lead to multiple market-clearing interest rates in the domestic bond market. Moreover, any equilibrium necessarily generates a 'crash', whereby the equilibrium interest rate discontinuously changes with fundamentals, triggering a discrete change in the probability of a devaluation.

Now, when the central bank is concerned mostly about the cost of high domestic interest rates, traders care about each others' actions only to the extent that this affects the domestic interest rate. The demand for domestic assets is uniquely determined, but non-monotonic: As the domestic interest rate increases, it may reach a point, at which traders shift their portfolio to foreign assets because they expect that the central bank is likely to devalue. This gives rise to multiple market-clearing interest rates, with different self-fulfilling expectations about the devaluation outcome. Moreover, the absence of an explicit coordination problem carries over directly into the model with incomplete information, and there are multiple equilibria, even when the domestic interest rate becomes infinitely noisy, i.e. when the shocks in the domestic bond supply are large.

In contrast, when the devaluation outcome is mainly determined by the loss of foreign reserves, multiplicity results from an explicit coordination problem among traders, and the domestic interest rate merely adjusts to clear the domestic bond market. In this case, it becomes important the the domestic interest rate be sufficiently effective at aggregating private information, i.e. that the supply shocks not be too large. Furthermore, multiplicity also requires that the domestic bond supply be sufficiently elastic: If the bond supply is completely inelastic, there is a unique equilibrium, in which the central bank's foreign reserve losses and the devaluation outcome are uniquely determined by fundamentals and domestic supply shocks.

In summary, our results show that many features of multiple equilibrium models of financial crises, most notably the unpredictability of speculative attacks and the sudden jumps in domestic interest rates, are robust to a lack of common knowledge and a formal model of information aggregation. Indeed, the logic behind the multiplicity results in Obstfeld (1986, 1996) is driven not so much by the assumption that fundamentals are common knowledge, but by the dual role that interest rates play in coordinating individual investment decisions, along with directly or indirectly determining the ultimate devaluation outcome. Our results further highlight the difference between foreign reserve losses and interest rates as the driving forces behind the central bank's decision to maintain or abandon a fixed exchange rate. All this is more appropriately captured within a rational expectations equilibrium than by a stylized coordination game, which abstracts from the role of domestic interest rates.

Related Literature: Following the original papers of Carlsson and van Damme (1993) and Morris and Shin (1998), several papers have studied the robustness of equilibrium selection to exogenous public information and the effect of public information on coordination outcomes (see for example, Morris and Shin 2003 and 2004, and Hellwig 2002). We build on their insights, but endogenize the information structure by considering the informational role of interest rates.

Atkeson (2000) is the first to discuss the potential problems that the lack of a theory of prices poses for global coordination games. In response to his critique, the idea of using a noisy REE approach to introduce prices into global games also appears in Tarashev (2003) and in Angeletos and Werning (2004).[2] Tarashev analyzes a version of Morris and Shin's currency crises game with endogenous interest rate determination, in which he establishes the existence of a unique equilibrium. His result appears as a special case of our model, in which the devaluation outcome is determined by reserve losses, and the domestic bond supply is inelastic.

[2] Chari and Kehoe (2000) use a noisy REE approach to introduce prices in herding models.

The paper most closely related to ours is Angeletos and Werning (2004). Angeletos and Werning study the effects of information aggregation through noisy public signals of aggregate activity and prices in global coordination games, and show that equilibrium multiplicity may be restored by the endogenous public signal, when private signals are sufficiently precise.[3] However, their analysis focuses on the interaction between a global coordination game, and a 'derivative' asset market, in which traders trade assets whose payoffs depend either on the same exogenous fundamentals as the payoffs in the coordination game, or directly on the outcome of the coordination game. In their environment, prices affect coordination outcomes only through the information that they provide.

Finally, the idea of multiple equilibria in asset pricing models due to non-monotone asset demand and supply schedules also arises in traditional REE asset pricing models in which coordination problems are absent, such as Genotte and Leland's (1990) analysis of stock market crashes. More recently, it appears in Barlevy and Veronesi (2003), where multiple market-clearing prices and discontinuities in the equilibrium price function are due to the interaction between informed and uninformed traders. Our discussion here shows, how a similar argument underlies multiplicity of equilibria in models of financial crises.

2 Model description

Players, actions and payoffs: We consider an economy populated by a measure one continuum of risk-neutral traders, indexed by $i \in [0, 1]$, and a central bank (CB). Initially, each agent is endowed with one unit of domestic currency. Traders can invest their endowment either in a domestic bond, or they can go to the central bank and exchange the domestic currency one-for-one for a dollar. The investment in the domestic bond yields a safe market-determined net interest rate r. The return to exchanging the domestic currency for a dollar is determined by whether a devaluation occurs. If there is no devaluation, and the dollar is converted back into domestic currency at the same level, its net return 0. However, if the CB decides to abandon the fixed exchange rate, the exchange rate drops to 2 units of domestic currency for the dollar, and the net return on the dollar is 1. These investment returns are summarized in the following table:

[3]In this environment, they are the first to point out that multiplicity of rational expectations equilibria may arise from the price function.

	Devaluation	No devaluation
Dollar	1	0
Domestic Bond	r	r

Devaluation decision: The central bank's decision to devalue the domestic currency depends on the market-determined domestic interest rate r, its loss of foreign reserves $A \in [0,1]$, which measures the total of dollars withdrawn by traders, and an unobserved fundamental θ, which measures the strength of the CB's commitment to maintain a fixed exchange rate. The net value of maintaining the fixed exchange rate is given by $\theta - U(r, A)$, and the central bank will devalue, if and only if

$$\theta \leq U(r, A). \tag{1}$$

θ may be interpreted as the value of the peg in the absence of any reserve losses or interest rate increases, and $U(r, A)$ measures the cost of having to defend the exchange rate in the event of high interest rates, or losses of foreign reserves. We normalize $U(0,0) = 0$, and assume $\frac{\partial U}{\partial r} \geq 0$ and $\frac{\partial U}{\partial A} \geq 0$, so that the value of maintaining the fixed exchange rate is non-increasing in both the domestic interest rate and the loss of foreign reserves. This general formulation embeds two special cases that are of interest: $U(r, A) = r$ allows for a scenario, in which the CB is concerned exclusively by high domestic interest rates, such as in Obstfeld (1996). On the other hand, $U(r, A) = A$ represents the case in which a devaluation is purely determined by the CB's loss of foreign reserves. This corresponds to the modeling assumptions in Krugman (1979), Flood and Garber (1984) or Obstfeld (1986).

Information structure and timing: The currency crisis game has three stages. In stage 1, nature selects $\theta \in \mathbb{R}$, according to a common prior distribution characterized by absolutely continuous cdf $H(\cdot)$ and pdf $h(\cdot)$. Then, each trader observes an idiosyncratic, private signal about θ, denoted x_i. Conditional on θ, private signals are independent, and identically distributed according to cdf $F(\cdot \mid \theta)$ and pdf $f(\cdot \mid \theta)$. We assume that the support of x_i is $\mathbb{R}$, and $F(\cdot \mid \theta)$ satisfies the monotone likelihood ratio property, implying that $F(\cdot \mid \theta)$ is first-order stochastically increasing in θ.

In stage 2, the domestic bond market and the central bank open. Traders submit contingent bids $a_i(r) \in [0,1]$, which indicate, conditional on the market-determined domestic interest rate r, what fraction of their wealth they wish to invest in the dollar. $1 - a_i(r)$ is then the bid submitted to the domestic bond market. The supply of dollars is guaranteed by the central bank. The supply of domestic bonds is exogenously given by $S(s, r)$, a continuous function of the realized interest rate r, and an exogenous supply shock s. $s \in \mathbb{R}$ is independent of θ and the private signals, and is distributed according to absolutely continuous cdf $G(\cdot)$ and pdf $g(\cdot)$. Once all bids are submitted and the supply shock is realized, a Walrasian auctioneer selects an interest rate r to clear the domestic bond market.

In stage 3, the CB decides whether or not to maintain the fixed exchange rate, after observing θ, r, and the total of dollar withdrawals A.

Strategies and Equilibrium: In stage 2, each trader submits a contingent bid $a_i(r)$, conditional on his private signal x_i. We let $a(x, r)$ denote the traders' *bidding strategy*, which, conditional on a private signal x and interest rate r, indicates a trader's dollar withdrawal.[4]

Integrating individual bidding strategies over x, we find the total demand for dollars, or equivalently, the CB's reserve losses, as a function of θ and r, denoted $A(\theta, r)$:

$$A(\theta, r) \equiv \int a(x, r) f(x|\theta)\, dx. \tag{2}$$

The demand for domestic bonds is then given by $1 - A(\theta, r)$, and clearing the domestic bond market requires

$$1 - A(\theta, r) = S(s, r). \tag{3}$$

Therefore, the auctioneer selects an *interest rate function* $R(\theta, s)$, such that for each θ and s, $R(\theta, s)$ clears the domestic bond market.

Now, suppose that the CB's reserve losses are $A(\theta, r)$ and the auctioneer selects an interest rate function $R(\theta, s)$. Let $p(x, r)$ denote the posterior belief that a devaluation occurs, conditional on observing a signal x, and conditional on the market-clearing interest rate being r. A bidding strategy $a(x, r)$ is optimal, if and only if

$$\begin{aligned} a(x, r) &= 1, \text{ if } p(x, r) > r \\ a(x, r) &\in [0, 1], \text{ if } p(x, r) = r \\ a(x, r) &= 0, \text{ if } p(x, r) < r \end{aligned} \tag{4}$$

[4] Note that we are restricting attention to *symmetric* bidding strategies, in which conditional on having observed identical signals, two traders submit identical bids. It is straight-forward to rule out equilibria with asymmetric bidding strategies.

(4) may be interpreted as an *uncovered interest parity condition*: r is the excess return on domestic bonds. $p(x,r)$ is the probability of devaluation, which here corresponds to the expected depreciation of the domestic currency. (4) thus states that optimal investment decisions trade off the expected depreciation against the domestic interest rate premium. For any r, such that $\{(\theta,s) : r = R(\theta,s)\}$ is non-empty, Bayes' Law implies that $p(x,r)$ is given by

$$p(x,r) = \frac{\int_{\theta \leq U(r,A(\theta,r));r=R(\theta,s)} f(x|\theta)\,h(\theta)\,g(s)\,d\theta ds}{\int_{r=R(\theta,s)} f(x|\theta)\,h(\theta)\,g(s)\,d\theta ds}. \tag{5}$$

On the other hand, if $\{(\theta,s) : r = R(\theta,s)\}$ is empty for some r, then r is never realized as a market-clearing interest rate, and Bayes' Law no longer determines $p(x,r)$. We have the following equilibrium definition:

Definition 1 *A Perfect Bayesian Equilibrium consists of a bidding strategy $a(x,r)$, an interest rate function $R(\theta,s)$, a reserve loss function $A(\theta,r)$, and posterior beliefs $p(x,r)$ such that*

(i) $a(x,r)$, $A(\theta,r)$ and $R(\theta,s)$ satisfy, respectively, (4), (2) and (3), given beliefs $p(x,r)$; and

(ii) for all r such that $\{(\theta,s) : r = R(\theta,s)\}$ is non-empty, $p(x,r)$ satisfies (5).

The ability to submit bids contingent on r enables the traders to take into account the information conveyed by the market-clearing interest rate. The interest rate r affects optimal bidding strategies through two channels: On the one hand, there is a *payoff effect*, since the return on the domestic bond is increasing r. This is captured by the RHS of the optimality condition $p(x,r) \gtreqless r$. But r also appears on the LHS of this optimality condition, capturing the *expectations effect* of r: The market-clearing interest rate conveys information about the likelihood of a devaluation and thereby affects the expected return on investing in a dollar. If this expectations effect becomes sufficiently strong and positive, a marginal increase in r may raise the return on the dollar by more than the return on domestic bonds, which in turn implies that the demand for domestic bonds becomes decreasing in r. On the other hand, since $p(x,r) \in [0,1]$, the payoff effect dominates, whenever $r < 0$ or $r > 1$.

Functional form assumptions: We conclude the description of the environment with a series of functional form assumptions for the information structure, the supply of domestic bonds and the CB preferences that will enable us to arrive at closed-form solutions for our model.

(A1) *Common prior:* nature draws θ from an improper uniform distribution over the entire real line.[5]

(A2) *Private signals:* $x_i|\theta \sim \mathcal{N}\left(\theta; \beta^{-1}\right)$. β thus denotes the precision of private signals about θ.

(A3) *Central Bank preferences:* $U(r, A) = \Phi\left(\lambda\Phi^{-1}(A) + (1-\lambda)\Phi^{-1}(r)\right)$,

where $\Phi(\cdot)$ denotes the cdf of the standard normal distribution, and $\lambda \in [0,1]$ is a parameter that determines the CB's weighting between the cost of high interest rates and reserve losses. If $\lambda = 1$, $U(r, A) = A$ and the CB cares only about reserve losses. If $\lambda = 0$, $U(r, A) = r$ and the CB cares only about interest rates.

(A4) *Domestic bond supply:* $S(s, r) = \Phi\left(s - \gamma\Phi^{-1}(r)\right)$,

where $s \sim \mathcal{N}(0, \delta^{-1})$, i.e. the supply shock is normally distributed with the mean of zero, and variance $1/\delta$.[6] As we will discuss below, δ determines how much noise there is in the trading process (equivalently, to what extent the interest rate is efficient at aggregating private information). In the limiting case where $\delta \to \infty$, r becomes fully revealing of the state; when $\delta \to 0$, the supply shocks become so big that r becomes totally uninformative. The parameter γ reflects the interest rate elasticity of the domestic bond supply. Together δ and γ determine to what extent the bond supply and, as a consequence of market-clearing, foreign reserve losses, are driven by interest rate movements vs. exogenous supply shocks.

3 Obstfeld (1986, 1996) vs. Morris & Shin (1998)

In this section, we review main ideas of the second-generation currency crises models developed by Obstfeld (1986 and 1996) and others in the context of our model, and we contrast them with the model of Morris and Shin (1998). Let's suppose for the moment that $\theta \in (0,1]$ is common knowledge among all traders in stage two.[7] With a slight abuse of notation, we let $p(\theta, r)$ denote the probability of a devaluation, $a(\theta, r)$ individual bidding strategies and $A(\theta, r)$ the central bank's

[5] This improper prior assumption is not essential for our results.

[6] At $r \in \{0,1\}$ and/or $A \in \{0,1\}$, $U(r, A)$ and $S(s, r)$ are defined by extention to the limit.

[7] If $\theta > 1$ were common knowledge, it is easy to check that there always exists a unique equilibrium, in which $r = A = 0$, and no devaluation occurs. Likewise, if $\theta \leq 0$ is common knowledge, there exists a unique equilibrium, in which $r = A = 1$, and a devaluation does occur.

reserve losses, conditional on θ and r. At the center of the analysis is the uncovered interest parity condition according to which agents bidding strategies depend on whether $p(\theta, r) \gtreqless r$. As we will show next, the arguments for multiplicity all rely critically on the fact that $p(\theta, r)$ is a non-monotone function of r. The models differ however in the economic mechanisms that deliver this property. We look at each one of them separately.

3.1 Obstfeld (1996): multiplicity caused by interest rates

Obstfeld (1996) argues that self-fulfilling devaluations may be triggered by the cost of high interest rates. High interest rates become self-fulfilling, because they make a devaluation more likely: In one equilibrium, investors expect a devaluation, which leads to a high domestic interest rate premium, whose political and economic costs are unsustainable. In an alternative equilibrium, investors do not expect a devaluation, and hence the resulting low interest rate becomes sustainable.

Within the context of our model, consider the case, where the central bank has preferences only over the interest rates, i.e. $\lambda = 0$, and a devaluation occurs, if and only if $\theta \leq r$. In that case, the probability of a devaluation has a particularly simple form:

$$p(\theta, r) = \begin{cases} 1 \text{ if } \theta \leq r, \\ 0 \text{ if } \theta > r \end{cases} . \tag{6}$$

Therefore, optimal bidding strategies are characterized as follows: if $r > 1$, the domestic bond strictly dominates the dollar, and $a(\theta, r) = 1$. If $r < 0$, the dollar strictly dominates the domestic bond, and $a(\theta, r) = 0$. If $r \in (0, 1)$, agents convert their endowment of domestic currency into dollars, if and only if $\theta \leq r$. Finally, when $r = 0$, $\theta > r$, a devaluation does not occur, and traders are indifferent between the dollar and the domestic bond. Similarly, when $r = 1$, a devaluation does occur, and again traders are indifferent. To summarize, optimal bidding strategies are characterized as:

$$a(\theta, r) = A(\theta, r) \in \begin{cases} \{0\} \text{ if } r > 1 \\ [0,1] \text{ if } r = 1 \\ \{1\} \text{ if } r \in (\theta, 1) \\ \{0\} \text{ if } r \in (0, \theta] \\ [0,1] \text{ if } r = 0 \\ \{1\} \text{ if } r < 0 \end{cases} . \tag{7}$$

Therefore, for each θ and r (with the exception of $r = 0$ and $r = 1$), the demand schedule for domestic bonds, $1 - A(\theta, r)$, is uniquely pinned down, and is non-monotone in r. Moreover,

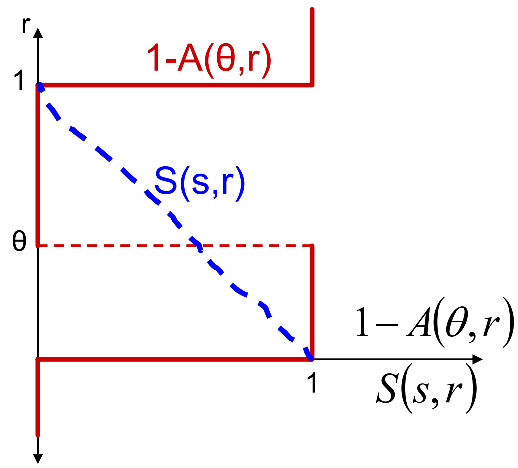

Figure 1: Obstfeld (1996), multiplicity caused by high interest rates

the domestic bond supply is exogenously given by $S(s,r)$. We illustrate the equilibrium in the domestic bond market in Figure 1, which plots the supply curve $S(s,r)$ and the correspondence for the domestic bond supply, $1-A(\theta,r)$, as characterized by (7). We can see from this figure that, unless the domestic bond supply is perfectly elastic at some exogenous r, there are two market clearing prices. Given the functional form assumption for $S(s,r)$, there are multiple equilibria, irrespective of s. For any s and $\theta\in[0,1]$, $r=0$ and $r=1$ both clear the domestic bond market. If $r=0$, then $S(s,0)=1-A(\theta,0)=1$, and no devaluation will take place. On the other hand, if $r=1$, $S(s,1)=1-A(\theta,1)=1$, and a devaluation will take place.

When only interest rates matter for the CB's devaluation decision, multiplicity of equilibria arises from the existence of multiple market-clearing prices. This multiplicity results, because the demand for domestic bonds is locally decreasing in r; at the point of this non-monotonicity, the increase in r leads to a discrete increase in the expected devaluation premium, which more than offsets the increase in the domestic bond return r. This argument does not in any way require that traders have an explicit motive to coordinate individual trading strategies, and conditional on r, traders do not need to make any forecast on what actions the other traders are likely to take.

3.2 Obstfeld (1986): multiplicity caused by reserve losses

We next consider a case in which the central bank's devaluation decision is driven only by reserve losses. In this case, we show that multiplicity is driven by an explicit coordination motive: In one equilibrium, traders expect a devaluation, the interest rate premium is high, and there is a large loss of foreign reserves which validates the traders' expectations of a devaluation. In another equilibrium, traders do not expect a devaluation, the interest rate premium is low and the loss of reserves small, again validating the traders' expectations.

Formally, suppose that $\lambda = 1$, i.e. a devaluation occurs, if and only if $\theta \leq A$. In that case, the probability of a devaluation is given by

$$p(\theta, r) = \begin{cases} 1 \text{ if } \theta \leq A(\theta, r), \\ 0 \text{ if } \theta > A(\theta, r) \end{cases} \tag{8}$$

Hence, $r \in [0, 1]$ affects individual decisions only to the extent that it enables them to coordinate on either all attacking (in which case a devaluation occurs), or on not attacking; in other words, r serves as a coordination device. Unlike the previous case, $a(\theta, r)$ and $A(\theta, r)$ are no longer uniquely pinned down. In fact, for any $r \in [0, 1]$, if all agents attack, a devaluation will occur, and it is indeed optimal to attack, while, if no agent attacks, no devaluation will occur, and it is optimal not to attack, i.e. for $r \in [0, 1]$, both $A(\theta, r) = 0$ and $A(\theta, r) = 1$ are part of the best response correspondence for the demand for domestic bonds. If $r > 1$, agents strictly prefer the domestic bond, and if $r < 0$, agents strictly prefer to invest in the dollar. Finally, if $r = 0$, agents are indifferent between the domestic bond and the dollar, as long as $\theta > A(\theta, r)$; hence any $A < \theta$ can be sustained as part of the demand correspondence. Similarly, if $r = 1$, agents are indifferent, as long as $\theta \leq A(\theta, r)$, and hence any $A \geq \theta$ is sustainable. Thus, the best-response correspondence for optimal bidding strategies is given by:

$$a(\theta, r) = A(\theta, r) \in \begin{cases} \{0\} \text{ if } r > 1 \\ \{0\} \cup [A, 1] \text{ if } r = 1 \\ \{0, 1\} \text{ if } r \in (0, 1) \\ [0, A) \cup \{1\} \text{ if } r = 0 \\ \{1\} \text{ if } r < 0 \end{cases}. \tag{9}$$

In figure 2, we again plot the demand correspondence for domestic bonds, $1 - A(\theta, r)$, and the supply $S(s, r)$, for given s and $\theta \in (0, 1]$. As long as $\gamma > 0$ (i.e. unless the bond supply is perfectly

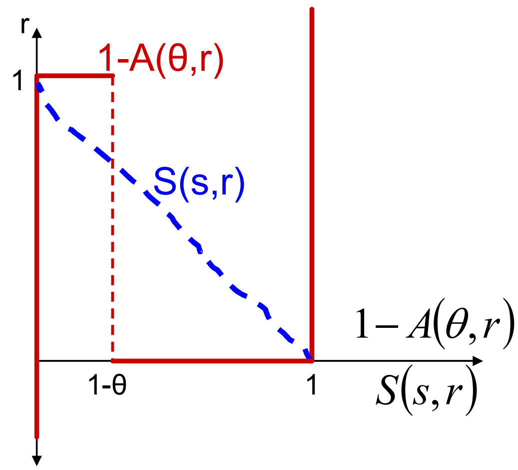

Figure 2: Obstfeld (1986), multiplicity caused by reserve losses

inelastic), it is immediate that there are multiple equilibria: If $r = 1$, $S(s, r) = 0$, which clears the bond market when $A(\theta, r) = 1$, while if $r = 0$, $S(s, r) = 1$, which clears the bond market when $A(\theta, r) = 0$. Therefore, there are two equilibria, one in which the interest rate is high, reserve losses large, a devaluation occurs, and all traders attack, and one, in which the interest rate is low, reserve losses are low, no devaluation occurs, and noone attacks.

Figure 2 highlights the importance of the coordination motive among traders in generating multiplicity in this environment. This coordination motive implies that the demand correspondence is no longer uniquely pinned down for a given $r \in [0, 1]$. Multiple equilibria arise, because traders can coordinate on multiple best responses to a given r. Market-clearing then requires that for different responses by traders, different values of r must be selected to clear the market. The multiplicity argument is thus quite different from the previous one, in which conditional on r, bidding strategies were uniquely pinned down, but there were multiple market-clearing interest rates.

The figure also reveals the role played by the interest rate elasticity. Indeed, if the domestic bond supply was infinitely inelastic, i.e. $\gamma = 0$, and $S(s, r) = S \in (0, 1)$ for all r, there exists a unique equilibrium, in which $r \in \{0, 1\}$ adjusts so that $1 - A = S$, and in equilibrium, either $1 - S \geq \theta$, in which case there is a devaluation and $r = 1$, or $1 - S < \theta$, in which case there is no devaluation, and $r = 0$. Thus, if the domestic bond supply is perfectly inelastic, there is a unique equilibrium, in which the ultimate devaluation outcome is purely driven by the fundamentals θ, and by the shocks to the domestic bond supply s, and r adjusts to clear the domestic bond market. On the other hand, if $\gamma > 0$, there is room for multiple equilibria. In that case, as r increases, there is a collapse in the domestic supply of bonds, and an increase in the loss of foreign reserves, which makes a devaluation more likely, and thereby validates the initial increase in the interest rate.

While this basic argument is present in many currency crises models with multiple equilibria, models may differ in what leads to a self-fulfilling collapse of the domestic bond supply. To our knowledge, the argument is made first in Obstfeld (1986), where the domestic supply of bonds has positive interest elasticity because of a time consistency problem in monetary policy: After a devaluation, an inflationary policy is anticipated, which leads to a self-fulfilling collapse in domestic credit and an increase in the domestic interest rate. However, there are other forces that give rise to similar arguments: Higher domestic interest rates lead to a collapse of domestic investment, and thereby reduce the demand for domestic credit, and the supply of bonds. To the extent that these or similar forces are present, the analysis will give rise to results similar to the ones presented here.

3.3 Morris & Shin (1998)

In their influential (1998) paper, Morris and Shin argue that multiplicity of equilibria in models of financial crises may be the artefact of assuming that fundamentals are common knowledge. Morris and Shin's argument is based on an equilibrium selection result for coordination games by Carlsson and van Damme (1993). This argument relies in particular on the assumption that conditional on the state, payoffs in the coordination game are exogenously fixed. Translated into our environment, this condition requires that the domestic interest rate must be exogenously fixed at some predetermined level $r \in (0, 1)$, at which the supply of domestic bonds is infinitely elastic. Fixing r exogenously further requires that the CB cares about reserve losses. Morris and Shin then show that there is a unique equilibrium, if the information structure is characterized by assumptions (A1) and (A2).

Proposition 1 (Morris & Shin, 1998) *Under assumptions (A1) and (A2), with the domestic bond supply infinitely elastic at $r \in (0, 1)$, and a devaluation occurring iff $\theta \leq A$, there exist thresholds x_{MS}, and θ_{MS}, such that in the unique equilibrium, agents attack, if and only if $x \leq x_{MS}$, and buy the domestic bond otherwise, and a devaluation occurs, if and only if $\theta \leq \theta_{MS}$. x_{MS} and*

θ_{MS} *are characterized by*

$$\theta_{MS} = 1 - r = \Phi\left(\sqrt{\beta}\left(x_{MS} - \theta_{MS}\right)\right) \tag{10}$$

This uniqueness result not only requires a departure from common knowledge in (A1) and (A2), but also relies on specific assumptions about the nature of the domestic bond supply and the CB's objective.

4 Equilibrium Characterization with heterogeneous information

In this section, we characterize equilibria of our currency crises game with heterogeneous information and we derive conditions under which there are multiple equilibria. We will restrict attention to *monotone strategy equilibria*, which are characterized by thresholds $x^*(r)$ and $\theta^*(r)$ such that

$$a(x,r) = \begin{cases} 1, \text{ if } x \leq x^*(r); \\ 0, \text{ if } x > x^*(r) \end{cases}$$

and the currency is devalued if and only if $\theta \leq \theta^*(r)$. With these bidding strategies, $A(\theta, r) = F(x^*(r)|\theta)$ is decreasing in θ. This in turn implies that for any r, there exists a unique $\theta^*(r)$, for which

$$\theta^*(r) = U(r, A(\theta^*(r), r)) = U(r, F(x^*(r)|\theta^*(r))), \tag{11}$$

and a devaluation occurs if and only if $\theta \leq \theta^*(r)$. By standard representation theorems (Milgrom, 1981), $p(x,r) = \Pr(\theta \leq \theta^*(r)|x, r = R(\theta, s))$ is strictly decreasing in x, and there exists a unique $x^*(r)$, such that

$$r = p(x^*(r), r) \tag{12}$$

and $r \lesseqgtr p(x^*(r), r)$ whenever $x \lesseqgtr x^*(r)$. Thus, if the devaluation outcome is characterized by a threshold rule $\theta^*(r)$, optimal bidding strategies are also characterized by a threshold rule, and equilibrium thresholds $x^*(r)$ and $\theta^*(r)$ must jointly solve (11) and (12), given posterior beliefs $p(x,r)$.

To complete the equilibrium characterization, we need to determine the information conveyed by r in equilibrium, and the conditional beliefs $p(x,r)$. If agents use a threshold rule characterized by $x^*(r)$, we have $A(\theta, r) = \Phi\left(\sqrt{\beta}\left(x^*(r) - \theta\right)\right)$, and $S(\theta, r) = \Phi\left(s - \gamma\Phi^{-1}(r)\right)$. Since market-clearing implies $1 - A(\theta, r) = S(\theta, r)$, we have

$$1 - \Phi\left(\sqrt{\beta}\left(x^*(r) - \theta\right)\right) = \Phi\left(s - \gamma\Phi^{-1}(r)\right)$$

or

$$z \equiv x^*(r) - \frac{\gamma}{\sqrt{\beta}}\Phi^{-1}(r) = \theta - \frac{1}{\sqrt{\beta}}s. \tag{13}$$

Therefore, $R(\theta, s) = r$ is admissible in equilibrium, if and only if θ, s and r satisfy condition (13), for all θ, s and r. The LHS of this equation only depends on r, on which agents can condition their bids. The RHS only depends on the unobservable shocks θ and s. Moreover, conditional on θ, z is uncorrelated with private signals. Therefore, if the Walrasian auctioneer conditions r on $z \equiv \theta - s/\sqrt{\beta}$, selecting the same $R(z)$ for any θ, s, s.t. $\theta - s/\sqrt{\beta} = z$, z becomes a sufficient statistic for the information conveyed by r on the equilibrium path.[8] We thus have the following lemma:

Lemma 1 *Suppose that all other agents follow a threshold rule characterized by $x^*(r)$, and a devaluation occurs, whenever $\theta \leq \theta^*(r)$. Then,*

(i) the information conveyed by r is summarized by

$$z \equiv x^*(r) - \frac{\gamma}{\sqrt{\beta}}\Phi^{-1}(r), \tag{14}$$

where $z \sim \mathcal{N}(\theta, (\beta\delta)^{-1})$; and

(ii) If $\{z : r = R(z)\}$ is non-empty, the probability of devaluation $p(x, r)$ is given by

$$p(x, r) = \Pr\left(\theta \leq \theta^*(r) \,|\, x, z\right) = \Phi\left(\sqrt{\beta + \beta\delta}\left(\theta^*(r) - \frac{\beta x + \beta\delta z}{\beta + \beta\delta}\right)\right). \tag{15}$$

Part (i) immediately follows from the preceding arguments. From this, it is immediate that conditional on θ, z is normally distributed with mean θ and precision $\beta\delta$, $z \sim \mathcal{N}(\theta, (\beta\delta)^{-1})$. Part ($ii$) is a consequence of the fact that devaluation occurs, iff $\theta \leq \theta^*(r)$, and the conditional posterior of θ is normal, given x and z.

[8] A technical problem arises if for given θ, s, there are multiple market-clearing interest rates. In that case, if the auctioneer were to condition $R(\theta, s)$ on θ and s separately, z would no longer suffice as a sufficient statistic for the information conveyed by r.

Lemma 1 highlights the role of r in aggregating private signals. In equilibrium, r, or equivalently, z, provides a normally distributed *public* signal of θ. Moreover, its precision increases with the precision of exogenous private signals: an increase in β increases the the precision of z. Thus, as in Angeletos and Werning (2004), we have information aggregation: the more precise exogenous private signals are, the more precise the endogenous public signal becomes. At the same time, bigger shocks in the domestic bond supply (a smaller δ) make r less informative.

Any monotone strategy equilibrium is thus characterized by an interest rate function $R(z)$, and thresholds $\{x^*(r), \theta^*(r)\}$, s.t. for every z, (11), (12), and (14) are all satisfied, and $p(x,r)$ is given by (15). Solving these conditions, we provide a complete equilibrium characterization in the theorem 1.

Theorem 1 (Equilibrium characterization) *Under the functional form assumptions (A1)-(A4), $\theta^*(r)$, $x^*(r)$ and $R(z)$ characterize a monotone strategy equilibrium if and only if they satisfy the following conditions.*

(1) On the equilibrium path, $\theta^*(r)$ and $x^*(r)$ are uniquely characterized by

$$\theta^*(r) = \Phi\left(\left[1-\lambda+\lambda\frac{\gamma\delta-\sqrt{1+\delta}}{1+\delta}\right]\Phi^{-1}(r)\right) \tag{16}$$

$$x^*(r) = \theta^*(r)+\frac{\gamma\delta-\sqrt{1+\delta}}{\sqrt{\beta}(1+\delta)}\Phi^{-1}(r) \tag{17}$$

(2) The equilibrium interest rate function $R(z)$ is selected from a correspondence $\widehat{R}(z)$, which is defined as the set of interest rates r, which solve

$$r=\Phi\left(\sqrt{\beta(1+\delta)}\left[\theta^*(r)-\frac{x^*(r)+\delta z}{1+\delta}\right]\right). \tag{18}$$

Theorem 1 highlights important equilibrium properties: First, for any r, optimal bidding strategies and the devaluation outcome are uniquely pinned down on the equilibrium path. However, the interest rate function $R(z)$ is selected from a correspondence $\widehat{R}(z)$. Therefore, to establish uniqueness vs. multiplicity of equilibria, one must examine whether $\widehat{R}(z)$ is single-valued for all z, or whether there is a non-empty subset of values z, for which (18) has multiple solutions. (18) is characterized by the uncovered interest parity condition which determines the traders' optimal bidding strategies; this condition must hold with equality for an agent who is just indifferent between

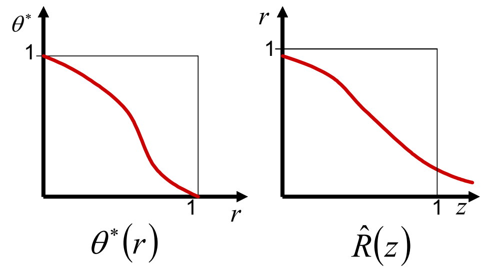

Figure 3: Unique equilibrium for all β

converting to the dollar and buying the domestic bond. As in the game with common knowledge, the scope for multiplicity arises from a trade-off between the payoff effect of r for the domestic bond, and its effect on the expected devaluation premium: when the latter is increasing in r, there is scope for multiple market-clearing interest rates for a given z, and hence multiple equilibria. The following corollary provides necessary and sufficient conditions, under which $\widehat{R}(z)$ is single-valued and the equilibrium is unique.

Corollary 1 (Uniqueness) *There is a unique equilibrium, if and only if*

$$\frac{\sqrt{\beta}\,(1+\delta)}{\sqrt{1+\delta}+\gamma}\left[1-\lambda+\lambda\frac{\gamma\delta-\sqrt{1+\delta}}{1+\delta}\right]\leq\sqrt{2\pi}. \tag{19}$$

Condition (19) allows us to distinguish two different scenarios:

(i) If $\frac{1-\lambda}{\lambda}\sqrt{1+\delta}+\frac{\gamma\delta}{\sqrt{1+\delta}}\leq 1$, the equilibrium is unique, irrespective of the precision of private signals, β. This requires that γ be small, δ be small, and λ sufficiently large. In figure 3, we graphically represent $\theta^*(r)$ and $R(z)$ for this case. Note that $\theta^*(r)$ is decreasing in r. Therefore, the expected devaluation premium is decreasing in r, and there is a unique market-clearing interest rate function $R(z)$, which is decreasing in z.

This case mirrors the results in the benchmark game, when $\lambda=1$ (i.e. the central bank cares only about reserve losses), and $\gamma=0$, i.e. the bond supply is perfectly inelastic; note that if $\gamma=0$ and $\lambda=1$, uniqueness is obtained irrespective of the size of supply shocks δ. However, if either $\lambda<1$ or $\gamma>0$, the LHS of (19) is strictly increasing in δ, and becomes positive if δ is sufficiently large.

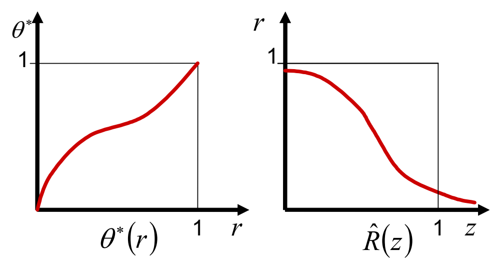

Figure 4: Unique equilibrium, if β is sufficiently small

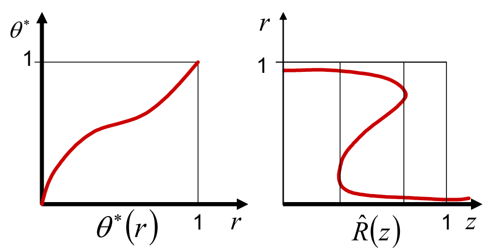

Figure 5: Multiple equilibria, if β is sufficiently large

(ii) If $\frac{1-\lambda}{\lambda}\sqrt{1+\delta}+\frac{\gamma\delta}{\sqrt{1+\delta}}>1$, the LHS of (19) is strictly increasing and unbounded in both β and δ, and therefore, there will necessarily be multiple equilibria, if β and/or δ are large enough. For this case, figures 4 and 5 represent $\theta^*(r)$ and $R(z)$, for different values of the parameters β and δ. $\theta^*(r)$ is increasing in r, i.e. a higher interest rate leads to a higher devaluation threshold, and a higher expected devaluation premium. If β and/or δ are large enough, this leads to the possibility of multiple market-clearing interest rates, and multiple equilibrium interest rate functions. Moreover, any such equilibrium necessarily leads to a crash, i.e. the interest rate function must have a discontinuity at some value of z. At that point, small changes in the underlying shocks lead to large, discrete changes in the realized interest rate, and a discrete change in the probability of devaluation.

This case mirrors our benchmark game, when λ is sufficiently small, i.e. when the central bank is concerned only about high interest rates, or when λ is large, but $\gamma>0$, provided that δ is also large enough, in which case the central bank is concerned about the loss of reserves, and there is a positive supply elasticity in the bond market. Moreover, if $\lambda<1/2$, the LHS of (19) is strictly increasing in β, even if $\delta=0$, and multiplicity results even in the limiting case, where the supply shocks in the domestic bond market become so large as to make the interest rate completely uninformative.

That noisy rational expectations models of asset prices may give rise to multiple market-clearing price functions is not unique to this model. Moreover, the basic intuition for multiplicity in such environments is generally based on a similar trade-off between the payoff and informational roles of prices. In traditional asset pricing models that abstract from coordination issues, this argument is at the heart of Genotte and Leland's (1990) analysis of stock-market crashes; more recently, it appears in Barlevy and Veronesi (2003), where multiplicity and asset price crashes come as the result of the interaction between informed and uninformed traders.

Most closely related to our analysis, Angeletos and Werning (2004) establish multiplicity of rational expectations equilibria in an environment, in which asset prices interact with a coordination game. In their environment, prior to playing a coordination game, traders have the possibility to trade an asset whose dividend depends either on the same fundamentals, or on actions taken in the coordination game; the only connection between the asset market and the coordination game is through the information conveyed by the price. Multiplicity is then a result of information aggregation: as private information becomes more precise, this also improves the public signal provided by the price, which improves the agents' ability to coordinate their decisions, and thereby restores multiplicity, once private signals are sufficiently precise.[9] When the asset is conditioned exogenously on the same fundamentals, its price is uniquely determined and independent of the coordination game; and multiplicity arises from strategies in the coordination game. In contrast, when the asset's dividend is a function of the agents' actions in the coordination game, strategies in the latter are uniquely pinned down; but there are multiple equilibrium price functions. This may appear surprising at first, since the source of multiplicity remains the coordination game; however, the authors discuss the underlying reason: The equilibrium price function reflects the agents' expectations about what strategies will likely be played in the coordination game. These expectations become self-fulfilling and pin down uniquely the agents' strategies in the coordination game. In other words, the price provides a noisy public observation, not so much of the state θ, but of the average action taken by other agents, A. Conditional on having precise information about θ and A, however, the agents' actions are uniquely pinned down. By the same logic, bidding

[9]See Hellwig (2002) for a discussion of the role of exogenous public information in restoring equilibrium multiplicity in global coordination games.

strategies in our model are uniquely pinned down, but multiplicity results from the interest rate function, even if a devaluation is caused by reserve losses, because r acts as an endogenous public signal of A.

Finally, note that as in Angeletos and Werning (2004), and in contrast to Morris and Shin (1998), there are multiple equilibria, when β is sufficiently *high*, i.e. both private and public information are very precise. In Angeletos and Werning, this conclusion follows purely from endogenous information aggregation: as β increases, the higher precision of the endogenous public signal more than outweighs the underlying increase in β. In contrast, in our model, the interest rate plays multiple roles besides aggregating information. In the next section, we examine to what extent our multiplicity results are driven by information aggregation or other factors.

5 Special cases of the general model

In this section, we reexamine the special cases of our model that we considered earlier, to examine the economic reasoning behind the previous uniqueness vs. multiplicity results.

5.1 Special Case I: Obstfeld (1996)

We begin with the case where a devaluation is triggered by the cost of high domestic interest rates. Suppose that $\lambda = 0$, i.e. the central bank devalues, if and only if $r \geq \theta$. In this case, the equilibrium characterization is particularly simple, since the devaluation threshold is given by $\theta^*(r) = r$. Substituting into the uncovered interest parity condition, we find the following special case of our main theorem:

Proposition 2 *When $\lambda = 0$, $\theta^*(r)$, $x^*(r)$ and $R(z)$ characterize a monotone strategy equilibrium, if and only if*

(1) on the equilibrium path, $\theta^*(r)$, $x^*(r)$ are given by

$$\theta^*(r) = r \quad \text{and} \quad x^*(r) = r + \frac{\gamma\delta - \sqrt{(1+\delta)}}{\sqrt{\beta}\,(1+\delta)}\Phi^{-1}(r)$$

(2) $R(z)$ is selected as a solution to:

$$r = \Phi\left(\frac{\sqrt{\beta}\,(1+\delta)}{\sqrt{1+\delta+\gamma}}\,(r-z)\right). \tag{20}$$

The equilibrium is unique if and only if $\frac{\sqrt{\beta}(1+\delta)}{\sqrt{1+\delta+\gamma}} \leq \sqrt{2\pi}$.

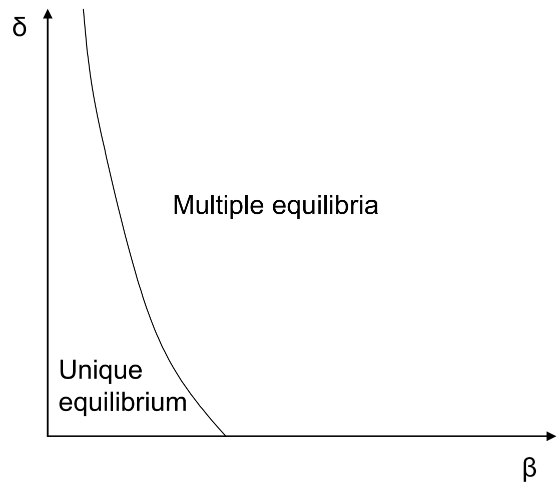

Figure 6: Reconsidering Obstfeld (1996) with heterogeneous information

When the CB is concerned only about the interest rate, demand and supply schedules were uniquely pinned down under common knowledge, but there were multiple market-clearing prices. With incomplete, heterogeneous information, we have a similar result. The devaluation outcome is uniquely pinned down by θ and r, since $\theta^* (r) = r$, and given r optimal bidding strategies are uniquely pinned down for any z and x. However, there may be multiple equilibria because there are multiple market-clearing price functions, due to a demand for domestic bonds that is locally decreasing in the domestic interest rate.

As in the benchmark model, the expected devaluation premium is locally increasing in r, for given z and x, since an increase in r increases the range of states for which a devaluation occurs. Furthermore, as long as the selection $R(z)$ is monotone decreasing, an increase in r leads to inference that z is lower, which lowers expectations about θ, and increases the probability that $\theta \leq r$. This second effect is largest, when $\gamma = 0$; when $\gamma > 0$, the inference that results from an increase in r is partly offset by the fact that the equilibrium loss of foreign reserves is increasing in r. The first effect on its own may already be sufficient to generate a demand for domestic bonds that is decreasing in the interest rate, if β is sufficiently large. In that case, even if $\delta = 0$, i.e. when the interest rate is infinitely noisy, there may be multiple equilibria. Finally, as in the common knowledge benchmark, there is a unique equilibrium, when the domestic bond supply becomes perfectly elastic. We plot the uniqueness conditions in Figure 6.

If a devaluation is solely triggered by unsustainably high domestic interest rates, then the argument for equilibrium multiplicity that arises in Obstfeld (1996) is maintained, provided that private information is sufficiently precise. In contrast to Angeletos and Werning (2004), this result does not rely on information aggregation by prices: if β is sufficiently large, multiplicity arises for any value of δ, i.e. even if the domestic bond market is infinitely noisy, so that little or no public information is provided by prices.

5.2 Special Case II: Reconsidering Obstfeld (1986)

Next, we reconsider the model where devaluations are triggered by reserve losses. Setting $\lambda = 1$, we have the following special case of Theorem 1 for the game with incomplete, heterogeneous information:

Proposition 3 *When $\lambda = 1$, $\theta^*(r)$, $x^*(r)$ and $R(z)$ characterize a monotone strategy equilibrium, if and only if*

(1) on the equilibrium path, $\theta^*(r)$, $x^*(r)$ are given by

$$\theta^*(r) = \Phi\left(\frac{\gamma\delta - \sqrt{1+\delta}}{1+\delta}\Phi^{-1}(r)\right) \quad \text{and} \quad x^*(r) = \theta^*(r) + \frac{\gamma\delta - \sqrt{1+\delta}}{\sqrt{\beta}(1+\delta)}\Phi^{-1}(r)$$

(2) $R(z)$ is selected as a solution to:

$$r = \Phi\left(\frac{\sqrt{\beta}(1+\delta)}{\sqrt{1+\delta}+\gamma}(\theta^*(r) - z)\right).$$

The equilibrium is unique if and only if $\dfrac{\sqrt{\beta}(1+\delta)}{\sqrt{1+\delta}+\gamma}\dfrac{\gamma\delta - \sqrt{1+\delta}}{1+\delta} \leq \sqrt{2\pi}$.

Here, the possibility of having multiple equilibria depends on whether $\gamma \gtreqless \frac{\sqrt{1+\delta}}{\delta}$, i.e. whether the domestic bond supply is sufficiently elastic, relative to the variance of the bond supply shocks, δ^{-1}. If $\gamma \leq \frac{\sqrt{1+\delta}}{\delta}$, the domestic bond supply is very inelastic, and supply shocks are large, i.e. the equilibrium quantity of bonds is mostly driven by supply shocks. In that case, the devaluation threshold $\theta^*(r)$ is decreasing in r, the expected devaluation premium is decreasing in r, and there is a unique market-clearing interest rate, irrespective of β. On the other hand, if $\gamma > \frac{\sqrt{1+\delta}}{\delta}$, the domestic bond supply is mostly driven by interest rate movements. In that case, $\theta^*(r)$ is increasing in r, the probability of a devaluation is increasing in r and there are multiple market-clearing

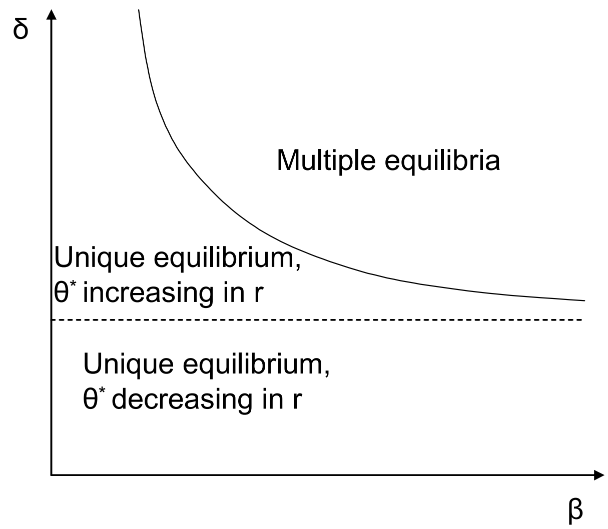

Figure 7: Reconsidering Obstfeld (1986) with heterogeneous information

interest rates, provided that β is sufficiently large. We plot the uniqueness condition graphically in figure 7.

In contrast to the previous case, the role of interest rates in aggregating information is critical for establishing multiplicity: under common knowledge, multiplicity of equilibria resulted from the fact that for any given r, traders could coordinate on multiple best responses. r then serves as an endogenous signal of the total loss of foreign reserves: when r is low, traders anticipate that there are few withdrawals, and hence a devaluation is unlikely, while when r is large, there is a large withdrawal, and a devaluation is likely to occur. For this argument, it is important that the elasticity of the domestic bond supply be sufficiently large: if the elasticity is low, variations in r only have a small effect on the equilibrium level of reserve losses. But then, the payoff effect of r in raising the return on domestic bonds always outweighs the information effect of raising the expected devaluation premium, and there is a unique equilibrium interest rate. In the extreme case when $\gamma = 0$, the devaluation outcome is entirely determined by the level of fundamentals, and by the shocks in the domestic bond market, and the interest rate merely adjusts to clear the domestic bond market.[10] $\theta^* (r)$ in turn is decreasing in r.

[10] The uniqueness result in Tarashev (2002) is based on this special case. However, this result does not rely in any way on global games reasoning and a lack of common knowledge, as it would have been obtained in identical form in

5.3 Revisiting Morris & Shin (1998) and Angeletos & Werning (2004)

Finally, we return to the relation between our results and the results obtained by Morris and Shin (1998), and we compare the effects of information aggregation in our model with Angeletos and Werning (2004).

Previously, we argued that, in order to apply the global games equilibrium selection results, Morris and Shin (1998) have to assume that r is fixed exogenously and the central bank is concerned about reserve losses only. Their model can be generalized to allow for the presence of exogenous public and private information. Assuming that agents observe an exogenous public signal $y \sim \mathcal{N}(\theta, \eta^{-1})$ with mean θ and precision η, the main result of the global games literature with public and private information (Morris and Shin, 2003, 2004; Hellwig 2002) shows that there exists a unique equilibrium, if and only if $\eta/\sqrt{\beta} \leq \sqrt{2\pi}$, that is, if the precision of private signals is sufficiently high, relative to the public signal precision. Furthermore, the devaluation threshold θ^* is characterized as:

$$r = \Phi\left(\frac{\eta}{\sqrt{\beta+\eta}}\left(\theta^* - y\right) - \sqrt{\frac{\beta}{\beta+\eta}}\Phi^{-1}\left(\theta^*\right)\right) \tag{21}$$

To compare this to our model, we consider the same payoff assumptions as Morris and Shin. First, we set $\lambda = 1$, so that the central bank is concerned only about reserve losses. In that case, substituting out r, and writing θ^* directly as a function of z, we have:

$$\theta^* = \Phi\left(\sqrt{\beta}\frac{\gamma\delta - \sqrt{(1+\delta)}}{\gamma + \sqrt{(1+\delta)}}\left(\theta^* - z\right)\right) \tag{22}$$

Second, note that as $\gamma \to \infty$, the domestic asset supply becomes infinitely elastic at $r = 1/2$. Therefore, we compare (22), taking the limit as $\gamma \to \infty$, to (21), where we equate r to $1/2$, set the exogenous public signal y equal to the endogenous public signal z, and set its signal precision η equal to the endogenous signal precision $\beta\delta$. It follows immediately that the two equilibrium conditions are equivalent. Moreover, taking the limit of (19), as $\gamma \to \infty$, and $\lambda = 1$, there exists a unique equilibrium, if and only if $\sqrt{\beta}\delta \leq \sqrt{2\pi}$. Since the precision of our endogenous public signal z is $\beta\delta$, this condition exactly mirrors the uniqueness condition of Morris and Shin.

Our model thus embeds Morris and Shin's analysis as a special case, albeit with a key difference: In our environment, the precision of the public signal is endogenously determined by the precision of private signals; as in Angeletos and Werning (2004), we have information aggregation. In the

the game with common knowledge

limiting case that we have considered here, this information aggregation is completely unmitigated: Since the infinite supply elasticity exogenously pins down r, the domestic interest rate purely serves to aggregate private information, but has no direct payoff effects on traders' strategies.

It is useful to further compare this limiting case to the results in Angeletos and Werning (2004). In their model as in ours, an asset price serves as an endogenous public signal, which aggregates private information. However, their model focuses on the role of prices in "derivative" markets, in which agents first trade assets, and then participate in a speculative attack, with the asset dividends dependent on the outcome of the speculative attack. The speculative attack is modelled as in Morris and Shin, with the domestic interest rate fixed, and the devaluation outcome triggered by reserve losses. Consequently, just as in our limiting case, when $\gamma \to 1$, the only impact that the security trading in the first stage has on the second stage currency run, is through the aggregation of private information by the price. Therefore, in their model as in our limiting case, uniqueness vs. multiplicity is determined purely by the effect of information aggregation, according to the above condition, which compares the precision of endogenous public and exogenous private information.

Since $\sqrt{\beta}\frac{\gamma\delta-\sqrt{(1+\delta)}}{\gamma+\sqrt{(1+\delta)}} \leq \sqrt{\beta}\delta$, the condition for multiplicity becomes more stringent away from the limit. With a finite supply elasticity, the domestic interest rate retains its role as an aggregator of private information, but this effect is now partly offset by the payoff effect of interest rates for the return on domestic bonds. In other words, the same interest rate change is indicative of a larger change in foreign reserves, when the domestic bond supply becomes more elastic. Thus, the scope for multiplicity decreases, as the domestic bond supply becomes more inelastic. If the domestic bond supply is sufficiently inelastic, the payoff effect of r always dominates the expectations effect, and there is a unique equilibrium.

6 Conclusion

In this paper, we have studied the role of domestic interest rates in a stylized global game of currency crises. We have taken a noisy REE approach with heterogeneously informed traders, in which the market-clearing interest rate serves to aggregate private information. Our analysis shows that multiple equilibria in models of financial crises are not the artefact of assuming that fundamentals are commonly known, but result from the dual role that interest rates play in coordinating individual investment decisions, along with directly or indirectly determining the ultimate devaluation outcome. This, however, is not captured by a stylized global coordination game, which abstracts from the role of interest rates.

At the same time, our analysis also reveals new insights that would not have been possible under common knowledge, and it suggests new avenues for future research. For example, we have argued that information aggregation through interest rates tends to be most destabilizing and induce multiple equilibria, when it is not mitigated by direct payoff effects, i.e. when the domestic bond supply is perfectly elastic. This insight may be useful for understanding the informational connections between primary and derivative markets: A comparison between our results and Angeletos and Werning suggests a potential argument why derivative markets may have a destabilizing effect on primary markets, since derivative prices aggregate information without the mitigating payoff effects that results from price movements in the primary market. Another question, which our model may be apt to address is the effects of public information disclosures in the context of financial crises. We leave an analysis of these questions for future work.

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7 Appendix

Proof of Theorem 1. Substituting the market-clearing condition $z = x^*(r) - \frac{\gamma}{\sqrt{\beta}}\Phi^{-1}(r)$ into the interest parity condition, we find

$$r = \Phi\left(\sqrt{\beta(1+\delta)}\,(\theta^*(r) - x^*(r)) + \frac{\gamma\delta}{\sqrt{1+\delta}}\Phi^{-1}(r)\right),$$

or, after solving for $x^*(r)$,

$$x^*(r) = \theta^*(r) + \frac{1}{\sqrt{\beta}}\frac{\gamma\delta - \sqrt{1+\delta}}{1+\delta}\Phi^{-1}(r).$$

Now, the devaluation condition is

$$\theta^*(r) = \Phi\left(\lambda\Phi^{-1}(A) + (1-\lambda)\Phi^{-1}(r)\right), \text{ where } A = \Phi\left(\sqrt{\beta}\,(x^*(r) - \theta^*(r))\right)$$

Therefore, substituting the previous expression for $x^*(r)$, we find $\theta^*(r)$ as a function of r:

$$\begin{aligned}\theta^*(r) &= \Phi\left(\lambda\sqrt{\beta}\,(x^*(r) - \theta^*(r)) + (1-\lambda)\Phi^{-1}(r)\right)\\ &= \Phi\left(\left[\lambda\frac{\gamma\delta - \sqrt{1+\delta}}{1+\delta} + 1 - \lambda\right]\Phi^{-1}(r)\right)\end{aligned}$$

Thus, we have solved for $x^*(r)$ and $\theta^*(r)$ as functions of r. ■

Proof of Corollary 1. To establish uniqueness, substitute $x^*(r)$ and $\theta^*(r)$ into the interest parity condition to find:

$$\begin{aligned}\frac{1}{\sqrt{\beta(1+\delta)}}\Phi^{-1}(r) &= \frac{\delta}{1+\delta}(\theta^*(r) - z) - \frac{1}{\sqrt{\beta}}\frac{\gamma\delta - \sqrt{1+\delta}}{(1+\delta)^2}\Phi^{-1}(r)\\ \Phi^{-1}(r) &= \frac{\sqrt{\beta}(1+\delta)}{\gamma + \sqrt{1+\delta}}(\theta^*(r) - z)\\ \Phi^{-1}(r) &= \frac{\sqrt{\beta}(1+\delta)}{\gamma + \sqrt{1+\delta}}\left(\Phi\left(\left[\lambda\frac{\gamma\delta - \sqrt{1+\delta}}{1+\delta} + 1 - \lambda\right]\Phi^{-1}(r)\right) - z\right)\end{aligned}$$

This implicitly describes the correspondence $\widehat{R}(z)$ of market-clearing prices. Necessarily $\widehat{R}(z)$ is single-valued whenever the derivative of the RHS w.r.t. $\Phi^{-1}(r)$ is smaller than 1, for all z. However, when the slope of the RHS locally exceeds 1, there exists values of z, s.t. $\widehat{R}(z)$ takes on multiple values, and hence there are multiple equilibria. Taking the derivative of the RHS w.r.t. $\Phi^{-1}(r)$, we find

$$\frac{\sqrt{\beta}\,(1+\delta)}{\gamma+\sqrt{1+\delta}}\left[\lambda\frac{\gamma\delta-\sqrt{1+\delta}}{1+\delta}+1-\lambda\right]\phi\left(\left[\lambda\frac{\gamma\delta-\sqrt{1+\delta}}{1+\delta}+1-\lambda\right]\Phi^{-1}(r)\right)$$

Since $\phi(\cdot)$ is bounded above by $\frac{1}{\sqrt{2\pi}}$, the equilibrium is unique, whenever

$$\frac{\sqrt{\beta}\,(1+\delta)}{\gamma+\sqrt{1+\delta}}\left[\lambda\frac{\gamma\delta-\sqrt{1+\delta}}{1+\delta}+1-\lambda\right]\leq\sqrt{2\pi}$$

■